CSC165 Assignment 1

Axel Steingrimsson
999707143,
axel.steingrimsson@mail.utoronto.ca

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1 Question 1

1.1 a)

C: Carol, D: Dan

$B(C) \wedge B(D) \Rightarrow P(C, D) \wedge P(D, C)$

Negation: If Carol and Dan don't play this game, then they haven't played it with each other.

$\neg(B(C) \wedge B(D)) \Rightarrow \neg(P(C, D) \wedge P(D, C))$

1.2 b)

C: Class, B: Bob

$\forall_x \in C, \neg P(x, B)$

Negation: Everyone in the class has played this 2 person game with Bob

$\forall_x \in C, P(x, B)$

1.3 c)

C: Class

$\exists_x \in C, \neg B(x)$

Negation: There is no one in the class who does not play this 2 person game

$\exists_x \in C, \neg B(x), x = 0$

1.4 d)

C: Class

$\exists_x \in C, \exists_y \in C, \exists_z \in C, P(x, y) \Rightarrow P(x, z), y \neq z$

Negation: Everyone in your class who plays this 2-person board game has played the game with only one person.

$\exists_x \in C, \exists_y \in C, \exists_z \in C, \neg P(x, y) \Rightarrow \neg P(x, z), y = z$

1.5 e)

C: Class

$\exists_x \in C, \forall_y \in C, P(x, y)$

Negation: There is no student in your class has played this 2-person board game with everyone in your class.

$\exists_x \in C, \forall_y \in C, \neg P(x, y)$

2 Question 2

2.1 a)

Bob has not traveled outside of the country.

2.2 b)

Some of the students have traveled to Denmark, but all of the students have traveled to France.

2.3 c)

Pete and Mona have both traveled to some countries, but they have not traveled to the same ones.

2.4 d)

None of the students have traveled to the same countries.

2.5 e)

There is more than one student that has traveled to all of the countries.

2.6 f)

All of the students have been to some countries.

3 Question 3

3.1 a)

Converse: If Socrates is Mortal, and all men are Mortal, then Socrates is a man
Contrapositive: If Socrates isn't mortal, and all men are Mortal, then Socrates isn't a man.

3.2 b)

Converse: If the sailing race is held and the lifesaving demonstration will goes on, then it does not rain or it is not foggy.
Contrapositive: If the sailing race is not held and the lifesaving demonstration does not go on, then it rains or it is foggy.

3.3 c)

Converse: If I'm sore today, then I was playing hockey yesterday
Contrapositive: If I'm not sore today, then I didn't play hockey yesterday.

4 Question 4

4.1 a)

$S1 \Rightarrow S2$ Because There are no Natural numbers that satisfy S2, but the implication is still valid if S1 is False and S2 is True.

4.2 b)

$S3 \Rightarrow S2$ For the elements in S3, 2 is the only prime number that divides into them, therefore if S3 is true, then S2 is true.

4.3 c)

P: Prime number

$\forall_x \in P, x^n = y \Leftrightarrow y \div x = n, S1 \Rightarrow S4$

There are cases for which S1 can be false, so the statement can remain true if S1 implies S4

4.4 d)

$S1 \Rightarrow S5$

There are an infinite possibility of number which i and j could represent, and S1 does not satisfy that set.

5 Question 5

5.1 a)

$((P \Rightarrow Q_1) \wedge (P \Rightarrow Q_2) \Leftrightarrow (P \Rightarrow (Q_1 \wedge Q_2))$
$\Leftrightarrow ((P \Rightarrow Q_1) \wedge (P \Rightarrow Q_2))$
$\Leftrightarrow ((\neg P \vee Q_1) \wedge (\neg P \vee Q_2))$ Implication
$\Leftrightarrow (\neg P \vee (Q_1 \wedge Q_2))$ Distribution Law
$\Leftrightarrow (P \Rightarrow (Q_1 \wedge Q_2))$ Implication

5.2 b)

$((\exists_x \in D, P(x) \vee Q(x)) \Rightarrow R(y)) \Leftrightarrow (((\exists_x \in D, P(x) \Rightarrow R(y)) \wedge ((\exists_x \in D, Q(x)) \Rightarrow R(y)))$
$\Leftrightarrow ((\exists_x \in D, P(x) \vee Q(x)) \Rightarrow R(y))$
$\Leftrightarrow ((\forall_x \in D, \neg P(x) \wedge \neg Q(x)) \vee R(y))$ Implication
$\Leftrightarrow ((\forall_x \in D, \neg P(x)) \vee R(y)) \wedge ((\forall_x \in D, \neg Q(x)) \vee R(y))$ Distribution Law
$\Leftrightarrow ((\neg(\exists_x \in D, P(x)) \vee R(y)) \wedge ((\neg(\exists_x \in D, Q(x)) \vee R(y)))$ DeMorgans Law
$\Leftrightarrow (((\exists_x \in D, P(x)) \Rightarrow R(y)) \wedge ((\exists_x \in D, Q(x)) \Rightarrow R(y)))$ Implication