Optimal Redistribution: Rising Inequality vs. Rising Living Standards

Axelle Ferriere¹ Philipp Grübener² Dominik Sachs³

¹Sciences Po, CNRS & CEPR

²Washington University in St. Louis

³University of St. Gallen & CEPR

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- Large increase in **income inequality** in the US from 1950 to 2010
 - Larger top income shares, thicker Pareto tail

Piketty and Saez (2003)

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 - Income per capita tripled, spending share on necessities dropped
 - Workhorse models feature homothetic preferences: changes in levels are irrelevant
- \Rightarrow How does the standard of living affect the optimal tax-and-transfer (t&T) system?

- This paper: Optimal taxation with non-homothetic preferences
 - Heterogeneous income elasticities of demand across sectors (Engel's law)

NH CES Comin, Lashkari, and Mestieri (2021), IA Preferences Alder, Boppart, and Müller (2022)

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 - Optimal 2010 t&T system: 1 if only rising inequality; and 2 when also accounting for growth

What We Find

- lacktriangle Non-homotheticities \Rightarrow decreasing relative risk aversion (DRRA)
 - More curvature in utility function of the poor
- Mirrlees formula: two main effects of growth
 - Growth lowers dispersion in marginal utilities ⇒ Lower distribution gains from redistribution
 - Growth lowers income effects \Rightarrow Ambiguous effects on efficiency costs of redistribution

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- Quantitatively large effects of rising living standards
 - Growth calls for less redistribution
 - Dampens by at least 25% the optimal increase in redistribution due to rising inequality

Literature

■ Optimal taxation

- Stationary economies and business cycle fluctuations in homothetic one sector economies Mirrlees (1971), Diamond (1998), Saez (2001); Ramsey (1927), Werning (2007), Heathcote, Storesletten, and Violante (2017)
- Optimal tax system over time in homothetic economies
 Mankiw, Weinzierl, and Yagan (2009), Diamond and Saez (2011), Scheuer and Werning (2017), Heathcote,
 Storesletten, and Violante (2020), Brinca, Duarte, Holter, and Oliveira (2022)
- Optimal taxation with non-homothetic preferences
 Oni (2023), Jaravel and Olivi (2024)

■ Consumption patterns, Engel curves, and non-homothetic preferences

Geary (1950), Herrendorf, Rogerson, and Valentinyi (2013), Boppart (2014), Herrendorf, Rogerson, and Valentinyi (2014), Aguiar and Bils (2015), Comin, Lashkari, and Mestieri (2021), Alder, Boppart, and Müller (2022)

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Mirrleesian Optimal Nonlinear Income Taxation

with Non-Homothetic Preferences

Households

- \blacksquare Continuum of heterogeneous households with labor productivity θ
 - Pre-tax labor income $y=\theta n$, where n is labor; distribution $f(\theta)$

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- lacktriangle Separable utility over consumption and leisure: U(c)-v(n)
 - Isoelastic labor preferences $v(n) = Bn^{1+\varphi}/(1+\varphi)$
 - $c=(c_1,\ldots,c_J)$ denotes a basket of consumption goods

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 - $-c = (c_1, \ldots, c_J)$ denotes a basket of consumption goods
- Let u denote the indirect utility function

$$u(e;p,\Lambda) \equiv \max_{\{c_j\}_j} U(c)$$
 s.t. $\sum_j p_j c_j = e,$ where $p_j \equiv \frac{\hat{p}_j}{\Lambda}$

- e: nominal expenditures
- $-\hat{p}$: vector of relative prices, kept constant (drop it!)
- Λ: level of the economy ⇒ aggregate growth

Optimal Taxation Problem

■ Household's static maximization problem:

$$V(\theta; \mathcal{T}(\cdot), \Lambda) \equiv \max_{e,n} u(e; \Lambda) - B \frac{n^{1+\varphi}}{1+\varphi} \quad \text{s.t.} \quad e = n\theta - \mathcal{T}(n\theta)$$

- $\mathcal{T}(\cdot)$: fully nonlinear tax-and-transfer schedule
- Let $n(\theta; \mathcal{T}(\cdot), \Lambda)$ denote the labor policy

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- $-\mathcal{T}(\cdot)$: fully nonlinear tax-and-transfer schedule
- Let $n(\theta; \mathcal{T}(\cdot), \Lambda)$ denote the labor policy
- Government's maximization problem:

$$\max_{\mathcal{T}(\cdot;\Lambda)} \int_{\underline{\theta}}^{\overline{\theta}} V(\theta;\mathcal{T}(\cdot;\Lambda),\Lambda) w(\theta) f(\theta) d\theta \quad \text{s.t.} \quad \int_{\underline{\theta}}^{\overline{\theta}} \mathcal{T}(n(\theta;\mathcal{T}(\cdot;\Lambda),\Lambda)\theta;\Lambda) f(\theta) d\theta \geq 0$$

- Pareto weights distribution $\{w(\theta)\}$, balanced budget with no spending

Nonlinear Taxes: General Formula

lacktriangle Optimal marginal rate equates efficiency costs of taxation to distribution gains $\forall heta^*$

Heathcote and Tsujiyama (2021)

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Heathcote and Tsujiyama (2021)

$$\underbrace{1 - \frac{1 - \frac{\mathcal{T}'(y(\theta^*; \Lambda); \Lambda)}{1 - \mathcal{T}'(y(\theta^*; \Lambda); \Lambda)} \frac{1}{1 + \varphi} \frac{\theta^* f(\theta^*)}{1 - F(\theta^*)} + \int_{\theta^*}^{\bar{\theta}} \mathcal{T}'(y(\theta; \Lambda); \Lambda) \eta(\theta; \Lambda) \frac{dF(\theta)}{1 - F(\theta^*)}}_{E(\theta^*; \mathcal{T}, \Lambda)}}_{E(\theta^*; \mathcal{T}, \Lambda)} = \underbrace{1 - \underbrace{\int_{\theta}^{\bar{\theta}} u_e(\theta; \Lambda) w(\theta) \frac{dF(\theta)}{1 - F(\theta^*)}}_{\int_{\underline{\theta}}^{\bar{\theta}} u_e(\theta; \Lambda) w(\theta) dF(\theta)}}_{D(\theta^*; \mathcal{T}, \Lambda)}$$

- Let $\eta(\theta;\Lambda) \equiv dy(\theta;\Lambda)/d\mathcal{T}(0;\Lambda)$ denote the income effect of type- θ worker
- Let $u_e(\theta;\Lambda)$ denote the marginal utility of expenditure of type- θ worker

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- Let $\eta(\theta;\Lambda) \equiv dy(\theta;\Lambda)/d\mathcal{T}(0;\Lambda)$ denote the income effect of type- θ worker
- Let $u_e(\theta;\Lambda)$ denote the marginal utility of expenditure of type- θ worker
- Changes in Λ can alter: $\eta(\theta; \Lambda)$, $u_e(\theta; \Lambda)$; $y(\theta; \Lambda)$, $e(\theta; \Lambda)$

$$E(\theta^*; \mathcal{T}, \Lambda) = 1 - \frac{1 - \frac{\mathcal{T}'(y(\theta^*; \Lambda); \Lambda)}{1 - \mathcal{T}'(y(\theta^*; \Lambda); \Lambda)} \frac{1}{1 + \varphi} \frac{\theta^* f(\theta^*)}{1 - F(\theta^*)} + \int_{\theta^*}^{\bar{\theta}} \mathcal{T}'(y(\theta; \Lambda); \Lambda) \eta(\theta; \Lambda) \frac{dF(\theta)}{1 - F(\theta^*)}}{1 + \int_{\underline{\theta}}^{\bar{\theta}} \mathcal{T}'(y(\theta; \Lambda); \Lambda) \eta(\theta; \Lambda) dF(\theta)}$$

lacktriangle Efficiency costs of taxes and transfers depend on elasticities $arphi^{-1}$ and income effects η

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- **Numerator:** Fiscal effect of higher marginal rate at $y(\theta^*)$...

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 - + Decreases labor supply of worker with $y(\theta^*)$: elasticity φ^{-1}

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- **Denominator:** Effects of higher lump-sum transfer. . .

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- No behavioral responses: $\eta = 0$, $\varphi^{-1} = 0 \Rightarrow E = 0$

В

Nonlinear Taxes: Distribution Gains $D(\theta^*; \mathcal{T}, \Lambda)$

lacktriangle Distribution gains of taxes and transfers depend on dispersion of marginal utilities u_e

$$D(\theta^*; \mathcal{T}, \Lambda) = 1 - \frac{\int_{\theta^*}^{\bar{\theta}} u_e(\theta; \Lambda) w(\theta) \frac{dF(\theta)}{1 - F(\theta^*)}}{\int_{\underline{\theta}}^{\bar{\theta}} u_e(\theta; \Lambda) w(\theta) dF(\theta)}$$

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- **Numerator:** Welfare loss from taxing workers with $y > y(\theta^*)$
- Denominator: Welfare gains from increasing lump-sum transfer
- No heterogeneity: $\mathbb{E}[u_e(\theta;\Lambda)|\theta \geq \theta^*] = \mathbb{E}[u_e(\theta;\Lambda)] \ \forall \theta^* \Rightarrow D = 0$

■ Assume homothetic CRRA preferences

$$U(c) = \frac{\mathcal{C}(c)^{1-\gamma}}{1-\gamma}, \text{ where } \mathcal{C}(c) = \left(\sum_{j} \Omega_{j}^{\frac{1}{\sigma}} c_{j}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1-\sigma}{\sigma}}$$

Indirect utility function reads

$$\frac{\left(e/p^{\star}\right)^{1-\gamma}}{1-\gamma}-B\frac{n^{1+\varphi}}{1+\varphi}, \text{ with } p^{\star}=\frac{1}{\Lambda}\left(\sum_{j}\Omega_{j}\hat{p}_{j}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

 $\hfill\Box$ Proposition: The level Λ is irrelevant to the optimal level of redistribution.

 \square **Proposition:** The level Λ is irrelevant to the optimal level of redistribution.

Under the optimal tax-and-transfer system:

- Expenditures and incomes grow at constant rate $\alpha \equiv (1-\gamma)/(\gamma+\varphi) \ \forall \theta$

$$y(\theta; \Lambda(1+g)) = (1+\alpha g)y(\theta; \Lambda), \ e(\theta; \Lambda(1+g)) = (1+\alpha g)e(\theta; \Lambda),$$

- Marginal and average tax rates are constant $\forall \theta$:

$$\begin{split} \mathcal{T}'(y(\theta;\Lambda(1+g));\Lambda(1+g)) &= \mathcal{T}'(y(\theta;\Lambda);\Lambda),\\ \frac{\mathcal{T}(y(\theta;\Lambda(1+g));\Lambda(1+g))}{y(\theta;\Lambda(1+g))} &= \frac{\mathcal{T}(y(\theta;\Lambda),\Lambda)}{y(\theta;\Lambda)}. \end{split}$$

- T also grows at rate α .

 \square **Proposition:** The level Λ is irrelevant to the optimal level of redistribution.

Under the optimal tax-and-transfer system:

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- T also grows at rate α .
- Sketch of a proof: Ratios of marginal utilities are constant; Income effects are constant
- lacksquare Extend to G>0 as long as G also grow at constant rate lpha

Non-Homothetic Preferences

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 - Service shares are rising over time and with income in the cross-section
- Nonlinear Engel curves ⇒ non-constant relative risk aversion Stiglitz (1969), Hanoch (1977), Crossley and Low (2011)
- + Decreasing relative risk aversion (DRRA), or "Luxuries Are Easier to Postpone" Atkeson and Ogaki (1996), Browning and Crossley (2000)

Non-Homothetic CES Comin, Lashkari, and Mestieri (2021)

■ Utility from aggregated consumption:

$$\frac{\mathcal{C}(c)^{1-\gamma}}{1-\gamma}$$

 \blacksquare Consumption aggregator $\mathcal{C}(c)$ implicitly defined by

$$\sum_{j}^{J} \left(\Omega_{j}(\mathcal{C}(c))^{\varepsilon_{j}}\right)^{\frac{1}{\sigma}} c_{j}^{\frac{\sigma-1}{\sigma}} = 1$$

- $arepsilon_{j}$ governs income elasticity of demand for good j, σ is elasticity of substitution btw. goods

$$\Rightarrow \frac{\partial c_j}{\partial e} = \sigma + (1 - \sigma) \frac{\varepsilon_j}{\bar{\varepsilon}}$$

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- ε_j governs income elasticity of demand for good j, σ is elasticity of substitution btw. goods $\Rightarrow \frac{\partial c_j}{\partial e} = \sigma + (1-\sigma)\frac{\varepsilon_j}{\bar{\varepsilon}}$
- Focus on gross complements $\sigma < 1$

IA Preferences Alder, Boppart, and Müller (2022)

lacktriangle Preferences defined over expenditures e

$$u(e;\Lambda) = \frac{1}{1-\gamma} \left(\frac{1}{\mathbf{B}(\Lambda)} \left(e - \underbrace{\sum_{j} \hat{p}_{j}}_{\mathbf{A}(\Lambda)} \bar{c}_{j} \right) \right)^{1-\gamma} - \mathbf{D}(\Lambda)$$

$$- \text{ Price function } \mathbf{B}(\Lambda) = \Big(\textstyle \sum_j \Omega_j p_j^{1-\sigma} \Big)^{\frac{1}{1-\sigma}} = \Lambda^{-1} \Big(\textstyle \sum_j \Omega_j \hat{p}_j^{1-\sigma} \Big)^{\frac{1}{1-\sigma}} = p^\star$$



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- $\ \, \mathsf{Price function} \ \, \mathbf{B}(\Lambda) = \Big(\textstyle\sum_{j} \Omega_{j} p_{j}^{1-\sigma}\Big)^{\frac{1}{1-\sigma}} = \Lambda^{-1} \Big(\textstyle\sum_{j} \Omega_{j} \hat{p}_{j}^{1-\sigma}\Big)^{\frac{1}{1-\sigma}} = p^{\star}$
- Generalized Stone-Geary $\mathbf{A}\left(\Lambda\right)$



IA Preferences Alder, Boppart, and Müller (2022)

lacktriangle Preferences defined over expenditures e

$$u(e;\Lambda) = \frac{1}{1-\gamma} \left(\frac{1}{\mathbf{B}(\Lambda)} \left(e - \sum_{j} \frac{\hat{p}_{j}}{\Lambda} \bar{c}_{j} \right) \right)^{1-\gamma} - \mathbf{D}(\Lambda)$$

- $\ \, \mathsf{Price function} \ \, \mathbf{B}(\Lambda) = \Big(\textstyle\sum_{j} \Omega_{j} p_{j}^{1-\sigma}\Big)^{\frac{1}{1-\sigma}} = \Lambda^{-1} \Big(\textstyle\sum_{j} \Omega_{j} \hat{p}_{j}^{1-\sigma}\Big)^{\frac{1}{1-\sigma}} = p^{\star}$
- Generalized Stone-Geary $\mathbf{A}(\Lambda)$
- Price function $\mathbf{D}(\Lambda)$ is independent of expenditures e (PIGL)
 - + Regularity condition when $\mathbf{D} \neq 0$: $\gamma < 1$



$$\mathsf{RRA}(e;\Lambda) = \gamma \times \underbrace{\frac{\mathcal{C}_e(e;\Lambda)e}{\mathcal{C}(e;\Lambda)}}_{\mathsf{Elasticity of }\mathcal{C} \; \mathsf{w.r.t.} \; e} - \underbrace{\frac{\mathcal{C}_{ee}(e;\Lambda)e}{\mathcal{C}_e(e;\Lambda)}}_{\mathsf{Elasticity of }\mathcal{C}_e \; \mathsf{w.r.t.} \; e}$$

■ Non-Homothetic CES preferences

$$\mathsf{RRA}(e;\Lambda) = \gamma \times \underbrace{\frac{\mathcal{C}_e(e;\Lambda)e}{\mathcal{C}(e;\Lambda)}}_{\mathsf{Elasticity of }\mathcal{C} \; \mathsf{w.r.t.} \; e} - \underbrace{\frac{\mathcal{C}_{ee}(e;\Lambda)e}{\mathcal{C}_e(e;\Lambda)}}_{\mathsf{Elasticity of }\mathcal{C}_e \; \mathsf{w.r.t.} \; e}$$

- Homothetic: $\mathcal{C}(e;\Lambda) \propto e \Rightarrow \mathsf{RRA} = \gamma$

■ Non-Homothetic CES preferences

$$\mathsf{RRA}(e;\Lambda) = \frac{\gamma}{\gamma} \times \underbrace{\frac{\mathcal{C}_e(e;\Lambda)e}{\mathcal{C}(e;\Lambda)}}_{\mathsf{Elasticity of } \mathcal{C} \; \mathsf{w.r.t.} \; e} - \underbrace{\frac{\mathcal{C}_{ee}(e;\Lambda)e}{\mathcal{C}_e(e;\Lambda)}}_{\mathsf{Elasticity of } \mathcal{C}_e \; \mathsf{w.r.t.} \; e}$$

- Homothetic: $C(e; \Lambda) \propto e \Rightarrow \mathsf{RRA} = \gamma$
- Lemma: $\varepsilon_i \neq \varepsilon_j \Rightarrow$ Elasticity of \mathcal{C} w.r.t. e decreasing in e
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 - · Continuum of goods, $\{\varepsilon_j\}$ follow a gamma distribution
 - · Proposition: DRRA ⇔ labor supply falls with growth



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- DRRA in quantitative model with 3 goods and falling labor supply



■ IA preferences

$$\mathsf{RRA}(e;\Lambda) = \gamma \times \frac{e}{e - \mathbf{A}(\Lambda)}$$

- Proposition: Decreasing in $e \Leftrightarrow A > 0$
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$$\mathsf{RRA}(e;\Lambda) = \gamma \times \frac{e}{e - \mathbf{A}(\Lambda)}$$

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■ IA preferences

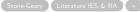
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■ Taking stock:

Dynamics of consumption baskets & dynamics of labor supply ⇒ DRRA

■ Evidence for DRRA/increasing IES Ogaki and Zhang (2001). Blundell. Browning, and Meghir (1994). Attanasio and Browning (1995). . . .



$$\underbrace{1 - \frac{1 - \frac{\mathcal{T}'(y(\theta^*; \Lambda); \Lambda)}{1 - \mathcal{T}'(y(\theta^*; \Lambda); \Lambda)} \frac{1}{1 + \varphi} \frac{\theta^* f(\theta^*)}{1 - F(\theta^*)} + \int_{\theta^*}^{\bar{\theta}} \mathcal{T}'(y(\theta; \Lambda); \Lambda) \eta(\theta; \Lambda) \frac{dF(\theta)}{1 - F(\theta^*)}}_{E(\theta^*; \mathcal{T}, \Lambda)}}_{E(\theta^*; \mathcal{T}, \Lambda)} = \underbrace{1 - \frac{\int_{\theta^*}^{\bar{\theta}} u_e(\theta; \Lambda) w(\theta) \frac{dF(\theta)}{1 - F(\theta^*)}}{\int_{\underline{\theta}}^{\bar{\theta}} u_e(\theta; \Lambda) w(\theta) dF(\theta)}}_{D(\theta^*; \mathcal{T}, \Lambda)}$$

- 1. **DRRA** ⇒ Dispersion of **marginal utilities** decreases with growth
 - → Redistribution should decrease with growth

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 - (a) Efficiency cost of taxes increases \rightarrow Redistribution should decrease with growth
 - (b) Efficiency cost of lump-sum transfer decreases → Redistribution should increase with growth

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- Proposition [1+3]
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- Quantitatively, decreasing dispersion in marginal utilities dominates

Quantification in a Dynamic Model

with Private Insurance

Quantification in a Dynamic Model

- Dynamic incomplete markets model with private saving
 - To disentangle inequality in expenditure, income, and wealth
 - To discipline DRRA with dynamic savings decisions
- Parametric tax-and-transfer system

Ferriere, Grübener, Navarro, and Vardishvili (2023)

Households: Value Function

■ Household's value function with productivity θ and assets a:

$$V(a,\theta) = \max_{e,a',n} \left\{ u(e;\Lambda) - B \frac{n^{1+\varphi}}{1+\varphi} + \beta \mathbb{E}_{\theta'} \left[V(a',\theta') | \theta \right] \right\}$$

 $e+a' < \theta n + (1+r)a - \mathcal{T}(\theta n)$. a' > 0

- Productivity θ follows a stochastic process
- Discount factor β

s.t.

- Fixed interest rate r (partial equilibrium)

Calibration Overview

- Calibration to the US economy in 1950 and 2010 with three sectors
 - Preferences; Government; Growth; Inequality

Calibration Growth

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 - Preferences; Government; Growth; Inequality
- Growth: Fall in prices
 - Aggregate growth in GDP per capita: 3.3
 NIPA
 - Prices relative to goods

Computed based on Herrendorf, Rogerson, and Valentinyi (2013) [NIPA]

- + Agriculture (food) \rightarrow 1.00, 1.87
- + Services \rightarrow 1.00, 3.16

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- + Agriculture (food) \rightarrow 1.00, 1.87 + Services \rightarrow 1.00, 3.16
- Interest rate fixed at 2%; discount factor to match wealth-to-income ratio of 4 in 2010 Piketty and Zucman (2014) [NIPA]
 - Untargeted wealth-to-income ratio in 1950 of 3 [data: 3.65]

Calibration Government

Functional form

- Parametric tax function plus lump-sum transfer

Ferriere, Grübener, Navarro, and Vardishvili (2023)

$$\mathcal{T}(y) = \exp\left[\log(\lambda)\left(y^{-2\tau}\right)\right]y - T$$

 $+\lambda$: level of the tax rates; τ : progressivity; T: transfers



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■ Changes over time

- T to match spending on income security

White House Office of Management & Budget

$$+$$
 $T/Y=1.1\%$ in 1950 \rightarrow 3.6% in 2010

- au to match difference in average marginal tax rate between top 10% and bottom 90% Mertens and Montiel Olea (2018)

$$+$$
 AMTR is 13% in 1950 \rightarrow 9% in 2010

- Exogenous government spending to capture all remaining federal spending
 - + Constant over time: $G/Y \approx 14\%$



Calibration Inequality

- Wages follow AR(1) in logs, with appended Pareto tail
 - Persistence ρ fixed at 0.9
 - Shock innovation set to match variance of log income from SCF+ Kuhn, Schularick, and Steins (2020)
 - Time-varying Pareto tail parameter Aoki and Nirei (2017)

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1950	Income Share by Quintile					
Model	6%	11%	13%	21%	49%	
Data (SCF+)	6%	11%	15%	21%	48%	
2010	Income Share by Quintile					
Model	4%	9%	11%	19%	56%	
Data (SCF+)	4%	9%	13%	21%	53%	

Calibration Inequality

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1950	Wealth Share by Quintile						
Model	0%	2%	6%	17%	76%		
Data (SCF+)	0%	1%	4%	11%	84%		
2010	Wealth Share by Quintile						
Model	0%	1%	5%	13%	81%		
Data (SCF+)	-1%	1%	3%	10%	87%		

Calibration Preferences

- Non-homothetic CES parameters
 - Income elasticities of demand and elasticity of substitution between goods Estimates of Comin, Lashkari, and Mestieri (2021) based on CEX micro data

$$+ \ \sigma = 0.3; \ \varepsilon_A = 0.1, \varepsilon_G = 1.0, \varepsilon_S = 1.8$$

Calibration Preferences

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- Income elasticities of demand and elasticity of substitution between goods Estimates of Comin, Lashkari, and Mestieri (2021) based on CEX micro data $+ \sigma = 0.3$; $\varepsilon_A = 0.1$, $\varepsilon_C = 1.0$, $\varepsilon_S = 1.8$

- $-\Omega_{j}$: match aggregate sector shares in 2010 Computed based on Herrendorf, Rogerson, and Valentinyi (2013) [NIPA]
 - + Agriculture (food) 8%, goods 26%, services 67%
 - + Untargeted 1950: agriculture 17% [data 22%], goods 49% [39%], services 34% [39%]

Calibration Preferences

■ Non-homothetic CES parameters

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■ Remaining preference parameters

- Fix Frisch elasticity $1/\varphi$ to standard value of 0.5
- Consumption curvature γ to match RRA ≈ 1 in 2010

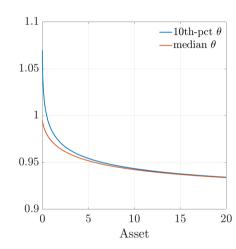
Implied RRA in the Model Decreasing RRA

■ Calibrated non-homothetic preferences imply DRRA



Implied RRA in the Model Decreasing RRA

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 - RRA falls from 1.07 in 1950 to 1, small dispersion

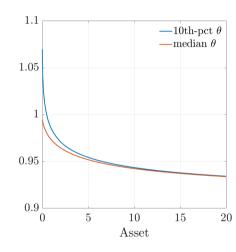




Implied RRA in the Model Decreasing RRA

- Calibrated non-homothetic preferences imply DRRA
 - RRA falls from 1.07 in 1950 to 1, small dispersion
- Implied labor supply dynamics
 - Falling labor supply over time
 - + Fall in average hours by 7%

 Ramey and Francis (2009), Boppart and Krusell (2020)
 - Cross-section correlation between hours and wages
 - + Roughly flat hours profile in 1950, positive in 2010 Costa (2000), Mantovani (2022)



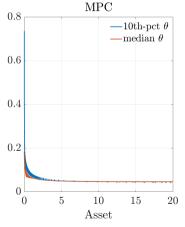
Implied RRA in the Model MPCs and Wealth Effects

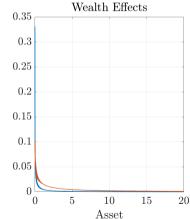
■ Relation between RRA, wealth effects, and MPC: RRA × MPC = $\eta\left(\varphi\frac{e}{y} + \frac{e\mathcal{T}''(y)}{\mathcal{T}'(y)}\right)$



Implied RRA in the Model MPCs and Wealth Effects

lacktriangleq Relation between RRA, wealth effects, and MPC: $\frac{\mathsf{RRA}}{\mathsf{RRA}} \times \mathsf{MPC} = \eta \left(\varphi \frac{e}{y} + \frac{e \mathcal{T}''(y)}{\mathcal{T}'(y)} \right)$





- Model MPC: 18% in 2010 Johnson, Parker, and Souleles (2006), Fagereng, Holm, and Natvik (2021), Kaplan and Violante (2022)
- Wealth effects: 0.02 in 2010 Golosov, Graber, Mogstad, and Novgorodsky (2023)

Rising Living Standards vs. Rising Inequality

■ Use dynamic model to quantify effect of rising living standards relative to rising inequality



Rising Living Standards vs. Rising Inequality

- Use dynamic model to quantify effect of rising living standards relative to rising inequality
- Pareto weights
 - Inverse optimum in 1950
 - Bourguignon and Spadaro (2012), Lockwood and Weinzierl (2016), Hendren (2020)
 - Weights as a function of the expenditure percentile

$$\omega(p_i) = \mu + p_i(e_i)^{\nu}$$
, with $\mu = 0.05$, $\nu = 116.4$



Rising Living Standards vs. Rising Inequality

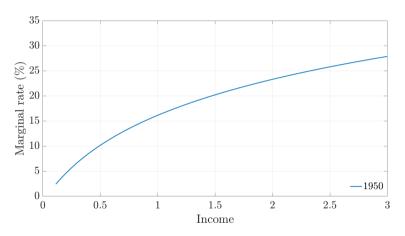
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$$\omega(p_i) = \mu + p_i(e_i)^{\nu}$$
, with $\mu = 0.05$, $\nu = 116.4$

- Experiment in two steps
 - First add inequality only
 - Second compare optimal 2010 with inequality and growth
 - + Growth: fall in prices and changes in relative prices

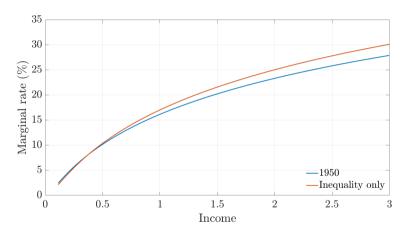


Optimal Marginal Rates



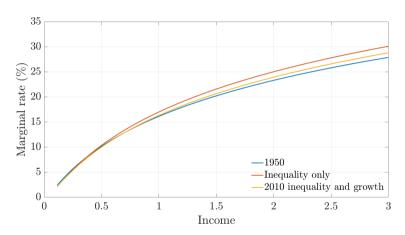
■ Calibration in 1950: $T/Y \approx 1\%$

Optimal Marginal Rates



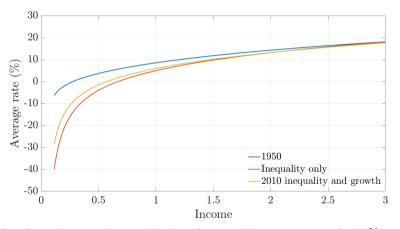
 \blacksquare Calibration in 1950: $T/Y\approx 1\%~\Rightarrow T/Y=4.6\%$ with higher inequality

Optimal Marginal Rates



- \blacksquare Calibration in 1950: $T/Y \approx 1\% \ \Rightarrow T/Y = 3.3\%$ with higher inequality and growth
 - Growth reduces increase in T/Y by 35%

Optimal Average Rates



■ Growth reduces increase in top-10 minus bottom-10 average rates by 30%

Quantitative Mirrlees Setup

- Calibration following a partial-insurance approach
 - Target consumption dispersion of the quantitative model in 1950 and 2010

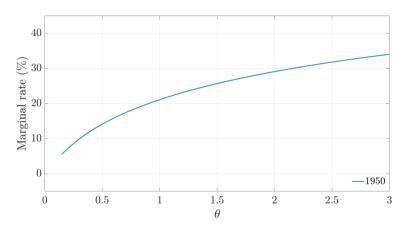


Quantitative Mirrlees Setup

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 - Target consumption dispersion of the quantitative model in 1950 and 2010
- Replicate the main quantitative exercise
 - Obtain similar effects of rising living standards relative to rising inequality

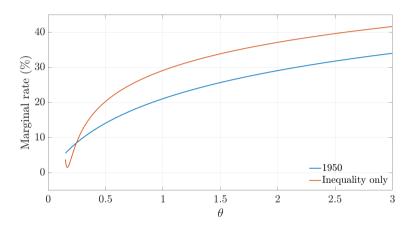


Optimal Marginal Rates Mirrlees



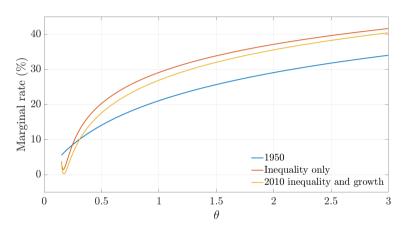
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 \blacksquare Calibration in 1950: $T/Y\approx 1\% \ \Rightarrow T/Y=6.7\%$ with higher inequality

Optimal Marginal Rates Mirrlees



- \blacksquare Calibration in 1950: $T/Y \approx 1\% \ \Rightarrow T/Y = 4.5\%$ with higher inequality and growth
 - Growth reduces increase in T/Y by 40%

Quantitative Mirrlees Setup

- Calibration following a partial-insurance approach
 - Target consumption dispersion of the quantitative model in 1950 and 2010
- Replicate the main quantitative exercise
- Decompose the different channels using the optimal tax formula
 - Decomposition into effects of marginal utilities, income effects, and the hours distribution

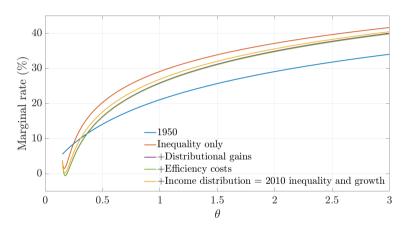
Optimal Marginal Rates Decomposition

- \blacksquare In 1950, calibrated/optimal $T/Y\approx 1\%$
- \blacksquare Optimal T/Y in 2010
 - Accounting for inequality only: T/Y = 6.7%
 - Accounting for growth as well: T/Y=4.5% \Rightarrow -2.2 p.p.

Optimal Marginal Rates Decomposition

- \blacksquare In 1950, calibrated/optimal $T/Y\approx 1\%$
- \blacksquare Optimal T/Y in 2010
 - Accounting for inequality only: T/Y = 6.7%
 - Accounting for growth as well: $T/Y = 4.5\% \Rightarrow$ -2.2 p.p.
 - + Fall in dispersion in marginal utilities: -2.9 p.p.
 - + Also accounting for lower income effects: -0.1 p.p.
 - + Also accounting for the more compressed distribution of hours: +0.8 p.p.

Optimal Marginal Rates Decomposition



■ 1950: T/Y=1.2% \Rightarrow T/Y=6.7% with inequality, T/Y=4.5% with growth \Rightarrow T/Y=3.8% with marginal utilities only, T/Y=3.7% adding efficiency concerns

Quantitative Mirrlees Setup

- Calibration following a partial-insurance approach
- Replicate the main quantitative exercise
- Decompose the different channels using the optimal tax formula
- Robustness

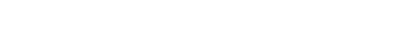


Conclusion

- Optimal taxation with rising living standards
 - Affect efficiency and distribution concerns
- Dampen optimal increase in redistribution due to rising inequality



Appendix



Literature

Evidence: Risk Aversion and IES

- DRRA supported by consumption data from Indian villages Ogaki and Zhang (2001), Zhang and Ogaki (2004)
- IES increasing in consumption/wealth, based on estimating consumption Euler equation Blundell, Browning, and Meghir (1994), Attanasio and Browning (1995), Atkeson and Ogaki (1996)
- DRRA powerful in matching portfolio choices across the wealth distribution Wachter and Yogo (2010), Straub (2019), Cioffi (2021), Meeuwis (2022)



Cardinalization

- Infer intertemporal properties of utility from intratemporal allocations
 - Cardinalization?
 - One can always add a monotonic V(.) function to $u(e;\Lambda)-B\frac{n^{1+\varphi}}{1+\varphi}$

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Stiglitz (1969), Hanoch (1977), Crossley and Low (2011)

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- Quantitative: Dynamic model with dynamic policy functions

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Data Appendix

SCF+

- Long-run data on income and wealth inequality in the US Compiled by Kuhn, Schularick, and Steins (2020)
 - Based on historical waves of the Survey of Consumer Finances (SCF)
 - Time period 1949-2016
- Income components
 - Wages and salaries
 - Income from professional practice and self-employment
 - Business and farm income
 - Excluded: rental income, interest, dividends, transfers

SCF+ (cont.)

- Net worth/wealth components (assets debt)
 - Assets
 - + Liquid assets: checking, savings, call/money market accounts, certificates of deposit
 - + Housing and other real estate
 - + Bonds, stocks and business equity, mutual funds
 - + Cash value of life insurance
 - + Defined-contribution retirement plans
 - + Cars
 - Debt
 - + Housing debt: debt on owner-occupied homes, home equity loans and lines of credit
 - + Other debt: car loans, education loans, consumer loans

SCF+ (cont.)

- Sample selection
 - Head of household aged 25 to 60
 - Minimum income restriction
 - + \$5,000 for 2010 (in 2016 dollars)
 - + In 1950 such that ratio of minimum income to median is the same (\$2,700)



Government Spending

■ Programs included in transfers

White House Office of Management & Budget

- General retirement and disability insurance (excluding social security)
- Federal employee retirement and disability; Unemployment compensation
- Housing assistance; Food and nutrition assistance; Other income security

■ Government spending

- Supposed to capture all remaining federal spending
- Purposefully chosen such that G/Y constant
 - + Spending has risen in the data, but largely deficit-financed



Model Appendix

Non-Homothetic Preferences Non-Homothetic CES

Comin, Lashkari, and Mestieri (2021)

- Conditions for DRRA with two goods: $\varepsilon_1 < \varepsilon_2 = 1$
 - Necessary condition: $\gamma > \varepsilon_1$
 - Sufficient condition: $\gamma + \varepsilon_1 \geq 2$
- Typical calibration with three goods ⇒ quantitatively true



Non-Homothetic Preferences Stone-Geary Preferences

Geary (1950)

■ One-sector Stone-Geary preferences

$$u(c) = \frac{(c - \bar{c})^{1 - \gamma}}{1 - \gamma}$$

- Subsistence consumption level $\bar{c} > 0$
- ⇒ Implies increasing elasticity of intertemporal substitution (DRRA)
 - Counterfactual: vanishing non-homotheticities

IA Preferences Alder, Boppart, and Müller (2022)

 \blacksquare **D**(.) term defined as:

$$\mathbf{D}(\Lambda) = \frac{\nu}{\eta} \left(\left[\left(\sum_{j \in J} \theta_j p_j^{1-\iota} \right)^{\frac{1}{1-\iota}} \mathbf{B}(\Lambda)^{-1} \right]^{\eta} - 1 \right)$$

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 \blacksquare Consumption share $cs_j \equiv p_j c_j / e$

$$cs_{j} = \frac{\mathbf{A}_{j}p_{j}}{e} + \frac{\mathbf{B}_{j}p_{j}}{\mathbf{B}} \left(1 - \frac{\mathbf{A}}{e} \right) + \frac{\mathbf{D}_{j}}{\gamma} p_{j} \left(\frac{e}{\mathbf{B}} - \frac{\mathbf{A}}{\mathbf{B}} \right)^{\gamma} \left(\frac{e}{\mathbf{B}} \right)^{-1}$$

$$cs_{j} = \frac{\mathbf{A}_{j}p_{j}}{e} + \frac{\mathbf{B}_{j}p_{j}}{\mathbf{B}} \left(1 - \frac{\mathbf{A}}{e} \right) + \frac{\mathbf{D}_{j}}{\gamma} p_{j} \frac{\mathbf{B}^{1-\gamma}}{e^{1-\gamma}} \left(1 - \frac{\mathbf{A}}{e} \right)^{\gamma}$$

$$\mathbf{E} \mathbf{X} / \partial p_{j}.$$

where $\mathbf{X}_j = \partial \mathbf{X}/\partial p_j$.

Calibration Aggregates

- Prices for all goods p_A, p_G, p_S pinned down by growth and relative price changes
 - Aggregate growth in GDP per capita: 3.3
 NIPA
 - Prices relative to goods
 Computed based on Herrendorf, Rogerson, and Valentinyi (2013) [NIPA]

```
+ Agriculture (food) \rightarrow 1.00, 1.87 + Services \rightarrow 1.00, 3.16
```

- Interest rate fixed at 2%; discount factor to match wealth-to-income ratio of 4 in 2010 Piketty and Zucman (2014) [NIPA]
 - Untargeted wealth-to-income ratio in 1950 of 3 [data: 3.65]

Calibration Preferences

■ Non-homothetic CES parameters

- $\{\varepsilon_j\}$ and σ : estimates of Comin, Lashkari, and Mestieri (2021) with CEX micro data + $\sigma=0.3;$ $\varepsilon_A=0.1,$ $\varepsilon_G=1.0,$ $\varepsilon_S=1.8$

Calibration Preferences

■ Non-homothetic CES parameters

- $\{\varepsilon_j\}$ and σ : estimates of Comin, Lashkari, and Mestieri (2021) with CEX micro data + $\sigma=0.3;$ $\varepsilon_A=0.1,$ $\varepsilon_G=1.0,$ $\varepsilon_S=1.8$
- Ω_j : match aggregate sector shares in 2010 Computed based on Herrendorf, Rogerson, and Valentinyi (2013) [NIPA]
 - + Agriculture (food) 8%, goods 26%, services 67%
 - + Untargeted 1950: agriculture 17% [data 22%], goods 49% [39%], services 34% [39%]

Labor Supply in the Time Series and Cross-Section

■ Fall in average hours across time: 7%
Ramey and Francis (2009), Boppart and Krusell (2020)

- Correlation between hours and hourly wage in the cross-section
 - Roughly flat hours profile in 1950
 - Positive in 2010

costa2000wage, Mantovani (2022)



Calibration Income inequality

- Wages follow AR(1) in logs, with appended Pareto tail
 - Time-varying Pareto tail parameter Aoki and Nirei (2017)
 - Time-varying innovation to AR(1) set to match variance of log income from SCF+ Kuhn, Schularick, and Steins (2020)

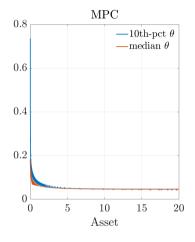
1950	Income Share by Quintile						
Model	6%	11%	13%	21%	49%		
Data (SCF+)	6%	11%	15%	21%	48%		
2010	Income Share by Quintile						
Model	4%	8%	12%	19%	56%		
Data (SCF+)	4%	9%	13%	21%	53%		

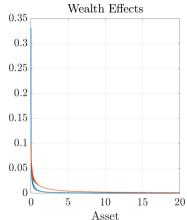
Calibration Expenditure Inequality

- Variance of log consumption in 2010: 0.46, top-quintile expenditure share of 45%
- Less expenditure inequality in 1950
- Variance of log consumption in 1950: 0.33, top-quintile expenditure share of 39%



Implied RRA in the Model MPCs and Wealth Effects





- Model MPC: 18% in 2010 Johnson, Parker, and Souleles (2006), Fagereng, Holm, and Natvik (2021), Kaplan and Violante (2022)
- Wealth effects: 0.02 in 2010 Golosov, Graber, Mogstad, and Novgorodsky (2023)

Wealth Effects: Evidence Golosov, Graber, Mogstad, and Novgorodsky (2023)

- How does income respond to unexpected wealth shocks?
 - Golosov et al. merge US tax data with data on lottery winnings
 - Compute earnings change over five years after lottery win
 - Earnings drop by on average 2.3\$ per 100\$ of win
- Replicate in model using mean post-tax win
 - Earnings drop by on average 2.1\$ per 100\$ of win



Calibration: Inequality

- A partial-insurance approach
 - Calibrate f(.) as exponentially modified Gaussian (EMG) to match dispersion in expenditures

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- In 2010, data on income and expenditure inequality
 - Dispersion: $\mathbb{V}[\log y] = 0.78$; $\mathbb{V}[\log e] \approx 0.35$ SCF+ (Kuhn, Schularick, and Steins 2020); Attanasio and Pistaferri (2014), Heathcote, Perri, and Violante (2010)
 - Pareto tail: $\lambda_y=1.65$; $\lambda_e\approx 3.3$ Aoki and Nirei (2017); Toda and Walsh (2015)

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 - Pareto tail: $\lambda_y=1.65;~\lambda_epprox 3.3$ Aoki and Nirei (2017); Toda and Walsh (2015)
- In 1950, data on income inequality only
 - Dispersion: $\mathbb{V}[\log y] = 0.57$; \Rightarrow infer $\mathbb{V}[\log e] \approx 0.25$ SCF+ (Kuhn, Schularick, and Steins 2020)
 - Pareto tail: $\lambda_y = 2.2 \Rightarrow \text{infer } \lambda_e = 4.4$ Aoki and Nirei (2017)

Calibration: Expenditure Inequality

1950	Expenditure Share by Quintile						
Dynamic model	8%	13%	17%	23%	39%		
Static model	9%	13%	17%	23%	38%		
2010	Expenditure Share by Quintile						
Dynamic model	7%	11%	16%	21%	45%		
Static model	7%	12%	16%	23%	43%		

- Use Mirrlees formula to quantify how growth changes efficiency vs. distribution concerns
 - Static "partial insurance" setup with expenditure distribution as in dynamic model



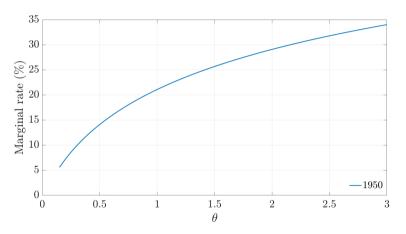
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 - + Pareto weights such that calibrated 1950 tax system is optimal



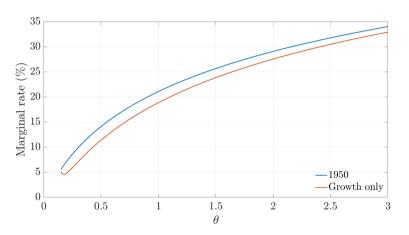
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 - Optimal taxes with growth of 2010
 - + Decomposition into effects of marginal utilities, income effects, and the hours distribution





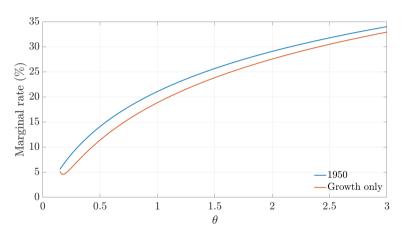
■ Optimal 1950 transfers: T/Y = 1.2%





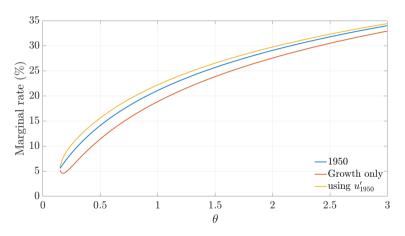
■ Optimal 1950 transfers: T/Y = 1.2% \Rightarrow With 2010 growth, T/Y = -0.7%





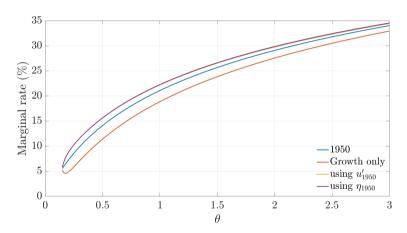
■ With 2010 growth, T/Y = -0.7%





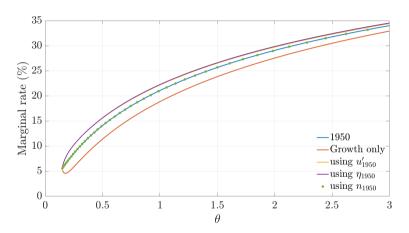
■ With 2010 growth, T/Y = -0.7% \Rightarrow With 1950 marg. u dispersion, T/Y = 2.4%





■ With 2010 growth, T/Y = -0.7% \Rightarrow With 1950 income effects, T/Y = 2.4%





■ With 2010 growth, T/Y = -0.7% \Rightarrow With 1950 hours worked, T/Y = 1.2% (1950 level)



■ Decomposition into effects of marginal utilities, income effects, and the hours distribution

$$\underbrace{1 - \frac{1 - \frac{\mathcal{T}'(y(\theta^*; \Lambda); \Lambda)}{1 - \mathcal{T}'(y(\theta^*; \Lambda); \Lambda)} \frac{1}{1 + \varphi} \frac{\theta^* f(\theta^*)}{1 - F(\theta^*)} + \int_{\theta^*}^{\bar{\theta}} \mathcal{T}'(y(\theta; \Lambda); \Lambda) \eta(\theta; \Lambda) \frac{dF(\theta)}{1 - F(\theta^*)}}_{E(\theta^*; \mathcal{T}, \Lambda)}}_{E(\theta^*; \mathcal{T}, \Lambda)} = \underbrace{1 - \frac{\int_{\theta^*}^{\bar{\theta}} u_e(\theta; \Lambda) w(\theta) \frac{dF(\theta)}{1 - F(\theta^*)}}{\int_{\underline{\theta}}^{\bar{\theta}} u_e(\theta; \Lambda) w(\theta) dF(\theta)}}_{D(\theta^*; \mathcal{T}, \Lambda)}$$

■ Starting from optimal taxes with growth



$$\underbrace{1 - \frac{1 - \frac{\mathcal{T}'(y(\theta^*; \Lambda); \Lambda)}{1 - \mathcal{T}'(y(\theta^*; \Lambda); \Lambda)} \frac{1}{1 + \varphi} \frac{\theta^* f(\theta^*)}{1 - F(\theta^*)} + \int_{\theta^*}^{\bar{\theta}} \mathcal{T}'(y(\theta; \Lambda); \Lambda) \eta(\theta; \Lambda) \frac{dF(\theta)}{1 - F(\theta^*)}}_{E(\theta^*; \mathcal{T}, \Lambda)}}_{E(\theta^*; \mathcal{T}, \Lambda)} = \underbrace{1 - \frac{\int_{\theta^*}^{\bar{\theta}} u_e(\theta; \Lambda) w(\theta) \frac{dF(\theta)}{1 - F(\theta^*)}}{\int_{\underline{\theta}}^{\bar{\theta}} u_e(\theta; \Lambda) w(\theta) dF(\theta)}}_{D(\theta^*; \mathcal{T}, \Lambda)}$$

- Starting from optimal taxes with growth
 - 1. Optimal taxes with $u_e(\cdot)$ computed using p_{1950}



$$\underbrace{1 - \frac{1 - \frac{\mathcal{T}'(y(\theta^*; \Lambda); \Lambda)}{1 - \mathcal{T}'(y(\theta^*; \Lambda); \Lambda)} \frac{1}{1 + \varphi} \frac{\theta^* f(\theta^*)}{1 - F(\theta^*)} + \int_{\theta^*}^{\bar{\theta}} \mathcal{T}'(y(\theta; \Lambda); \Lambda) \eta(\theta; \Lambda) \frac{dF(\theta)}{1 - F(\theta^*)}}_{E(\theta^*; \mathcal{T}, \Lambda)}}_{E(\theta^*; \mathcal{T}, \Lambda)} = \underbrace{1 - \frac{\int_{\theta^*}^{\bar{\theta}} u_e(\theta; \Lambda) w(\theta) \frac{dF(\theta)}{1 - F(\theta^*)}}{\int_{\underline{\theta}}^{\bar{\theta}} u_e(\theta; \Lambda) w(\theta) dF(\theta)}}_{D(\theta^*; \mathcal{T}, \Lambda)}$$

- Starting from optimal taxes with growth
 - 1. Optimal taxes with $u_e(\cdot)$ computed using p_{1950}
 - 2. Adding $\eta(\cdot)$ using p_{1950}



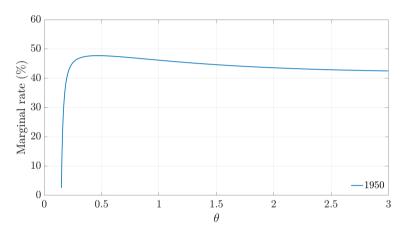
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- Starting from optimal taxes with growth
 - 1. Optimal taxes with $u_e(\cdot)$ computed using p_{1950}
 - 2. Adding $\eta(\cdot)$ using p_{1950}
 - 3. Adding $n(\cdot)$ using p_{1950}

$$\underbrace{1 - \frac{1 - \frac{\mathcal{T}'(y(\theta^*;\Lambda);\Lambda)}{1 - \mathcal{T}'(y(\theta^*;\Lambda);\Lambda)} \frac{1}{1 + \varphi} \frac{\theta^* f(\theta^*)}{1 - F(\theta^*)} + \int_{\theta^*}^{\bar{\theta}} \mathcal{T}'(y(\theta;\Lambda);\Lambda) \eta(\theta;\Lambda) \frac{dF(\theta)}{1 - F(\theta^*)}}_{E(\theta^*;\mathcal{T},\Lambda)}}_{E(\theta^*;\mathcal{T},\Lambda)} = \underbrace{1 - \frac{\int_{\theta^*}^{\bar{\theta}} u_e(\theta;\Lambda) w(\theta) \frac{dF(\theta)}{1 - F(\theta^*)}}{\int_{\underline{\theta}}^{\bar{\theta}} u_e(\theta;\Lambda) w(\theta) dF(\theta)}}_{D(\theta^*;\mathcal{T},\Lambda)}$$

- Starting from optimal taxes with growth
 - 1. Optimal taxes with $u_e(\cdot)$ computed using p_{1950}
 - 2. Adding $\eta(\cdot)$ using p_{1950}
 - 3. Adding $n(\cdot)$ using p_{1950}
 - \Rightarrow Back to 1950

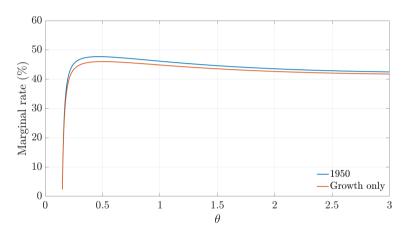
Optimal Marginal Rates with Growth Utilitarian



■ Optimal 1950 transfers: T/Y = 25.2%



Optimal Marginal Rates with Growth Utilitarian



■ Optimal 1950 transfers: T/Y = 25.2% \Rightarrow With 2010 growth, T/Y = 24.0%



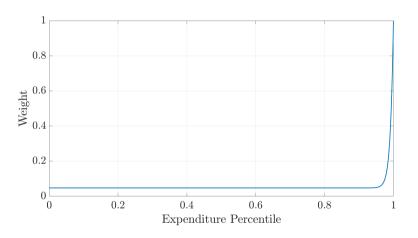
Weights

- More degrees of freedom in finding inverse optimum weights
- Restriction to functional form motivated by instruments: lump sum and progressivity
- Weights as function of percentiles of the expenditure distribution

$$\omega\left(p_i\right) = \mu + p_i(e_i)^{\nu}$$

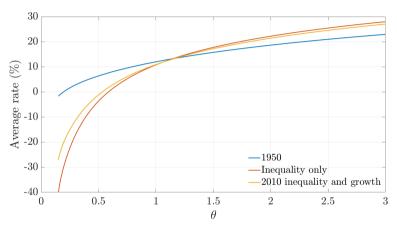
 $\mu = 0.05, \ \nu = 116.4$

Weights





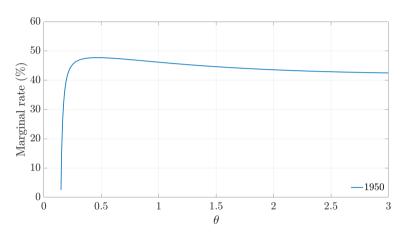
Optimal Average Rates Mirrlees



■ Growth reduces increase in top-10 minus bottom-10 average rates by almost 30%



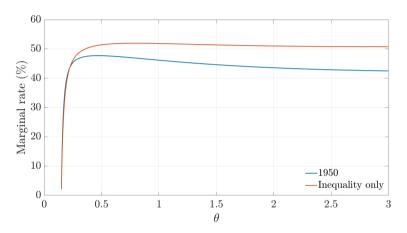
Optimal Marginal Rates Mirrlees Utilitarian



■ Optimum in 1950: T/Y = 25.2%



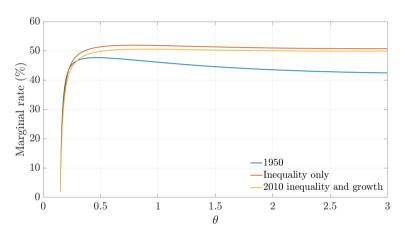
Optimal Marginal Rates Mirrlees Utilitarian



■ Optimum in 1950: $T/Y = 25.2\% \Rightarrow T/Y = 29.2\%$ with higher inequality

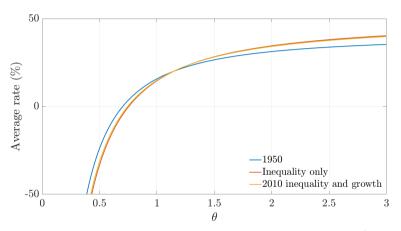


Optimal Marginal Rates Mirrlees Utilitarian



- Optimum in 1950: T/Y = 25.2% \Rightarrow T/Y = 27.6% with higher inequality and growth
 - Growth reduces increase in T/Y by 39%

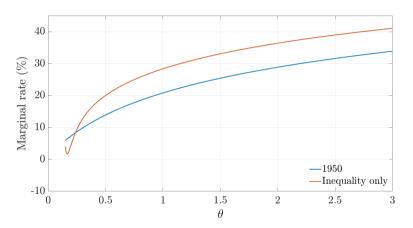
Optimal Average Rates Mirrlees Utilitarian



lacktriangle Growth reduces increase in top-10 minus bottom-10 average rates by 9%

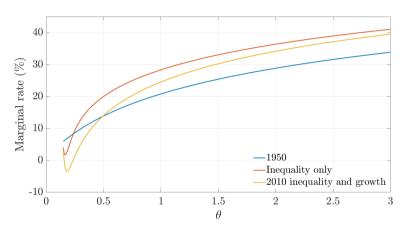


Optimal Marginal Rates Mirrlees IA Preferences



■ Calibration in 1950: T/Y = 1.1% $\Rightarrow T/Y = 5.6\%$ with higher inequality

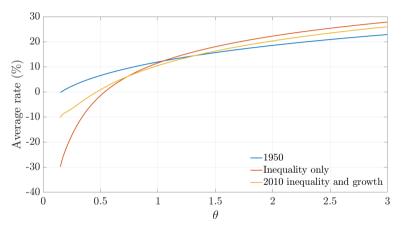
Optimal Marginal Rates Mirrlees IA Preferences



- Calibration in 1950: T/Y = 1.1% \Rightarrow T/Y = 2.0% with higher inequality and growth
 - Growth reduces increase in T/Y by more than 80%



Optimal Average Rates Mirrlees IA Preferences



■ Growth reduces increase in top-10 minus bottom-10 average rates by almost 50%



IA Parameters

$$1 - \eta = \gamma = 0.9$$

■ A-term

$$-\bar{c}_A = 0.03$$
, $\bar{c}_G = 0.00$, $\bar{c}_S = 0.005$

■ B-term

- $-\sigma = 0.001$
- $\omega_A=0.06$, $\omega_G=0.4$, $\omega_S=1-\omega_A-\omega_G$

■ D-term

- $\nu = 15$
- $\iota = 2$
- $-\theta_A = 0.22, \ \theta_G = 0.62, \ \theta_S = 1 \theta_A \theta_G$

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