Master's Thesis

Optimal Order Placement when trading bitcoins at orderbook level, using Reinforcement Learning

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Declaration

I hereby declare, that I am the sole author and composer of my Thesis and that no other sources or learning aids, other than those listed, have been used. Furthermore, I declare that I have acknowledged the work of others by providing detailed references of said work.
I hereby also declare, that my Thesis has not been prepared for another examination or assignment, either wholly or excerpts thereof.

Freiburg, July 4, 2017
Place, date

Axel Perschmann

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Abstract

English abstract

Zusammenfassung

Deutsche Zusammenfassung

Contents

1 Introduction

1.1 Motivation

In the domain of computational finance, much research is performed to find and improve algorithms that help maximize revenue. One possibility to maximize revenue is to minimize inevitably occurring costs.

In the first place, investors participating in exchange markets, must expect fees, charged by the respective market place organizer in return for granting access to their infrastructure. Additionally, there are hidden costs to be considered as well. Most markets function after the microeconomic supply and demand [**todo**] model, where a universe of opposing trading interests determines the current price of a commodity.

While trades with little capital (relative to the whole market liquidity) usually cause minor impact on the current market situation, large-scale investors must be cautious when it comes to order placement. Large orders can have a major impact on supply and demand, which leads to diminishing availability, worsening prices and as such this so called slippage must be seen as hidden costs.

Well considered trading strategies help large-scale investors to reduce their impact and avoid costly market turbulences by unwinding large orders of shares over time. Nevmyvaka et al. [3] applied reinforcement learning to optimally distribute the trading activity over a fixed time horizon.

1.2 Objectives

This thesis tackles the important problem of optimized trade execution, which frequently occurs in the domain of financial computing. In its simplest form, the problem is defined by a particular financial instrument (here: bitcoins), which must be bought or sold within a fixed time horizon, while minimizing the expenditure (share price) for doing so.

The scope of this thesis is to transfer Nevmyvakas [3] reinforcement learning approach from traditional stock markets with expensive, proprietary data sources, to the relatively young market of bitcoin trading and to improve it's general ability to solve the important problem of optimized trade execution. In contrast to their experiments this thesis builds on inexpensive, publicly retrievable bitcoin exchange data. Snapshots of the current market situation are retrieved on a low-resolution, minute-scale basis from the open bitcoin exchange platform Poloniex. As such the usability of the retrieved dataset remains to be shown.

Additional market features, describing the current market situation as well as historic market performance, are evaluated in terms of cost impact. An Orderbook Trading Simulator (OTS), which simulates the individual traders influence on the current market situation, is implemented and used in order to learn and evaluate various trading strategies.

1.3 Related Work

x

1.4 Contributions

- An orderbook trading simulator framework is presented which takes into account the individual traders influence on the current market situation.

1.5 Outline of Contents

The remainder of this thesis is structured as follows:
Section 2 gives a general introduction into the vocabulary of financial computing and the machine learning techniques employed, Section 3 describes the Orderbook Trading Simulator, developed within the scope of this thesis, and Section 4 covers the machine learning part. Section 6 closes with a conclusion and discussion.

2 Background

This chapter gives a general introduction into the vocabulary of financial computing and the machine learning techniques employed.

2.1 Exchange Markets, Bitcoins and Trading Basics

An exchange is a market, where financial instruments are sold and bought. It is typically organized by a broker, which can be both, an individual or a firm, executing buy and sell orders on behalf of dealers for a certain fee or commission. The respective prices are determined by the current market situations, in particular by supply and demand.

Specialized exchanges concentrate on certain sub-types of financial instruments and offer a trading venue for those willing to buy and sell these instruments. Some of them are listed below:

Stock Exchange Market A stock exchange or bourse provides companies access to investment capital in exchange to a share of ownership. Especially in times with notoriously low interest rates, investors tend to accept the greater risk of business development over a risk free, but faint investment, to grow their assets.
E. g. NASDAQ, Deutsche Börse, . . .

Commodity exchange market Commodity exchange markets allow for speculations with goods like oil, gold, corn, . . .
E. g. Eurex, . . .

Foreign exchange market Foreign exchange (short: forex) is considered the largest financial market in the world. The forex market is responsible for determining currency exchange rates.

Bitcoin exchange market x
E. g. Poloniex, . . .

Most modern markets are usually fully electronic.......

2.1.1 Ask and Bid

Most exchange markets function after the so called auction market model, where the exchange acts as a mediator between buyers and sellers to ensure fair trading. Here buyers can *bid* a price they are willing to pay for a certain number of shares and sellers can *ask* a price they are aiming to make with a number of shares. The highest of all bids is called the *bid price*, the lowest of all offers ist called the *ask price*. Together they represent the current price at which an instrument is traded.

2.1.2 Limit Order Book and Market Depth

A limit orderbook reflects supply (asks) and demand (bids) for a particular financial instrument. It is usually maintained by the trading venue and lists the number of shares being bid or offered, organized by price levels in two opposing books. Incoming orders are constantly appended to this highly dynamic list, while a matching engine cautiously resolves any inconsistencies (i. e. overlaps) between asks and bids by mediating between the involved parties.

It is usually not before the matching engine has arranged an actual trade, that a trading venue claims a certain percentage of the turnover as a service fee. To encourage active market participation, the pure submission, revision and cancelation of orders is typically free of charge.

	Amount	Type	Volume	VolumeAcc	norm_Price
31.00	200.0	ask	6200.0	8425.0	1.074533
30.00	50.0	ask	1500.0	2225.0	1.039871
29.00	25.0	ask	725.0	725.0	1.005208
28.85	NaN	center	NaN	NaN	NaN
28.70	200.0	bid	5740.0	5740.0	0.994810
28.50	100.0	bid	2850.0	8590.0	0.987877
28.00	300.0	bid	8400.0	16990.0	0.970546

Table 1: Exemplary snapshot of a limit orderbook for stocks of AIWC[1]

Table 1 shows a limit orderbook snapshot up to a market depth of 3, as seen by market participants. Here Alice offers 25 shares per 29$, Bob and Cedar offer 20 and 30 shares respectively per 30$ and David offers 200 shares per 31$.

Based on their trading needs, traders can typically choose between multiple levels of real-time market data.

Level 1 Market Data Basic informations only:
Bid price + size, Ask price + size, Last price + size

Level 2 Market Data Additional access to the orderbook.
Usually data providers display the orderbook only up to a certain market depth m, i. e. the lowest m asks and the highest m bids.

Level 3 Market Data Full data access.
Typically only accessible for the market maker.

1 Acme Internet Widget Company

2.1.3 Slippage

Slippage is defined as the difference between expected and achieved price at which a trade is executed. Slippage may occur due to delayed trade execution. Especially during periods of high volatility, markets might change faster than the order takes to be executed. Slippage is also liked to the order size, as larger orders tend to *eat* into the opposing book and are fulfilled at successively worse price levels.
Slippage can be both positive or negative, depending on the current market movements and must be taken into account by serious investors.

2.1.4 Order Types

Investors can execute orders of different types, of which the most common ones are described below:

Market Orders are the most simple form of orders. Here, the investor only specifies the number of shares he want's to buy/sell and the full order is executed immediately, at any price. Especially for large-scale traders or traders with level 1 data access only, these simple market orders are rather hazardous, since the achieved price can significantly differ from the expected price due to sparse supply and demand.

Limit Orders additionally feature a worst price, i. e. the highest price a buyer is willing to pay per share or, respectively the lowest price a seller is willing to make per share. Limit orders are immediately placed into the orderbook and (partially) executed, once the matching engine finds a corresponding trade in the opposing book.
Limit orders reduce the risk of slippage, but do not guarantee execution.

Hidden Orders are placed into the market makers internal orderbook, but not displayed to other market participants with level 2 market data access. They represent a simple solution to large-scale investors seeking anonymity in the market, aiming to obfuscate their trading intention from other market participants.

2.1.5 Trading strategies

An order placement typically originates from a carefully considered *trading strategy*. An *active* trading strategy buys and sells instruments frequently based on short-term price movements, whereas a *passive* trading strategy such as *Buy-And-Hold* believes in long-term price movements eventually outweighting any short-term fluctuations.

As the execution of trades typically implies trading costs and slippage, these have to be taken into account. Particularly active traders with a high order quantity and large-scale investors with high order volumes are concerned with this burden. The order type chosen has a major impact on speed of execution and slippage generated.

While *limit orders* reduce the risk of slippage, they do not guarantee full order execution. This leads to the important problem of *optimized trade execution*, which frequently occurs in the domain of financial computing. In its simplest form, the problem is defined by a particular financial instrument (here: Bitcoins), which must be bought or sold within a fixed time horizon, for the best achievable share price.

In [4] Nevmyvaka et al. introduce a Submit & Leave Strategy (S&L), which cleverly combines market and limit orders: After an initial limit order submission, the order is left on the market for a predefined time horizon, after which it's unexecuted part is transformed into a market order and thus executed completely. They later extended their strategy to a Submit & Revise Strategy (S&R) [3], where the order limit may be revised at discrete time steps, depending on trade progress and market changes.

2.2 Bitcoins

[placeholder]

Figure 1: bitcoin volatility
High frequency tick data, compared to minutely snapshots.

2.2.1 Marktreaktionen

"Bitcoin ist eine Währung, die äußerst sensibel auf Nachrichten reagiert. Begründet wird dies vor allem durch die Möglichkeit, ständig am Markt teilnehmen zu können: Es gibt keine zentrale Ausgabestelle mit geregelten Handelszeiten, an die man gebunden ist."

"Auch die Tatsache, dass viele Anfänger in Bitcoins investieren, führte bereits in der Vergangenheit zu den ein oder anderen Panikverkäufen. Wer sein Geld in Bitcoins investieren möchte, kann die meist lukrative Möglichkeit nutzen, sollte sich jedoch regelmäßig über Marktveränderungen informieren.

Da viele Investoren schnell auf Meldungen reagieren, kann es innerhalb von Stunden zu großen Kursverlusten oder Gewinnen kommen." https://www.btc-echo.de/bitcoin-trading-tipps-prinzipien-des-bicoin-handels_2015022502/

2.3 Supervised Learning

Supervised learning is a subdomain of machine learning, where a function is learned from labeled training data $\{(x_1, y_1), ..., (x_N, y_N)\}$. Each training sample maps a feature vectors $x_i \in X$ to a desired target value or label $y_i \in Y$. Target values may either be categorial, making the learning task a *classification* problem, or continuous, making the learning task a *regression* problem.

A supervised learning algorithms seeks to find a general function $g_w()$ (or it's parameters w), such that $g_w(x_i) \approx y_i | i \leq N$. The learned function should ideally avoid overfitting by finding a generalization to previously unseen data.

Markow Decision Process

Value Function and Bellmann Equation

Value Iteration

Q-Learning

2.3.1 Logistic Regression

bla

2.3.2 Random Forest

bla

2.4 Reinforcement Learning

Reinforcement learning is a subdomain of machine learning, where strategies are learned by an *agent* interacting with its environment. Rather than learning from labeled training data, the agent applies a *trial and error* pattern and exploits external rewards to find actions, maximizing his expected future reward.

2.4.1 Dynamic backward programming

bla

2.4.2 Tree-Based Batch Mode Reinforcement Learning

bla

2.4.3 Neural Fitted Q-learning

bla

3 Orderbook Trading Simulator

This chapter describes the Orderbook Trading Simulator (OTS) and it's underlying OrderbookContainers, implemented within the scope of this thesis. Fed with historic orderbook data it serves as a backtesting framework for testing out various trading strategies. The OTS provides detailed feedback in terms of trading progress, achieved prices and accrued costs.

3.1 Data Origin

Since typical financial data providers must make an earning from their treasures, they typically only deliver delayed market data on a complimentary basis. Investors dependent on real time or level 2 market data (see Section 2.1.2) are usually charged horrendous monthly subscription fees.

A costless alternative exists in open cryptocurrencies, like bitcoins (see Section 2.2). The digital asset exchange platform Poloniex [5] provides an open API for querying detailed market data in real time. As their push API, to receive live order book updates and trades, was rather error-prone and buggy when this project started, the decision was made, to query full orderbooks on a minutely basis.

On Nov, 10th 2016, 10:00 am, a daemon was started, to fetch orderbook snapshots up to a market depth of 5000 from Poloniex via HTTP GET requests (see Listing 1). The volume of recorded orderbook snapshots for nine distinct currency pairs[2] has since grown to roughly 100GB (as per 2017-06-20). This thesis is based on a condensed version of the currency pair USDT/Bitcoin.

Listing 1: Data fetched from Poloniex via HTTP GET request

```
# https://poloniex.com/public?command=returnOrderBook&currencyPair=USDT_BTC&depth=5000
{"asks" :[[ "705.450000" ,2.772181], [ "705.450196", 0.139212] ,["706.170000" ↩
    ,0.052838] , ... ], "bids":[["705.000000",0.158232],["703.700000" ,0.001250], ↩
    ... ], "isFrozen": 0, "seq": 63413296}
```

Figure 2 shows the price evolution of currency pair USDT/Bitcoin over the period of recording. Prices have burst from roughly 700$ to more than 2.000$. Red and green lines depict the percentual slippage induced by immediate market orders in buy and sell direction respectively.

2 Recorded currency pairs include USDT/BTC, BTC/ETH, BT/XMR, BTC/XRP, BTC/FCT, BTC/NAV, BTC/-DASH, BTC/MAID, BTC/ZEC

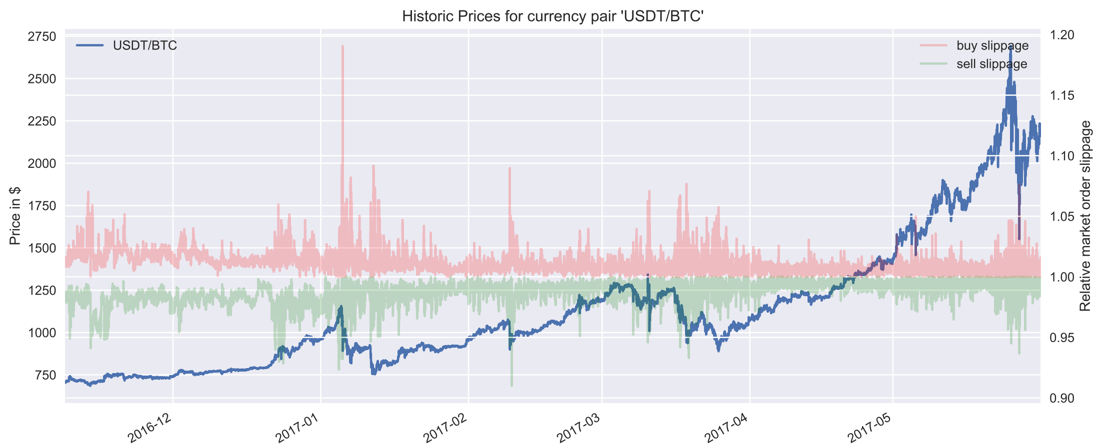


Figure 2: Historic center prices between Nov, 10th 2016 and May, 31 2017, as fetched from Poloniex.

3.2 Data preprocessing

The python `class OrderbookContainer` aggregates all informations contained in an individual orderbook snapshot. It enforces correct price ordering in the two opposing bid and ask books and provides additional methods for market visualization and feature extraction. To restrict wasteful memory usage, orderbook snapshots are condensed in several ways:

- Almost identically price levels are round to the second decimal and their respective order volumes merged.

$$\left.\begin{array}{r} 0.139212 * 705.450000 \\ 2.632969 * 705.450196 \end{array}\right\} = 2.772181 * 705.45$$

- Market depth is capped just above the threshold of 100 bitcoins, roughly corresponding to a market depth of 100-140 prices levels in both books. This threshold allows to simulate trades up to a market order price of 70.000 $ at any time throughout the whole recording period.
- Erroneous orderbook snapshots have been discarded. Occasional errors may occur, due to Poloniex API failures.

These measurements reduce the average individual orderbook snapshots size from 30KB to approximately 6.6KB. As for the december 2016, this results into 44.640 snapshots with a total size of 295MB instead of 1.35GB, clearly reducing the memory consumption.

Listing 2 shows the most important functions, provided by the OrderbookContainer class. OrderbookContainer instances are vigorously used by the OTS. Figure 3 shows a plain visualization of an individual orderbook snapshot.

Listing 2: OrderbookContainer

```
ob = OrderbookContainer(timestamp="2016-11-08T10:00",
                        bids=pd.DataFrame([200., 100., 300.],
                        columns=['Amount'], index=[28.7, 28.5, 28]),
                        asks=pd.DataFrame([25., 50., 200.],
                        columns=['Amount'], index=[29., 30., 31.]))
# Available methods
ob.plot(outfile='sample.pdf')  # plt.show or plt.savefig
ob.asks  # pd.DataFrame
ob.bids  # pd.DataFrame
ob.features  # returns a dict of precomputed features
ob.get_bid(), ob.get_ask(), ob.get_center()  # float
ob.get_current_price(volume=100)  # achievable cashflow by market order
ob.get_current_sharecount(cash=70000) # number of shares aquirable by market order
ob.compare_with(other_ob) # returns orderbook deltas used by the OTS
ob.enrich()  # computes Volume, VolumeAcc and norm_Price
ob.head(depth=3)  # returns the orderbook, capped at a market depth of 3
ob.plot()
```

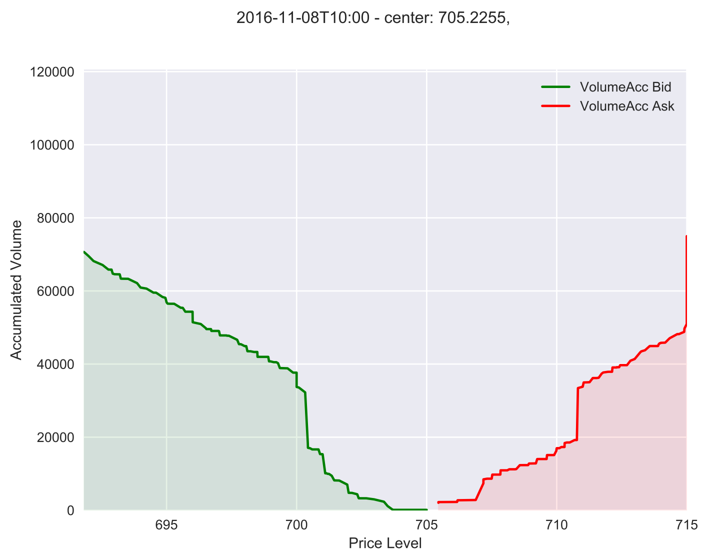


Figure 3: A simple visualization of an limit orderbook.

3.3 Simulator

The OTS framework serves as base for all preceding experiments and evaluations. Each simulator instance is fed with an array of subsequent OrderbookContainers (*orderbook windows*) and a targeted trading volume V, which it pretends to trade into cash or vice versa within a fixed time horizon H, according to an external strategy.

In the rare case of missing orderbook snapshots (see Section 3.2), the *real* time horizon may be larger than usual, since always H subsequent orderbooks are selected. By default, the OTS refuses to work with orderbook windows, whose actual length differs from the presumed length by more than two minutes.

Limit orders may be placed at predefined, discrete time steps within the trading horizon H. This is done though the simulators main interface method `trade(limit=...)`. The simulator is done, once the remaining trading volume is zero, which is enforced at the very last time point. Any remaining volume at *H-1* is transformed into a simple market order and executed immediately, at any price. Additional parameters control the simulators precise behavior:

volume : The targeted trading volume V.
Positive values indicate buy orders, negative values indicate sell orders.

consume : `'cash'` or `'volume'`
Defines whether `volume` should be interpreted as *cash* (goal: buy/sell shares for V dollars), or as *sharecount* (goal: buy/sell V shares).

period_length : `default=15`
Defines the duration at which a limit order is executed. After a trade has been placed, the simulator iterates over the next `period_length` orderbooks, before the results are reported and a reviewed order may be placed.

tradingperiods : `default=4`
Defines the number of trade reviews, that can be made within the time horizon $H =$ `period_length * tradingperiods`.

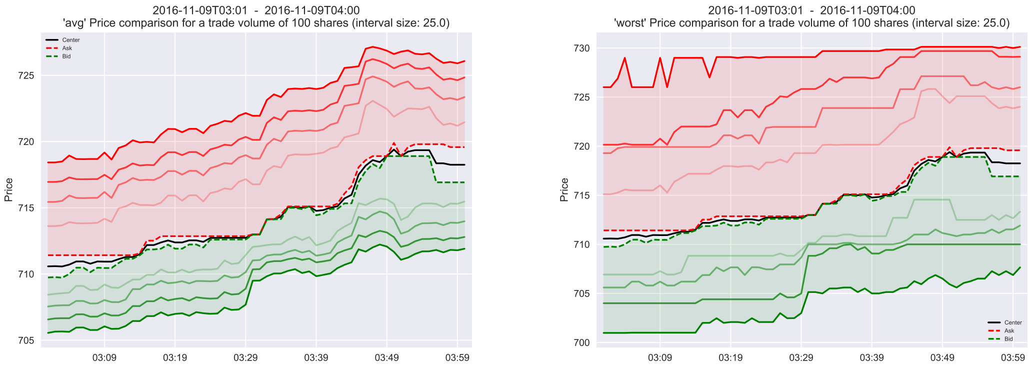
max_lengh_tolerance : `default=2`
Defines the accepted tolerance between actual and presumed trading horizon in minutes. Throws `ValueError`, if exceeded.

costtype : `default='slippage'`
Defines which of multiple cost functions to use in the reports returned.

3.3.1 Orderbook and strategy visualization

Figure 4 visualizes a 60 minutes long orderbook window, where the solid red lines mark the *average* (4a) and *worst* (4b) price, that has to be paid at a given time point, in case of an immediate market order of 100, 75, 50 and 25 bitcoins respectively. Analogously, the solid green lines represent achieved prices for sale orders of -25, -50, -75 and -100 bitcoins.

As can be seen in this graph, ask prices deviate more from the center price than bid prices. An plausible inference might be, that imbalances between demand and supply might serve as a valuable indicator for future price trends.



(a) Average market price.

(b) Worst market price.

Figure 4: An orderbook window over a period of 60 minutes.

3.3.2 Masterbook

During instantiation, the OTS creates a copy of the first orderbook, called the *masterbook*. Hereinafter executed trades do only affect this internal *masterbook*. The remaining orderbooks are then converted into *deltabooks*, containing only changes between subsequent orderbooks.

The OTS may be reset to its initial state at any time via `ots.reset()`. This avoids computational overhead, when testing out multiple strategies on the same *window*, as only *masterbook* and *history* are reset, while *deltabooks* need not be recomputed and *orderbooks* need not be retransferred. `ots.reset()` provides optional parameters for modifying the simulators start conditions. As such the simulator might be instructed to start at a `custom_starttime` or with a `custom_startvolume`, to simplify the exploration of possible strategies.

3.3.3 Trade execution

The simulated trade execution is triggered by an external call to `trade(limit=...)`. The OTS expects a `limit` and iterates over the next `period_length` orderbooks, matching all eligible orders. The `limit` represents the highest accepted price level for buy orders and the lowest accepted price level for sell orders respectively. If `limit=None`, a simple market order is performed.

In a first step, all eligible orders, bound by the given limit and the total trade_volume, are cut from the internal masterbook and pasted into the `ots.trade_history`, as such these orders are assumed be fulfilled. The `volume` and `cash` variables are updated accordingly. The simulator then moves to the next time point and adds the corresponding *deltabook* to the masterbook.

In case of order size reductions[3], negative order sizes may appear in the masterbook. They are silently dropped, assuming they where matched before they actually vanished from the market. This is a possible source of trouble, as this assumption can not be proven to be valid. As such, matching orders that are simultaneously updated to another price level, are virtually doubled. They are perceived as a new order, even though the responsible market participant has already realized an execution and no basis to submit another one.

(1) `master += diff(ob[t] - ob[t-1])`, drop negative share counts.
(2) perform trade: buy until given limit
(3) `master -= bought bitcoins`
(4) done if `volume==0` or $t == T - 1$ (`forced=True` a)

Figure 5: Figure describing the masterbook adjustments as a graph?!

The masterbook is then ready to be queried again. Any eligible *new* orders are cut from the masterbook and past into the `ots.trade_history`. After `period_length` steps, a detailed trading report, as shown in Table 2 is returned. The upper case columns represent internal variables and orderbook statistics observed at the particular period start. The lower case columns summarize the actual trade in terms of highest, lowest and average price achieved, traded volume and cash and observed costs.

	ASK	BID	CENTER	SPREAD	LIMIT	T	VOLUME	volume_traded	CASH	cash_traded	...	avg	forced	initialMarketAvg	low	high	cost
03:01	711.42	709.74	710.58	1.68	713.0	15	100.00	46.77	0.00	-33280.72	...	711.51	False	718.42	711.42	713.00	43.48
03:16	712.52	711.86	712.19	0.66	715.0	15	53.23	28.90	-33280.72	-20630.99	...	713.99	False	718.42	712.52	715.00	98.53
03:31	715.10	712.98	714.04	2.12	717.5	15	24.33	6.68	-53911.71	-4780.95	...	716.16	False	718.42	715.10	717.41	37.28
03:46	718.60	717.77	718.18	0.83	720.0	15	17.65	17.65	-58692.66	-12706.15	...	719.73	False	718.42	718.60	720.00	161.57

Table 2: Trading history, as returned after four consecutive calls of `ots.trade()`.

3.3.4 Visualization

A visual representation of the same trading strategy, which underlies Table 2, is shown in Figure 6. Here, the OTS was instructed to buy 100 bitcoins within a period of 60 minutes and called with four consecutive limits prices 713, 715, 717.5 and 720, which are shown as grey step function in the upper subplot.

The second subplot displays trade executions and declining trade volume over time and the bottom subplot displays induced costs. Expected costs conform to the respective accumulated costs, extrapolated to the full trade volume.

3 order size may be reduced, due to order fulfillment, order updates or order cancelations through the other market participants

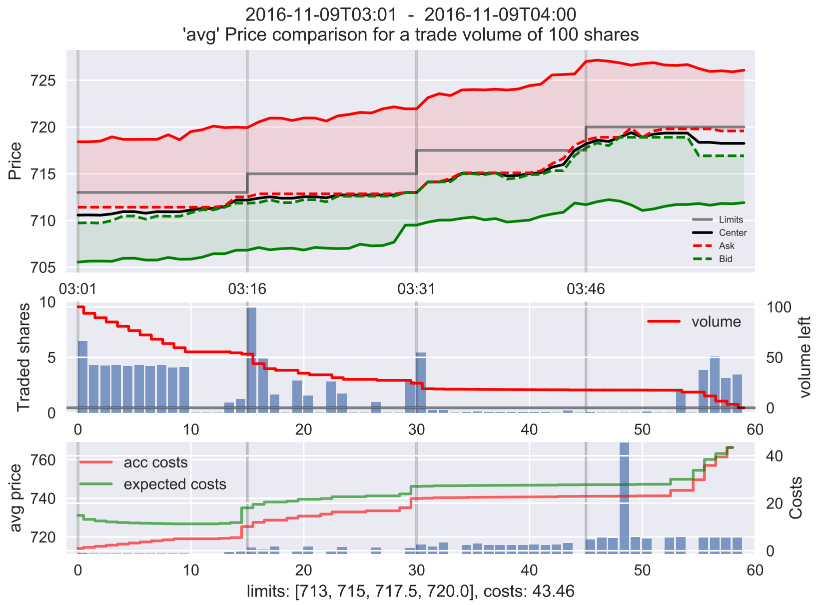


Figure 6: Visualization of an exemplary trading strategy.

3.3.5 Model correctness

The presented model does not account for market reactions, induced by the currently executed trading strategy. It moronically follows the market trend, as it would have evolved without any intervention. Some sources of troubles are pictures below:

- As stated in Section 3.3.3, the presented model can not distinguish between a limit order being removed from the market makers orderbook and a limit order essentially only being updated to another price level. In the latter case, the order presumably wouldn't have reappeared in the orderbooks after the simulator matched it.
- Other market participants typically monitor market activity thoroughly, which is particularly true for purely electronic markets of digital assets, like bitcoins. It is delusive to assume, that no other market participants or trading bots react to large orders, that eat significantly into the orderbook.

[placeholder]

Figure 7: sample of a *curious masterbook shape*
large part of order fulfilled, eaten deeply into the orderbook, deltabook brings in new orders close to original centerprice $\Rightarrow$ uncommon gap.

- Orderbooks on a minutely basis miss a great part of the markets volatility.
  In addition to orderbook snapshots, professional data providers typically grant access to market level 2 data (see Section 2.1.2) in form of log files as well. The log files consist of timestamped orderbook updates (typically of type *remove* and

modify), which allow the reconstruction of the orderbook for an arbitrary time point. As mentioned in Section 3.1, Poloniex push API, to receive live order book updates and trades, was rather error-prone and occasionally failed to report important orderbook updates. As the valid reconstruction of orderbooks is highly vulnerable to missing logs, inconsistencies arose and the decision was made, to query orderbooks on a minutely basis only.

- Hidden Orders, as introduced in Section 2.1.4, are not accounted for.

As a consequence, trading on the masterbook can only be seen as an rough approximation of true market behaviour. Curious masterbook shapes, resulting from the described simulation process, encourage to perform actual feature extraction on the original orderbooks, examining the market as it would have evolved without any interventions.

4 Reinforcement Learning Approach

This chapter describes an Reinforcement Learning (RL) approach, used to tackle the important problem of optimized trade execution. To a large extend, it is based on the RL formulation, as described by Nevmyvaka et al. [3], but modified in detail.

4.1 Original Algorithm

The original algorithm claims to find optimal limits from a discretized state space, describing trade progress (i. e. *remaining time* and *remaining inventory*) and market situation. While the obtained strategies achieved an impressive 50% gain over the more simple Submit & Leave Trading Strategy (see Section 2.1.5), they left some room for improvements. As their work was based on a rather large, proprietary dataset of 1.5 years of millisecond time-scale limit order data from NASDAQ, it was furthermore intriguing to evaluate its performance on a smaller, self-recorded dataset of limited resolution.

Nevmyvaka et al. [3] fused Q-learning and dynamic programming to learn a state-based strategy over the first year of their data in a brute force manner.

4.1.1 State space

The state space consists of various discrete variables, describing the current trade progress (*private variables*) and the current market situation as observable from orderbook data (*market variables*). The two private variables `time` and `volume` make the base for all subsequent experiments, while various additional market variables were enclosed to examine their impact on a valuable decision making.

As such, each `state` $s \in < time, volume, o_1, o_2, ... >$ forms a vector of at least two private variables, plus a variable number of market variables. More specifically, the following market variables were evaluated in terms of improvement over a state space based on two private variables only.

Bid-Ask Spread : spread between best bid price and best ask price.

Bid-Ask Volume Misbalance : volume imbalance between orders at the best bid price and the best ask price.

Immediate Market Order Cost : costs, if remaining volume would be executed immediately, at the current market price.

Signed Transaction Volume : signed volume of all trades executed within last 15 seconds. A positive value indicates more buy orders, while a negative value complies to more sell orders being executed.

All market variables were discretized into 0 (low), 1 (medium) and 2 (high), while the concrete category mapping process was not further described. Market variables are extracted from the original orderbooks, as if the market had evolved without our impact.

4.1.2 Action space

Actions define the level of trading aggression to be performed. An action $a \in \mathbb{R}$ defines the deviation between current best price and chosen limit price, as $bid + a$ (for buy orders) and $ask - a$ (for sell orders). As such, actions must cross the full bid-ask-spread to find matching orders in the opposing book. Large actions a result in larger fractions of the order volume being matched immediately. Smaller or even negative actions a help to prolong the actual execution, and may help to profit from opportune market movements.

In case of the market situation as shown in Table 3, a buy order with an aggressive action $a = 1.4$ would translate into $limit = bid + a = 28.7 + 1.4 = 30.1$. This limit would allow trading up to 75 shares instantaneously.

	Amount	Type	Volume	VolumeAcc	norm_Price
31.00	200.0	ask	6200.0	8425.0	1.074533
30.00	50.0	ask	1500.0	2225.0	1.039871
29.00	25.0	ask	725.0	725.0	1.005208
28.85	NaN	center	NaN	NaN	NaN
28.70	200.0	bid	5740.0	5740.0	0.994810
28.50	100.0	bid	2850.0	8590.0	0.987877
28.00	300.0	bid	8400.0	16990.0	0.970546

Table 3: Action $a = 1.4$ translates into $limit = 28.7 + 1.4 = 30.1$.

The employed number of selectable actions and their actual value range was not further specified.

4.1.3 Costs

Costs are defined as the slippage induced from the previously chosen actions. The baseline is given by the initial center price. The following formula is used to compute (partial) costs in terms of price deviation from the idealized case of buying all shares at the initial center price:

$$cost_{im} = (avg\_paid - initial\_center) * volume\_traded \tag{1}$$

$$initial\_center = (\frac{ask_t + bid_t)}{2}|t = 0 \tag{2}$$

Since the complete trade execution happens within a finite time horizon and full execution of the `volume` is mandatory, partial costs, as observed within the individual trading_periods, can simply be summed up without any discounting.

It is impossible to compute occurring costs in advance, as they depend on how the orderbook evolves within the subsequent trading_period.

4.1.4 Backward learning

In order to learn the optimal limit for each possible situation, orderbook windows are examined in a backward, brute-force manner as described in Listing 3. Each orderbook window from the training data set is sampled $T * I * L$ times, where T is the number of performed limit revisions, I is the number of discrete volume states and L is the number of available actions.

Listing 3: Brute-Force strategy learning approach as described in [3].

```
Optimal_strategy(V, H, T, I, L)
    For t=1 to T
        While(not end of data)
            Transform (orderbook) -> o_1 ... o_R
            For i =0 to I
                For a = 0 to L
                    Set x = [t, i, o_1, ..., o_R]
                    Simulate transition x -> y
                    Calculate immediate cost_im(x, a)
                    Look up argmax cost(y, p)
                    Update cost([t, v, o_1, ..., o_R], a)
    Select the highest-payout action argmax cost(y, p) in every state y to output ↩
        optimal policy
```

While the algorithms running time depends solely on the resolution of the two private variables, the chosen action space and the size of the training data, it is approximately independent of the number of market variables chosen. The transition simulation was not further described, but for this thesis the model described in Section 3.3.3 is utilized. The cost update rule is given below:

$$cost(x_t, a) = \frac{n}{n+1} cost(x_t, a) + \frac{1}{n+1}[cost_{im}(x_t, a) + \arg\min_{p} cost(x_{t-1}, p)] \tag{3}$$

The algorithm assumes the individual trading_periods to be of an (approximately) Markovian nature, where the optimal action to choose at state x_t with $t = \tau$ is completely independent of actions chosen previously ($t \geq \tau$).

As such, the state-action function can be computed inductively via dynamic programming. In a first round, expected costs for all actions in states being immediate predecessors of end states (i. e. $t = 1$) are computed according to Equation (3). Figure 8 visualizes the resulting optimal costs (right) and corresponding actions (left) after the first round has finished.

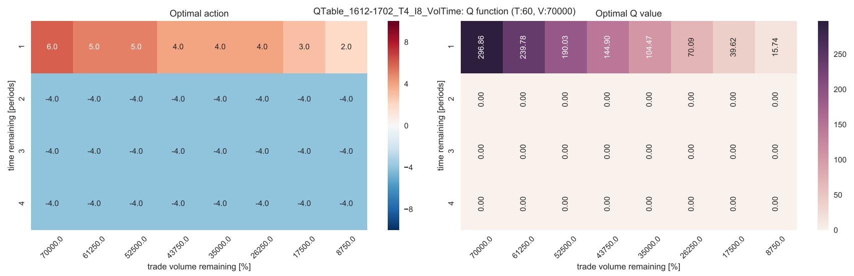


Figure 8: State-Action function, visualized after the first training round.
$T = 4$, $I = 8$, L=15

Knowing the optimal state-action values for all states with $t = 1$, all informations are given to compute the optimal state-action values for their predecessor states with $t = 2$ (see Figure 9).

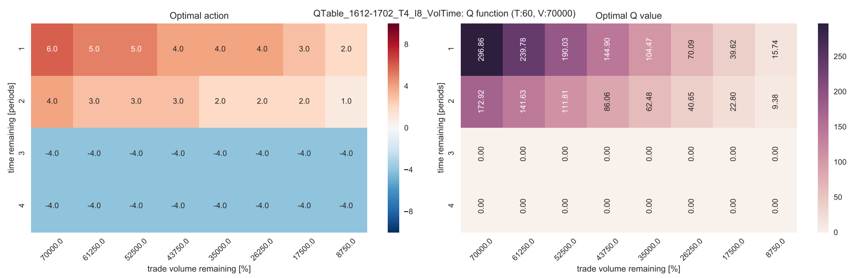


Figure 9: State-Action function, visualized after the second training round.
$T = 4$, $I = 8$, L=15

After T iterations a globally optimal policy, as shown in Figure 10, has been found. The annotated q values (right) denote the corresponding minimum over all available actions. The visualized state-action function was trained over orderbook snapshots from Nov, 10th 2017 10am to May, 31st 2017, partitioned into 4.154 orderbook windows of 60 minutes length each. Specifying $T = 4$ limit prices, 70.000$ cash (discretized in 8 intervals) had to be traded into Bitcoins. The state space included the two private variables `time` and `volume` only, while the action space comprised 15 actions. As such, a total of $4.154 * 4 * 8 * 15 = 1.993.920$ transition tuples were generated.

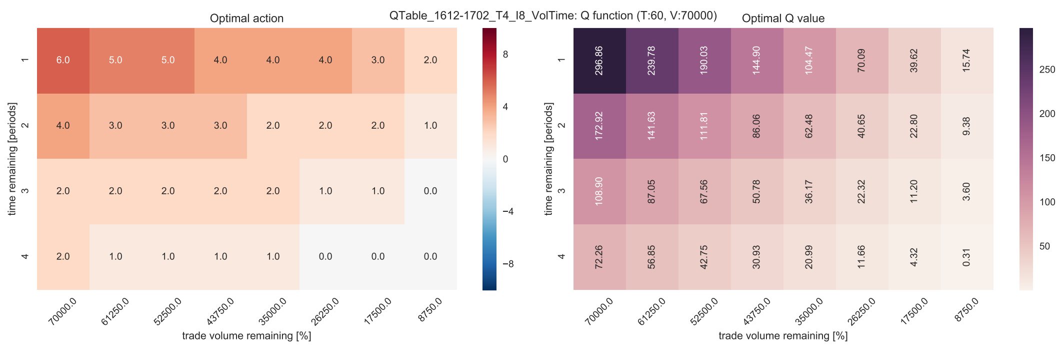


Figure 10: Final State-Action function.
$T = 4$ (60min), $I = 8$ (70.000$), L=15 ([-4, -3, . . ., 9, 10])

Figure 10 illustrates clearly, how the optimal strategy becomes more aggressive as time ceases and a large portion of the trading volume remains unexecuted.

4.2 Discussion

While the presented algorithm exploits the available data profoundly in a brute force manner, it comes with some room for improvements.

4.2.1 Discrete State Space

The algorithms most obvious weakness lies in its vulnerability to seldomly observed market situations. Since the state-action function is implemented as a simple lookup table without any generalization capabilities, it is strictly dependent on a thorough exploration of the underlying state space. As exhaustive exploration is enforced for the value range of private variables only, market variables are entrusted to chance. Especially when increasing the state space dimension by adding multiple market variables simultaneously, the explanatory power of a learned state-action mapping depends crucially on the number of underlying observations. There exists even the chance of certain states never been monitored at all during the training phase. The concrete discretization process was not further described or questioned. Particularly it is not stated, how boundaries between 0 (low), 1 (medium) and 2 (high) have been chosen, and how the trading performance may be influenced by a higher market variable resolution.

Potential Improvements are described and tested in Section 5.2.

Firstly, the discretization is automatized, such that boundaries between the bins are automatically derived from the training data. Performances of multiple discretization resolutions are compared. Secondly, the underlying lookup tables are replaced by function approximations, making discretization unnecessary.

Alternative Market Variables may be considered.

The market variables exploited (see Section 4.1.1) basically came from market level 1 data only. Section 5.2.3 evaluates the impact of additional market level 2 features.

4.2.2 Cost scaling

While discretization of market variables mainly affects the strategies explanatory power, the resolution of private variable `volume` leads to considerable rounding issues in regards to the cost function employed. As observed successor states x_{t-1} must be discretized equally to allow looking up the corresponding minimal costs, the immediate costs, as computed in Equation (1), must

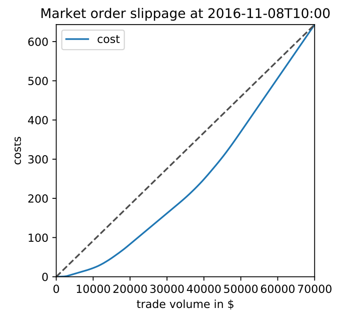


Figure 11: Non-linear slippage growth.

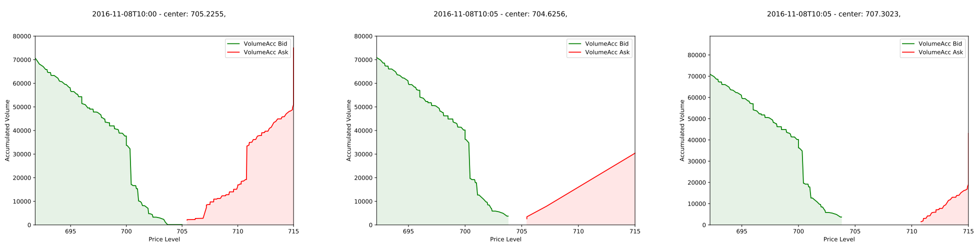
be scaled accordingly. Simply replacing `volume_traded` with `round(volume_traded)` falsifies the actual costs, as they correlate to the accomplished trade volume in an unpredictable, non-linear manner as indicated in Figure 11. Nevmyvaka et al. [3] did not mention this problematic and presumably did not perform any cost scaling at all.

Potential Improvements A simple, but imprecise approach is to perform cost scaling, as described above. In general, this problem should be void, when function approximations are trained from the original float values, instead of rounded values.

4.2.3 Markovian Assumption

The proposed backward algorithm assumes individual `trading_periods` to be of an (approximately) Markovian nature. This is in fact not true, as the OTS's internal masterbook shape depends drastically from the preceding trade history.

Figure 12 shows differing masterbook shapes, that can build the base of a simulated `trading_period`, e. g. if starting at $t = 45min$ and a remaining trade volume of 17.500$.
12a shows the original orderbook, as if no orders had been matched by our strategy. This version of the masterbook is queried by the original backward algorithm, potentially resulting in more passive trading aggressions, as it does not account for the own impact on the market at all. 12c and 12b show two ideas, how the market impact can be incorporated. Both lead to different masterbook shapes and to different subsets of attainable prices.



(a) Original Orderbook. **(b)** Assuming 52.500 shares being matched at `t=0`, then no further matches. **(c)** Assuming 52.500 shares being matched evenly at 1.166 shares per minute.

Figure 12: Different shaped masterbooks at `t=45`.
The shapes differ, depending on the preceding trade history.

Potential Improvements Initializing the OTS outside the original start point ($t = 0, V = 100\%$), the supposedly traded volume must be incorporated, e. g. as done in Figure 12c. Other than that, a more realistic sampling method my be applied: The forward sampling process, described in **??**, approaches sampling from the other side. Rather than evaluating actions on individual `trading_periods`, the OTS always starts with $V = 100\%$ and keeps going until the full trade is executed. As such, more realistic samples are generated. This does not give the problem a Markovian nature, but the internal masterbook is always of a realistic shape. In contrast to the backward approach, exhaustive exploration of the state space is not self-evident and must be enforced. As a side benefit, the forward sampling approach generates more versatile samples for training the function approximates, as the generated samples do not necessarily start at *discrete* start points only.

4.2.4 Action Space

The mapping from actions to limits deserves some reflection as well. The proposed method (see Section 4.1.2) adds the value of the chosen action directly to the current best price of the opposing book, such that the bid-ask spread must be crossed before any orders may be matched.

On the one hand, it seems pointless to fix the origin at the best price of the *opposing* book. By doing so, it appears obvious, that decisions derived from state spaces including a variable for the current spread size, outperform decisions derived from state spaces not including this market variable. Indeed, the bid-ask spread was posed as the market variable causing the greatest individual impact on cost reduction, namely -7.97%, which is a major fraction of the maximum achievement of -12.85%[4].

On the other hand, the proposed mapping method does not necessarily fit to the Bitcoin data at hand. As is shown in Figure 2, Bitcoin prices have burst from roughly 700$ to more than 2.000$ in the period of recording, i. e. interpreting actions as the absolute difference to the current best prices, maps to significantly different levels of aggression as time passes.

Potential Improvements lay in alternative action-limit mappings.

On the one hand side, the limit base may be fixed to the other side of the bid-ask-spread, reducing pointless dependencies on the speads size. On the other hand, actions should be interpreted as factors, rather than summands to allow for a consistent aggression interpretation. The mapping functions are thus redefined as follows:

`limit_buy = ask * (1 + (a/1000))` instead of `limit_buy = bid + a`

`limit_sell = bid * (1 - (a/1000))` instead of `limit_sell = ask - a`.

Actions a now represent the deviation from the limit base in per mile. The influence of the chosen limit base is analyzed in Section 5.2.1.

4 Strategies derived from five dimensional state spaces including the market variables Spread, ImmCost & Signed Vol, as described in Section 4.1.3, outperformed strategies derived from two dimensional state spaces, containing the private variables only, by -12.85% in average.

5 Orderbook Agents

This chapter describes several *Orderbook Agents*, implemented to learn optimal *Submit & Revise* trading strategies from a given training data set.

5.1 Implementation

Each *Orderbook Agents* inherits some commonly shared functionality from the super class `class RL_Agent_Base`. This base class provides a consistent framework, allowing to swap the logic behind `learn_from_samples` and `predict` only, while general functionality like `load`, `save`, `plot_heatmap_Q` and `collect_samples` may be reused.

An *Orderbook Agent* holds the complete list of environment parameters, used to initialize the OTS. As such, different definitions for the optimal trading problem require different Orderbook Agents.

5.1.1 Backward Sampling & Learning

The process of sampling from training data and learning optimal state-action values has been split into two independent phases, mostly due to performance[5] reasons. In the *sampling phase*, all possible combinations of actions and private variables are evaluated and stored as 5-tuples $sample = (state, action, cost, timestamp, new\_state)$. As market variables are supposedly not influenced by the agents behavior (see Section 4.1.1), it suffices to remember the orderbooks time point, to allow adjacent substitution of market variables.

Samples are stored as DataFrame and may be exported and reused by other agents[6], as wished. This allows to skip the time consuming process of backward sampling. Before the action learning phase starts, the collected samples may be enhanced by a variety of market variables, discretized or untouched. The helper function `addMarketFeatures_toSamples()` retrospectively adds market variables to the agents samples DataFrame.

If desired, features are discretized according to their individual value range.
E. g. a chosen resolution of 3, evenly transforms features o_m into o_{m_disc3}, according to automatically determined boundaries:
0 (if $o_m < 1/3$ quantile), 1 (if $1/3 \leq o_m < 2/3$ quantile) and 2 (if $o_m \geq 2/3$ quantile). Quantiles arise from the training data and are applied to test data unaltered.

5 The backward sampling phase for the example mentioned in Section 4.1.4, took roughly 10 hours, even though the work load was distributed among 24 CPU's.

6 Both agents should refer to the same environment settings, as samples collected from different environments (e. g. trading horizon of $H = 8min$ vs. $H = 60min$) may not be compatible.

The learning phase starts hereinafter, whereas three types of orderbook agents have been implemented and experimented with, each of which employs different techniques:

QTable_Agent : Dynamic Programming as described in Section 4.1.4.
Expected state-values are stored in a simple lookup table, hence discretization of the state space is required. For each state, a vector of length $L = num\_actions$ is maintained, of which the first argmin refers to the optimal action. The *QTable* does not generalize to previously unobserved states and returns the very first action (here -4) in such a case. For proper cost-updates an additional *NTable* is maintained, referencing the particular number of observations made.
QTable and NTable entries are computed in an iterative manner. In the first round, all samples with `time_left=1` are evaluated, in the second round all samples with `time_left=2`, etc. The private variable `volume` is discretized according to the specified resolution and linear cost scaling (see Section 4.2.2) is performed.

BatchTree_Agent : Tree-Based Batch Mode Reinforcement Learning [1].
An ensemble of 100 decision trees, with a maximum allowed depth of 20 is fed with the full batch of samples simultaneously, no discretization required. $T*2$ learning rounds are performed, to completely allow expected costs to unfold over the given time horizon.
The state space is enlarged by an extra dimension for the chosen action. Predicting the optimal action to choose for a given state, the ensemble must be queried $L = num\_actions$ times, from which the argmin refers to the optimal action.

NN_Agent : Neural fitted Q Iteration [6].
A simple multi layer perceptron with 400 hidden units and 1 output neuron is trained in mini batches of size 4.096, optimized over multiple epochs by Adam[2] with a learning rate of 0.01. As with the BatchTree_Agent, no state space discretization is required and actions are included into the state space. L predictions are required, to obtain the optimal action.

5.1.2 Forward Sampling & Learning

In order to create more realistic orderbook shapes (see Section 4.2.3), an forward sampling method is presented.

Listing 4: Forward sampling approach.

```
collect_samples_forward(V, H, T, I, L, E)
    While(not end of data)
        init OTS with V=100%
            For epoch=0 to E
                reset OTS at random startpoint
               Transform (orderbook) -> o_1 ... o_R
               Apply e-greedy strategy until V=0%
          Store samples in DataFrame
        occasionally retrain strategy with new collected samples (+ experience replay)
```

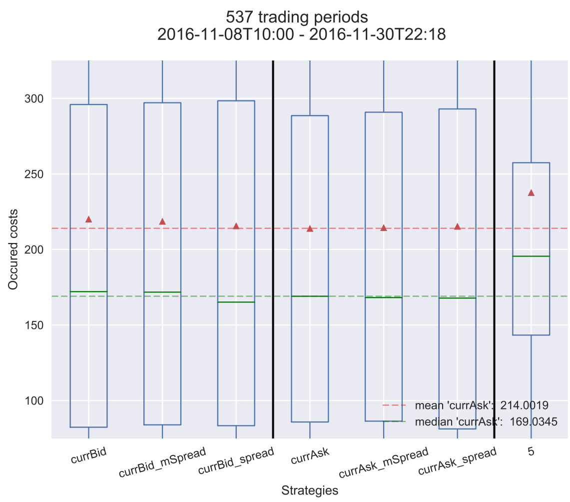
5.2 Backward Sampling Experiments

Various experiments have been conducted to examine the agents ability to find optimal solutions to the problem of optimized trade execution. The recorded orderbook snapshots for currency pair USDT/BTC (see Section 3.1) have been split into training period (Nov, 10th 2016 - Apr, 30th 2017) and test period (May 2017).

The studied agents refer to the very same environment settings. Their common task is to buy Bitcoins worth of 70.000$ within a trading horizon of 60 minutes. As such the training set translates into 4.154 orderbook windows, while the test set gives 724 orderbook windows. If not stated otherwise, a period_length of 15 minutes is assumed, such that the OTS expects up to 4 order limit prices. After an initial backward sampling phase, the obtained samples DataFrame makes the base for varying learning phases.

5.2.1 Action-Limit Mapping

As mentioned in Section 4.2.4, it seems pointless to force actions to cross the bid-ask-spread before any orders can be matched. Figure 13 shows exemplary for November 2016, how the choice of the limit base affects the agents trading performance. While buy orders forcing agents to cross the bid-ask-spread did indeed benefit from the two market variables `spread` and `marketSpread`, this was not necessarily the case for agents which had the limit base fixed to the other best price. As the latter agents consistently showed better performance, choice has been made to avoid crossing the spread henceforth.



(a) Slippage.

	slippage	performance
currBid	220.19	92.6%
currBid_mSpread	218.71	91.9%
currBid_spread	215.76	90.7%
currAsk	214.00	90.0%
currAsk_mSpread	214.53	90.2%
currAsk_spread	215.38	90.6%
S&L: 5	237.75	100.0%
MarketOrder	737.58	310.2%

(b) Mean slippage.

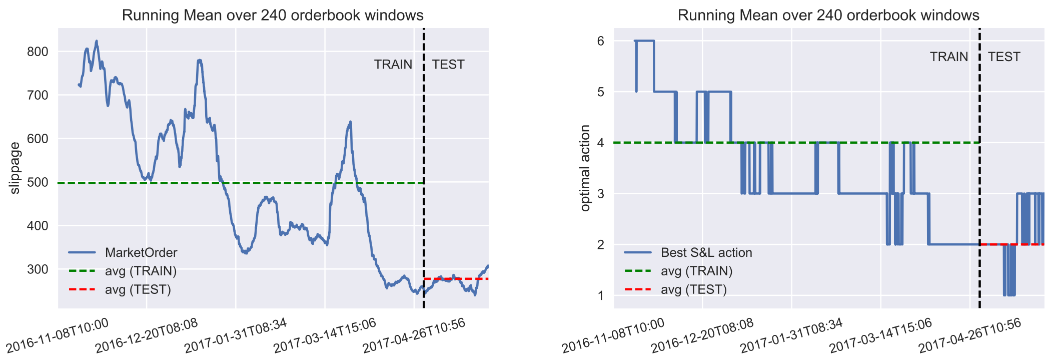
Figure 13: Evaluating the impact of different limit base levels.

In order to produce comparable results, all subsequent experiments are based on the same set of 15 actions: $[-4, -3, \ldots, 8, 9, 10]$. In line with the formulas presented in Section 4.2.4, these actions translate into order limits deviating by -0.4% to +1.0% from the current best price.

5.2.2 Baseline

Simple S&L strategies (see Section 2.1.5) and immediate market order placements serve as benchmark for the performance measurement.

Due to bursting Bitcoin prices (see Figure 2), the investigated sum of 70.000$ constitutes a declining contingent of the total market volume. Figure 14a shows a running average over the amount of slippage, as induced by immediate market orders. Concurrent to declining slippage, the optimal S&L actions become less aggressive as time passes. The green, dashed lines show the respective average over the train period, which significantly differs from the red, dotted line referring to the average over the test period. Figure 15 shows the average costs, induced by varying S&L strategies within the training period.



(a) Observed Market Order Slippage. **(b)** Best S&L action.

Figure 14: Concurrent to declining slippage, the optimal S&L actions become less aggressive as time passes.

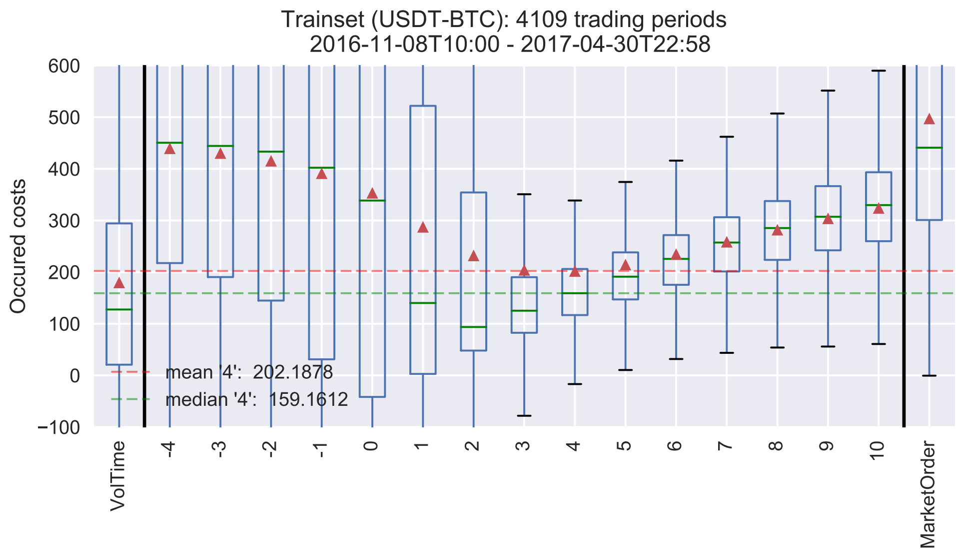


Figure 15: Average costs induced by the varying S&L actions over the full training period. In average, action 4 performed best.

In order to provide a more realistic and unexaggerated baseline, the optimal S&L action is estimated from the test period. As such, performance of subsequents experiments are compared to the performance of a simple market order and the S&L strategy "2", with the initial order limit fixed to 1.002 times the initial ask (i. e. $+ = 0.2\%$).

5.2.3 Additional Market Variables

In addition to the market variables proposed in Section 4.1.2, the following market level 2 features have been examined: `marketPrice_*` describes the relative difference between current center_price and the worst price that must be paid in case of simple market orders. `marketPrice_buy_worst` Table 4 shows the performance of individual QTable agents trained on private variables plus one discretized market variable attached at a time. While observed trading costs undercut costs induced by simple market orders by almost -47.13%, the gain in comparison to simple S&L strategies is far smaller: -7.43%. In contrast to the results reported by Nevmyvaka et al. [3], the market variable `spread` yields only an improvement of -2.14% (compared to -7.97%) over the plain `VolTime`-agent.

	slippage	std	perf_2	perf_4	perf_M	perf_VolTime
center_orig_disc3	149.57	420.14	94.37%	84.81%	53.90%	99.30%
ImmCost_buy_worst_disc3	148.35	361.17	93.61%	84.11%	53.46%	98.49%
ImmCost_sell_worst_disc3	146.72	385.13	92.57%	83.19%	52.87%	97.41%
ImmCost_spread_disc3	150.27	355.46	94.82%	85.20%	54.16%	99.77%
ImmCost_imbalance_disc3	148.42	358.49	93.64%	84.15%	53.49%	98.53%
sharecount_buy_disc3	149.57	420.14	94.37%	84.81%	53.90%	99.30%
sharecount_imbalance_disc3	148.57	353.91	93.74%	84.24%	53.54%	98.64%
sharecount_sell_disc3	149.57	420.14	94.37%	84.81%	53.90%	99.30%
sharecount_spread_disc3	147.57	350.33	93.11%	83.67%	53.18%	97.97%
spread_disc3	147.41	370.96	93.01%	83.58%	53.12%	97.86%
ob_direction_disc3	65.80	335.76	41.52%	37.31%	23.71%	43.68%
future_center5_disc3	133.58	377.88	84.29%	75.74%	48.14%	88.69%
future_center15_disc3	107.74	364.92	67.98%	61.09%	38.83%	71.53%
future_center60_disc3	139.20	398.05	87.83%	78.93%	50.17%	92.42%
VolTime	150.63	358.66	95.04%	85.40%	54.28%	100.00%
2	158.49	400.59	100.00%	89.86%	57.12%	105.22%
4	176.37	273.58	111.28%	100.00%	63.56%	117.09%
MarketOrder	277.49	158.66	175.08%	157.33%	100.00%	184.22%

Table 4: Average trading costs, as observed within the test period, show the effect of adding individual, discretized market variables to the state space of QTable agents.

The exact reason, why individual market variables have such little effect, is unclear, but one possible explanations lies in the data set employed. The original experiment assessed a different data quality. Due to the low resolution of the available orderbook snapshots, a large fraction of the market activity is inaccessible and consequently the majority of trading opportunities are missed by the agents. Furthermore, the minute time-scaled limit order book data inevitably requires the trading horizon to be rather long. Experiments with shorter time horizons nullified the achievable savings, while longer time horizons led to unacceptable computation times. Consequently, the agents where trained on 4.109 sixty-minute orderbook windows, while Nevmyvaka et al. invoked 45.000 two-minute orderbook windows.

In order to proof the algorithms general ability to find costs reducing order limits, look-ahead features[7] were added to the universe of market variables. The hypothetical knowledge about future price trends (i. e. percentual changes between the current center price and the center price in 5, 15 and 60 minutes respectively) reduced observed trading costs by -32.02%.

The largest impact (-58.48%) was observed for the look-ahead feature `ob_direction`, which marks the general price trend of the currently observed orderbook window. In contrast to the `future_enter*` variables, it's value stays constant within individual orderbook windows: `orderbook[-1].get_center()/ orderbook[0].get_center()`, which seems to provoke more stable strategies.

Constant Market Variables

The findings from the look-ahead features encourage a tailing experiment. Rather than observing market variables at the actual time, always the market situation, observed at `t=0` is selected.

5.2.4 Cost function

default: slippage based on initial_center
experimental: Improvement over MarketPrice

Discretization Resoution

Comparison of disc3, disc5, disc9

Function Approximation

- RandomForest (BatchTree Agent)
- NN Agent

5.2.5 Forward Sampling

- Growing batch learning
- Experience Replay (NN_Agent)
- Exploration vs avoidance of repeatedly trying same actions.
- Markov Property violated

7 look-ahead features provide a glance into the future, and are thus equivalent to cheating.

- Realistic samples, no rounding. Better fit for *curious* masterbook shapes as described in Section 3.3.5?!

6 Conclusion

Due to the volume of data, the real world example shown in **??** would have been a tough job on any single local workstation or students notebook. When analyzing big data, it is a great relief or even an inevitable thing to use a cluster of computer for distributed storage and data processing.

Hadoop is a great and powerful cluster framework and R is a highly popular and well-advanced programming language for statisticians. In a world of ever growing data, Hadoop and R make a perfect fit. Both combined, the mightful analytic capabilities of R can be applied to big data.

A Glossary

OTS Orderbook Trading Simulator

RL Reinforcement Learning

S&L Submit & Leave Strategy

S&R Submit & Revise Strategy

NASDAQ National Association of Securities Dealers Automated Quotations (American Stock Exchange))

B References

[1] D. Ernst, P. Geurts, and L. Wehenkel. *Tree-Based Batch Mode Reinforcement Learning*. In: *J. Mach. Learn. Res.* Vol. 6 (Dec. 2005), pp. 503–556. ISSN: 1532-4435.

[2] D. P. Kingma and J. Ba. *Adam: A Method for Stochastic Optimization.* In: *CoRR*, Vol. abs/1412.6980 (2014).

[3] Y. Nevmyvaka, Y. Feng, and M. Kearns. *Reinforcement Learning for Optimized Trade Execution*. In: *Proceedings of the 23rd International Conference on Machine Learning*. ICML '06. Pittsburgh, Pennsylvania, USA: ACM, 2006, pp. 673–680. ISBN: 1-59593-383-2.

[4] Y. Nevmyvaka et al. *Electronic Trading in Order-Driven Markets: Efficient Execution*. In: *CEC*. 2005.

[5] Poloniex. *Poloniex Digital Asset Exchange*. http://www.poloniex.com. [Online; accessed 20-June-2017]. 2010.

[6] M. Riedmiller. *Neural Fitted Q Iteration – First Experiences with a Data Efficient Neural Reinforcement Learning Method*. In: *Proceedings of the 16th European Conference on Machine Learning*. ECML'05. Porto, Portugal: Springer-Verlag, 2005, pp. 317–328. ISBN: 3-540-29243-8, 978-3-540-29243-2.

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