Introduction to the Theory of Computing 1.
Second Retake of the First Midterm Test
December 16, 2024

1. How many integers are there between 1 and 10000 which give a remainder of 15 when divided by 32 and a remainder of 11 when divided by 38? Determine the middle one (in increasing order) among those.

2. Let a, b_1, b_2, m be integers and $m > 1$. Decide whether the statements below are true or false.(You have to justify your decisions.)
   a) If the linear congruences $ax \equiv b_1 \pmod{m}$ and $ax \equiv b_2 \pmod{m}$ are solvable then the linear congruence $ax \equiv b_1 + b_2 \pmod{m}$ is solvable as well.
   b) If the linear congruence $ax \equiv b_1 \cdot b_2 \pmod{m}$ is solvable then the linear congruences $ax \equiv b_1 \pmod{m}$ and $ax \equiv b_2 \pmod{m}$ are solvable as well.

3. Determine whether the plane through the points $A(1,0,2)$, $B(2,1,1)$ and $C(5,-2,-4)$ intersects the z axis or not. If yes, then determine the point of intersection.

4. Let the set V consist of the vectors $\underline{v}$ in $\mathbf{R}^4$ for which it holds that the set of vectors $\underline{v}, \underline{w}_1, \underline{w}_2$ is linearly dependent, where $\underline{w}_1 = (1,1,0,0)^T$ and $\underline{w}_2 = (0,0,1,1)^T$. (E.g. $\underline{v} = (1,1,1,1)^T$ is in V.) Decide whether V forms a subspace in $\mathbf{R}^4$ or not.

5. Let $\underline{a} = (1,0,2)^T$, $\underline{b} = (0,-1,1)^T$, $\underline{c} = (-2,1,-4)^T$ be vectors in $\mathbf{R}^3$. Do they form a generating system in $\mathbf{R}^3$?

6. * How many integers a are there between 1 and 2024 (including 1 and 2024) for which $a^{\varphi(1012)} \equiv 1 \pmod{2024}$ holds?

Please work on stapled sheets only, and submit all of them at the end of the midterm, including drafts. Write your name on every sheet you work on, and write your Neptun code on the first page.

You have 90 minutes to work on the problems. Each of them is worth 10 points. The problem marked with an * is supposed to be more difficult.

The details of the solutions must be explained, giving the result only is not worth any points. Notes, calculators, phones (or similar devices) cannot be used.