Amrit Gautam

CMPE 480 – Matlab Project 1
Sampling and Reconstruction

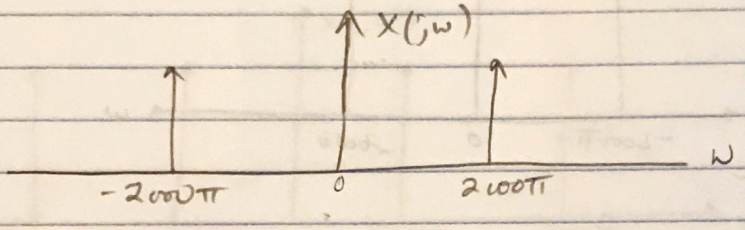
(2) (a)

$x(t) = \cos(2\pi 1000 t)$ at Sampling $F_s = 5000$ Hz

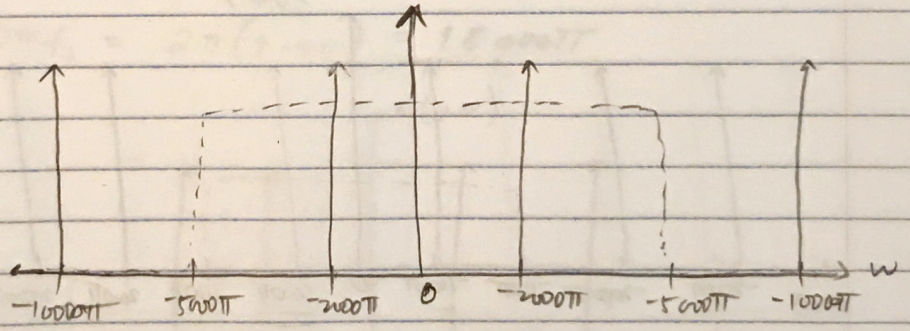
$x(t)\cos(w_0 t) = \pi\delta(w - w_0) + \pi\delta(w + w_0)\ X(jw)$

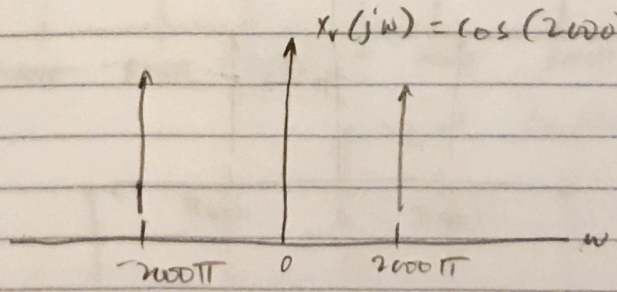
$\cos(2\pi 1000 t) = \pi\delta(w - 2000\pi) + \pi\delta(w + 2000\pi)$

$X(jw) = \pi\delta(w - 2000\pi) + \pi\delta(w + 2000\pi)$

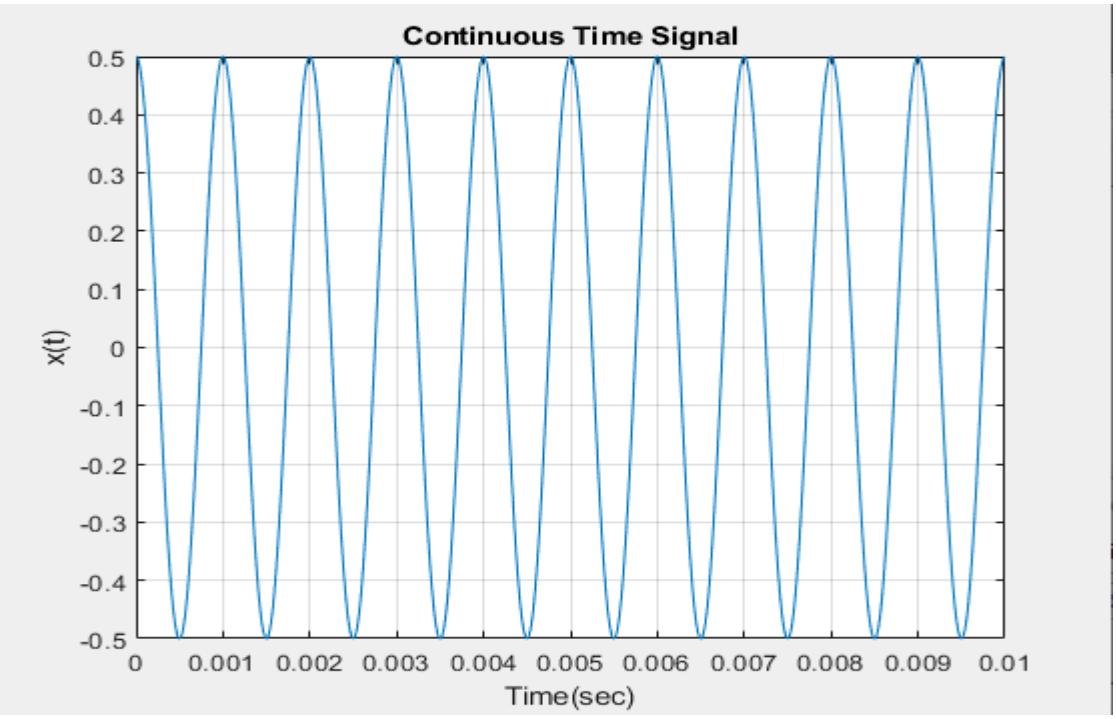


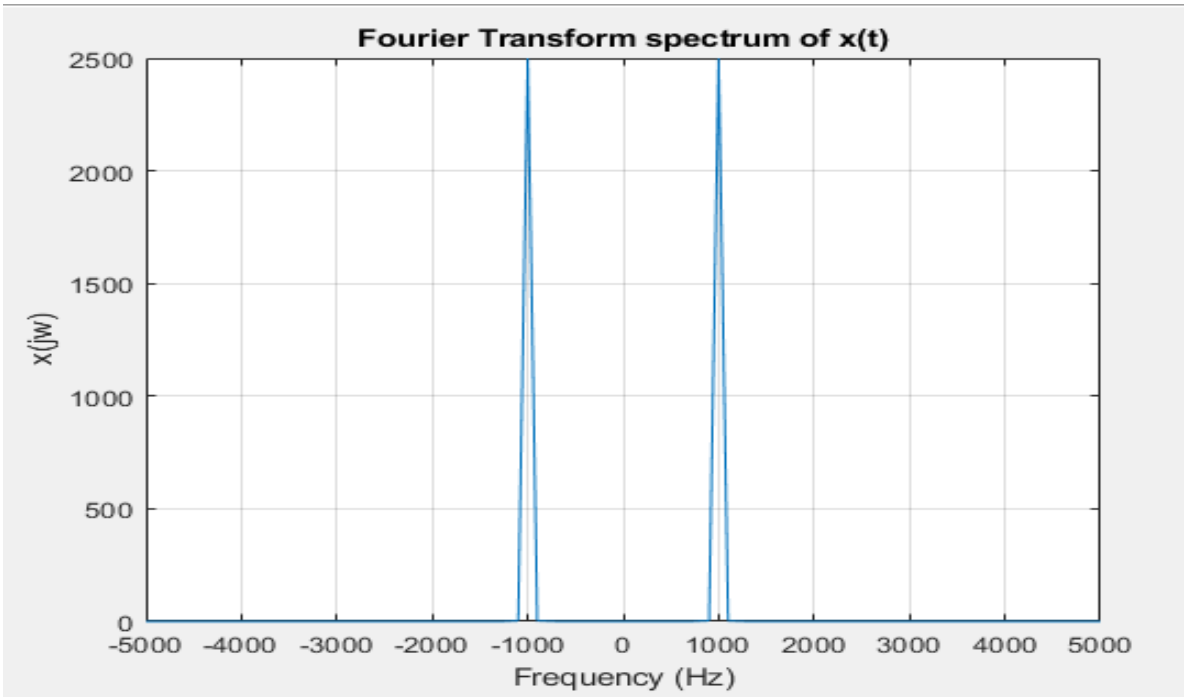
(b) $w_s = 2\pi f_s \Rightarrow 2\pi(5000) = 10000\pi$

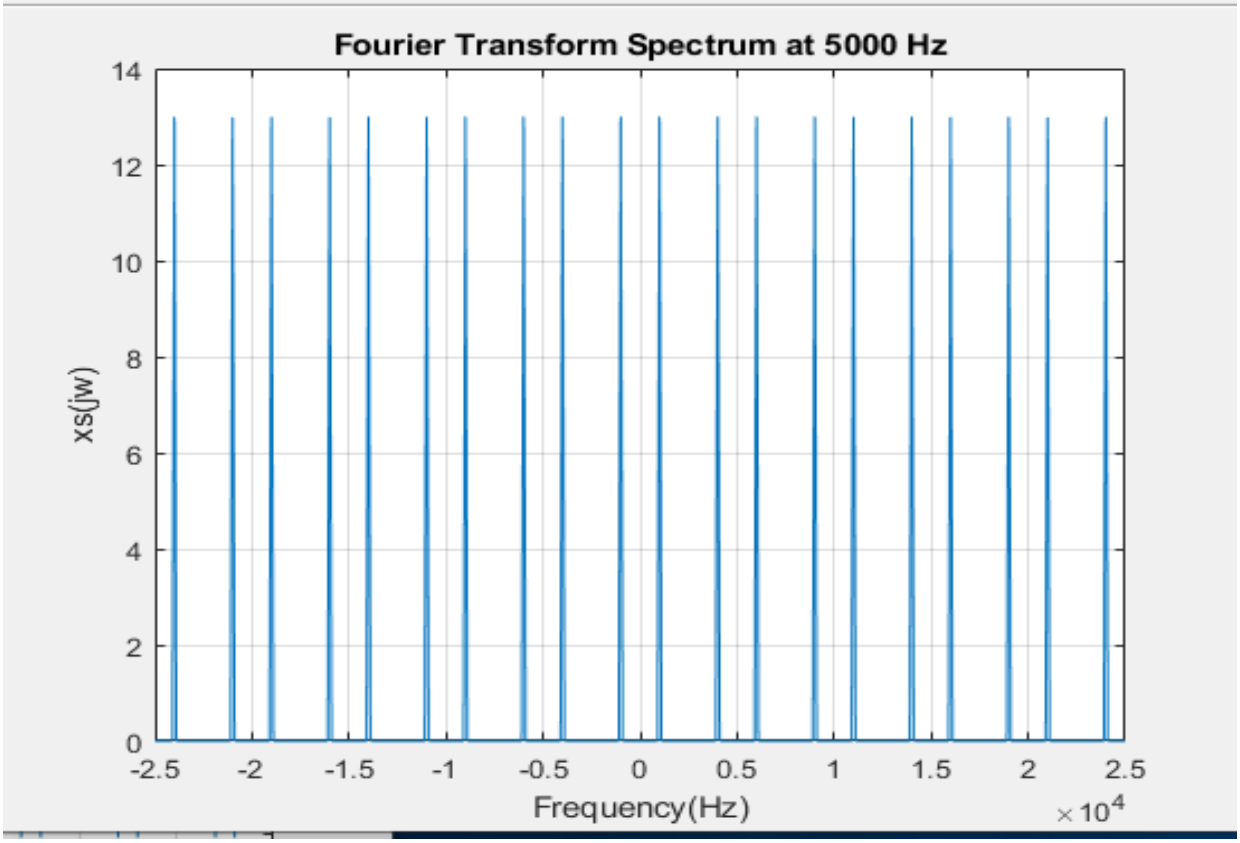


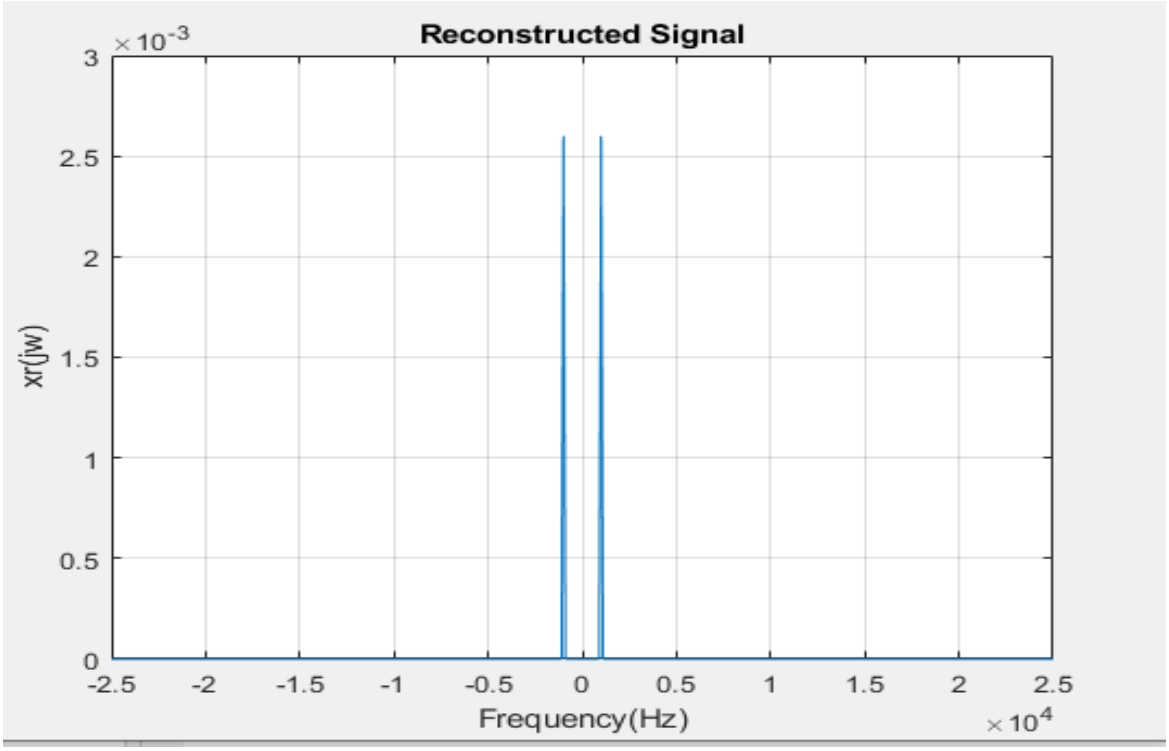


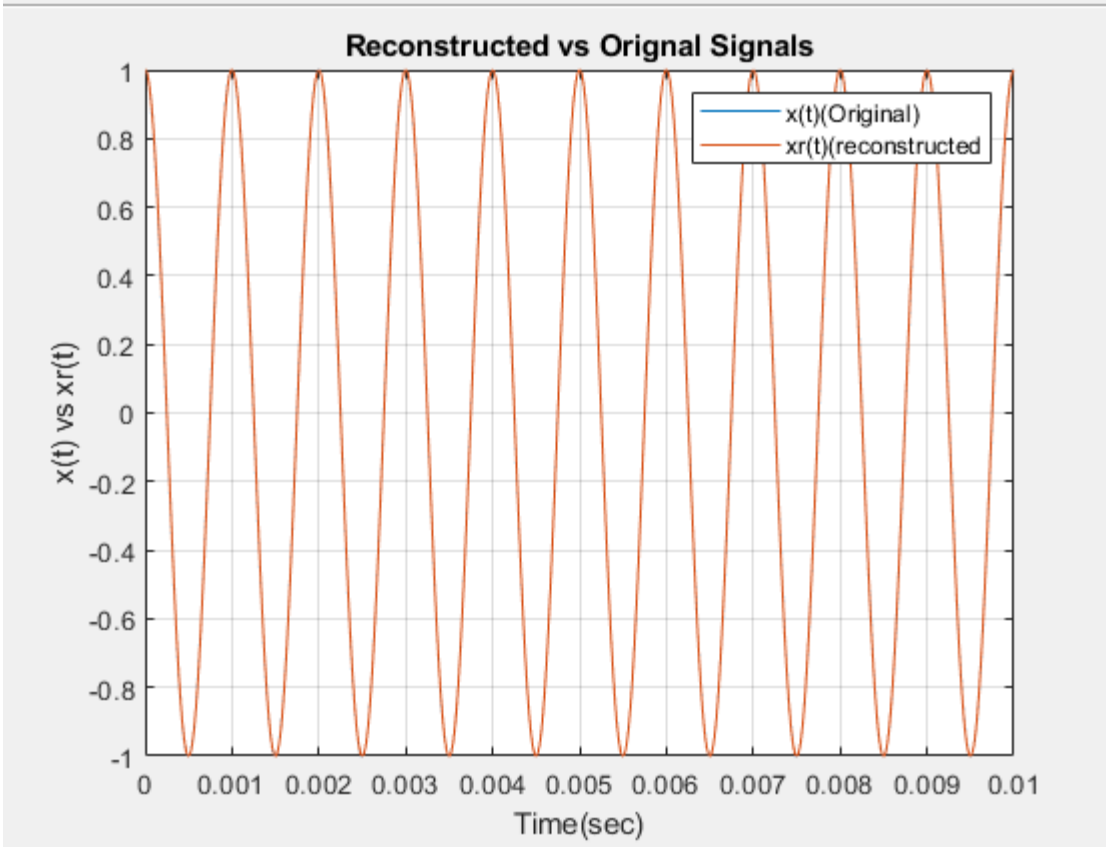
Problem 2


Continuous Time Signal
x(t)
Time(sec)
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
0.009
0.01


Fourier Transform spectrum of x(t)
x(jw)
Frequency (Hz)
2500
2000
1500
1000
500
0
-5000
-4000
-3000
-2000
-1000
0
1000
2000
3000
4000
5000


Fourier Transform Spectrum at 5000 Hz
14
12
10
8
6
4
2
0
xs(jw)
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
Frequency(Hz)
×10^4


Reconstructed Signal
×10^-3
3
2.5
2
1.5
1
0.5
0
xr(jw)
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
Frequency(Hz)
×10^4



d) The results from Matlab and the results from above calculations are as expected and similar after being reconstructed.

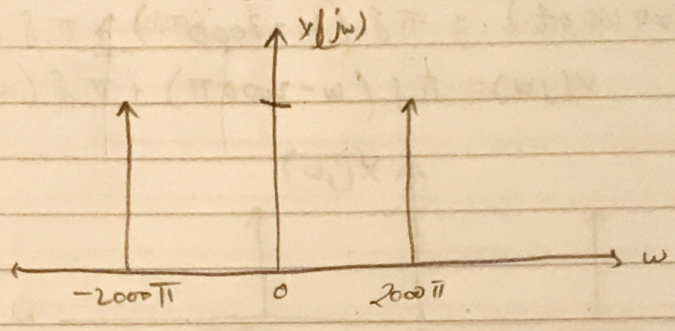
e) As we can see from both calculation and Matlab plot, there is no aliasing. In this case the reconstructed signal matched the original signal.

f) The original signal and reconstructed signal look the same. Looking at the previous statement the signals look to be identical, which shows that there is no aliasing.

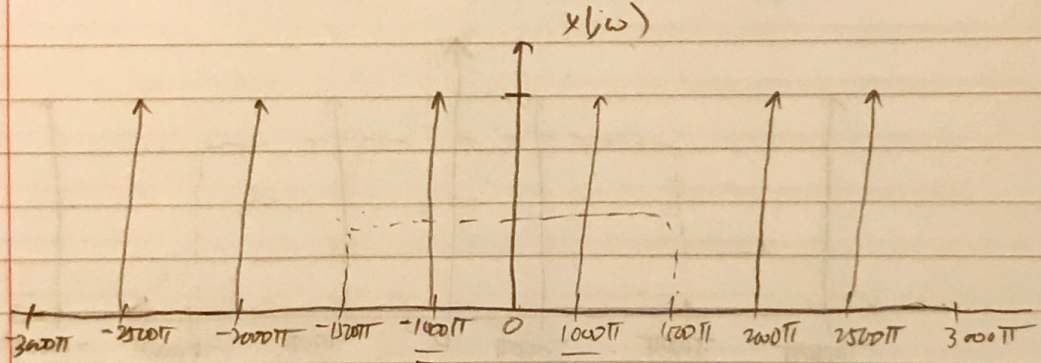
(3) (a) $x(t) = \cos(2\pi 1000 t)$ at Sampling $f_s = 1500\ Hz$

$$x(t)\cos(\omega_0 t) = \pi\delta(\omega-\omega_0) + \pi\delta(\omega+\omega_0) \quad X(j\omega)$$

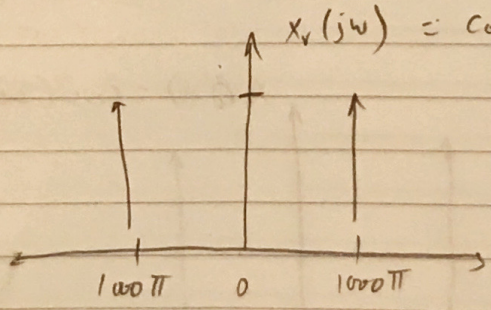
$$X(j\omega) = \pi\delta(\omega - 2000\pi) + \pi\delta(\omega + 2000\pi)$$



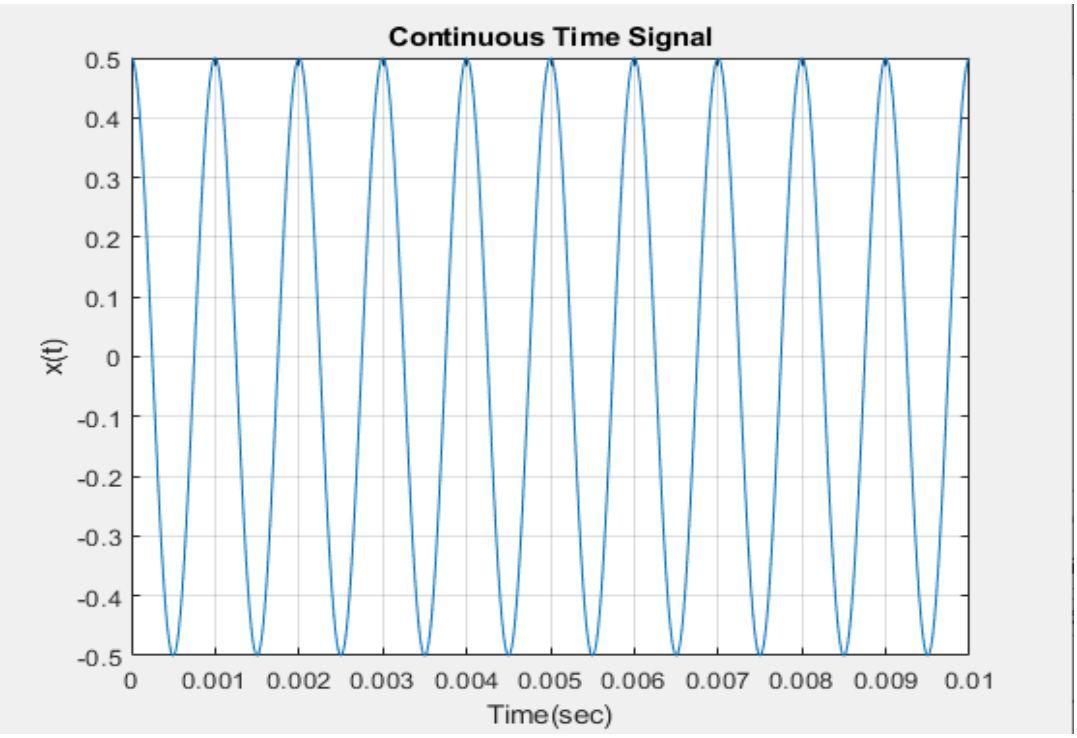
(b) $\omega_s = 2\pi f_s = 2(1500)\pi = 3000\pi$

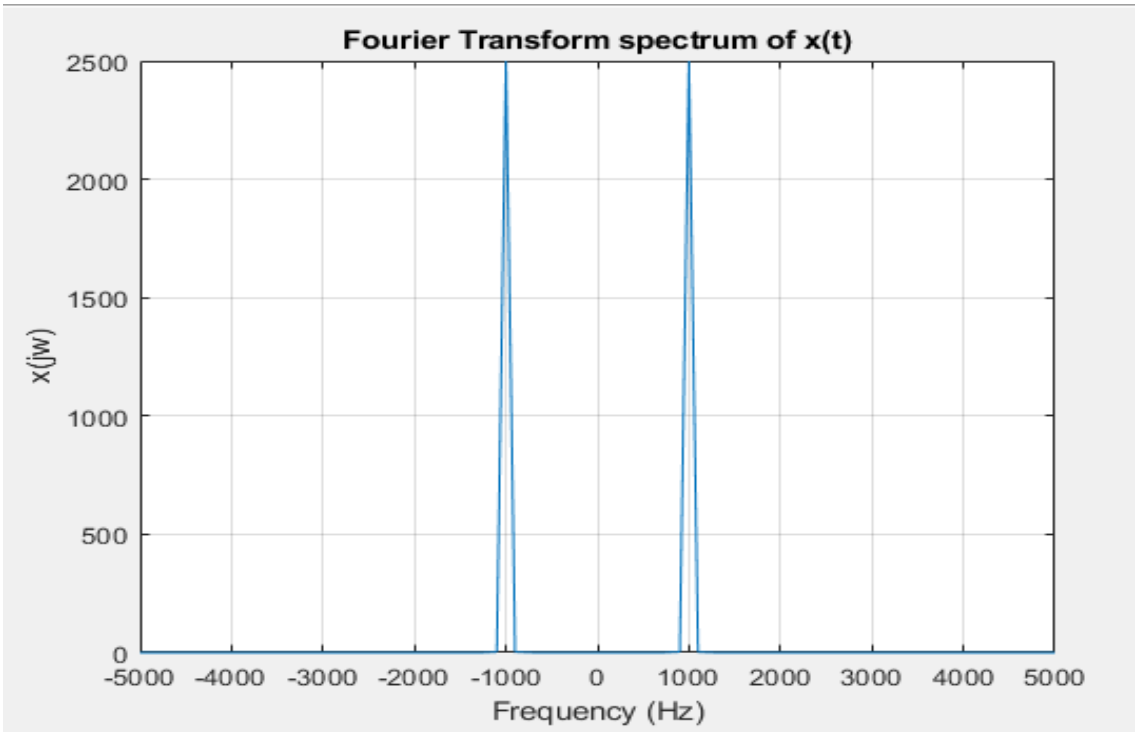


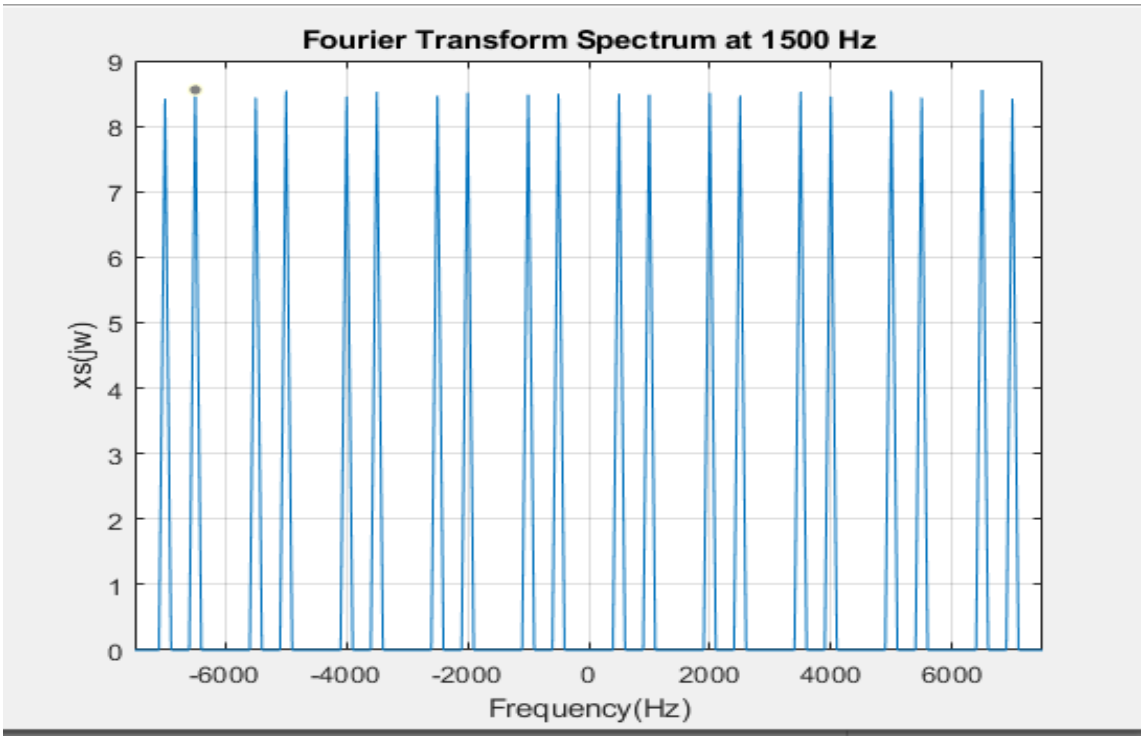
$X_r(j\omega) = \cos(1000\pi t)$

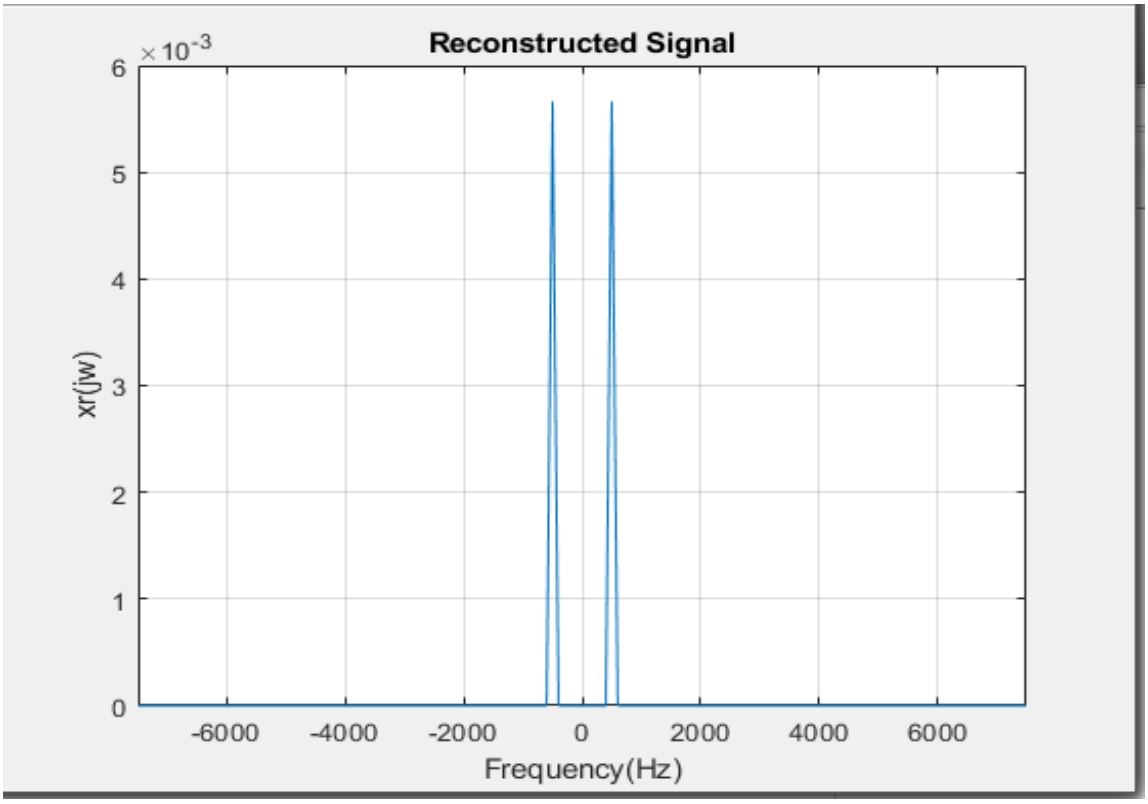


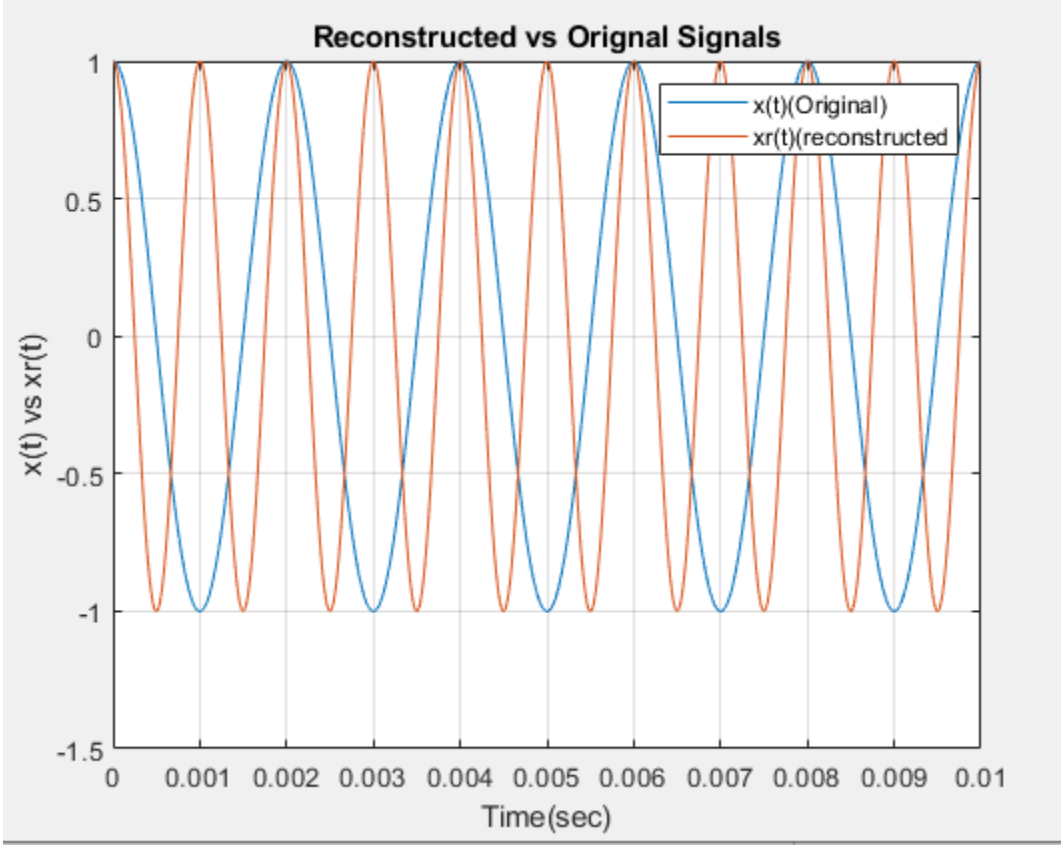
Problem 3


Continuous Time Signal
x(t)
Time(sec)
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
0.009
0.01


Fourier Transform spectrum of x(t)
X(jw)
Frequency (Hz)
2500
2000
1500
1000
500
0
-5000
-4000
-3000
-2000
-1000
0
1000
2000
3000
4000
5000


Fourier Transform Spectrum at 1500 Hz
xs(jw)
Frequency(Hz)
0
1
2
3
4
5
6
7
8
9
-6000
-4000
-2000
0
2000
4000
6000


Reconstructed Signal
×10^-3
xr(jw)
Frequency(Hz)
0
1
2
3
4
5
6
-6000
-4000
-2000
0
2000
4000
6000



d) The results from Matlab and the results from above calculations are as expected and similar after being reconstructed.

e) As we can see from both calculation and Matlab plot, there is aliasing this time. We can see signals distorted which could have caused by having a low frequency.

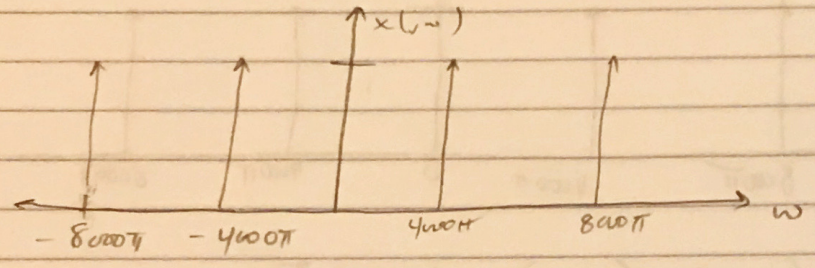
f) The original signal and reconstructed signal look different. Looking at the previous statement the signals look to be not overlapping, which shows that there is aliasing.

(4)

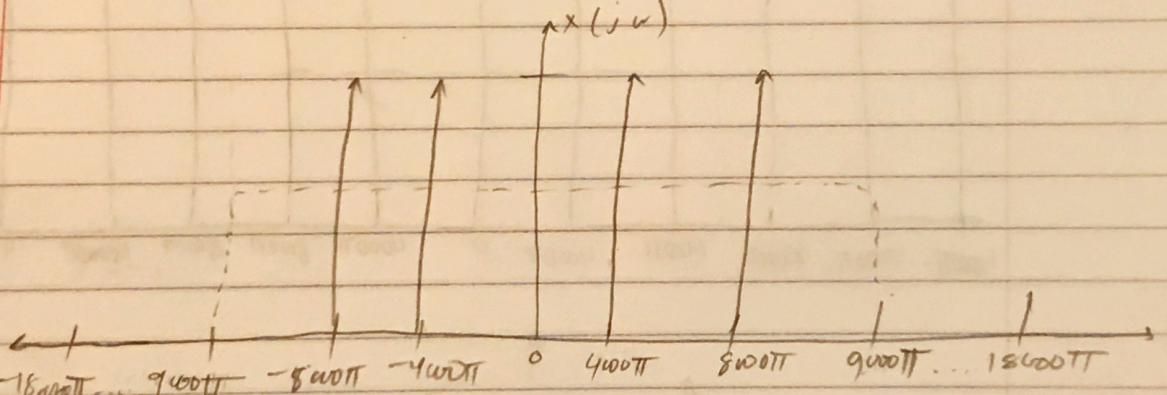
(a) $x(t) = \cos(2\pi 1000t)\cos(2\pi 3000t)$ at Sampling $f_s = 9000\ Hz$

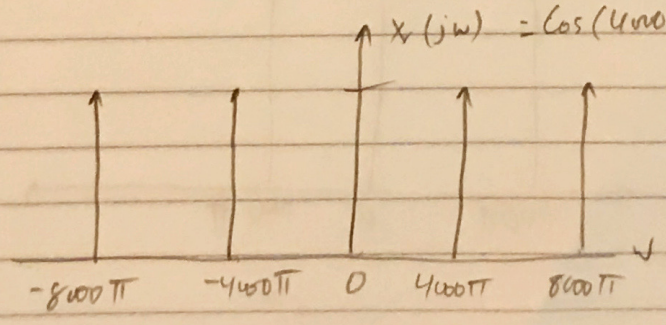
$$X(j\omega) = \mathcal{F}\{x(t)\}$$

$$= \frac{\pi}{2}\delta(\omega - 8000\pi) + \frac{\pi}{2}\delta(\omega - 4000\pi) + \frac{\pi}{2}\delta(\omega + 4000\pi) + \frac{\pi}{2}\delta(\omega + 8000\pi)$$

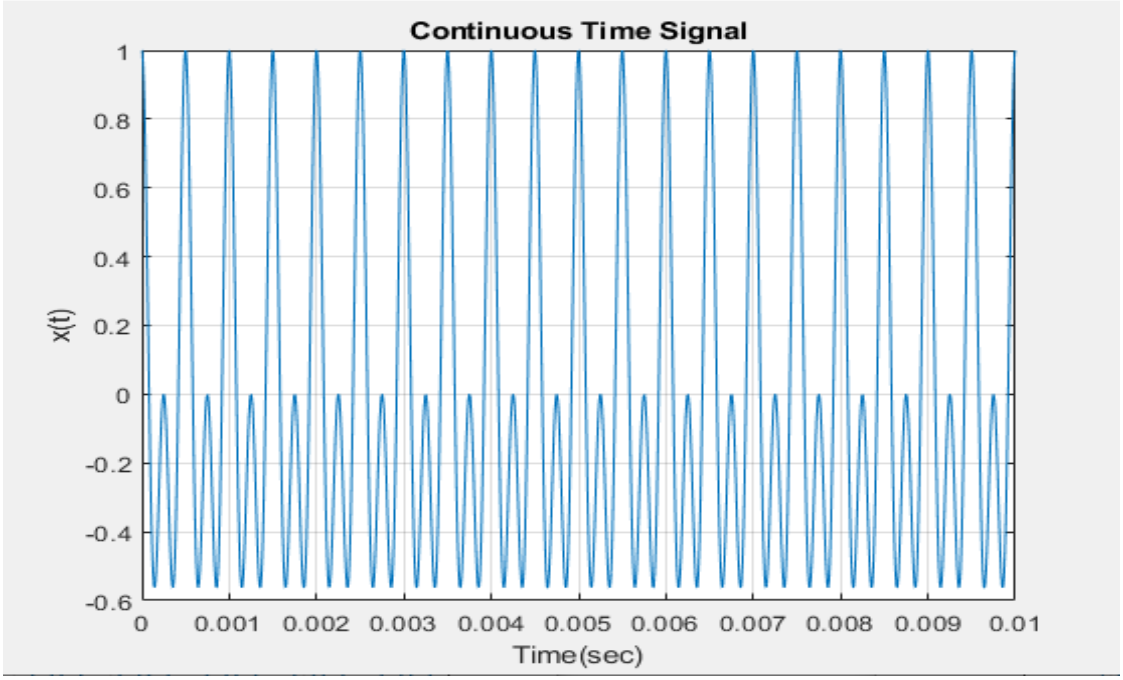


(b) $\omega_s = 2\pi f_s = 2\pi(9000) = 18000\pi$

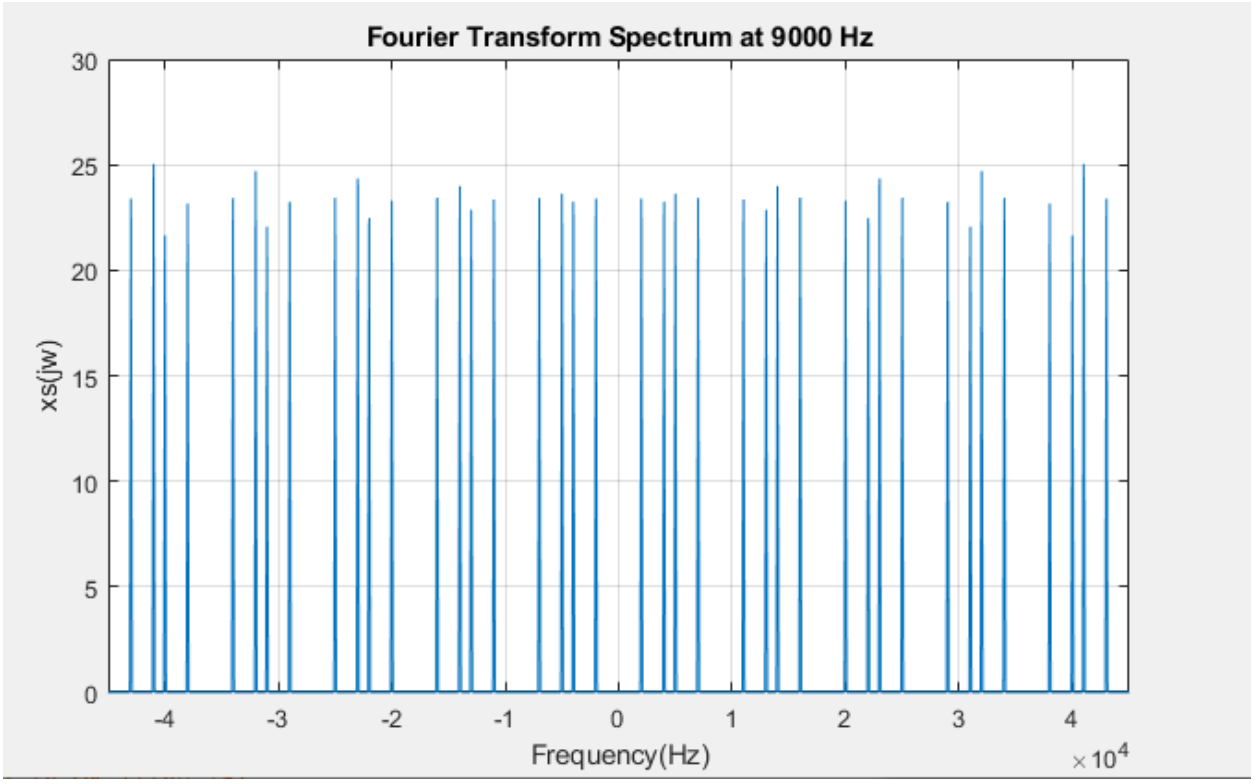


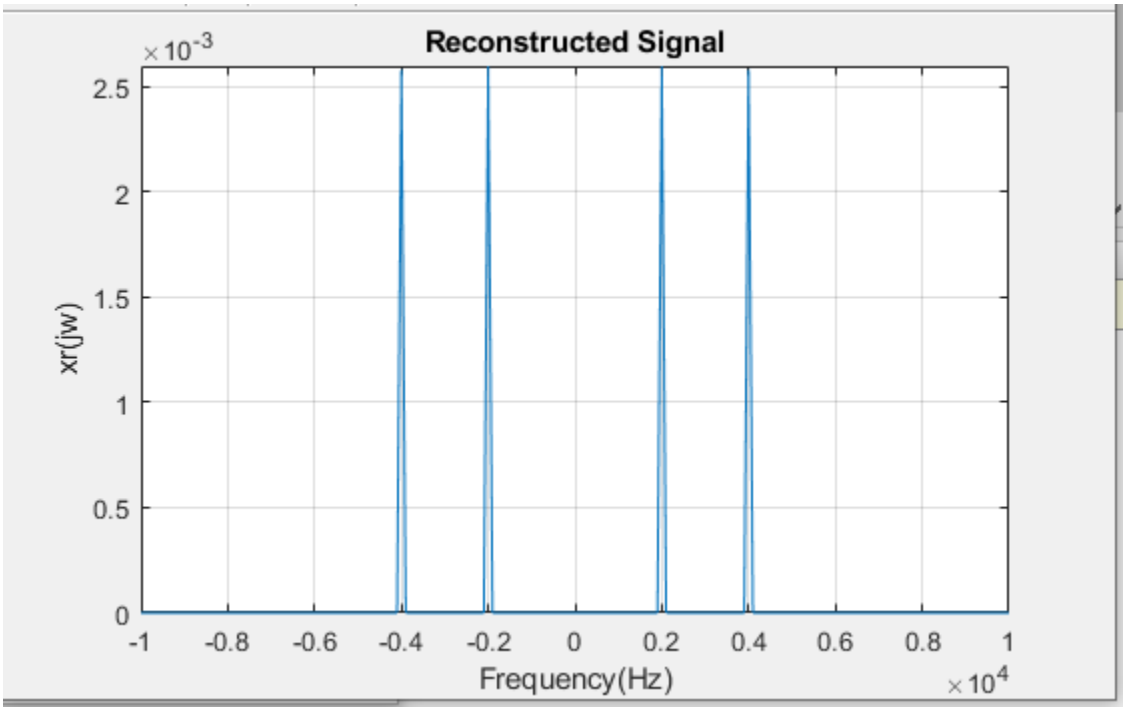


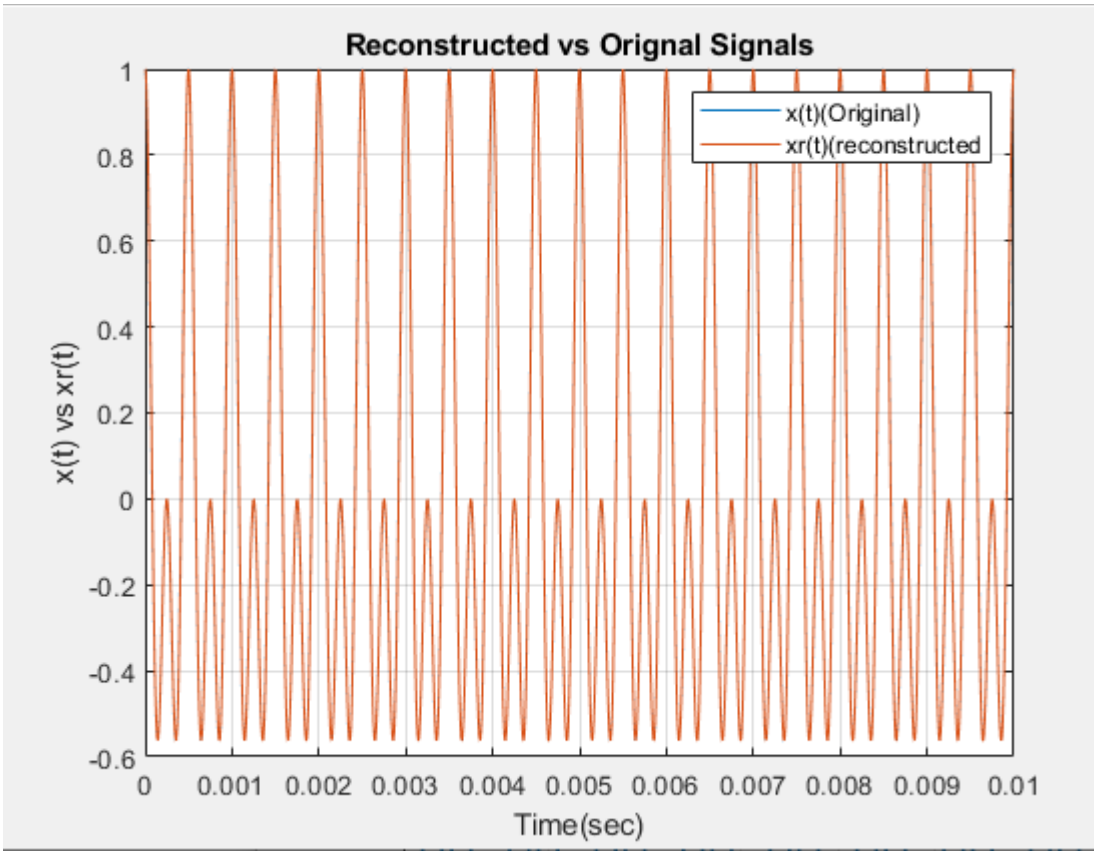
Problem 4


Continuous Time Signal
x(t)
Time(sec)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
0.009
0.01


Fourier Transform spectrum of x(t)
x(jw)
Frequency (Hz)
2500
2000
1500
1000
500
0
-5000
-4000
-3000
-2000
-1000
0
1000
2000
3000
4000
5000


Fourier Transform Spectrum at 9000 Hz
xs(jw)
Frequency(Hz)
×10^4
30
25
20
15
10
5
0
-4
-3
-2
-1
0
1
2
3
4


Reconstructed Signal
×10^-3
xr(jw)
Frequency(Hz)
×10^4
2.5
2
1.5
1
0.5
0
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1



d) The results from Matlab and the results from above calculations are as expected and similar after being reconstructed.

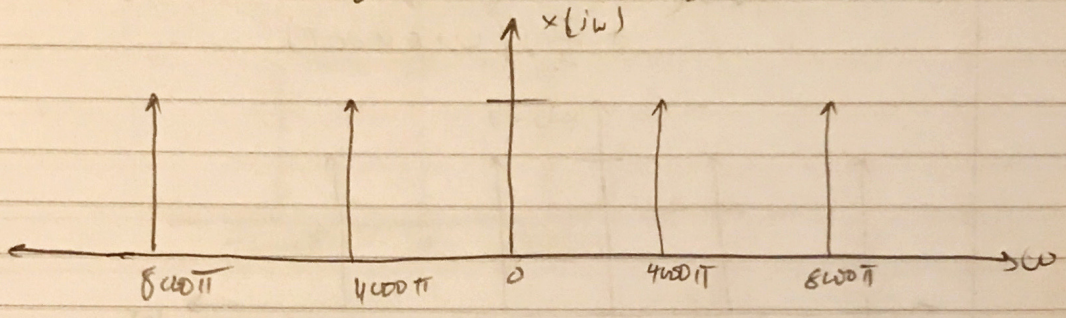
e) As we can see from both calculation and Matlab plot, there is no aliasing. In this case the reconstructed signal matched the original signal.

f) The original signal and reconstructed signal look similar with overlapping signals. Looking at the previous statement the signals look to be identical, which shows that there is no aliasing.

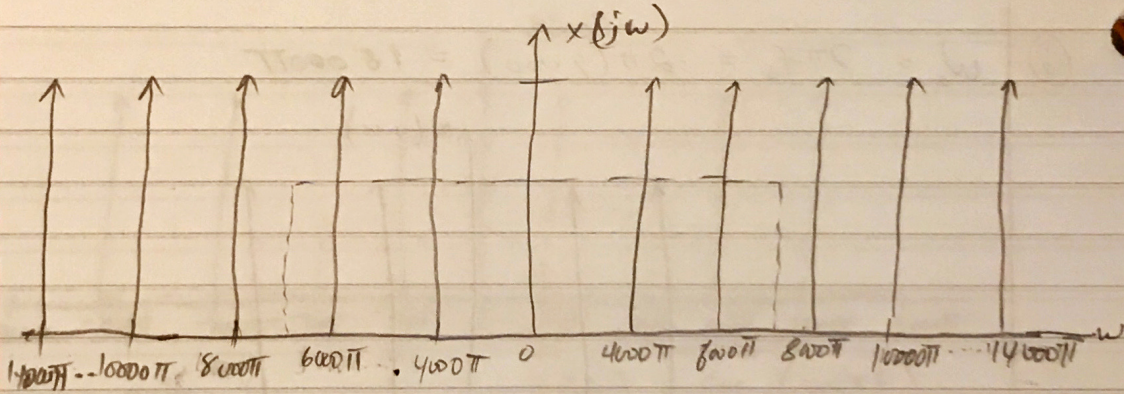
(5) (a) $x(t) = \cos(2\pi 1000t)\cos(2\pi 3000t)$ at sampling $f_s = 7000$ Hz

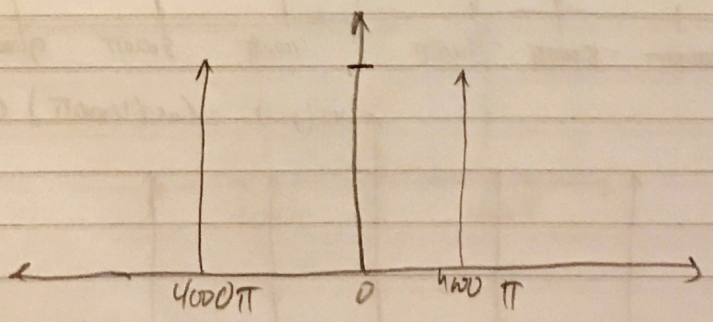
$X(jw) = F\{x(t)\} \Rightarrow \frac{\pi}{2}\delta(w - 8000\pi) + \frac{\pi}{2}\delta(w - 4000\pi) +$

$\frac{\pi}{2}\delta(w + 4000\pi) + \frac{\pi}{2}\delta(w + 8000\pi)$

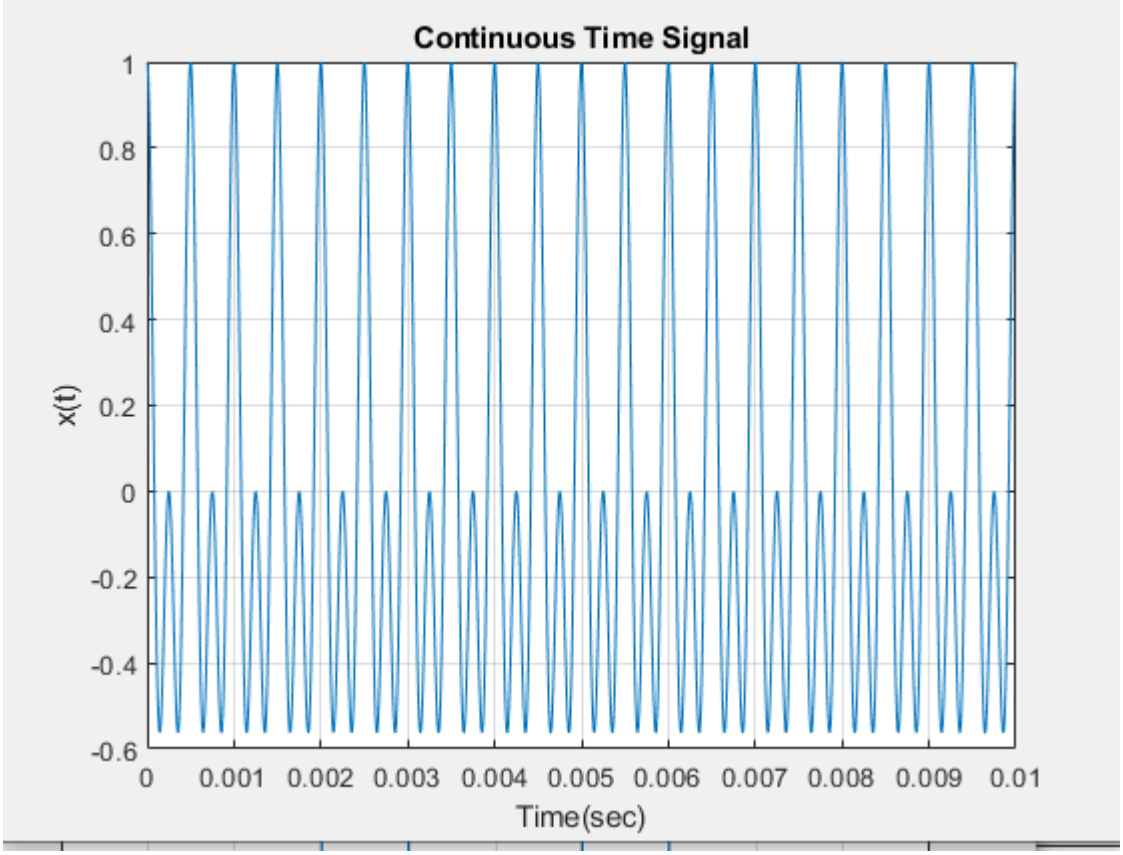


(b) $w_s = 2\pi f_s = 2\pi(7000) = 14000\pi$

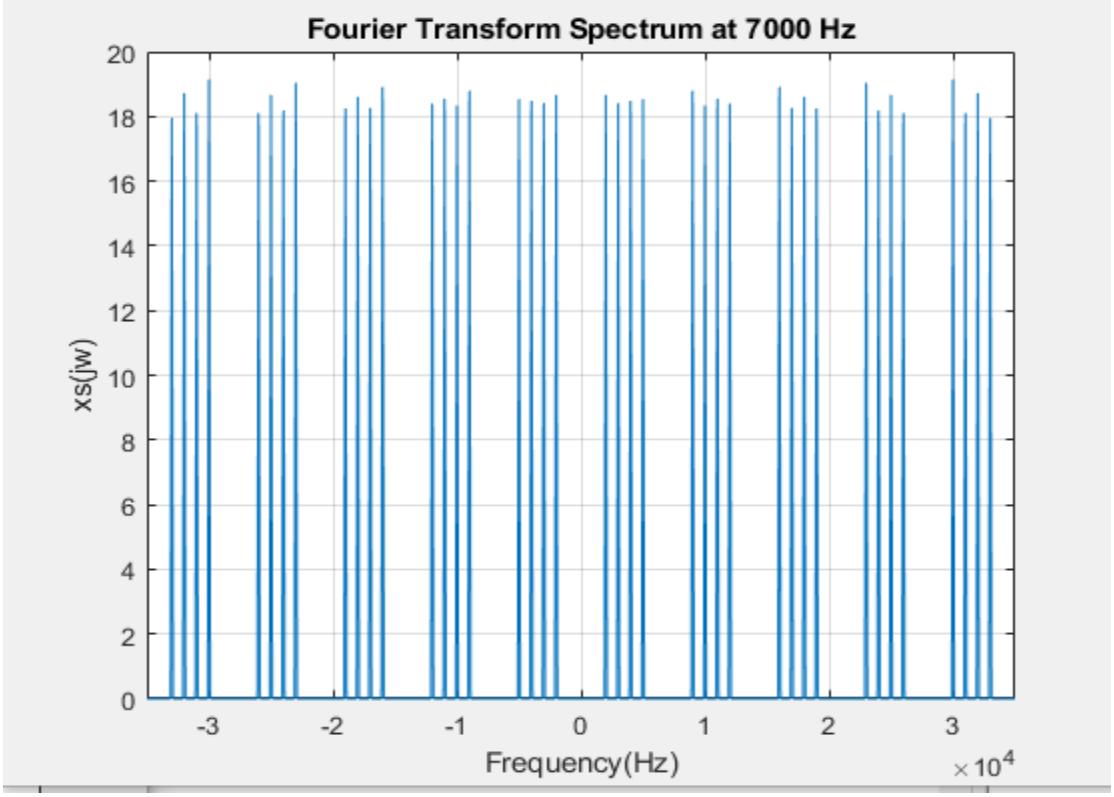


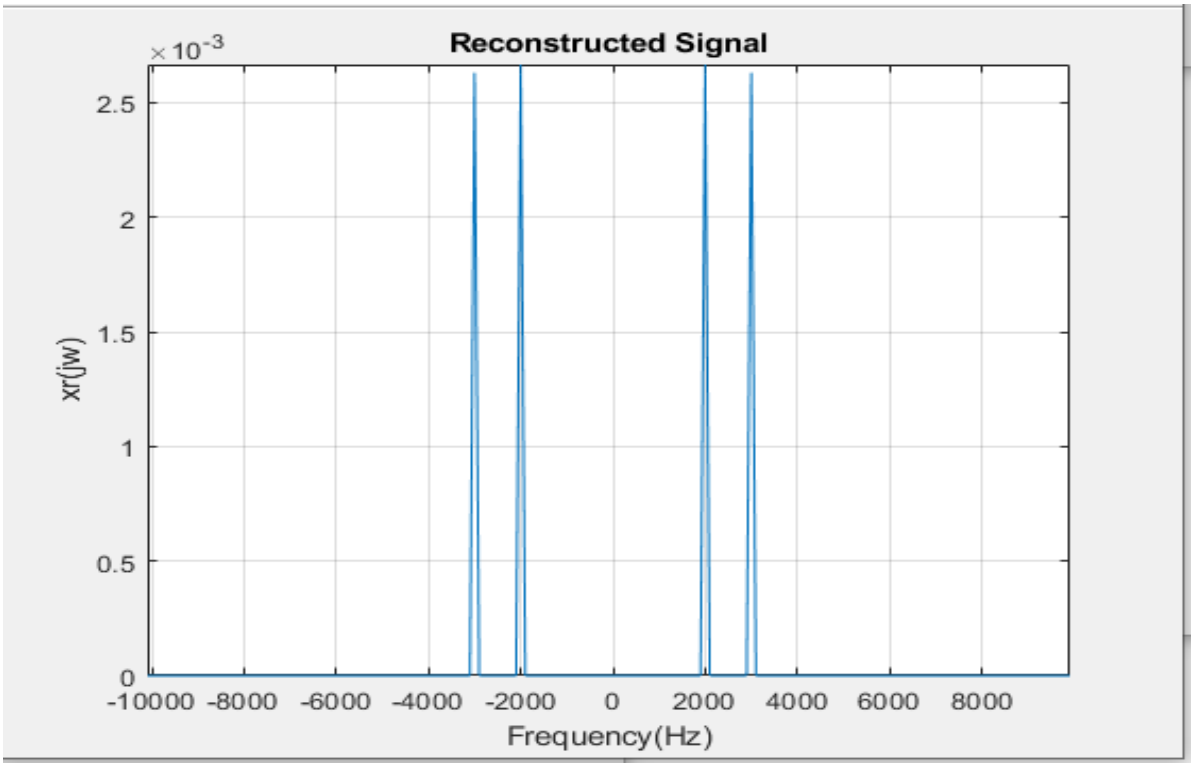


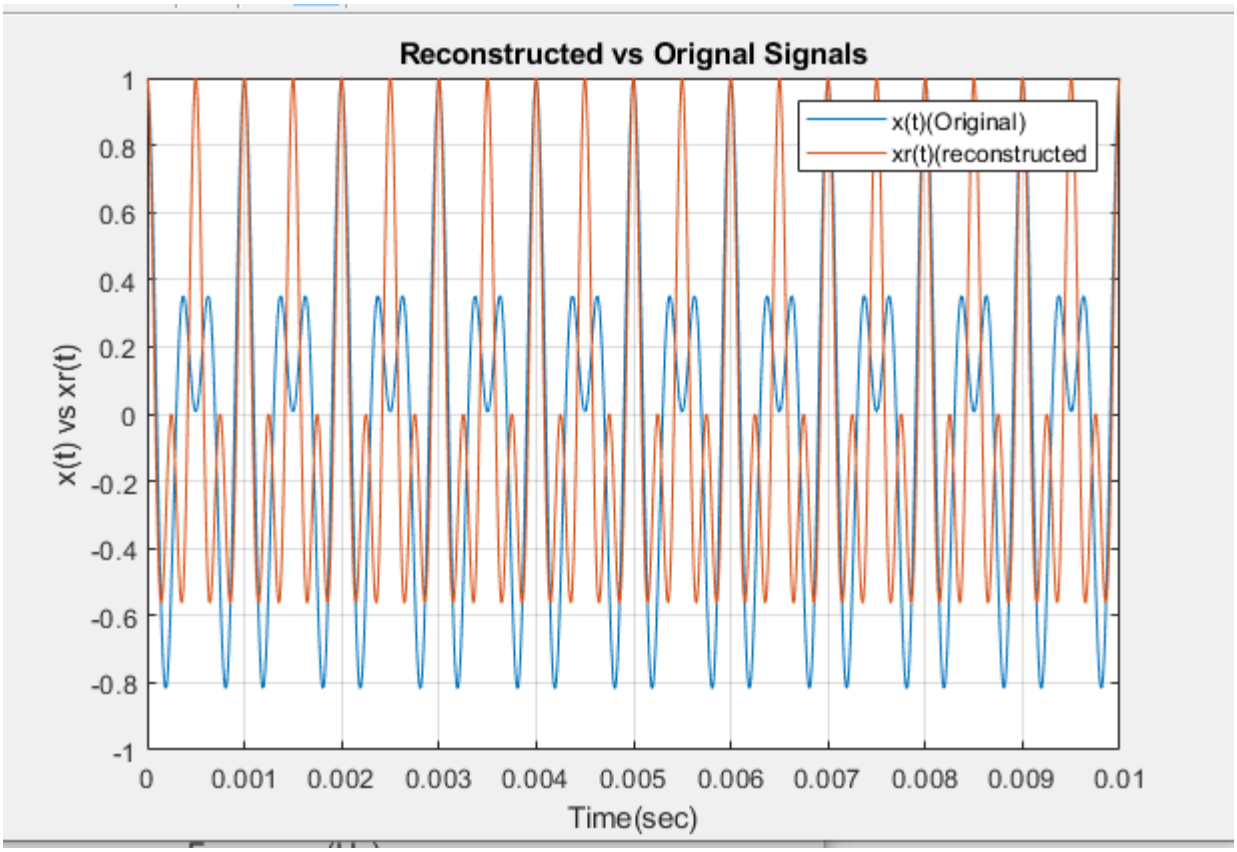
Problem 5


Continuous Time Signal
x(t)
Time(sec)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
0.009
0.01


Fourier Transform spectrum of x(t)
x(jw)
Frequency (Hz)
2500
2000
1500
1000
500
0
-5000
-4000
-3000
-2000
-1000
0
1000
2000
3000
4000
5000


Fourier Transform Spectrum at 7000 Hz
xs(jw)
Frequency(Hz)
×10^4
0
2
4
6
8
10
12
14
16
18
20
-3
-2
-1
0
1
2
3


Reconstructed Signal
×10^-3
xr(jw)
Frequency(Hz)
0
0.5
1
1.5
2
2.5
-10000
-8000
-6000
-4000
-2000
0
2000
4000
6000
8000

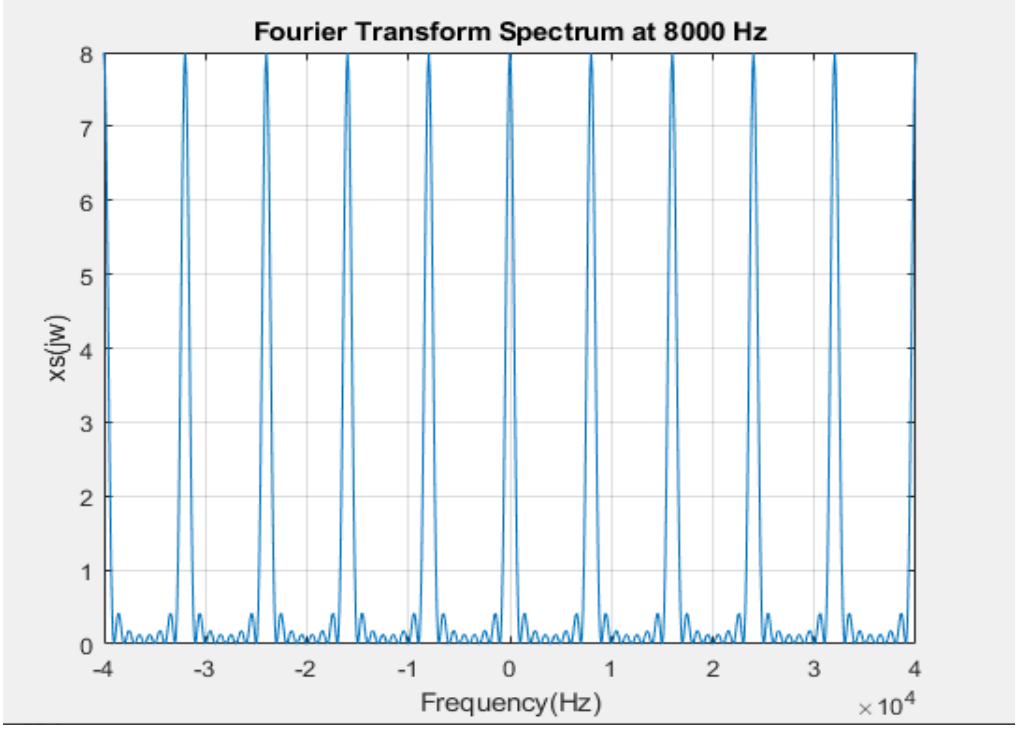


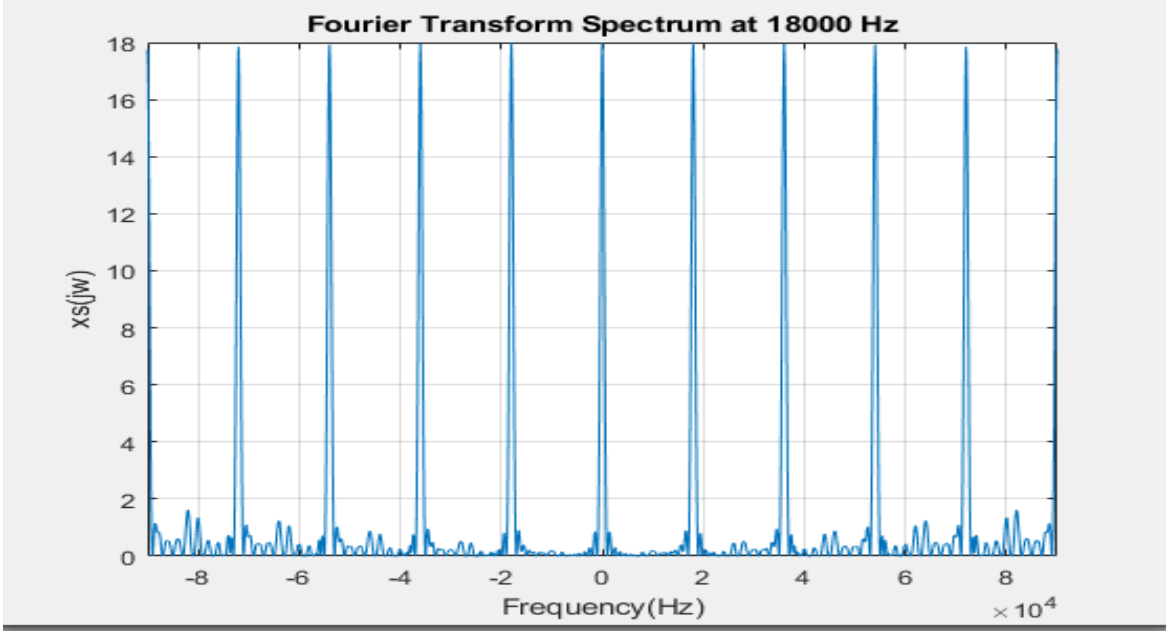
d) The results from Matlab and the results from above calculations are as expected and similar after being reconstructed.

e) As we can see from both calculation and Matlab plot, there is aliasing this time. We can see signals distorted which could have caused by having a lower frequency than what is actually needed.

f) The original signal and reconstructed signal look different. Looking at the previous statement the signals look to be not overlapping, which shows that there is aliasing.

Problem 6

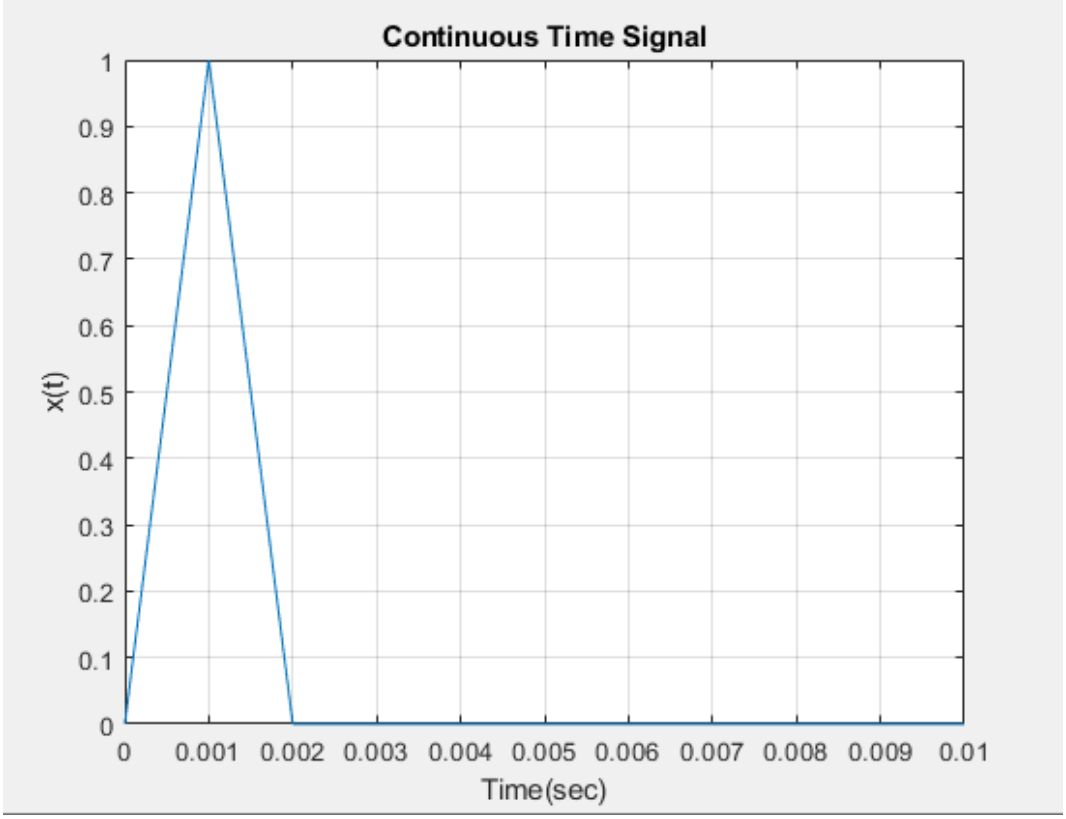


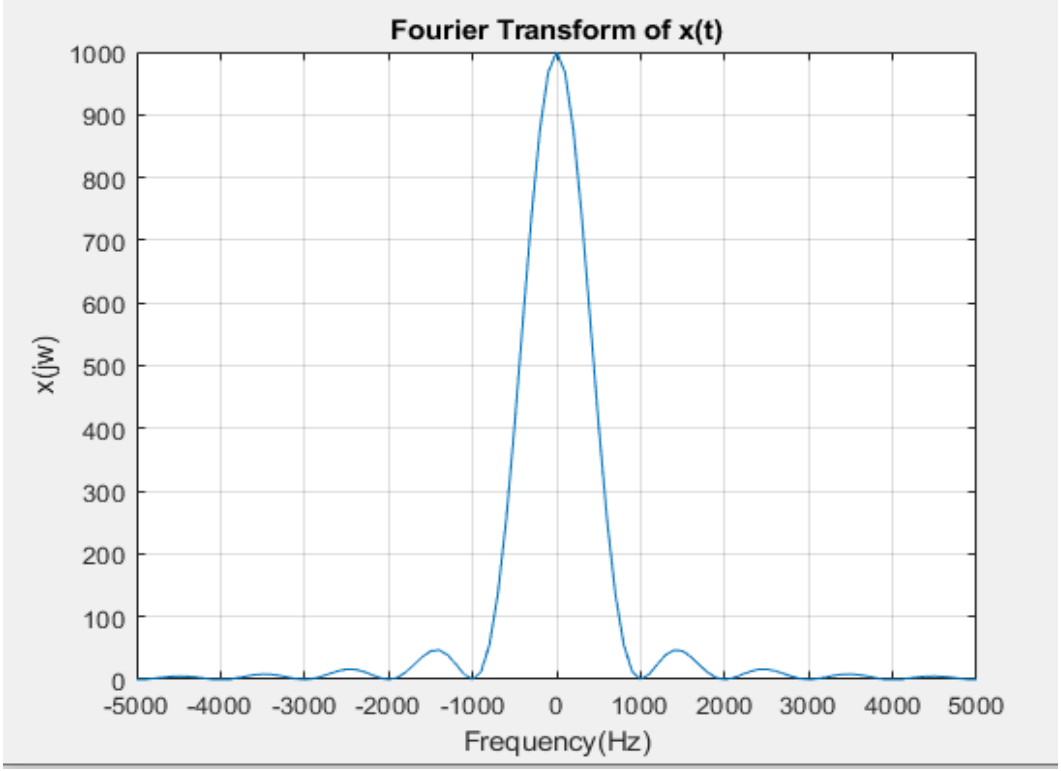


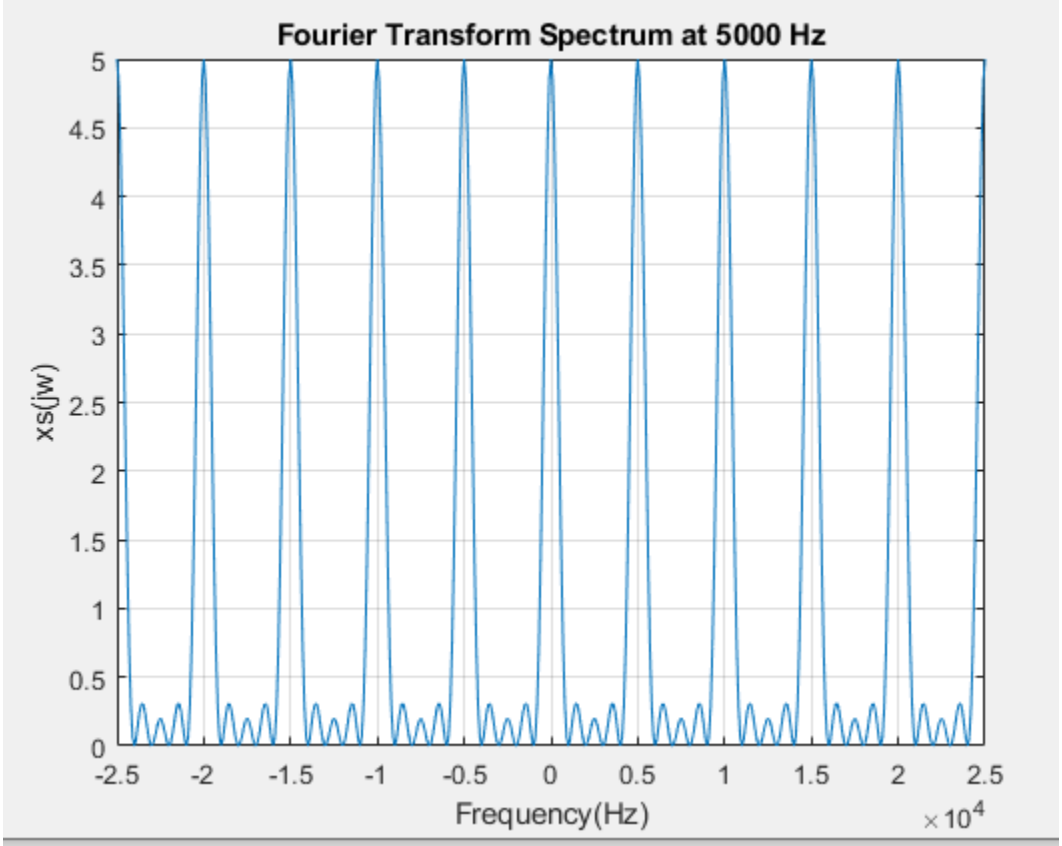
The sampled signal is not bandlimited since there is no finite limit to its frequency and the signal could be above a specified frequency. There is a high possibilities that there will be aliasing in this case.

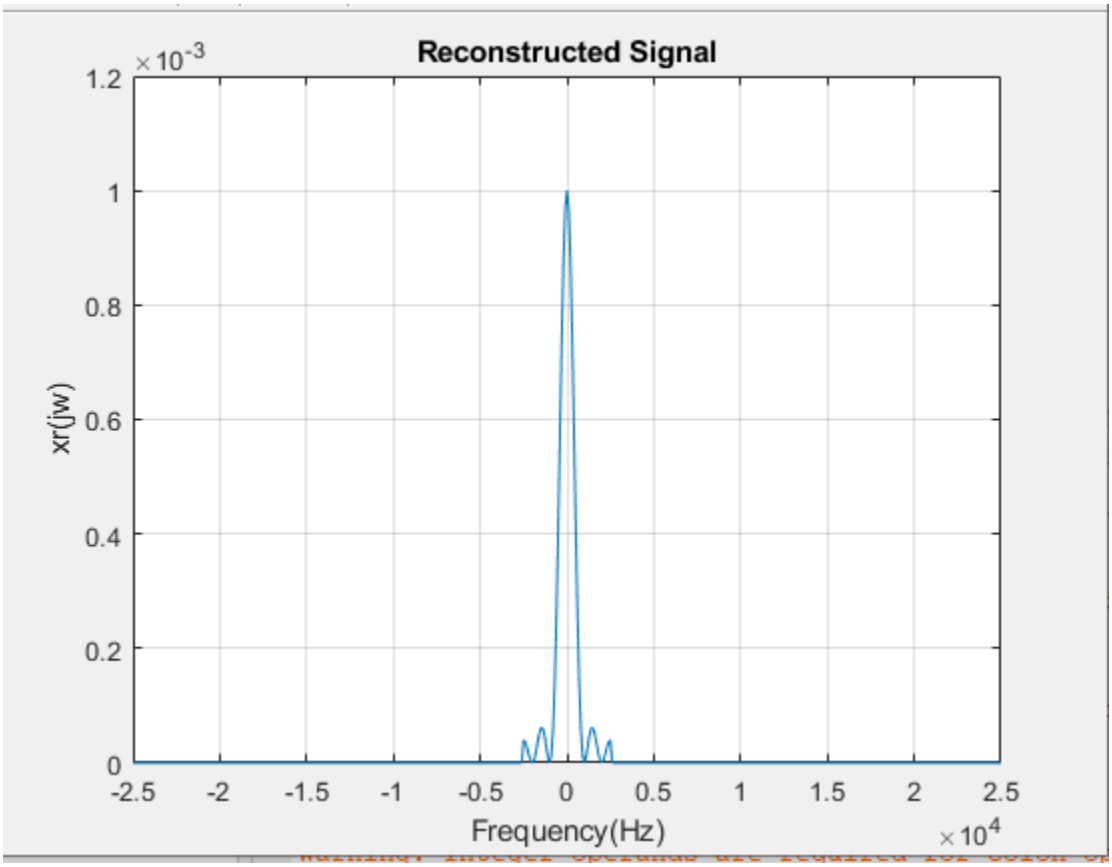
Problem 7

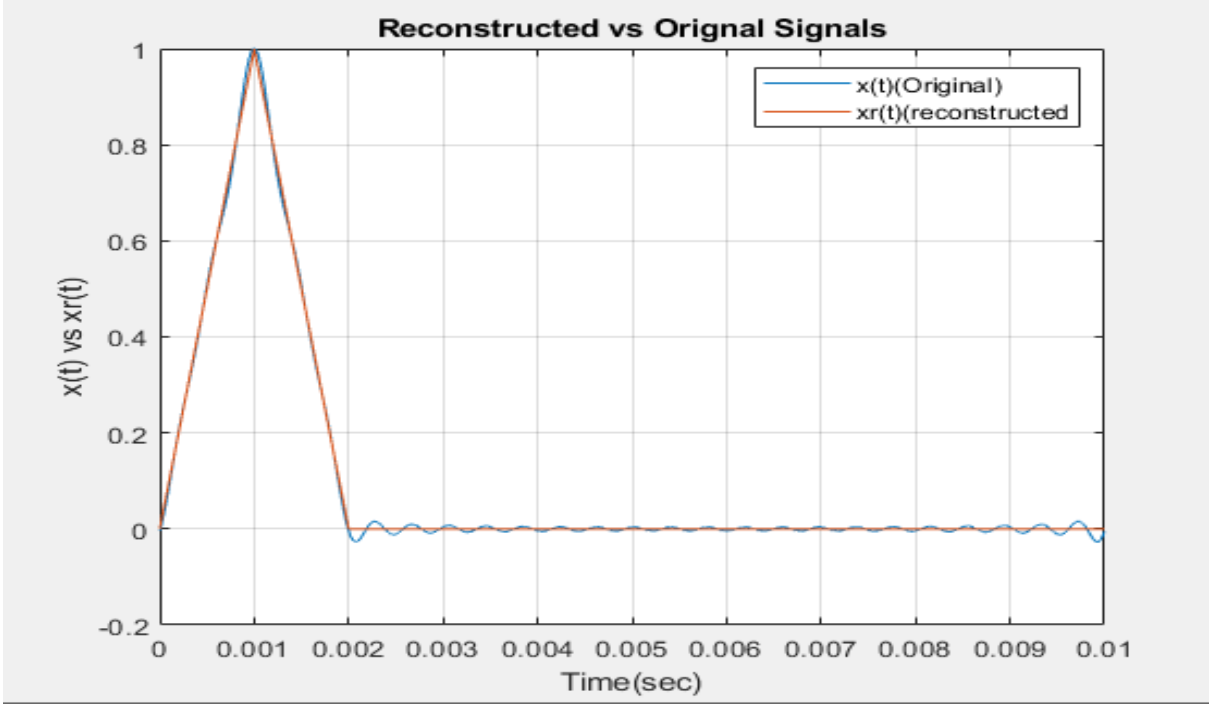
a)


Continuous Time Signal
x(t)
Time(sec)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
0.009
0.01


Fourier Transform of x(t)
x(jw)
Frequency(Hz)
1000
900
800
700
600
500
400
300
200
100
0
-5000 -4000 -3000 -2000 -1000 0 1000 2000 3000 4000 5000


Fourier Transform Spectrum at 5000 Hz
xs(jw)
Frequency(Hz)
×10^4
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
-2.5 -2 -1.5 -1 -0.5 0 0.5 1 1.5 2 2.5


Reconstructed Signal
×10^-3
1.2
1
0.8
0.6
0.4
0.2
0
xr(jw)
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
Frequency(Hz)
×10^4



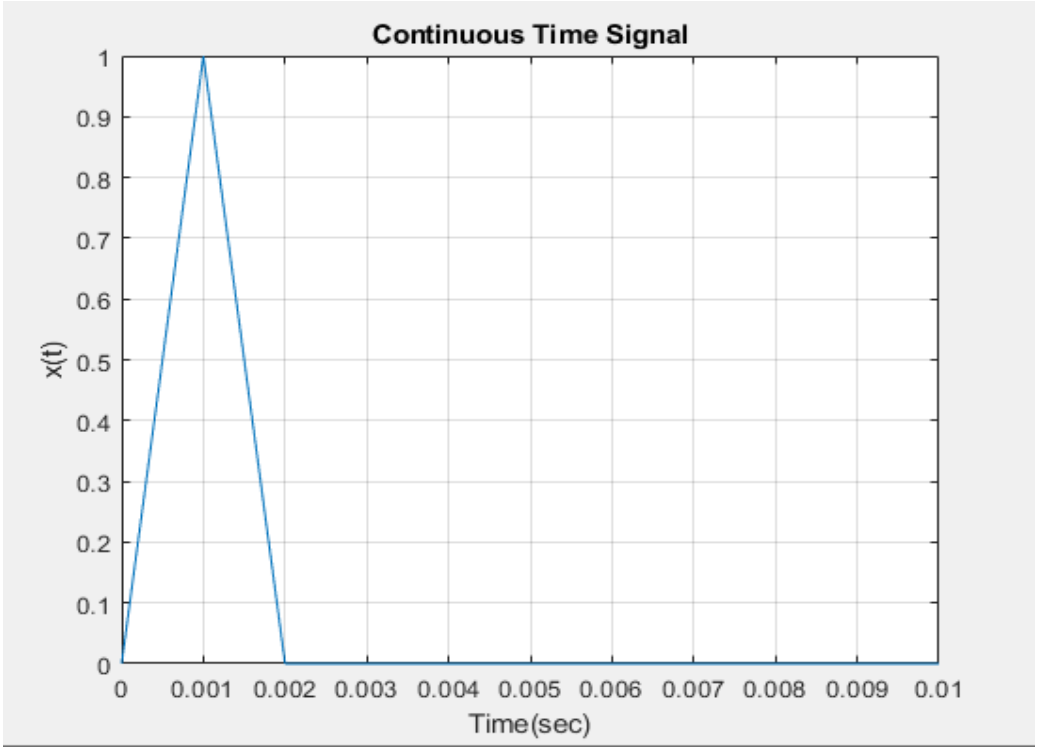
b) Based on the Matlab plot, there seem to be minimal aliasing, low frequency and signal not being bandlimited could be the cause.

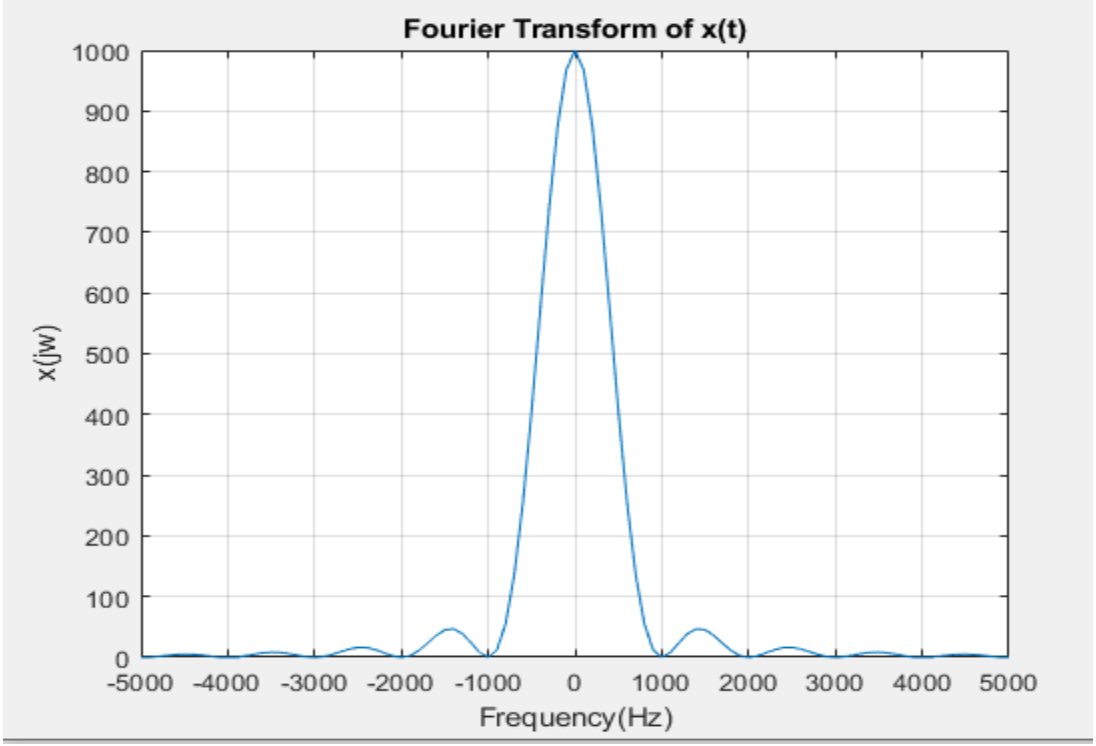
c) As mentioned above there is a small aliasing, which can be seen in the plot with slight breakdown of each signals.

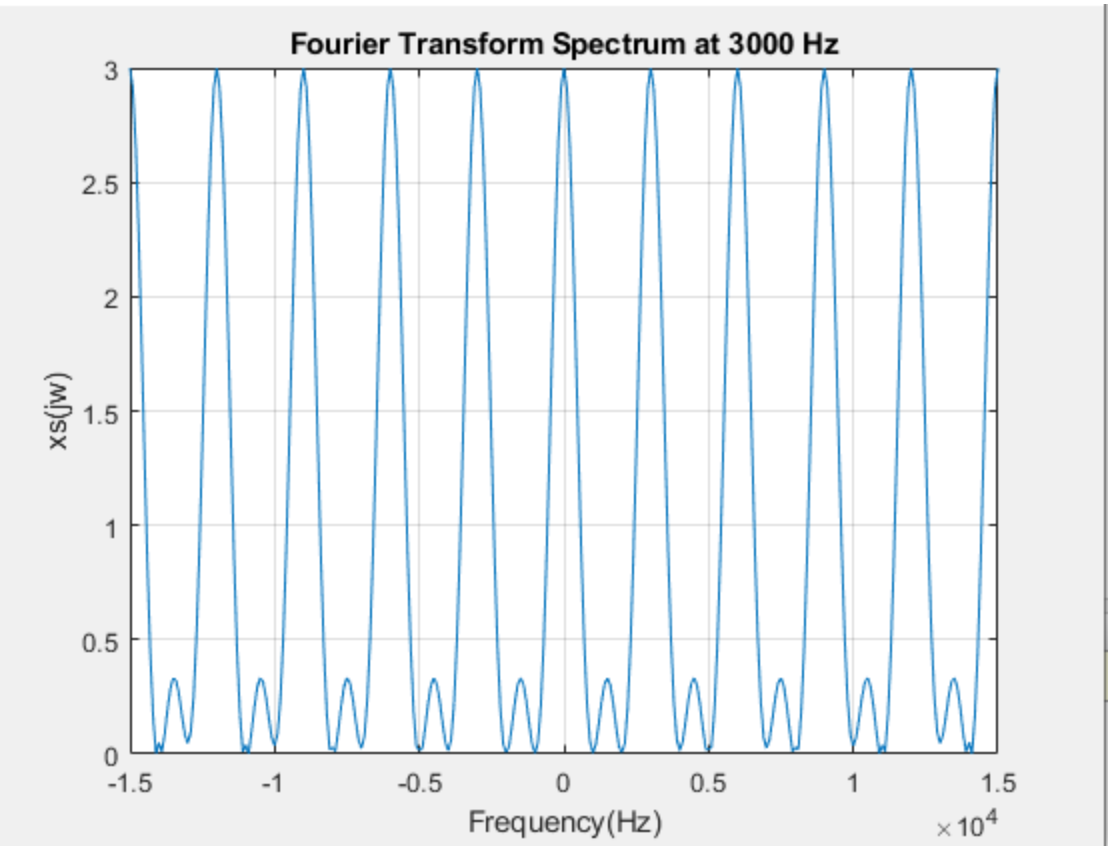
d) When comparing the original signal and constructed signal they look very similar but they are in fact bit different because of tiny aliasing.

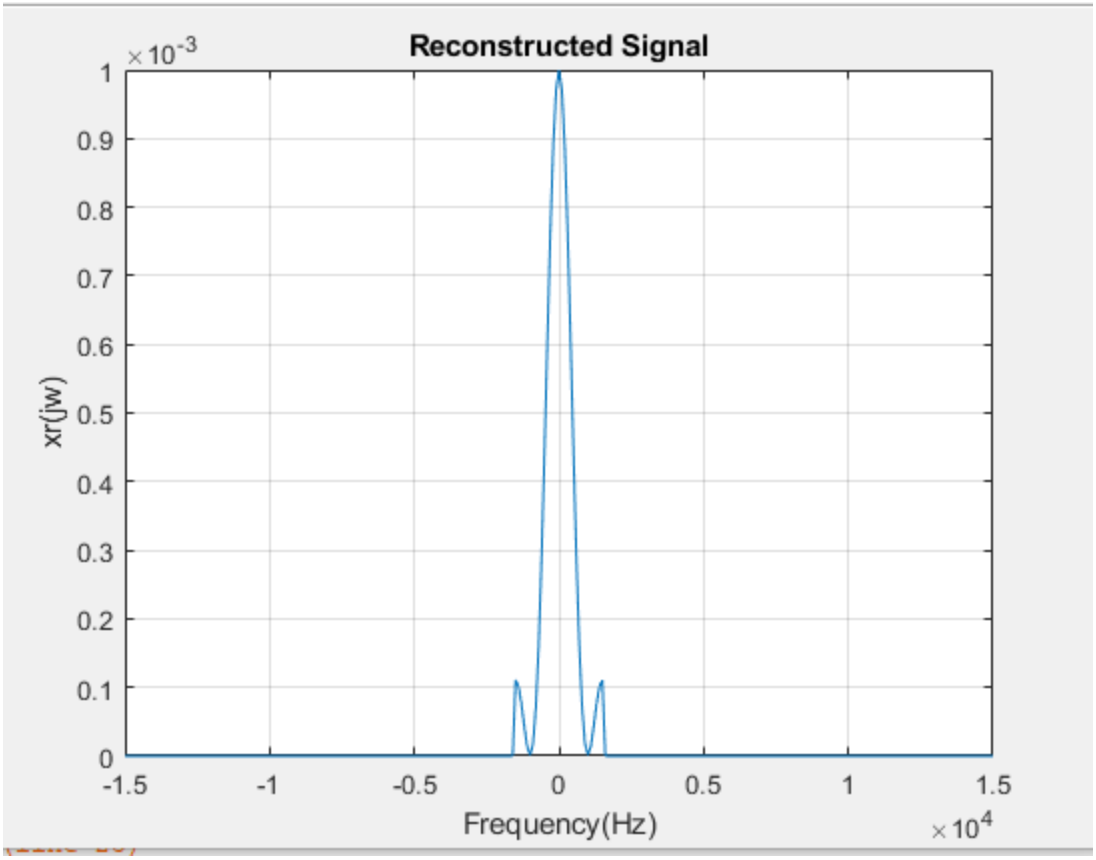
Problem 8

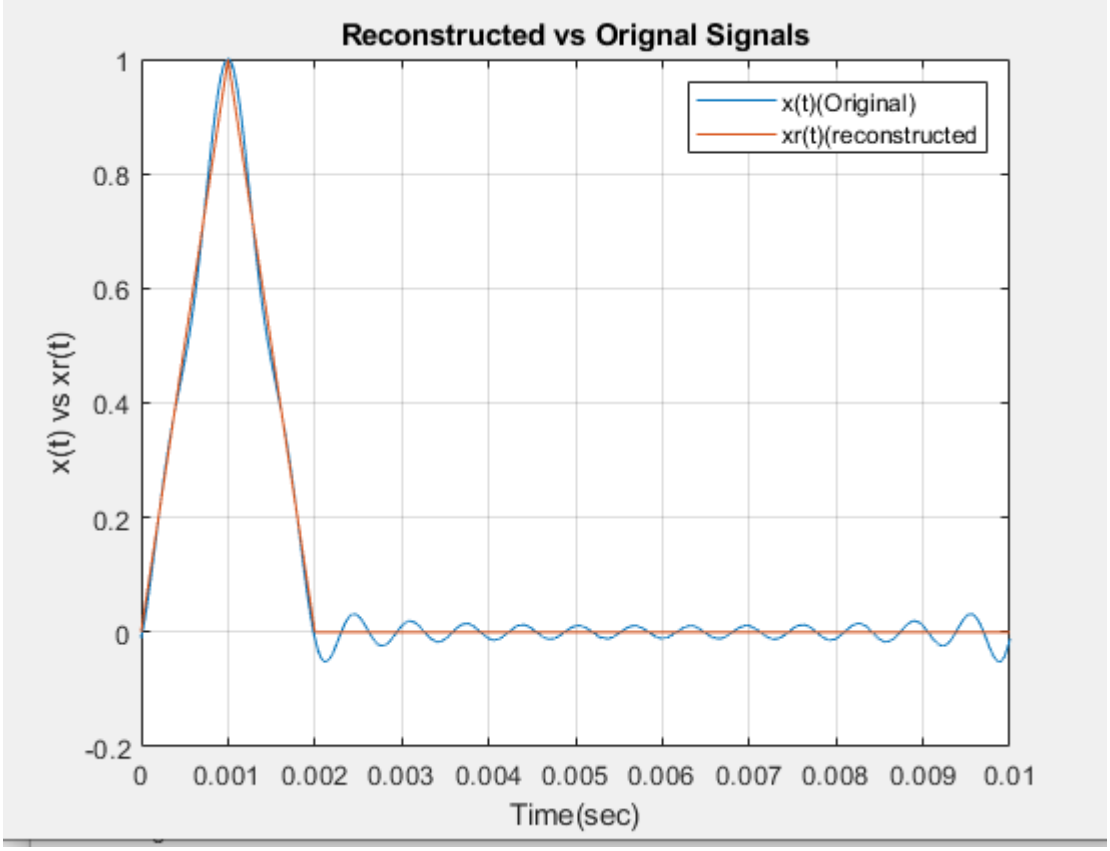
a)


Continuous Time Signal
x(t)
Time(sec)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
0.009
0.01


Fourier Transform of x(t)
x(jw)
Frequency(Hz)
0
100
200
300
400
500
600
700
800
900
1000
-5000
-4000
-3000
-2000
-1000
1000
2000
3000
4000
5000


Fourier Transform Spectrum at 3000 Hz
xs(jw)
Frequency(Hz)
×10^4
3
2.5
2
1.5
1
0.5
0
-1.5
-1
-0.5
0
0.5
1
1.5


Reconstructed Signal
×10^-3
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
xr(jw)
-1.5
-1
-0.5
0
0.5
1
1.5
Frequency(Hz)
×10^4



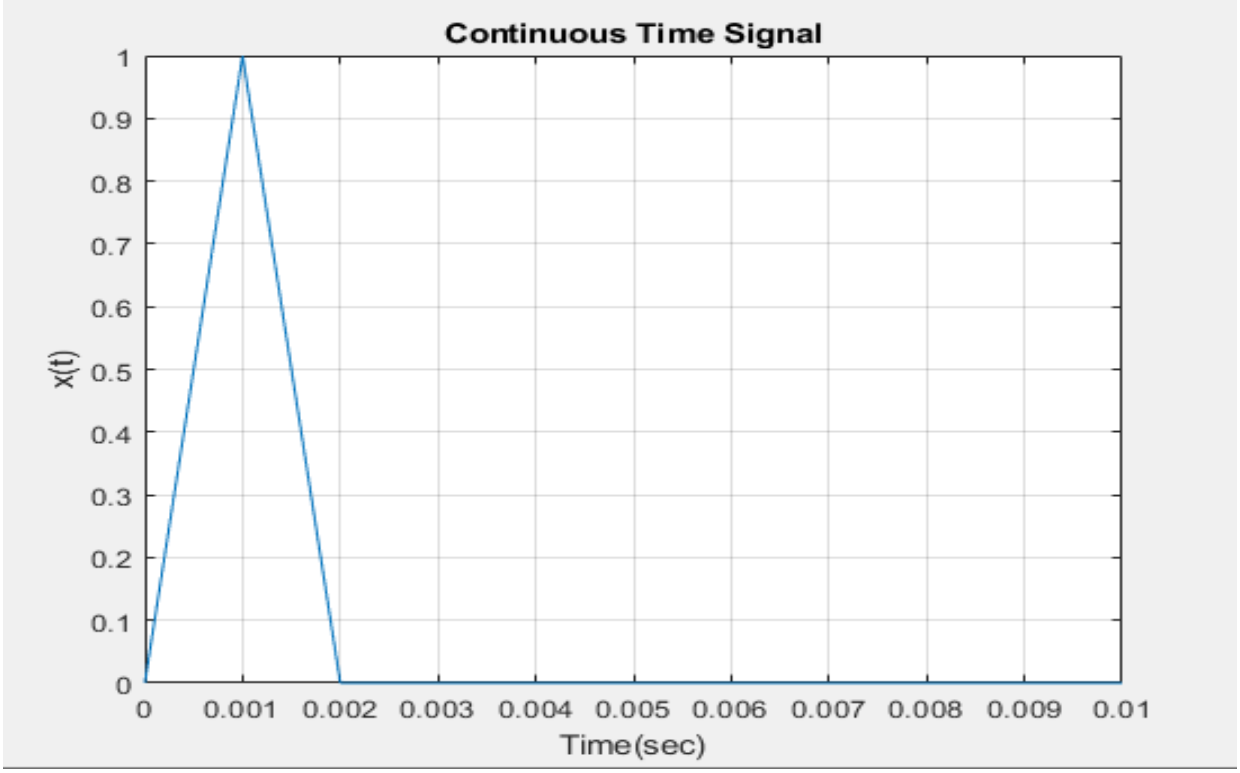
b) Based on the Matlab plot, there seem to be some aliasing, low frequency and signal not being bandlimited could be the cause.

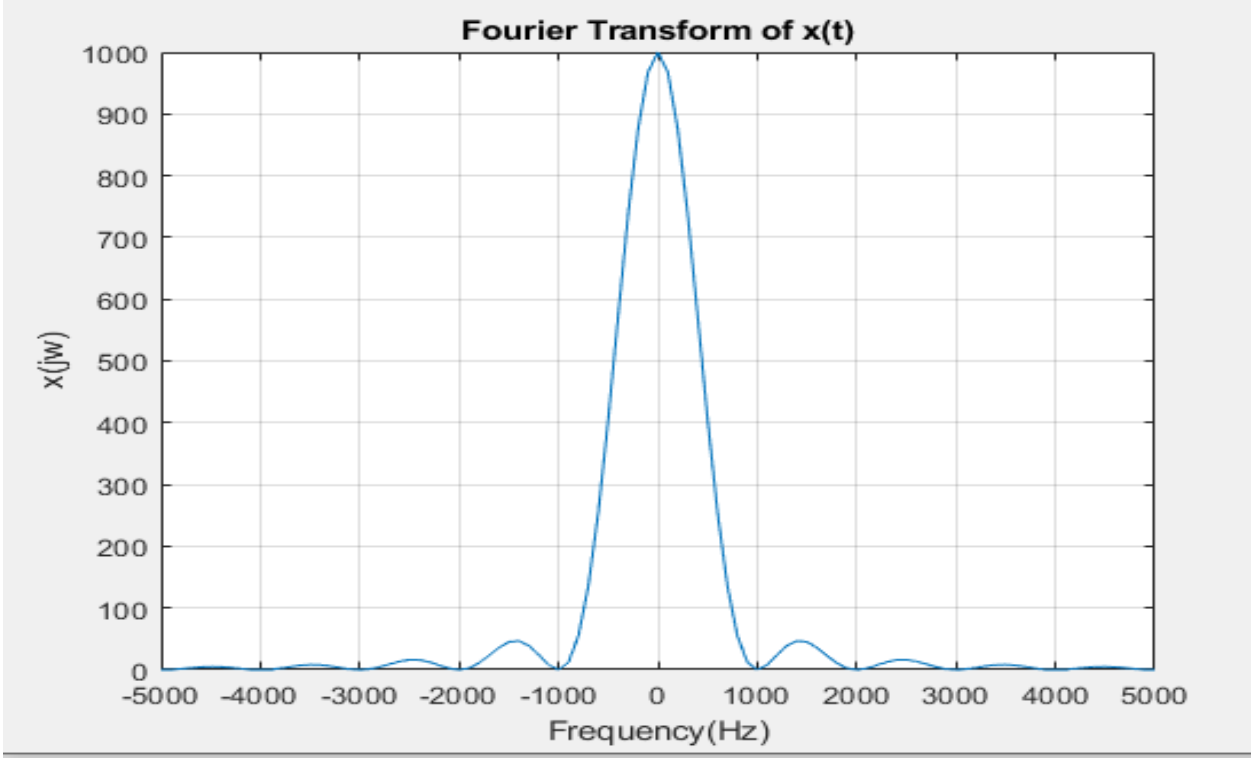
c) As mentioned above there is a small aliasing which can be seen in the plot with slight breakdown of each signals.

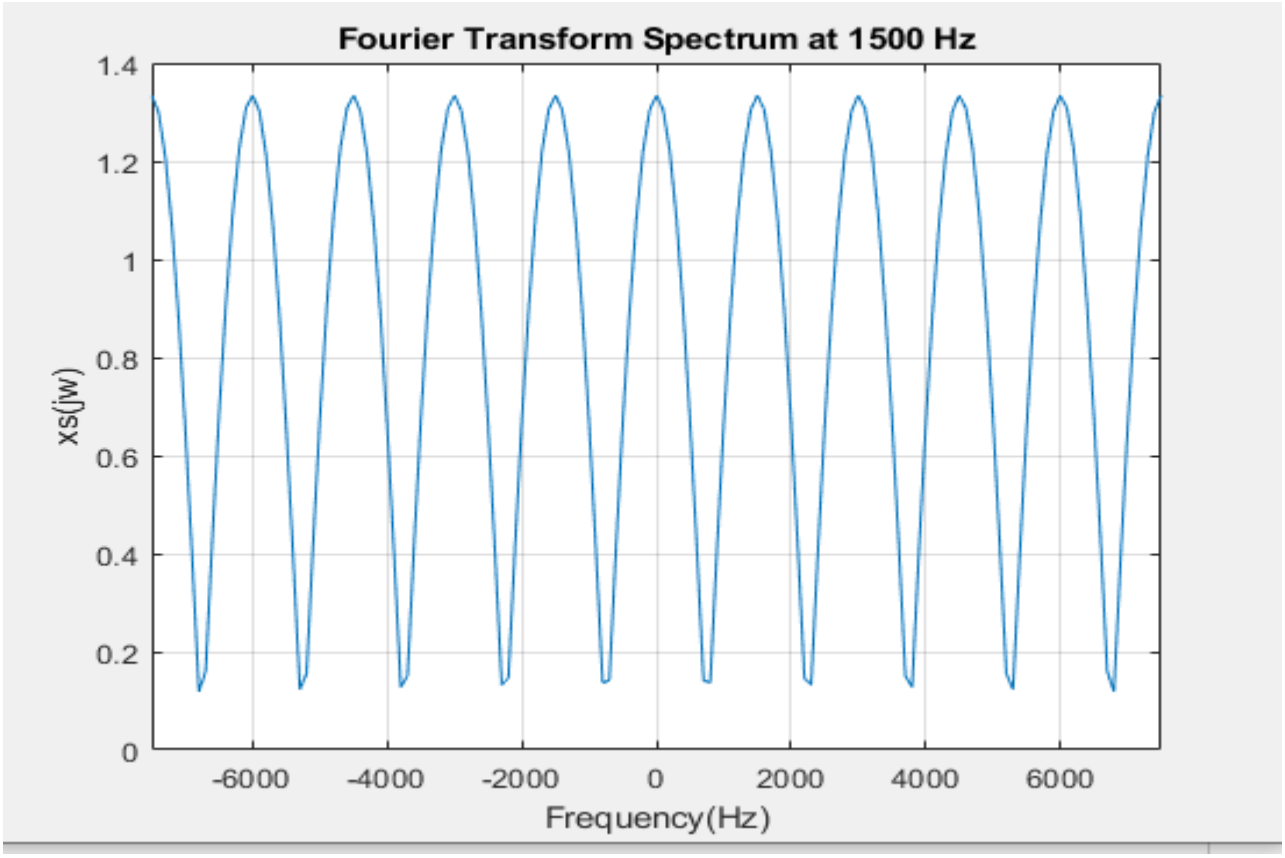
d) When comparing the original signal and constructed signal they look very similar with some signal distorted which shows that we have small aliasing going on here.

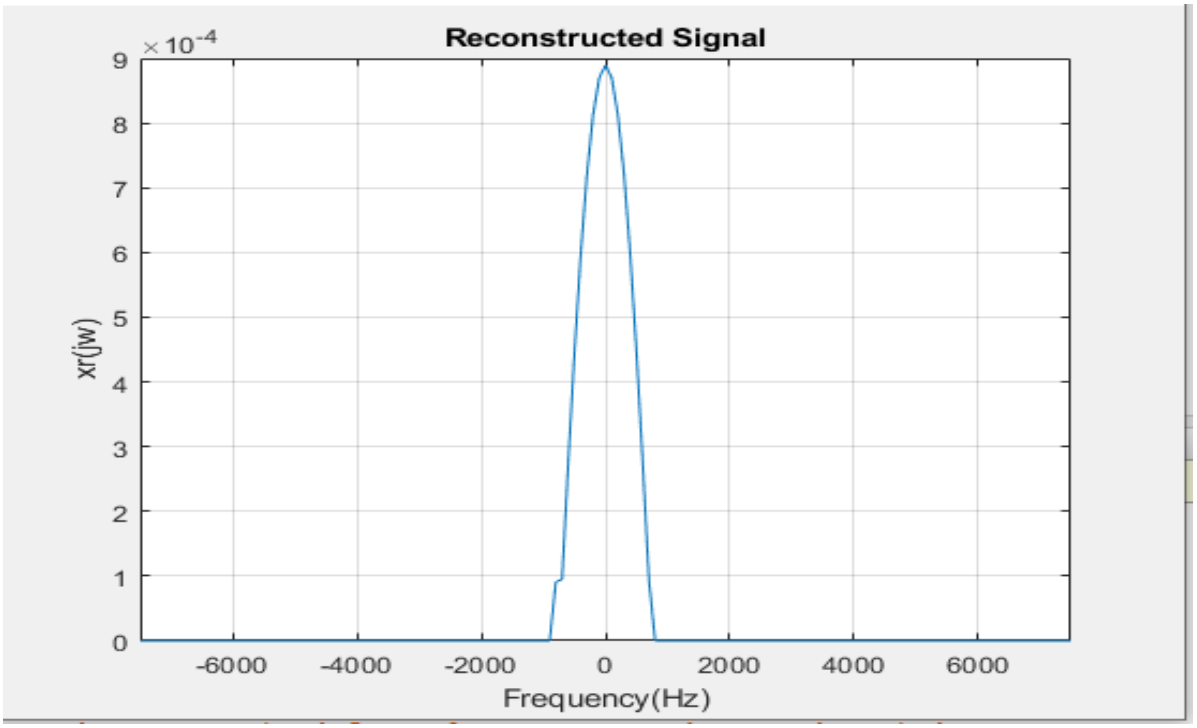
9)

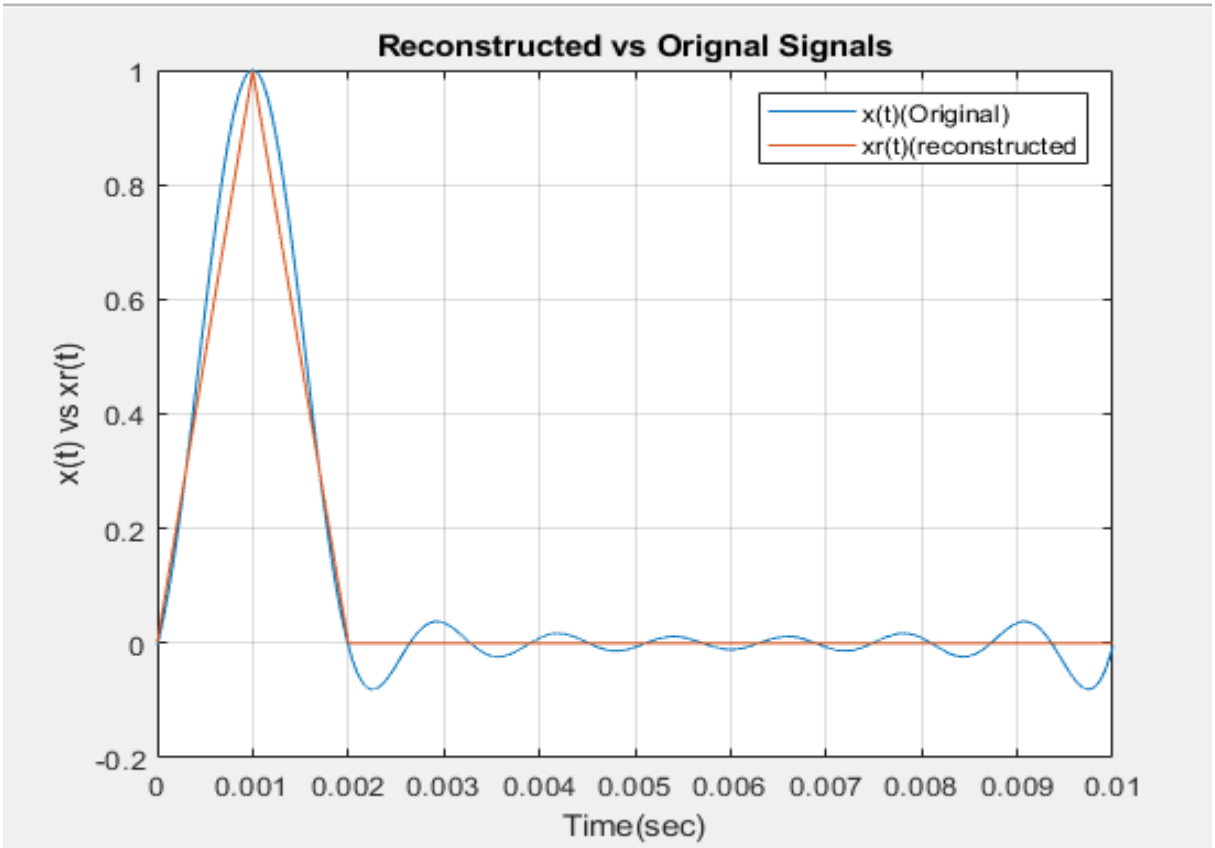
a)


Continuous Time Signal
x(t)
Time(sec)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
0.009
0.01


Fourier Transform of x(t)
1000
900
800
700
600
500
400
300
200
100
0
X(jw)
-5000
-4000
-3000
-2000
-1000
0
1000
2000
3000
4000
5000
Frequency(Hz)


Fourier Transform Spectrum at 1500 Hz
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Xs(jw)
-6000
-4000
-2000
0
2000
4000
6000
Frequency(Hz)


Reconstructed Signal
×10^-4
9
8
7
6
5
4
3
2
1
0
xr(jw)
-6000
-4000
-2000
0
2000
4000
6000
Frequency(Hz)



b) Based on the Matlab plot, there seem to be bigger aliasing here, significantly low frequency and signal not being bandlimited could be the cause.

c) As mentioned above there is aliasing, which can be seen in the plot with bigger breakdown of each signals.

d) When comparing the original signal and constructed signal they look very similar with some signal distorted which shows that we have small aliasing going on here.