2.1

Sunday, October 15, 2023 1:31 PM

$$y(t) = G_0(q)u(t) + e(t)$$

$$G_0 = \frac{Y(t)}{U(t)} = q^{-2}\frac{b_0 + b_1 q^{-1}}{1 - a_0 q^{-1}}$$

$$Y(t)(1 - a_0 q^{-1}) = U(t) q^{-2}(b_0 + b_1 q^{-1})$$

$$Y(t) - a_0 Y(t-1) = U(t-2)b_0 + U(t-3)b_1$$

$$Y(t) = a_0 Y(t-1) + U(t-2)b_0 + U(t-3)b_1$$

$$= a_0 q^{-1} Y(t) + b_0 q^{-2} U(t) + b_1 q^{-3} U(t)$$

$$Y(t) = q^{-2}(b_0 + b_1 q^{-1})$$

$$= q^{-2}b_0 + q^{-3}b_1$$

$$U(t) = 1 - a_0 q^{-1}$$

$$Y(t) = a_0 q^{-1} Y(t) + b_0 q^{-2} U(t) + b_1 q^{-3} U(t)$$

2.2

Sunday, October 15, 2023 1:37 PM

$$y(t) - a y(t-1) = u(t-2) b_0 + u(t-3) b_1 + e(t)$$

$$y(t) = a y(t+1) + u(t-2) b_0 + u(t-3) b_1 + e(t)$$

$$\theta_0^T = [a \quad b_0 \quad b_1]$$

$$\gamma = [y(t+1) \quad u(t+2) \quad u(t-3)]$$

$$w(t) = e(t)$$

$$\underline{y(t) = \theta_0^T \gamma(t)} + w(t)$$

2.3

Sunday, October 15, 2023 12:33 PM

$$y(t) = G_0(q)\,u(t) + e(t)$$

$$E(y(t)) = E(G_0(q)\,u(t) + \underbrace{e(t)}_{\to 0})$$

$$= E\left(q^{-2}\frac{b_0 + b_1 q^{-1}}{1 - a_0 q^{-1}}\; u(t)\right)$$

$$= (q^{-2} b^0 + q^{-3} b_1)u = (1 - a_0 q^{-1})\, y(t)$$

$$b_0\, u(t-2) + b_1\, u(t-3 = y(t) - a_0\, y(t-1)$$

$$E(y(t) = a_0\; y(t-1) + b_0 \underbrace{u(t-2)}_{\mu} + b_1 \underbrace{u(t-3)}_{\mu} + e(t)$$

$$E(y) = a_0 E(y(t-1)) + \mu b_0 + \mu b_1$$

$$E(y) - a_0 E(y(t-1))$$

$$E(y) = \frac{\mu b_0 + \mu b_1}{1 - a_0}$$

Variance

$$= E\left((y(t) - E(y(t)))^2\right) \qquad (y - Ey)$$

$$= E\left(\left(y - \frac{\mu b_0 + \mu b_1}{1-a}\right)^2\right)$$

$$= E\left(b_0 u(t-2) + b_1 u(t-3) + a_0\, y(t-1) - \frac{\mu b_0 + \mu b_1}{1-a} + e(t)\right)^2$$

$$= E(y^2(t) - 2(E y(t))y(t) + (E y(t))^2)$$

$$y: (a_0 q^{-1} y + b_0 q^{-2} u + b_1 q^{-3} u + e(t)$$

$$= E\left(y^2(t) - 2\left(\frac{\mu b_0 + \mu b_1}{1-a_0}\right)(y(t)) + \left(\frac{\mu b_0 + \mu b_1}{1-a_0}\right)^2\right)$$

$$= E(y^2(t)) - E\left(2\left(\frac{\mu b_0 + \mu b_1}{1-a_0}\right) y(t)\right) + E\left(\frac{\mu b_0 + \mu b_1}{1-a_0}\right)^2$$

$$= E(y^2(t)) + \frac{\mu b_0 + \mu b_0}{1 - a_0}$$

3.1

Sunday, October 15, 2023 3:12 PM

$$v(t) = \frac{1}{5}\left(e(t) + e(t-1) + \cdots + e(t-4)\right)$$

$$\rightarrow H(q) = \frac{1}{5}\left(q^{0} + q^{-1} + q^{-2} + q^{-3} + q^{-4}\right)$$

$$= \sum_{k=0}^{4} \frac{1}{5} q^{-k}$$

3.2

Sunday, October 15, 2023 6:55 PM

$$Z(x(t+n)) = Z\{q^n x(t)\} = z^n X(z)$$

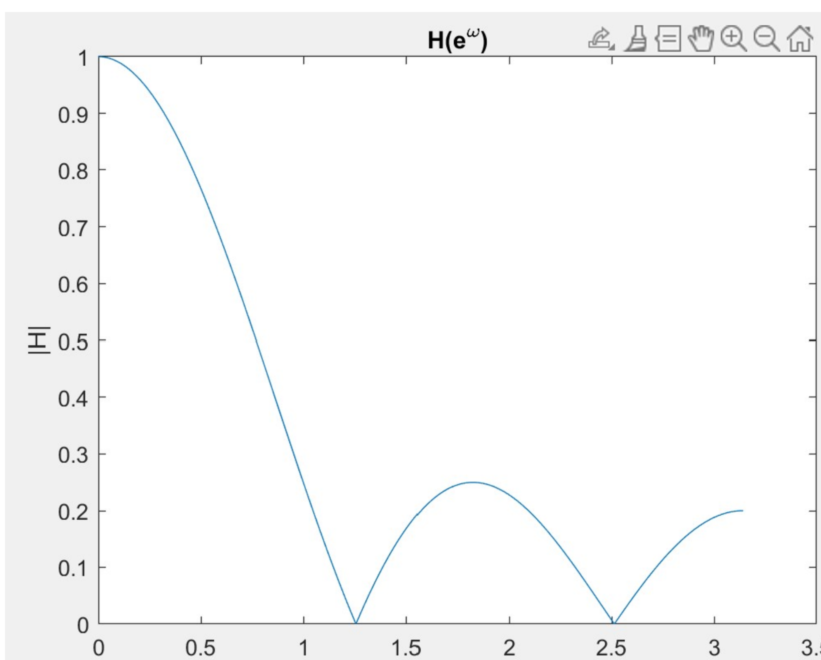
$$H(t) = \sum_{n=0} \frac{1}{5} q^{-n} \rightarrow \frac{1+q^{-1}+q^{-2}+q^{-3}+q^{-4}}{5}$$

$$Z(e) = \sum_{n=0}^{4} \frac{1}{5} z^{-n}$$

$$= \frac{1 + z^{-1} + z^{-2} + z^{-3} + z^{-4}}{5}$$

$$z = e^{j\omega}$$

$$= \frac{1}{5}\left[1 + e^{-j\omega} + e^{-2j\omega} + e^{-3j\omega} + e^{-4j\omega}\right]$$



The freq with $|H(e^{i\omega})| = 0$ are at about $\omega = 1.25$ and 2.5

These points are zeros and at these points the signal is suppressed through this function

3.3 ✓

Sunday, October 15, 2023 3:14 PM

$R_v(\tau) = E\{v(t)\, v(t-\tau)\}$ $\quad \tau \in N$

$v(t) = \frac{1}{5} e(q^{0} + q^{-1} + q^{-2} + q^{-3} + q^{-4})$

$= \Big\{ \frac{1}{5}(e(t) + e(t-1) + e(t-2) + e(t-3) + e(t-4)) \cdot$

$\cdot \frac{1}{5}(e(t-\tau) + e(t-\tau-1) + e(t-\tau-2) + e(t-\tau-3) + e(t-\tau-4)) \Big\}$

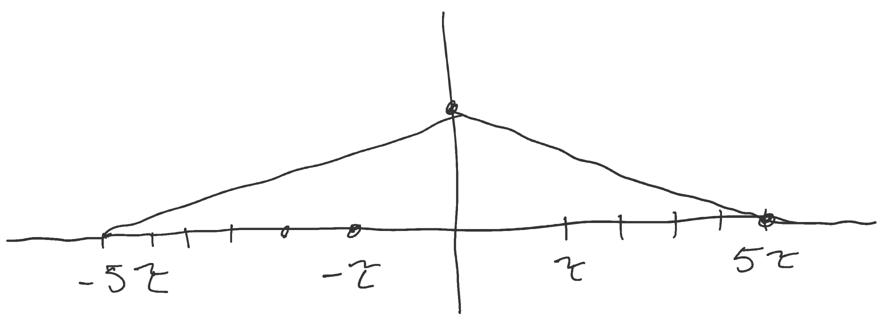
@ $\tau = 0$ $\quad E(v(t)^2) = \lambda$

$\tau = 1$

$E\{ v(t)\,(t-\tau) \}$

$e(t) + e(t-1)$

$+ e(t-5)$



If we multiplied it out we would see that

$E(v(t)^2)$ is close to $E(v(t)\,v(t-1))$ because

$E\Big\{ \frac{1}{5}(e(t) + e(t-1) + e(t-2) + e(t-3) + e(t-4)) \cdot$

$\cdot \frac{1}{5}(e(t-1) + e(t-2) + e(t-3) + e(t-4) + e(t-5)) \Big\}$

only the terms on the end no longer have a pair the rest is the same as $E(v(t))^2$

So! as we get further → τ is increasing there become fewer pairs and the expected values are nolonger similar

1.1 ✓

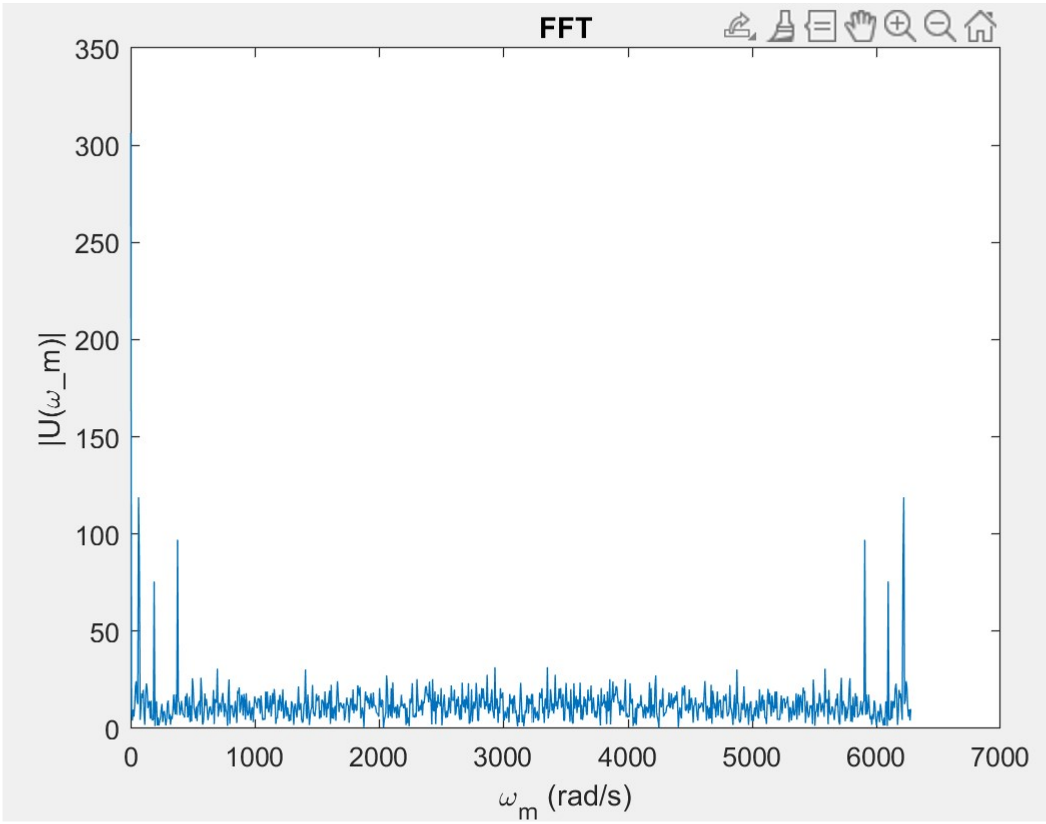
Sunday, October 15, 2023 4:35 PM

Frequency Res : $\frac{1}{N \cdot \Delta T} = 1 \text{ Hz}$

largest frequency : 1000 Hz

1.2

Sunday, October 15, 2023 5:02 PM



1.3

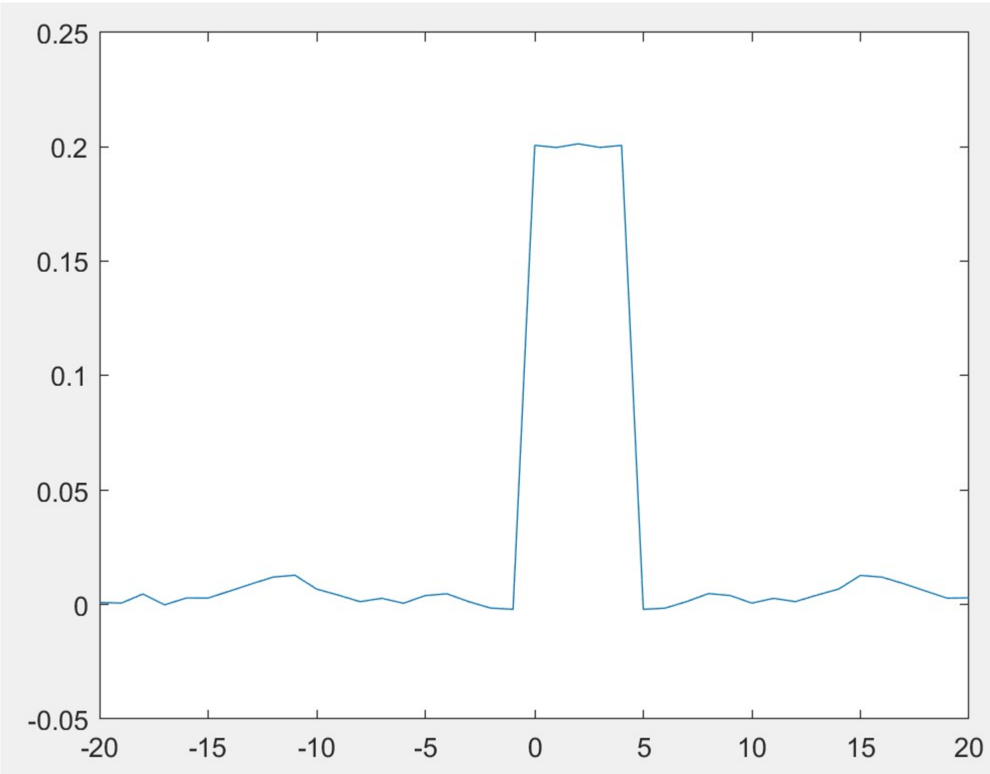
Sunday, October 15, 2023 5:04 PM

As we increased the number of samples to N -> infinity

- The resolution of the signal analysis gets better because T = N*deltaT increases
- We get finer frequency as T increases because frequency resolution = 1/T

3.4

Sunday, October 15, 2023 8:42 PM



The square shape has features of a flat top around the middle sharp vertical lines towards the end of the top
This shape comes from our transfer function
The top of the square has a length of about 5
This is because in our average estimation we consider the past 5 discrete samples
Beyond the past 5 samples, the signals will no longer be similar
I expect that my average function will be close to the expected signal because of the 5 previous samples which is accurate over 10000 samples