HANK's Response to Aggregate Uncertainty in an Estimated Business Cycle Model

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Motivation

- Time-varying uncertainty is key for the study of business cycles and asset prices
- Both aggregate and idiosyncratic uncertainty matter, but are studied in isolation so far
- This is for technical reasons: With expected utility preferences, changes in uncertainty have only third-order effects on utility and choice
- HANK models are typically solved at first-order

This Paper: Aggregate + Idiosyncratic Uncertainty

- Develop and estimate a HANK model with **ambiguity** about TFP, idiosyncratic income risk, and illiquid assets
  - Very tractable when aggregate uncertainty is modeled as ambiguity using multiple priors preference
  - Aggregate uncertainty then has first-order effects on utility and is reflected in the equations for the steady state and linear dynamics
- Savings and portfolio choice by households respond to aggregate and idiosyncratic uncertainty (level and shocks)
- Moreover, there is a unique first-order effect of aggregate uncertainty on intertemporal choices by firms

This Paper: Main Findings

- Aggregate uncertainty shocks interact with portfolio frictions to generate a powerful comovement mechanism
- Ambiguity about TFP jointly explains more than 60% of cyclical variation in key macroeconomic aggregates as well as in the excess return on capital and the real interest rate
- Mechanism: Capital owners' countercyclical substitution away from capital, an asset that is not only illiquid (1.3% premium on average) but also uncertain (4.7% premium).
- Strong substitution also distinguishes aggregate from idiosyncratic uncertainty shocks that play only a small role in our estimation.

Literature

Uncertainty shocks in business cycles: Justiniano and Primiceri (2008), Ilut and Schneider (2014), Leduc and Liu (2016), Basu and Bundick (2017),...
Fernandez-Villaverde and Guerron-Quintana (2020) give an overview.

Estimating HANK: Hagedorn, Manovskii and Mitman (2018), Auclert, Rognlie and Straub(2020), Bayer, Born and Luetticke (2020), Bilbiie, Primiceri and Tambalotti (2022),...

Asset prices and real activity: Gertler and Kiyotaki (2010), Jermann and Quadrini (2012), Brunnermeier and Sannikov (2014), Bocola and Lorenzoni (2023),...

Ambiguity

Preferences: Ambiguity Aversion

- Exogenous state for household i: vector $s_{i,t} \in S$, with history $s_i^t = (s_{i,1}, ..., s_{i,t}) \in S^t$
- Consumption plan (over goods and leisure) $C_i = C_{i,t}(s_i^t)$
- Recursive multiple-priors utility (Epstein and Schneider (2003))

$$U_t\left(C_i; s_i^t\right) = u\left(C_{i,t}\left(s_i^t\right)\right) + \beta \min_{p \in \mathcal{P}_t\left(s_i^t\right)} E^p\left[U_{t+1}\left(C_i; s_i^t, s_{i,t+1}\right)\right]$$

- Primitives
  - felicity u (eg. GHH), discount factor β, one-step-ahead belief sets $\mathcal{P}_t\left(s_i^t\right)$
  - larger $\mathcal{P}_t\left(s_i^t\right) \to$ more ambiguity about $s_{i,t+1}$
  - state dependence of $\mathcal{P}_t\left(s_i^t\right)$ captures e.g. arrival of information
- Why this functional form?
  - preference for knowing the odds (Ellsberg Paradox)
  - worst case belief endogenous – depends on C_i

Ambiguity about Aggregate TFP

- Parameterize one-step ahead belief sets $\mathcal{P}_t\left(s_i^t\right)$ by mean of TFP innovations

$$\begin{aligned} \log Z_{t+1} &= \rho_z \log Z_t + \mu_t + \epsilon_{t+1}^Z; \quad \epsilon^Z \sim i.i.d N(0, \sigma_z) \\ \mu_t &\in [-a_t, a_t] \end{aligned}$$

- Higher $a_t \rightarrow$ larger belief set $\rightarrow$ more ambiguity about TFP in $t+1$
- Stochastic process for a_t:

$$a_t - \bar{a} = \rho_a(a_{t-1} - \bar{a}) + \epsilon_t^a$$

  - long run mean $\bar{a} > 0$, persistence $0 \leq \rho_a < 1$, and $\epsilon_t^a \sim i.i.d\ N(0, \sigma_a)$

- Times of high (low) $a_t - \bar{a}$ represent unusually high (low) uncertainty about TFP

Ambiguity in Equilibrium

- Perception of endogenous variables
    - have defined ambiguity about exogenous TFP shocks
    - agents understand law of motion of economy, as usual
    - also perceive ambiguity about wages, returns etc.
- Objective of the firm
    - to first order, all agents agree on optimal production plan
    - shareholder value maximization well defined
    - precautionary motive in firm's intertemporal decisions

Characterizing the Equilibrium

- Need to find (endogenous) equilibrium belief together with optimal choices
- This model: worst case belief is always low mean TFP
  - after any history, for any agent $\mu_t^* = -a_t, \forall t, \forall i$
  - observational equivalence to model of pessimistic agents
  - pessimism in mean = first order effects of uncertainty!
- Interpretation: precautionary motive in all intertemporal decisions
  - household agents save & choose portfolios *as if* future expected wages & returns are low
  - firms invest & set prices *as if* future cost is high
  - correlated wedges driven by ambiguity shock a_t matter jointly for all decisions
- Given equilibrium law of motion, characterize path of variables under true DGP

$$\log Z_t = \rho_z \log Z_{t-1} + \epsilon_t^Z$$

Estimating a two-asset HANK model with aggregate and idiosyncratic uncertainty

Application: A two asset HANK model (Bayer et al., 2024)

Households		Production	Government
Obtain income	Trade Assets	Produce and differentiate goods	Monetary & fiscal authority
Wages • idiosyn. risk • taxes and transfers • Sticky wages **Interest on bonds** • set by monetary authority **Illiquid asset** • earns net MPK **All non-wage rents** • go to rich entrepreneurs			

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Households

- Households face time-varying productivity risk

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$$
\begin{aligned}
\log h_{it} &= \rho_h \log h_{it-1} + \epsilon^h_{it}, \quad \epsilon^h_{it} \sim N\left(0, \sigma_{ht}\right) \\
\sigma^2_{h,t} &= \bar{\sigma}^2_h \exp s_t, \\
s_{t+1} &= \rho_s s_t + \epsilon^s_t, \quad \epsilon^s_t \sim N\left(0, \sigma_s\right)
\end{aligned}
$$

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- A fraction λ_t of households are randomly selected to participate in the market for illiquid capital
- A fraction of households becomes "entrepreneurs" and earns all other pure rents. Stochastic transition into and out of this state
- A union differentiates labor, driving a wedge between MPL and wages paid to workers. It distributes related profits among workers.

Embedded in an otherwise standard NK model

- Factor Prices equal marginal products:

$$w_t^F = \alpha mc_t Z_t \left(\frac{u_t K_t}{N_t}\right)^{1-\alpha}, \qquad r_t^F + q_t^F \, \delta(u_t) = u_t(1-\alpha) mc_t Z_t \left(\frac{N_t}{u_t K_t}\right)^{\alpha},$$

where $\delta\left(u_t\right) = \delta_0 + \delta_1\left(u_t - 1\right) + \delta_2/2\left(u_t - 1\right)^2$

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where $\delta\left(u_t\right) = \delta_0 + \delta_1\left(u_t - 1\right) + \delta_2/2\left(u_t - 1\right)^2$

- Capital Price equals costs of production of capital.

$$\begin{aligned} 1 =& q_t^F \left[1 - \frac{\phi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \phi\left(\frac{I_t}{I_{t-1}} - 1\right)\frac{I_t}{I_{t-1}}\right] \\ &+ \beta q_{t+1}^F \phi\left(\frac{I_{t+1}}{I_t} - 1\right)\left(\frac{I_{t+1}}{I_t}\right)^2 \end{aligned}$$

Embedded in an otherwise standard NK model

- Phillips Curve under quadratic price adjustment costs
- Wage Phillips Curve under quadratic price adjustment costs

Monetary Policy

- Monetary policy follows Taylor rule

$$\log \frac{R^b_{t+1}}{\bar{R}^b} = \rho_{TR} \log \frac{R^b_t}{\bar{R}^b} + (1-\rho_{TR})\theta_\pi \log \frac{\pi_t}{\bar{\pi}_t} + (1-\rho_{TR})\theta_y \log \frac{Y_t}{\bar{Y}} + \varepsilon^R_t$$

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- with exogenous shocks:

$$\begin{aligned} \bar{\pi}_t &= \rho_\pi \bar{\pi}_{t-1} + \epsilon^\pi_t; \quad \epsilon^\pi \sim i.i.dN(0,\sigma_\pi) \\ \varepsilon^R_t &= \rho_R \varepsilon^R_{t-1} + \epsilon^R_t; \quad \epsilon^R \sim i.i.dN(0,\sigma_R) \end{aligned}$$

Fiscal Policy

- Government debt accumulation rule as in Woodford (1995):

$$\Delta \log B_{t+1} \quad = \quad \gamma_B \log \frac{B_t}{\bar{B}_t} + \gamma_Y \log \frac{Y_t}{\bar{Y}}$$

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- Government spending determined by government budget constraint

$$G_t = B_{t+1} + T_t - R_t^b B_t / \pi_t \ ,$$

where $T_t = \tau(N_t w_t + \Pi_t^U + \Pi_t^F)$

Sources of Fluctuations

Aggregate and idiosyncratic productivity as well as time-varying uncertainty about both

- ambiguous total factor productivity
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Monetary shocks

- Interest rate
- Inflation target

Estimation of Aggregate Uncertainty

Modeling aggregate uncertainty as ambiguity has major computational advantages:

- Computational time as the first-order perturbation solutions in Bayer et al. (2024) or Auclert et al. (2020)
- Bayer et al. (2024) gives an upper bound on the achievable dimensionality reduction for first-order solutions. These results apply to our model even though it has aggregate uncertainty

Estimation: 2 Step Procedure

Implementation:

- [1] a) Solve steady state under the worst-case realization of TFP
  b) Solve the first-order dynamics with ambiguity about TFP,
  which yields the ergodic distribution of the model with aggregate uncertainty
- [2] Do Bayesian estimation based on the dynamics

This allows to take into account the effect of aggregate uncertainty on the ergodic distribution and the effect of the latter on aggregate dynamics

Estimation Targets

Ergodic distribution pins down average:

- Wealth-to-output ratio
- Fraction of hand-to-mouth households
- Capital premium

Parameters:

- Discount factor
- Illiquidity
- Worst-state TFP

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Ergodic distribution pins down average:

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Parameters:

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Model dynamics pin down:

- Relative importance of shocks
- Monetary and fiscal rules
- Nominal and real frictions

Observables

US data, 1985Q1 – 2019Q4

In first-differences

- Hours, Consumption, Investment

In log-levels

- GDP deflator based inflation rates
- Federal funds rate (shadow)
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Measurement error on all observables

- 6 observables, 5 shocks
- iid error
- Focus on internal propagation

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Measurement error on all observables

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- Focus on internal propagation

All demeaned except for capital premium

- No permanent TFP shocks
- Inflation indexation
- Focus on uncertainty vs liquidity for asset distribution and prices

Estimation Results

Capital premium = 6%

- 1.3% comes from illiquidity
- 4.7% comes from ambiguity

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- 16% comes from ambiguity

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Capital to output = 300%

- 320% comes from illiquidity
- -20% comes from ambiguity

Estimation Results

Capital premium $= 6\%$

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Capital to output $= 300\%$

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Parameter estimates:

- Discount factor: $\beta = 0.98$
- Iliquidity: $\lambda = 0.05$
- Worst-state TFP: $\underline{A} = 0.99$

Estimated Shock Processes

Parameter	Distribution	Prior Mean	Prior Std. Dev.	Posterior HANK with Ambiguity
		TFP		
ρ_Z	Beta	0.50	0.20	0.869 (0.828, 0.905)
		Ambiguity		
$\underline{A}$	Beta	0.985	0.01	0.991 (0.989, 0.994)
ρ_A	Beta	0.50	0.20	0.942 (0.923, 0.959)
σ_A	Inv.-Gamma	20.00	20.00	44.910 (33.066, 59.253)
		Idiosyncratic risk		
ρ_S	Beta	0.50	0.20	0.504 (0.354, 0.653)
σ_S	Inv.-Gamma	20.00	20.00	15.823 (6.163, 33.199)

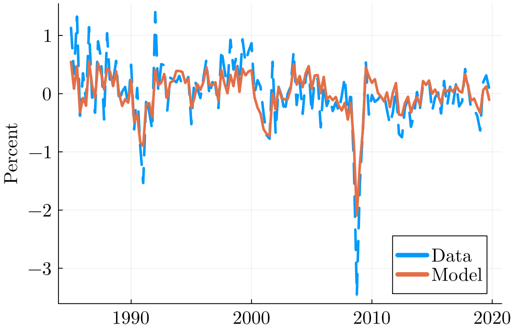
Prior and posterior distributions of estimated parameters

		Prior		Posterior
Parameter	Distribution	Mean	Std. Dev.	HANK with Ambiguity
		Nominal and real frictions		
δ_s	Gamma	5.00	2.00	5.885 (5.249, 6.544)
ϕ	Gamma	4.00	2.00	0.446 (0.407, 0.493)
κ	Gamma	0.10	0.03	0.129 (0.086, 0.181)
κ_w	Gamma	0.10	0.03	0.117 (0.073, 0.172)

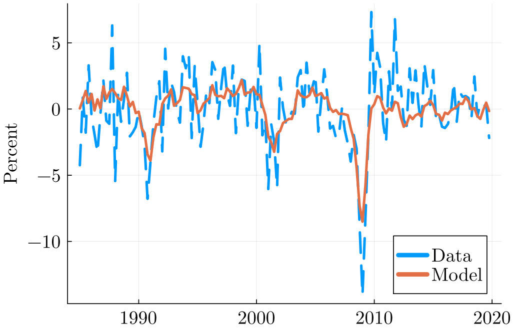
Prior and posterior distributions of estimated parameters

		Prior		Posterior
Parameter	Distribution	Mean	Std. Dev.	HANK with Ambiguity
		Monetary policy		
ρ_R	Beta	0.50	0.20	0.162 (0.103, 0.231)
θ_π	Inv.-Gamma	0.10	2.00	2.822 (2.555, 3.094)
θ_Y	Normal	1.70	0.30	-0.004 (-0.047, 0.045)
ρ_R^ϵ	Beta	0.50	0.20	0.947 (0.897, 0.982)
σ_R^ϵ	Inv.-Gamma	0.10	2.00	0.078 (0.054, 0.106)
ρ_π^ϵ	Beta	0.50	0.20	0.635 (0.540, 0.727)
$\sigma_p i^\epsilon$	Inv.-Gamma	0.10	2.00	0.046 (0.028, 0.060)
		Fiscal policy		
γ_B	Gamma	0.10	0.08	0.067 (0.057, 0.079)
γ_Y	Normal	0.00	1.00	1.073 (0.950, 1.189)

Results: Model Fit Quantities

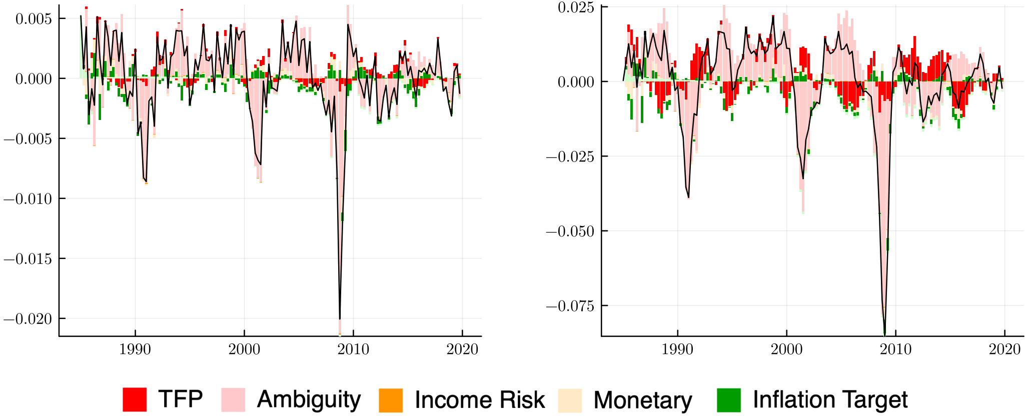


Consumption Growth

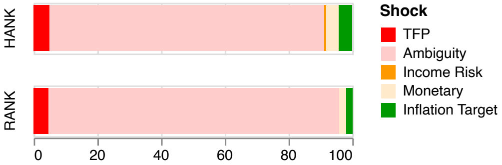


Investment Growth

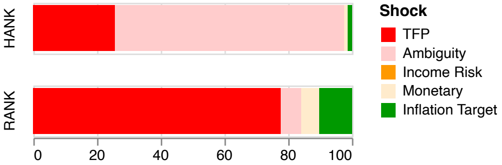
Results: Historical Decomposition of Consumption & Investment



Results: Variance Decomposition

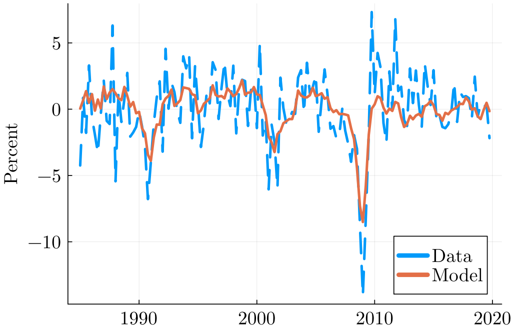


Consumption C_t

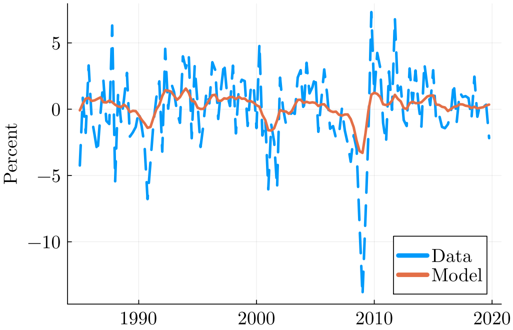


Investment I_t

Results: Model Fit HANK vs RANK

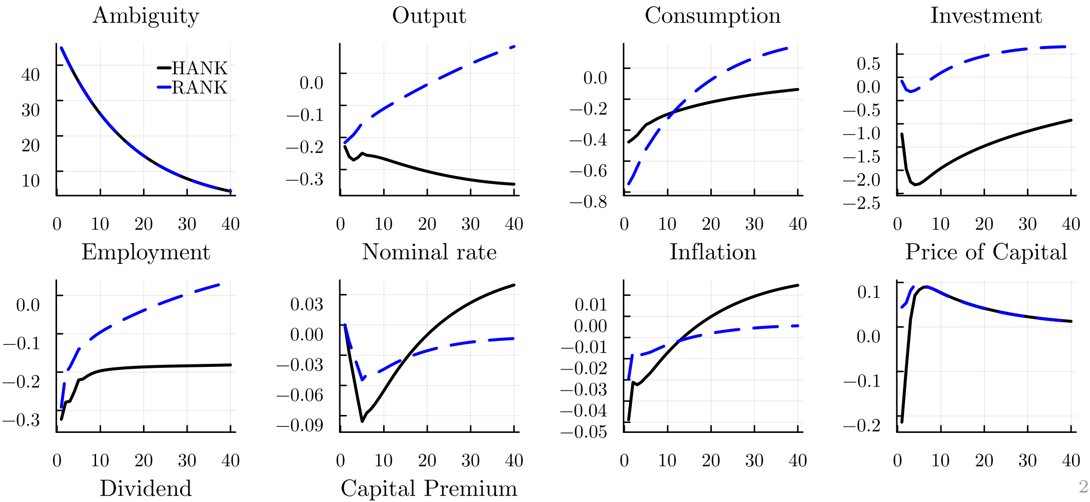


Investment Growth in HANK

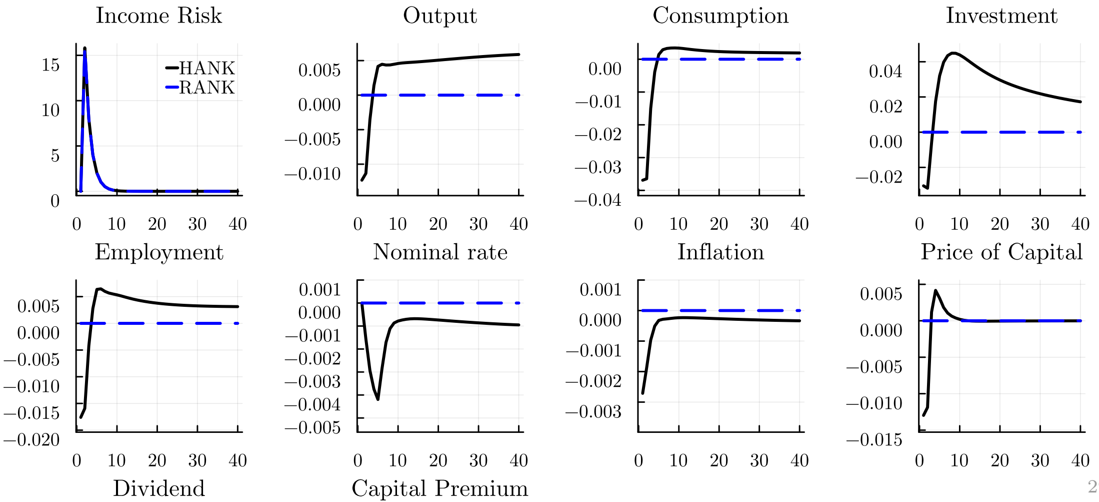


Investment Growth in RANK

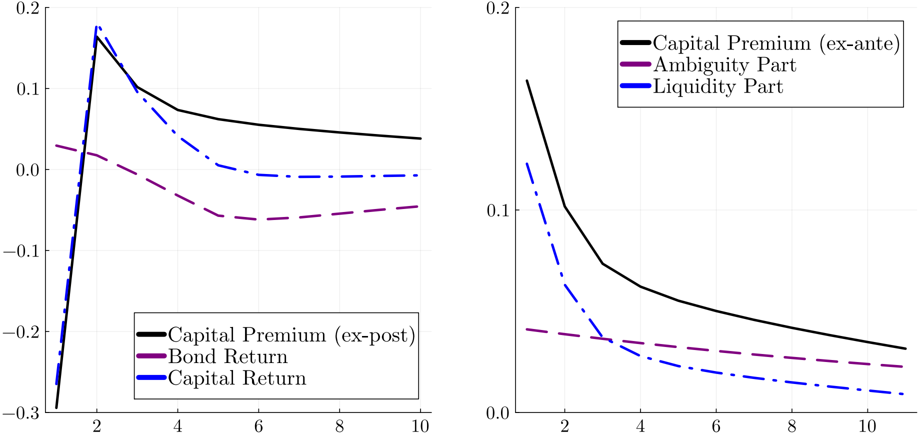
Results: Impulse Responses to Ambiguity Shock



Results: Impulse Responses to Idiosyncratic Income Risk Shock



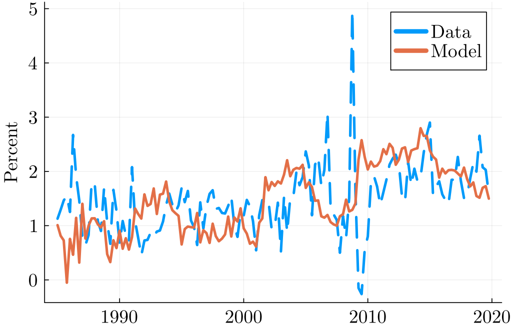
Results: Capital Premium



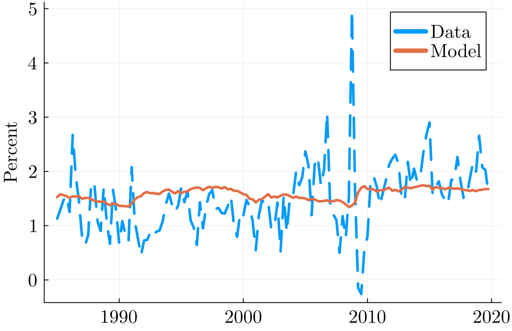
Price Decomposition

Mechanism Decomposition

Results: Model Fit HANK vs RANK

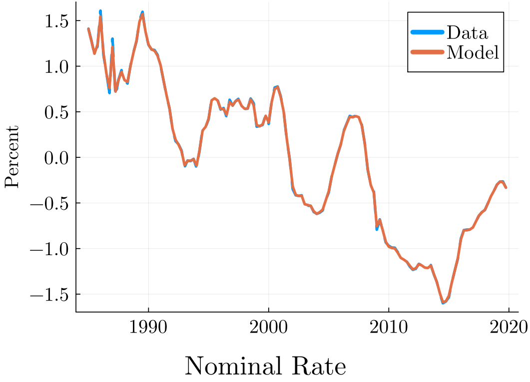


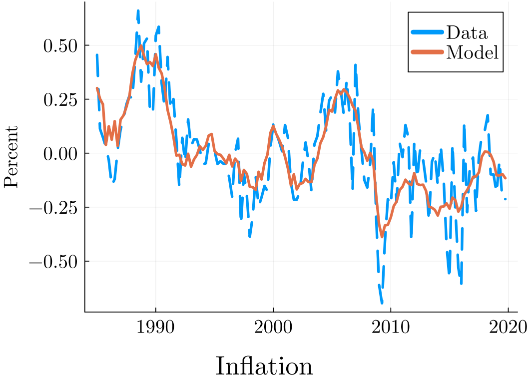
Capital Premium in HANK



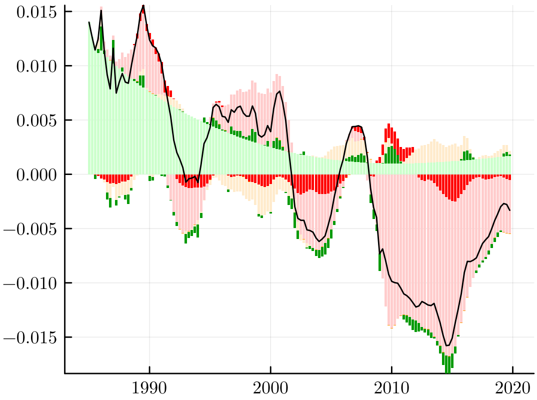
Capital Premium in RANK

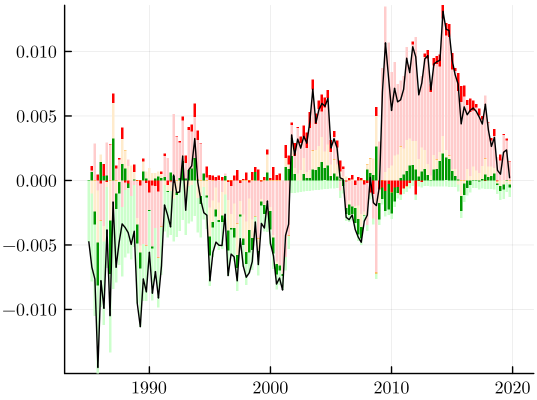
Results: Model Fit Prices





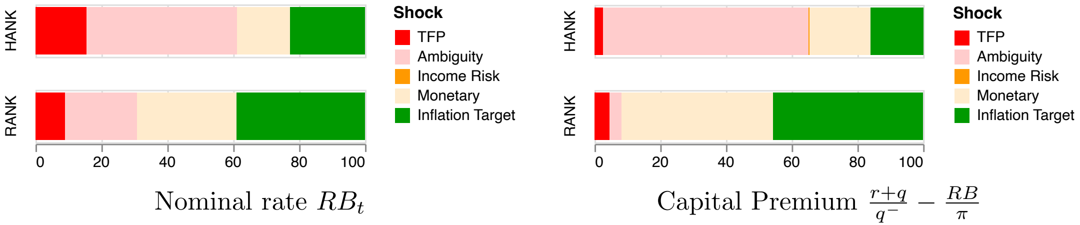
Results: Historical Decomposition of Nominal Rate & Premium





TFP Ambiguity Income Risk Monetary Inflation Target

Results: Variance Decomposition



Conclusion

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- Capital premium mainly reflects compensation for aggregate uncertainty, but liquidity considerations become important when aggregate uncertainty is high
- Aggregate uncertainty generates HtM households with less portfolio frictions
- Ambiguity about TFP jointly explains more than 60% of cyclical variation in key macroeconomic aggregates as well as in the excess return on capital and the real interest rate
- Strong substitution distinguishes aggregate from idiosyncratic uncertainty shocks

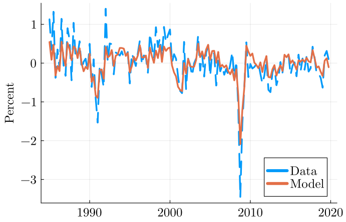
Beliefs vs Data

- True DGP
$$\log Z_{t+1} = \rho_z \log Z_t + \mu_t^* + \sigma^* \epsilon_{t+1}$$
    - deterministic sequence $\{\mu_t^*\}$ unknown
      empirical moments same as iid normal process with mean zero & variance σ_μ^2
    - cannot identify μ_t^*, σ^* without further assumptions
- Econometrician
    - resolve uncertainty probabilistically by assuming stationarity
    - represent uncertainty as risk, with $\sigma^2 = (\sigma^*)^2 + \sigma_\mu^2$
$$\log Z_{t+1} = \rho_z \log Z_t + \sigma \epsilon_{t+1}$$
- Agents
    - consider nonstationary models given by different $\widetilde{\mu}_t$s and $\tilde{\sigma}$
    - treat one-step ahead mean as ambiguous: *as if* minimizing over $[-a_t, a_t]$

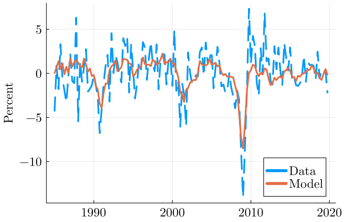
Steady State Parameters

Par.	Value	Description	Par.	Value	Description
Households: Income process					
ρ_h	0.980	Persistence income	σ_h	0.120	Std. income
ι	0.063	Trans. prob. E. to W.	ζ	2.0E-5	Trans. prob. W. to E.
Households: Financial frictions					
λ	0.05	Portfolio adj. prob.	$\bar{R}$	0.021	Borrowing penalty
Households: Preferences					
β	0.98	Discount factor	ξ	4.000	Relative risk aversion
γ	0.500	Frisch elasticity	α	0.680	Share of labor
Firms					
δ_0	0.018	Depreciation rate	$\bar{\eta}$	11.000	Elasticity of sub.
$\bar{\zeta}$	11.000	Elasticity of sub.	$\bar{\tau}^L$	0.180	Tax rate level
Government					
$\bar{\tau}^P$	0.102	Tax progressivity	$\bar{R}^b$	1.020	Gross nominal rate
$\bar{\pi}$	1.000	Gross inflation			

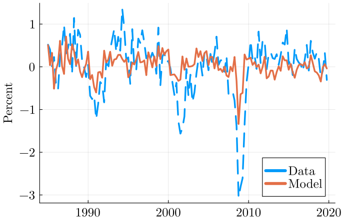
Results: Model Fit Quantities



Consumption Growth

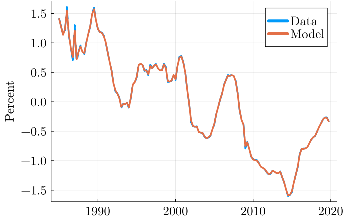


Investment Growth

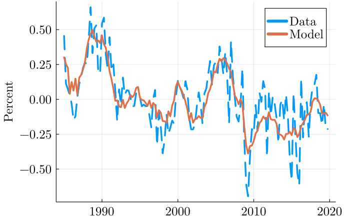


Hours Growth

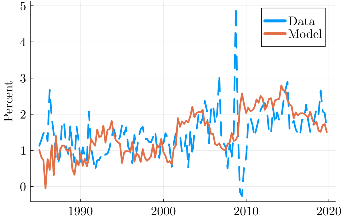
Results: Model Fit Prices



Nominal Rate



Inflation



Capital Premium

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