Lab 4: Complex systems

SI1136 Simulation and Modeling

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4.1

4.1 Project

Implement the cellular automaton traffic model with periodic boundary conditions. Run it for 10 cars on a road of length 50 with $v_{\mathrm{max}} = 2, p = 0.5$.[1] Allow the system to evolve before recording the flow rate. Repeat the simulation with a different initial configuration to estimate the uncertainty in the data. Repeat for 2, 5, 20, 30 and 40 cars. Plot the flow rate versus the density. This plot is called the fundamental diagram. Explain its qualitative shape. At what density do traffic jams begin to occur?

[1] Plotting suggestions: plot road on x-axis, time on y-axis and dots for the cars; for a movie plot symbols running along the edge of a square or circle.

4.1 Method

Firstly, the traffic model is simulated when 10 cars start at rest on positions $0, 1, \dots, 9$. The simulation runs for 100 steps and then the positions of the cars are plotted to the time indexes.

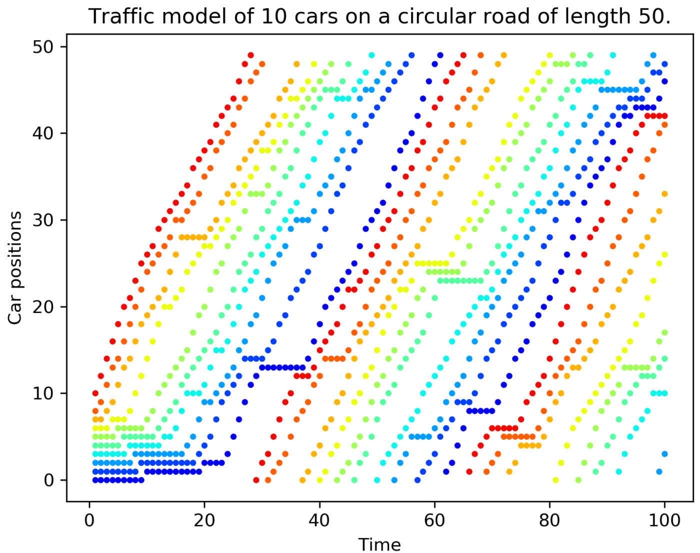
Secondly, 1000 simulations are executed for each $n_{\text{cars}} \in \{2, 5, 10, 15, 20, 25, 30, 35, 40, 45, 50\}$. The flow rate, `sum(carVelocities)/roadlength`, is determined for each simulation when $t = 100$. The 1000 values for each density is used to plot a box plot. The function

$$f(x) = cx^k e^{-ax}$$

if fitted to the means of the flow rates by the least-squares method.[2]

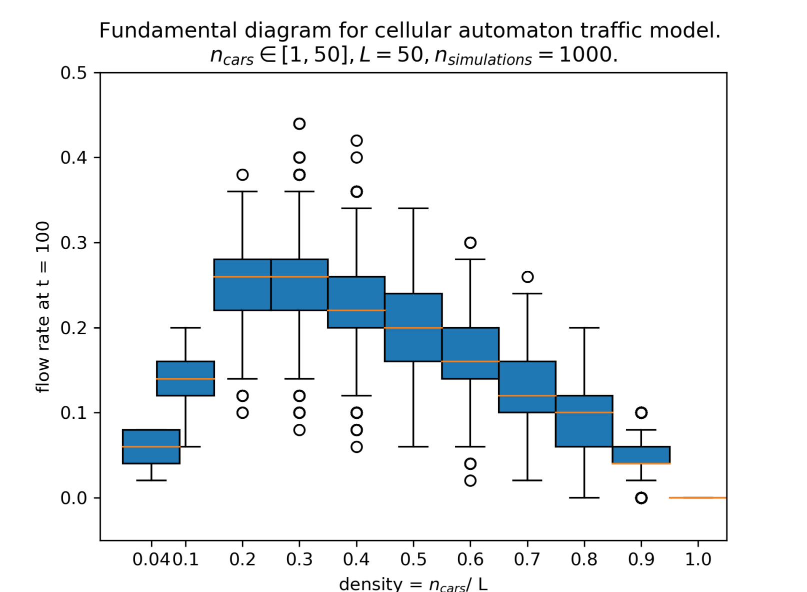
[2] An alternative approach is to reason that the flow rate increases linearly until traffic jam occurs and from here it decreases linearly.

4.1 Results i

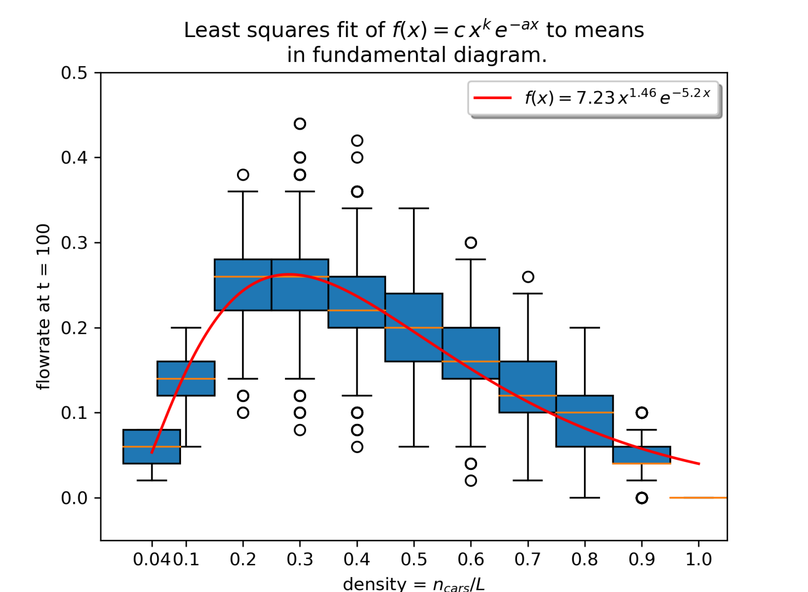


Traffic model of 10 cars on a circular road of length 50.
Car positions
Time
0
10
20
30
40
50
0
20
40
60
80
100

4.1 Results ii



4.1 Results iii



4.1 Results iv

qualitative shape: The qualitative shape of the fundamental diagram follows the function

$$f(x) = cx^k e^{-ax}$$

where

$$c = 7.13, \quad k = 1.46, \quad a = 5.17 \ .$$

An approximation with d as the density is thus

$$\mathit{flow\ rate} = 7.1\sqrt{d^3}e^{-5.2} \ .$$

traffic jams: Traffic jams begin to occur when $d = 0.3$. As $d \to 1$ the flow rate drastically reduces. Of course, when $d = 1$ the flow rate is zero since no cars can move forward.

4.2

4.2 Project

Implement 4.1 with a road length of 500 and the same car densities. Use other road lengths to determine the minimum road length needed to obtain results that are independent of the length of the road.

4.2 Method

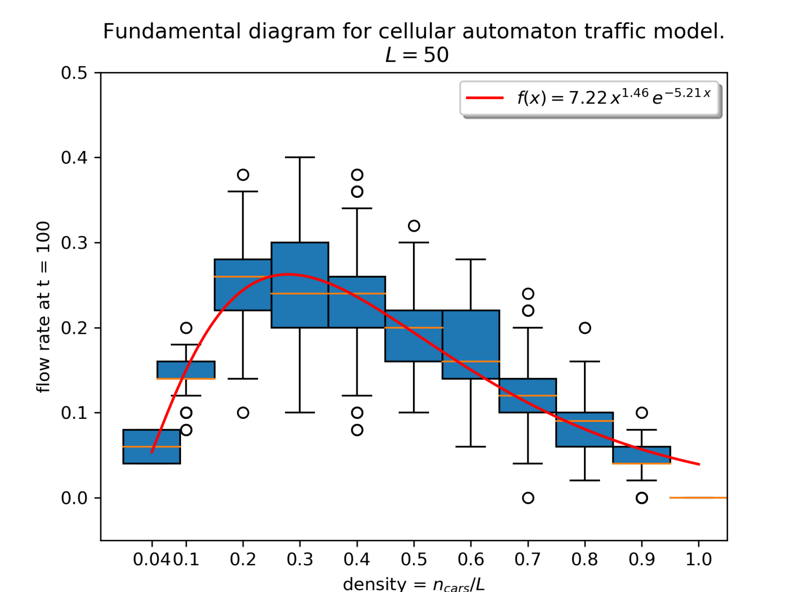
100 simulations are executed for each density,

$$d \in \{0.04, 0.1, 0.2, \dots, 1.0\}$$

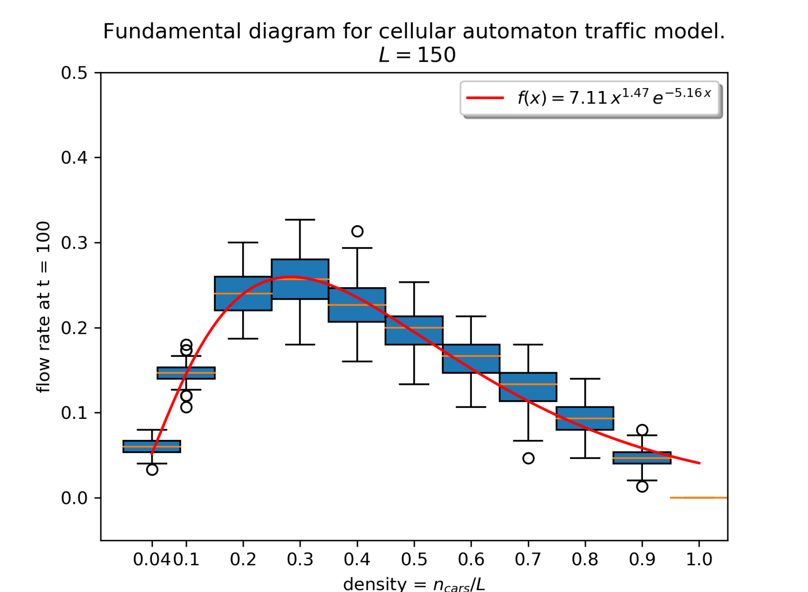
for each road length,

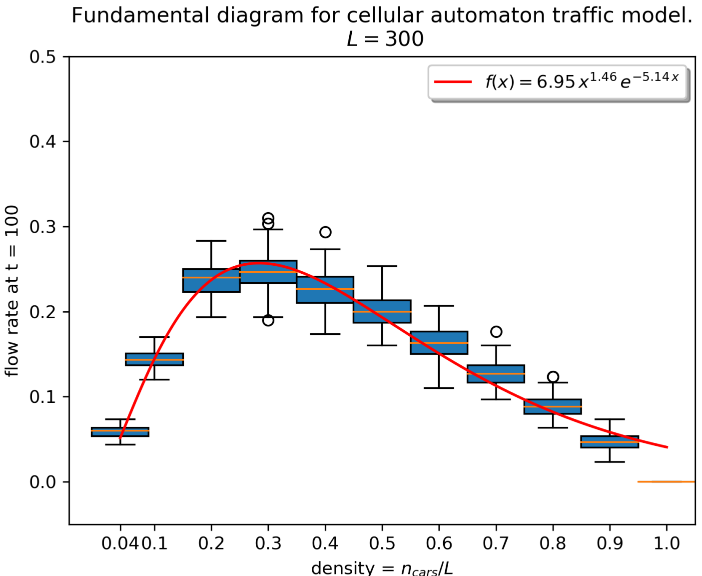
$$L \in \{50, 150, 300, 500\} \ .$$

The flow rate, `sum(carVelocities)/roadlength`, is determined for each simulation at $t = 100$. The 100 values for each d, for each L, is used to plot one box plot for each L.

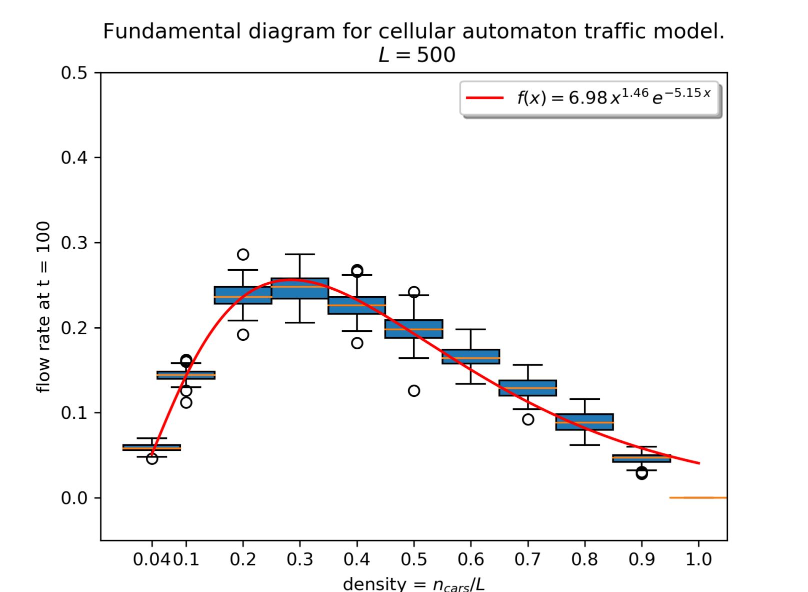

Fundamental diagram for cellular automaton traffic model.
$L = 50$
$f(x) = 7.22\,x^{1.46}\,e^{-5.21\,x}$
flow rate at t = 100
0.0
0.1
0.2
0.3
0.4
0.5
0.04
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
density = n_{cars}/L

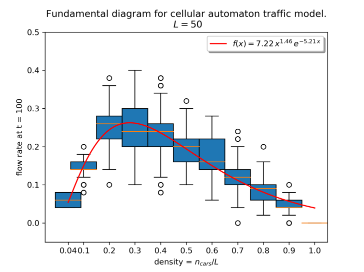
4.2 Results ii

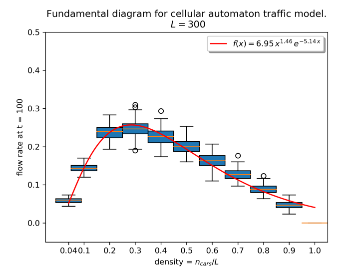


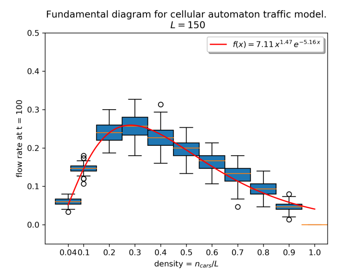

Fundamental diagram for cellular automaton traffic model.
$L = 300$
$f(x) = 6.95\,x^{1.46}\,e^{-5.14\,x}$
flow rate at t = 100
0.5
0.4
0.3
0.2
0.1
0.0
0.04
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
density = n_{cars}/L

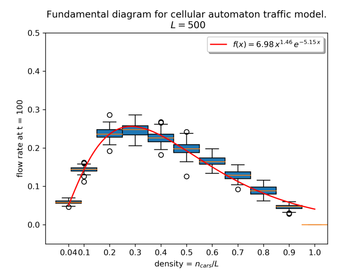
4.2 Results iv




Fundamental diagram for cellular automaton traffic model.
$L = 50$
$f(x) = 7.22\,x^{1.46}\,e^{-5.21\,x}$
flow rate at t = 100
density = n_{cars}/L
0.0 0.1 0.2 0.3 0.4 0.5
0.04 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0


Fundamental diagram for cellular automaton traffic model.
$L = 300$
$f(x) = 6.95\,x^{1.46}\,e^{-5.14\,x}$
flow rate at t = 100
density = n_{cars}/L
0.0 0.1 0.2 0.3 0.4 0.5
0.04 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0


Fundamental diagram for cellular automaton traffic model.
$L = 150$
$f(x) = 7.11\,x^{1.47}\,e^{-5.16\,x}$
flow rate at t = 100
density = n_{cars}/L
0.0 0.1 0.2 0.3 0.4 0.5
0.04 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0


Fundamental diagram for cellular automaton traffic model.
$L = 500$
$f(x) = 6.98\,x^{1.46}\,e^{-5.15\,x}$
flow rate at t = 100
density = n_{cars}/L
0.0 0.1 0.2 0.3 0.4 0.5
0.04 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0

4.2 Results vi

mean: The mean for the four subplots are similar when $L \geq 150$. When $L = 50$ the flow rate is dependent on the length of the road. The primary effect can be seen when $d = 0.2$.

variance: The range of the quartiles are large when $L = 50$. Thus the variance is large here as well.

The road minimum road length to obtain results independent of the length of the road is $L \geq 150$. The variance is of course smaller when $L = 500$, however, to compromise with computer power it's better to use $L = 150$ and the mean of many simulations.

4.3

4.3 Project

For a fixed road length, compare your results for $v_{\text{max}} = 1$ with your results for $v_{\text{max}} = 2$. Also consider $v_{\text{max}} = 5$. Are there any quantitative differences in the behavior of the cars?

Firstly, the traffic model is simulated when 20 cars start at rest on positions $0, 1, \ldots, 19$ for each

$$v_{\text{max}} \in \{1, 2, 5\}$$

with the parameter values

$$L = 100, \quad p_{\text{brake}} = 0.5 \ .$$

The simulation runs for 300 steps and then the positions of the cars are plotted to the time indexes.

Secondly, 100 simulations are executed for each $n_{\text{cars}} \in \{4, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100\}$. The flow rate,

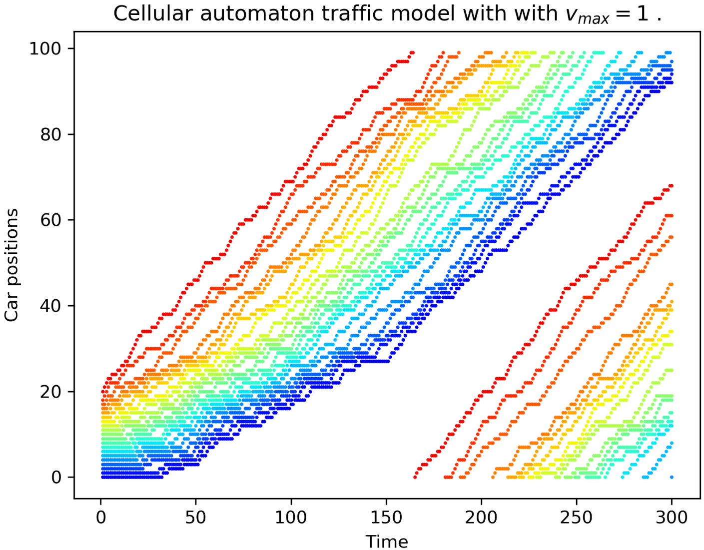
4.3 Method ii

`sum(carVelocities)/roadlength`, is determined for each simulation when $t = 300$. The 100 values for each density is used to plot a box plot. The function

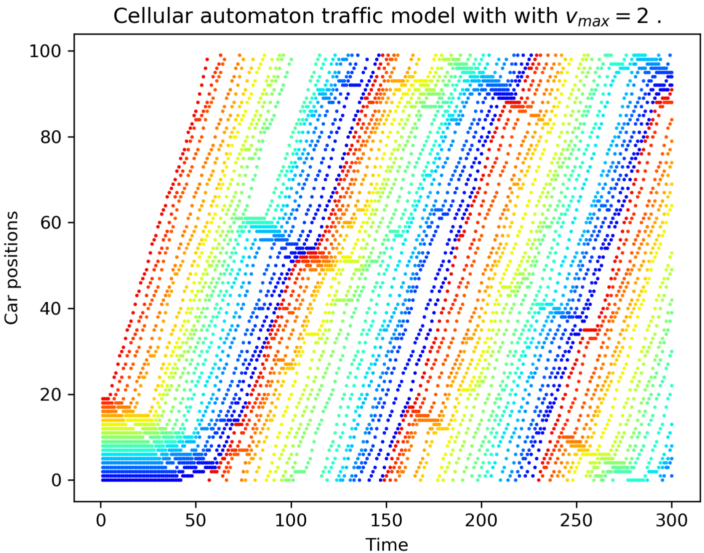
$$f(x) = cx^k e^{-ax}$$

if fitted to the means of the flow rates by the least-squares method.

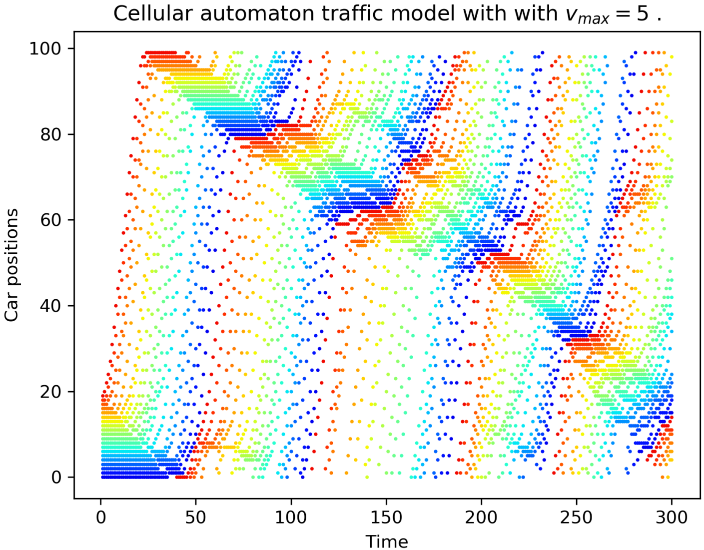
4.3 Results i

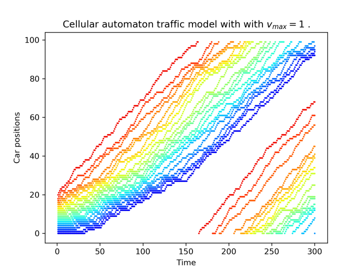

Cellular automaton traffic model with with $v_{max} = 1$.
Car positions
Time
0
20
40
60
80
100
0
50
100
150
200
250
300

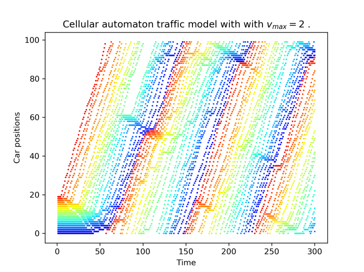
4.3 Results ii

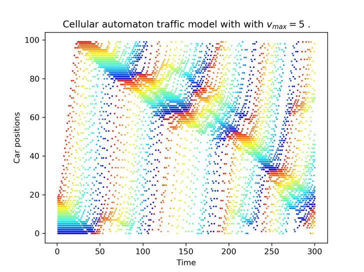


4.3 Results iii

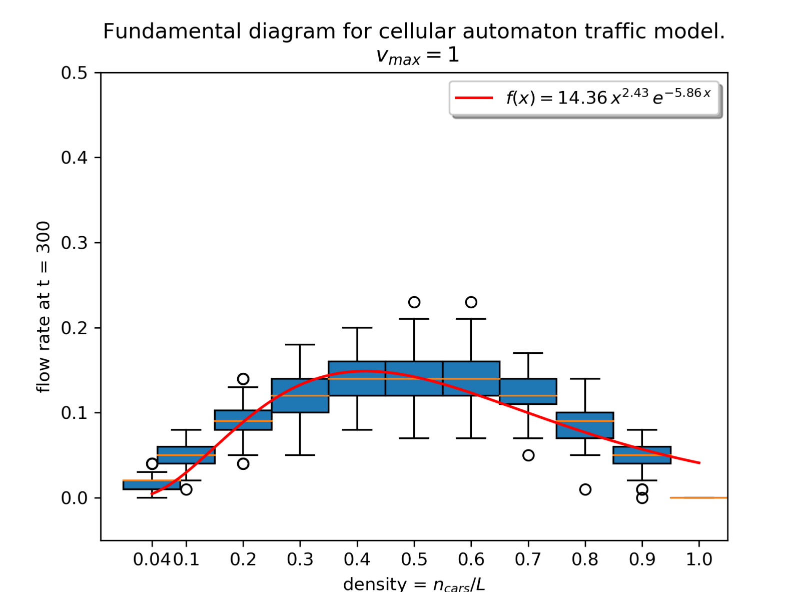



Cellular automaton traffic model with with $v_{max} = 1$.
Car positions
Time
0
20
40
60
80
100
0
50
100
150
200
250
300


Cellular automaton traffic model with with $v_{max} = 2$.
Car positions
Time
0
20
40
60
80
100
0
50
100
150
200
250
300

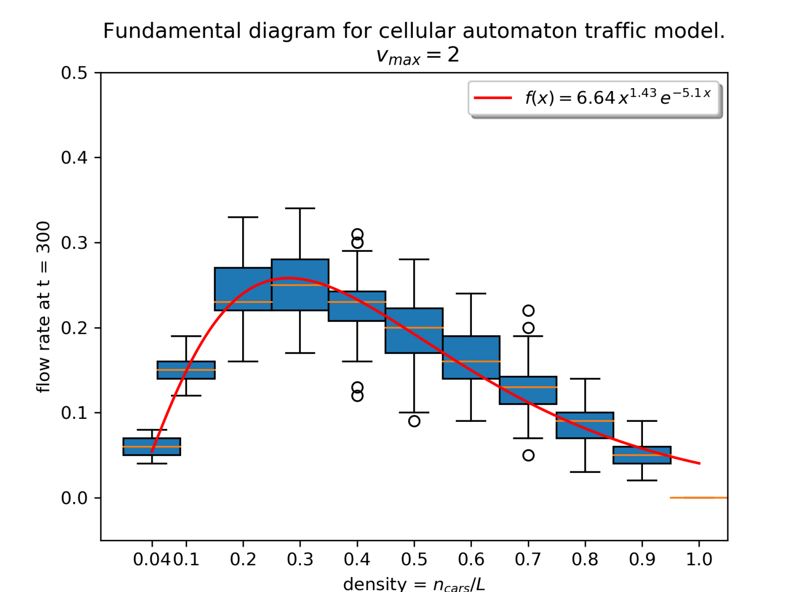

Cellular automaton traffic model with with $v_{max} = 5$.
Car positions
Time
0
20
40
60
80
100
0
50
100
150
200
250
300

4.3 Results v

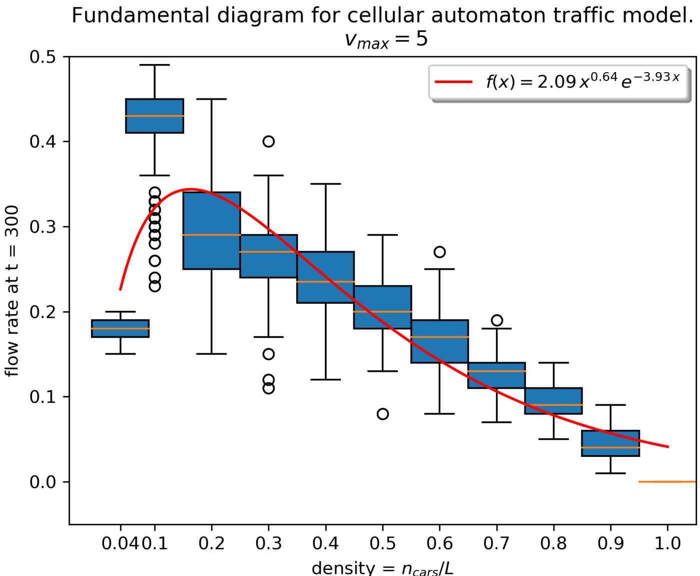


Fundamental diagram for cellular automaton traffic model.
$v_{max} = 1$
$f(x) = 14.36\,x^{2.43}\,e^{-5.86\,x}$
flow rate at t = 300
density = n_{cars}/L
0.0
0.1
0.2
0.3
0.4
0.5
0.04
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

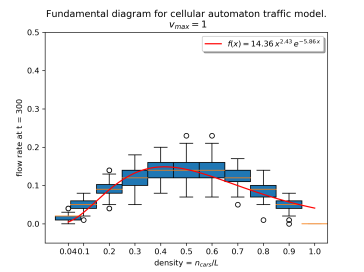
4.3 Results vi

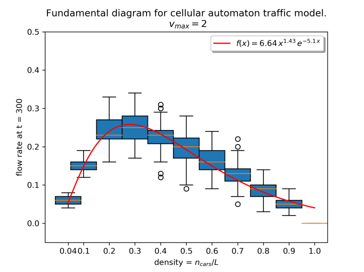


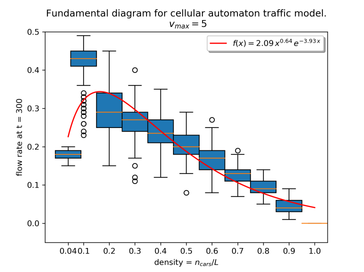
4.3 Results vii



4.3 Results viii







4.3 Results ix

Even though it causes large, indefinite traffic jams, $v_{\text{max}} = 5$ had the highest flow rate when $d = 0.1$.

A general rule for maximum flow rate is decrease v_{max} as d increases. This reduces traffic jams, the propagation of traffic jams as well as increasing the safety on the road. Even though $v_{\text{max}} = 5$ had the best results it's not doesn't reflect human reaction time and traffic accidents.

4.4

4.4 Project

Explore the effect of the speed reduction probability by considering $p=0.2$ and $p=0.8$.

4.4 Method i

Firstly, the traffic model is simulated when 20 cars start at rest on positions $0, 1, \ldots, 19$ for each

$$p_{\text{brake}} \in \{0.2, 0.5, 0.8\}$$

with the parameter values

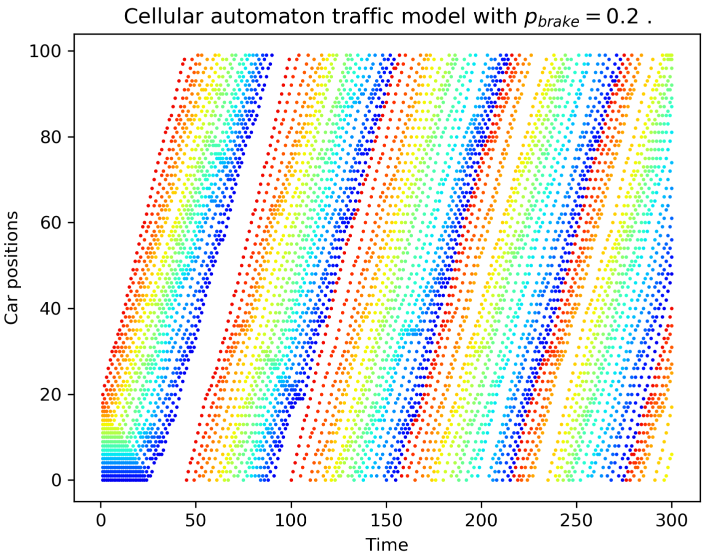
$$L = 100, \quad v_{\text{max}} = 2 \ .$$

The simulation runs for 300 steps and then the positions of the cars are plotted to the time indexes.

Secondly, 100 simulations are executed for each $n_{\text{cars}} \in \{4, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100\}$. The flow rate, `sum(carVelocities)/roadlength`, is determined for each simulation when $t = 300$. The 100 values for each density is used to plot a box plot. The function

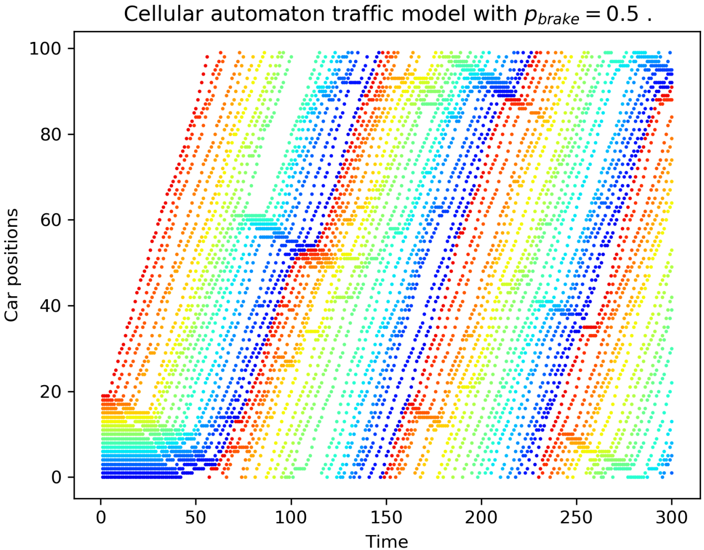
$$f(x) = cx^k e^{-ax}$$

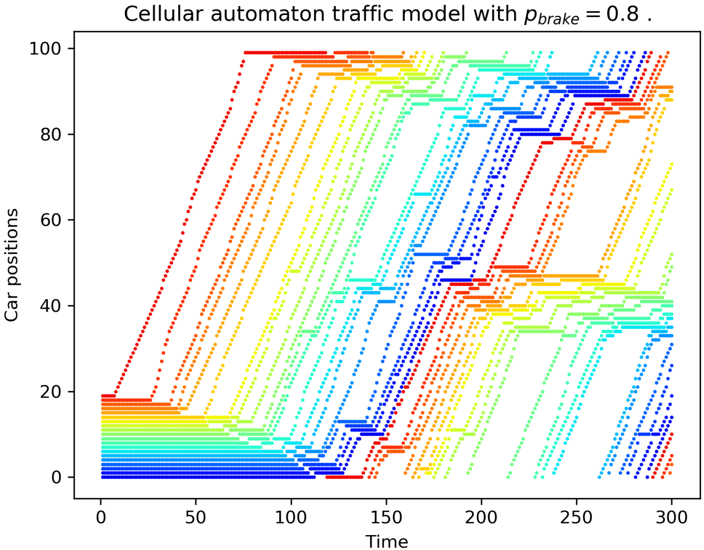
if fitted to the means of the flow rates by the least-squares method.



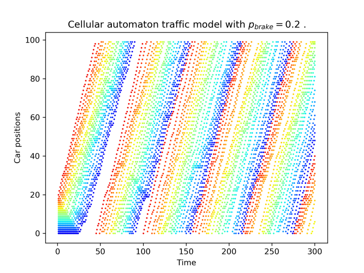
Cellular automaton traffic model with $p_{brake} = 0.2$.
Car positions
Time
0
20
40
60
80
100
0
50
100
150
200
250
300

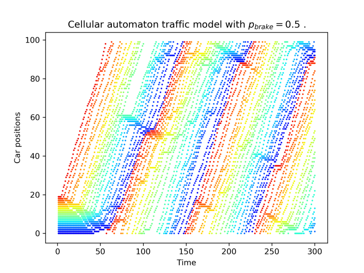
4.4 Results ii

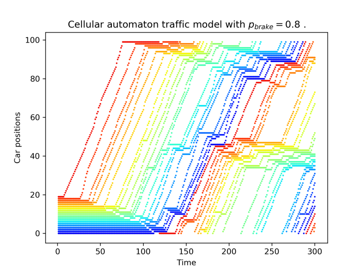



Cellular automaton traffic model with $p_{brake} = 0.8$.
Car positions
100
80
60
40
20
0
0
50
100
150
200
250
300
Time

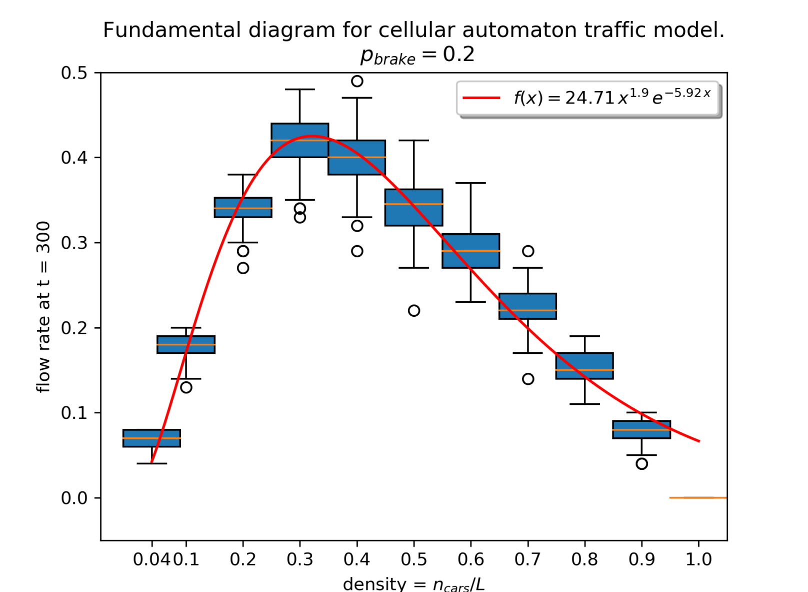
4.4 Results iv





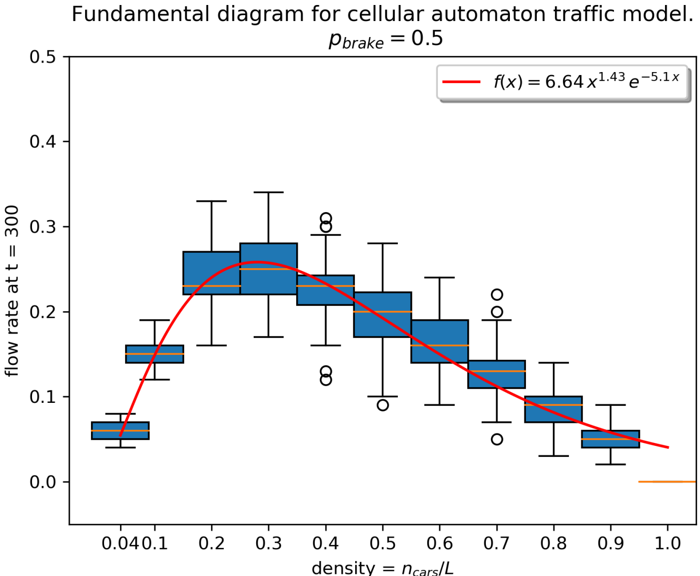


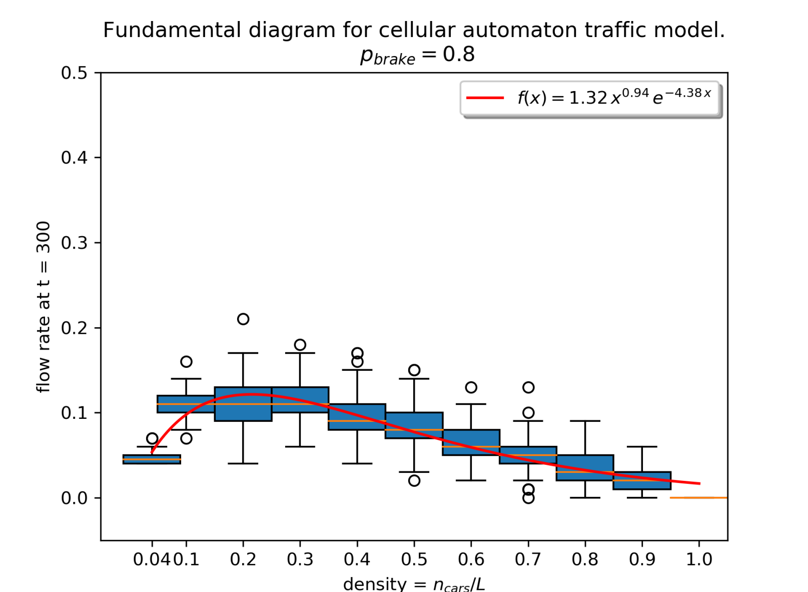
4.4 Results v



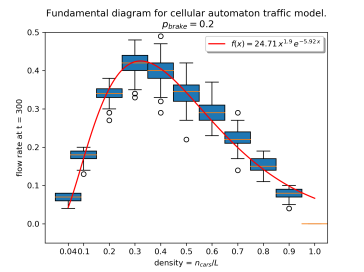
Fundamental diagram for cellular automaton traffic model.
$p_{brake} = 0.2$
$f(x) = 24.71\,x^{1.9}\,e^{-5.92\,x}$
flow rate at t = 300
density = n_{cars}/L
0.0
0.1
0.2
0.3
0.4
0.5
0.04
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

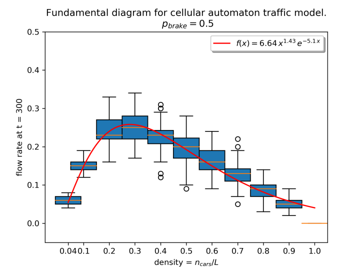
4.4 Results vi

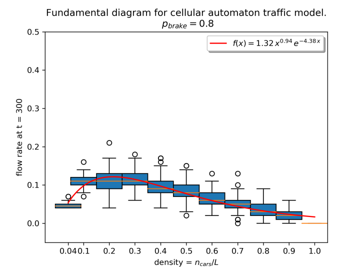



Fundamental diagram for cellular automaton traffic model.
$p_{brake} = 0.8$
$f(x) = 1.32\,x^{0.94}\,e^{-4.38\,x}$
flow rate at t = 300
0.5
0.4
0.3
0.2
0.1
0.0
0.04
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
density = n_{cars}/L

4.4 Results viii







flow rate: Increasing the speed reduction probability decreases the flow rate for all densities.

traffic jams: Increasing the speed reduction probability increases the frequency, size and propagation of traffic jams.

density for maximum flow rate: Increasing the speed reduction probability decreases the density required for maximum flow rate.

4.5

Determine $N(s)$, the number of clusters of trees of size s that catch fire in each iteration. Two trees are in the same cluster if they are nearest neighbors. Is the behavior of $N(s)$ consistent with $N(s) \sim s^{-\alpha}$? (hint: also try $f \ll g$) If so, estimate the exponent α for several values of g and f.

Critical phenomenon

A large, rare event. A system is critical if

$$N(s) \sim s^{-\alpha}$$

where $N(s)$ is the number of events of size s.

For each

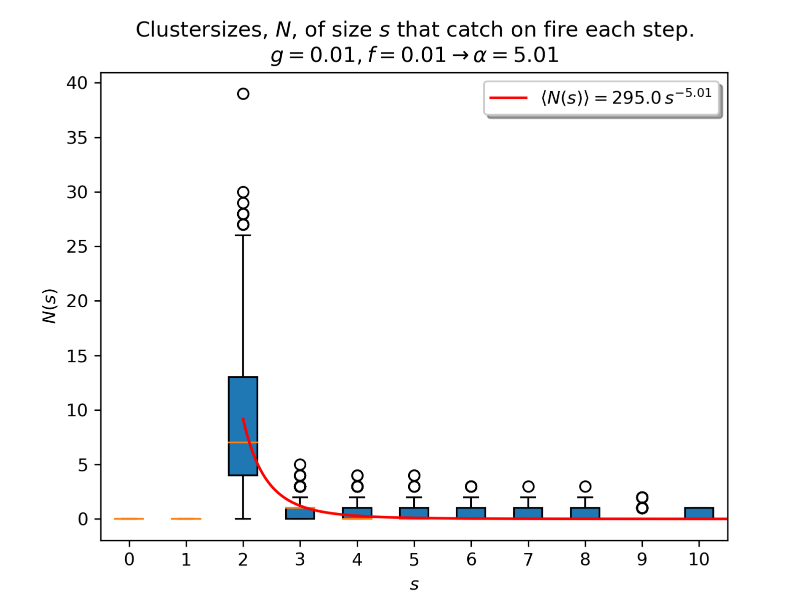
$$g \in \{0.1, 0.2\}, \quad f \in \{0.01, 0.1, 0.3\}$$

the forest fire is simulated during 500 time steps. $N(s)$ is found by counting the depth of the recursive spreading of the fire. $N(s)$ is then plotted as a box plot and a function

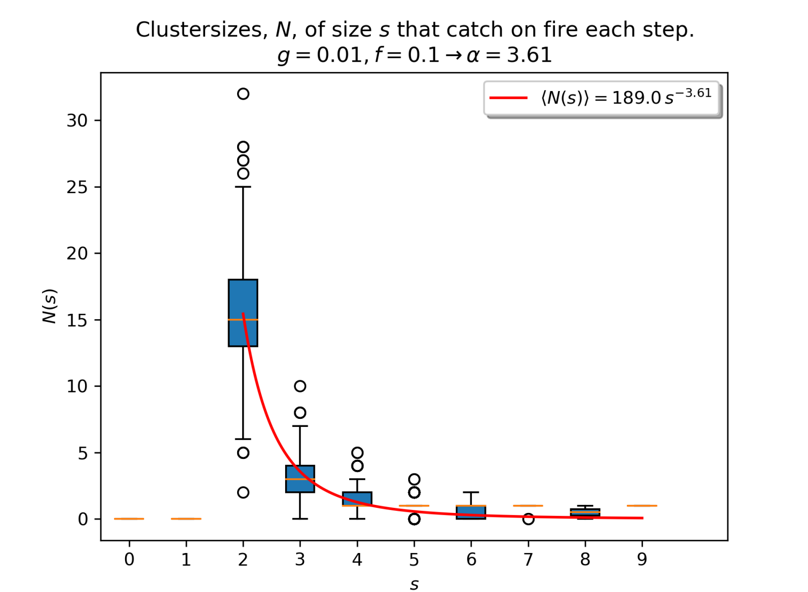
$$f(s) = cs^{-\alpha}$$

is fitted to the mean data points when $2 \leq s \leq 10$. Lastly, $\langle N(s) \rangle$ is plotted for all combinations of g and f.

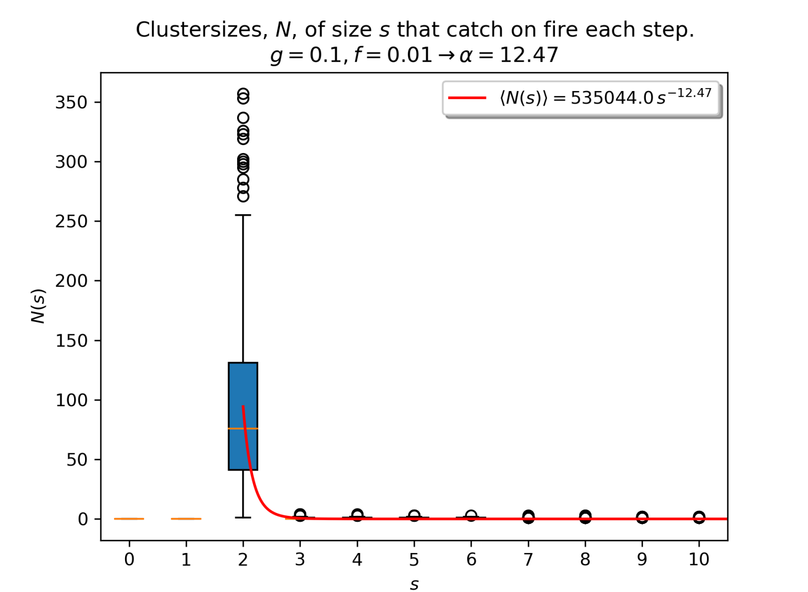
4.5 Results i

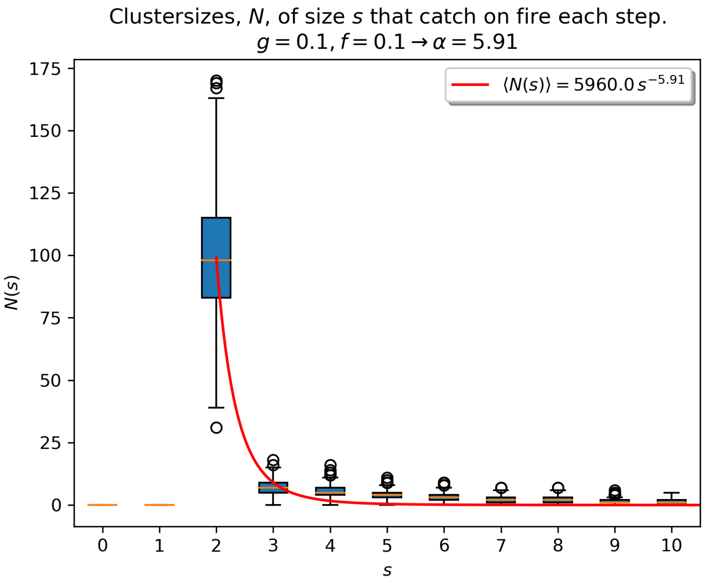


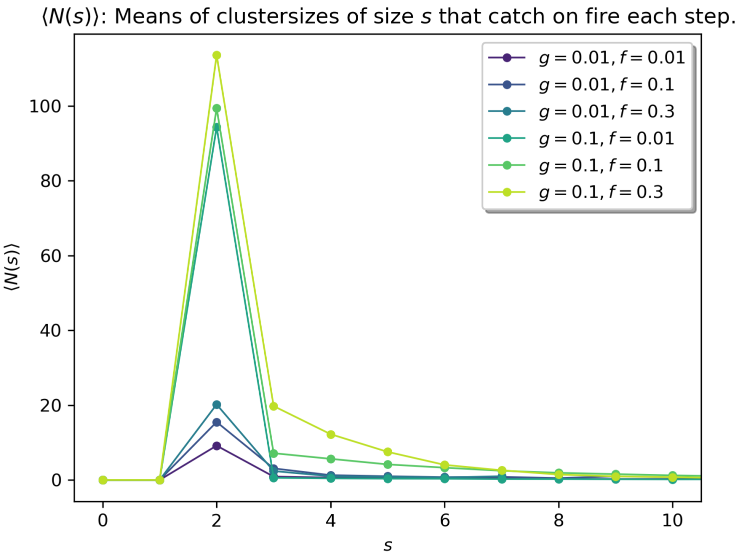
Clustersizes, N, of size s that catch on fire each step.
$g = 0.01, f = 0.01 \rightarrow \alpha = 5.01$
$\langle N(s) \rangle = 295.0\, s^{-5.01}$
$N(s)$
s
0
5
10
15
20
25
30
35
40
0
1
2
3
4
5
6
7
8
9
10


Clustersizes, N, of size s that catch on fire each step.
$g = 0.01, f = 0.1 \rightarrow \alpha = 3.61$
$\langle N(s) \rangle = 189.0\, s^{-3.61}$
$N(s)$
s
0
5
10
15
20
25
30
0
1
2
3
4
5
6
7
8
9

4.5 Results iii




Clustersizes, N, of size s that catch on fire each step.
$g = 0.1, f = 0.1 \rightarrow \alpha = 5.91$
$\langle N(s) \rangle = 5960.0\, s^{-5.91}$
$N(s)$
s
0
25
50
75
100
125
150
175
0
1
2
3
4
5
6
7
8
9
10


⟨N(s)⟩: Means of clustersizes of size s that catch on fire each step.
g = 0.01, f = 0.01
g = 0.01, f = 0.1
g = 0.01, f = 0.3
g = 0.1, f = 0.01
g = 0.1, f = 0.1
g = 0.1, f = 0.3
100
80
60
40
20
0
⟨N(s)⟩
0
2
4
6
8
10
s

g	f	α
0.01	0.01	5.01
0.01	0.1	3.61
0.01	0.3	4.92
0.1	0.01	12.47
0.1	0.1	5.91
0.1	0.3	3.78

Table 1: Table over some combinations of g and f and the corresponding α.

Appendix

Source code

Available at:

`https://github.com/axelstr/SI1336-Simulations-and-Modeling/tree/master/4%20Complex%20systems`