

Master's Thesis

A forward oriented reinforcement learning approach to the problem of optimized trade execution

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July 24, 2017

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Writing period

12.01.2016 – 31.07.2017

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Declaration

I hereby declare, that I am the sole author and composer of my Thesis and that no other sources or learning aids, other than those listed, have been used. Furthermore, I declare that I have acknowledged the work of others by providing detailed references of said work.

I hereby also declare, that my Thesis has not been prepared for another examination or assignment, either wholly or excerpts thereof.

Freiburg, August 7, 2017

Place, date

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Acknowledgements

I like to thank the following people for their help and support:

- Manuel Blum, who directed me and provided me with great feedback throughout the whole project.
- All employees of the data science company *PSIORI GmbH*, for the pleasant working atmosphere and their occasional feedback on my topic.

Abstract

This thesis tackles the important problem of optimized trade execution, which aims to obtain the best possible price for a trade instructed by higher level investment decisions. In its simplest form the problem is defined by a particular financial instrument, which must be bought or sold within a fixed time horizon, while minimizing the expenditure for doing so.

Modifications to an existing reinforcement learning approach are made and a novel forward-learning algorithm is proposed, outperforming the cost avoiding capabilities of all previous methods.

Zusammenfassung

Diese Thesis befasst sich mit dem Problem der Optimalen Handelsausführung, welches darin besteht den bestmöglichen Preis für ein Finanzprodukt zu erzielen, dessen Handel von einer höheren Investitionsstrategie angeordnet wurde. Am leichtesten lässt sich dieses Problem beschreiben durch ein bestimmtes Finanzprodukt, welches innerhalb eines festgelegten Zeitraumes zum bestmöglichen Preis gekauft oder verkauft werden soll.

Hierfür wurde ein bestehender Reinforcement Learning Ansatz verbessert und ein neuartiger, vorwärts lernender Algorithmus vorgestellt, der die kostenreduzierenden Fähigkeiten der vorherigen Methoden übertrifft.

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1 Introduction

1.1 Motivation

Modern markets have, to a large extent, become computerized technological systems. Opportunistic investors and market makers likewise interact within these high-frequency marketplaces with the use of electronic algorithms. While generally aiming for (large) profits, the typical portfolio manager diversifies the available capital in a variety of assets simultaneously in order to reduce risk. *Technical analysis*, which is the analysis of past stock prices, and *fundamental analysis*, which is the analysis of a companies intrinsic value, are used to estimate whether individual assets are currently under- or overvalued.

Based on these insights, high level strategies optimize which assets shall be bought or sold at what share count. These investment decisions are then entrusted to a *trader*, which is specialized to the task of *optimized trade execution*. While the high level strategy aims to maximize revenue through exploitation of profitable opportunities, the subsequent trading algorithm aims to maximize revenue through minimization of inevitably occurring costs.

In most exchange markets, asset prices are determined by supply and demand: While large demand for an asset results in higher prices, oversupply has the opposite effect of lowering prices. A *fair* price is automatically derived from the universe of opposing trading interests. This observation is consolidated in Fama's *efficient market hypothesis* [10], which assumes markets to be extremely *efficient*. In efficient markets news are assumed to spread so fast, that all relevant informations are incorporated into the respective assets price instantaneously and unimpeded. Future price changes reflect only future news and are independent of todays price changes. Consequently, as news are per definition unpredictable, future price changes are unpredictable and random. According to this theory, it should neither be possible for investors to earn above average returns from the overall market by purchasing undervalued shares and selling shares for inflated prices, nor to profit from optimized trade execution deliberately.

In [19] Malkiel shows empirically and theoretically that the efficient market hypothesis is only valid under certain preconditions. While the paper concludes that most markets are very, but not extremely, efficient and that prices are far less predictable than some (at that time) recent academic papers claimed, there remains a certain degree at which asset prices are indeed predictable.

Investors participating in exchange markets must expect fees, charged by the respective market place organizer in return for granting access to their infrastructure. Other than

that, there are hidden costs to be considered as well: While trades involving little capital¹ usually cause minor impact on the current market situation, large-scale investors must be cautious when it comes to order placements. Large orders can have a major impact on supply and demand, leading to diminishing availability and consequently *hidden costs* in form of worsening prices.

While direct costs are contractually regulated and thus predictable to a large extend, this is not the case for hidden costs. The unpredictable nature of hidden costs bears a significant risk that grows exponentially with the trading volume pursued. This motivates for well considered trading strategies, that help to reduce the investors impact and avoid causing costly market turbulences by unwinding large orders of shares over time.

1.2 Objectives

This thesis tackles the important problem of *optimized trade execution*, which aims to obtain the best possible price for a trade instructed by higher level investment decisions.

Figure 1 shows exemplary how a high level investment strategy delegates individual orders to independent instances of a specialized trading algorithm. While trade-execution algorithms may be offered as a service by banks or brokers, they are often proprietary and withheld to internal use of large institutional investors.

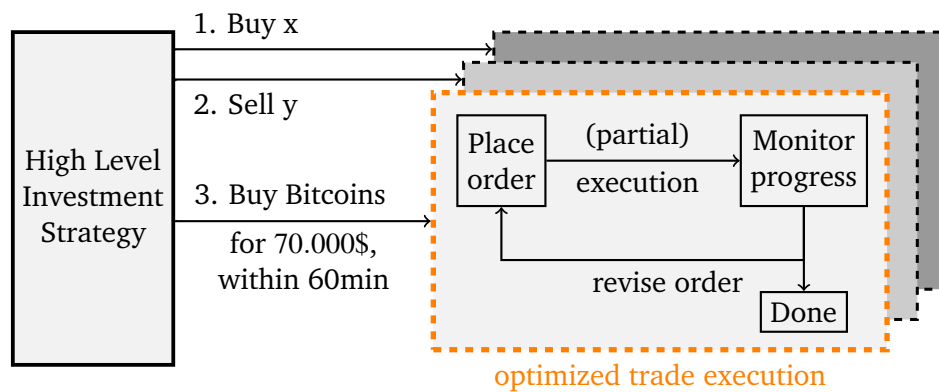


Figure 1: Optimized Trade Execution.

In its simplest form, the problem of optimized trade execution is defined by a particular financial instrument (here: Bitcoins), which must be bought or sold within a fixed time horizon, while minimizing the expenditure (share price) for doing so.

This problem constitutes an ideal playground for reinforcement learning approaches, which aim to learn optimal behavior based on observed states. Regularly monitoring

¹ Little capital, relative to the whole market liquidity.

trade progress and market situations, reinforcement learning agents hold the potential to automatically unwind large orders over time in an advantageous way.

The scope of this thesis is to examine and improve the reinforcement learning approach proposed by Nevmyvaka et al. [22]. While the authors derived valuable strategies from a discretized state space in a brute force manner, a novel forward learning algorithm is proposed, which samples from a continuous state space and outperforms the foremost algorithm. As reinforcement learning requires interacting with an environment, a trading simulation framework is implemented. The accrued Orderbook Trading Simulator (OTS) provides a full-featured reinforcement learning environment, that simulates the execution of orders on historic Limit Order Book (LOB) data and returns vital feedback to steer the agent towards optimal decisions.

While most research is performed on traditional stock markets with expensive, proprietary data sources, this was simply out of budget for this work. Instead, the relatively young market of bitcoin trading is assessed, as it offers inexpensive access to real-time prices through publicly retrievable interfaces. Snapshots of the current market situations are retrieved on a low-resolution, minute-scale basis from the open bitcoin exchange platform Poloniex². Consequently, the usability of the retrieved dataset is analyzed.

1.3 Related Work

Much research has been performed to study the optimal execution of buy and sell orders in exchange markets.

In 1970 Fama published his *efficient market hypothesis* [10], that assumed news to spread so fast, that all relevant informations are incorporated into the respective assets price without delay, making it impossible to profit from optimized trade execution deliberately. By the start of the twenty-first century, many statisticians and financial economists started to believe in (partially) predictability of stock prices. In 2003 Malkiel [19] examined the critics on the efficient market hypothesis and delivered empirical and theoretical proof that the efficient market hypothesis is only valid under certain preconditions. While the paper concludes that most markets are very, but not extremely, efficient and that prices are far less predictable than some (at that time) recent academic papers claimed, there remains a certain degree at which asset prices are indeed predictable.

Parlour (1998) [25] and Rosu (2009) [27] both proposed equilibrium models of price formation in limit order markets. They showed that price evolutions in exchange markets are rather complex and depend critically on the state of the LOB. Cont, Stoikov & Talreja

² <http://www.poloniex.com>

(2010) [6] showed empirically, that over short time intervals, stock price changes are mainly driven by imbalances between supply and demand and as such indeed partially predictable. In 2011 Stokov, Avellaneda & Reed [30] proposed a stochastic model, that computes the probability of an upward move in the price, conditional on the level 1-quotes, such as best bid/ask sizes. The price impact of unexecuted orders, given the current **LOB**, was studied inter alia by Bertsimas & Lo (1998) [5], Almgren & Chriss (2000) [2], Alfonsi & Fruth (2010) [1] and Cont & Larrard (2011) [7] who proposed a stochastic model that exploits the state of the **LOB**. Maglaras, Moallemi & Zheng (2015) [18] modeled **LOBs** as multi-class queueing systems under fluid dynamics to optimize how to execute a block of shares over a predetermined time horizon.

While most approaches are base on stochastic methods, relatively little literature about machine learning approaches to the problem of optimal order execution has been published. In 2006, Nevmyvaka, Feng & Kearns [22] used reinforcement learning to unwind large orders over time though a mixture of limit and market orders. Park & Van Roy (2012) [24] used reinforcement learning to explore and learn a price impact model alongside trading. Kercheval & Zhang (2015) [14] used support vector machines to capture the dynamics of high frequency **LOBs** and predict future price movements. Kempf & Korn (1998) [13] used neural networks and found that the assumption of a linear impact of orders on the price is highly questionable. Fletcher & Shawe-Taylor (2013) [11] used fisher kernel learning for the prediction of currency exchange rates and achieved an average accuracy of 55% when classifying the direction of price movements for a 200s predictive time horizon.

In 2016 Sirignano [28] developed a new neural network architecture for modeling spatial distributions of **LOBs**. Deep learning models were used to model the joint distribution of the state of the **LOB** at a future time conditional on its current state. The proposed models were trained and tested on nearly 50 terabytes of raw **LOB** data and run on a cluster with 50 GPUs, outperforming naive empirical models, logistic regression and standard neural network architectures.

1.4 Contributions

The following contributions to the problem of optimized trade execution have been made by this work:

- (1) With the **OTS**, a full-featured reinforcement learning environment for the simulation of trade execution is presented. It simulates the execution of orders on historic orderbook data and returns vital feedback to steer reinforcement learning agents towards optimal decisions. Additionally, it serves as a backtesting framework in order to evaluate the cost reducing capabilities of different trading strategies.

- (2) An existing reinforcement learning approach, deriving valuable strategies from a discretized state space in a brute force manner, has been examined and was improved in detail. As an example, the proposed backward-sampling method neglected the agents own impact on the market situation. It was found, that the agents cost reducing capabilities may be further improved, if this self-induces impact is incorporated properly.
- (3) A novel reinforcement learning approach is presented, that applies growing batch reinforcement learning on the problem of optimal trade execution. The proposed forward-sampling method samples from a more realistic, continuous state space and outperforms the foremost algorithm.

1.5 Outline of Contents

The remainder of this thesis is structured as follows:

Section 2 gives a general introduction into the vocabulary of financial computing, the machine learning techniques employed and the problem of optimized trade execution.

Section 3 describes the Orderbook Trading Simulator, that provides a full-featured reinforcement learning environment.

Section 4 describes and discusses both reinforcement learning approaches, i.e. the original brute-force method and the novel forward learning method.

Section 5 describes various experiments, that have been conducted in order to improve the reinforcement learning approaches and to find better trading strategies.

Section 6 closes with a conclusion and suggestions for future work.

2 Background

This chapter gives a general introduction into the vocabulary of financial computing and the machine learning techniques employed.

2.1 Trading Basics

In order to understand the objective of this thesis, fundamental knowledge about the domain of financial computing is obligatory. This section provides a quick overview.

2.1.1 Financial Instruments

Financial instruments represent legal agreements of monetary value between parties and can be traded as assets. These assets span from actual cash, through evidence of entity or share ownership, to contractual rights to receive or deliver another type of financial instrument.

2.1.2 Markets

Financial instruments are typically traded through one of two basic market types:

Over-The-Counter (OTC) In an OTC or off-exchange market, transactions take place directly between two parties, without a mediator. Prices are negotiated directly between buyer and seller and typically not published to the public.

Exchange In exchange markets, transactions are executed by a so called broker. A broker, which can be both, an individual or a firm, executes buy and sell orders on behalf of traders for a certain fee or commission. The respective prices are determined by the current market situation, in particular by supply and demand.

2.1.3 Exchange Markets

Specialized exchanges concentrate on certain sub-types of financial instruments and offer a trading venue for those willing to buy and sell these instruments. Some of them are listed below:

Stock Exchange Market A stock exchange or bourse provides companies access to investment capital in exchange to a share of ownership. Especially in times with notoriously low interest rates, investors tend to accept the greater risk of business development over a risk free, but faint investment, to grow their assets.

E. g. [NASDAQ](#), Deutsche Börse, ...

Commodity exchange market Commodity exchange markets allow for speculations with goods like oil, gold, corn, ...

E. g. [Eurex](#), ...

Foreign exchange market Foreign exchange (short: forex) is considered the largest financial market in the world. The forex market is responsible for determining currency exchange rates.

E. g. FXCM, ...

Digital assets exchange market Digital asset exchange markets allow for speculations with virtual goods, like digital documents, motion picture, audible content, or digital currencies (i. e. electronic money).

E. g. Poloniex, ...

In the past, exchanges were physical locations. In many cases, these physical locations have been replaced by fully electronically organized markets, accessible through the internet. As a result, more traders can execute orders in a higher frequency.

2.1.4 Ask and Bid

Most exchange markets function after the so called auction market model [15], where the exchange acts as a mediator between buyers and sellers to ensure fair trading. Here, buyers can *bid* a price they are willing to pay for a certain number of shares and sellers can *ask* a price they are aiming to make with a number of shares. The highest of all bids is called the *bid price*, the lowest of all offers is called the *ask price*. Together they represent the current price at which an instrument is traded.

The difference between bid price and ask price is called *bid-ask spread*, often abbreviated to *spread*.

2.1.5 Limit Order Book and Market Depth

A Limit Order Book (LOB) reflects supply (asks) and demand (bids) for a particular financial instrument. It is usually maintained by the trading venue and lists the number of shares being bid or offered, organized by price levels in two opposing books. Incoming orders are constantly appended to this highly dynamic list, while a matching engine cautiously resolves any inconsistencies (i. e. overlaps) between asks and bids by mediating between the involved parties.

It is usually not before the matching engine has arranged an actual trade, that a trading venue claims a certain percentage of the turnover as a service fee. To encourage active market participation, the pure submission, revision and cancelation of orders is typically free of charge.

	Amount	Type	Volume	VolumeAcc	norm_Price
31.00	200.0	ask	6200.0	8425.0	1.074533
30.00	50.0	ask	1500.0	2225.0	1.039871
29.00	25.0	ask	725.0	725.0	1.005208
28.85	NaN	center	NaN	NaN	NaN
28.70	200.0	bid	5740.0	5740.0	0.994810
28.50	100.0	bid	2850.0	8590.0	0.987877
28.00	300.0	bid	8400.0	16990.0	0.970546

Table 1: Exemplary snapshot of a limit orderbook for stocks of AIWC³.

Table 1 shows a limit orderbook snapshot up to a market depth of 3, as seen by market participants. Here Alice offers 25 shares per 29\$, Bob and Cedar offer 20 and 30 shares respectively per 30\$ and David offers 200 shares per 31\$.

Based on their trading needs, traders can typically choose between multiple levels of real-time market data.

Level 1 Market Data Basic informations only:

Bid price + size, Ask price + size, Last price + size

Level 2 Market Data Additional access to the orderbook.

Usually data providers display the orderbook only up to a certain market depth m , i. e. the lowest m asks and the highest m bids.

Level 3 Market Data Full data access.

Typically only accessible for the market maker.

2.1.6 Slippage

Slippage is defined as the difference between expected and achieved price at which a trade is executed. Slippage may occur due to delayed trade execution. Especially during periods of high volatility, markets might change faster than the order takes to be executed. Slippage is also linked to the order size, as larger orders tend to eat into the opposing book and are fulfilled at successively worse price levels. Slippage can be both positive or negative, depending on the current market movements and must be taken into account by serious investors.

2.1.7 Order Types

Investors can execute orders of different types, of which the most common ones are described below:

Market Orders are the most simple form of orders. Here, the investor only specifies the number of shares he want's to buy/sell and the full order is executed immediately, at any price. Especially for large-scale traders or traders with level 1 data access

³ Acme Internet Widget Company

only, these simple market orders are rather hazardous, since the achieved price can significantly differ from the expected price due to sparse supply and demand.

Limit Orders additionally feature a worst price, i. e. the highest price a buyer is willing to pay per share or respectively the lowest price a seller is willing to make per share. Limit orders are immediately placed into the [LOB](#) and (partially) executed, once the matching engine finds a corresponding trade in the opposing book.

Limit orders reduce the risk of slippage, but do not guarantee execution.

Hidden Orders are placed into the market makers internal orderbook, but not displayed to other market participants with level 2 market data access. They represent a simple solution to large-scale investors seeking anonymity in the market, aiming to obfuscate their trading intention from other market participants.

2.1.8 Trading strategies

An order placement typically originates from a carefully considered *trading strategy*. An *active* trading strategy buys and sells instruments frequently based on short-term price movements, whereas a *passive* trading strategy such as *Buy-And-Hold* believes in long-term price movements eventually outweighing any short-term fluctuations.

As the execution of trades typically implies trading costs and slippage, these have to be taken into account. Particularly active traders with a high order quantity and large-scale investors with high order volumes are concerned with this burden. The order type chosen has a major impact on speed of execution and slippage generated.

While *limit orders* reduce the risk of slippage, they do not guarantee full order execution. This leads to the important problem of *optimized trade execution*, which frequently occurs in the domain of financial computing. In its simplest form, the problem is defined by a particular financial instrument (here: Bitcoins), which must be bought or sold within a fixed time horizon, for the best achievable share price.

In [23] Nevmyvaka et al. introduce a Submit & Leave Strategy ([S&L](#)), which cleverly combines market and limit orders: After an initial limit order submission, the order is left on the market for a predefined time horizon, after which it's unexecuted part is transformed into a market order and thus executed completely. They later extended their strategy to a Submit & Revise Strategy ([S&R](#)) [22], where the order limit may be revised at discrete time steps, depending on trade progress and market changes.

2.2 Bitcoin

Bitcoin is a digital cryptocurrency, released in 2009 as open-source software under the name Satoshi Nakamoto [8]. Motivated in part by anger over the foregone financial

crisis, the underlying peer-to-peer system[21], provides a decentralized payment system. To this point, electronic transactions always required involving a third party to validate transactions, an inevitable necessity to forestall double-spending bitcoin tokens. Double-spending is a problem unique to digital currencies, as digital informations, in contrast to physical currencies, may be replicated arbitrarily. Rather than entrusting a central authority or central server with this validation task, a distributed public transaction log, known as the *block chain*[29] is maintained among all participants.

The block chain is a continuously growing list of confirmed and timestamped transaction records, called *blocks*. Every block includes a complex proof-of-work and a cryptographic hash of the previous block, making it extremely resistant against retrospective modifications. To motivate others to validate transactions, newly created bitcoins are rewarded to the first miner (or mining party), successfully serving the computationally demanding proof-of-work.

Through a manifold of legal and black markets, bitcoins may be exchanged for other currencies, products and services. In contrast to traditional exchanges, no regular trading hours exist. Transactions can be executed at all times, mostly unregulated and pseudonymous⁴[17]. As a consequence, Bitcoin are particularly sensitive to extraordinary events or news, frequently causing immediate (panic) reactions. This effect is possibly leveraged by relatively low entry barriers, attracting amateur and professional traders likewise.

2.2.1 Volatility

Bitcoin is a highly volatile currency, capable of massive price changes towards both directions within short periods of time. A high volatility provides profitable opportunities, but comes at the price of higher risk. Due to a very high trading volume⁵, limit orderbooks are typically more condensed around the current best price than limit orderbooks of less frequently traded stocks. As a consequence, less slippage is to be expected from eating into the orderbook, while more slippages originates from general price fluctuations.

Figure 2 compares volatilities of currency pairs USDT/BTC (blue line) and USDT/Pound (red line).

4 Owners of bitcoin tokens are not explicitly identified, but bitcoin exchanges may collect personal data on their customers if required by law.

5 ~ 40.000.000\$ per 24h for currency pair BTC/USDT on Poloniex.

Source: <https://coinmarketcap.com/exchanges/volume/24-hour> (accessed 12-July-2017)

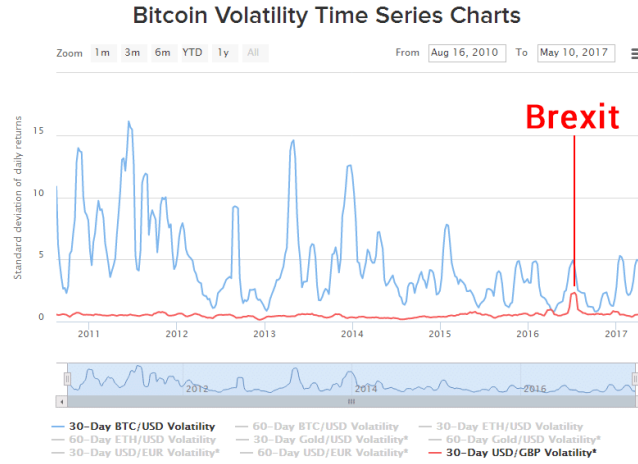


Figure 2: USDT/BTC and USDT/Pound volatility compared.

Source: <https://99bitcoins.com/bitcoin-volatility-explained>

2.3 Reinforcement Learning

Reinforcement Learning (RL) is a subdomain of machine learning. It was first defined by Sutton and Barto[31] as the problem of learning from interactions with an environment in order to achieve long-term goals. In contrast to Supervised Learning (SL), no labeled training data is present, such that sub-optimal actions can not be corrected explicitly. Instead, in each time step t , the agent takes the action a_t which maximizes⁶ his future expected reward R_t , based on empirical knowledge.

Figure 3 shows the typical RL scenario. An agent (partially) observes a state s_t and a reward r_t at every discrete time step t . It takes an action a_t and hereon receives a new state s_{t+1} and reward r_{t+1} .

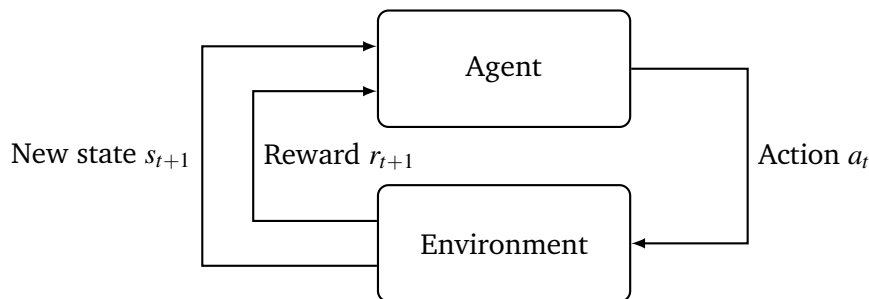


Figure 3: The discrete RL scenario defined by Sutton and Barto[31].

Rather than learning from labeled training data, the agent applies a *trial and error* pattern and exploits (delayed) external rewards to find actions, maximizing the expected future reward. As actions may not necessarily show an immediate effect, rewards are

⁶ Alternatively, it is possible to *minimize* the future expected cost: $C_t = -R_t$

often delayed and must be accounted for in the so called *credit assignment problem* [20].

The expected future reward R_t can be expressed by the cumulative reward of all successive rewards. The discounting factor $\gamma \in [0,1]$ represents the importance between long-term and short-term rewards. With a high γ , the agent aims to maximize long-term rewards, whereas a small γ leads to greedy actions, favouring short-term rewards.

$$R_t = \sum_{i=0}^{\infty} \gamma^i r_{t+i+1} \quad (1)$$

2.3.1 Markov Decision Process

In reinforcement learning, the environment is typically formulated as Markov Decision Process (MDP) [3], which provides a mathematical framework for optimization problems.

An MDP is a discrete time stochastic control process, written as 5-tuple (S, A, P, R, γ) . It consists of a (finite) set of environment states $s \in S$, a (finite) set of actions $a_t \in A$, a transition probability matrix $P : S \times A \rightarrow \mathbb{P}(S)$, which maps from state-action pairs (s_t, a_t) to a probability distribution over all successor states $s_{t+1} \in S$,

$$P_{ss'}^a = \mathbb{P}[s_{t+1} = s' | s_t = s, a_t = a] \quad (2)$$

and a reward (or cost) function $R : S \times A \rightarrow \mathbb{R}$, that computes the immediate reward received after applying action a_t in state s_t .

$$R_s^a = \mathbb{E}[r_{t+1} | s_t = s, a_t = a] = -\mathbb{E}[c_{t+1} | s_t = s, a_t = a] \quad (3)$$

The discount factor $\gamma \in [0,1]$ eventually defines the level of foresightedness. It is obligatory in MDPs with an infinite time horizon, to impede infinite values for accumulated long-term returns.

The general goal in reinforcement learning is to find optimal policies $\pi^* : S \rightarrow A$, that map states to the best applicable actions, such that the (discounted) expected long-term reward is maximized, or respectively the (discounted) expected long-term cost is minimized.

$$\pi^*(s) = \arg \max_{a \in A(s)} \mathbb{E}[R_t | s_t = s] = \arg \min_{a \in A(s)} \mathbb{E}[C_t | s_t = s] \quad (4)$$

2.3.2 Markov Property

An important condition of MDPs is the *Markov Property*, which defines the probability distribution of future states to be independent of the past (s_{t-n}) , given the present (s_t) .

As such, states must capture all relevant information from the history, to deliver sufficient statistics of the future. A state s_t is *Markov* if and only if:

$$\mathbb{P}(S_{t+1} = s' | s_t, a_t) = \mathbb{P}(s_{t+1} = s' | s_t, a_t, s_{t-1}, a_{t-1}, \dots) \quad (5)$$

2.3.3 Value Function and Bellmann Equation

In order to find the optimal strategy π^* , the value of states and actions must be estimated. The state-value function $V^\pi : S \rightarrow \mathbb{R}$ returns the expected reward (or cost) when following π from state s :

$$\begin{aligned} V^\pi(s) &= \mathbb{E}[R_t | s_t = s] \\ &= \mathbb{E}_\pi \left[\sum_{k=0}^{\infty} \gamma^k r_{t+k+1} | s_t = s \right] \\ &= \mathbb{E}_\pi \left[r_{t+1} + \gamma \sum_{k=1}^{\infty} \gamma^k r_{t+k+1} | s_t = s \right] \\ &= \sum_{s' \in S} P(s' | s, \pi(s)) [r(s, \pi(s), s') + \gamma V^\pi(s')] \end{aligned} \quad (6)$$

where $0 \leq \gamma \leq 1$ is the discount rate. Assuming an optimal policy π^* , the optimal value function may be rewritten as $V^{\pi^*} = V^*$ and Bellmann's *principle of optimality* holds:

"An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision."

Richard Bellman, 1957 [4]

$$V^*(s) = \max_{a \in A} \sum_{s' \in S} P(s' | s, a) [r(s, a, s') + \gamma V^*(s')] \quad (7)$$

The *Bellman equation* shown in Equation (7) rewrites the value estimation problem as a recursive definition of the value function. This recursion breaks the complex optimization problem into simpler subproblems, that can be solved via *Dynamic Programming*[4].

2.3.4 Value Iteration

Bellmann's *value iteration algorithm* recursively applies the Bellman equation to an arbitrarily initialized value function, as shown in Algorithm 1. The algorithm repeatedly computes V^{k+1} for all states $s \in S$, until convergence: $\lim_{k \rightarrow \infty} V^k = V^*$. In practice, value iteration is stopped, once the Bellmann residual (i. e. the difference between both sides

of the equation) falls below a predefined threshold θ .

Algorithm 1: Value Iteration.

Input: MDP, $\theta > 0$
Output: approximate of V^*
 $V^0(s) \leftarrow$ arbitrary values
 $k \leftarrow 0$
repeat
 $k \leftarrow k + 1$
 foreach state $s \in S$ **do**
 $V^k(s) = \max_{a \in A} \sum_{s' \in S} P(s' | s, a) [r(s, a, s') + \gamma V^{k-1}(s')]$
 end
until $\forall s |V^k(s) - V^{k-1}(s)| < \theta$

Once the optimal value function $V^*(s)$ is known, the optimal policy π^* can simply be extracted thereof:

$$\pi^*(s) = \arg \max_{a \in A} \sum_{s' \in S} P(s' | s, a) [r(s, a, s') + \gamma V^*(s')] \quad (8)$$

2.3.5 Q-learning

A more challenging problem arises, if transition probability matrix $P : S \times A \rightarrow \mathbb{P}(S)$ and reward function $R : S \times A \rightarrow \mathbb{R}$ are unknown a priori. In such case, the *model-free* reinforcement learning technique *Q-learning* [32] can be utilized to learn an action-value function Q , based on previously observed state transitions. The action value function $Q^\pi : S \times A \rightarrow \mathbb{R}$ returns the expected reward (or cost) starting from state s , taking action a , and following π thereafter:

$$\begin{aligned} Q^\pi(s, a) &= \mathbb{E}[R_t | s_t = s, a_t = a] \\ &= \mathbb{E}_\pi \left[\sum_{k=0}^{\infty} \gamma^k r_{t+k+1} | s_t = s, a_t = a \right] \\ &= \mathbb{E}_\pi \left[r_{t+1} + \gamma \sum_{k=1}^{\infty} \gamma^{k-1} r_{t+k+1} | s_t = s, a_t = a \right] \\ &= \sum_{s' \in S} P(s' | s, a) [r(s, a, s') + \gamma Q^\pi(s', \pi(s'))] \end{aligned} \quad (9)$$

Similar to above, the optimal action-value function may be rewritten as $Q^{\pi^*} = Q^*$:

$$Q^*(s, a) = \sum_{s' \in S} P(s' | s, a) [r(s, a, s') + \gamma \max_{a' \in A} Q^*(s', a')] \quad (10)$$

The *Q-learning algorithm*, shown in Algorithm 2, repeatedly computes Q^{k+1} from a set of previously observed transition tuples $D = \{(s, a, r, s') | t = 1..n\}$, until convergence: $\lim_{k \rightarrow \infty} Q^k = Q^*$. Convergence is guaranteed if all state-actions pairs are visited an unbounded

number of times and an appropriate learning rate is chosen [33].

Algorithm 2: Q-learning [32].

Input: $D = \{(s, a, r, s'), \dots\}$, $\alpha > 0$, $\theta > 0$
Output: approximate of Q^*
 $Q^0(s, a) \leftarrow$ arbitrary values
 $k \leftarrow 0$
repeat
 $k \leftarrow k + 1$
 foreach sample $(s, a, r, s') \in D$ **do**
 $Q^k(s, a) = (1 - \alpha)Q_{k-1}(s, a) + \alpha(r + \gamma \max_{a' \in A} Q^{k-1}(s', a'))]$
 end
until $\forall s \forall a |Q^k(s, a) - Q^{k-1}(s, a)| < \theta$

Once the optimal state-action function $Q^*(s, a)$ is known, the optimal policy π^* can simply be extracted thereof:

$$\pi^*(s) = \arg \max_{a \in A} Q^*(s, a) \quad (11)$$

2.3.6 Exploration

In environments with a large (or infinite) number of states, it is not feasible to visit all state-action pairs an unbounded number of times. Instead, the environment must be examined at the best possible rate. The agent has to carefully balance between *exploration* and *exploitation*, i. e. between testing out previously unknown paths and refining of promising actions. As the latter may get stuck in local optima, exploration helps to find the global optimum.

This balancing act can be tackled by a so called ε -greedy strategy. With a probability of $p = \varepsilon \in [0, 1]$ the agent takes a random action and with the remaining probability $q = 1 - \varepsilon$ the agent takes the presumably optimal action, as deduced from his current model. As the agent consolidates his model over time, ε may be slowly reduced.

2.3.7 Function Approximation

In small environments, V and Q -values are usually stored in a lookup table. For continuous or (infinitely) large environments function approximators are used to generalize the V and Q -functions to previously unseen states-action pairs. Besides scalability towards arbitrarily large environments, this may even speed up the learning process, as earlier experiences are generalized to previously unseen states.

More formally V and Q are replaced as follows:

$$V(s) = \hat{V}(s; \theta) \quad (12)$$

$$Q(s, a) = \hat{Q}(s, a; \theta) \quad (13)$$

where θ refers to function parameters, that must be learned through supervised learning methods⁷.

2.3.8 Batch Reinforcement Learning

In the general reinforcement learning problem, the agent observes a state s_t , executes an action a_t and incrementally improves its policy π in conformity with the obtained reward r_t and successor state s'_t in an infinite loop. In *batch reinforcement learning* [16], an optimal strategy is learned from a fixed set of known transition samples $D = \{(s, a, r, s') \mid t = 1..n\}$. As shown in Figure 4, the samples are collected a priori in an exploration phase, that is not part of the actual batch learning task.

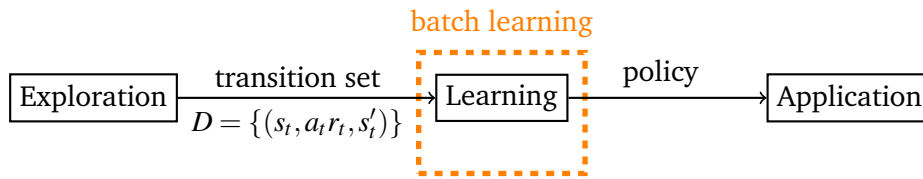


Figure 4: Batch Reinforcement Learning.

Because exploration has an important impact on the achieved strategy quality, more modern *growing batch reinforcement learning* approaches frequently alternate between exploration and learning phase as shown in Figure 5. In doing so, the agent gets a rough idea of a good policy in order to focus on *interesting* regions of the state space.

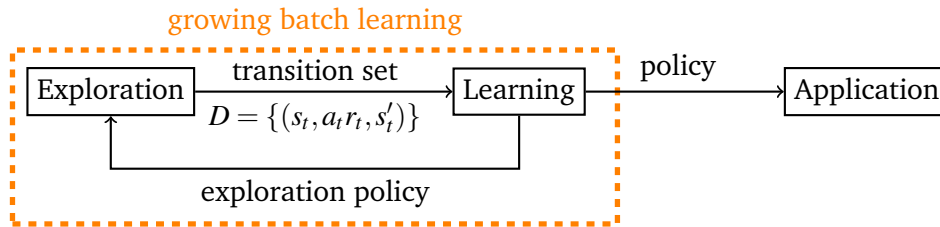


Figure 5: Growing Batch Reinforcement Learning.

2.3.9 Tree-Based Batch Mode Reinforcement Learning

Randomized decision trees can be used to approximate the parameters of $Q(s, a; \theta)$, as proposed by Damien Ernst [9]. The batch reinforcement learning algorithm is shown in

⁷ Supervised learning is a subdomain of machine learning, where a function is learned from labeled training data $\{(x_1, y_1), \dots, (x_N, y_N)\}$. Each training sample maps a feature vector $x_i \in X$ to a desired target value or label $y_i \in Y$. Target values may either be categorical, making the learning task a *classification* problem, or continuous, making the learning task a *regression* problem.

A supervised learning algorithms seeks to find a general function g_θ (or it's parameters θ), such that $g_\theta(x_i) \approx y_i \mid i \leq N$. The learned function should ideally avoid overfitting by finding a generalization to previously unseen data.

Algorithm 3.

Algorithm 3: Fitted Q Iteration [9].

Input: $D = \{(x_t, a_t, s_t, r_t), \dots | t = 1..n\}$
 $\hat{Q} \leftarrow 0$
 $k \leftarrow 0$
repeat
 $k \leftarrow k + 1$
 generate pattern set $P = \{(i^t, o^t) | t = 1, \dots, n\}$ where
 $i^t = (x^t, a^t), \quad o^t = r_t + \gamma * \max_u \hat{Q}^k(s_t, u)$
 $\hat{Q}^{k+1} \leftarrow \text{fit } P$
until stopping condition reached

2.4 Optimized Trade Execution

As outlined in Section 1.2, the problem of *optimized trade execution* is defined by a particular financial instrument, which must be bought or sold within a fixed time horizon, while minimizing the expenditure (share price) for doing so. In order to achieve the best possible price, proper timing is crucial to reduce hidden costs like slippage (see Section 2.1.6).

The general task of this work is to apply reinforcement learning to successfully learn state-based strategies for optimal order placements. Based on the observed market situation and trade progress the obtained Submit & Revise Strategy (see Section 2.1.8) may become more or less aggressive as time passes.

The examined agents derive their strategies from a (growing) batch of sample transitions stemming from interactions with a simulated trading environment, that is described in the next chapter.

3 Orderbook Trading Simulator

As reinforcement learning requires agents to interact with an environment, a trading simulation framework is implemented. The accrued Orderbook Trading Simulator (OTS) provides a full-featured reinforcement learning environment, that simulates the execution of orders on historic data and returns vital feedback to steer the agent towards optimal decisions. Additionally, it serves as a backtesting framework in order to evaluate the cost reducing capabilities of different trading strategies.

This chapter describes the origin and quality of the assessed orderbook data, as well as the OTS and it's underlying OrderbookContainers.

3.1 Data Origin

Since typical financial data providers must make an earning from their treasures, they typically only deliver delayed market data on a complimentary basis. Investors dependent on real time or level 2 market data (see Section 2.1.5) are usually charged horrendous monthly subscription fees.

A costless alternative exists in open cryptocurrencies, like bitcoins (see Section 2.2). The digital asset exchange platform Poloniex⁸ provides an open API for querying detailed market data in real time. As their push API, to receive live order book updates and trades, was rather error-prone and buggy when this project started, the decision was made, to query full orderbooks on a minutely basis.

On Nov, 10th 2016, 10:00 am, a daemon was started, to fetch orderbook snapshots up to a market depth of 5.000 from Poloniex via HTTP GET requests (see Listing 1). The volume of recorded orderbook snapshots for nine distinct currency pairs⁹ has since grown to roughly 100GB (as per 2017-06-20). This thesis is based on a condensed version of the currency pair USDT/Bitcoin.

Listing 1: Data fetched from Poloniex via HTTP GET request.

```
# https://poloniex.com/public?command=returnOrderBook&currencyPair=USDT_BTC&depth=5000
{"asks": [[ "705.450000" ,2.772181], [ "705.450196" , 0.139212] ,["706.170000" ←
,0.052838] , ... ], "bids": [[ "705.000000" ,0.158232],[ "703.700000" ,0.001250], ←
... ], "isFrozen": 0, "seq": 63413296}
```

Figure 6 shows the price evolution of currency pair USDT/Bitcoin over the period of recording. Prices have burst from roughly 700\$ to more than 2.000\$. Red and green lines

⁸ <http://www.poloniex.com>

⁹ Recorded currency pairs include USDT/BTC, BTC/ETH, BT/XMR, BTC/XRP, BTC/FCT, BTC/NAV, BTC/-DASH, BTC/MAID, BTC/ZEC

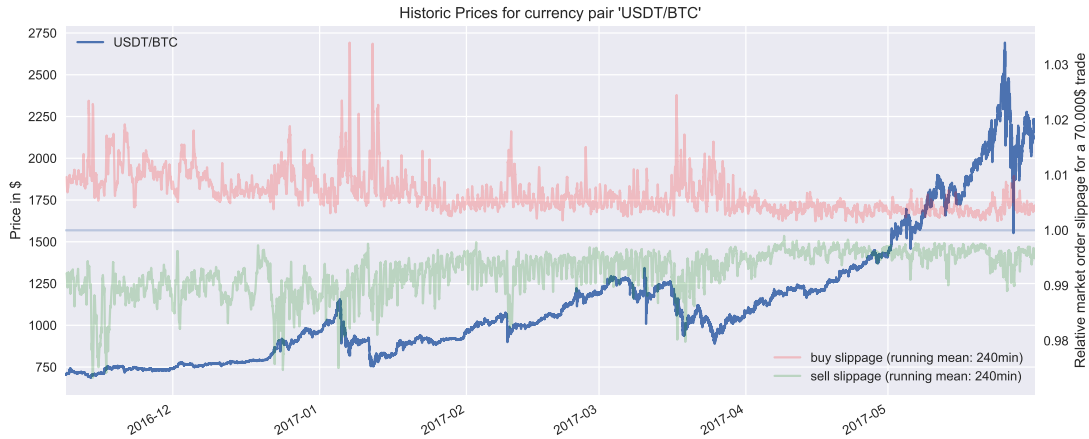


Figure 6: Historic center prices between Nov, 10th 2016 and May, 31 2017, as fetched from Poloniex.

depict the relative difference between the worst price paid/received for an immediate market order of $\pm 70.000\$$ respectively¹⁰.

3.2 Data Preprocessing

The python `class OrderbookContainer` aggregates all informations contained in an individual orderbook snapshot. It enforces correct price ordering in the two opposing bid and ask books and provides additional methods for market visualization and feature extraction. To restrict wasteful memory usage, orderbook snapshots are condensed in several ways:

- Almost identically price levels are round to the second decimal and their respective order volumes merged.

$$\left. \begin{array}{l} 0.139212 * 705.450000\$ \\ 2.632969 * 705.450196\$ \end{array} \right\} = 2.772181 * 705.45\$$$

- Market depth is capped just above the threshold of 100 bitcoins, roughly corresponding to a market depth of 100-140 prices levels in both books. This threshold allows to simulate trades up to a market order price of 70.000\$ at any time throughout the whole recording period.
- Erroneous orderbook snapshots have been discarded. Occasional errors may occur, due to Poloniex API failures.

¹⁰ With a total trading volume of roughly 40.000.000\$ per day (see Section 2.2.1), these $\pm 70.000\$$ refer to roughly 4.2% thereof.

These measurements reduce the average individual orderbook snapshots size from 30KB to approximately 6.6KB. As for december 2016, this results into 44.640 snapshots with a total size of 295MB instead of 1.35GB, clearly reducing the memory consumption.

Listing 2 shows the most important functions, provided by the OrderbookContainer class. OrderbookContainer instances are vigorously used by the OTS. Figure 7 shows a plain visualization of an individual orderbook snapshot.

Listing 2: OrderbookContainer.

```

ob = OrderbookContainer(timestamp="2016-11-08T10:00",
                        bids=pd.DataFrame([200., 100., 300.],
                                           columns=['Amount'], index=[28.7, 28.5, 28]),
                        asks=pd.DataFrame([25., 50., 200.],
                                           columns=['Amount'], index=[29., 30., 31.]))
5
# Available methods
ob.plot(outfile='sample.pdf') # plt.show or plt.savefig
ob.asks # pd.DataFrame
ob.bids # pd.DataFrame
10 ob.features # returns a dict of precomputed features
ob.get_bid(), ob.get_ask(), ob.get_center() # float
ob.get_current_price(volume=100) # achievable cashflow by market order
ob.get_current_sharecount(cash=70000) # number of shares aquirable by market order
ob.compare_with(other_ob) # returns orderbook deltas used by the OTS
15 ob.enrich() # computes Volume, VolumeAcc and norm_Price
ob.head(depth=3) # returns the orderbook, capped at a market depth of 3
ob.plot()

```

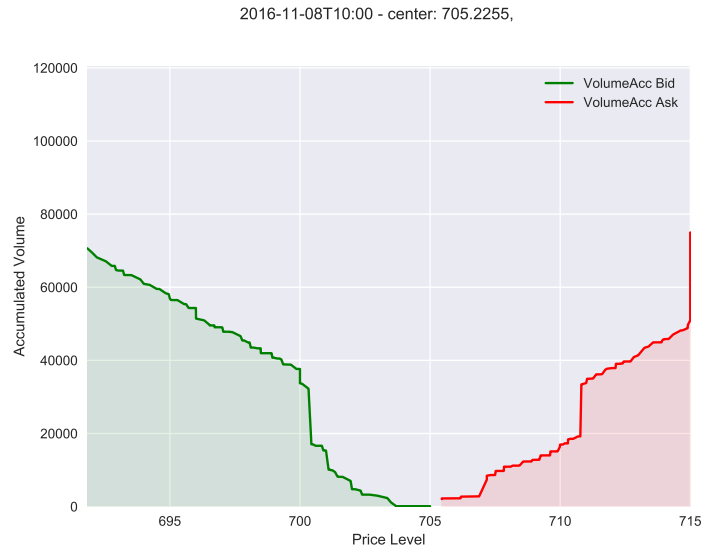


Figure 7: A simple visualization of an limit orderbook.

3.3 Simulator

The OTS framework serves as base for all preceding experiments and evaluations. Each simulator instance is fed with an array of subsequent OrderbookContainers (*orderbook*

windows) and a targeted trading volume V , which it pretends to trade into cash or vice versa within a fixed time horizon H , according to an external strategy.

In the rare case of missing orderbook snapshots (see Section 3.2), the *real* time horizon may be larger than usual, since always H subsequent orderbooks are selected. By default, the **OTS** refuses to work with orderbook windows, whose actual length differs from the presumed length by more than two minutes.

Limit orders may be placed at predefined, discrete time steps within the trading horizon H . This is done through the simulator's main interface method `trade(limit=...)`. The simulator is done, once the remaining trading volume is zero, which is enforced at the very last time point. Any remaining volume at $H - 1$ is transformed into a simple market order and executed immediately, at any price. Additional parameters control the simulator's precise behavior:

volume : The targeted trading volume V .

Positive values indicate buy orders, negative values indicate sell orders.

consume : `'cash'` or `'volume'`

Defines whether `volume` should be interpreted as *cash* (goal: buy/sell shares for V dollars), or as *sharecount* (goal: buy/sell V shares).

period_length : `default=15`

Defines the duration at which a limit order is executed. After a trade has been placed, the simulator iterates over the next `period_length` orderbooks, before the results are reported and a reviewed order may be placed.

tradingperiods : `default=4`

Defines the number of trade reviews, that can be made within the time horizon $H = \text{period_length} * \text{tradingperiods}$.

max_lengh_tolerance : `default=2`

Defines the accepted tolerance between actual and presumed trading horizon in minutes. Throws `ValueError`, if exceeded.

costtype : `default='slippage'`

Defines which of multiple cost functions to use in the reports returned.

3.3.1 Orderbook and strategy visualization

Figure 8 visualizes a 60 minutes long orderbook window, where the solid red lines mark the *average* (8a) and *worst* (8b) price, that has to be paid at a given time point, in case of an immediate market order of 100, 75, 50 and 25 bitcoins respectively. Analogously, the solid green lines represent achieved prices for sale orders of -25 , -50 , -75 and -100 bitcoins respectively.

As can be seen in this graph, ask prices (red) deviate slightly more from the center price than bid prices (green) do. A plausible inference is, that imbalances between demand and supply serve as a valuable indicator for future price trends.

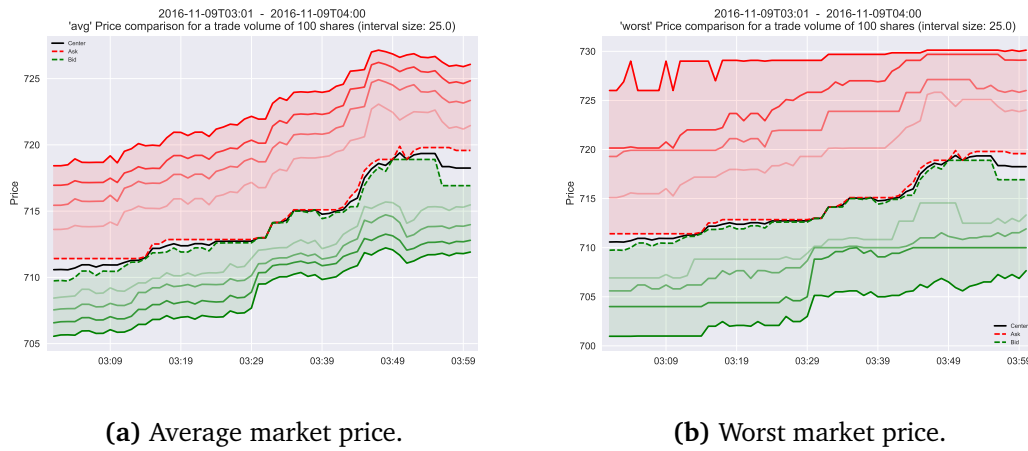


Figure 8: An orderbook window over a period of 60 minutes.

3.3.2 Masterbook

During instantiation, the `OTS` creates a copy of the first orderbook, called the *masterbook*. Hereinafter, executed trades do only affect this internal *masterbook*. The remaining orderbooks are then converted into *deltabooks*, containing only changes between subsequent orderbooks.

The `OTS` may be reset to its initial state at any time via `ots.reset()`. This avoids computational overhead, when testing out multiple strategies on the same *window*, as only *masterbook* and *history* are reset, while *deltabooks* need not be recomputed and *orderbooks* need not be retransferred. `ots.reset()` provides optional parameters for modifying the simulators start conditions. As such, the simulator might be instructed to start at a `custom_starttime` or with a `custom_startvolume`, to simplify the exploration of possible strategies.

3.3.3 Trade execution

The simulated trade execution is triggered by an external call to `trade(limit=...)`. The `OTS` expects a `limit` and iterates over the next `period_length` orderbooks, matching all eligible orders. The `limit` represents the highest accepted price level for buy orders and the lowest accepted price level for sell orders respectively. If `limit=None`, a simple market order is performed.

In a first step, all eligible orders, bound by the given limit and the total `trade_volume`, are cut from the internal *masterbook* and pasted into the `ots.trade_history`, as such these

orders are assumed be fulfilled. The `volume` and `cash` variables are updated accordingly. The simulator then moves to the next time point and adds the corresponding *deltabook* to the masterbook.

In case of order size reductions¹¹, negative order sizes may appear in the masterbook. They are silently dropped, assuming they were matched before they actually vanished from the market. This is a possible source of trouble, as this assumption can not be proven to be valid. As such, matching orders that are simultaneously updated to another price level, are virtually doubled. They are perceived as a new order, even though the responsible market participant has already realized an execution and no basis to submit another one.

- (1) `master += diff(ob[t] - ob[t-1])`, drop negative share counts.
- (2) perform trade: buy until given limit
- (3) `master -= bought bitcoins`
- (4) done, if `volume==0` or `t == T - 1` (`forced=True`)

Figure 9: The circuit of masterbook adjustments.

The masterbook is then ready to be queried again. Any eligible *new* orders are cut from the masterbook and past into the `ots.trade_history`. After `period_length` steps, a detailed trading report, as shown in Table 2 is returned. The upper case columns represent internal variables and orderbook statistics observed at the particular period start. The lower case columns summarize the actual trade in terms of highest, lowest and average price achieved, traded volume, cash flow and observed costs.

	ASK	BID	CENTER	SPREAD	LIMIT	T	VOLUME	volume_traded	CASH	cash_traded	...	avg	forced	initialMarketAvg	low	high	cost
03:01	711.42	709.74	710.58	1.68	713.0	15	100.00	46.77	0.00	-33280.72	...	711.51	False	718.42	711.42	713.00	43.48
03:16	712.52	711.86	712.19	0.66	715.0	15	53.23	28.90	-33280.72	-20630.99	...	713.99	False	718.42	712.52	715.00	98.53
03:31	715.10	712.98	714.04	2.12	717.5	15	24.33	6.68	-53911.71	-4780.95	...	716.16	False	718.42	715.10	717.41	37.28
03:46	718.60	717.77	718.18	0.83	720.0	15	17.65	17.65	-58692.66	-12706.15	...	719.73	False	718.42	718.60	720.00	161.57

Table 2: Trading history, as returned after four consecutive calls of `ots.trade()`.

3.3.4 Visualization

A visual representation of the same trading strategy, which underlies Table 2, is shown in Figure 10. Here, the *OTS* was instructed to buy 100 bitcoins within a period of 60 minutes and called with four consecutive limits prices 713, 715, 717.5 and 720, which are shown as grey step function in the upper subplot.

¹¹ Order size may be reduced due to order fulfillment, order updates or order cancelations through the other market participants.

The second subplot displays trade executions and declining trade volume over time and the bottom subplot displays induced costs. Expected costs conform to the respective accumulated costs, extrapolated to the full trade volume.

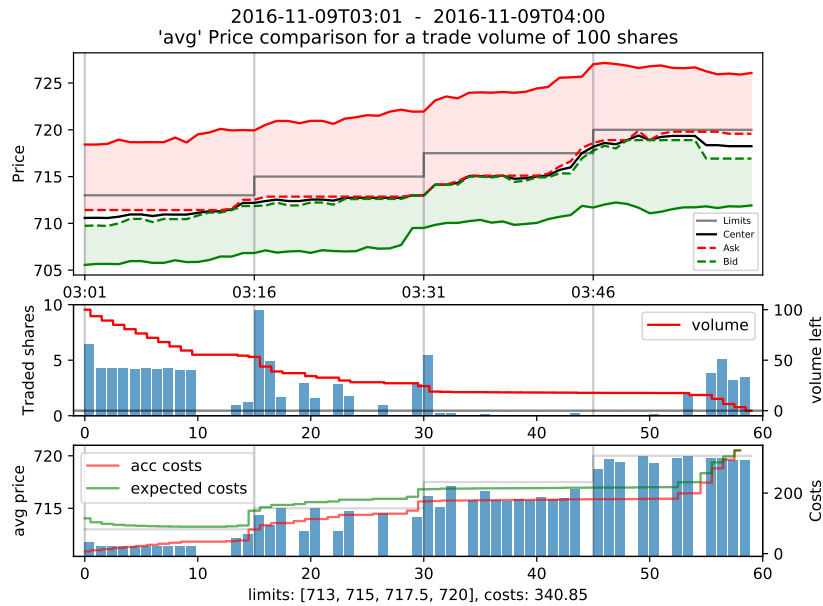


Figure 10: Visualization of an exemplary trading strategy.

A total slippage of 340.85\$ was generated, whereas a simple market order would have cost 784.32\$.

3.3.5 Model correctness

The presented model does not account for market reactions induced by the currently executed trading strategy. It moronically follows the market trend, as it would have evolved without any intervention. Some sources of troubles are pictures below:

- As stated in Section 3.3.3, the presented model can not distinguish between a limit order being removed from the market makers orderbook and a limit order essentially only being updated to another price level. In the latter case, the order presumably wouldn't have reappeared in the orderbooks after the simulator matched it.
- Other market participants typically monitor market activity thoroughly, which is particularly true for purely electronic markets of digital assets, like bitcoins. It is delusive to assume, that no other market participants or trading bots react to large orders, that eat significantly into the orderbook.
- Orderbooks on a minutely basis miss a great part of the markets volatility. In addition to orderbook snapshots, professional data providers typically grant access to market level 2 data (see Section 2.1.5) in form of log files as well. The log files consist of timestamped orderbook updates (typically of type *remove*

and *modify*), which allow the reconstruction of the orderbook for an arbitrary time point. As mentioned in Section 3.1, Poloniex push API to receive live order book updates and trades was rather error-prone and occasionally failed to report important orderbook updates. As the valid reconstruction of orderbooks is highly vulnerable to missing logs, inconsistencies arose and the decision was made, to query orderbooks on a minutely basis only.

- Hidden Orders, as introduced in Section 2.1.7, are not accounted for.

As a consequence, trading on the masterbook can only be seen as a rough approximation of true market behaviour. Curious masterbook shapes, resulting from the described simulation process, encourage to perform actual feature extraction on the original orderbooks, examining the market as it would have evolved without any interventions.

4 Reinforcement Learning Approaches

This chapter describes two RL approaches used to tackle the important problem of optimized trade execution. Based upon the RL formulation described by Nevmyvaka et al. [22], Q-learning and dynamic programming are fused to find an optimal, state-based strategy from a training set of historic orderbook data. While sampling the discretized state space¹² in a backward, brute force manner, the authors achieved an impressive 50% gain over the more simple Submit & Leave Strategy (see Section 2.1.8). The novel, forward-learning approach proposed in this work, outperforms the original method, as it samples from the continuous state space through forward exploration in order to generate more realistic transitions.

Some of the assumptions made in the original, backward approach deserve a closer look and motivate for modifications. As the authors assessed a rather large, proprietary dataset of 1.5 years of millisecond time-scale limit order data from NASDAQ, it is furthermore intriguing to evaluate its performance on a smaller, self-recorded dataset of limited resolution.

4.1 General Framework

In the first place, the general framework of the reinforcement learning problem is described.

4.1.1 State space

The state space consists of various variables, describing the current trade progress (*private variables*) and the current market situation as observable from orderbook data (*market variables*). The two private variables *remaining time* (`time`) and *remaining inventory* (`volume`) make the base for all subsequent experiments, while optional market variables may be enclosed to potentially assist the process of decision finding.

Each `state` $s \in \langle \text{volume}, \text{time}, o_1, o_2, \dots \rangle$ forms a vector of at least both private variables, plus a variable number of market variables. More specifically, the following market variables may be added to the state space to further improvement the cost reducing capabilities of the strategies obtained over plain `VolTime`-strategies.

Bid-Ask Spread : Spread between best bid price and best ask price.

Bid-Ask Volume Misbalance : Volume imbalance between orders at the best bid price and the best ask price.

¹² The state space describes actual trade progress (i. e. *remaining time* and *remaining inventory*) as well as current market situation.

Immediate Market Order Cost : Costs, if remaining volume would be executed immediately, at the current market price.

Signed Transaction Volume : Signed volume of all trades executed within the last 15 seconds. A positive value indicates more buy orders, while a negative value implies to more sell orders being executed.

Except for *Immediate Market Order Cost*, the market variables originally exploited basically stem from market level 1 data only. In addition to the above mentioned market variables, the impact of additional market level 2 features, describing potential imbalances between ask and bid book and forecasting estimated action consequences is examined.

marketPrice : Relative difference between the current center price and the worst price that must be paid (or is received) in case of simple market orders.

sharecount : Number of Bitcoins, immediately available for the specified amount of cash.

center_price : Current center price.

_a*_effects : Minimum cash volume immediately tradeable by individual actions. The value is a lower bound estimate of the true volume, because only the current order-book is retrievable. New opportunities, arising within the forthcoming `trading_period` are unacquainted and thus not included.

In the original approach, market variables are discretized into 0 (low), 1 (medium) and 2 (high), while the concrete category mapping process is not further described. In this work, market variables are evenly transformed into N bins $o_m \rightarrow o_{m_discN}$, where boundaries between bins are automatically derived from the respective training data quantiles. Private variables are discretized in 4 to 8 bins, while greater resolutions lead to strictly better results and linearly higher computation times.

4.1.2 Action space

Actions define the level of trading aggression to be performed. In the original approach, action $a \in \mathbb{R}$ defines the deviation between current best price and chosen limit price, as $bid + a$ (for buy orders) and $ask - a$ (for sell orders). As such, actions must cross the full bid-ask-spread to find matching orders in the opposing book. Large actions a result in larger fractions of the order volume being matched immediately. Smaller or even negative actions a help to prolong the actual execution, and may help to profit from opportune market movements.

In case of the exemplary market situation shown in Table 3, a buy order with an aggressive action $a = 1.4$ would map into $limit = bid + a = 28.7 + 1.4 = 30.1$. This limit would allow trading up to 75 shares instantaneously.

	Amount	Type	Volume	VolumeAcc	norm_Price
31.00	200.0	ask	6200.0	8425.0	1.074533
30.00	50.0	ask	1500.0	2225.0	1.039871
29.00	25.0	ask	725.0	725.0	1.005208
28.85	<i>NaN</i>	center	<i>NaN</i>	<i>NaN</i>	<i>NaN</i>
28.70	200.0	bid	5740.0	5740.0	0.994810
28.50	100.0	bid	2850.0	8590.0	0.987877
28.00	300.0	bid	8400.0	16990.0	0.970546

Table 3: Action $a = 1.4$ translates into $limit = 28.7 + 1.4 = 30.1$.

The employed number of selectable actions and their actual value range was not further specified.

As the proposed mapping method does not necessarily fit well to the bitcoin data at hand, two modifications are made: On the one hand, Bitcoin prices have burst from roughly 700\$ to more than 2.000\$ in the period of recording (see Figure 6), i. e. interpreting actions as the absolute difference to the current best prices, maps to significantly different levels of aggression as time passes. On the other hand, the large volatility of Bitcoins (see Figure 2) causes rapid fluctuations in the bid-ask spread as a side effect. Fixing the limit base to the opposing best price, the spread must no longer be crossed and action $a = 0$ now generally refers to the first action that causes a guaranteed execution volume larger than zero.

The mapping functions are thus redefined as follows:

`limit_buy = ask * (1 + (a/1000))` instead of `limit_buy = bid + a`
`limit_sell = bid * (1 - (a/1000))` instead of `limit_sell = ask - a`

Typically the following set of fifteen actions is used: $L = [-4, -3, \dots, 9, 10]$, where e. g. a buy order with action $a = 4$ translates into $limit = ask * 1.004 = ask + 0.4\%$.

4.1.3 Costs

Costs are defined as the slippage induced from the previously chosen actions. The baseline is given by the initial center price. The following formula is used to compute (partial) costs in terms of price deviation from the idealized case of buying all shares at the initial center price:

$$cost_{im} = (avg_paid - initial_center) * volume_traded \quad (14)$$

$$initial_center = \left(\frac{ask_t + bid_t}{2} \right) |_{t=0} \quad (15)$$

Since the complete trade execution happens within a finite time horizon and full execution of the `volume` is mandatory, partial costs can simply be summed up without any discounting. Utilizing the `OTS`, it is not possible to compute occurring costs in advance

exactly, as they depend on how the market evolves within the subsequent `trading_period`.

While discretization of market variables mainly affects the strategies explanatory power, the resolution of private variable `volume` leads to considerable rounding issues in regards to the cost function employed. As observed successor states x_{t-1} must be discretized equally to allow looking up the corresponding minimal costs, the immediate costs, as computed in Equation (14), must be scaled accordingly. Simply replacing `volume_traded` with `round(volume_traded)` falsifies the actual costs, as they correlate to the accomplished trade volume in an unpredictable, non-linear manner as indicated in Figure 11.

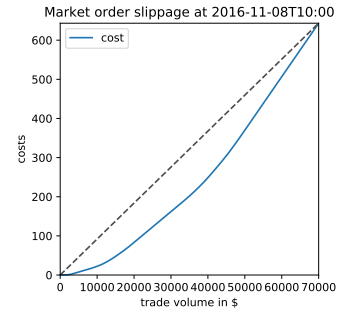


Figure 11: Non-linear slippage growth.

Nevmyvaka et al. [22] did not mention this problematic and presumably did not perform any cost scaling at all. Despite the non-linear relationship between trading volume and observed costs, this simple, but imprecise approach of *linear cost scaling* is used in all discrete state spaces examined. In continuous state spaces, this problem is void.

4.1.4 Subject of Trade

Nevmyvaka et al. [22] tackled the problem of trading a certain amount of shares within a fixed time period. A more practicable problem definition must distinguish between the concrete trading direction. While the original problem definition fits to the case of selling shares that are in the investors possessions, it does not mirror the opposite case properly. If the subject of trade is to obtain shares, it is more realistic to define a fixed amount of disposable *cash*, rather than the number of *shares*, that shall be acquired. Subsequently, all experiments in buy direction are instructed to commute a fixed amount of cash into as many shares as possible.

If cash is the subject of trade, the optional market variable *Immediate Market Order Cost* (e. g. 72.321\$) becomes meaningless, and is replaced by *Immediate Market Order Shares* (e. g. 92 BTC), quoting the number of shares acquirable through an immediate market order.

4.2 Backward Learning

In order to learn the optimal limit for each possible situation, orderbook windows are examined in a backward, brute-force manner as described in Algorithm 4. Each orderbook window from the training data set is sampled $T * I * L$ times, where T is the number of performed limit revisions, I is the number of discrete volume states and L is the number

of available actions.

Algorithm 4: Optimal_strategy[22].

Input: $V=70.000\$, H=60\text{min}, T=4, I=[12.5\%..100\%], L=[-4..10]$

```

for  $t=1$  to  $T$  do
  while not end of data do
    Transform (orderbook)  $\rightarrow o_1..o_R$ 
    for  $i=0$  to  $I$  do
      for  $a=0$  to  $L$  do
        Set  $x = [t, i, o_1, \dots, o_R]$ 
        Simulate transition  $x \rightarrow y$ 
        Calculate immediate cost  $cost_{im}(x, a)$ 
        Look up  $\text{argmax cost}(y, p)$ 
        Update cost  $([t, v, o_1, \dots, o_R], a)$ 
      end
    end
    Select the highest-payout action  $\text{argmax cost}(y, p)$  in every state  $y$  to output optimal policy
  end
end

```

While the algorithms running time depends solely on the resolution of the two private variables, the chosen action space and the size of the training data set, it is approximately independent of the number of market variables chosen. For the transition simulation, the model described in Section 3.3.3 is utilized. The cost update rule is given below:

$$cost(x_t, a) = \frac{n}{n+1} cost(x_t, a) + \frac{1}{n+1} [cost_{im}(x_t, a) + \arg \min_p cost(x_{t-1}, p)] \quad (16)$$

The algorithm assumes the individual trading periods to be of an (approximately) Markovian nature, where the optimal action to choose at state x_t with $t = \tau$ is completely independent of actions chosen previously ($t \geq \tau$).

As such, the state-action function can be computed inductively via dynamic programming. In a first round, expected costs for all actions in states being immediate predecessors of end states (i. e. $t = 1$) are computed according to Equation (16). Figure 12 visualizes the resulting optimal costs (right) and corresponding actions (left) after the first round has finished.

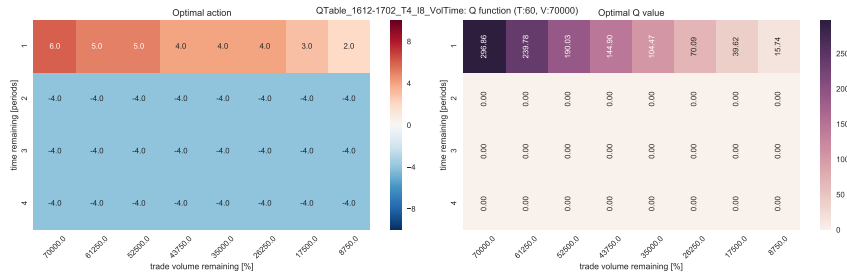


Figure 12: State-Action function, visualized after the first training round.

Knowing the optimal state-action values for all states with $t = 1$, all informations are given to compute the optimal state-action values for their predecessor states with $t = 2$ (see Figure 13).

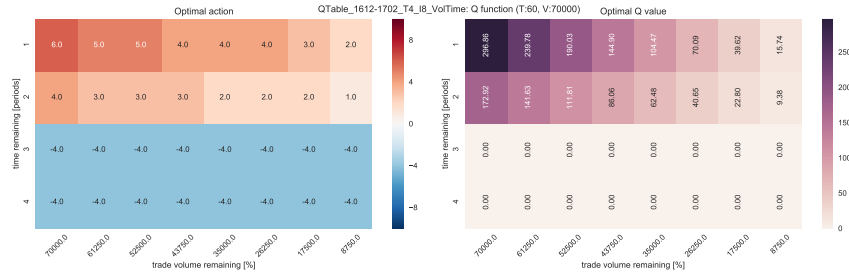


Figure 13: State-Action function, visualized after the second training round.

After T iterations a globally optimal policy, as shown in Figure 14, has been found. The annotated q values (right) denote the corresponding minimum over all available actions. The visualized state-action function was trained over orderbook snapshots from Nov, 10th 2017 10am to April, 30st 2017, partitioned into 4.154 orderbook windows of 60 minutes length each. Over a trading horizon of $T = 4$ `trading_periods`, 70.000\$ cash (discretized in $I = 8$ intervals) had to be traded into Bitcoins. The state space included the two private variables `time` and `volume` only, while the action space comprised $L = 15$ actions. As such, a total of $4.154 * 4 * 8 * 15 = 1.993.920$ transition tuples were generated.

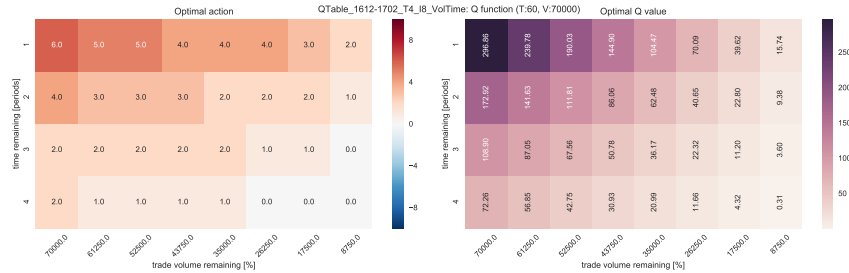


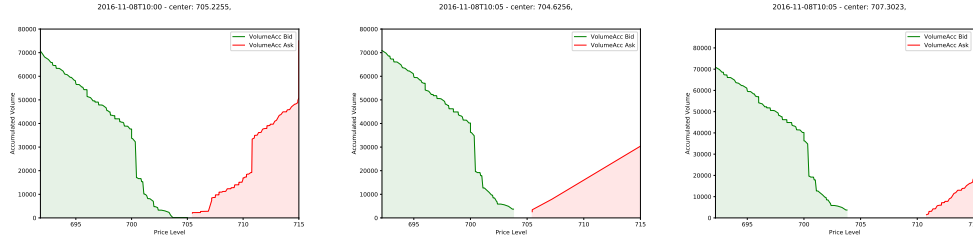
Figure 14: Final State-Action function.
 $T = 4$ (60min), $I = 8$ (70.000\$), $L = 15$ ($[-4, -3, \dots, 9, 10]$)

Figure 14 illustrates clearly, how the optimal strategy becomes more aggressive as time ceases and a large portion of the trading volume remains unexecuted.

4.2.1 Incorporating preceding trades

In order to strengthen the Markov Property, i. e. the assumption that states capture all relevant information from the history, the preceding trade history (i. e. the self-induced market impact) must be taken into account. Figure 15 shows differing masterbook shapes, that can build the base of a simulated `trading_period`, e. g. if starting at $t = 45min$ and a remaining trade volume of 17.500\$. 15a shows the original orderbook, as if no orders had been matched by our strategy. This version of the masterbook is queried by the original backward algorithm, potentially resulting in more passive trading aggressions,

as it does not account for the self-induced impact on the market at all. 15b and 15c show two ideas, how the market impact of the supposedly consumed trading volume of $70.000\$ - 17.500\$ = 52.500\$$ may be incorporated. Both lead to different masterbook shapes and as such to different subsets of attainable prices.



(a) Original Orderbook. (b) Assuming 52.500 shares being matched at $t=0$, then no further matches. (c) Assuming 52.500 shares being matched evenly at 1.166 shares per minute.

Figure 15: Different shaped masterbooks at $t=45$.
The shapes differ, depending on the preceding trade history.

Algorithm 5 shows a modified version of the original backward-learning algorithm. Prior to the actual trade simulations, the supposedly consumed trading volume is removed from the masterbook. The orders affected are matched and removed from the masterbook evenly along the elapsed time, as indicated in Figure 15c.

Algorithm 5: Optimal_strategy, improved.

Input: $V=70.000\$, H=60\text{min}, T=4, I=[12.5\%..100\%], L=[-4..10]$

for $t=1$ **to** T **do**

while not end of data **do**

 Transform (orderbook) $\rightarrow o_1..o_R$

for $i=0$ **to** I **do**

for $a=0$ **to** L **do**

 Incorporate preceding trade history

 Set $x = [t, i, o_1, \dots, o_R]$

 Simulate transition $x \rightarrow y$

 Calculate immediate cost $_{im}(x, a)$

 Look up $\text{argmax cost}(y, p)$

 Update cost $([t, v, o_1, \dots, o_R], a)$

end

end

 Select the highest-payout action $\text{argmax cost}(y, p)$ in every state y to output optimal policy

end

end

4.2.2 Separation of Sampling & Learning Phases

The process of sampling from training data and learning optimal state-action values is split into two independent phases, mostly due to performance¹³ reasons. In the *exploration phase* all possible combinations of actions and private variables are evaluated and stored as 5-tuples $sample = (state, action, cost, timestamp, new_state)$. As market variables are supposedly not influenced by the agents behavior (see Section 4.1.1), it suffices to remember the orderbooks time point, to allow adjacent substitution of market variables.

Samples are stored as DataFrame and may be exported and reused by other agents¹⁴, as wished. This allows to skip the time consuming process of backward sampling. Before the action learning phase starts, the collected samples may be enhanced by a variety of market variables, discretized or untouched. The helper function `addMarketFeatures_toSamples()` retrospectively adds market variables to the agents samples DataFrame.

If desired, features are discretized according to their individual value range. E. g. a chosen resolution of 3 evenly transforms feature o_m into o_{m_disc3} , according to automatically determined boundaries:
0 (if $o_m < 1/3$ quantile), 1 (if $1/3 \leq o_m < 2/3$ quantile) and 2 (if $o_m \geq 2/3$ quantile). Boundaries arise from the training data and are applied to test data unaltered.

The learning phase starts hereinafter, whereas two types of *orderbook agents* have been implemented and experimented with, each of which employ different techniques:

QTable_Agent : Dynamic Programming as described in Section 4.2.

Expected state-values are stored in a simple lookup table, hence discretization of the state space is required. For each state, a vector of length $L = num_actions$ is maintained, of which the first argmin refers to the optimal action. The *QTable* does not generalize to previously unobserved states and returns the very first action (here -4) by default. For proper cost-updates an additional *NTable* is maintained, referencing the particular number of observations made.

QTable and NTable entries are computed in an iterative manner. In the first round, all samples with `time_left=1` are evaluated, in the second round all samples with `time_left=2`, etc. The private variable `volume` is discretized according to the specified resolution and linear cost scaling (see Section 4.1.3) is performed.

BatchTree_Agent : Tree-Based Batch Mode Reinforcement Learning [9].

An ensemble of 400 decision trees, with a maximum allowed depth of 15 is fed

¹³ The backward sampling phase for the example mentioned in Section 4.2, took roughly 10 hours, even though the work load was distributed among 24 CPU's.

¹⁴ Agents should refer to the same environment settings, as samples collected from different environments (e. g. trading horizon of $H = 8min$ vs. $H = 60min$) may not be compatible.

with the full batch of samples simultaneously, no discretization required. T Fitted-Q-learning rounds (see Section 2.3.9, Algorithm 3) are performed, to completely allow expected costs to unfold over the given time horizon.

The state space is enlarged by an extra dimension for the chosen action. Predicting the optimal action to choose for a given state, the ensemble must predict $L = \text{num_actions}$ q-values, from which the argmin refers to the optimal action.

Each *orderbook agents* inherits some commonly shared functionality from the super class `RL_Agent_Base`. This base class provides a consistent framework, allowing to swap the logic behind `learn_from_samples` and `predict` only, while general functionality like `load`, `save`, `plot_heatmap_Q` and `collect_samples` may be reused.

An *orderbook agent* holds the environment parameters, used to initialize the OTS. As such, different definitions of the optimal trading problem require different orderbook agents.

4.3 Forward Learning

In order to collect more realistic transition samples, an alternative, *forward sampling* method is presented. In a growing batch reinforcement learning approach the trained agent alternates between exploration and learning phase on a regular basis.

4.3.1 Forward Sampling & Learning

Rather than examining all available actions on the great variety of combinational possibilities the state space has to offer, the forward approach repeatedly simulates trades in their entirety. The once again utilized OTS always starts with $V = 100\%$, but may be initialized at random start points (e. g. `t=0`, `t=15`, `t=30`, `t=45`) and keeps going until the full trade is executed.

As such, the obtained transitions stem from a more natural and furthermore fully continuous environment. Only actually attainable states, that could arise in the (simulated) reality, are used as transition start points, potentially increasing their significance. As major disadvantage, this forward sampling approach does not assure exhaustive exploration of the state space. A thorough exploration is not self-evident and must be supervised.

The pseudo-code of the forward sampling approach is shown in Algorithm 6. The algorithm iterates over the training data, starts E exploration phases per orderbook window and follows an ϵ -greedy exploration strategy. Each exploration phase starts with

$\varepsilon = 1.0$ which slowly decays to $\varepsilon = 0.05$ according to $\varepsilon = 0.05^{e/E}$.

Algorithm 6: Forward sampling and learning approach.

Input: data, $V=70.000\%$, $H=60\text{min}$, $T=4$, $L=[-4..10]$, $E=60$, $\text{retrain}=256$
 Shuffle(data)
 Split data into chunks of length *retrain*
while not end of chunks **do**
 for orderbook window in chunks **do**
 Init OTS(orderbook_window, V, H, T, L)
 for epoch=0 to E **do**
 Reset OTS to $V = 100\%$ and random time point (in H)
 repeat
 Set $x_t = [\text{time_left}, \text{volume_left}, o_1, \dots, o_R]$
 Enquire ε -greedy action from model
 if action chain led to an end state previously **then**
 | choose other action
 end
 Apply action
 Remember transition $\{x_t, \text{action}, \text{cost}, x_{t+1}\}$
 until $V = 0\%$
 end
 Retrain model from collected transitions (growing batch)
end
end

If ε -greedy proposes an actions, that previously led to an end state (i. e. trade completed), an alternative action is sampled according to a gaussian distribution with its peak at the originally proposed action. Before the actual random draw, probabilities of all known dead-end-actions are set to zero, to forestall pointless resampling. In case of repeatedly running into dead-ends (e. g. `max_dead_ends=4`) the exploration phase is aborted and continues with the next orderbook.

After every `retrain` exploration phases, the new samples are fit into the agents model (e. g. `retrain=256`) and the updated model is used for future ε -greedy exploration. Prior to the sampling phase, the training set is shuffled.

In contrast to the backward sampling method, market variables may be queried on the go, such that they entail the strategies impact. Because sample trajectories stem from a continuous state space, function approximations prove handy. In this work, a RandomForestRegressor (i. e. BatchTree (BT)) is utilized and retrained from scratch every `retrain` exploration phases. The alternative approach of exploiting Neural Fitted Q-iteration [26], has been postponed to future experiments.

5 Discussion and Experiments

This chapter summarizes various experiments, that have been conducted to examine the agents ability to find optimal solutions to the problem of optimized trade execution. The recorded orderbook snapshots for currency pair USDT/BTC (see Section 3.1) have been split into training period (Nov, 10th 2016 - Apr, 30th 2017) and test period (May 2017).

All experiments refer to the very same environment settings and build up on each other to some extend. The agents common task is to buy Bitcoins worth of 70.000\$ within a trading horizon of 60 minutes. The problem definition is opposed to the task evaluated by Nevmyvaka et al. [22], where the number of shares (respectively Bitcoins) is fixed, rather than the amount of cash.

The training set translates into 4.154 orderbook windows, while the test set gives 724 orderbook windows. A period length of 15 minutes is assumed, such that the **OTS** expects up to $T = 4$ order limit prices. Private variable `volume` is discretized in 8 intervals, i.e. $I = [8.750\$, \dots, 70.000\$]$, and the action space consists of fifteen actions $L = [-4, -3, \dots, 8, 9, 10]$. In line with the formulas presented in Section 4.1.2, these actions translate into order limits deviating by -0.4% to $+1.0\%$ from the current best price.

5.0.1 Baseline

Simple **S&L** strategies (see Section 2.1.8) and immediate market order placements serve as benchmark for the performance measurement.

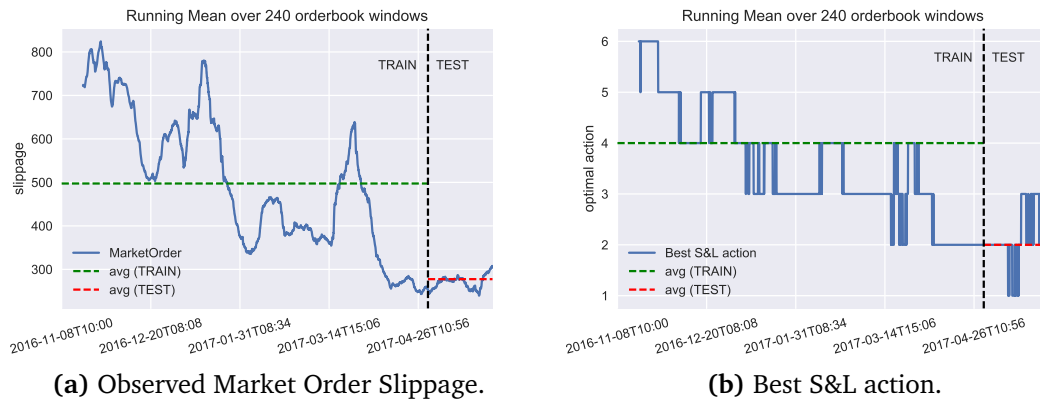


Figure 16: Concurrent to declining slippage, the optimal **S&L** actions become less aggressive as time passes.

Due to bursting Bitcoin prices (see Section 3.1, Figure 6), the investigated sum of 70.000\$ constitutes a declining contingent of the total market volume. Figure 16a shows a running average over the amount of slippage, as induced by immediate market orders. Concurrent to declining slippage, the optimal **S&L** actions become less aggressive as time

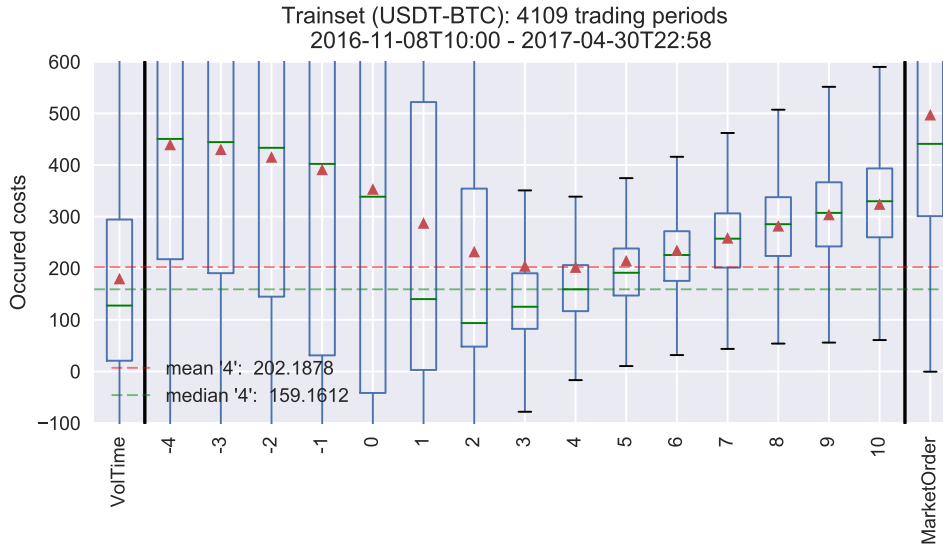


Figure 17: Average costs induced by the varying S&L actions over the full training period. In average, action 4 performed best.

passes. The green, dashed lines show the respective average over the training period, which significantly differs from the red, dotted line referring to the average over the test period. Figure 17 shows the average costs, induced by varying S&L strategies within the training period.

In order to provide a more realistic and unexaggerated baseline, the optimal S&L action is estimated from the test period. As such, performances of subsequents experiments are compared to the performance of a simple market order and the S&L strategy "2", with the initial order limit fixed to 1.002 times the initial ask (i. e. +0.2%).

5.1 Examined Issues

While the presented backward algorithm exploits the available data profoundly in a brute force manner, the underlying assumptions deserve a short discussion. Alternative formulations are tested out, to potentially improve the algorithms cost saving capabilities.

5.1.1 Action-Limit Mapping

By fixing the limit base at the best price of the *opposing* book, it appears obvious, that decisions derived from state spaces including a variable for the current spread size, out-perform decisions derived from state spaces not including this market variable. Indeed, in the original paper[22] the bid-ask spread was posed as the market variable causing the greatest individual impact on cost reduction, namely -7.97%, which is a major fraction of

the maximum achievement of -12.85% ¹⁵.

The positive impact of moving the limit base to the opposite book, as proposed in Section 4.1.2, is shown in Section 5.2.1.

5.1.2 Discrete State Space

The most obvious weakness of the discrete backward approach lies in its vulnerability to seldomly observed market situations. Since the state-action function is implemented as a simple lookup table without any generalization capabilities, it is strictly dependent on a thorough exploration of the underlying state space. As exhaustive exploration is enforced for the value range of private variables only, market variables are entrusted to chance. Especially when increasing the state space dimension by adding multiple market variables simultaneously, the explanatory power of a learned state-action mapping depends crucially on the number of underlying observations. There exists even the chance of certain states never been monitored at all during the training phase.

Consequently, the impact of different market variables at varying levels of discretization is examined, starting in Section 5.2.2. Eventually, the effect of replacing the underlying lookup table with function approximators is studied in Section 5.2.5.

5.1.3 Markov Property

The proposed backward algorithm (see Section 4.2, Algorithm 4) assumes individual `trading_periods` to be of an (approximately) Markovian nature. As explained in Section 4.2.1, the OTS's internal masterbook shape depends drastically from the preceding trade history. Since all relevant information from the history must be taken into account, two approaches are examined, that aim to generate more realistic samples and as such to strengthen the Markov Property of states:

The advantage of the improved backward algorithm, that incorporates preceding trades prior to actual trade simulations (see Section 4.2.1, Algorithm 5), is shown in Section 5.2.3. Section 5.3 demonstrates, that the novel forward sampling algorithm described in Section 4.3 outperforms all other methods, as strategies are derived from a more realistic set of sample transitions.

¹⁵ Strategies derived from five dimensional state spaces including the market variables Spread, ImmCost & Signed Vol, as described in Section 4.1.3, outperformed strategies derived from two dimensional state spaces, containing the private variables only, by -12.85% in average.[22]

5.2 Backward Learning

In the following, backward learning experiments that paved the way towards improved trade execution strategies are presented.

5.2.1 Action-Limit Mapping

As mentioned in Section 5.1.1, it seems pointless to force actions to cross the bid-ask-spread before any orders can be matched. Table 4 shows exemplary for November 2016, how the choice of the limit base affects the agents trading performance.

	<i>slippage</i>	<i>performance</i>
currBid	220.19	92.6%
currBid_mSpread	218.71	91.9%
currBid_spread	215.76	90.7%
currAsk	214.00	90.0%
currAsk_mSpread	214.53	90.2%
currAsk_spread	215.38	90.6%
S&L: 5	237.75	100.0%
MarketOrder	737.58	310.2%

Table 4: Evaluating the impact of different limit base levels.
Results stem from applying the respective strategies on all 537 orderbook windows recorded between Nov, 10th 2016 and Nov, 30th 2016.

While buy orders, forcing agents to cross the bid-ask-spread (i. e. `currBid*`), typically benefit from the two market variables `spread` and `marketSpread`, this is not necessarily the case for agents which have the limit base fixed to the opposing best price (i. e. `currAsk*`). As the latter agents consistently showed better performance, the limit base is henceforth fixed to the best price of the corresponding trading direction.

5.2.2 Additional Market Variables

In addition to the originally proposed market variables (see Section 4.1.1), the following orderbook features have been examined:

marketPrice* describes the relative difference between current center price and the worst price that must be paid (or is received) in case of simple market orders.

sharecount_* quotes the number of Bitcoins, immediately available for 70.000\$.

`sharecount_buy` is tantamount to the originally proposed `ImmCost` (See Section 4.1.4).

center_price was added as a consequence of the findings from Section 5.0.1, Figure 16:

Apparently, as the ratio between current Bitcoin price and the investable 70.000\$ increases, an optimal strategy may have to place less aggressive order limits.

_a*_effects quotes the minimum volume immediately obtainable by individual actions. The value is a lower bound of the truly obtainable volume, because only the current orderbook is retrievable. New opportunities, arising within the forthcoming `trading_period` are unacquainted and not included.

Table 5 shows the performance of individual QTable agents trained on both private variables plus one discretized market variable attached at a time. While observed trading costs undercut those induced by simple market orders by -49.16% , the gain in comparison to the optimal S&L strategy lies at -8.93% .

Adding the market variable `spread` yields a notable improvement of -2.14% over the plain `VolTime`-agent. By increasing the discretization level to 5, the performance can be further improved to -6.34% , verifying the findings of Nevmyvaka et al. [22]. However, in contrast to their reported results, the market variable `sharecount_buy` falls flat with a worsening of $+6.70\%$ (compared to -4.26%).

Disappointing is the futility of `center_price`. For a discretization level of 5, the corresponding agent underperforms the plain `VolTime`-agent by $+7.35\%$. Potentially, this may be explained by a compulsorily underexploited training set: Due to generally rising Bitcoin prices, the majority of prices in the test period will map to the highest discretization value available, which, in a simple lookup-table, resembles only a fraction of the assessed training set.

	slippage	med	std	perf_2	perf_4	perf_M	perf_VolTime
center_price_disc3	149.57	46.15	420.14	-3.44%	-15.19%	-46.10%	-0.70%
center_price_disc5	161.70	37.83	450.36	+4.39%	-8.32%	-41.73%	+7.35%
marketPrice_buy_worst_disc5	146.83	39.62	351.77	-5.21%	-16.75%	-47.08%	-2.52%
marketPrice_sell_worst_disc5	148.94	42.71	388.86	-3.85%	-15.55%	-46.33%	-1.12%
marketPrice_spread_disc5	150.46	44.15	371.32	-2.87%	-14.69%	-45.78%	-0.11%
marketPrice_imbalance_disc5	150.10	68.68	336.60	-3.10%	-14.90%	-45.91%	-0.35%
sharecount_buy_disc3	149.57	46.15	420.14	-3.44%	-15.19%	-46.10%	-0.70%
sharecount_buy_disc5	160.73	37.78	449.22	+3.76%	-8.87%	-42.08%	+6.70%
sharecount_imbalance_disc5	148.03	40.49	372.86	-4.44%	-16.07%	-46.65%	-1.73%
sharecount_sell_disc5	151.50	47.49	425.35	-2.20%	-14.10%	-45.40%	+0.58%
sharecount_spread_disc5	148.93	40.84	352.12	-3.86%	-15.56%	-46.33%	-1.13%
spread_disc3	147.41	35.48	370.96	-4.84%	-16.42%	-46.88%	-2.14%
spread_disc5	141.07	37.39	349.72	-8.93%	-20.02%	-49.16%	-6.34%
spread_disc9	142.80	36.69	364.76	-7.81%	-19.03%	-48.54%	-5.20%
VolTime	150.63	33.83	358.66	-2.76%	-14.60%	-45.72%	0.00%
2	154.90	68.62	389.15	0.00%	-12.17%	-44.18%	+2.84%
4	176.37	141.66	273.58	+13.86%	0.00%	-36.44%	+17.09%
MarketOrder	277.49	246.48	158.66	+79.14%	+57.33%	0.00%	+84.22%

Table 5: Evaluating the impact of additional market variables.
Average performance over the test period may 2017, currency pair USDT/BTC.
See Appendix A.1, Table 12 for complete results.

The different discretization levels yield rather mixed results. While `spread` typically performed best in case of 5 levels, no clear statement can be made for the other variables.

The exact reason, why factually only adding `spread` causes a beneficial effect, is unclear, but one possible explanation lies in the data set employed. The original experiment assessed a different data quality. Due to the low resolution of the available orderbook snapshots, a large fraction of the market activity remains inaccessible and consequently the majority of trading opportunities are missed by the agents. Furthermore, the minute time-scaled data inevitably requires the trading horizon to be rather long. Experiments

with shorter time horizons nullified the achievable savings, while longer time horizons led to unacceptable computation times. Consequently, the agents were trained on 4.109 sixty-minute orderbook windows, while Nevmyvaka et al. invoked 45.000 two-minute orderbook windows.

Look-Ahead Features

In order to proof the algorithms general ability to find costs reducing strategies, look-ahead features¹⁶ were added to the universe of market variables. `future_center*` quotes percentual changes between the current center price and the center price in 5, 15 and 60 minutes respectively. This hypothetical knowledge about future price trends reduces observed trading costs by -12.47% (i. e. -14.88% over simple S&L strategy).

	slippage	med	std	perf_2	perf_4	perf_M	perf_VolTime
ob_direction_disc5	61.99	97.52	319.84	-59.98%	-64.85%	-77.66%	-58.84%
future_center5_disc5	141.61	41.79	389.58	-8.58%	-19.71%	-48.97%	-5.98%
future_center15_disc5	133.78	40.90	405.88	-13.63%	-24.15%	-51.79%	-11.18%
future_center60_disc5	131.85	47.63	391.46	-14.88%	-25.25%	-52.49%	-12.47%
VolTime	150.63	33.83	358.66	-2.76%	-14.60%	-45.72%	0.00%
2	154.90	68.62	389.15	0.00%	-12.17%	-44.18%	$+2.84\%$
4	176.37	141.66	273.58	$+13.86\%$	0.00%	-36.44%	$+17.09\%$
MarketOrder	277.49	246.48	158.66	$+79.14\%$	$+57.33\%$	0.00%	$+84.22\%$

Table 6: Evaluating the impact of look-ahead features.
Average performance over the test period may 2017, currency pair USDT/BTC.

A vastly larger impact (-58.84% respectively -59.98%) is caused by the look-ahead feature `ob_direction`, quoting the general price trend of the currently observed orderbook window. In contrast to the `future_center*` variables, its value stays constant within individual orderbook windows: `ob_direction = orderbook[-1].get_center() / orderbook[0].get_center()`, which seems to provoke more stable strategies.

Constant Market Variables

The findings from the look-ahead features encouraged for a supplementary experiment. Rather than considering the actual market situation, the market variables are observed once (at $t=0$), and kept frozen for the remaining trading horizon. Hope was, to provoke more stable strategies, as the agents could potentially decide on a *major* strategy and consequently only adapt the limits to the private variables representing the actual trade progress.

While Table 7 shows improved performance for previous bad performers, the leading `VolTimeSpread`-agent dropped from -8.93% to -7.32% . Contrary to initial hopes, this approach did not lead to superior performance and was not further pursued.

¹⁶ look-ahead features provide a glance into the future, and are thus equivalent to cheating.

	slippage	med	std	perf_2	perf_4	perf_M	perf_VolTime
center_orig_disc3	158.96	48.93	431.17	2.62%	-9.87%	-42.71%	+5.54%
center_orig_disc5	150.37	47.01	422.14	-2.92%	-14.74%	-45.81%	-0.17%
marketPrice_buy_worst_disc5	150.74	50.15	402.13	-2.69%	-14.54%	-45.68%	+0.07%
marketPrice_sell_worst_disc5	157.56	43.95	379.33	+1.72%	-10.66%	-43.22%	+4.61%
marketPrice_spread_disc5	152.19	53.07	398.10	-1.75%	-13.71%	-45.15%	+1.04%
marketPrice_imbalance_disc5	147.16	47.03	383.19	-5.00%	-16.57%	-46.97%	-2.30%
sharecount_buy_disc3	149.57	46.15	420.14	-3.44%	-15.19%	-46.10%	-0.70%
sharecount_buy_disc5	150.37	47.01	422.14	-2.92%	-14.74%	-45.81%	-0.17%
sharecount_imbalance_disc5	152.44	53.41	400.89	-1.59%	-13.57%	-45.06%	+1.20%
sharecount_sell_disc5	150.37	47.01	422.14	-2.92%	-14.74%	-45.81%	-0.17%
sharecount_spread_disc5	149.23	53.41	402.65	-3.66%	-15.39%	-46.22%	-0.93%
spread_disc3	148.92	41.70	361.12	-3.86%	-15.57%	-46.33%	-1.14%
spread_disc5	143.56	40.78	350.74	-7.32%	-18.60%	-48.26%	-4.69%
spread_disc9	145.54	43.25	352.05	-6.04%	-17.48%	-47.55%	-3.38%
VolTime	150.63	33.83	358.66	-2.76%	-14.60%	-45.72%	0.00%
2	154.90	68.62	389.15	0.00%	-12.17%	-44.18%	+2.84%
4	176.37	141.66	273.58	+13.86%	0.00%	-36.44%	+17.09%
MarketOrder	277.49	246.48	158.66	+79.14%	+57.33%	0.00%	+84.22%

Table 7: Evaluating the impact of constant market variables.
Average performance over the test period may 2017, currency pair USDT/BTC.
See Appendix A.2, Table 13 for complete results.

5.2.3 Simulation of preceding trades

As described in Section 5.1.3, the OTS’s internal masterbook shape depends drastically from the preceding trade history. In the following, the effect of incorporating the own impact on the market is investigated.

	slippage	med	std	perf_2	perf_4	perf_M	perf_VolTime	perf: Table 5
center_orig_disc3	158.97	48.93	431.16	+2.62%	-9.87%	-42.71%	+5.54%	-0.70%
center_orig_disc5	150.47	47.49	422.12	-2.86%	-14.69%	-45.78%	-0.11%	+7.35%
marketPrice_buy_worst_disc5	148.33	47.46	400.73	-4.24%	-15.90%	-46.55%	-1.53%	-2.52%
marketPrice_sell_worst_disc5	154.06	42.10	370.91	-0.54%	-12.65%	-44.48%	+2.28%	-1.12%
marketPrice_spread_disc5	151.12	53.07	395.68	-2.44%	-14.32%	-45.54%	+0.33%	-0.11%
marketPrice_imbalance_disc5	143.86	47.05	375.43	-7.13%	-18.44%	-48.16%	-4.49%	-0.35%
sharecount_buy_disc3	149.57	46.15	420.14	-3.44%	-15.19%	-46.10%	-0.70%	-0.70%
sharecount_buy_disc5	150.47	47.49	422.12	-2.86%	-14.69%	-45.78%	-0.11%	+6.70%
sharecount_imbalance_disc5	151.92	53.41	400.69	-1.92%	-13.86%	-45.25%	+0.86%	-1.73%
sharecount_sell_disc5	149.93	47.01	420.70	-3.21%	-14.99%	-45.97%	-0.47%	+0.58%
sharecount_spread_disc5	149.36	53.41	402.86	-3.58%	-15.31%	-46.17%	-0.84%	-1.13%
spread_disc3	145.63	41.89	346.00	-5.98%	-17.43%	-47.52%	-3.32%	-2.14%
spread_disc5	139.87	41.22	336.30	-9.70%	-20.69%	-49.59%	-7.14%	-6.34%
spread_disc9	141.30	42.80	336.88	-8.78%	-19.89%	-49.08%	-6.19%	-5.20%
ob_direction_disc5	62.03	97.52	320.76	-59.95%	-64.83%	-77.64%	-58.82%	-58.84%
VolTime_simulatedTrades	147.86	42.10	346.17	-4.55%	-16.17%	-46.72%	-1.84%	
VolTime	150.63	33.83	358.66	-2.76%	-14.60%	-45.72%	0.00%	
2	154.90	68.62	389.15	0.00%	-12.17%	-44.18%	+2.84%	
4	176.37	141.66	273.58	+13.86%	0.00%	-36.44%	+17.09%	
MarketOrder	277.49	246.48	158.66	+79.14%	+57.33%	0.00%	+84.22%	

Table 8: Evaluating the impact of incorporating preceding trades.
Average performance over the test period may 2017, currency pair USDT/BTC.
See Appendix A.3, Table 14 for complete results.

The backward sampling phase is modified, such that prior to the actual trade simulations, the supposedly consumed trading volume is removed from the masterbook. The trading volume affected is matched and removed from the masterbook evenly along the elapsed time, as is indicated in Section 4.2.1, Figure 15c.

Table 8 attests a clear improvement over the original sampling method, if preceding trades are incorporated into the **OTS** masterbook. Figure 18 shows, how the plain **VolTime**-agent becomes slightly more aggressive, which improves the performance over the optimal **S&L** strategy from -2.76% to -4.55% .

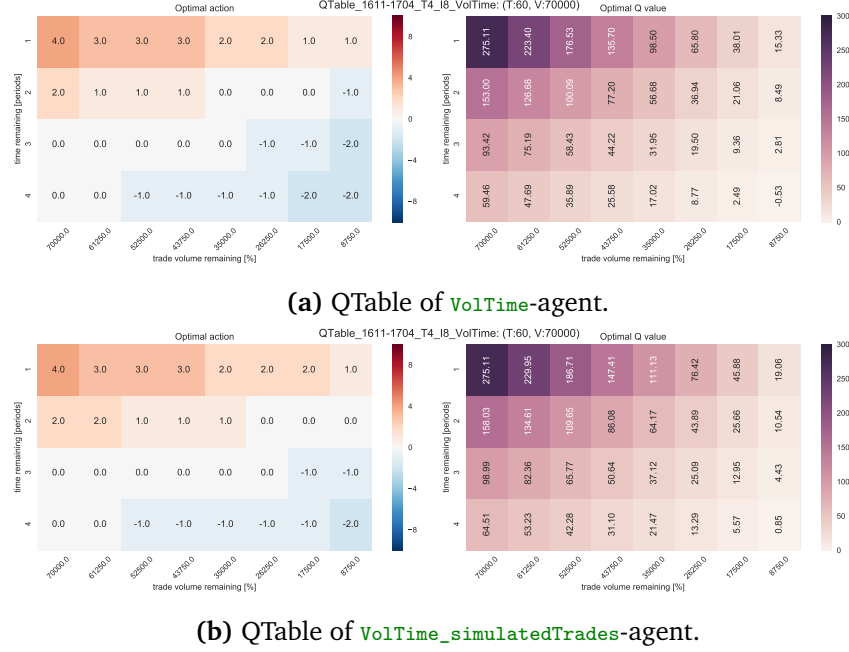


Figure 18: A slight increase in aggression, if preceding trades are incorporated.
 $T = 4, I = 8, L = 15$

The best performing **VolTimeSpread_disc5**-agent now undercuts costs induced by simple market orders by -49.59% (before: -49.16%), the gain in comparison to the optimal **S&L** strategy enlarged to -9.70% (before: -8.93%).

5.2.4 Estimating Action Effects

Table 9 evaluates the impact of adding (lower bound) knowledge about consequences of individual actions to the state space. That way, **_a_4_** quotes the minimum volume immediately obtainable by a limit order placed $+0.4\%$ above the current ask price.

Potentially due to coarse discretization, these variables cause no major impact. Almost all **_a*_**-agents perform worse than the underlying **VolTime_simulatedTrades**-agent.

	slippage	med	std	perf_2	perf_4	perf_M	perf_VolTime
a-4_disc5	149.84	40.54	368.50	-3.27%	-15.04%	-46.00%	-0.52%
a-3_disc5	150.70	37.95	372.15	-2.71%	-14.55%	-45.69%	+0.05%
_a_0_disc5	153.01	45.17	375.98	-1.22%	-13.25%	-44.86%	+1.58%
_a_1_disc5	148.56	41.60	385.79	-4.10%	-15.77%	-46.46%	-1.37%
_a_2_disc5	149.47	43.47	368.49	-3.50%	-15.25%	-46.13%	-0.77%
_a_3_disc5	145.04	42.42	365.27	-6.37%	-17.76%	-47.73%	-3.71%
_a_4_disc5	155.52	40.84	373.98	+0.40%	-11.82%	-43.95%	+3.25%
_a_5_disc5	152.05	38.53	372.03	-1.84%	-13.79%	-45.21%	+0.94%
_a_6_disc5	151.80	40.39	371.41	-2.00%	-13.93%	-45.29%	+0.78%
_a_7_disc5	148.61	40.39	363.67	-4.06%	-15.74%	-46.44%	-1.34%
_a_8_disc5	152.46	41.26	363.02	-1.57%	-13.56%	-45.06%	+1.22%
_a_9_disc5	151.59	40.84	363.68	-2.14%	-14.05%	-45.37%	+0.64%
_a_10_disc5	150.28	40.84	363.70	-2.99%	-14.80%	-45.84%	-0.23%
VolTime_simulatedTrades	147.86	42.10	346.17	-4.55%	-16.17%	-46.72%	-1.84%
VolTime	150.63	33.83	358.66	-2.76%	-14.60%	-45.72%	0.00%
2	154.90	68.62	389.15	0.00%	-12.17%	-44.18%	+2.84%
4	176.37	141.66	273.58	+13.86%	0.00%	-36.44%	+17.09%
MarketOrder	277.49	246.48	158.66	+79.14%	+57.33%	0.00%	+84.22%

Table 9: Evaluating the impact of action-consequence knowledge.
Average performance over the test period may 2017, currency pair USDT/BTC.

5.2.5 Function Approximation: BatchTree

In a final experiment the look-up table is replaced by an approximation thereof. This eliminates any need for discretization, avoids the cost scaling problem and as such, potentially helps to exploit the underlying sample transitions better. Various BT-Agents are trained on the sample transitions collected by the improved backward exploration algorithm, that incorporates preceding trades (see Section 5.2.3). Results are shown in Table 10.

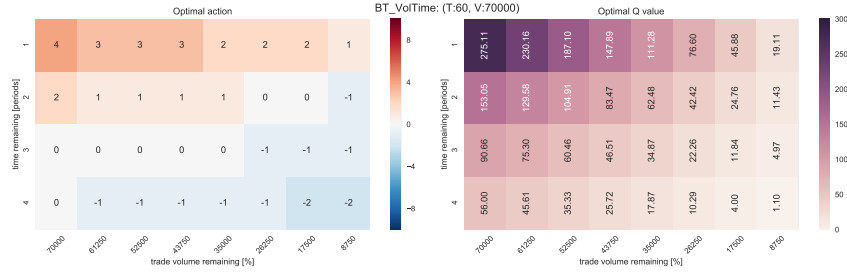
	slippage	med	std	perf_2	perf_4	perf_M	perf_VolTime
BT_VolTime_a4_	163.21	25.48	466.45	+5.36%	-7.46%	-41.18%	+8.35%
BT_VolTime_a*_	143.24	38.53	373.35	-7.53%	-18.78%	-48.38%	-4.90%
BT_VolTime	151.08	47.39	391.02	-2.47%	-14.34%	-45.55%	+0.30%
BT_VolTimeSpread	249.07	237.29	142.11	+60.79%	+41.22%	-10.24%	+65.36%
VolTime_simulatedTrades	147.86	42.10	346.17	-4.55%	-16.17%	-46.72%	-1.84%
VolTime	150.63	33.83	358.66	-2.76%	-14.60%	-45.72%	0.00%
2	154.90	68.62	389.15	0.00%	-12.17%	-44.18%	+2.84%
4	176.37	141.66	273.58	+13.86%	0.00%	-36.44%	+17.09%
MarketOrder	277.49	246.48	158.66	+79.14%	+57.33%	0.00%	+84.22%

Table 10: Evaluating the performance of function approximators.
Average performance over the test period may 2017, currency pair USDT/BTC.

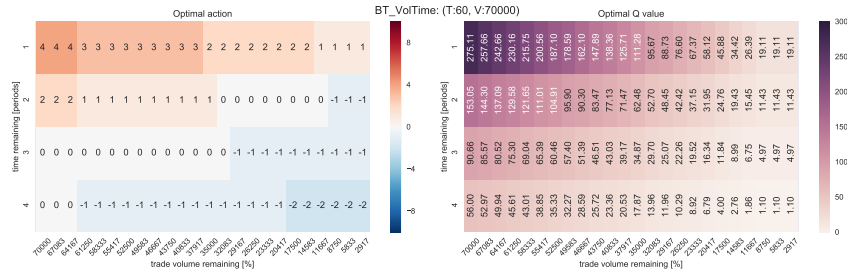
While the plain BT_VolTime-agent performs de facto worse than the original, discrete VolTime-agent (-2.47% vs. -4.55%), agents employing individual market variables perform terribly if fed with continuous values. The worst of all, BT_VolTimeSpread-agent, performs +60.79% worse than the optimal S&L strategy!

Conversely, a BT-agent trained on all fifteen _a*_effects-variables simultaneously lifts the BT performance from -2.47% to -7.53%, which is a good result. Unfortunately, it does not catch up with the previously attained -9.7% (see Section 5.2.3).

Figure 19 shows exemplary for the plain **VolTime**-agent, how the **BT**-agent is able to deliver a more accurate resolution of q-values and optimal actions.



(a) QTable by **VolTime**-agent.



(b) QTable approximation by **BT_VolTime**-agent (High resolution).

Figure 19: The **BT**-agent generalizes to continuous states.

5.3 Forward Learning

Based on the same training data and simulator settings as used in the preceding backward learning experiments, the novel forward sampling approach, described in Section 4.3, is examined.

The **BT_Forward**-agent samples from the previously shuffled training data, following an ϵ -greedy strategy. In each exploration phase the respective orderbook window is run through up to 60 times, while only disparate paths are traced. After every 256 exploration phases, the underlying RandomForestRegressor is retrained from scratch, using all transition samples collected to this point. Intermediate models are stored, to document the learning progress over time. The continuous state space consists of both private variables and the fifteen action-effect-variables `_a*_`, as introduced in Section 5.2.2.

The performance of all sixteen intermediate models is shown in Table 11. Starting vaguely halfway through the training data, the **BT_Forward**-agent outperforms all previously examined agents, before the final model constitutes the overall winner with an improvement of -11.41% over the optimal **S&L** strategy.

Figure 20 visualized the **BT_Forward**-agents performance evolution over the training phase. The horizontal lines refer to baselines and previously achieved performances. This

	slippage	med	std	perf_2	perf_4	perf_M	perf_VolTime
BT_Forward_samples_025.078	143.72	74.69	300.93	-7.22%	-18.52%	-48.21%	-4.59%
BT_Forward_samples_043.334	147.92	82.24	310.49	-4.51%	-16.13%	-46.69%	-1.80%
BT_Forward_samples_061.103	149.56	77.33	288.56	-3.45%	-15.20%	-46.10%	-0.71%
BT_Forward_samples_079.809	146.67	77.20	277.43	-5.31%	-16.84%	-47.14%	-2.62%
BT_Forward_samples_097.615	143.45	73.18	281.71	-7.40%	-18.67%	-48.31%	-4.77%
BT_Forward_samples_116.008	144.26	77.94	285.80	-6.87%	-18.21%	-48.01%	-4.23%
BT_Forward_samples_134.011	138.22	73.30	269.71	-10.77%	-21.63%	-50.19%	-8.24%
BT_Forward_samples_152.352	136.90	69.59	264.95	-11.62%	-22.38%	-50.66%	-9.11%
BT_Forward_samples_170.605	137.75	77.50	261.17	-11.07%	-21.90%	-50.36%	-8.55%
BT_Forward_samples_188.224	137.64	73.71	253.37	-11.14%	-21.96%	-50.40%	-8.62%
BT_Forward_samples_206.409	137.10	74.22	257.54	-11.49%	-22.27%	-50.59%	-8.98%
BT_Forward_samples_224.083	142.90	75.43	265.78	-7.75%	-18.98%	-48.50%	-5.13%
BT_Forward_samples_242.342	129.85	74.43	239.96	-16.17%	-26.37%	-53.20%	-13.79%
BT_Forward_samples_261.018	137.98	75.69	248.07	-10.92%	-21.77%	-50.27%	-8.39%
BT_Forward_samples_279.347	138.79	75.28	258.49	-10.40%	-21.31%	-49.98%	-7.86%
BT_Forward_samples_298.020	137.23	78.71	242.88	-11.41%	-22.19%	-50.55%	-8.89%
VolTime	150.63	33.83	358.66	-2.76%	-14.60%	-45.72%	0.00%
2	154.90	68.62	389.15	0.00%	-12.17%	-44.18%	+2.84%
4	176.37	141.66	273.58	+13.86%	0.00%	-36.44%	+17.09%
MarketOrder	277.49	246.48	158.66	+79.14%	+57.33%	0.00%	+84.22%

Table 11: Performance growth of the final **BT_Forward-agent**.
Average performance over the test period may 2017, currency pair USDT/BTC.

performance has been achieved with a significantly smaller set of transitions: Whereas the backward learning strategies are learned from 1.999.920 transitions, the **BT_Forward-agent** is pleased with 298.020 samples. Consequently, the **BT_Forward-agent** exhibits a significantly accelerated exploration phase and learns faster.

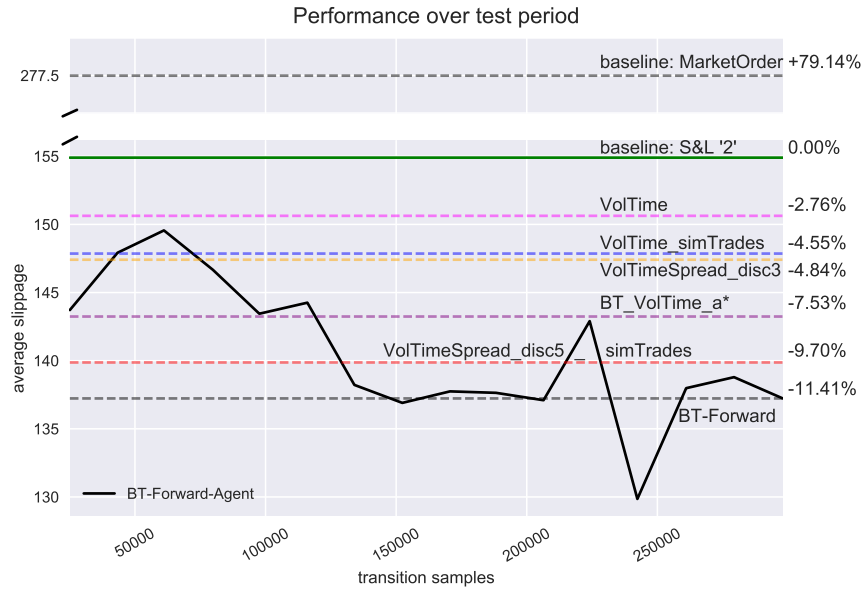


Figure 20: The **BT_Forward-agent** performance on the test period.

6 Conclusion

The goal of this thesis was to reduce trade execution costs by applying reinforcement learning on the problem of optimized trade execution. While higher level trading strategies analyze a universe of assets and decide on how many shares of individual assets shall be bought (or sold), optimized trading execution tackles the consecutive phase. Here, the trading decision itself is not questioned, but simply executed at the best possible rate.

As reinforcement learning requires agents to interact with an environment, a trading simulation framework was implemented. The accrued [OTS](#) provides a full-featured reinforcement learning environment, that simulates the execution of orders on historic data and returns vital feedback to steer the agent towards optimal decisions. Additionally, it serves as a backtesting framework in order to evaluate the cost reducing capabilities of different trading strategies.

This thesis assessed a data set of self recorded bitcoin [LOB](#) snapshots on a rather low minute-scale basis, which stands in contrast to most other works, which had the advantage of large, proprietary data sets on millisecond basis from traditional stock exchanges. The discrepancies between data quality and observed price volatility make it difficult to compare achieved performances to other studies. Very dense [LOBs](#) and large price fluctuations in the bitcoin data set suggest, that large parts of the observed slippage stem from price fluctuations, which are harder to predict than slippage that originates from eating into the respective [LOBs](#).

An existing reinforcement learning approach, deriving valuable strategies from a discretized state space in a brute force manner, has been replicated and transferred to the data set at hand. If trained on private variables only, the replicated algorithm could achieve an improvement of -2.76% over the more simple [S&L](#) strategies. If compared to the [S&L](#) strategy that performed best on the training set (rather than on the test set: see Section [5.0.1](#)), this value increases to -14.60% .

In the scope of this thesis, the exploration phase of the backward learning agent has been overhauled, such that it now accounts for self-induced impact on the market situation. By incorporating preceding trades properly, the agents cost reducing capabilities could be raised from -2.76% to -4.55% . As in the original approach, adding (one) additional market variable(s) further increased this performance: Knowledge about the current spread led to performance gains of -4.84% for the replicated and -9.70% for the overhauled algorithm.

Finally, a novel reinforcement learning approach was presented, that applies growing batch reinforcement learning on the problem of optimal trade execution. The proposed

forward-sampling algorithm samples from a more realistic, continuous state space and outperforms the aforementioned algorithms with an improvement of -11.41% over the more simple S&L strategy. Similar to above, this translates into an average improvement of -22.19% over the S&L strategy that performed best on the training set.

In absolute values, the slippage for a total order volume of 70.000\$ could be halved from 277\$ (as achieved by simple market orders) to $\sim 137\%$ (-50.55%). As high level strategies typically instruct many such (or even larger) trades in parallel, this scales to considerable amounts of saved money.

6.1 Future Work

The most promising start point for future work lies in further improvements to the novel forward learning algorithm. The utilization of (deep) neural networks and replay memory may help to find better generalizations of the state-action functions. The generalizing of Q-learning to work with a continuous action space potentially allows for more appropriate actions.

As the experiments with look-ahead features revealed, a great potential may be found in specialized time series prediction methods as e. g. Prevedex STaR [12]. An improved ability to (partially) predict future price trends possibly helps to deal with highly volatile markets.

A Appendix

A.1 Additional Market Variables

	slippage	med	std	perf_2	perf_4	perf_M	perf_VolTime
center_orig_disc3	149.57	46.15	420.14	-3.44%	-15.19%	-46.10%	-0.70%
center_orig_disc5	161.70	37.83	450.36	+4.39%	-8.32%	-41.73%	+7.35%
center_orig_disc9	161.65	38.83	450.40	+4.36%	-8.35%	-41.74%	+7.32%
marketPrice_buy_worst_disc3	148.35	41.26	361.17	-4.23%	-15.89%	-46.54%	-1.51%
marketPrice_buy_worst_disc5	146.83	39.62	351.77	-5.21%	-16.75%	-47.08%	-2.52%
marketPrice_buy_worst_disc9	150.53	39.66	368.95	-2.82%	-14.65%	-45.75%	-0.07%
marketPrice_sell_worst_disc3	146.72	41.91	385.13	-5.28%	-16.81%	-47.13%	-2.59%
marketPrice_sell_worst_disc5	148.94	42.71	388.86	-3.85%	-15.55%	-46.33%	-1.12%
marketPrice_sell_worst_disc9	149.28	45.05	390.19	-3.63%	-15.36%	-46.20%	-0.89%
marketPrice_spread_disc3	150.27	40.95	355.46	-2.99%	-14.80%	-45.84%	-0.23%
marketPrice_spread_disc5	150.46	44.15	371.32	-2.87%	-14.69%	-45.78%	-0.11%
marketPrice_spread_disc9	151.98	47.12	372.53	-1.89%	-13.83%	-45.23%	+0.90%
marketPrice_imbalance_disc3	148.42	50.44	358.49	-4.19%	-15.85%	-46.51%	-1.47%
marketPrice_imbalance_disc5	150.10	68.68	336.60	-3.10%	-14.90%	-45.91%	-0.35%
marketPrice_imbalance_disc9	150.50	64.44	357.22	-2.84%	-14.67%	-45.76%	-0.09%
sharecount_buy_disc3	149.57	46.15	420.14	-3.44%	-15.19%	-46.10%	-0.70%
sharecount_buy_disc5	160.73	37.78	449.22	+3.76%	-8.87%	-42.08%	+6.70%
sharecount_buy_disc9	161.65	38.83	450.40	+4.36%	-8.35%	-41.74%	+7.32%
sharecount_imbalance_disc3	148.57	40.68	353.91	-4.08%	-15.76%	-46.46%	-1.36%
sharecount_imbalance_disc5	148.03	40.49	372.86	-4.44%	-16.07%	-46.65%	-1.73%
sharecount_imbalance_disc9	153.27	47.28	380.38	-1.05%	-13.10%	-44.77%	+1.75%
sharecount_sell_disc3	149.57	46.15	420.14	-3.44%	-15.19%	-46.10%	-0.70%
sharecount_sell_disc5	151.50	47.49	425.35	-2.20%	-14.10%	-45.40%	+0.58%
sharecount_sell_disc9	150.37	46.15	424.22	-2.93%	-14.75%	-45.81%	-0.17%
sharecount_spread_disc3	147.57	40.95	350.33	-4.73%	-16.33%	-46.82%	-2.03%
sharecount_spread_disc5	148.93	40.84	352.12	-3.86%	-15.56%	-46.33%	-1.13%
sharecount_spread_disc9	146.61	39.62	373.00	-5.35%	-16.87%	-47.16%	-2.66%
spread_disc3	147.41	35.48	370.96	-4.84%	-16.42%	-46.88%	-2.14%
spread_disc5	141.07	37.39	349.72	-8.93%	-20.02%	-49.16%	-6.34%
spread_disc9	142.80	36.69	364.76	-7.81%	-19.03%	-48.54%	-5.20%
ob_direction_disc3	65.80	81.55	335.76	-57.52%	-62.69%	-76.29%	-56.32%
ob_direction_disc5	61.99	97.52	319.84	-59.98%	-64.85%	-77.66%	-58.84%
ob_direction_disc9	57.20	86.64	321.42	-63.07%	-67.57%	-79.38%	-62.02%
future_center5_disc3	133.58	40.40	377.88	-13.76%	-24.26%	-51.86%	-11.31%
future_center5_disc5	141.61	41.79	389.58	-8.58%	-19.71%	-48.97%	-5.98%
future_center15_disc3	107.74	31.91	364.92	-30.44%	-38.91%	-61.17%	-28.47%
future_center15_disc5	133.78	40.90	405.88	-13.63%	-24.15%	-51.79%	-11.18%
future_center15_disc9	121.11	47.41	411.10	-21.82%	-31.33%	-56.36%	-19.60%
future_center60_disc3	139.20	36.02	398.05	-10.14%	-21.07%	-49.83%	-7.58%
future_center60_disc5	131.85	47.63	391.46	-14.88%	-25.25%	-52.49%	-12.47%
future_center60_disc9	131.57	48.52	422.35	-15.06%	-25.40%	-52.59%	-12.65%
VolTime	150.63	33.83	358.66	-2.76%	-14.60%	-45.72%	0.00%
2	154.90	68.62	389.15	0.00%	-12.17%	-44.18%	+2.84%
4	176.37	141.66	273.58	+13.86%	0.00%	-36.44%	+17.09%
MarketOrder	277.49	246.48	158.66	+79.14%	+57.33%	0.00%	+84.22%

Table 12: Evaluating the impact of additional market variables.

Average performance over the test period may 2017, currency pair USDT/BTC.

See Section 5.2.2 for the actual experiment description. This is the full version of Table 5.

A.2 Constant Market Variables

	slippage	med	std	perf_2	perf_4	perf_M	perf_VolTime
center_orig_disc3	158.96	48.93	431.17	2.62%	-9.87%	-42.71%	+5.54%
center_orig_disc5	150.37	47.01	422.14	-2.92%	-14.74%	-45.81%	-0.17%
center_orig_disc9	149.76	46.15	420.67	-3.32%	-15.09%	-46.03%	-0.57%
marketPrice_buy_worst_disc3	150.27	44.67	398.78	-2.99%	-14.80%	-45.85%	-0.24%
marketPrice_buy_worst_disc5	150.74	50.15	402.13	-2.69%	-14.54%	-45.68%	+0.07%
marketPrice_buy_worst_disc9	155.99	44.67	416.56	+0.70%	-11.56%	-43.79%	+3.56%
marketPrice_sell_worst_disc3	150.13	41.26	352.48	-3.08%	-14.88%	-45.90%	-0.33%
marketPrice_sell_worst_disc5	157.56	43.95	379.33	+1.72%	-10.66%	-43.22%	+4.61%
marketPrice_sell_worst_disc9	147.90	48.20	357.93	-4.52%	-16.14%	-46.70%	-1.81%
marketPrice_spread_disc3	152.15	51.32	391.87	-1.78%	-13.74%	-45.17%	+1.01%
marketPrice_spread_disc5	152.19	53.07	398.10	-1.75%	-13.71%	-45.15%	+1.04%
marketPrice_spread_disc9	150.74	41.60	382.24	-2.69%	-14.53%	-45.68%	+0.08%
marketPrice_imbalance_disc3	148.04	51.06	379.25	-4.43%	-16.06%	-46.65%	-1.71%
marketPrice_imbalance_disc5	147.16	47.03	383.19	-5.00%	-16.57%	-46.97%	-2.30%
marketPrice_imbalance_disc9	146.85	54.71	375.38	-5.20%	-16.74%	-47.08%	-2.51%
sharecount_buy_disc3	149.57	46.15	420.14	-3.44%	-15.19%	-46.10%	-0.70%
sharecount_buy_disc5	150.37	47.01	422.14	-2.92%	-14.74%	-45.81%	-0.17%
sharecount_buy_disc9	149.76	46.15	420.67	-3.32%	-15.09%	-46.03%	-0.57%
sharecount_imbalance_disc3	151.20	52.13	395.90	-2.39%	-14.27%	-45.51%	+0.38%
sharecount_imbalance_disc5	152.44	53.41	400.89	-1.59%	-13.57%	-45.06%	+1.20%
sharecount_imbalance_disc9	154.90	57.56	395.50	-0.00%	-12.18%	-44.18%	+2.84%
sharecount_sell_disc3	149.57	46.15	420.14	-3.44%	-15.19%	-46.10%	-0.70%
sharecount_sell_disc5	150.37	47.01	422.14	-2.92%	-14.74%	-45.81%	-0.17%
sharecount_sell_disc9	149.53	46.15	420.50	-3.47%	-15.22%	-46.11%	-0.73%
sharecount_spread_disc3	150.78	48.23	419.23	-2.66%	-14.51%	-45.66%	+0.10%
sharecount_spread_disc5	149.23	53.41	402.65	-3.66%	-15.39%	-46.22%	-0.93%
sharecount_spread_disc9	148.65	52.13	408.66	-4.04%	-15.72%	-46.43%	-1.31%
spread_disc3	148.92	41.70	361.12	-3.86%	-15.57%	-46.33%	-1.14%
spread_disc5	143.56	40.78	350.74	-7.32%	-18.60%	-48.26%	-4.69%
spread_disc9	145.54	43.25	352.05	-6.04%	-17.48%	-47.55%	-3.38%
VolTime	150.63	33.83	358.66	-2.76%	-14.60%	-45.72%	0.00%
2	154.90	68.62	389.15	0.00%	-12.17%	-44.18%	+2.84%
4	176.37	141.66	273.58	+13.86%	0.00%	-36.44%	+17.09%
MarketOrder	277.49	246.48	158.66	+79.14%	+57.33%	0.00%	+84.22%

Table 13: Evaluating the impact of constant market variables.

Average performance over the test period may 2017, currency pair USDT/BTC.

See Section 5.2.2 for the actual experiment description. This is the full version of Table 7.

A.3 Simulation of preceding trades

	slippage	med	std	perf_2	perf_4	perf_M	perf_VolTime
center_orig_disc3	158.97	48.93	431.16	+2.62%	−9.87%	−42.71%	+5.54%
center_orig_disc5	150.47	47.49	422.12	−2.86%	−14.69%	−45.78%	−0.11%
center_orig_disc9	149.76	46.15	420.66	−3.32%	−15.09%	−46.03%	−0.57%
marketPrice_buy_worst_disc3	148.49	45.78	396.35	−4.14%	−15.81%	−46.49%	−1.42%
marketPrice_buy_worst_disc5	148.33	47.46	400.73	−4.24%	−15.90%	−46.55%	−1.53%
marketPrice_buy_worst_disc9	154.36	39.16	414.24	−0.35%	−12.48%	−44.37%	+2.48%
marketPrice_sell_worst_disc3	150.23	40.84	351.30	−3.02%	−14.82%	−45.86%	−0.26%
marketPrice_sell_worst_disc5	154.06	42.10	370.91	−0.54%	−12.65%	−44.48%	+2.28%
marketPrice_sell_worst_disc9	152.07	49.16	362.68	−1.83%	−13.78%	−45.20%	+0.96%
marketPrice_spread_disc3	151.10	52.13	390.08	−2.45%	−14.33%	−45.55%	+0.31%
marketPrice_spread_disc5	151.12	53.07	395.68	−2.44%	−14.32%	−45.54%	+0.33%
marketPrice_spread_disc9	148.45	41.60	380.40	−4.16%	−15.83%	−46.50%	−1.44%
marketPrice_imbalance_disc3	145.82	47.88	374.09	−5.86%	−17.32%	−47.45%	−3.19%
marketPrice_imbalance_disc5	143.86	47.05	375.43	−7.13%	−18.44%	−48.16%	−4.49%
marketPrice_imbalance_disc9	144.90	51.39	365.94	−6.45%	−17.84%	−47.78%	−3.80%
sharecount_buy_disc3	149.57	46.15	420.14	−3.44%	−15.19%	−46.10%	−0.70%
sharecount_buy_disc5	150.47	47.49	422.12	−2.86%	−14.69%	−45.78%	−0.11%
sharecount_buy_disc9	149.76	46.15	420.67	−3.32%	−15.09%	−46.03%	−0.57%
sharecount_imbalance_disc3	153.26	53.41	398.42	−1.06%	−13.10%	−44.77%	+1.75%
sharecount_imbalance_disc5	151.92	53.41	400.69	−1.92%	−13.86%	−45.25%	+0.86%
sharecount_imbalance_disc9	154.33	57.43	394.25	−0.37%	−12.49%	−44.38%	+2.46%
sharecount_sell_disc3	149.57	46.15	420.14	−3.44%	−15.19%	−46.10%	−0.70%
sharecount_sell_disc5	149.93	47.01	420.70	−3.21%	−14.99%	−45.97%	−0.47%
sharecount_sell_disc9	149.53	46.15	420.50	−3.47%	−15.22%	−46.11%	−0.73%
sharecount_spread_disc3	150.35	48.23	418.91	−2.94%	−14.76%	−45.82%	−0.19%
sharecount_spread_disc5	149.36	53.41	402.86	−3.58%	−15.31%	−46.17%	−0.84%
sharecount_spread_disc9	148.47	50.52	410.27	−4.15%	−15.82%	−46.50%	−1.43%
spread_disc3	145.63	41.89	346.00	−5.98%	−17.43%	−47.52%	−3.32%
spread_disc5	139.87	41.22	336.30	−9.70%	−20.69%	−49.59%	−7.14%
spread_disc9	141.30	42.80	336.88	−8.78%	−19.89%	−49.08%	−6.19%
ob_direction_disc3	69.01	101.49	323.00	−55.45%	−60.87%	−75.13%	−54.18%
ob_direction_disc5	62.03	97.52	320.76	−59.95%	−64.83%	−77.64%	−58.82%
ob_direction_disc9	57.74	87.92	321.20	−62.73%	−67.26%	−79.19%	−61.67%
VolTime_simulatedTrades	147.86	42.10	346.17	−4.55%	−16.17%	−46.72%	−1.84%
VolTime	150.63	33.83	358.66	−2.76%	−14.60%	−45.72%	0.00%
2	154.90	68.62	389.15	0.00%	−12.17%	−44.18%	+2.84%
4	176.37	141.66	273.58	+13.86%	0.00%	−36.44%	+17.09%
MarketOrder	277.49	246.48	158.66	+79.14%	+57.33%	0.00%	+84.22%

Table 14: Evaluating the impact of incorporating preceding trades.

Average performance over the test period may 2017, currency pair USDT/BTC.

See Section 5.2.3 for the actual experiment description. This is the full version of Table 8.

B Glossary

OTS Orderbook Trading Simulator

LOB Limit Order Book

RL Reinforcement Learning

SL Supervised Learning

BT BatchTree

MDP Markov Decision Process

S&L Submit & Leave Strategy

S&R Submit & Revise Strategy

Eurex European Exchange

NASDAQ National Association of Securities Dealers Automated Quotations (American Stock Exchange))

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