P 1

Thursday, November 2, 2023 11:38 AM

1.1 $G(s) = \frac{\alpha(s)}{U_{yaw}(s)}$ $\qquad I_3 \frac{d^2}{dt^2}\alpha \approx u_{yaw}$

$I_3 s^2 \alpha \approx u_{yaw}(s)$

$u_{yaw} = \frac{(u_{T_1} + u_{T_2})}{2}$ $\qquad G(s) = \frac{1}{I_3 s^2}$

1b. $D(s) = K$ $\qquad T(s) = \frac{G(s)\,D(s)}{1+G(s)\,D(s)} = \frac{K\frac{1}{I_3 s^2}}{1+\frac{K}{I_3 s^2}} = \frac{K}{s^2+K}$

$-1 = \frac{K}{s^2}$

$-s^2 = K$

$s = \sqrt{-K}$

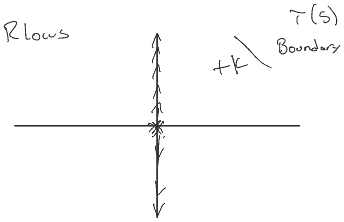
zeros: $K = 0$

poles: $s^2 + \frac{K}{I_3} = 0$ $\qquad s = \pm\sqrt{-\frac{K}{I_3}}$

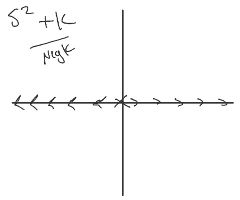
$s = \pm\sqrt{\frac{-1}{I_3 K \alpha}}$ $\qquad I_3 = \frac{m w^2}{12}$ $\qquad$ m = mass, w = pipe diam

$T(s) = \frac{K}{s^2+K}$

RLocs

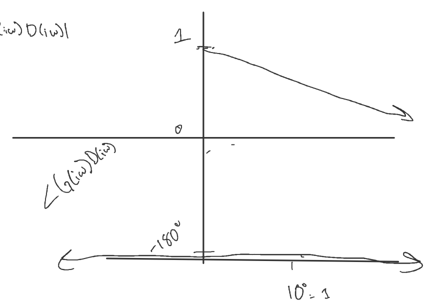


NegK



Bode

$|G(i\omega)D(i\omega)|$



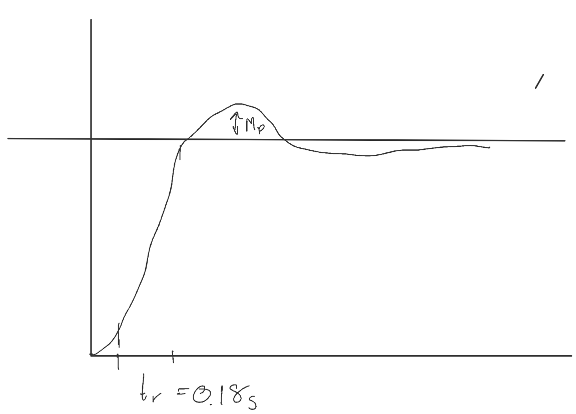
$\frac{K}{s^2}$

poles @ 0 (2)

-180

1c. $t_r = 0.18$ sec $\qquad$ Yaw control loop $D(s)$

$M_p = 10\%$



known

$t = 1.8/\omega_n$

$\frac{0.18}{1.8} = \frac{1}{\omega_n}$

$10 = \omega_n$

$M_p = 10\%$

$\zeta = 0.6$

$\frac{b_0}{s^2 + a_1 s_1 + a_0}$ $\qquad a_1 = 2\zeta\omega_n = 2 \cdot 0.6 \cdot 10 = 12$

$a_0 = \omega_n^2 = 120$

$$Pol_{\pm} = -\sigma \pm i\omega_d$$

$$= -\frac{a_1}{2} \pm i\sqrt{a_0 - \frac{a_1^2}{4}}$$

$$= -\frac{10}{2} \pm i\sqrt{120 - \frac{12^2}{4}}$$

$$= -5 \pm i$$

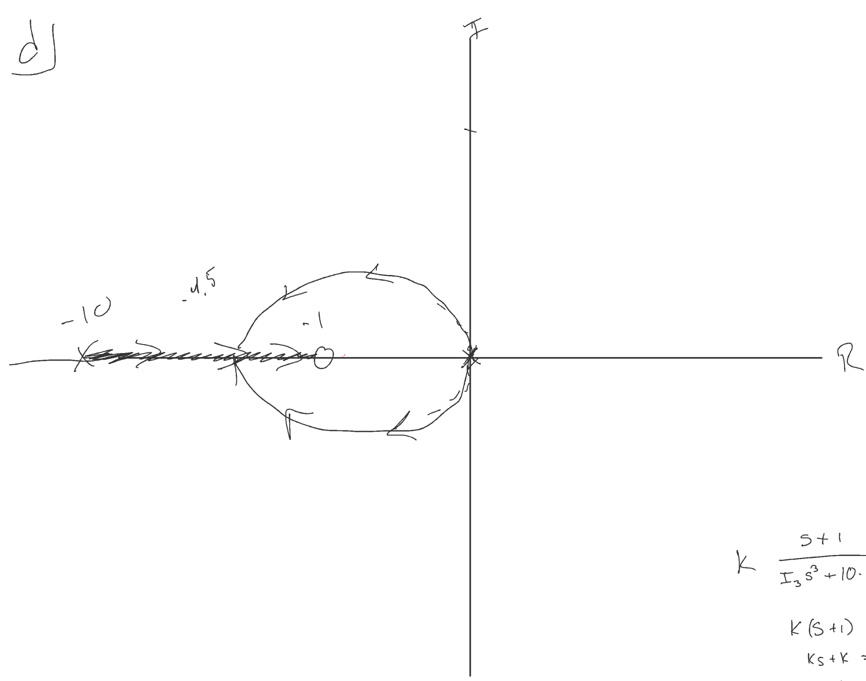
$$\boxed{P_{\pm} = -6 \pm 8i}$$

$8 \cdot 8 = 64$

$9 \cdot 9 = 81$

$8.5 \cdot 8.5$

1d)



$z = 1$

$$D(s) = K\frac{s+z}{s+p}$$

$p = 10$

$$G(s) = \frac{1}{I_3 s^2}$$

2) K.

$$L(s) = K\frac{s+1}{s+10} \cdot \frac{1}{I_3 s^2}$$

closed loop

$$L(s) + 1 = 0$$

$$K\frac{s+1}{s+10} \cdot \frac{1}{I_3 s^2} + 1 = 0$$

$$K\frac{s+1}{I_3 s^3 + 10 \cdot I_3 s^2} = -1$$

$$K(s+1) = I_3 s^3 + 10 \cdot I_3 s^2$$

$$Ks + K = I_3 s^3 + 10 I_3 s^2$$

$$K = I_3 s^3 + 10 I_3 s^2 - Ks$$

$$= s(I_3 s^2 + 10 I_3 s - K)$$

$$= s \quad \frac{-10I_3 \pm \sqrt{(10I_3)^2 - 4I_3(-K)}}{2I_3}$$

Zeros

$$D(s)G(s) = 0$$

$$K\frac{s+1}{s+10} \cdot \frac{1}{I_3 s^2} = 0$$

$$K \cdot \frac{s+1}{I_3 s^3 + 10 I_3 s^2} = 0$$

$$K \cdot s + 1 = 0$$

$$s = -1$$

Zeros : -1

Poles : 0, 0, -10

Poles $K = s$ and

$K = -5 \pm \frac{1}{2I_3}\sqrt{100 I_3^2 + 4 I_3 K}$

$-10 + 1 = -9$

$m = 2 \, 1 -$

^ Cross at

$$\frac{0 + 0 + -10 - -1}{2 - 1} = \frac{-9}{2} = -4.5$$

$$\frac{0+0+-10--1}{3-1} = \frac{-9}{2} = -4.5$$ "Cross at"

Stable - the system has moved into the LHP

$\zeta = 0.6$

1e) $\zeta \leq \frac{PM}{100}$ $L(s) = \frac{s+1}{s+10} \cdot \frac{1}{I_3 s^2} = 0$

$PM \geq 60$

$$K \cdot \frac{s+1}{s+12} \cdot \frac{1}{I_3 s^2} = 0$$

1f) $D_{lag} = \frac{s+2.17}{s+0.18}$ | Add a lag filter

$0.2\pi = \sqrt{12z^2}$

$12 = \frac{P}{z}$ $\sqrt{\frac{0.04\pi}{12}} = P = 0.18$

$0.2\pi = \sqrt{Pz}$ $P = 12z$ $z = 2.17$

1g) @ $\omega = 100$ Hz Add a low pass filter to sopress high freq

$$D(s) = \frac{\omega_c^2}{s^2 + 2\zeta s \omega_c + \omega_c^2}$$

ζ damping ω_c corner freq

P2

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2. W_g = crossover freq

$$\sqrt{pz} = w_g$$

$$\sqrt{\alpha z^2} = W_g$$

maybe up to 40

$W_g = \frac{1.8}{t_r}$ $\quad W_{N_y} = 10 \cdot w_g$ $\quad W_{N_y} = \frac{\pi}{h}$

$$W_{N_y} = 10 \cdot W_g$$

$$10 w_g = \frac{\pi}{h}$$

$$h = \frac{\pi}{10 \cdot \frac{1.8}{t_r}} = \frac{\pi}{100}$$

2b. Time delay $= \frac{h}{2}$ by zoh $= \frac{\pi}{200}$

$w_g = 10$

phaseloss $= \frac{\frac{\pi}{100} \cdot 10}{2}$ $\quad = \frac{\pi}{2}$ at nyquist

$= \frac{hw}{2}$

h = Sample period

By holding points constant until the next point recorded, us th ZOH, DAQ does. There is a lag between continuous and discrete. The continuous experiment continues moving but we can only sample at discrete time. There will be a difference in time associated with with this called the delay a delay of $\frac{h}{2}$ for your sampling period h is not bad at low frequencies. but at higher frequencies around the nyquist, the phaseloss will be $\frac{\pi}{2}$ this is significant and can nolonger operate our controller

When working from continuous to discrete we can add a pade approx. to add enough phase lead at our cross over freq.

2c. To account for phase loss in the system we must add phase lead.

$$\frac{P}{Z} > 1$$

Apply a lead controller that looks like

$$\frac{s+b}{s+a}$$ where $|a| > |b|$ to the cascade so that there is phase gain towards nyquist frequency. We can add an additional to sum phase lead on the controller

2d. ① Given an optimized CT controller $D(s)$

② - Start with a tustin prewarp factor f

where $f = \frac{2[1-\cos(\bar{\omega}h)]}{\bar{\omega}h \cdot \sin(\bar{\omega}h)}$ h = time step
$s = i\bar{\omega}$ $\bar{\omega} < \frac{\pi}{h}$

- Use Pade to compensate for phase loss by adding phase lead

②a Tustin's approx using $s = \frac{2}{fh}\frac{z-1}{z+1}$ in of $D(s)$ tf. ← useful near notch freq

③ Take the inverse z transform:

$$f_k = \frac{1}{2\pi i}\oint_T F(z)z^{k-1}dz$$ → Use the tables Pg 214 to convert from $F(z)$ to f_k

$k = 1, 2, 3\ldots$

difference fn

④ Watch for phaseloss

3

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$$D \xrightarrow{F_{thrust}} G_1 \xrightarrow{\Theta(s)} G_2 \xrightarrow{X(s)}$$

BR $\quad (I_p + m_p l^2) s^2 \Theta(s) - m_p g l \Theta(s) = m_p l s^2 X(s)$

T $\quad (m_c + m_p) s^2 X(s) = m_p l s^2 \Theta(s) + F(s)$

$$G_2 \stackrel{?}{=} \frac{(I_p + m_p l^2)s^2 - m_p g l}{m_p l s^2} = \frac{X(s)}{\Theta(s)} \rightarrow \frac{\left(\frac{I_p + m_p l^2}{m_p l}\right)s^2 - \frac{\cancel{m_p} g \cancel{l}}{\cancel{m_p l}}}{s^2}$$

$$① \left((I_p + m_p l^2) s^2 \Theta(s) - m_p g l \Theta(s)\right) \cdot \frac{1}{m_p l s^2} = X(s)$$

$$\downarrow$$

$$② (m_c + m_p)\cancel{s^2}\left[\frac{1}{m_p l \cancel{s^2}}\left((I_p + m_p l^2)s^2 \Theta(s) - m_p g l \Theta(s)\right)\right] - m_p l s^2 \Theta(s) = F(s)$$

$$\frac{m_c + m_p}{m_p l}\left[(I_p + m_p l^2)s^2 - m_p g l\right] - m_p l s^2 = \frac{F(s)}{\Theta(s)}$$

$$s^2\left[\frac{m_c + m_p}{m_p l}\left[(I_p + m_p l^2)\right] - m_p l\right] - \frac{m_c + m_p}{\cancel{m_p l}}\left(\cancel{m_p} g \cancel{l}\right) = \frac{F(s)}{\Theta(s)}$$

$$s^2\left[\frac{m_c + m_p}{m_p l} I_p + (m_c + m_p) l - m_p l\right] - (m_c + m_p) g =$$

$$\frac{\Theta(s)}{F(s)} = \frac{1}{s^2\left[\frac{m_c + m_p}{m_p l} I_p + (m_c + m_p) l - m_p l\right] - (m_c + m_p) g}$$

$$G_1(s) = \frac{\dfrac{1}{\left[\frac{m_c + m_p}{m_p l} I_p + (m_c + m_p) l - m_p l\right]}}{s^2 - \dfrac{m_c + m_p}{\left[\frac{m_c + m_p}{m_p l} I_p + (m_c + m_p) l - m_p l\right]}}$$

$$b_0 = \frac{1}{\left[\frac{m_c + m_p}{m_p \ell} I_p + (m_c + m_p)\ell - m_p \ell\right]}$$

$$a_0 = \frac{(m_c + m_p) g}{\left[\frac{m_c + m_p}{m_p \ell} I_p + (m_c + m_p)\ell - m_p \ell\right]}$$

$$\bar{b}_2 = \frac{I_p + m_p \ell^2}{m_p \ell}$$

$$\bar{b}_0 = g$$

3b. $D_1 = K \frac{s + z}{s + p}$ $\quad p > z$

$\sqrt{pz} = w_y$ $\quad \frac{p}{z} = \alpha$

$G_1 = \frac{b_0}{s^2 - a_0}$ $\quad$ Root locus

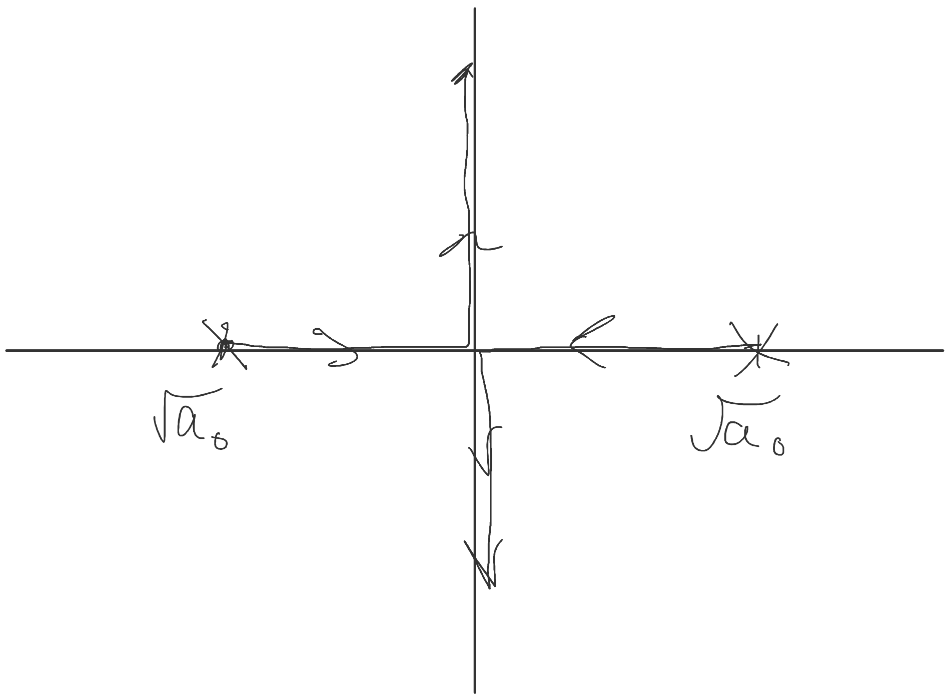
$p = \pm\sqrt{a_0}$ $\quad$ RHS pole @ $p = +\sqrt{a_0}$

We can add a lead compensator to cancel the pole which intialy looks ok. But if we observe the step response of longer time period the response ... It's not stable

The step response of longer time peri...
will not settle. It's not stable

Since our RHP pole $= +\sqrt{a_0}$ our
lead compensator can be

$$D_{lead} = \frac{s - \sqrt{a}}{s + 10\sqrt{a}}$$



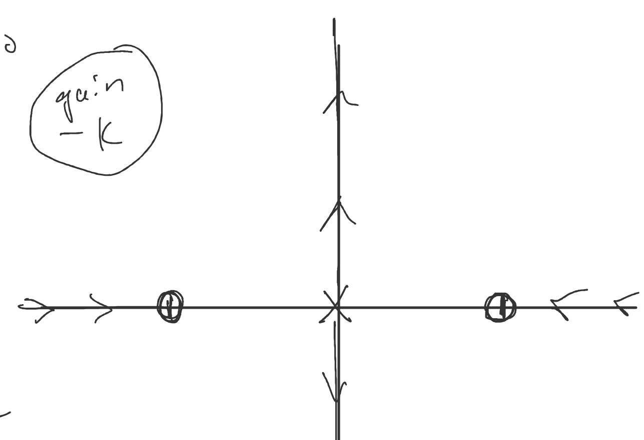
3c)

$$G_2 = \frac{\bar{b}_2 s^2 - \bar{b}_0}{s^2}$$ gain $-K$

Pole @ 0, 0

$z: \ \bar{b}_2 s^2 = \bar{b}$

$$s = \pm\sqrt{\frac{\bar{b}}{\bar{b}_2}}$$



$$S = \pm \sqrt{\frac{b}{b_2}}$$

To cancel the RHP zero lets add

$$D_{lead}(s) = \frac{s + \frac{1}{10}\sqrt{\frac{b}{b_2}}}{s - \sqrt{\frac{b}{b_2}}}$$