Exercises for Chapter 5

1 True or false?

For each of the following statements, decide whether it is true or false, and **explain your answer in at most two sentences**.

Question 1.1. There is no conceptual difference between ideas and physical goods, so economically, they should be treated in the same way.

Question 1.2. A patent is a monopoly right. Monopolies are socially destructive. We should therefore get rid of patents.

Question 1.3. During the coronavirus pandemic in July 2020, the government started Operation Warp Speed which, among other things, promised to buy 100 milion doses of the Pfizer vaccine at a pre-agreed price if Pfizer is able to deliver the vaccine by December 2020. This is an example how other mechanisms than patents may incentivize R&D.

Question 1.4. In the Romer model, the society can trade off short-term losses with long-term gains in output by shifting people from the production sector to the research sector.

2 Exercise: Combining Romer and Solow**

In this section, we will combine the Solow and Romer models together and derive a model that will exhibit both transition dynamics (catching up of poor countries) as well as persistent growth. The derivation of the model closely follows the textbook and the slides.

Time is discrete and indexed by $t = 0, 1, 2, \ldots$. The economy starts at time 0 with an initial stock of ideas A_0 and stock of capital K_0, and is described by the following five equations, with variables that we defined in class.

$$
\begin{aligned}
1 &: \quad Y_t = A_t K_t^{\alpha} L_{yt}^{1-\alpha} \\
2 &: \quad \Delta A_{t+1} = \bar{z} A_t L_{at} \\
3 &: \quad \Delta K_{t+1} = sY_t - \delta K_t \\
4 &: \quad \bar{L} = L_{yt} + L_{at} \\
5 &: \quad L_{at} = \bar{l}\bar{L}
\end{aligned}
$$

Question 2.1. Give names to each of the five equations, representing their economic meaning.

Question 2.2. Solve for the growth rate of ideas $g_{A,t} \equiv \frac{\Delta A_{t+1}}{A_t}$.

We now introduce a method how to solve separately for the transition dynamics. The method will consist of rescaling output Y_t and capital K_t by the level of technology. Define so-called **scaled variables**

$$\tilde{Y}_t = \frac{Y_t}{(A_t)^{\frac{1}{1-\alpha}}} \quad \text{and} \quad \tilde{K}_t = \frac{K_t}{(A_t)^{\frac{1}{1-\alpha}}}. \tag{1}$$

We can interpret $\tilde{Y}_t$ and $\tilde{K}_t$ as output and capital levels **relative** to the level of technology. You may wonder why A_t has the exponent $\frac{1}{1-\alpha}$ in this scaling, but for now taking it as a 'good guess' which will become clearer very soon.

Question 2.3. Express Y_t and K_t from equations (1), and plug them into the output production function (first equation of Solow–Romer model above). Show that you can simplify the equation so that the technology level A_t drops out of the equation.

Question 2.4. In the same way, plug in Y_t and K_t from equations (1) into the capital accumulation equation (third equation of Solow–Romer model above). Simplify the equations and show that the production function will now not depend on A_t directly, and the capital accumulation equation will only contain A_t as a ratio $\frac{A_{t+1}}{A_t}$, which is equal to $1 + g_{A,t}$.

Hint: Before you start, it is advantageous to write the capital accumulation equation as

$$K_{t+1} = sY_t + (1-\delta)\, K_t.$$

You should obtain

$$\tilde{K}_{t+1} = \frac{s}{(1+g_{A,t})^{\frac{1}{1-\alpha}}}\tilde{Y}_t + \frac{1-\delta}{(1+g_{A,t})^{\frac{1}{1-\alpha}}}\tilde{K}_t. \tag{2}$$

Question 2.5. Show that we can write this capital accumulation equation as

$$\Delta\tilde{K}_{t+1} = \tilde{s}\tilde{Y}_t - \tilde{\delta}\tilde{K}_t$$

where $\tilde{s}$ and $\tilde{\delta}$ are parameters that can be called 'adjusted saving rate' and 'adjusted depreciation rate'. Express these parameters as a function of the original parameters of the model and the growth rate $g_{A,t}$.

Observe that we can now write the production function and the capital accumulation equation as

$$\begin{aligned} \tilde{Y}_t &= \tilde{K}_t^{\alpha} L_{yt}^{1-\alpha} \\ \tilde{K}_{t+1} - \tilde{K}_t &= \tilde{s}\tilde{Y}_t - \tilde{\delta}\tilde{K}_t \end{aligned}$$

Question 2.6. Express L_{yt} in terms of the parameters of the model. Now determine the 'steady state' in variables $\tilde{Y}_t$, $\tilde{K}_t$.

Question 2.7. When $\tilde{K}_t$ and $\tilde{Y}_t$ are in steady state, at what rate do capital K_t and output Y_t grow? Notice that you will obtain that K_t and Y_t grow at constant rates — such a situation is called the **balanced growth path**.

Question 2.8. Verify that $g_Y > g_A$. Why does output and capital grow at a faster rate than technology?