

Introduction to the Theory of Computing 1.
First Midterm Test
October 25, 2024

1. Determine all the integers between 1 and 500 for which it holds that 48 times the integer gives 1 more remainder when divided by 277 than the integer itself.
which, when multiplied by 48, give the remainder 11 if we divide them by 2020. Determine all the integers n between 1 and 2023 for which both the fraction $\frac{n-2}{21}$ and the fraction $\frac{n-5}{166}$ has an integer value.
2. Determine all the integers between 1 and 2024 whose last two digits in both the numeral system of base 6 and in the numeral system of base 8 are 11 as well.
3. A regular rectangular chest stands on a flat, sloping ground. The corner (vertex) of the chest $A(1, 4, 2)$ is on the ground, but the vertex $B(4, 2, 1)$ which is adjacent to A is not on the ground. Does the plane of the ground intersect the z axis? If yes, where?
4. We know of the vectors $\underline{a}, \underline{b}, \underline{c}, \underline{d}$ that $\underline{d} \in \text{span } \underline{a}, \underline{b}, \underline{c}$, but $\underline{a} \notin \text{span } \underline{b}, \underline{c}, \underline{d}$. Decide that which possibility holds from the following ones for the two statements below.
 - i) the statement is always true,
 - ii) the statement is always false,
 - iii) the statement can be true or false (depending on the choice of $\underline{a}, \underline{b}, \underline{c}, \underline{d}$).
 - a) $\underline{d} \in \text{span } \underline{b}, \underline{c}$,
 - b) $\underline{b} \in \text{span } \underline{c}, \underline{d}$.
5. Decide whether the vectors in \mathbf{R}^4 below are linearly independent or not.
$$\underline{u} = (1, -3, 1, 1)^T, \underline{v} = (-2, 6, 1, 4)^T, \underline{w} = (0, 0, 4, 9)^T$$
6. * Determine all the Fermat liars for $m = 1024$, that is, all the positive integers less than 1024 which in the Fermat test don't show that 1024 is composite.

Please work on stapled sheets only, and submit all of them at the end of the midterm, including drafts.

Write your name on every sheet you work on, and write your Neptun code and the number of the group you are registered to in Neptun (A1, A2 or A3) on the first page.

You have 90 minutes to work on the problems. Each of them is worth 10 points. To obtain a signature you have to achieve at least 24 points on each of the two midterm tests.

The details of the solutions must be explained; giving the result only is not worth any points. Notes, calculators or any additional tools cannot be used. The problem marked with an * is supposed to be more difficult.