Agglomeration in Purely Neoclassical and Symmetric Economies*

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Abstract

This article demonstrates the emergence of agglomeration unaccompanied by conventional drivers such as scale economies, externalities or comparative advantages. We construct a two-region general equilibrium model with four types of households; there are four commodities and the same linear production functions in each region. Households migrate in search of commodities they lack in their endowment. A type sorts disassortatively toward another type who holds such commodities, resulting in intense agglomerations of diverse types. In contrast, a type sorts assortatively away from another type when they compete for endowments that cannot be transported or produced, resulting in moderate agglomerations dominated by selected types. We identify type complementarity and endowment portability as the primary causative factors behind spatial sorting and the resultant equilibrium agglomeration.

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"Therefore, it follows that if assumptions a1-a4 are upheld, there exists either a trivial solution, or no (price taking) competitive equilibrium. In short, the spatial impossibility theorem says that the smooth market mechanism alone cannot generate spatial agglomeration of activities." (Fujita [Fuj86], pp. 113–114).

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1 Introduction

1.1 Overview

Here we examine the circumstances underlying equilibrium population agglomeration in the context of a completely standard economy, namely without externalities or imperfect competition, but with ordinary utility functions and constant returns to scale production. Whatever equilibria there are will clearly be Pareto efficient. And symmetric equilibria will be present. In such a situation, what force can possibly cause population to agglomerate, and importantly, can this force complement or substitute for the agglomerative forces more commonly used in the literature, such as the New Economic Geography or externalities?

As we shall explain, it is a bit puzzling and surprising that agglomeration can be generated in such a simple neoclassical model, starting with a completely symmetric situation. In fact, transportation cost can be zero or positive; the results are identical. In equilibrium, the regions or locations are autarkic, but the population distributions can be asymmetric. In the end, it is complementarity of types of consumers through their endowments that causes agglomeration. Next, we detail the strategy for our analysis.

Our focus is on a very specific example for tractability and expository reasons. We adopt and then adapt the example of Kehoe [Keh85]. This classical example is aspatial, so it is best to imagine it to have only one region. There are four commodities and four consumers with different Cobb-Douglas utilities, but two different producers with constant returns to scale technologies. Constant returns to scale simplifies matters, since equilibrium profits must be zero. Thus, there is no need to worry about profit distribution and the zero profit conditions yield restrictions on equilibrium prices, useful for computational purposes. The key properties of this example are that it is quite simple **but features three equilibria**. Heterogeneous income effects play a big role both in Kehoe's example and in our work.

Next, we adapt Kehoe's model to the spatial context. There will be two identical regions or locations. There will be measure 1 of each of the four types of consumer. The same production technologies are available in each region. There are now eight commodities, four in each region. Consumers can move between regions at no cost, as is standard in the literature. Commuting between regions to work is not allowed.

We consider three versions of the model with differing portability of endowments. In the first version, endowments move with the consumers. An example of a portable endowment is labor. In the second, endowments are not portable but income derived from endowments moves with the consumers. Notice that land is an example of an endowment that is not portable. In the third, both types of endowments are present.

All three versions support a uniform distribution of population in equilibrium, as well as deviations from it. These deviations stem from how types interact: mutual benefits from each other's endowments lead to strong co-location, while competition for endowments leads to separation and milder agglomeration.

Our model and results are perfectly consistent with the spatial impossibility theorem as stated by Fujita and Thisse ([FT13], p. 39), even though we have a continuum of agents:[1]

> **The Spatial Impossibility Theorem**. Assume a two-region economy with a finite number of consumers and firms. If space is homogeneous, transport is costly, and preferences are locally nonsatiated, there is no competitive equilibrium involving transportation.

Our results are consistent with the theorem because, in equilibrium, no trade takes place.

1.2 Consumer-Driven Agglomeration

Although our setup is stripped of conventional agglomeration drivers such as scale economies, externalities, or comparative advantage, it still produces spatial concentration. At the core of our model lie endowments and the spatial sorting they induce. Their influence on the spatial concentration of economic activity unfolds through three interrelated facets. We illustrate them using a rudimentary example below.

Consider an economy with two commodities, two types of consumers (each with unit mass), and two regions, a and b. Assume that inter-regional trade is prohibitively costly, while inter-regional relocation of consumers is costless. Consumer types are distinguished by their endowments: A type-1 consumer is endowed with a unit of commodity 1, and a type-2 with a unit of commodity 2.

Suppose that a type-1 consumer seeks to obtain commodity 2. If she can do so only by having type 2 move into her region, type 2 is said to be **indispensable** to type 1. If she can do so without having type 2 in the same region, type 2 is said to be **replaceable** to type 1. If her consumption of commodity 2 declines if type 2 moves into her region, type 2 is said to be **rivalrous** with type 1. These relationships are determined by whether endowments move with their owners and whether they can be produced. Table 1 summarizes the terms.

	producible	non-producible
portable	replaceable indifferent	indispensable disassortative
non-portable	rivalrous assortative	rivalrous assortative

Table 1. Type j's endowment characteristics, type k's relationship to j and type k's sorting behavior toward j ($k \neq j$). Portable means that endowments move with their owners. For instance, if type j is endowed with portable, non-producible goods, they are indispensable to type k, and type k prefers to locate in the same region as type j.

First, consider a pure exchange economy. Consumers take their endowment with them when they relocate. Let $\lambda_j \in [0, 1]$ denote the fraction of type-j consumers in region

[1] We will discuss this point in more detail in Section 3.5.

a. There is no agglomeration if both regions house equal numbers of consumers, namely, $\lambda_1 + \lambda_2 = 1$. Let us identify conditions under which this value deviates from 1.

In this setup, equilibrium $\lambda = \begin{bmatrix} \lambda_1 & \lambda_2 \end{bmatrix}$ must take the form $\begin{bmatrix} c & c \end{bmatrix}$, where $c \in [0, 1]$. Suppose, instead, $\lambda = \begin{bmatrix} .1 & .9 \end{bmatrix}$. Then

in region a

- only 10% of type 1 compete for 90% of commodity 2 available in the economy
- 90% of type 2 compete for only 10% of commodity 1

in region b

- 90% of type 1 compete for only 10% of commodity 2
- only 10% of type 2 compete for 90% of commodity 1.

This imbalance drives type 1 out of region b into a and type 2 in the opposite direction, until two types make a proportional presence in each region, i.e., $\lambda = \begin{bmatrix} c & c \end{bmatrix}$. Any regional price differential will be eliminated through inter-regional migration.

Equilibria involve agglomeration, except when $c = .5$. Agglomeration arises from the requirement that both regions have identical type compositions in equilibrium. Otherwise, type j faces regional differences in the amount of available commodity $k\,(\neq j)$, prompting relocation until the compositions align, a process that naturally gives rise to agglomeration. In this example, the two types are **mutually indispensable**, in which case they are **complementary** to each other. Each type prefers to reside where the other type is, leading to jointly disassortative sorting. Convex preferences compel the co-location of different types, making type complementarity a key driver of agglomeration patterns.

In truth, the preceding example still remains limited in what it reveals. Notably, types only see each other as indispensable trading partners, and there is no regional difference in composition of types, i.e., no spatial sorting is observed in the end.[2] This offers little insight into how different types interact or how regional composition shapes economic outcomes.

Now suppose that in addition to simply **consuming** the other type's endowment, they can also **produce** the good they lack from the good they are endowed with. Type 2 can repurpose part of their endowment for the **production of commodity 1** instead of selling it to type 1 in **exchange for commodity 1**. Type 1 then becomes **replaceable** to type 2. Namely, type 1 becomes interchangeable with a firm from type 2's standpoint so that type 2 is now indifferent between assortative and disassortative sorting. Production substitutes for migration, opening the possibility of assortative sorting. This still leads to agglomeration[3] albeit in a milder form. There will be no inter-regional price differentials in this case either so long as each region has the access to the same production technology.

[2]As a result, the equilibrium is scalable.

[3]With unlikely exceptions detailed in Proposition 3.2.

As the preceding example illustrates, a workable model of consumer-driven agglomeration must contain at least three elements:

1. heterogeneity in how endowments are allotted to consumers,
2. preferences for goods that a consumer is not endowed with (to generate type indispensability), and
3. production (to generate type replaceability).

Let us reiterate that production is introduced solely to generate type replaceability, not to serve as an independent source of agglomeration. We keep technology linear to avoid conflating it with consumer-led agglomeration that the current article aims to highlight.

The illustration above assumes endowments are portable. If endowments do not move with their owners, an incoming type simply increases local demand without adding anything to the local pool of endowments, and becomes rivalrous with the other type. In such an economy, consumers' locations are constrained by the exogenous locations of endowments. If endowments are evenly distributed, so are the consumers. This is indeed the case in a pure exchange economy.[4]

Nevertheless, we include non-portable endowments in our analysis for two reasons. First, not all endowments are portable in practice. Second, production can offset the rigidity of fixed endowment locations by substituting for migration, effectively replicating an economy where endowments are portable. For example, even if commodities 1 and 2 are evenly distributed as non-portable endowments, residents may still consume more commodity 1 and less commodity 2 through production. We outline next how this process unfolds.

In fact, it would be unusual for agglomeration **not** to occur. Suppose some non-portable endowments are distributed evenly over two regions. Let $\lambda = \begin{bmatrix} .5 & .5 \end{bmatrix}$. If one type-1 leaves region a and is replaced by exactly one type-2, the distribution remains uniform. However, such one-for-one replacement is rare: for instance, if type 1 consumes commodities 1 and 2 in a 1:1 ratio, while type 2 consumes them in a 1:10 ratio, replacing the one with the other will not clear the markets. Instead, one type-1 will typically be replaced by fewer or more than one type-2, with production adjusting commodity ratios to accommodate the new type composition in each region, breaking the initial uniform distribution. Agglomeration emerges because the two types disagree on how much of the endowed commodity should be converted into the produced commodity.

Note that Table 1 describes only **unilateral** sorting behavior. The overall sorting pattern is determined by the **bilateral** relationship. If two types are mutually

- indispensable, they are **complementary**, resulting in disassortative sorting.
- replaceable, they are **substitutable**, with an indeterminate sorting outcome.

[4]We will detail this in Appendices A.3 and A.8.

- rivalrous, they are **antagonistic**, resulting in assortative sorting.

For discordant cases, such as type 1 being indispensable to type 2 and type 2 being rivalrous to type 1, a model is needed to determine which force prevails. The remainder of the paper develops such a framework. We identify the conditions under which different types are drawn to or repelled by one another, determined by their multilateral relationships. This, in turn, determines agglomerative forces.

1.3 Related Literature

We base our model on the production economy founded by Kehoe [Keh85]. His work follows a line of research started in the 1970s on the uniqueness, stability, and smoothness of general equilibrium as a function of exogenous parameters. Debreu [Deb70] introduced the notion of regular equilibria and showed that regularity is a generic property. Dierker [Die72] developed the index theorem to characterize the number of equilibria in regular economies. Mas-Colell [MC85] offers a comprehensive treatment of the index theorem and regular economies. Kehoe [Keh85] operationalized this theory by constructing computable general equilibrium examples that feature multiple equilibria. His model is both tractable and sufficiently rich to examine questions of equilibrium multiplicity and robustness. Though he uses a numerical example to establish the existence of multiple equilibria, regularity ensures that these results are robust. This makes it well suited for our spatial adaptation, where regularity ensures that observed agglomeration and sorting patterns are not artifacts of knife-edge parameterizations.

Whereas earlier contributions focused on pure exchange economies, Kehoe [Keh80] was the first to extend the index theorem to economies with production. In particular, his model features linear production technology. This is an ideal setup for our purposes. A pure exchange economy does not suffice to replicate spatial sorting, while non-linear technology will risk conflating the consumer-driven agglomeration with firm-oriented agglomerative forces. His framework is simple enough to maintain our computational burden to a manageable level; and still it is rich enough to produce non-trivial results that enable us to analyze how agglomeration emerges from consumer-led sorting behavior.

In this regard, our model relates to a line of work set forth by Diamond [Dia16]. Her model features two types of endowments, high- and low-skilled labor. A composition of types changes the amount of inputs available in a region both in her and our models. However, in her model, this further changes individual type's productivity by way of agglomeration externality effects and skill complementarities in a city. Amenities are impacted by the composition as well, encompassing another source of externalities.

This line of inquiry could also address a broader gap in the literature, as noted by Berliant and Mori [BM17], who observe that existing models often explain city-size distributions largely through residual variation, suggesting room for more structurally grounded approaches. In the same vein as Diamond [Dia16], Giannone [Gia17] incorporates local agglomeration spillovers directly into the production function, where productivity rises with the density or share of high-skilled labor. Eeckhout et al. [EPS14] model households' location choices as depending on wages, housing prices, and amenities, with amenities entering the utility function as technical externalities, while endogenous prices generate pecuniary externalities. In equilibrium, households sort disassortatively due to skill complementarities. Behrens et al. [BDRN14] examine the interaction of sorting, selection, and agglomeration in explaining productivity differences across cities, where externalities similarly enter the production function via productivity gains linked to labor density. Baum-Snow et al. [BSFP18] highlight skill-biased agglomeration economies, with productivity gains driven by the local skill mix, again operating through the production function. These models also generate pecuniary externalities, such as higher rents in tighter land markets, but those effects stem from underlying technical externalities. Helsley and Strange [HS14] provide the theoretical foundation for the production-based externalities employed above. The productivity of an industry depends on the composition of other industries in the same city, capturing co-agglomeration spillovers. They emphasize that urban productivity depends not only on city size but also on its industrial composition, as complementary industries mutually reinforce one another's efficiency.

In contrast, our model can be viewed as a stripped-down, pecuniary-only counterpart to these richer frameworks: we retain the role of composition in shaping the allocation of inputs but deliberately abstract from productivity- or amenity-driven externalities. Productivity is identical across workers and scales of production; sorting arises solely from the distribution of types. Consumers position themselves to avoid direct competition with others seeking the same endowment for consumption or production.[5] With heterogeneous endowments and preferences, such sorting occurs even if productivity does not depend on the population size. All interactions between types occur exclusively through market exchange, so the only externalities we allow are pecuniary in nature.

This parallels a similar contrast in the literature on the producer-driven agglomeration. Whereas the earlier literature on producer-driven agglomeration relied on externalities (see for example Henderson [Hen74]), New Economic Geography (Fujita et al. [FKV99]) builds on pecuniary externalities like us, but with imperfect competition. In contrast, we employ perfect competition and our equilibrium allocations are Pareto optimal.

We situate our work within the broader fabric of agglomeration research in Appendix A.1.

The remainder of the paper proceeds as follows. In Section 2 we lay out our model. Section 3 outlines baseline findings with a downscaled version of the model with only two

[5] Having two regions enables them to sort spatially. However, there will be no welfare gains in the end for the reasons we will explain.

types and two commodities. In particular, we discuss inter-regional trades in Section 3.5. Three versions of the full-fledged model will be presented afterwards: Endowments are portable in Section 4.1, not portable in Section 4.2, and partially portable in Section 4.3, followed by a discussion on robustness in Section 4.4. Section 5 concludes.

2 The Model

We build our model on the production economy analyzed by Kehoe [Keh85]. His model features a single region with four commodities $i = 1, \cdots, 4$, four consumers $j = 1, \cdots, 4$, and linear technology. We add one more region to it. As a result, there are eight commodities, four in each region.

There is a unit mass of each of four types of consumers, who take up residence in either region a or b. There is no preference for location. Consumers can freely choose their location of residence, in other words there is no relocation cost. We denote the population distribution by $\lambda = \begin{bmatrix} \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 \end{bmatrix}$, where $\lambda_j \in [0, 1]$ is a fraction of type-j consumers in region a. In what follows, we use a superscript to denote a row and commodity i, and a subscript to denote a column and consumer type j and/or region a or b as is standard in general equilibrium models.

A consumer of type j maximizes Cobb-Douglas utility function $u_j(x_j) = \prod_{i=1}^{4} \left(x_j^i\right)^{\alpha_j^i}$ subject to $\pi \cdot x_j \le \pi \cdot w_j$, where $x_j = \begin{bmatrix} x_j^1 & x_j^2 & x_j^3 & x_j^4 \end{bmatrix}^\top$ is his consumption bundle, $w_j = \begin{bmatrix} w_j^1 & w_j^2 & w_j^3 & w_j^4 \end{bmatrix}^\top$ is his endowment, and $\pi = \begin{bmatrix} \pi^1 & \pi^2 & \pi^3 & \pi^4 \end{bmatrix}^\top$ is a price vector. Let $z_j := x_j - w_j$ be his net demand. **As we will show in Proposition 4.1, the equilibrium price vector will be the same in both regions**. Expenditure share α and endowment w are specified as

$$\alpha = \begin{bmatrix} .52 & .86 & .5 & .06 \\ .4 & .1 & .2 & .25 \\ .04 & .02 & .2975 & .0025 \\ .04 & .02 & .0025 & .6875 \end{bmatrix} \quad \text{and} \quad w = \begin{bmatrix} 50 & 0 & 0 & 0 \\ 0 & 50 & 0 & 0 \\ 0 & 0 & 400 & 0 \\ 0 & 0 & 0 & 400 \end{bmatrix}. \tag{1}$$

For example, type-4 consumers allocate .06 of their expenditure to commodity 1, and they are endowed with zero units of it. Each type j is endowed exclusively with commodity j. This stylization allows us to isolate causal mechanisms and trace type relationships with clarity.

Technology is linear and specified by technological process

$$A = \begin{bmatrix} -1 & 0 & 0 & 0 & 6 & -1 \\ 0 & -1 & 0 & 0 & -1 & 3 \\ 0 & 0 & -1 & 0 & -4 & -1 \\ 0 & 0 & 0 & -1 & -1 & -1 \end{bmatrix}. \tag{2}$$

The supply is Ay, where y is a 6×1 non-negative vector indicating how much of each column of A is deployed in production. The first four columns are free disposal technologies. Since all commodities are goods, the first four entries of y are zero in equilibrium.

Inter-regional trade does not occur in equilibrium so long as transport costs are strictly positive, however small (see Section 3.5 to follow). Markets are cleared by moving people between the regions, not commodities. That is because equilibrium prices equate across regions.

The distribution of endowments is $\mu \in [0,1]^4$, where μ^i denotes a fraction of w^i_j located in region a for every j. This is exogenous if cross-regional movement of endowment is not allowed. We define equilibrium next.

Definition 2.1 Equilibrium

Intra-regional equilibrium *Region a is in intra-regional equilibrium when each resident maximizes their utility level subject to their budget constraint, and excess demand*

- $z_a \lambda^\top - Ay_a = \mathbb{0}$ *if the endowment travels with the consumer, or*
- $x_a \lambda^\top - \mu \circ (w\mathbb{1}) - Ay_a = \mathbb{0}$ *if fraction μ^i of endowment w^i is allocated to region a.*[6]

Inter-regional equilibrium *Populated regions a and b are in inter-regional equilibrium if*

1. *each region is in intra-regional equilibrium, and*
2. $u\left(x_{j,a}\right) = u\left(x_{j,b}\right)$ *for any j.*

Remark. Whereas the first requirement for inter-regional equilibrium guarantees that the gains from trade are exhausted in each region, the second requirement guarantees that the utility gains from relocation are exhausted across regions.[7]

Proposition 2.1 Orthogonal Prices

Given (1) and (2), the set of potential intra-regional equilibrium price vectors is

$$\Pi^\perp = \left\{ \pi \in \mathbb{R}^4_{++} : \ \pi = \begin{bmatrix} \pi^1 & \frac{1}{4} & \frac{7\pi^1 - 1}{3} & \frac{-10\pi^1}{3} + \frac{13}{12} \end{bmatrix}^\top \text{ and } \pi^1 \in \left(\frac{1}{7}, \frac{13}{14} \right) \right\}. \tag{3}$$

Proof. Firms earn zero profits in equilibrium because of constant returns to scale. Thus, the intra-regional equilibrium price vector must be orthogonal to the column space of A. In addition, Walras' law enables the normalization of prices, $\sum \pi^i = 1$. Combined, π must be of the form (3) in intra-regional equilibrium. □

Remark. In what follows, if a function is defined over π, we will plot it over π^1 alone, with the understanding that the remaining three prices are functions of π^1 according to (3) in equilibrium.

[6] We denote an entry-wise product by a circle: $x \circ y = \begin{bmatrix} x_1 y_1 & x_2 y_2 & \cdots & x_n y_n \end{bmatrix}$.

[7] If a type resides only in one region, then Item 2 in Definition 2.1 may not be satisfied with equality. Appendix A.2 discusses such cases.

Despite its simplicity, Kehoe [Keh85]'s model entails three equilibria with distinct prices and allocations due to income effects. We list them below, along with its pure-exchange counterpart to clarify the role of production:[8]

- Equilibrium #1

$$
\begin{aligned}
\pi &= \begin{bmatrix} 0.159 & 0.250 & 0.0387 & 0.552 \end{bmatrix}^\top \\
\begin{array}{c}\text{excess demand}\\ (x-w)\mathbb{1}-Ay\end{array} &= \begin{bmatrix} 26-50 & 67.43 & 48.49 & 83.09 \\ 12.75 & 5-50 & 12.37 & 220.77 \\ 8.25 & 6.47 & 119-400 & 14.28 \\ 0.58 & 0.45 & 0.07 & 275-400 \end{bmatrix}\mathbb{1} - \begin{bmatrix} 6 & -1 \\ -1 & 3 \\ -4 & -1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 42.7 \\ 81.2 \end{bmatrix} \quad (= \mathbb{0}) \\
\text{income } \pi^\top w &= \begin{bmatrix} 8.0 & 12.5 & 15.5 & 220.8 \end{bmatrix} \\
u &= \begin{bmatrix} 16.0 & 44.9 & 47.4 & 240.5 \end{bmatrix}
\end{aligned}
$$

- Equilibrium #2

$$
\begin{aligned}
\pi &= \begin{bmatrix} 0.250 & 0.250 & 0.250 & 0.250 \end{bmatrix}^\top \\
(x-w)\mathbb{1}-Ay &= \begin{bmatrix} 26-50 & 43 & 200 & 24 \\ 20 & 5-50 & 80 & 100 \\ 2 & 1 & 119-400 & 1 \\ 2 & 1 & 1 & 275-400 \end{bmatrix}\mathbb{1} - \begin{bmatrix} 6 & -1 \\ -1 & 3 \\ -4 & -1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 52 \\ 69 \end{bmatrix} \quad (= \mathbb{0}) \\
\pi^\top w &= \begin{bmatrix} 12.5 & 12.5 & 100 & 100 \end{bmatrix} \\
u &= \begin{bmatrix} 19.1 & 29.8 & 140.8 & 181.9 \end{bmatrix}
\end{aligned}
$$

[8] A number in script font denotes a column or row vector or a matrix (whichever is appropriate) consisting of repeated entries of a same number, e.g., $\mathbb{0} = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^\top$. We will use $\mathbb{0}$, $.\mathbb{5}$, and $\mathbb{1}$ in this article.

- Equilibrium #3

$$\begin{aligned}
\pi &= \begin{bmatrix} 0.275 & 0.250 & 0.309 & 0.166 \end{bmatrix}^\top \\
(x - w)\mathbb{1} - Ay &= \begin{bmatrix} 26 - 50 & 39.07 & 224.36 & 14.50 \\ 22.01 & 5 - 50 & 98.77 & 66.49 \\ 1.78 & 0.81 & 119 - 400 & 0.54 \\ 3.31 & 1.50 & 1.86 & 275 - 400 \end{bmatrix} \mathbb{1} - \begin{bmatrix} 6 & -1 \\ -1 & 3 \\ -4 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 53.2 \\ 65.1 \end{bmatrix} \quad (= \mathbb{0}) \\
\pi^\top w &= \begin{bmatrix} 13.8 & 12.5 & 123.5 & 66.5 \end{bmatrix} \\
u &= \begin{bmatrix} 20.1 & 27.6 & 155.8 & 159.1 \end{bmatrix}
\end{aligned}$$

- Equilibrium without production (the equilibrium is unique)[9]

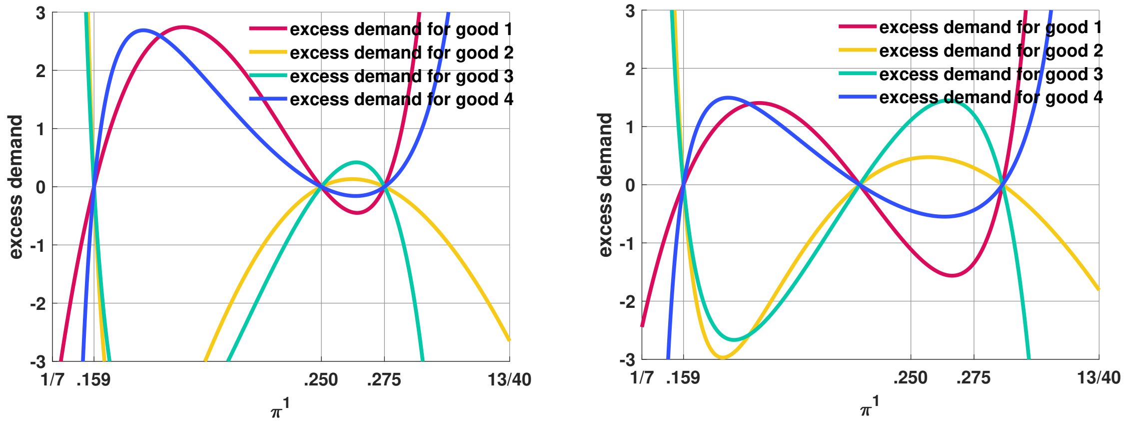
$$\begin{aligned}
\pi &= \begin{bmatrix} 0.652 & 0.329 & 0.00586 & 0.0131 \end{bmatrix}^\top \\
(x - w)\mathbb{1} &= \begin{bmatrix} 26 - 50 & 21.72 & 1.79 & 0.48 \\ 39.59 & 5 - 50 & 1.42 & 3.98 \\ 222.55 & 56.21 & 119 - 400 & 2.24 \\ 99.44 & 25.11 & 0.45 & 275 - 400 \end{bmatrix} \mathbb{1} \quad (= \mathbb{0}) \\
\pi^\top w &= \begin{bmatrix} 32.6 & 16.5 & 2.3 & 5.2 \end{bmatrix} \\
u &= \begin{bmatrix} 35.4 & 19.2 & 6.0 & 64.4 \end{bmatrix}.
\end{aligned}$$

Figure 1(a) plots excess demand in region a over π^1. His model is a special case of our model where endowments are portable and everyone congregates in one region, i.e., $\lambda = \mathbb{0}$ or $\mathbb{1}$. By adding another region, aggregate excess demand in region a becomes the sum of individual demand with weight $\lambda \in [0, 1]^4$ rather than $\lambda = \mathbb{0}$ or $\mathbb{1}$. The **intra-regional** markets may clear at a price outside Kehoe's as plotted in Figure 1(b).

In rendering Kehoe's aspatial economy spatial, in essence, we are dividing it into two parts with a desired level of interconnection. In the extreme where two regions have no connections, there are simply two aspatial economies. The polar opposite is where there is no friction of any kind between the two. This too is in effect aspatial. We examine the cases that fall between these two extremes, with differing degrees of inter-regional linkage described in Table 2.

In practice, there are four ways in which two regions may be spatially connected. To prevent any confusion, we specify our terminologies below and do not use them inter-

[9]The owner of abundant endowments (types 3 and 4) are better off with production in this case. They can take advantage of type replaceability made possible by production, as outlined in Section 1.2. With production, they can opt for furnishing scarce commodities 1 and 2 from their endowments rather than from exchanges.



(a) $\lambda = \mathbb{1}$, namely Kehoe's economy.

(b) $\lambda = \begin{bmatrix} .972 & .126 & .569 & .641 \end{bmatrix}$.

Figure 1. Excess demand in region a. Price is on the x-axis and quantity is on the y-axis.

term	portable	tradable	mobile	transferable
applicable to	endowment	output	people	income
Sections 3.2 and 4.1	●		●	●
Sections 3.3 and 4.2			●	●
Sections 3.4 and 4.3	partially		●	●
Section 3.5	either	●	●	●

Table 2. Types of cross-border movements allowed in each section.

changeably in this article. Term "portable" indicates whether a consumer must or cannot take his endowments with him when he relocates,[10] and "tradable" indicates whether outputs can be shipped to another region. Income from endowments is "transferable" if it can be cashed in at a different region than where it was generated. We reserve the term "mobile" to refer to worker's geographic mobility. With the possible exception of tradability, any cross-border movements incur no cost. Table 2 summarizes these four terms.

We assume throughout the paper that workers are perfectly mobile and income is transferable at no cost. Perfect mobility enables us to characterize equilibrium as a state where utility levels equate type by type across regions. It also rationalizes portability: We will not be able to discuss portability unless workers are mobile in the first place. Transferability makes no difference when endowments are portable. However, a lack of it would add inessential constraints to the analysis when endowments are not portable.

[10] In our context, portability does not mean that a consumer decides whether he takes endowment with him at his discretion. Rather, he **must** move with it and **put it to use where he resides**. This eliminates an inter-regional commute as labor cannot be employed outside where a worker lives.

For example, labor is portable and land is not portable. Neither one of them is tradable in our sense. Their portability or tradability notwithstanding, their owners are perfectly mobile and income generated from them is freely transferable.

Since $u_j(x_j) > u_j(w_j) = 0$ for any $x_j \in \mathbb{R}^4_{++}$ for any j, type j's location decision is strongly motivated by where endowed commodity $i\ (\neq j)$ is located, whether needed for direct consumption or as inputs for producing their preferred outputs. In general, interaction among types is less intense and responsive when endowments are not portable because the migration decision has no impact on μ.

Despite the simple setup, the model above is not analytically solvable. We will preface full-fledged versions to be introduced in Section 4 with a downscaled but tractable variant below. Agglomeration is said to occur when the equilibrium populations of the two regions are not the same; type plays no role here. Any consumer counts as one resident regardless of type.

3 Economy with Two Types and Two Commodities

3.1 Overview

Define an $I \times J$ economy to be an economy consisting of I commodities and J types. The current section considers 2×2 economies. We isolate types 1 and 2, and commodities 1 and 2 from Section 2, and overwrite A with a 2×1 vector $\hat{A} := \begin{bmatrix} A_5^1 & A_5^2 \end{bmatrix}^\top = \begin{bmatrix} 6 & -1 \end{bmatrix}^\top$. Commodity 1 is produced using commodity 2 as an input, but not vice versa. Allowing a technology that converts 1 into 2 in a 2×2 economy would violate the zero-profit condition, because no price vector could support both activities without yielding positive profits. As in Proposition 2.1, the firm generates zero profits so that $\pi \cdot \hat{A} = 0$, resulting in a unique (up to normalization) equilibrium price vector. If there is only one region (call it E^{1R}), the equilibrium aggregate net demand $(x-w)\mathbb{1} = \begin{bmatrix} 28.3-50 & 268.8 \\ 3.6 & 5.2-50 \end{bmatrix}\mathbb{1} = \begin{bmatrix} 247.02 \\ -41.17 \end{bmatrix}$, aggregate supply $\hat{A}y = \begin{bmatrix} 6 \\ -1 \end{bmatrix} 41.17 = \begin{bmatrix} 247.02 \\ -41.17 \end{bmatrix}$, $\pi = \begin{bmatrix} \frac{1}{7} & \frac{6}{7} \end{bmatrix}^\top$, income $\pi^\top w = \begin{bmatrix} 7.1 & 42.9 \end{bmatrix}$, and $u = \begin{bmatrix} 11.7 & 178.2 \end{bmatrix}$. Appendix A.3 presents its pure exchange counterpart to isolate the role of type replaceability enabled by production.

Consider two-region economies. We sort them by portability as follows: E^P with portable endowments, E^{NP} with non-portable endowments, and E^{MP} with mixed portability where w^1 is portable and w^2 is not.

PROPOSITION 3.1 AGGLOMERATION IN 2×2 ECONOMIES

Suppose that endowments, if not portable, are allocated evenly over two regions. In a 2×2 economy with parameters inherited from (1) and technology $\hat{A}$, the equilibrium price vector is $\pi = \begin{bmatrix} \frac{1}{7} & \frac{6}{7} \end{bmatrix}^\top$, *shared between two regions, and the equilibrium distribution is*

$$\left\{\lambda \in [0,1]^2 : \lambda_2 \in \left[\frac{||z_1||}{||z_2||}\lambda_1, 1 - \frac{||z_1||}{||z_2||}(1-\lambda_1)\right]\right\} \qquad \text{in } E^P, \tag{4}$$

$$\left\{\lambda \in [0,1]^2 : \lambda_2 = .5\right\} \qquad \text{in } E^{MP}, \text{ and} \tag{5}$$

$$\left\{\lambda \in [0,1]^2 : \frac{w_1^1}{\hat{A}^1}(\lambda_1 - .5) = \frac{w_2^2}{\hat{A}^2}(\lambda_2 - .5)\right\} \qquad \text{in } E^{NP}. \tag{6}$$

Proof. Since there is no regional variation in technology, both regions meet the same condition $\pi \cdot \hat{A} = 0$, leading to $\pi = \begin{bmatrix} \frac{1}{7} & \frac{6}{7} \end{bmatrix}^\top$. Solve $z(\pi)\lambda^\top = Ay_a$ in E^P or $x(\pi)\lambda^\top = Ay_a + \mu \circ (w\mathbb{1})$ in E^{NP} and the corresponding equality in region b for λ, with the constraint $y_a, y_b \geq 0$. For E^{MP}, replace μ^1 with λ_1. □

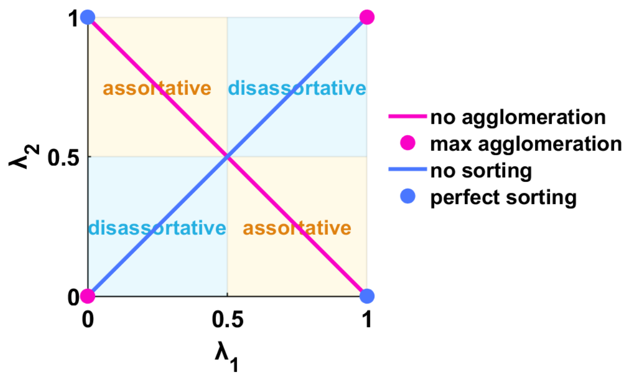
Figure 2 illustrates 2×2 economies. The measures of types 1 and 2 in region a are on the axes. Figure 2(a) classifies the distribution $\lambda \in [0,1]^2$ across two dimensions: agglomeration and spatial sorting. The fraction of the total population in region a is $(\lambda_1 + \lambda_2)/2$. The economy involves agglomeration if this value differs from .5, namely if the distribution is found off the red diagonal in Figure 2(a). The farther the distribution lies from the diagonal, the deeper the degree of agglomeration, maximized at the northeast and southwest corners, where the entire population concentrates in one region (see Appendix A.2).

Although all markets are perfectly competitive and utility functions are not defined over λ, we may retrospectively interpret interactions among types as either cooperative and complementary, or competitive and assortative to better understand the resulting equilibrium sorting patterns. There is no spatial sorting along the blue diagonal in Figure 2(a). Each region may have a different total population but the proportion of types is the same. The economy involves spatial sorting if λ is found off the blue diagonal. In particular, points in the northeast and southwest quadrants reflect disassortative sorting, while points in the northwest and southeast quadrants reflect assortative sorting. At the extremes, namely at the northwest and southeast corners, they exhibit perfect sorting pattern: each region houses only one type.

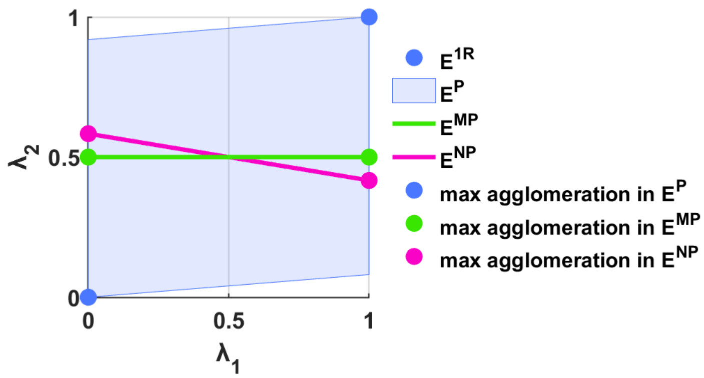
Overall sorting patterns described above are the product of bilateral relationships. In predicting sorting patterns and, by extension, agglomeration, we must consider not only a unilateral sorting behavior, but also the magnitude of their sorting tendencies on each side and the surrounding market conditions that mediate their interaction.

Even though agglomeration and sorting are distinct concepts, disassortative sorting typically results in stronger agglomeration than assortative sorting.[11]

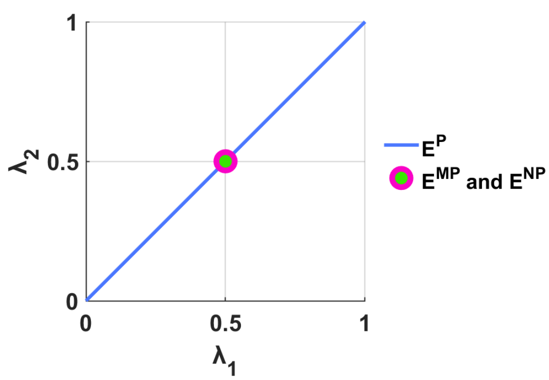
[11] In particular, any agglomeration where one region hosts more than three quarters of the total population cannot arise out of assortative sorting.



(a) Schematic diagram of agglomeration and spatial sorting. There is no spatial sorting along the blue diagonal, and no agglomeration along the red diagonal. Agglomeration becomes more pronounced away from the diagonal, towards the northeast or southwest corner, at which point the distribution becomes degenerate.



(b) The set of inter-regional equilibria in production economies. The area shaded in blue represents the set of equilibria in E^P. E^{1R} is a special case of E^P where $\lambda = \mathbb{0}$ or $\mathbb{1}$, indicated by blue dots. The green horizontal line shows the set of equilibria in E^{MP}. The red downward sloping line corresponds to E^{NP}. Table 3 breaks down the interaction between types.



(c) The set of inter-regional equilibria in pure exchange economies. No spatial sorting takes place. Furthermore, no agglomeration takes place in E^{MP} and E^{NP}. They are all subsets of their respective counterparts in production economy in Figure 2(b). See Appendix A.3 for details.

Figure 2. The population distribution $\lambda \in [0, 1]^2$ in 2×2 economies. The population of type 1 in region a is on the horizontal axis and that of type 2 is on the vertical axis. These are the only types in Section 3.

Figure 2(b) accompanies the equilibrium distributions (4), (5) and (6). The set of inter-regional equilibrium λ is up slanted in E^P (area shaded in blue), flat in E^{MP} (green line) and downward sloping in E^{NP} (red line). We also filled the equilibria in E^{1R} in blue dots for comparison.

In addition, we indicated the maximum agglomeration possible in each set of equilibria. For instance, in E^P, the maximum agglomeration possible is $(\lambda_1 + \lambda_2)/2 = 0$ or 1, as represented with blue dots.[12] Among three economies, E^P has the largest agglomeration possible and E^{NP} has the least.

We include equilibria of pure exchange economies in Figure 2(c) to serve as a reference point, isolating the role of type replaceability. Assortative sorting no longer occurs in the absence of this feature, which is only enabled by production (see Section 1.2). Appendix A.3 provides further detail on the consequences of removing production.

All three production economies inherit the same value of **individual** demand x_j from E^{1R} because there is only one equilibrium price and the budget constraints are thus identical over two regions, as are the utility levels. Their differences arise from the construction of **region-wide** net demand. They all realize agglomeration in equilibrium but for different reasons.

Table 3 summarizes sorting patterns using the terms introduced in Table 1. We will detail them in order in the subsequent sections.

	for type j	type k ($\neq j$) is	j's sorting	overall sorting	agglomeration
E^P	1	2 is indispensable	disassortative	either	strong
	2	1 is replaceable	either		
E^{NP}	1 (or 2)	2 (or 1) is rivalrous	assortative	assortative	weak
E^{MP}	1	2 is rivalrous	assortative	neither*	intermediate
	2	1 is replaceable	either*		

Table 3. Spatial sorting by endowment portability. See Table 4 for pure exchange counterparts. *Type 2 can be assortative or disassortative for type 1's replaceability. In equilibrium, however, λ_2 must be .5 regardless of λ_1 to ensure market clearance.

3.2 Portable Endowments (E^P)

As displayed in Figure 2(b), there is a two-dimensional continuum of equilibria. The set of equilibria entails both assortative and disassortative sorting. The irreversibility of production creates asymmetry in type behaviors.

Type 1 is replaceable to type 2. Type 2 can settle in any region, in any number, regardless of type 1's presence, because they have two avenues for acquiring commodity 1: either by trading with type 1 or by producing it from their endowment of commodity 2. They can therefore be self-sufficient, with the firm effectively replacing type 1 wherever type 2 outnumbers them.

By contrast, type 2 is indispensable to type 1. Type 1 needs to be accompanied by a sufficient number of type 2 to secure the (unproducible) commodity 2. If $\lambda_2 < \frac{||z_1||}{||z_2||}\lambda_1$, the

[12] This happens to coincide with the set of equilibria in E^{1R}, also represented with blue dots.

supply of commodity 2 will fall short of type 1's net demand z_1^2, lowering type 1's utility. Production cannot fill this gap because it is only designed to convert commodity 2 into 1, not the reverse. This shortfall prompts type 1 to move from region a to b, resulting in disassortative sorting.

Consequently, while perfect sorting (as represented by the northwest and southeast corners in Figure 2(b)) cannot take place because of type 1, both assortative and disassortative sorting are possible because of type 2. In region a, exchange prevails toward the southeast, where type 2 is inclined to sell their endowments to type 1, while production prevails toward the northwest, where they are inclined to sell their endowment to the firm instead.[13]

3.3 Non-Portable Endowments (E^{NP})

There is a one-dimensional continuum of equilibria. In order to neutralize the effect of spatial inhomogeneity, we shall set $\mu = .5$ in E^{NP}. Fujita [Fuj86] states:

> Given that the free mobility of agents is an essential feature of the long-run problem, Mills [Mil72, Mil80] and Kanemoto [Kan80] have suggested the following possible causes of spatial agglomerations in a city:
>
> b1. uneven distribution of natural resources.
>
> b2. proximity to economical transportation with the rest of the world.
>
> b3. increasing returns to scale or indivisibility.
>
> b4. externalities or public goods. (p. 114).

If we divide non-portable endowments unevenly, they may generate agglomeration for reason b1. above, obscuring the causal relationship between spatial sorting and agglomeration that we intend to showcase.

Unlike E^P, disassortative sorting disappears in E^{NP} because decoupling the location of an owner from that of her endowment removes the need to co-locate with specific **types**. Consumers now only need to be near the relevant **endowments** to maximize utility. In this setting, type 1 no longer provides commodity 1 to type 2 and only consumes endowment 2 that type 2 needs for its own production, making type 1 rivalrous. The resulting sorting pattern is therefore antagonistic for mutual rivalry.

Nevertheless, agglomeration still emerges as outlined in Section 1.2. Consumption ratios are $\frac{x_1^1}{x_1^2} = 7.8$ and $\frac{x_2^1}{x_2^2} = 51.6$. When a type-2 consumer moves into a region, two events take place to accommodate her. First, some type-1 consumers are displaced due to the fixed local supply of endowments. Second, production expands to modify the ratio

[13] Without production, type 1 becomes dispensable to type 2 as well. Appendix A.3 elaborates on the role of production and the replaceability it induces.

of endowed commodity 2 and produced commodity 1, as the ratio of aggregate demand tilts more toward 51.6 than 7.8.[14]

Agglomeration will not take place if one incoming type 2 drives out exactly one type 1 in the scenario above. However, this is a measure-zero case as the next proposition shows:

Proposition 3.2 Conditions for Agglomerations in E^{NP}

Suppose $\mu = .\bar{5}$ in E^{NP}. If $\frac{-\hat{A}^2}{\hat{A}^1} = \frac{w_2^2}{w_1^1}$, the economy does not support agglomeration in equilibrium.

Proof. Given the equality above, (6) implies $\lambda\mathbb{1} = 1$. □

Remark. Suppose that the economy is currently in inter-regional equilibrium. When a fraction δ of type 1 moves in, the region-wide demand rises by δx_1. Equilibrium will be restored if a specific amount of x_2 is removed. If $\frac{-\hat{A}^2}{\hat{A}^1} = \frac{w_2^2}{w_1^1}$, the amount to remove is δx_2. In this case, a fraction δ of type 2 leaves, keeping $\begin{bmatrix}\lambda_1 + \delta & \lambda_2 - \delta\end{bmatrix}\mathbb{1} = 1$ so that the distribution remains uniform. The red line in Figure 2(b) tilts to coincide with the red line in Figure 2(a).

$\hat{A}$ and w are exogenous. Unless we select very specific values, even assortative behavior, though not as prominent as disassortative behavior, results in agglomeration.

Therefore, even when types are rivalrous, moderate agglomeration arises because the types do not displace each other in a 1:1 ratio. In this case, one incoming type 2 displaces $\left|\frac{w_2^2}{w_1^1}\frac{\hat{A}^1}{\hat{A}^2}\right| = 6$ type 1s. Where type 2 is prevalent, production y_a rises to meet their higher demand for commodity 1, and vice versa where type 1 dominates.

This assortative sorting stems from externalities of a pecuniary nature. For example, with $\lambda = \begin{bmatrix}1 & \frac{5}{12}\end{bmatrix}$, like-minded type 1s cluster in region a because endowed commodity 2 there is converted into produced commodity 1 as much as market clearance permits. Their assortativeness arises entirely from market-clearing interactions in perfectly competitive settings, without local public goods (Tiebout [Tie56]) or local amenities (Rosen [Ros79]; Roback [Rob82]).

3.4 Mixed Portability (E^{MP})

There is a one-dimensional continuum of equilibria. Standing midway between the two preceding economies, interaction between the two types falls between that of E^P and E^{NP}. Type 1 is replaceable to type 2 and type 2 is rivalrous to type 1.

In fact, the economy can only support aggregate demand in a 6:1 ratio of commodities 1 to 2 due to the technological requirement ($\hat{A} = \begin{bmatrix}6 & -1\end{bmatrix}^\top$). Type 1's portable endowment

[14]Without production, the second event cannot take place. Then there is no way to reconcile $\frac{x_1^1}{x_1^2}$ with $\frac{x_2^1}{x_2^2}$ when type composition changes. See Appendix A.3 for its implication.

w_1^1 scales with λ_1, so their net demand $x_1 - w_1$ maintains this ratio for any λ_1. In contrast, type 2's non-portable endowment w_2^2 does not scale with λ_2, so their net demand $x_2\lambda_2 - w_2 \circ \mu$ departs from the ratio unless $\lambda_2 = \mu^2 = .5$.

Since exchange with type 1 or with the firm yields the same outcome, for type 2, it does not matter how many type 1s are in the region. Type 1's location is, in turn, determined by which combination of exchange and production type 2 chooses. The resultant sorting is neither assortative nor disassortative.[15]

3.5 Inter-Regional Trade

Agglomeration takes place whether inter-regional trade is allowed or not. As established in Proposition 3.1, the equilibrium price vector is shared between two regions, limiting the necessity for trade. Let $t \geq 1$ be units of any commodity required to be shipped from one region to receive one unit of it in the other region. Preceding sections have assumed $t \to \infty$. Let us consider two other cases: $t > 1$, and $t = 1$.

When $t > 1$, the inter-regional equilibria remain the same. Technology is not heterogeneous by region to warrant comparative advantages, nor does it exhibit increasing returns to scale to warrant exclusive production in a particular region. Imported goods are always priced higher than locally produced goods and thus no one buys them.

When $t = 1$, the set of equilibrium distributions remains unchanged. Free shipping only relaxes the location constraint for **output** 1; **endowment** of commodity 1, used for consumption, and endowment of commodity 2, used for either consumption or production, are still subject to portability restrictions. They move only when their owner relocates in E^P, and not at all in E^{NP}. Consequently, while equilibrium may feature costless inter-regional flows of outputs, the distribution λ remains unaffected.

This result is consistent with the spatial impossibility theorem (Fujita and Thisse [FT13]). When consumers relocate with endowments in E^P and E^{MP}, they change not only the population but also the regional allocation of endowments, introducing asymmetries between regions.

We should clarify, however, that our model is not designed to address this case in a fully coherent manner. Commodity 1 can be shipped if it is produced, but it cannot be shipped if it is endowed: Regardless of tradability,[16] if an endowment is portable, it must move with its owner (see footnote 10); if it is not portable, it cannot be moved at all. To

[15]Reversing the attributes (making w^1 non-portable and w^2 portable) yields the set of equilibrium distribution $\left\{\lambda \in [0,1]^2 : \lambda_1 = .5,\ \lambda_2 \in \left[\frac{.5||z_1||}{||z_2||},\ 1 - \frac{.5||z_1||}{||z_2||}\right]\right\}$ as opposed to (5). Unlike the case above, the admissible range of λ_2 is restricted because the unproducible endowment is now portable. As in E^P, a minimum presence of its owners (type 2) is necessary in both regions to ensure market clearance. However, agglomeration still emerges, indicating that it does not hinge on a particular combination of endowment attributes.

[16]We need to distinguish between the cross-regional movement of outputs and that of endowments. Whereas the former is independent of consumer mobility, the latter is tied to it in E^P, and thus requires separate treatment (see Table 2).

preclude this scenario, it suffices to impose an arbitrarily small transport cost that deters all commodity flows.

This intuition extends to 4×4 economies to follow as the equilibrium price will not differ by region either.

3.6 Limitations of 2×2 Economies

In 2×2 economies, to type 2, type 1 can be replaceable with the firm. Alternatively, type 1 can be replaceable with type 3, but we cannot address this scenario because there are **only two types**.

Furthermore, types cannot be mutually replaceable because production is irreversible: the firm can only make commodity 1 from 2. The introduction of a technology to produce commodity 2 from 1 will violate the zero-profit condition, unless there are more than two commodities. Thus, we cannot thoroughly address replaceability and resultant assortativeness either, because there are **only two commodities**.

With that, we now turn to economies with **four types and four commodities**, the two-region version of the Kehoe economy.

4 Economy with Four Types and Four Commodities

4.1 Portable Endowments (E^P)

This section parallels Section 3.2 in a 4×4 setting. The equilibrium price does not differ by region nor from Kehoe's values. We shall establish this in a general setting first.

Consider a two-region economy with an arbitrary number of commodities and types. Both regions have access to technology A with rank$(A) = K < I$.[17] An allocation is a list $(x_a, x_b; y_a, y_b; \lambda) \in \mathbb{R}_+^{I \times J} \times \mathbb{R}_+^{I \times J} \times \mathbb{R}_+^K \times \mathbb{R}_+^K \times [0, 1]^J$.

Proposition 4.1 Inter-Regional Equilibrium

Suppose that each type has a strictly quasi-concave, strictly monotone, differentiable utility function. If an inter-regional equilibrium exists and every type is present in both regions, $\pi_a = \pi_b$ and $x_a = x_b$.

Proof. Consider an inter-regional equilibrium allocation $(x_a, x_b; y_a, y_b; \lambda)$ with $\lambda \in (0, 1)^J$. Suppose that type 1 has different bundles by region: $x_{1,a} \neq x_{1,b}$; and the remaining types have the identical bundle: $x_{2+,a} = x_{2+,b} (=: x_{2+})$, where x_{2+} is a matrix with the first column of x removed. Let $\hat{x}_1 := \lambda_1 x_{1,a} + (1 - \lambda_1) x_{1,b}$ and $\hat{z}_1 := \hat{x}_1 - w_1$. If we aggregate

[17] Recall that I is the number of commodities.

each region's material balance conditions,

$$\begin{array}{rrcl} & z_a\lambda^\top + z_b\left(\mathbb{1}-\lambda^\top\right) - A(y_a+y_b) & = & \mathbb{0} \\ \Rightarrow & \hat{z}_1 + z_{2+}\mathbb{1} - A(y_a+y_b) & = & \mathbb{0} \\ \Rightarrow & .5\left\{\hat{z}_1 + z_{2+}\mathbb{1} - A(y_a+y_b)\right\} & = & \mathbb{0}. \end{array} \tag{7}$$

Thus, an allocation $\left(\begin{bmatrix}\hat{x}_1 & x_{2+}\end{bmatrix}, \begin{bmatrix}\hat{x}_1 & x_{2+}\end{bmatrix}; .5\,(y_a+y_b),\ .5\,(y_a+y_b);\ .\mathbb{5}\right)$ is feasible. By strict quasi-concavity, $u_1(\hat{x}_1) > u_1\left(x_{1,a}\right)$. This runs counters to the first fundamental theorem of welfare economics as the allocation involving $\hat{x}_1$ Pareto improves upon the existing equilibrium allocation. Thus, $x_{1,a} = x_{1,b}$, and in consequence, $x_a = x_b$.

Furthermore, strict monotonicity and strict convexity imply $x_{j,a} \gg \mathbb{0}$ for any j so that π_a points in the same direction as $\nabla u_j\left(x_{j,a}\right) = \nabla u_j\left(x_{j,b}\right)$, as does π_b. Therefore, $\pi_a = \pi_b$. □

Remark. The proposition applies to E^{MP} and E^{NP} as well. In these cases, the first line in (7) is replaced with $x_a\lambda^\top + x_b\left(\mathbb{1}-\lambda^\top\right) + w\mathbb{1} + A(y_a+y_b)$.

Since $\pi_a = \pi_b$ in equilibrium and w_j does not depend on λ, type j faces the same budget constraint no matter which region they choose. Consequently, $x_{j,\ a} = x_{j,\ b}$, and thus $u_j\left(x_{j,\ a}\right) = u_j\left(x_{j,\ b}\right)$ for any j. Therefore, the equilibrium utility level is the same regardless of λ for a given equilibrium price vector $\pi_a (= \pi_b)$.

Our model employs a Cobb-Douglas utility function, which satisfies the assumptions on preferences invoked by this proposition. The next proposition applies to our model as well.

Proposition 4.2 Equilibrium Prices in Single- and Two-Region Economies

Let Π^{1R} and Π^P be the set of (inter-regional) equilibrium price vectors in E^{1R} and E^P, respectively. Then $(\pi,\ \pi) \in \Pi^P \Leftrightarrow \pi \in \Pi^{1R}$.

Proof. For any $\pi \in \mathbb{R}^I_+$, net demand $z(\pi)$ is identical between E^{1R} and E^P. The budget constraint is the same in both.

(⇒) Consider a pair of price vectors $(\pi,\ \pi) \in \Pi^P$. The material balance condition in each region is $z(\pi)\lambda^\top = Ay_a$ and $z(\pi)(\mathbb{1}-\lambda)^\top = Ay_b$. Aggregate them to obtain $z(\pi)\mathbb{1} = A(y_a+y_b)$. Then material balance in E^{1R} can be met by setting $y^{1R} = y_a + y_b$. Therefore, $\pi \in \Pi^{1R}$.

(⇐) Consider some $\pi \in \Pi^{1R}$. The material balance condition is $z(\pi)\mathbb{1} = Ay^{1R}$. Let $\lambda = c\mathbb{1}$ with $c \in (0,\ 1)$ and set $y_a = cy^{1R}$ and $y_b = (1-c)y^{1R}$. Multiply both sides of the material balance condition by c and $(1-c)$ respectively to obtain $z(\pi)\lambda^\top = Ay_a$ and $z(\pi)(\mathbb{1}-\lambda)^\top = Ay_b$. Thus, material balance is met in each region under π by setting $\lambda = c\mathbb{1}$. Moreover, $u_j\left(x_j(\pi_a)\right) = u_j\left(x_j(\pi_b)\right)$ for any j because $\pi_a = \pi_b = \pi$. Therefore, $(\pi,\ \pi) \in \Pi^P$. □

Remark. The second part of the proof indicates that any λ of the form $c\mathbb{1}$ ($c \in (0, 1)$) constitutes an inter-regional equilibrium in E^P. Namely, the equilibria in E^{1R} are scalable.[18]

Outside $c\mathbb{1}$, the range of λ is restricted on two grounds. First, there is no equilibrium involving production in an economy that consists of only one type (see Appendix A.4). Thus, only select combinations of λ_j can clear the markets. Moreover, even if we find a linear combination $z(\pi)\lambda^\top$ that clears markets in region a, it does not necessarily do so in region b (see Appendix A.5).

Figure 3 displays the equilibria across all three types of economies, sorted according to the equilibrium price π^1. As defined in Section 3.1 with Figure 2(a), we use the terms "assortative" and "disassortative" to describe equilibrium allocation patterns across types, not coordinated behavior or joint location decisions by agents. We quantify regional differences in type compositions using Theil information index, shown on the horizontal axis (Reardon and Firebaugh [RF02]). A value of zero indicates that the two regions have identical type compositions, while higher values reflect stronger compositional differences across regions.

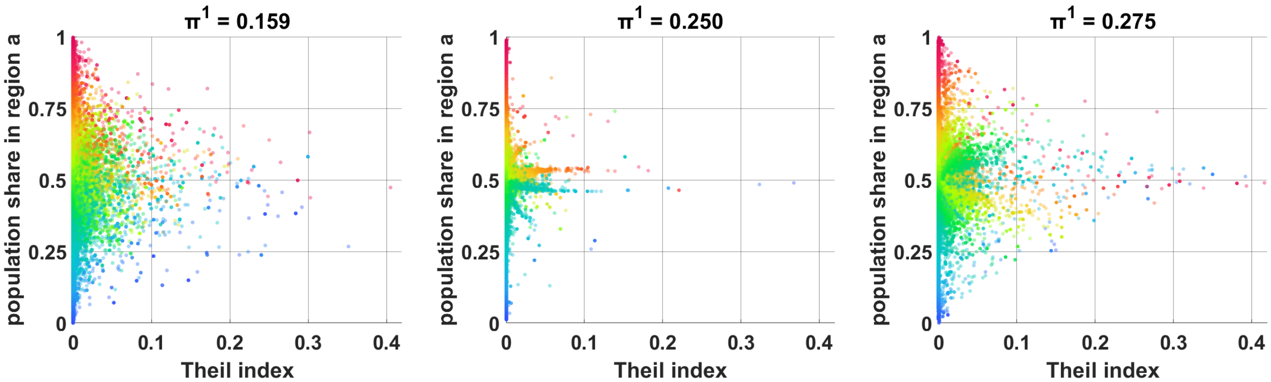
In our setting, the index is zero when $\lambda = c\mathbb{1}$ ($c \in [0, 1]$), in which case the equilibrium distribution is disassortative in the sense that all types co-locate across regions in the same proportion, generating no regional compositional differences.[19] At the other extreme, the index attains its maximum value of 0.5 in our two-region, four-type economy, when two types reside exclusively in one region and the remaining two types reside exclusively in the other. In this case, types separate into two groups across regions, yielding an assortative allocation **at the group level**, while within each region the coexisting types remain disassortatively paired **at the type level**.

The vertical axis represents the equilibrium population share in region a, given by $\lambda\mathbb{1}/4$, with points colored according to the value of λ_4 (red indicating $\lambda_4 \approx 1$ and blue indicating $\lambda_4 \approx 0$) to facilitate comparison with the corresponding figures in Appendix A.7. Whereas a population share of 0.5 indicates no agglomeration, values of 0 and 1 represent the maximal degree of agglomeration.

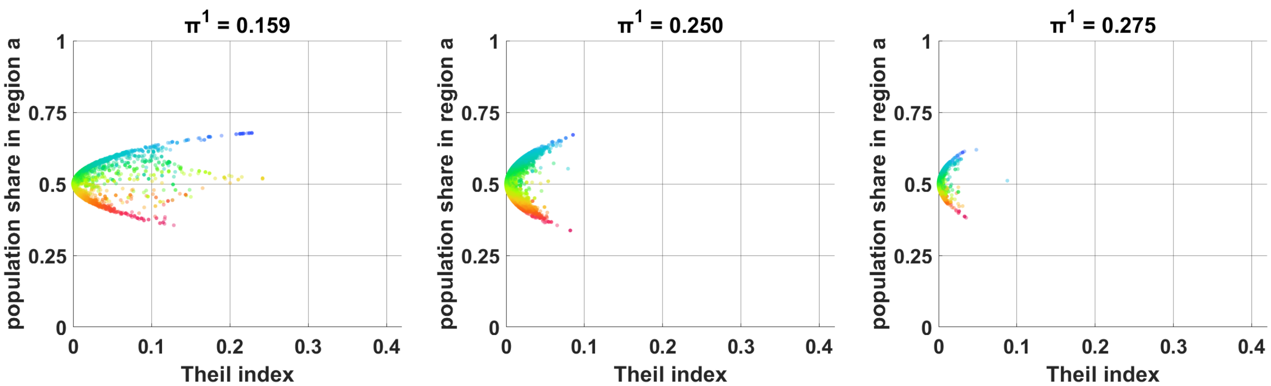
As in 2×2 E^P discussed in Section 3.2, both type indispensability and replaceability are at play. Yet, these do not map predictably onto the sorting behavior described in Table 1. We define type k as indispensable to type j if type j can access the endowment of commodity k only when type k is in the same region. In a two-commodity setting, this leads type j to behave unequivocally disassortatively. However, with more than two commodities, type k, while indispensable to type j, may compete with type j for commodity l. Then type j may not necessarily co-locate with type k. Below, we examine such interactions where one type behaves assortatively for one commodity and disassortatively

[18] See Appendix A.6 for E^{NP}.

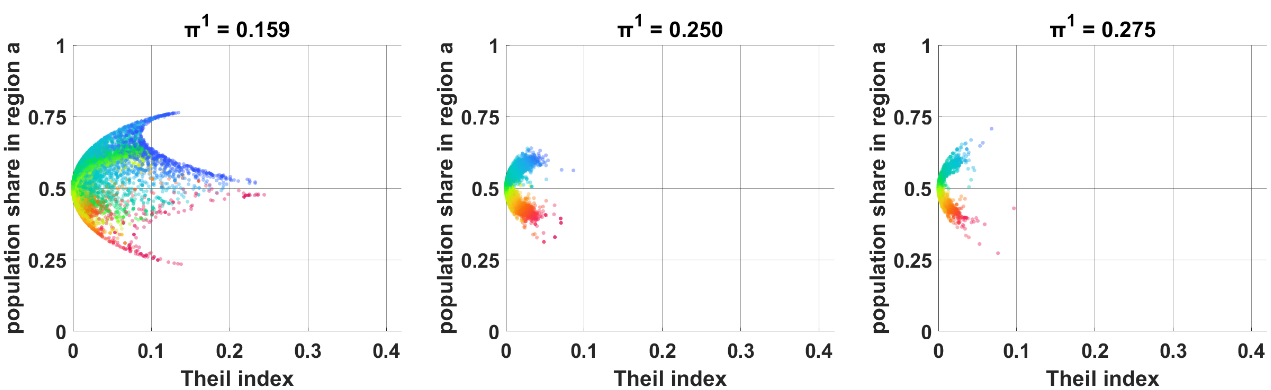
[19] By Proposition 4.2, any symmetric allocation of the form $\lambda = c\mathbb{1}$, $c \in (0, 1)$, constitutes an equilibrium in E^P. Such equilibria lie along the vertical axis (Theil index equal to zero) in Figure 3.



(a) E^P.



(b) E^{NP}.



(c) E^{MP}.

Figure 3. Agglomeration and assortativeness. Each dot represents an inter-regional equilibrium and is colored according to the value of λ_4 to allow comparison with Figures 6 to 8 in Appendix A.7. Whereas a low Theil index indicates that the two regions have similar type compositions, a population share of 0.5 indicates no agglomeration.

for another, and Appendix A.7 represents type-wise sorting patterns in more detail.

In E^P, types 1 and 2 form a disassortative distribution pattern, as do types 3 and 4. We shall focus on commodities 1 and 2, as variation in their production and exchange is most informative for understanding sorting patterns in this case. For these commodities, types 1 and 2 are mutually replaceable and therefore substitutable, as we outlined in Section 1.2. Type 1 can obtain commodity 2 either by trading with type 2 or by producing it from their own endowment of commodity 1, provided they can acquire the additional inputs, commodities 3 and 4. The same applies symmetrically to type 2. In the equilibrium considered here, they prefer exchange over production, resulting in the co-location of the two types.

On the other hand, types 3 and 4 are mutually indispensable and therefore complementary. Whereas type 3 requires commodity 4 to produce commodity 1, type 4 requires commodity 3 to produce commodity 2. This complementarity leads them to co-locate as well.

That said, the relationship between two groups, types 1-2 and types 3-4, is more involved. They are simultaneously indispensable, replaceable, and rivalrous along different margins, and may therefore settle in the same or in different regions depending on the equilibrium price. At $\pi^1 = 0.159$ and 0.275, the two groups are arranged disassortatively across regions, corresponding to a low Theil index and a high degree of agglomeration in Figure 3(a).[20] At $\pi^1 = 0.250$, they show a weakly assortative tendency. Equilibria at low Theil values are less frequently found at this price than at the other prices and agglomeration is correspondingly weaker as the two groups sort into different regions. This effect, however, is considerably less pronounced than at the other two prices. Appendix A.7.1 details the forces governing this outcome.

4.2 Non-Portable Endowments (E^{NP})

In parallel with Section 3.3, this section renders endowments non portable. As in Section 4.1, there is no regional variation in price. Agglomeration takes place but to a limited extent as it was the case in the 2×2 economies. Figure 3(b) lists inter-regional equilibria.

As in the 2×2 case, type indispensability disappears when endowments are not portable. Types 1 and 2 tend to agree on how much of commodities 3 and 4 should be converted into commodities 1 and 2, as do types 3 and 4 (see Appendix A.7.2 for details). As a consequence, types 1-2 tend to sort into one region, while types 3-4 sort into the other, resulting in an assortative behavior at the regional level. In such a case, the degree of agglomeration rises with the Theil index, as shown in Figure 3(b).

[20] As discussed in Section 1.2, sorting and agglomeration are related but distinct concepts. In this case, whereas high agglomeration always coincides with a low Theil index, a low Theil index does not necessarily imply strong agglomeration. Population can be either evenly split or highly concentrated in one region while yielding the same Theil value.

4.3 Mixed Portability of Endowments (E^{MP})

Finally we move on to an economy with mixed portability that corresponds to Section 3.4. We render endowments of commodities 2 and 4 non-portable, like land. The endowments of commodities 1 and 3 are portable, like labor.[21]

In inter-regional equilibrium, $\pi_a = \pi_b \in \Pi^{1R}$ because Proposition 4.1 applies. We present one of the inter-regional equilibria below.

$$
\begin{aligned}
\pi_a = \pi_b &= \begin{bmatrix} 0.159 & 0.250 & 0.0387 & 0.552 \end{bmatrix}^\top \\
\lambda &= \begin{bmatrix} 0.821 & 0.577 & 0.512 & 0.496 \end{bmatrix} \\
\begin{bmatrix} \lambda \mathbb{1} & 4 - \lambda \mathbb{1} \end{bmatrix} / 4 &= \begin{bmatrix} 0.602 & 0.398 \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
\begin{matrix} \text{aggregate net demand}_a \\ \left(x - w^P\right) \lambda^\top - \mu \circ (w^{NP} \mathbb{1}) \end{matrix} &= \begin{bmatrix} 26-50 & 67.43 & 48.49 & 83.09 \\ 12.75 & 5 & 12.37 & 220.77 \\ 8.25 & 6.47 & 119-400 & 14.28 \\ 0.58 & 0.45 & 0.07 & 275 \end{bmatrix} \begin{bmatrix} 0.821 \\ 0.577 \\ 0.512 \\ 0.496 \end{bmatrix} - .5 \begin{bmatrix} 0 \\ 50 \\ 0 \\ 400 \end{bmatrix} = \begin{bmatrix} 85.2 \\ 104.1 \\ -126.4 \\ -62.9 \end{bmatrix} \\
\begin{matrix} \text{aggregate supply}_a \\ A y_a \end{matrix} &= \begin{bmatrix} 6 & -1 \\ -1 & 3 \\ -4 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 21.2 \\ 41.8 \end{bmatrix} = \begin{bmatrix} 85.2 \\ 104.1 \\ -126.4 \\ -62.9 \end{bmatrix} \\
\begin{matrix} \text{income}_a \ \ \pi_a^\top w^P + \\ \pi_a^\top (\mu \circ w^{NP}) + \pi_b^\top \left((\mathbb{1} - \mu) \circ w^{NP}\right) \end{matrix} &= \begin{bmatrix} 8.0 & 12.5 & 15.5 & 220.8 \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
\begin{matrix} \text{aggregate net demand}_b \\ \left(x - w^P\right) (\mathbb{1} - \lambda)^\top - (\mathbb{1} - \mu) \circ (w^{NP} \mathbb{1}) \end{matrix} &= \begin{bmatrix} 26-50 & 67.43 & 48.49 & 83.09 \\ 12.75 & 5 & 12.37 & 220.77 \\ 8.25 & 6.47 & 119-400 & 14.28 \\ 0.58 & 0.45 & 0.07 & 275 \end{bmatrix} \begin{bmatrix} 0.179 \\ 0.423 \\ 0.488 \\ 0.504 \end{bmatrix} - .5 \begin{bmatrix} 0 \\ 50 \\ 0 \\ 400 \end{bmatrix} = \begin{bmatrix} 89.8 \\ 96.8 \\ -125.6 \\ -61.0 \end{bmatrix} \\
\begin{matrix} \text{aggregate supply}_b \\ A y_b \end{matrix} &= \begin{bmatrix} 6 & -1 \\ -1 & 3 \\ -4 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 21.5 \\ 39.4 \end{bmatrix} = \begin{bmatrix} 89.8 \\ 96.8 \\ -125.6 \\ -61.0 \end{bmatrix} \\
\begin{matrix} \text{income}_b \ \ \pi_b^\top w^P + \\ \pi_a^\top (\mu \circ w^{NP}) + \pi_b^\top \left((\mathbb{1} - \mu) \circ w^{NP}\right) \end{matrix} &= \text{same as income}_a
\end{aligned}
$$

$$
u_a = u_b = \begin{bmatrix} 16.0 & 44.9 & 47.4 & 240.5 \end{bmatrix},
$$

[21] Agglomeration takes place regardless of which endowments are rendered non-portable. See footnote 15.

where $w^P := \begin{bmatrix} w^1 \\ \mathbb{0} \\ w^3 \\ \mathbb{0} \end{bmatrix}$ and $w^{NP} := \begin{bmatrix} \mathbb{0} \\ w^2 \\ \mathbb{0} \\ w^4 \end{bmatrix}$ denote the matrices of portable and non-portable endowments, respectively.

Figure 3(c) summarizes equilibrium distributions. Relative to E^P and E^{NP}, the mixed-portability economy exhibits a non-monotonic and price-dependent relationship between agglomeration and regional differentiation. At $\pi^1 = 0.159$, agglomeration lies between that of E^P and E^{NP}, rising with the Theil index up to a point and then declining as compositional differences widen further. At $\pi^1 = 0.275$ agglomeration increases monotonically with the Theil index. By contrast, the case $\pi^1 = 0.250$ exhibits an intermediate pattern, with weaker agglomeration and limited sensitivity to variation in the Theil index. Overall, E^{MP} occupies an intermediate position between E^P and E^{NP}, with agglomeration shaped by the interaction of opposing forces rather than a single monotonic mechanism. The underlying mechanisms are detailed in Appendix A.7.3.

4.4 Robustness

Whereas we used specific parameter values to derive equilibria, they are regular and therefore robust. That is, there exists an open set of economies and an open set around each equilibrium in which the qualitative behavior remains similar. This is what Kehoe [Keh85] does in the simple one region economy. In general, this follows from the implicit function theorem.

We adapt the matrix from p. 1212 of Kehoe [Keh80] to our setting. Since individual demands $z_j(\pi)$ only matter through their aggregate in clearing markets, we focus on the vector of aggregate net demand, defined as $Z(\pi, \lambda) := z(\pi)\lambda^\top$. There are eight markets in total. The first four markets clear if $Z(\pi_a, \lambda) - Ay_a = \mathbb{0}$ and the remaining four clear if $Z(\pi_b, \mathbb{1} - \lambda) - Ay_b = \mathbb{0}$. Differentiating the system with respect to π_a, π_b, y_a, and y_b, we obtain the Jacobian

$$\begin{bmatrix} D_{\pi_a} Z(\pi_a, \lambda) & \mathbb{0} & -A & \mathbb{0} \\ \mathbb{0} & D_{\pi_b} Z(\pi_b, \mathbb{1} - \lambda) & \mathbb{0} & -A \\ A^\top & \mathbb{0} & \mathbb{0} & \mathbb{0} \\ \mathbb{0} & A^\top & \mathbb{0} & \mathbb{0} \end{bmatrix}. \tag{8}$$

To account for Walras' law in two regions with autarky, we remove the first and fifth rows. For homogeneity of the aggregate net demand function, we fix π_a^2 and π_b^2 and remove the corresponding second and sixth columns. The resulting matrix is non-singular at all equilibria we found. Therefore, the implicit function theorem applies: A small change in an exogenous parameter results in a well-defined nearby equilibrium.

4.5 Stability

The preceding section examined how the equilibrium responds to small changes in parameter values. This section focuses instead on small perturbations in prices.

In urban economic theory, for example the New Economic Geography, ad hoc notions of stability are used. In the NEG, small perturbations of population are employed to examine whether or not the economy returns to the original equilibrium. The dynamics are ad hoc, where the speed of adjustment of population is proportional to utility differences. In contrast, the reduced form of our model is a classical general equilibrium model, so tâtonnement stability using the Jacobian of aggregate excess demand can be examined to determine stability of equilibrium. That is how we proceed.

Let $J(\pi_a, \pi_b)$ be the Jacobian of aggregate excess demand $\begin{bmatrix} Z(\pi_a, \lambda) - Ay_a & Z(\pi_b, \mathbb{1} - \lambda) - Ay_b \end{bmatrix}^\top$. To account for Walras' law within each region, we remove the second and sixth rows and columns and denote the resultant matrix by $\tilde{J}$.

Similarly, we remove the second row of the technological process A. We further remove the first four columns, which are never deployed in equilibrium. Let $\tilde{A}^{1R}$ denote the matrix thus obtained. We then duplicate $\tilde{A}^{1R}$ to form $\tilde{A} := \begin{bmatrix} \tilde{A}^{1R} & \mathbb{0} \\ \mathbb{0} & \tilde{A}^{1R} \end{bmatrix}$, representing the technological process of the two-region economy as a whole.

According to Theorem 10 on p. 82 of McKenzie [McK02], if $\tilde{A}$ has full rank and $\tilde{J}$ is negative quasi-definite at an equilibrium price vector π, then the economy is locally stable at π.

A sufficient condition for negative quasi-definiteness is negative definiteness. All eigenvalues of $\tilde{J}$ are negative across all equilibria we found, and $\tilde{A}$ has full rank. Thus, our equilibria are locally stable.

5 Conclusions

We studied when and how heterogenous households agglomerate in a standard neoclassical and symmetric framework. Agglomeration is conventionally thought of as a producer-oriented phenomenon. Scale economies and positive externalities favor a concentration of inputs within close proximity. Barring scale economies, can there still be agglomeration? To examine whether agglomeration can be driven by consumers rather than producers, we adapted a general equilibrium model with constant returns to scale proposed by Kehoe [Keh85] to a spatial context. Our model demonstrates that heterogeneity among consumers creates asymmetry in population distribution and thus agglomeration does not necessitate the presence of scale economies.

Agglomeration forms out of spatial sorting in our model. We proposed three different types of economies based on the portability of endowments. Different types co-locate when one type's endowment is in demand by another type either as a consumption good or an input. They may as well settle into separate regions if their co-location would put them in direct competition for commodities that cannot be produced or relocated. Either sorting pattern breaks symmetry, more so under disassortative migration, and results in agglomeration to varying degrees.

Whereas our analysis focused on an economy with four commodities and four types to derive equilibrium, the underlying mechanisms are not specific to these numbers. The essential ingredients are type indispensability, replaceability, and rivalry. We anticipate that any model embedding these features will generate agglomeration. A full generalization of our framework is left to future research.

Our equilibria are not unique. All three economies in the 2×2 case feature a continuum of them, whereas only the portable endowment economy does so in the 4×4 case. Agglomeration can be augmented by the introduction of scale economies, externalities, amenities or imperfect mobility to further refine them.

It was suggested in the review process that our model might offer a theoretical foundation for the empirical literature on spatial sorting by skill and income. One possible approach would be to interpret endowments as the time endowments of high- and low-skill workers and to modify their distribution to distinguish landlords from renters. While we were able to identify equilibria featuring spatial sorting by both skill and housing tenure in this modified setting, we believe that fully developing this line of inquiry is best left to future research. Ideally, such extensions would incorporate inter-regional trade, dynamic considerations, and informational frictions (see, for example, Berliant and Yu [BY15] for a related framework).

One of the key assumptions is that consumers have preferences for goods they are not endowed with. This is ensured by the Inada condition that the Cobb-Douglas utility function satisfies. Without this assumption, the disassortative tendency would diminish, much like how production or non-portable endowments reduce the need to co-reside with other types. As a result, we expect agglomeration forces to weaken in this case.

Our model does not entail any externalities. The equilibria are Pareto efficient. Type complementarities are externalities of a pecuniary nature, that is, through market-mediated interactions. Beckmann [Bec76], and Mossay and Picard [MP11] also generate agglomeration from the consumers' side, but through social-interaction externalities. In Beckmann's framework, each individual's utility depends not only on their own choice but also on the average proximity to others, capturing a citywide social interaction effect. Mossay and Picard refine this idea by allowing the strength of interaction to decay with distance, so agents benefit more from being close to particular others. This spatial attenuation generates endogenous clustering even when preferences and endowments are homogeneous.

In our setting, by contrast, the value of interaction is priced in equilibrium, so what would appear as social-interaction utility in those models manifests instead as changes in excess demand. Incorporating their type of social-interaction term into our framework would likely amplify spatial sorting, making agglomeration patterns even more pronounced.

We are not intent on overriding the existing body of knowledge about producer-driven agglomeration. Rather, we cast light on the role heterogeneous consumers play in generating agglomeration, which, combined with other forces, should illustrate a more realistic mechanism behind agglomeration. An important open empirical question is: What percentage of agglomeration results from each force, including this one? There are welfare and policy implications due to the market failures embedded in other forces but not in ours.

A Appendix

A.1 Related Literature

We discussed the literature closely related to the current article in Section 1.3. This appendix relates our work to a broader context of agglomeration.

Whereas its impact on the economy is significant, modelling agglomeration poses some challenges. As we quoted at the beginning, agglomeration cannot be formed out of a standard setting. It usually requires exacting groundwork. Scale economies count among the most extensively studied sources of agglomeration. The New Economic Geography forgoes perfectly competitive markets. Increasing returns to scale, paired with low transport costs, give rise to agglomeration. Heterogeneity in preferences or endowments does not play a part in it because consumers are identical in this class of models. See Fujita and Thisse [FT13] for a comprehensive review.

In studying agglomeration, we typically assume that economic outcomes are non linear in population. The 10th and the 100th in-migrants contribute differently to the receiving city's economy. Their entry may manifest its impact within the industry they work in or spread across other industries that the city hosts. Jacobs [Jac69] advocates for the latter, namely, urbanization over localization externalities to promote urban growth. While her point of view pertains to the composition of producers, the current article examines the composition of consumers in a region. Externalities in our model are of a pecuniary nature.

Indeed it is useful to disaggregate agglomeration and examine what it is composed of. Agglomeration is exemplified by a non-uniform distribution of economic activities or population; spatial sorting is evidenced by varying compositions of heterogeneous agents by location. An economy may feature agglomeration without spatial sorting, and spatial sorting may take place without agglomeration.[22] Nevertheless, the analysis of sorting patterns enables us to identify the root cause of agglomeration in our model.

[22] Figure 2(a) visualizes these situations in an economy consisting of two types.

Agglomeration is not an exclusive product of firm-oriented factors. There are spatial models that examine mechanisms that give rise to agglomeration exclusive of scale economies. Berliant and Wang [BW93] identify the conditions under which a city or cities emerge as a place to trade location-specific commodity endowments. Berliant and Konishi [BK00] further introduce production to the framework. Their agglomeration also arises from gains from trade made possible through endogenously formed marketplaces, connected by a mass transport system if there are more than two of them. Both their and our models capitalize on preferences for a variety of goods in generating agglomeration rather than increasing returns to scale. The current article can be thought of as a distilled version of the antecedent works. The same technology is universally available and endowments are either evenly distributed or move with their owner. Regions are ex ante identical and cannot be a cause of agglomeration (see Section 3.3). Furthermore, as shown in Section 3.5, there are no gains from inter-regional trade. Agglomeration is purely an outcome of type indispensability, replaceability and/or rivalry.[23]

Mossay and Picard [MP11] also consider the emergence of agglomerations without imposing specific technology. Consumers derive utility from land and social interactions, whose net benefit fades with distance. Social interactions are not traded in markets and cause spatial externalities as they depend on the distribution of population. In our setup, the only way consumers interact with each other is through market transactions. Certain types benefit from the co-presence of other types as they increase the availability of a desired endowment, but they pay the price for it. Our equilibrium is thus Pareto optimal.

While our model features spatial sorting, it differs from Tiebout's [Tie56] foot voting, where relocation is driven by local public goods. Here, sorting is instead motivated by the spatial distribution of endowments and types. For example, one region may have higher production levels aligned with one type's consumption profile, while another preserves endowments instead of converting them into another commodity, matching another type's preferences. This sorting arises solely from market transactions, without externalities, and likewise differs from Rosen [Ros79] and Roback [Rob82], which rely on urban amenities that create exogenous asymmetry between regions.[24]

A.2 Degenerate Distribution

Degenerate distributions, where one region is completely vacated, cannot be realized when endowments are not portable. Any allocation leaving some endowments unconsumed can always be Pareto improved by relocating consumers to the vacant region to make use of them. Alternatively, their price is zero. By contrast, if all endowments are portable, complete regional vacancy becomes possible. Suppose $\lambda = \mathbb{1}$. This constitutes an equilibrium because the proof in the second part of Proposition 4.2 applies to the case

[23] Also of note, our model has only two discrete locations whereas the cited works build on a continuum.

[24] See Section 3.3 for a detailed discussion of this topic.

where $c = 1$. The equilibrium price vector in region a coincides with that of E^{1R}. In this case, utility levels do not change between $\lambda = \mathbb{1}$ and another equilibrium with the same price vector but $\lambda \neq \mathbb{1}$. As noted at the beginning of Section 2, there is no inherent preference for location. Region b will be left empty without residents or endowments.

A.3 2×2 Pure Exchange Economy

Section 3.1 presents the abridged production economy in E^{1R}, which, as noted in Section 1.2, entails both type dispensability and replaceability. We now present its pure-exchange counterpart, where the absence of production removes type replaceability, allowing us to isolate its effect.

The equilibrium is unique as in the production economy. The difference in equilibrium values between the production and pure exchange economies is $\Delta x := x\,(\text{production}) - x\,(\text{pure exchange}) = \begin{bmatrix} 0 & +247.0 \\ -41.2 & 0 \end{bmatrix}$, $\Delta\pi = \begin{bmatrix} -0.5304 & +0.5304 \end{bmatrix}^\top$, $\Delta\pi^\top w = \begin{bmatrix} -26.6 & +26.6 \end{bmatrix}$, and $\Delta u = \begin{bmatrix} -23.0 & +159.5 \end{bmatrix}$. Production works in favor of type 2, who prefer their endowment of commodity 2 converted into commodity 1 either through exchange or production. The terms of exchange are $\pi^2/\pi^1 = 0.4853$ in pure exchange as opposed to $\pi^2/\pi^1 = -A_5^1/A_5^2 = 6$ in production, underpinning type 2's utility gain.

In pure exchange, type 2 consumes $x_2^1 = 21.74$, all of which come from exchange with type 1. With production, $x_2^1 = 268.75$, of which only 8.1% comes from type 1, the reminder from the firm (ultimately via sales of w_2^2).[25] Namely, type 2's reliance on type 1 falls from 100% to 8.1% once type 1 becomes replaceable rather than indispensable. This percentage can be modified by the technology adopted, and in particular, by the ratio A_5^1/A_5^2.

Table 4 summarizes sorting patterns. In two-region economies, the equilibrium price

	for type j	type $k(\neq j)$ is	j's sorting	overall sorting	agglomeration
E^P	1 (or 2)	2 (or 1) is indispensable	disassortative	disassortative	exists
E^{NP}	1 (or 2)	2 (or 1) is rivalrous	assortative	neither*	none
E^{MP}	1	2 is rivalrous	assortative	neither	none
	2	1 is indispensable	disassortative		

Table 4. Spatial sorting by endowment portability in pure exchange economies. This corresponds to Table 3 for production economies. *Both types have assortative tendencies, but λ_j has to be .5 in the end to redress regional utility imbalances.

[25]Of $w_2^2 = 50$, type 2 sells 3.62 to type 1, 41.17 to the firm, and reserves the remaining 5.21 for their own consumption.

and allocation are the same as those of E^{1R} above. Without replaceability and assortativeness associated with it, the set of equilibria collapses to the one without spatial sorting as appears in Figure 2(c). Furthermore, in E^{NP} and E^{MP}, λ has to be $.\mathbb{5}$ regardless of sorting tendencies to equalize utility between two regions.

A.4 Limiting Factors of Distribution in E^P

If $z_j(\pi) - Ay$ were $\mathbb{0}$ for any j, then combining $z_j(\pi)$ with any arbitrary weight λ would make an equilibrium as it did in Section 3.2.[26] Figure 4 shows the minimum norm possible of excess demand when there is only type j in E^{1R}. None of them reaches zero on their own over $\Pi^{\perp}$. They do so only when aggregated with an equal weight $\mathbb{1}$ (in broken line) or with an equilibrium weight λ. If this was a 2×2 economy in Section 3.2, type 2 would reach zero at $\pi^1 = \frac{1}{7}$ on their own, resulting in a wide array of equilibrium λ.

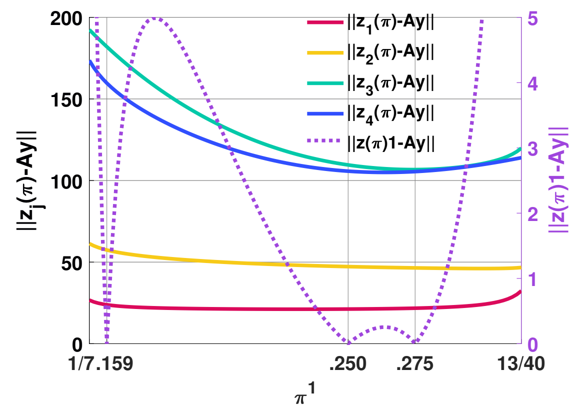


Figure 4. Minimum norm possible.

A.5 Equilibrium Prices in E^P

Expansion of a set of λ from $\{\mathbb{0}, \mathbb{1}\}$ of E^{1R} to $[0, 1]^4$ of E^P unleashes many market-clearing price vectors outside Π^{1R}, but most of them only clear the markets in one region. Figure 5 represents excess demand in region b that corresponds to Figure 1(b) in region a. Region a features one intra-regional equilibrium price vector in Π^{1R} and two outside Π^{1R}. The latter two will not make an inter-regional equilibrium because they only clear the markets in region a, but not in b.

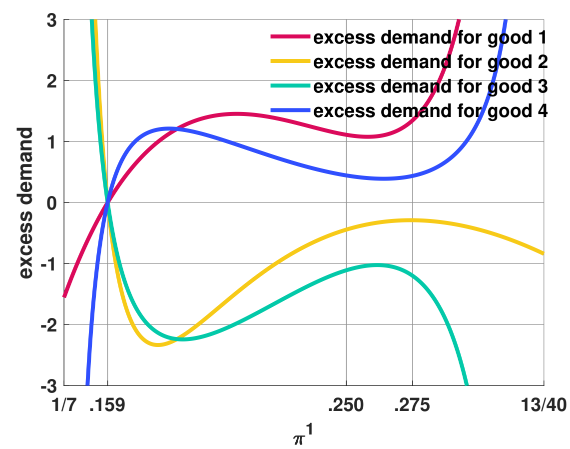


Figure 5. Excess demand in region b.

A.6 Equilibrim Prices in E^{NP}

The following is an E^{NP} counterpart to Proposition 4.2:

Proposition A.1 Equilibrium Prices in E^{1R} and E^{NP}

Suppose $\mu = .\mathbb{5}$. Let Π^{NP} be a set of equilibrium prices in E^{NP}. Then $(\pi, \pi) \in \Pi^{NP} \Leftrightarrow \pi \in \Pi^{1R}$.

Proof. The proof of Proposition A.1 is similar to that of Proposition 4.2 except that c needs to be equal to .5. At any $\pi \in \mathbb{R}^I_{++}$, optimal bundles $x(\pi)$ in E^{1R} and $x(\pi, \pi)$ in E^{NP} share

[26] Barring non-negativity constraints on y.

the same value because the budget constraint in E^{NP} does not involve λ nor μ. Income $\pi \cdot (\mu \circ w_j) + \pi \cdot \big([\mathbb{1} - \mu] \circ w_j\big)$ in E^{NP} reduces to $\pi \cdot w_j$ in E^{1R} for any j.

($\Rightarrow$) Consider a pair of price vectors $(\pi,\ \pi) \in \Pi^{NP}$. The material balance in each region is $x(\pi,\ \pi)\lambda^\top - \mu \circ (w\mathbb{1}) = Ay_a$ and $x(\pi,\ \pi)(\mathbb{1}-\lambda)^\top - (\mathbb{1}-\mu)\circ(w\mathbb{1}) = Ay_b$. Aggregate them and replace $x(\pi,\ \pi)$ with $x(\pi)$ to obtain $x(\pi)\mathbb{1} - w\mathbb{1} = A(y_a + y_b)$. Then the material balance in E^{1R} can be met by setting $y^{1R} = y_a + y_b$. Therefore, $\pi \in \Pi^{1R}$.

($\Leftarrow$) Consider some $\pi \in \Pi^{1R}$. The material balance is $x(\pi)\mathbb{1} - w\mathbb{1} = Ay^{1R}$. Let $\lambda = .\mathbb{5}$ and set $y_a = .5y^{1R}$ and $y_b = .5y^{1R}$. Multiply both sides of the material balance by .5 and replace $x(\pi)$ with $x(\pi,\ \pi)$ to obtain $x(\pi,\ \pi)\lambda^\top - \mu \circ (w\mathbb{1}) = Ay_a$ and $x(\pi,\ \pi)(\mathbb{1}-\lambda)^\top - (\mathbb{1}-\mu)\circ(w\mathbb{1}) = Ay_b$. Thus, the material balance is met in each region under $(\pi,\ \pi)$ by setting $\lambda = .\mathbb{5}$. Furthermore, $u_j\big(x_j(\pi_a,\ \pi_b)\big) = u_j\big(x_j(\pi_b,\ \pi_a)\big)$ for any j because $\pi_a = \pi_b$. Therefore, $(\pi,\ \pi) \in \Pi^{NP}$. □

Remark. The equilibrium is **not** scalable.

A.7 Pairwise Representation of Equilibria

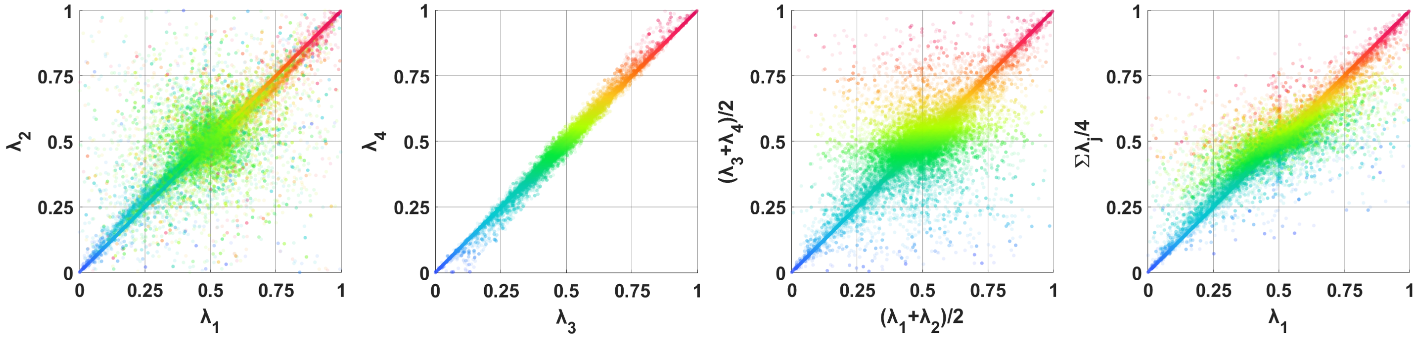
A.7.1 Portable Endowments (E^P)

This appendix provides a detailed discussion of sorting patterns summarized in Section 4.1. Figure 6 lists inter-regional equilibria sorted by price, which corresponds to Figure 3(a). We are interested in distribution λ in equilibrium. This contains four entries, $\lambda_1, \cdots, \lambda_4$. We represent this quadruple as pairwise projections in four plots at each price, since assortativeness is defined only between two types. We then interpret the patterns using the classifications introduced in Figure 2(a). Each dot represents one equilibrium, colored according to its value of λ_4 to track correspondence among four plots. As in Figure 3(a), a dot in a warmer color represents an equilibrium where λ_4 is higher, and a dot in a cooler color contains lower λ_4.

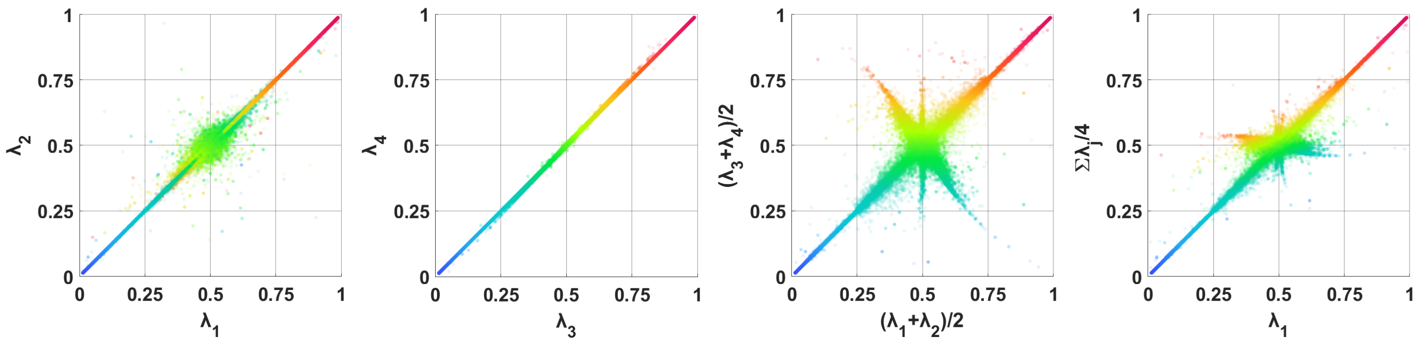
The first two plots represent equilibria with λ_1 or λ_3 on the horizontal axis and λ_2 or λ_4 on the vertical axis. These plots enable the assessment of assortativeness between two types.

The third plot visualizes the collective behavior of types 1 and 2 combined, in relation to types 3 and 4 combined. If the dots appear on the northeast or southwest quadrant, types 1 and 2 welcome the company of the other two types and thus they collectively behave disassortatively, yielding a low Theil index; otherwise, they sort into a different region than the other two types, resulting in a high Theil index.

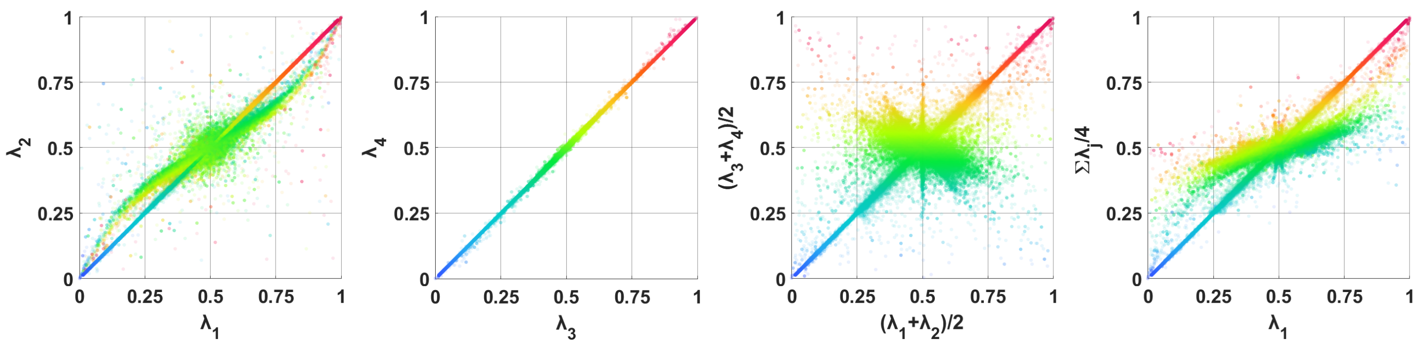
The fourth plot demonstrates the degree of agglomeration at each equilibrium. The fraction of residents who live in region a, $\lambda\mathbb{1}/4$, appears on the vertical axis. We use λ_1 on the horizontal axis to facilitate comparison with the three plots to the left, though any λ_j



(a) $\pi^1 = .159$.



(b) $\pi^1 = .250$.



(c) $\pi^1 = .275$.

Figure 6. Inter-regional equilibria by price in E^P. Each equilibrium is colored according to the value of λ_4 to show correspondence among plots at each price.

will serve our purpose. Dots located far from .5, either upwards and downwards, signify large agglomeration.[27]

In E^P, types 1 and 2 are mutually replaceable. In this example, they lean towards disassortative sorting, i.e., they prefer exchange over production (as shown in the first column of Figure 6). Their co-location is driven by **demand**. In contrast, types 3 and 4

[27]The 45° lines in these pairwise plots reflect symmetric equilibria of the form $\lambda = c\mathbb{1}$, $c \in (0, 1)$, which exist in E^P by Proposition 4.2. In such equilibria, all types are distributed across regions in the same proportion, so each pairwise projection lies on the diagonal.

are indispensable to each other. They also behave disassortatively (second column), but for a different reason: they seek to use each other's endowment mostly as an input in the production of commodities 1 and 2. Their location choices are therefore driven by **derived demand** rather than demand.[28]

Types 1 and 2 have a choice between exchange and production. Exchange does not involve co-location of types 3 and 4, whereas production does, which has a knock-on effect that was not visible in the 2×2 economy.[29]

Type 1 is replaceable to type 3. If type 3 opts for exchange and brings type 1 into their region, type 1 will also attract type 2 as noted above. Type 3 then has to compete with type 2 for consumption of commodity 1.

Switching to production avoids this route, but still requires type 2's endowment of commodity 2, again leaving less commodity 1 for type 3.

Type 2 is replaceable to type 3 by definition. However, they appear rivalrous to type 3 if they take commodity 1 into consideration.

Ultimately, whether types 1 and 3 sort into the same region depends on the net benefit of type 2's co-presence to type 3, which in turn depends on the price vector. As the third column of Figure 6 shows, interactions between types 1-2 and types 3-4[30] are sensitive to the price vector and can be assortative or disassortative. Accordingly, the intensity of agglomeration also varies with prices (last column).

A.7.2 Non-Portable Endowments (E^{NP})

Figure 7 represents the equilibria in E^{NP}. As in the 2×2 case, type indispensability disappears when endowments are not portable. However, unlike the 2×2 economy, disassortative behavior is present (first and second columns in Figure 7). Previously, an incoming type 2 displaced type 1 due to rivalry over fixed endowments. Here, an incoming type 2 still displaces someone but not necessarily type 1, as there are two more types in the economy. With more types than regions, some disassortative sorting is inevitable even without indispensability.[31]

Types 1 and 2 tend to agree on how much of commodities 3 and 4 should be converted into commodities 1 and 2, as do types 3 and 4. The former pair's demand for commodities 3 and 4 is small compared to the latter pair's as shown in Section 2.[32] As a result, their

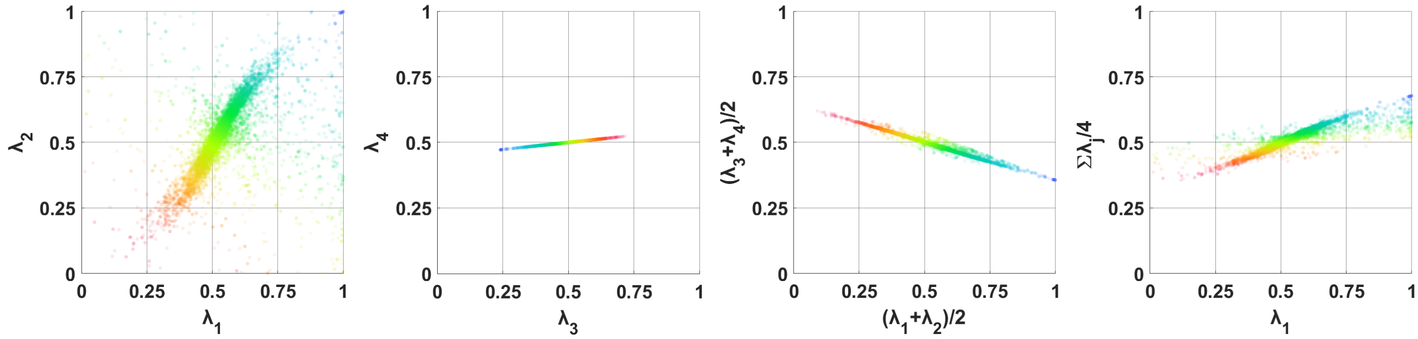
[28]We may interpret w_3^3 and w_4^4 as time endowments of high- and low-skill workers, respectively. Since neither of them is producible, both types are indispensable to each other. Therefore, they tend to sort into the same region. However, their co-location is not to consume each other's labor per se, but to combine their endowments in the production of commodities that require both.

[29]Appendix A.8 examines the case without production to isolate the role of replaceability.

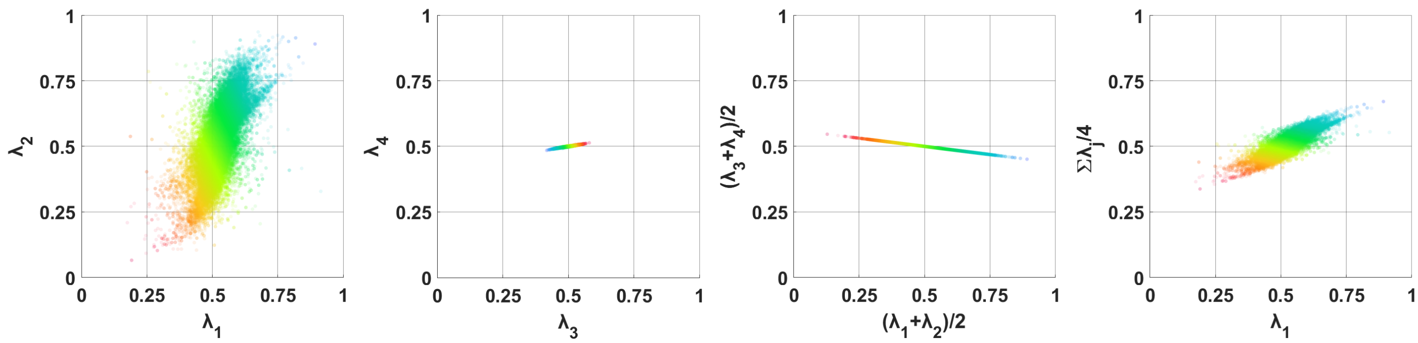
[30]The same tension that exists between types 2 and 3 also holds between types 1 and 4. For clarity, we group types 1 and 2 together against types 3 and 4 in the display.

[31]While all pairs can sort disassortatively, not all can sort assortatively. At least one region must host more than one type to make an allocation feasible.

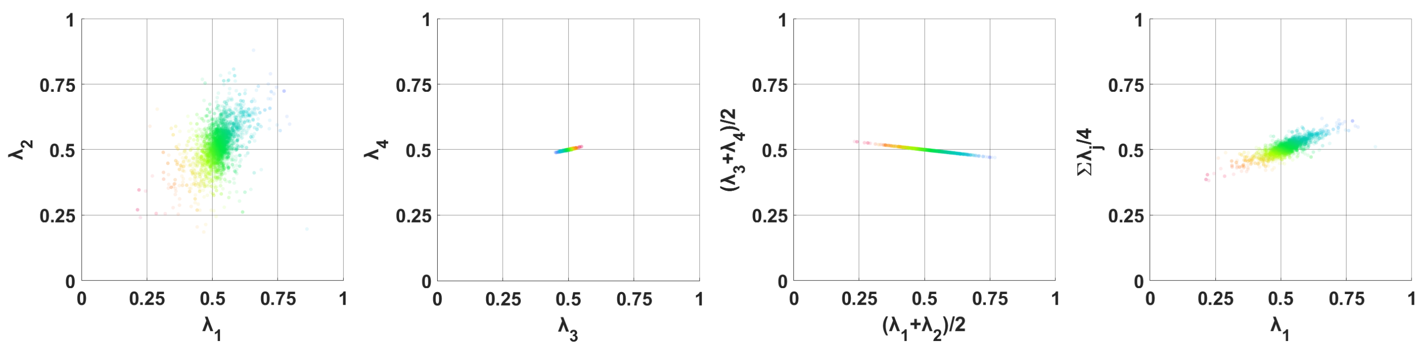
[32]Indeed, y_a is large when λ_1 and λ_2 are large.



(a) $\pi^1 = .159$.



(b) $\pi^1 = .250$.



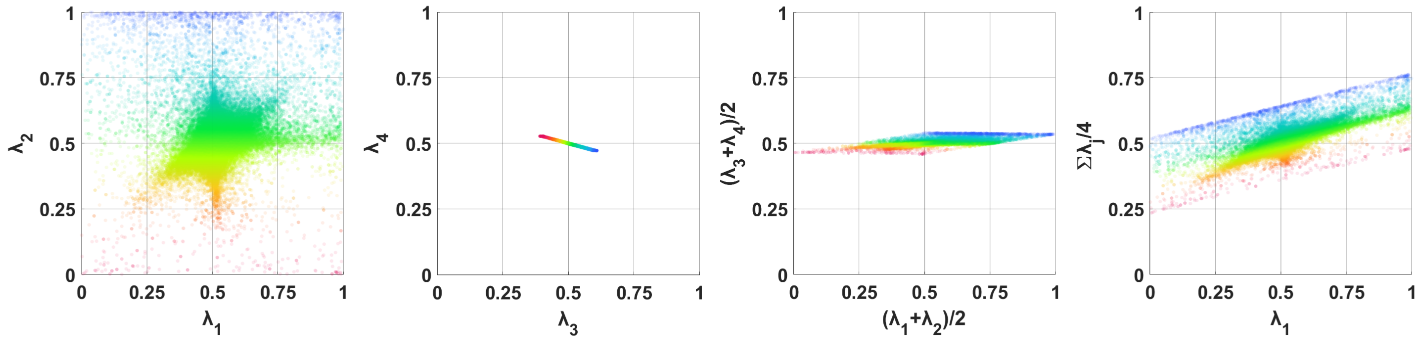
(c) $\pi^1 = .275$.

Figure 7. Inter-regional equilibria by price in E^{NP}. Each equilibrium is colored according to the value of λ_4 to show correspondence among plots at each price.

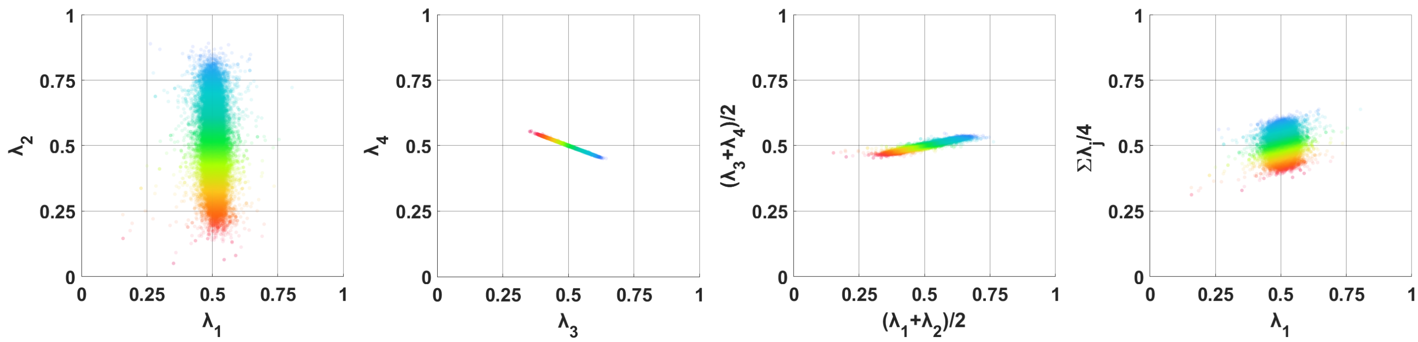
collective behavior tends to be assortative against types 3 and 4 as shown in the third column. Here, type rivalry still exists and leads to assortative behavior but discordance in the preferred level of production creates disassortative behavior between certain types. Due to the presence of assortative behavior, agglomeration is weaker than for E^P. Overall, agglomeration increases when regional differences in type compositions widen (i.e., with a high Theil index), as shown in Figure 3(b).

A.7.3 Mixed Portability (E^{MP})

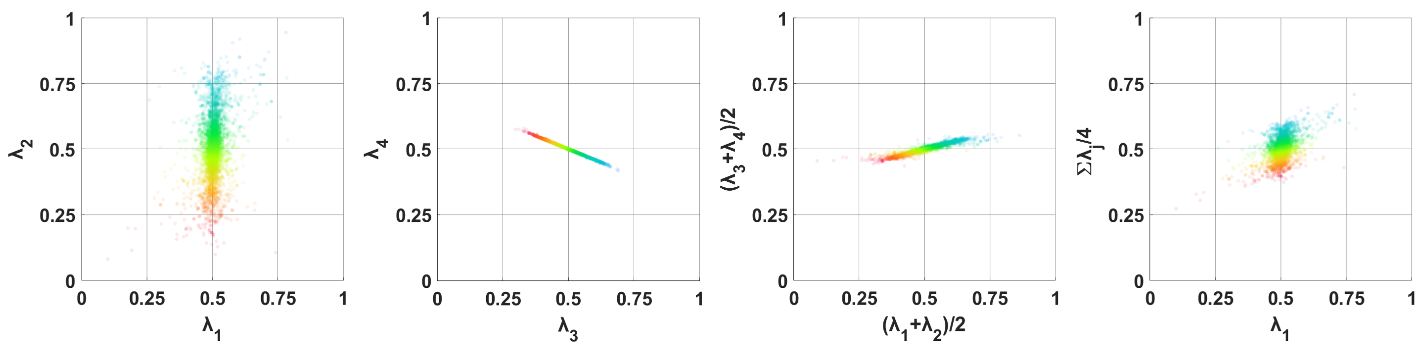
Figure 8 lists equilibria in E^{MP}. Unlike in its 2×2 counterpart in Section 3.4, not only type



(a) $\pi^1 = .159$.



(b) $\pi^1 = .250$.



(c) $\pi^1 = .275$.

Figure 8. Inter-regional equilibria by price in E^{MP}. Each equilibrium is colored according to the value of λ_4 to show correspondence among plots at each price.

replaceability and rivalry but also indispensability are present because of commodity 3, which is portable and not producible (as opposed to commodity 1, which is portable but producible). Types 2 and 3 behave disassortatively (third column in Figure 8); types 3 and 4 behave assortatively (second column). As a result, the intensity of agglomeration generally falls between that of E^P and E^{NP}.

In producing commodity 1, which types 2 and 3 prefer to consume, the firm employs a high volume of endowed commodity 3. Type 2 benefits from co-locating with type 3 for ease of procurement. Type 3 is therefore indispensable to type 2 for derived demand as it was the case between types 3 and 4 in Appendix A.7.1.

By the same token, type 4 prefers commodity 2. Its production requires in-migration of type 3 for portable endowments of commodity 3, which, in fact, puts them at odds for consumption of commodity 2 as type 3 consumes it as well. Consequently, type 4 may prefer to sort away from type 3. Whereas type 3 is indispensable to type 4 as a source of commodity 3 used to produce commodity 2, type 4 can still obtain commodity 2 without type 3, making type 3 appear replaceable in that respect.[33] Put differently, type 3 is not interchangeable with the firm (hence indispensable), but they are interchangeable with type 2 (hence replaceable in effect). Type 3's demand for commodity 2 is increasing in π^1, so type 4's assortativeness intensifies with π^1 as shown in the second column of Figure 8.

Their repulsion waters down the agglomerative force created from interaction between types 2 and 3, but still breaks symmetry (see Proposition 3.2).

These opposing forces are reflected in the behavior of the Theil index reported in Figure 3(c). Since assortative and disassortative sorting patterns coexist across different type pairs, regional differentiation is only partial, and the Theil index typically takes intermediate values. As a result, agglomeration in E^{MP} does not vary monotonically with the index.

As can be seen, the straightforward link between indispensability and disassortative sorting no longer holds. Production still substitutes for migration, but it requires three inputs with varied levels of impact on local demand. In 4×4 economies, sorting outcomes depend on the interplay between multiple, overlapping relationships across different commodities. A full account of these forces requires moving beyond simple pairwise analysis to consider the joint effects of type interactions, endowment portability, and production linkages.

A.8 4×4 Pure Exchange Economies

This appendix presents equilibria in 4×4 pure exchange economies, paralleling Appendix A.3 for 2×2 economies. The values below are for E^{MP}, which corresponds to those reporeted in Section 4.3. The differences, denoted by Δ, represent the changes from

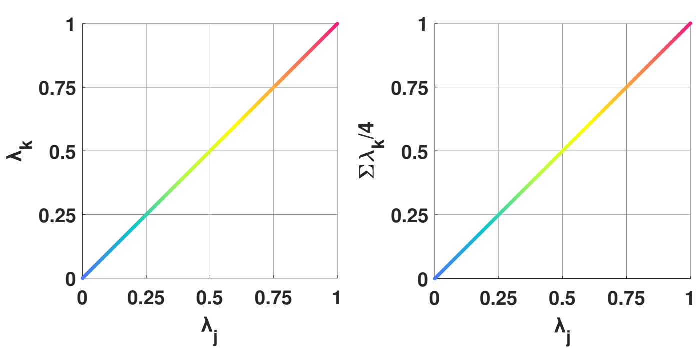
[33] In the 2×2 E^{MP} economy, this conflict did not arise because production and exchange were fully interchangeable. In the 4×4 setting, production of commodity 2 requires in-migration of type 3, increasing competition for commodity 2, whereas exchange does not.

the pure exchange to its production counterpart. The equilibrium price vector is unique.

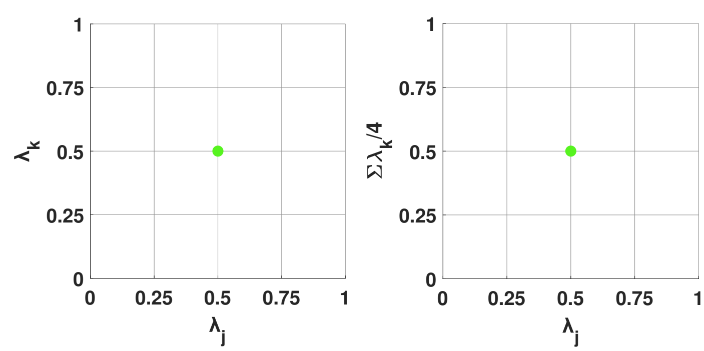
$$
\begin{aligned}
\Delta\pi_a = \Delta\pi_b &= \begin{bmatrix} -0.492 & -0.0792 & +0.0328 & +0.539 \end{bmatrix}^\top \\
\lambda &= \begin{bmatrix} .5 & .5 & .5 & .5 \end{bmatrix} \\
\begin{bmatrix} \lambda\mathbb{1} & 4-\lambda\mathbb{1} \end{bmatrix}/4 &= \begin{bmatrix} .5 & .5 \end{bmatrix} \\
\Delta x_a = \Delta x_b &= \begin{bmatrix} 0 & +45.71 & +46.69 & +82.61 \\ -26.84 & 0 & +10.95 & +216.79 \\ -214.30 & -49.74 & 0 & +12.04 \\ -98.86 & -24.66 & -0.38 & 0 \end{bmatrix} \\
\begin{array}{c}\text{aggregate net demand}_a \\ \left(x - w^P\right)\lambda^\top - \mu \circ (w^{NP}\mathbb{1})\end{array} &= \begin{bmatrix} 26-50 & 21.72 & 1.80 & 0.48 \\ 39.59 & 5 & 1.42 & 3.98 \\ 222.55 & 56.21 & 119-400 & 2.24 \\ 99.44 & 25.11 & 0.45 & 275 \end{bmatrix} \begin{bmatrix} .5 \\ .5 \\ .5 \\ .5 \end{bmatrix} - .5 \begin{bmatrix} 0 \\ 50 \\ 0 \\ 400 \end{bmatrix} (= \mathbb{0}) \\
\begin{array}{c}\text{aggregate net demand}_b \\ \left(x - w^P\right)(\mathbb{1}-\lambda)^\top - (\mathbb{1}-\mu) \circ (w^{NP}\mathbb{1})\end{array} &= \text{as above} \\
\Delta\text{income}_a = \Delta\text{income}_b &= \begin{bmatrix} -24.6 & -4.0 & +13.1 & +215.5 \end{bmatrix} \\
\Delta u_a = \Delta u_b &= \begin{bmatrix} -19.3 & +25.7 & +41.5 & +176.1 \end{bmatrix}.
\end{aligned}
$$

Production allows types 3 and 4 to convert their abundant commodities 3 and 4 into commodities 1 and 2, removing their reliance on the endowments of types 1 and 2.

Figure 9 visualizes equilibrium distributions in E^P, E^{MP} and E^{MP}, corresponding to Figures 6 to 8. Without replaceability, all types must present in equal numbers in each region to avoid regional utility imbalances in E^P, E^{MP} and E^{NP}. In addition, they need to split evenly in E^{NP} or E^{MP} for the same reason given in Appendix A.3.



(a) E^P.



(b) E^{MP} and E^{NP}.

Figure 9. Inter-regional equilibria in pure exchange economies. Each equilibrium is colored according to the value of λ_4 to show correspondence among plots.

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