Introduction to the Theory of Computing 1.
First Retake of the First Midterm Test

November 15, 2024

1. How many integers are there between 1 and 1000 which are divisible by both 7 and 8, and their remainder is 10 when divided by 22?

2. Let $n = 1122334455667788$. Use the algorithm we learnt to determine the g.c.d. of $48n + 12$ and $33n + 8$.

3. The system of equations of the line e is $\frac{2-x}{4} = \frac{y-7}{p} = \frac{z-6}{3}$, and that of the line f is $\frac{x+9}{2} = 2y = \frac{5-2x}{3}$, where $p \in \mathbf{R}$. Determine whose values of p for which
   a) e and f are parallel,
   b) e and f are perpendicular.

4. Let $\underline{u}$ and $\underline{v}$ be the vectors in $\mathbf{R}^4$ below. Determine all those elements of $\text{span}\{\underline{u}, \underline{v}\}$, the subspace spanned by $\underline{u}$ and $\underline{v}$, whose first and third coordinates are equal and the fourth coordinate is 5 larger than the second one.
$$\underline{u} = (2, 1, 0, -1)^T, \ \underline{v} = (-1, 0, 1, 3)^T$$

5. We know of the vectors $\underline{a}, \underline{b}, \underline{c}, \underline{d}$ in $\mathbf{R}^n$ that $\underline{a}, \underline{b}, \underline{c}$ are linearly independent, but $\underline{a}, \underline{b}, \underline{d}$ are linearly dependent. Decide which of the three statements below holds.
   i) The vectors $\underline{a}, \underline{b}, \underline{c} + \underline{d}$ are linearly independent for sure,
   ii) The vectors $\underline{a}, \underline{b}, \underline{c} + \underline{d}$ are linearly dependent for sure,
   iii) The vectors $\underline{a}, \underline{b}, \underline{c} + \underline{d}$ can be linearly independent or linearly dependent (depending on the choice of $\underline{a}, \underline{b}, \underline{c}, \underline{d}$).
   (You have to justify your decision.)

6. * Let a and b be coprime positive integers. Determine the remainder we get if we divide $a^{\varphi(b)} + b^{\varphi(a)}$ by ab.

Please work on stapled sheets only, and submit all of them at the end of the midterm, including drafts.

Write your name on every sheet you work on, and write your Neptun code and the number of the group you are registered to in Neptun (A1, A2 or A3) on the first page.

You have 90 minutes to work on the problems. Each of them is worth 10 points. To obtain a signature you have to achieve at least 24 points on each of the two midterm tests.

The details of the solutions must be explained; giving the result only is not worth any points. Notes, calculators or any additional devices cannot be used. The problem marked with an * is supposed to be more difficult.