Week 05, problems.

Recap/Practice of set 1-4, Lines, Planes.

1. a) (MT+'07) Determine the equation of the plane, which is perpendicular to the line $\frac{x-5}{2} = \frac{y-10}{-2} = \frac{z+8}{3}$ and goes through the point $P(1, 4, -1)$.
   b) Determine the system of equations of the line through the point $P(2, -5, -2)$ perpendicular to the plane $z = 4x + 7$.

2. a) (MT'10) Consider the plane which is parallel to the plane of equation $5x - 4y + 3z = 9$ and contains the point $P(1, 5, 5)$. Does this plane pass through the origin?
   b) Consider the line that is parallel to the line given by $\frac{x-5}{2} = \frac{1-y}{2} = -z - 9$ and goes through the point $P(4, -4, -2)$. Does this line pass through the origin?

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3. (MT'18) With the help of the extended Euclidean algorithm find all numbers between 0 and 301 for which its product with 222 gives a remainder of 34 when divided with 302.

4. (MT++'20) Let $n = 987654321$. Use the Euclidean algorithm done in class to find the greatest common divisor of $98n + 27$ and $76n + 21$.

5. (MT+'23) Let $\vec{d} = \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}$ be the direction vector of both the lines e and f. Line e contains the point $(3, 1, 2)$, and line f contains the point $(5, -1, 1)$. Determine the equation of the plane containing both the lines e and f.

6. (MT+'10) Determine the equation of the line passing through the point $P(12, 1, 7)$ and perpendicularly intersecting the line given by $x - 3 = \frac{y-2}{3} = \frac{-z-1}{4}$.

7. Use extended Euclidean algorithm to find the values of x for which the following congruences are true:
   a) $119x \equiv 2 \pmod{514}$? b) $158x \equiv 10 \pmod{346}$?

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8. (MT'23) A regular rectangular chest stands on a flat, sloping ground. The corner (vertex) of the chest $A(1, 4, 2)$ is on the ground, but the vertex $B(4, 2, 1)$ which is adjacent to A is not on the ground. Does the plane of the ground intersect the z axis? If yes, where?

9. (MT'21) Determine the value of the parameter p and the equation of the plane S if we know that S contains the points $A(1, 2, 2)$ and $B(3, 4, 1)$ and it is perpendicular to the line e given by $\frac{2x-7}{12} = \frac{8-y}{5} = \frac{z}{p}$.

10. (MT++'22) A ball (whose shape is a regular sphere) is rolling on a slope with a flat surface. In the moment when the ball touches the slope in the point $P(2, 5, -1)$ its center is in the point $C(16, 1, 7)$. Does the plane of the slope pass through the origin?

11. (MT'20) A plane that contains the origin also contains the line given by $\frac{x-4}{9} = \frac{3-y}{2} = \frac{z-1}{6}$. Does it contain the point $P(9, 5, 3)$?

12. (MT'19) The line e given by $\frac{x-11}{2} = \frac{z+19}{-5}$, $y = -1$ intersects the plane $2x + y - 2z = 3$ in the same point as the line f passing through the point $P(15, 2, -8)$. Determine the system of equations of f.

13. (MT++'11) Let the plane S_1 be given by $2x + y - 3z = 2$ and S_2 by the equation $x + 7y + 3z = 21$. Determine whether
    a) their line of intersection contains the point $P(5, 1, 3)$ or not;
    b) S_1 and S_2 are perpendicular to each other or not.

14. Use extended Euclidean algorithm to find the values of x for which the following congruences are true:
    a) $155x \equiv 7 \pmod{352}$? b) $122x \equiv 5 \pmod{166}$? c) $122x \equiv 6 \pmod{166}$?

Final Answers

1. a) $\vec{n} = \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}$, $2x - 2y + 3z = -9$, b) $\frac{x-2}{4} = \frac{z+2}{-1}$ and $y = -5$
2. a) yes, b) yes
3. 26,177
4. 3
5. $2x - y + 6z = 17$
6. $\frac{x-12}{5} = y - 1 = \frac{z-7}{2}$
7. a) $x \equiv 108 \pmod{514}$, b) $x \equiv 57 \pmod{346}$ or $x \equiv 230 \pmod{346}$
8. $3x - 2y - z = -7$, yes, $z = 7$
9. $p = 2$ and S is given by $6x - 5y + 2z = 0$
10. $7x - 2y + 4z = 0$
11. yes, equation of the plane is $4x - 3y - 7z = 0$
12. $\frac{x-15}{12} = \frac{y-2}{3} = \frac{z+8}{-9}$
13. yes, yes
14. a) $x \equiv 293 \pmod{352}$, b) no solutions, c) $x \equiv 64 \pmod{166}$ or $x \equiv 147 \pmod{166}$