Optimal Redistribution: Rising Inequality vs. Rising Living Standards

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Motivation

- Large increase in income inequality in the US from 1950 to 2010
  - Larger top income shares, thicker Pareto tail

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- How does the standard of living affect the optimal fiscal system?
  - Redistribution needs as well as efficiency concerns

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  - Heterogeneous income elasticities of demand across sectors
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  - Two approaches: static Mirrlees; Ramsey in Aiyagari-Bewley-Huggett-İmrohoroğlu setup

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  - Conservative calibration
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- Significant dampening of optimal increase in redistribution in quantitative set-up
  - Conservative calibration
    - Non-homothetic CES preferences with low curvature of utility function
  - Redistribution should be higher in 2010 than in 1950...
    - ...but the optimal increase is **at least 25% smaller** when accounting for growth

Literature

Mirrleesian Optimal Nonlinear Income Taxation with Non-Homothetic Preferences

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- Let u denote the indirect utility function

$$u(e; p) \equiv \max_{\{c_j\}_j} U(c) \;\; \text{s.t.} \;\; \sum_j p_j c_j = e$$

where e is nominal expenditures and p is the vector of prices

Optimal Taxation Problem

- Household's static maximization problem:

$$V(\theta; \mathcal{T}(\cdot), p) \equiv \max_{e,n} u(e; p) - v(n) \quad \text{s.t.} \quad e = n\theta - \mathcal{T}(n\theta)$$

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  - Balanced budget where G is exogenous spending

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- **Aggregate growth** modeled as a proportional fall in p

$$\hat{p} = p/(1+g)$$

Nonlinear Taxes: General Formula

- Optimal marginal rate equates efficiency costs of taxation to redistribution gains $\forall y(\hat{\theta})$

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  - Let $\eta(\theta) \equiv dy(\theta)/d\mathcal{T}(0)$ denote the income effect of type-θ worker
  - Let $u_e(\theta)$ denote the marginal utility of expenditure of type-θ worker

- Changes in p can alter: $\eta(\theta)$, $u_e(\theta)$, $y(\theta)$

Nonlinear Taxes: Efficiency Cost $E(g)$

- Efficiency costs of taxes and transfers depend on elasticities φ^{-1} and income effects η

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  - **Numerator:** Increasing revenues through higher marginal rate at $y(\hat{\theta})\ldots$
    - $+$ Decreases labor supply of worker with $y(\hat{\theta})$: elasticity φ^{-1}
    - $+$ Increases labor supply of workers with $y>y(\hat{\theta})$: income effect η

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  - **Denominator:** Increasing the lump-sum transfer...
    - $+$ Decreases labor supply of all workers: income effect η

Nonlinear Taxes: Redistribution Gains $R(g)$

- Redistribution gains of taxes and transfers depend on dispersion of marginal utilities u_e

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Homothetic Benchmark Neutrality Result

- Assume homothetic CRRA preferences

$$U(c) = \frac{[\mathcal{C}(c)]^{1-\gamma}}{1-\gamma}, \text{ where } \mathcal{C}(c) = \left(\sum_j \Omega_j^{\frac{1}{\sigma}} c_j^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1-\sigma}{\sigma}}$$

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- What about non-homothetic preferences?

Non-Homothetic Preferences Stone-Geary Preferences

Geary (1950)

- **One-sector** Stone-Geary preferences

$$u\left(c\right)=\frac{\left(c-\bar{c}\right)^{1-\gamma}}{1-\gamma}$$

- Subsistence consumption level $\bar{c}>0$

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- Counterfactual: vanishing non-homotheticities

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Comin, Lashkari, and Mestieri (2021)

- Consumption aggregator $\mathcal{C}(c)$ implicitly defined by

$$\sum_{j}^{J} \left(\Omega_j (\mathcal{C}(c))^{\varepsilon_j}\right)^{\frac{1}{\sigma}} c_j^{\frac{\sigma-1}{\sigma}} = 1.$$

  - Key parameters: ε_j governs income elasticity of demand for good j
  - Elasticity of substitution between goods σ

    $\Rightarrow$ $\partial c_j / \partial e = \sigma + (1-\sigma)\varepsilon_j / \bar{\varepsilon}$

- Utility from aggregated consumption: $\mathcal{C}(c)^{1-\gamma}/(1-\gamma)$

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$\Rightarrow$ Typically implies DRRA

  - Formal proof of necessary and sufficient conditions for two goods
  - Quantitatively true for typical calibration with three goods

DRRA Conditions

Non-Homothetic Preferences IA Preferences

Alder, Boppart, and Müller (2022)

- Preferences defined over expenditures $e = \sum_j p_j c_j$

$$v(e,p) = \frac{1-\varepsilon}{\varepsilon} \frac{1}{\mathbf{B}(p)^{\varepsilon}} \left(e - \underbrace{\sum_j p_j \bar{c}_j}_{\bar{\mathbf{A}}(p)} \right)^{\varepsilon} - \mathbf{D}(p)$$

  – Price function $\mathbf{B}(p) = \left(\sum_j \Omega_j p_j^{1-\sigma} \right)^{1/(1-\sigma)} (= p^{\star})$

D-Term

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$\Rightarrow$ Typically implies DRRA

  - u is DRRA $\Leftrightarrow \bar{\mathbf{A}}(p) > 0$
  - Typical calibration with three goods $\Rightarrow \bar{\mathbf{A}}(p) > 0$

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Non-Homothetic Preferences & Growth

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1. **DRRA** $\Rightarrow$ Dispersion of marginal utilities decreases with growth

   $\rightarrow$ redistribution should decrease with growth

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   - (b) efficiency cost of lump-sum transfer decreases $\rightarrow$ redistribution should increase with growth

Non-Homothetic Preferences & Growth

$$\underbrace{1-\frac{1-\frac{\mathcal{T}'(y(\hat{\theta}))}{1-\mathcal{T}'(y(\hat{\theta}))}\frac{1}{1+\varphi}\hat{\theta}f(\hat{\theta})+\int_{\hat{\theta}}^{\bar{\theta}}\mathcal{T}'(y(x))\eta(x)dF(x)}{1+\int_{\underline{\theta}}^{\bar{\theta}}\mathcal{T}'(y(x))\eta(x)dF(x)}}_{E(g)}=\underbrace{1-\frac{\int_{\hat{\theta}}^{\bar{\theta}}u_e(x)dF(x)}{\int_{\underline{\theta}}^{\bar{\theta}}u_e(x)dF(x)}}_{R(g)}$$

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Quantification in a Mirrlees Setup

Overview of Numerical Exercises

- Start from calibrated US economy in 1950
- Inverse optimum in 1950

  Bourguignon and Spadaro (2012), Lockwood and Weinzierl (2016), Hendren (2020)

  - Pareto weights such that calibrated 1950 tax system is optimal
- Two key changes until 2010
  - Reduce prices to achieve GDP per capita growth until 2010
  - Adjust skill distribution to capture rising inequality

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- Two key changes until 2010
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- Two exercises
  1. Isolate growth effect and decompose it based on the formula
  2. Quantify relative importance of growth vs. inequality

Calibration: Preferences

- Preferences: Comin, Lashkari, and Mestieri (2021)
  - Three goods: agriculture (food), goods, services
  - Micro-estimates using CEX data: $\varepsilon_a = 0.1$, $\varepsilon_g = 1$, $\varepsilon_g = 1.8$, $\sigma = 0.3$

Literature RA

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Literature RA

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- Frisch elasticity $\varphi^{-1} = 0.5$
- Low curvature of the utility function: $\gamma = 0.75$
  - Implies small dispersion in relative risk aversion, falling from on average 1.07 to 0.99
  - Consistent with small fall in labor supply from 1950 to 2010 ($\approx$ 5%)
    Ramey and Francis (2009), Boppart and Krusell (2020)

Literature RA

Calibration: Inequality

- A partial-insurance approach
  - Calibrate $f(.)$ as exponentially modified Gaussian (EMG) to match dispersion in expenditures

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  - Pareto tail: $\lambda_y = 1.65$; $\lambda_e \approx 3.3$

    Aoki and Nirei (2017); Toda and Walsh (2015)

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    Aoki and Nirei (2017); Toda and Walsh (2015)
- In 1950, data on income inequality only
  - Dispersion: $\mathbb{V}[\log y] = 0.57$; $\Rightarrow$ infer $\mathbb{V}[\log e] \approx 0.25$

    SCF+ (Kuhn, Schularick, and Steins 2020)
  - Pareto tail: $\lambda_y = 2.2 \Rightarrow$ infer $\lambda_e = 4.4$

    Aoki and Nirei (2017)

Calibration: Government

- For calibration, assume parametric tax function plus lump-sum transfer

  Ferriere, Grübener, Navarro, and Vardishvili (2023)

$$\bar{\tau}(y) = \exp\left[\log(\lambda)\left(y^{-2\tau}\right)\right]$$

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Spending Data

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- Government spending

  White House Office of Management & Budget

  – Transfer T: spending on income security, T/Y: 1.1% (1950), 3.6% (2010)

  – Exogenous spending G: all remaining spending, $G/Y \approx 14\%$ constant

- Difference in Average Marginal Tax Rate (AMTR) between top 10% and bottom 90%

  Mertens and Montiel Olea (2018)

  – 13% (1950), 9% (2010)

Spending Data

Calibration: Growth

- Level of standard of living as in 1950
  - Set preference parameters $\{\Omega_j\}$ to match aggregate expenditure shares, normalize $p = 1$
    Computed based on Herrendorf, Rogerson, and Valentinyi (2013) [NIPA]
    + 2010: agriculture (food) 7.5%, goods 25.6%, services 66.9%
    + 1950: agriculture (food) 21.5%, goods 39.2%, services 39.2%

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- Model growth as a fall in prices
  - Aggregate growth in GDP per capita: 3.3
    NIPA
  - Prices relative to goods
    Computed based on Herrendorf, Rogerson, and Valentinyi (2013) [NIPA]
    + Agriculture (food) $\rightarrow$ 1.00, 1.87
    + Services $\rightarrow$ 1.00, 3.16

Exercise 1: Understanding the Role of Growth

- First numerical exercise: start from 1950 and add only growth
  - Prices fall but skill inequality remains unchanged
- Implications for marginal and average tax rates

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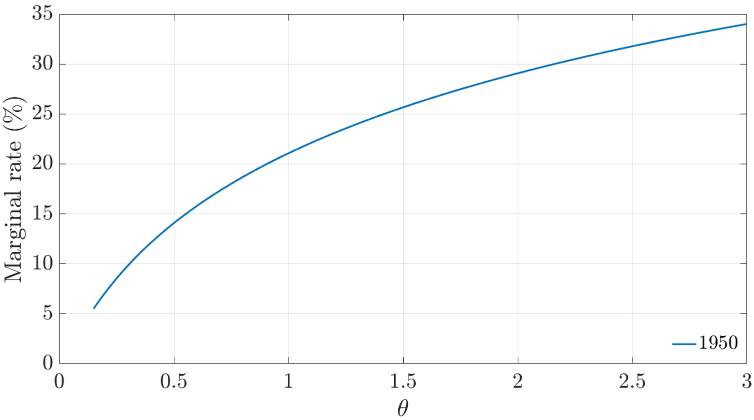
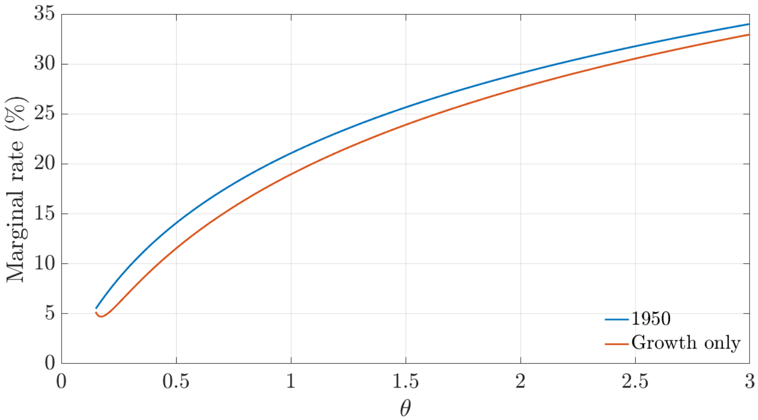
- Decomposition into effects of marginal utilities, income effects, and the income distribution

Optimal Marginal Rates with Growth



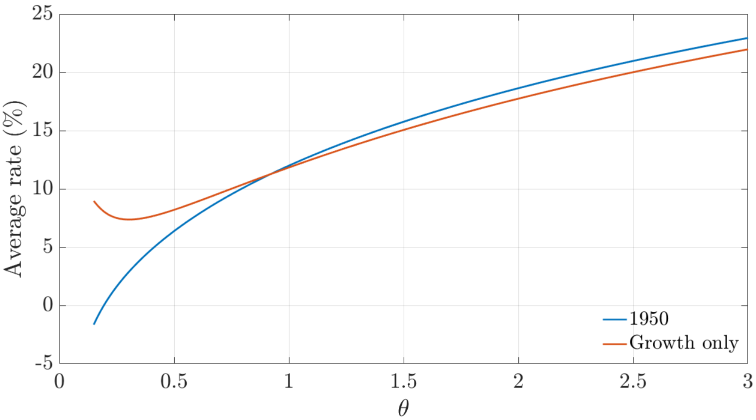
- In 1950, $T/Y = +1.2\%$

Optimal Marginal Rates with Growth



- In 1950, $T/Y = +1.2\%$ $\Rightarrow$ With 2010 growth, $T/Y = -0.6\%$
  - Marginal rates decrease by 2-3 p.p.

Optimal Average Rates with Growth



- In 1950, $T/Y = +1.2\%$ $\Rightarrow$ With 2010 growth, $T/Y = -0.6\%$
  - Averages rates increase for the bottom-50

Decomposition

- Decomposition into effects of marginal utilities, income effects, and the income distribution

$$\underbrace{1-\frac{1-\frac{\mathcal{T}'(y(\hat{\theta}))}{1-\mathcal{T}'(y(\hat{\theta}))}\frac{1}{1+\varphi}\hat{\theta}f(\hat{\theta})+\int_{\hat{\theta}}^{\bar{\theta}}\mathcal{T}'(y(x))\eta(x)dF(x)}{1+\int_{\underline{\theta}}^{\bar{\theta}}\mathcal{T}'(y(x))\eta(x)dF(x)}}_{E(p)}=\underbrace{1-\frac{\int_{\hat{\theta}}^{\bar{\theta}}u_e(x)dF(x)}{\int_{\underline{\theta}}^{\bar{\theta}}u_e(x)dF(x)}}_{R(p)}$$

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$$\underbrace{1 - \frac{1 - \frac{\mathcal{T}'(y(\hat{\theta}))}{1-\mathcal{T}'(y(\hat{\theta}))} \frac{1}{1+\varphi} \hat{\theta} f(\hat{\theta}) + \int_{\hat{\theta}}^{\bar{\theta}} \mathcal{T}'(y(x))\eta(x)dF(x)}{1 + \int_{\underline{\theta}}^{\bar{\theta}} \mathcal{T}'(y(x))\eta(x)dF(x)}}_{E(p)} = \underbrace{1 - \frac{\int_{\hat{\theta}}^{\bar{\theta}} u_e(x)dF(x)}{\int_{\underline{\theta}}^{\bar{\theta}} u_e(x)dF(x)}}_{R(p)}$$

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  1. Optimal taxes with $u_e(.)$ computed using p_{1950}

     $+$ But $\eta(.)$ and $y(.)$ decision using p_{2010}

     $\Rightarrow$ More redistribution

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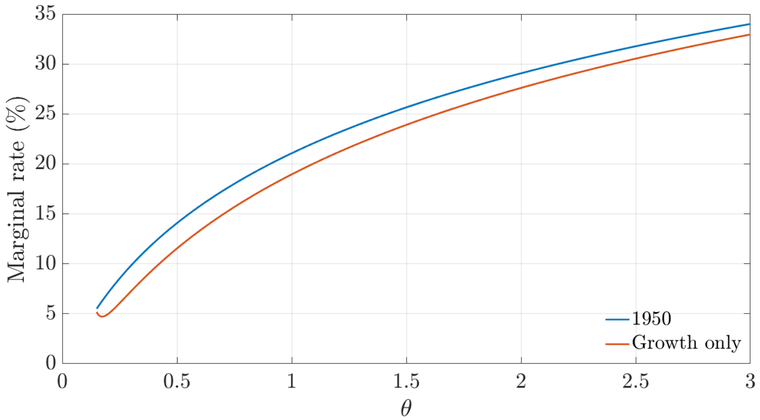
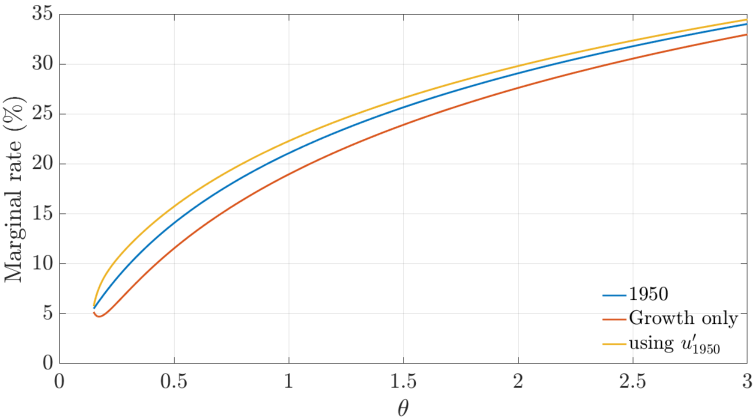
  $\Rightarrow$ Back to 1950

Optimal Marginal Rates with Growth: Decomposition



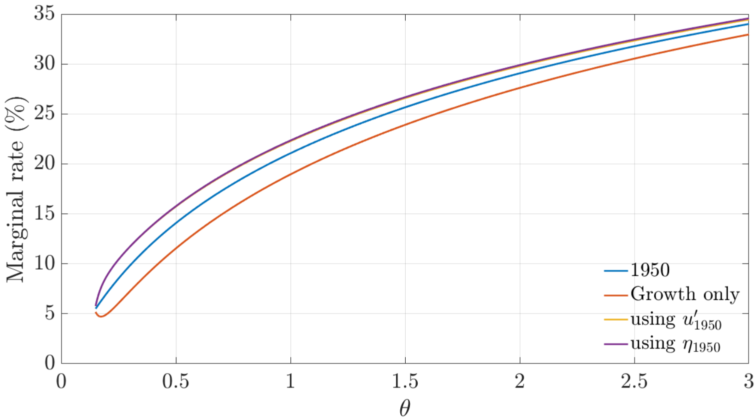
- With 2010 growth, $T/Y = -0.6\%$

Optimal Marginal Rates with Growth: Decomposition



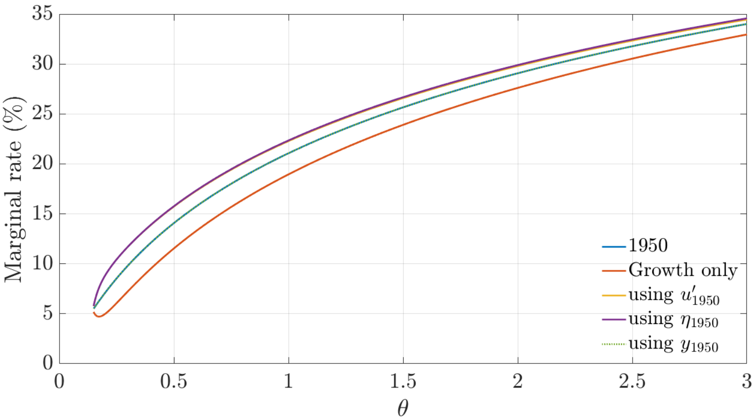
- With 2010 growth, $T/Y = -0.6\%$ $\Rightarrow$ With 1950 marg. u dispersion, $T/Y = 2.5\%$

Optimal Marginal Rates with Growth: Decomposition



- With 2010 growth, $T/Y = -0.6\%$ $\Rightarrow$ With 1950 income effects, $T/Y = 2.5\%$

Optimal Marginal Rates with Growth: Decomposition

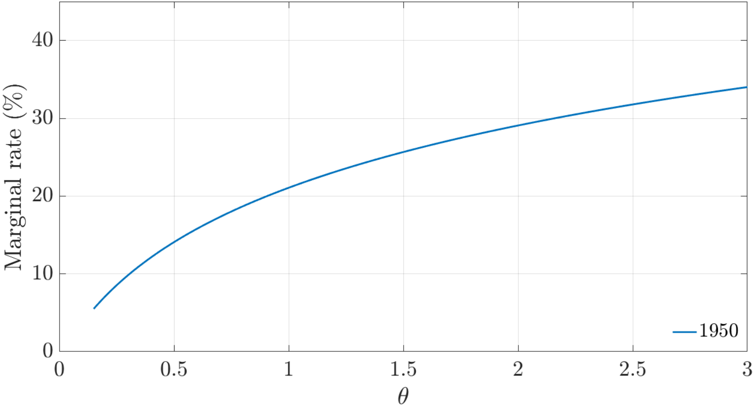


- With 2010 growth, $T/Y = -0.6\%$ $\Rightarrow$ With 1950 income dist, $T/Y = 1.2\%$ (1950 level)

Exercise 2: Growth vs. Inequality

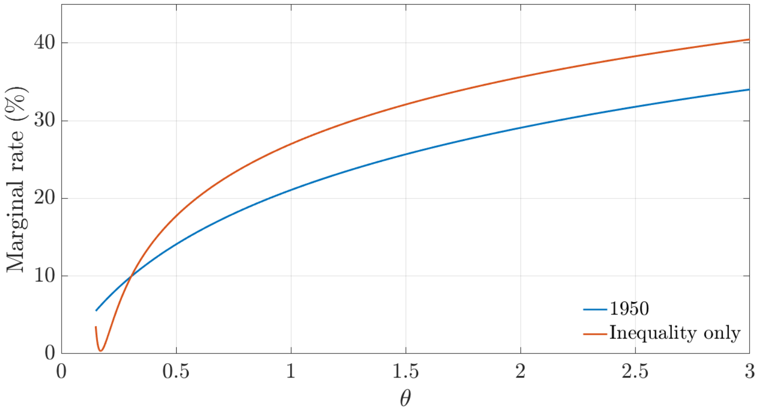
- Second numerical exercise: how important is growth relative to changing inequality?
- Starting from 1950, first change inequality, then account for growth
  - Pareto weights constant as a function of $F(\theta)$

Optimal Marginal Rates: Growth vs. Inequality



- In 1950, $T/Y = +1.2\%$

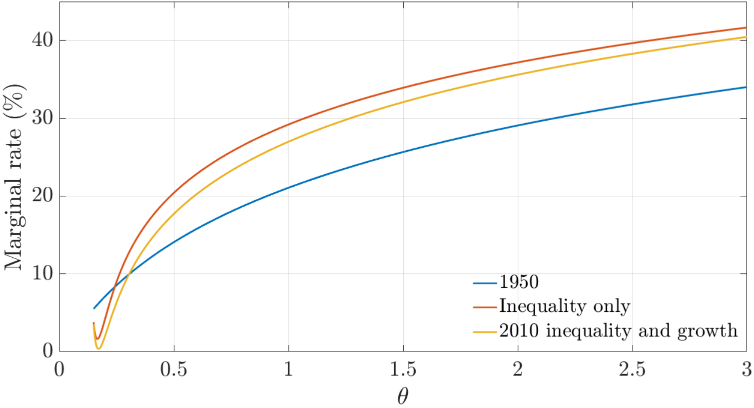
Optimal Marginal Rates: Growth vs. Inequality



- In 1950, $T/Y = +1.2\%$ $\Rightarrow$ With higher inequality, $T/Y = 6.8\%$

Utilitarian 2010 Calibration

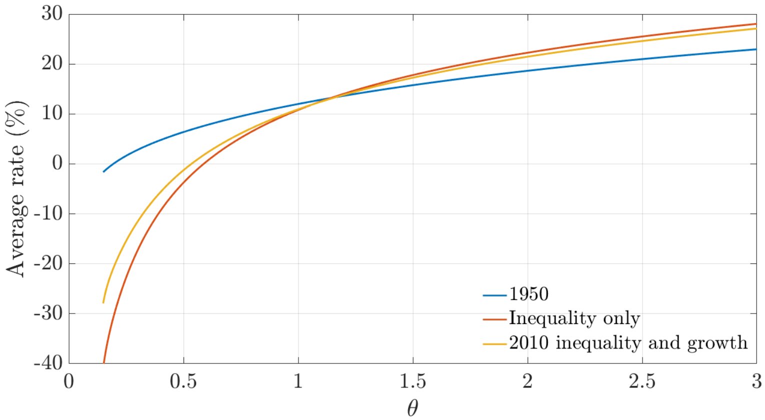
Optimal Marginal Rates: Growth vs. Inequality



- In 1950, $T/Y = +1.2\%$ $\Rightarrow$ With higher inequality **and growth**, $T/Y = 4.6\%$
  - Growth reduces increase in T/Y by 40%

Utilitarian 2010 Calibration

Optimal Average Rates: Growth vs. Inequality



- Growth reduces increase in top-10 minus bottom-10 average rates by 26%

Utilitarian 2010 Calibration

Taking Stock

- Growth dampens increase in redistribution driven by higher inequality
  - The optimal increase in T/Y is 40% smaller
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Taking Stock

- Growth dampens increase in redistribution driven by higher inequality
  - The optimal increase in T/Y is 40% smaller
  - The optimal increase in top-10 minus bottom-10 average rates is 26% smaller
  - Quantitatively conservative with low dispersion in risk aversion
- Next: dynamic incomplete markets model with self-insurance
  - Disentangle expenditure, income, and wealth
  - Dynamic household decisions with meaningful notion of risk aversion/EIS

Quantification in a Model with Private Insurance

A Model with Self-Insurance

- Richer quantitative model with self-insurance
  - Realistic distributions of expenditure, income, and wealth
  - Quantification of risk aversion, income effects, and MPCs
  - Parametric tax-and-transfer function

A Model with Self-Insurance

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  - Parametric tax-and-transfer function
- Similar exercise: optimal tax-and-transfer systems at two points in time
  - Calibration of 1950 steady state in partial equilibrium
  - Inverse optimum Pareto weights for 1950
  - 2010: new steady state with growth and higher inequality

Households

- Household's value function with productivity θ and assets a:

$$V\left(a,\theta\right)=\max_{e,a',n}\left\{u(e,p)-B\frac{n^{1+\varphi}}{1+\varphi}+\beta\mathbb{E}_{\theta'}\left[V\left(a',\theta'\right)|\theta\right]\right\}$$

s.t.

$$e+a'\leq\theta n+(1+r)a-\mathcal{T}\left(\theta n,ra\right),\quad a'\geq0$$

  - u is the indirect utility function
  - Productivity θ follows a stochastic process

Government

- Same parametric tax function as used before for calibration

$$\bar{\tau}(y) = \exp\left[\log(\lambda)\left(y^{-2\tau}\right)\right]$$

  - λ captures level of the tax rates
  - τ captures progressivity

- Lump-sum transfer

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$$\bar{\tau}\left(y\right) = \exp\left[\log(\lambda)\left(y^{-2\tau}\right)\right]$$

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- Lump-sum transfer

- Exogenous spending requirement
  - Spending in all sectors: G_a, G_g, G_s

- Balanced budget

Calibration Overview

- Calibration to the US economy in 1950 and 2010
- Strategy as before for ...
  - ... growth and relative prices
  - ... preference parameters
  - ... tax-and-transfer system
- New part: private saving
  - Distributions of expenditure, income, and wealth

Calibration Aggregates

- Interest rate fixed at 2%
- Discount factor to match wealth-to-income ratio of 4 in 2010
  Piketty and Zucman (2014) [NIPA]
  - Untargeted wealth-to-income ratio in 1950 of 3

Calibration Inequality

- Wages follow AR(1) in logs, with appended Pareto tail
  - Persistence ρ fixed at 0.9
  - Shock innovation set to match variance of log income from SCF+
    Kuhn, Schularick, and Steins (2020)
  - Time-varying Pareto tail parameter
    Aoki and Nirei (2017)

1950	Income Share by Quintile				
Model	5.7%	10.7%	13.2%	21.4%	49.0%
Data (SCF+)	5.5%	11.3%	14.9%	20.8%	47.5%

2010	Income Share by Quintile				
Model	4.2%	8.6%	11.3%	19.3%	56.5%
Data (SCF+)	4.1%	8.7%	12.9%	21.3%	53.0%

Calibration Inequality

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1950	Wealth Share by Quintile				
Model	0.0%	1.5%	6.2%	16.5%	75.7%
Data (SCF+)	-0.5%	1.3%	4.5%	10.5%	84.2%

2010	Wealth Share by Quintile				
Model	0.0%	1.0%	4.7%	13.3%	80.9%
Data (SCF+)	-1.1%	0.8%	3.3%	9.8%	87.2%

Calibration Inequality

- Wages follow AR(1) in logs, with appended Pareto tail
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  - Shock innovation set to match variance of log income from SCF+
    Kuhn, Schularick, and Steins (2020)
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    Aoki and Nirei (2017)

1950	**Expenditure** Share by Quintile				
Model	8.4%	12.8%	17.1%	22.8%	39.0%
Data	-	-	-	-	-

2010	**Expenditure** Share by Quintile				
Model	6.7%	11.1%	15.6%	21.3%	45.3%
Data (CEX)	9.4%	14.2%	18.1%	23.1%	35.2%

Implications for Risk Aversion and Wealth Effects

- Exploit relationship between risk aversion, wealth effects, and MPCs
  - Goal: validate the degree of DRRA in the model

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$$\eta \left(\varphi \frac{e}{\theta n} + \frac{e\mathcal{T}''(\theta n)}{\mathcal{T}'(\theta n)} \right) = \text{MPC} \times \text{RRA}$$

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- Exploit relationship between risk aversion, wealth effects, and MPCs
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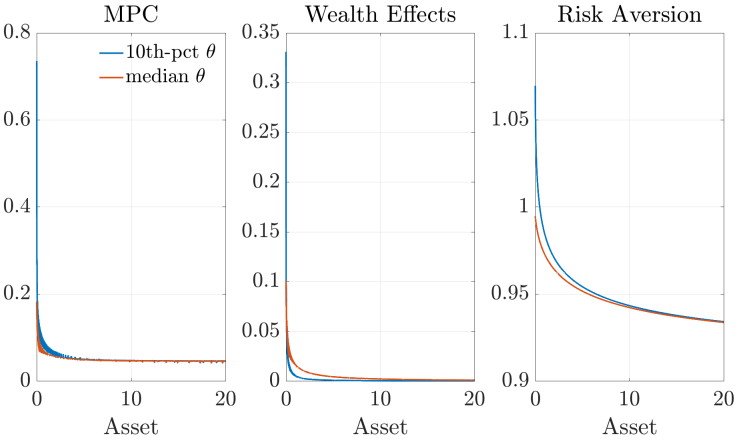
$$\eta \left(\varphi \frac{e}{\theta n} + \frac{e \mathcal{T}'' (\theta n)}{\mathcal{T}' (\theta n)} \right) = \text{MPC} \times \text{RRA}$$

  - Evidence for wealth effects and MPCs
  - Fuzzy evidence for risk aversion
  - Using the structure of the model to discipline expenditure shares and fiscal component

Implications for Risk Aversion and Wealth Effects

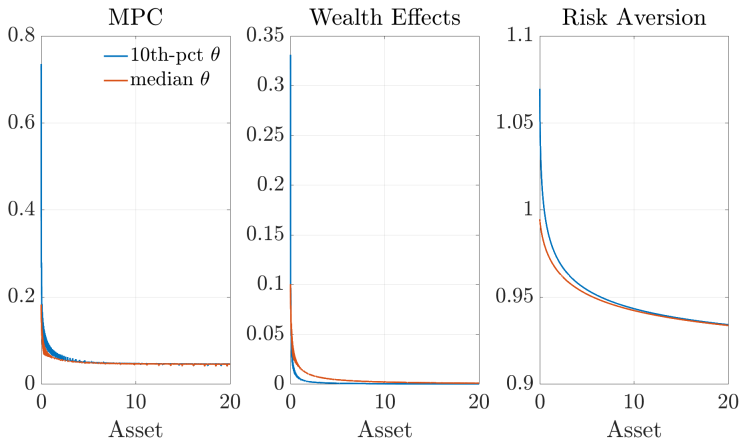


- Model implied MPC: average of 18% in 2010

  Johnson, Parker, and Souleles (2006), Fagereng, Holm, and Natvik (2021), Kaplan and Violante (2022)

Literature RA | Wealth Effects

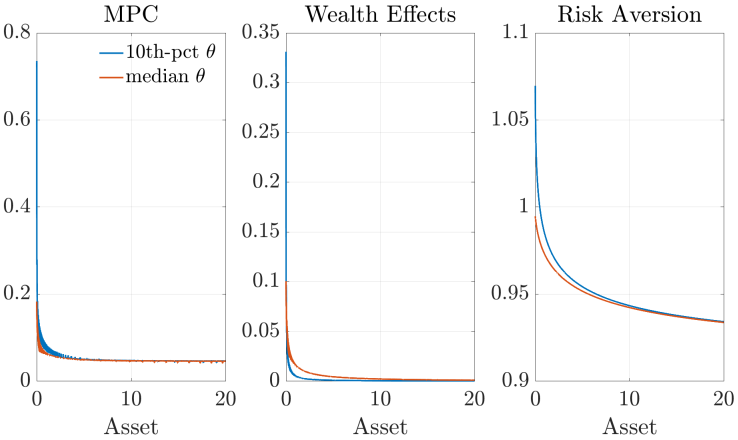
Implications for Risk Aversion and Wealth Effects



- Wealth effects: very good fit for income response to exogenous wealth shock

  Golosov, Graber, Mogstad, and Novgorodsky (2023)

Implications for Risk Aversion and Wealth Effects



- Risk aversion: very moderate decline in RRA (1.06 to 0.99)
  - Some empirical support from Euler equation estimation, portfolio choice, development

Literature RA Wealth Effects

Implications of Risk Aversion for Labor Decisions

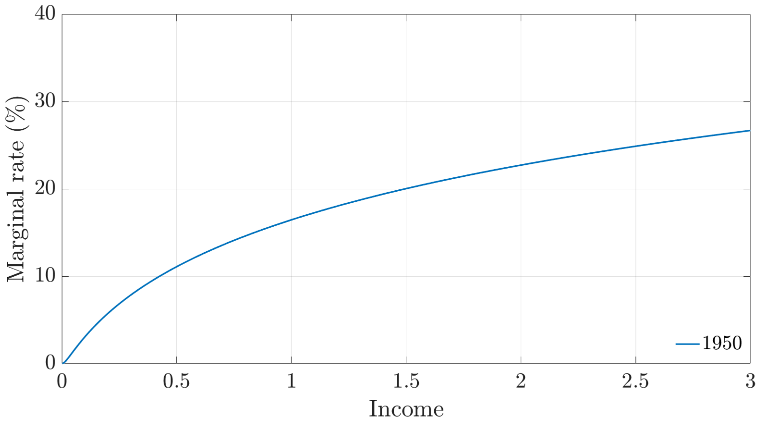
- Fall in average hours across time: 7%

  Ramey and Francis (2009), Boppart and Krusell (2020)

- Correlation between hours and hourly wage in the cross-section
  - Mildly negative in 1950
  - Positive in 2010

  Mantovani (2022)

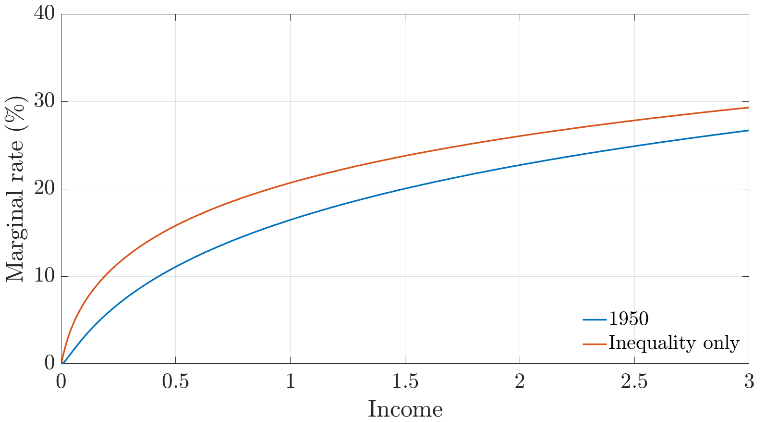
Optimal Marginal Rates



- Calibration in 1950: $T/Y = 1.0\%$

Weights

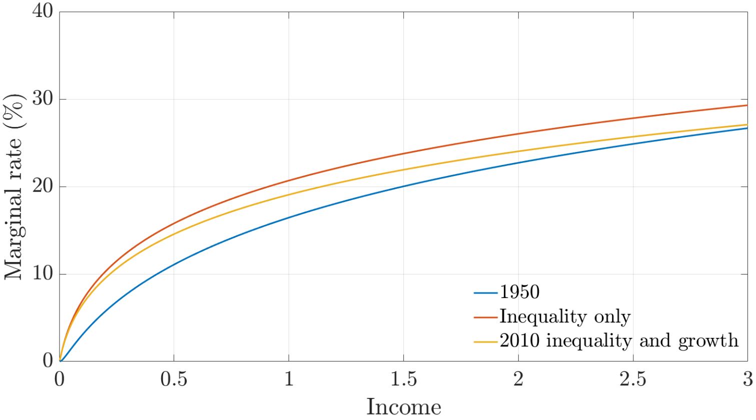
Optimal Marginal Rates



- Calibration in 1950: $T/Y = 1.0\% \ \Rightarrow T/Y = 6.2\%$ with higher inequality

Weights

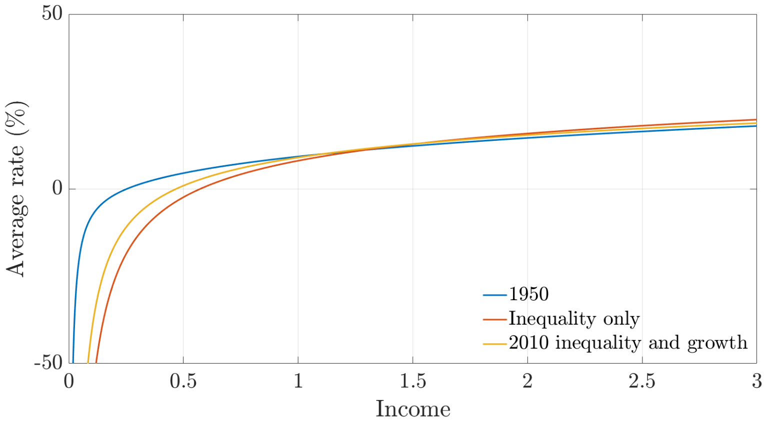
Optimal Marginal Rates



- Calibration in 1950: $T/Y = 1.0\% \;\Rightarrow T/Y = 4.4\%$ with higher inequality and growth
  - Growth reduces increase in T/Y by 35%

Weights

Optimal Average Rates



- Growth reduces increase in top-10 minus bottom-10 average rates by 34%

Robustness

- Utilitarian
  - 1950: $T/Y = 12.2\%$
  - 2010 inequality: $T/Y = 15.6\%$
  - 2010 inequality and growth: $T/Y = 14.9\%$
- $\gamma = 1.5$
  - 1950: $T/Y = 1.2\%$
  - 2010 inequality: $T/Y = 9.3\%$
  - 2010 inequality and growth: $T/Y = 4.8\%$
- IA preferences
  - Not implemented yet in this model
  - Usually larger effects in previous model versions

Conclusion

Conclusion

- **Non-homothetic preferences**: Growth matters for redistribution
  - Beyond the standard relative inequality
  - Standard of living affects income effects and dispersion of marginal utilities
- Quantification for US since 1950
  - Rising standard of living counteracts desired growth of welfare state due to inequality

Conclusion

2. In a 2017 interview with CNBC, economist Milton Friedman argued that the welfare state was less important today than in the past because "the standard of living of the ordinary person has risen enormously" and "the poor are much better off than they were before."

Conclusion

2. In a 2017 interview with CNBC, economist Milton Friedman argued that the welfare state was less important today than in the past because "the standard of living of the ordinary person has risen enormously" and "the poor are much better off than they were before."

ChatGPT, Feb 23

Appendix

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Literature

- **Optimal taxation**
  - Stationary economies and business cycle fluctuations in homothetic one sector economies
    Mirrlees (1971)-Diamond (1998)-Saez (2001), Ramsey (1927)-Werning (2007)-Heathcote, Storesletten, and Violante (2017)
  - Optimal tax system over time in homothetic economies
    Mankiw, Weinzierl, and Yagan (2009), Diamond and Saez (2011), Scheuer and Werning (2017), Heathcote, Storesletten, and Violante (2020), Brinca, Duarte, Holter, and Oliveira (2022)
  - Optimal taxation with non-homothetic preferences
    Kushnir and Zubrickas (2021), Jaravel and Olivi (2022)

- **Consumption patterns, Engel curves, and non-homothetic preferences**
  Geary (1950), Herrendorf, Rogerson, and Valentinyi (2013), Boppart (2014), Herrendorf, Rogerson, and Valentinyi (2014), Aguiar and Bils (2015), Comin, Lashkari, and Mestieri (2021), Alder, Boppart, and Müller (2022)

Back

Non-Homothetic Preferences Non-Homothetic CES

Comin, Lashkari, and Mestieri (2021)

- Conditions for DRRA with two goods: $\varepsilon_1 < \varepsilon_2 = 1$
  - Necessary condition: $\gamma > \varepsilon_1$
  - Sufficient condition: $\gamma + \varepsilon_1 \geq 2$
- Typical calibration with three goods $\Rightarrow$ quantitatively true

Back

Non-Homothetic Preferences IA Preferences

Alder, Boppart, and Müller (2022)

$$D(p) = \frac{(1-\varepsilon)\,\nu}{\kappa\gamma}\left[\left(\frac{\tilde{D}(p)}{B(p)}\right)^{\gamma} - 1\right]$$

$$\tilde{D}(p) = \left(\sum_{j \in J} \theta_j p_{j,t}^{1-\phi}\right)^{\frac{1}{1-\phi}}$$

Evidence: Risk Aversion and EIS

- EIS increasing in consumption/wealth, based on estimating consumption Euler equation
  Blundell, Browning, and Meghir (1994), Attanasio and Browning (1995), Atkeson and Ogaki (1996)
- DRRA powerful in matching portfolio choices across the wealth distribution
  Wachter and Yogo (2010), Cioffi (2021), Straub (2019), Meeuwis (2022)
- DRRA supported by consumption data from Indian villages
  Ogaki and Zhang (2001), Zhang and Ogaki (2004)

Back to Mirrlees Back to IM

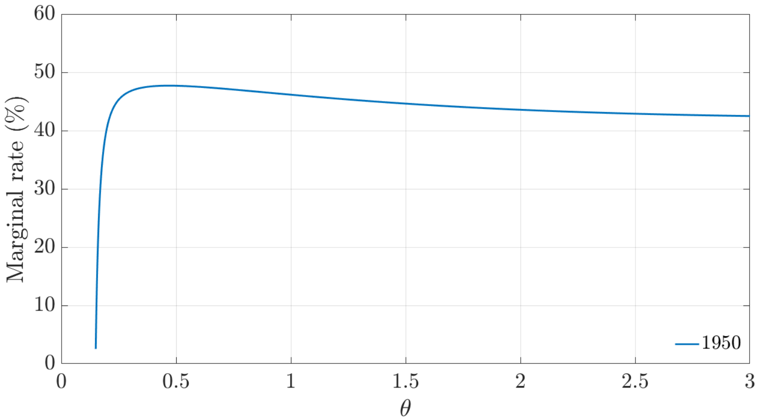
Calibration: Government

- Programs included in transfers
  - General retirement and disability insurance (excluding social security)
  - Federal employee retirement and disability
  - Unemployment compensation
  - Housing assistance
  - Food and nutrition assistance
  - Other income security

- Government spending
  - Supposed to capture all remaining federal spending
  - Purposefully chosen such that G/Y constant
    - Spending has risen in the data
    - Largely deficit financed, which cannot be captured in the model

Back

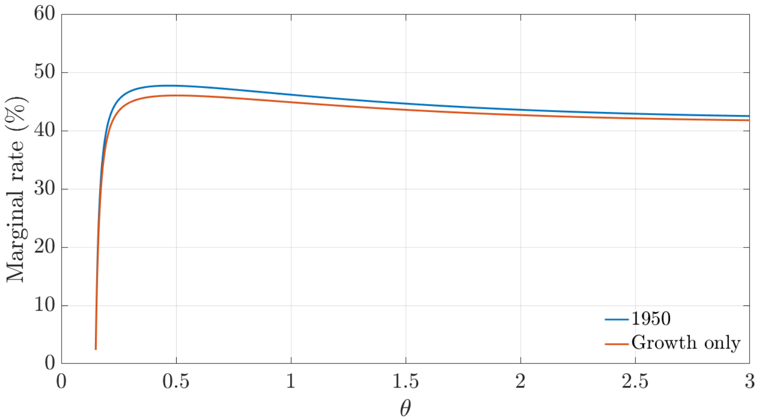
Optimal Marginal Rates with Growth **Utilitarian Planner**



- In 1950, $T/Y = +25.3\%$

Back

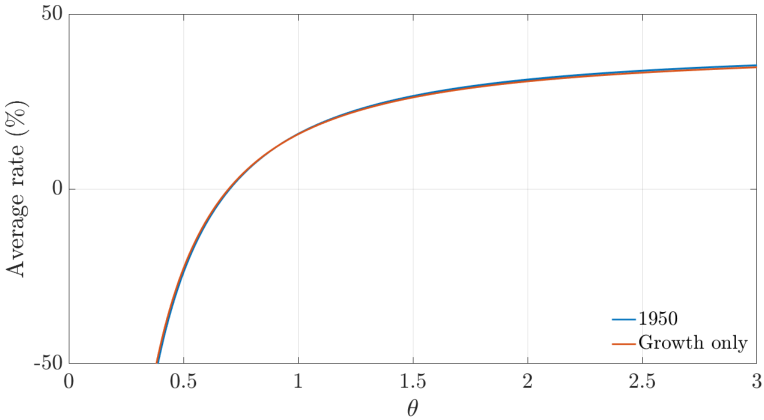
Optimal Marginal Rates with Growth Utilitarian Planner



- In 1950, $T/Y = +25.3\%$ $\Rightarrow$ With 2010 growth, $T/Y = 24.0\%$

Back

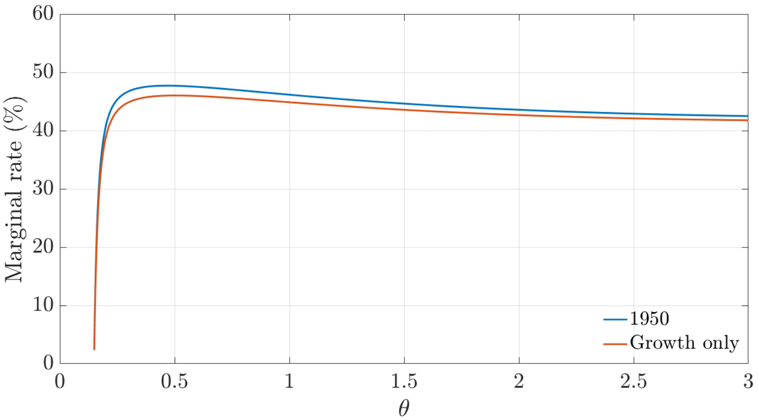
Optimal Average Rates with Growth Utilitarian Planner



- In 1950, $T/Y = +25.3\%$ $\Rightarrow$ With 2010 growth, $T/Y = 24.0\%$

Back

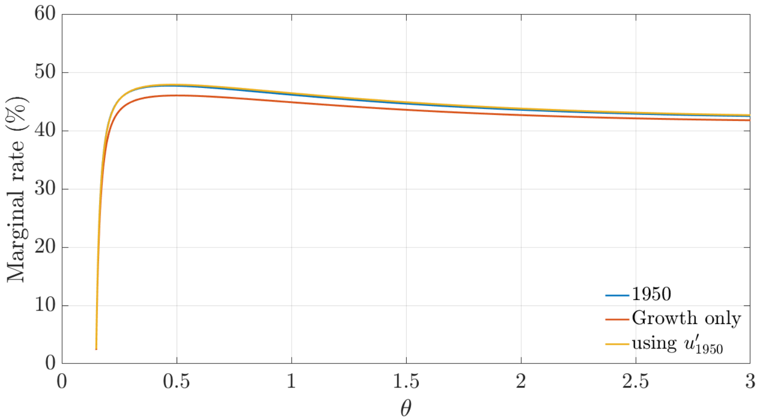
Optimal Marginal Rates with Growth: Decomposition **Utilitarian**



- With 2010 growth, $T/Y = 24.0\%$

Back

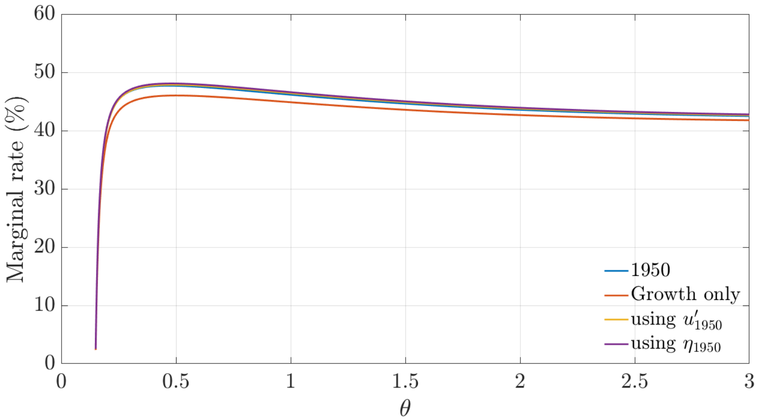
Optimal Marginal Rates with Growth: Decomposition **Utilitarian**



- With 2010 growth, $T/Y = 24.0\%$ $\Rightarrow$ With 1950 marg. u dispersion, $T/Y = 25.4\%$

Back

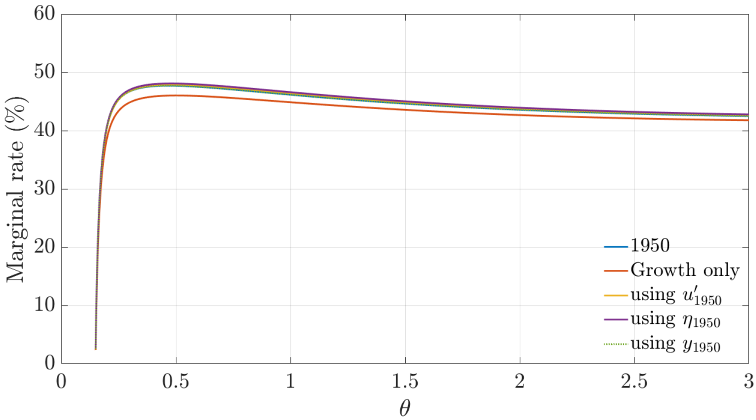
Optimal Marginal Rates with Growth: Decomposition Utilitarian



- With 2010 growth, $T/Y = 24.0\%$ $\Rightarrow$ With 1950 income effects, $T/Y = 25.6\%$

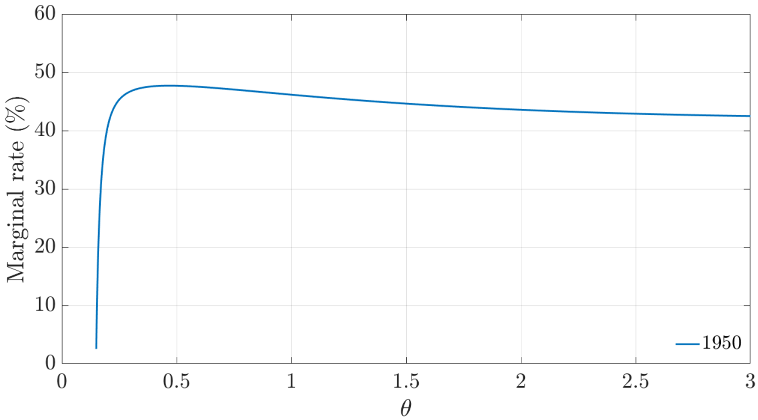
Back

Optimal Marginal Rates with Growth: Decomposition **Utilitarian**



- With 2010 growth, $T/Y = 24.0\%$ $\Rightarrow$ With 1950 income dist, $T/Y = 25.6\%$ (1950 level)

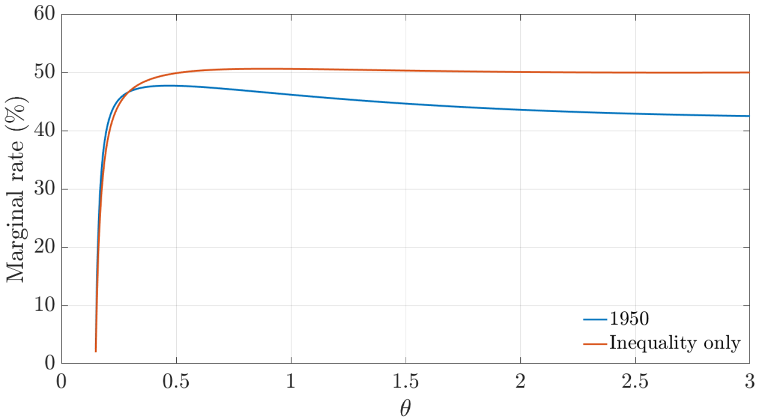
Optimal Marginal Rates: Growth vs. Inequality Utilitarian



- In 1950, $T/Y = +25.3\%$

Back

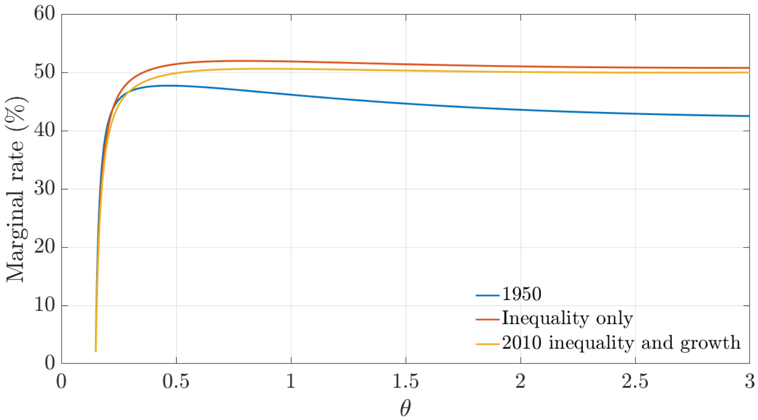
Optimal Marginal Rates: Growth vs. Inequality **Utilitarian**



- In 1950, $T/Y = +25.3\%$ $\Rightarrow$ With higher inequality, $T/Y = 29.2\%$

Back

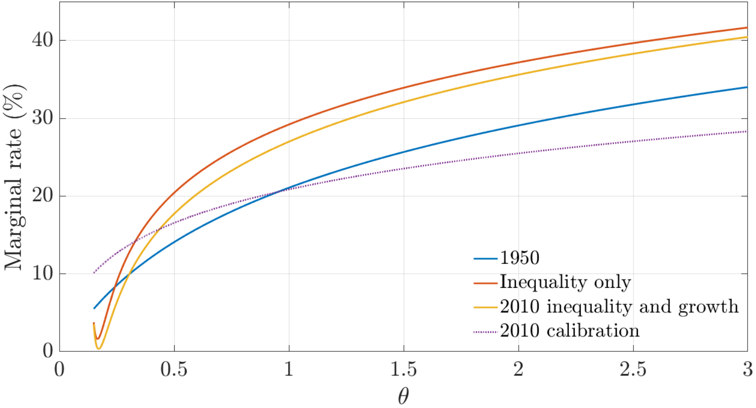
Optimal Marginal Rates: Growth vs. Inequality Utilitarian



- In 1950, $T/Y = +25.3\%$ $\Rightarrow$ With higher inequality **and growth**, $T/Y = 27.6\%$
  - Growth reduces increase in T/Y by 59%

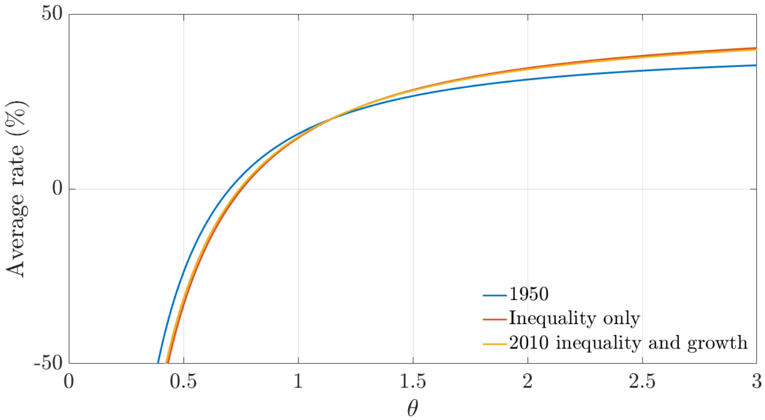
Back

Optimal Marginal Rates: Growth vs. Inequality



Back

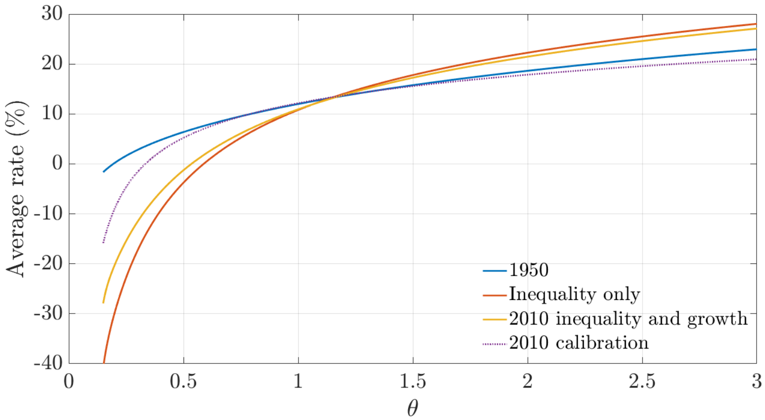
Optimal Average Rates: Growth vs. Inequality Utilitarian



- In 1950, $T/Y = +25.3\%$ $\Rightarrow$ With higher inequality **and growth**, $T/Y = 27.6\%$
  - Growth reduces increase in top-10 minus bottom-10 average rates by 9%

Back

Optimal Marginal Rates: Growth vs. Inequality



Back

Wealth Effects: Evidence Golosov, Graber, Mogstad, and Novgorodsky (2023)

- How does income respond to unexpected wealth shocks?
  - Golosov et al. merge US tax data with data on lottery winnings
  - Compute earnings change over five years after lottery win
  - Earnings drop by on average $-2.3\$$ per $100\$$ of win

- Replicate in model using mean post-tax win
  - Earnings drop by on average $-2.0\$$ per $100\$$ of win

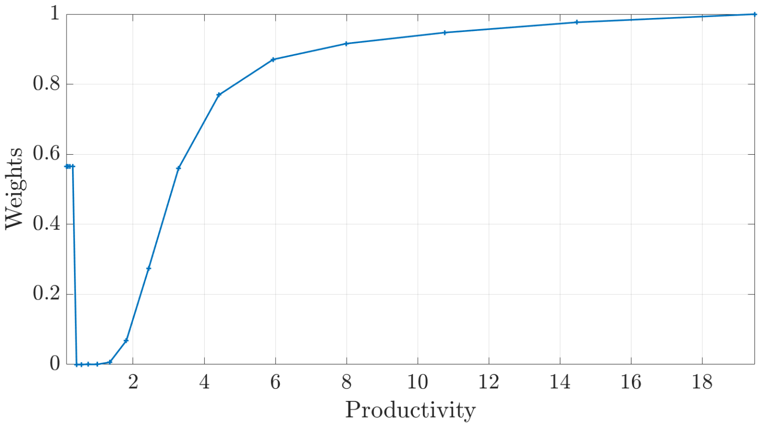
Weights

- More degrees of freedom in finding inverse optimum weights
- Restriction to functional form motivated by instruments: lump sum and progressivity
- Weights as function of percentiles of the skill distribution

$$\omega\left(p_i\right)=\begin{cases}\mu & \text{if } \theta_i \text{ s.t. } F\left(\theta_i\right)<0.10\\ (1-\mu)p_i(\theta_i)^{\nu} & \text{if } \theta_i \text{ s.t. } F\left(\theta_i\right)\geq 0.10\end{cases}$$

Back

Weights



Back