1.1

Monday, October 23, 2023 9:42 AM

1.1 $G_0(q) = \dfrac{b_1 q^{-1}}{1 + a_1 q^{-1}}$

$z = q$

$G_0(z) = \dfrac{b_1 z^{-1}}{1 + a_1 z^{-1}}$

$z = e^{jw}$

$= \dfrac{b_1 e^{-jw}}{1 + a_1 e^{jw}} = \dfrac{b}{e^{jw} + a}$

$|G_0(e^{jw})| = \dfrac{b}{e^{jw} + a}$

1.2

$\text{DFT} = U^N(\omega) = \sum_{t=1}^{N} u(t) e^{-jwt}$ ← watch for data before expir $|t| \in N$

$Y^N(\omega) = G_0(e^{jw}) U^N(\omega) + R^N(\omega)$ ← error of inital cond

$y(t) = \dfrac{b_1 q^{-1}}{1 + a_1 q^{-1}} u(t)$

$y(t) + a_1 y(t-1) = b_1 u(t-1)$

$F(\, y(t) = b_1 u(t-1) - a_1 y(t-1) \,)$

$\sum_{t=1}^{N} y(t) e^{-jwt} = b_1 \sum_{t=1}^{N} u(t-1) e^{-jwt} \quad a_1 \sum_{t=1}^{N} y(t-1) e^{-jwt}$

$Y^N(\omega) = b_1 \left(u(0) e^{-jw0} + b_1 \sum_{t=1}^{N} u(t) e^{-jw(t+1)} - u(N) e^{-jw(N+1)} \right) - a \left(y(0) e^{-jw0} + \sum_{t=1}^{N} y(t) e^{-jw(t+1)} - y(N) e^{-jwN+1} \right)$

$= b_1 u(0) + e^{-jw} b_1 \sum_{t=1}^{N} u(t) e^{-jwt} - b_1 u(N) e^{-jw(N+1)} - a y(0) - a e^{-jw} \sum_{t=1}^{N} y(t) e^{-jwt} + a y(N) e^{-jwN+1}$

$= b_1 u(0) + e^{jw} b_1 U(\omega) - b_1 e^{-jw} u(N) e^{-jwN} - a y(0) - a e^{-jw} Y(\omega) + a e^{-jw} y(N) e^{-jwN}$

$= e^{-jw} b_1 U(\omega) - a e^{-jw} Y(\omega) + b_1 u(0) - b_1 e^{-jw} u(N) e^{-jwN} - a y(0) + a e^{-jw} y(N) e^{-jwN}$

$[1 + a e^{-jw}] Y^N(\omega) = b_1 e^{-jw} U(\omega) + b_1 u(0) - b_1 e^{-jw} u(N) e^{-jwN} - a y(0) + a e^{-jw} y(N) e^{-jwN}$

$$Y^N(\omega) = \underbrace{\frac{b_1 e^{-jw}}{1 + a e^{-jw}}}_{G_0(e^{jw})} U(\omega) + \underbrace{\frac{b_1}{1 + a e^{-jw}} u(0) - \frac{b_1 e^{-jw}}{1 + a e^{-jw}} u(N) e^{-jwN} - \frac{a}{1 + a e^{-jw}} y(0) + \frac{a e^{-jw}}{1 + a e^{-jw}} y(N) e^{-jwN}}_{R^N}$$

2.1

Monday, October 23, 2023 9:57 AM

$y(t) = G_0(q)u(t) + e(t)$

$G_0(q) = \dfrac{b_1 q^{-1}}{1 + a_1 q^{-1}}$ $\quad \mu = 0, \quad \lambda = \lambda, \quad e \perp u$

$(1 + a_1 q^{-1}) y(t) = b_1 u(t-1) + (1 + a q^{-1}) e(t)$

$y(t) + a y(t-1) = b_1 u(t-1) + e(t) + a e(t-1)$

$y(t) = b_1 u(t-1) - a y(t-1) + \underbrace{e(t) + a e(t-1)}_{w(t)}$

$\theta^T = [b_1 \quad a]$

$\varphi = [u(t-1) \quad -y(t-1)]$

2.2 $\quad e(t) + a e(t-1)$

$\downarrow$

$(1 + a q^{-1}) e(t)$

$W(q) = (1 + a q^{-1})$

$\uparrow$

relying on past noise samples

ie - not white noise

$F(w) = 1 - a e^{-jw}$

2.3 $\quad \Phi_v(\omega) = F(R_v(\tau)) = \lim_{N \to \infty} E\{P_u^N(\omega)\}$

$R_v(\tau) = E[v(t) v(t-\tau)]$

$|W(e^{j\omega})|^2 \lambda$

xcorr

plot $\log \hat{\Phi}_v(\omega)$ vs $\log w$

$\log \Phi_v(\omega)$ vs $\log w$

@ $a = 0.9$

$N = 1000\,N$

$e(t)$: random

For white noise

$\Phi_v = (1 + a q^{-1})^2 \cdot 1$

$= (1 + a e^{+j\omega})^2 \cdot 1$

$DFT(R_v)$

might need $\frac{1}{N}$ when using routine xcor

xcorr

$\downarrow$

$\frac{1}{N} \sum_{t=0}^{N-1} v(t) v(t-\tau)$

$\sum_{\tau=-\infty}^{\infty} R_u^N(\tau) e^{-j\omega \tau}$

complx conj

$= W^N(e^{jw}) \cdot W^N(e^{jw})^*$

$(a + jb)(a - jb)$

$= |a + jb|^2$

fft()

Spectrum()

etfe()

spa()

$\hat{\Phi}^N_v(\omega) = \dfrac{\int_{\xi=-\pi}^{\pi} P_u^N(\xi) W_\gamma(\xi - \omega)\, d\xi}{\int_{\xi=-\pi}^{\pi} W_\gamma(\xi)\, d\xi}$

windowing choice

Convolution → Fourier → mult.

time $\qquad$ freq

$\Phi_v = |W(e^{jw})|^2 \lambda$

$= |1 + a e^{-jw}|^2$

$w = e(t) - ae(t-1)$

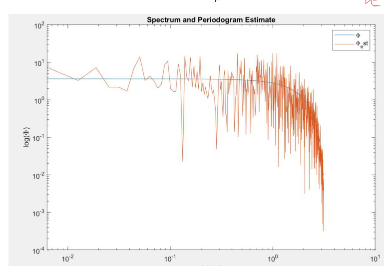
$\hat{\Phi}^N(\omega) = F(R_v)$

$R_v = \frac{1}{N} \sum_{t=0}^{N-1} w(t) w(t-\tau)$

or

plot $= DFT(xcorr(w))$

& use nyquist



2.4 $\quad \varphi_0^T = [u(t-1) \quad -y(t-1)]$

$2 \times 1 \qquad 1 \times 2$

$y(t) = [u(t-1) \quad -y(t-1)] \begin{bmatrix} b_1 \\ a_1 \end{bmatrix} + w(t)$

$] \; b\,u(t-1) - a y(t-1) + w(t)$

$\lim_{N \to \infty} E(\hat{\theta}^N_{LS}) = \left[\frac{1}{N} \sum_{t=1}^{N} \varphi_0(t) \varphi_0^T(t)\right]^{-1} \left[\frac{1}{N} \sum_{t=1}^{N} \varphi_0(t) y(t)\right]$

$[2 \times 2] \qquad [2 \times 1]$

$\begin{bmatrix} u(t-1) \\ -y(t-1) \end{bmatrix} (b\,u(t-1) - a y(t-1) + w(t))$

$R_u = \frac{1}{N} \sum u(t) u(t-\tau)$

$\frac{1}{N} \sum (u(t-1) u(t-1) + y(t-1) y(t-1))$

$[R_u(-1) \quad \cdot \quad R_y(-1)]$

answr $[2 \times 1]$

$\begin{bmatrix} u(t-1) \\ -y(t-1) \end{bmatrix} [u(t-1) \quad -y(t-1)]$

$\tau = N+1$

$t_0 = 2$

$\left(\sum^{N} u(t-1) u(t-1) \quad \sum_{t=1}^{N} -u(t-1) y(t-1) \right)^{-1} \frac{1}{N} \begin{bmatrix} \sum_{t=1}^{N} u(t-1) y(t) \\ \sum^{N} -u(t-1) u(t) \end{bmatrix}$

$$\frac{1}{N}\begin{bmatrix} \sum_{t=1}^{N} u(t-1)u(t-1) & \sum_{t=1}^{N} -u(t-1)y(t-1) \\ \sum_{t=1}^{N} -y(t-1)u(t-1) & \sum_{t=1}^{N} y(t-1)y(t-1) \end{bmatrix}^{-1} \frac{1}{N}\begin{bmatrix} \sum_{t=1}^{N} u(t-1)y(t) \\ \sum_{t=1}^{N} -y(t-1)y(t) \end{bmatrix}$$

$$\lim_{N\to\infty} \begin{bmatrix} R_u^{N-1,1}(0) & -R_{uy}^{N-1,1}(0) \\ -R_{yu}^{N-1,1}(0) & R_y^{N-1,1}(0) \end{bmatrix}^{-1} \begin{bmatrix} R_{yu}^{N,1}(1) \\ -R_y^{N,1}(1) \end{bmatrix}$$

$$\frac{1}{R_u(0)R_y(0) - (R_{uy}(0))(R_{yu}(0))} \begin{bmatrix} R_y(0) & R_{uy}(0) \\ R_{yu}(0) & R_u(0) \end{bmatrix}\begin{bmatrix} R_{yu}(1) \\ -R_y(1) \end{bmatrix}$$

2.5

assume $R_y(0) = 0$

2.5 $R_y(0)$ of R^N Break R_y into R_u

$R_y(1)$ input impulse resp coeff

comput $R_{yu}(\tau)$

impulse resp of system

Changing filter $\downarrow$ transfer fn $\downarrow$ frequency

2x2 find inverse

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

wont conv to θ_0 because equation noise is not white

$$\frac{1}{\mu R_y(0)}\begin{bmatrix} R_y(0) & 0 \\ 0 & \mu \end{bmatrix}\begin{bmatrix} b\mu \\ -R_y(1) \end{bmatrix}$$

$R_{yu}(0) = E(y(t)u(t))$

$R_e(0) = \lambda$

$R_u(0) = \mu$

$R_{eu}(\tau) = 0$

$y(t) + a_1 y(t-1) = b_1 u(t-1)$

$E(y(t) = b_1 u(t-1) - a_1 y(t-1))$

$y = \frac{b_1 q^{-1}}{1+aq^{-1}} u(t) + e(t)$

$R_y(0)$

$E\{yy\} = E\{bu(t-1)y(t) + e(t)y(t) + ae(t-1)y(t) - ay(t-1)y(t)\}$

$= bR_{uy}(-1) + R_{ey}(0) + aR_{ey}(-1) - aR_y(-1)$

$= bR_{yu}(1) + R_{ye}(0) + aR_{ye}(1) - aR_y(-1)$

$= b\cdot b\mu + \lambda + a\cdot 0 - aR_y(1)$

$E(y)$

$(1+aq^{-1})y = bu(t-1) + (1+aq^{-1})e(t)$

$y + ay(t-1) = bu(t-1) + e(t) + ae(t-1)$

$y = bu(t-1) + e(t) + ae(t-1) - ay(t-1)$

$R_y(1)$

$R_y(1) \Rightarrow E\{y\cdot y(t-1)\} = E\{bu(t-1)y(t-1) + e(t)y(t-1) + ae(t-1)y(t-1) - ay(t-1)y(t-1)\}$

$R_y(1) = bR_{uy}(0) + R_{ey}(1) + aR_{ey}(0) - aR_y(0)$

$= b\cdot 0 + 0 + a\lambda - a(b^2\mu + \lambda - aR_y(1))$

$R_y(1) = a\lambda - ab^2\mu - a\lambda + a^2 R_y(1)$

$(1-a^2)R_y(1) = a\lambda - ab^2\mu - a\lambda$

$R_y(1) = \frac{a\lambda - ab^2\mu - a\lambda}{(1-a)^2}$

$R_{ye}(0)$

$y\cdot e = E(bu(t-1)e(t) - ay(t-1)e(t) + e(t)e(t) + ae(t-1)e(t))$

$= b\underbrace{R_{ue}(-1)}_{0} - a\underbrace{R_{ye}(-1)}_{0} + \underbrace{R_e(0)}_{\lambda} + a\underbrace{R_e(-1)}_{0}$

$=$

$R_{ye}(0) = \lambda$

R_{yu}

$E(yu) = E(bu(t-1)u(t)) - ay(t-1)u(t) + e(t)u(t) + ae$

$\tau = 0 \Rightarrow = b\underbrace{R_u(-1)}_{0} - a\underbrace{R_{yu}(-1)}_{0} + \underbrace{R_{eu}(0)}_{0} + a\underbrace{R_{eu}(-1)}_{0}$

$R_{yu}(0) = 0$

$E\,yu(t-1) = E(bu(t-1)u(t-1) - ay(t-1)u(t-1) +$

$R_{yu}(1) = bR_u(0) - a\underbrace{R_{yu}(0)}_{0} + \underbrace{R_{eu}(1)}_{0}$

$R_{yu}(1) = b\mu$

$(t-1)u(t)$

$e(t)u(t-1) + a\,e(t-1)u(t-1)$

$+ a \underbrace{R_{eu}(0)}_{0}$

$R_{ye}(1)$

$y\,e(t-1) = E(b\,u(t-1)\,e(t-1) + e(t)\,e(t-1) + a\,e(t-1)\,e(t-1) - a\,y(t-1)\,e(t-1))$

$= b\,R_{ue}(0) + R_e(1) + a\,R_e(0) - a\,R_{ye}(0)$

$\underbrace{R_{ye}(1)}_{=0} = \quad 0 \quad + \quad 0 \quad + a\lambda - a\lambda = 0$

$$= \frac{1}{\mu R_y(0)} \begin{bmatrix} R_y(0) & 0 \\ 0 & \mu \end{bmatrix} \begin{bmatrix} b\mu \\ -R_y(1) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\mu} & 0 \\ 0 & \frac{1}{R_y(0)} \end{bmatrix} \begin{bmatrix} b\mu \\ -R_y(1) \end{bmatrix} =$$

$$= \begin{bmatrix} b \\ -\frac{R_y(1)}{R_y(0)} \end{bmatrix} = \begin{bmatrix} b \\ -\dfrac{\frac{a\lambda - ab^2\mu - a\lambda}{(1-a)^2}}{b^2\mu + \lambda - a\left(\frac{a\lambda - ab^2\mu - a\lambda}{(1-a)^2}\right)} \end{bmatrix}$$

3.1

Monday, October 23, 2023 6:56 PM

$X(s) = G(s, [m\ d\ k])\, F(s)$

$= \dfrac{1}{ms^2+ds+k} F(s)$

$\varepsilon = X(k) - \hat{X}(k)$ ← prediction

$\phi = \text{argmin} \sum_{k=n+1}^{N} \varepsilon(k,\theta)^2$

$= \min \sum_{k=n+1}^{N} (X(k) - \hat{X}(k))^2$

theta

$\dfrac{1}{ms^2+ds+k}$ $\quad z = e^{s\Delta T}$

$\dfrac{1/m}{s^2 + \frac{d}{m}s + \frac{k}{m}}$

$s = \dfrac{-\frac{d}{m} \pm \sqrt{\left(\frac{d}{m}\right)^2 - 4\left(\frac{k}{m}\right)}}{2}$

m d

$\dfrac{1}{ms^2+ds+k} \xrightarrow{\text{ZOH}} \underbrace{\dfrac{b's}{q^2+a_1q+a_2}}_{\text{poles}}$ ← simplify

2 poles 1 zero

ZOH

$\dfrac{\tau s+1}{ms^2+ds+k}$ hopefully τ is close to zero

$\dfrac{1}{ms^2+ds+k} \longrightarrow \dfrac{b_1 q^{-1} + b_2}{1+a_1 q^{-1} + a_2 q^{-2}} \rightarrow \dfrac{Y}{F}$

DC gain is same

$S = 0$

$1 + a_0 z^{-1} + a_1 z^{-2} = 0$

$y(k) + a_1 y(k-1) + a_2 y(k-2) = b_1 u(k-1) \quad b_2 u(k)$

$= b_1 u(k-1) \quad b_2 u(k) - a_1 y(k-1) - a_2 y(k-2)$

$u = F \qquad u = F$

$[\, b_1 \quad b_2 \quad a_1 \quad a_2 \,]$

$z = e^{s\Delta T}$

roots $\quad \ln z = s\Delta T$

$\dfrac{\ln z}{\Delta T} = s$

$\dfrac{-d \pm \sqrt{d^2 - 4mk}}{2m} = \text{roots}$

$-\dfrac{d}{2m} + \dfrac{1}{2m}\sqrt{d^2-4mk} =$

$-\dfrac{d}{2m} - \dfrac{1}{2m}\sqrt{d^2-4mk}$

$(2m \cdot r)^2$

gain

$s = 0$

$z = 1$

$\dfrac{1}{k} = \text{gain}$

$k = \dfrac{1}{\text{gain}}$

Think about model structure adding poles and zeros to see best fit

too many zeros = overfit or $z = 0$

$\dfrac{1}{ms^2+ds+k} = \dfrac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

ζ = damp ratio

ω_n = nat frq

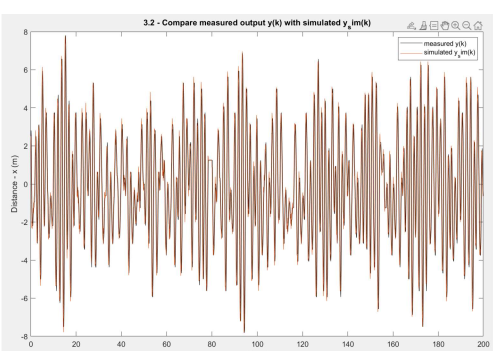
cont $\quad$ mapping $\quad$ discr

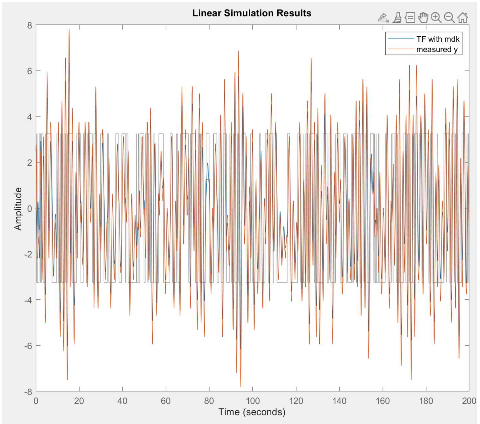
$-\zeta\omega_n \quad \omega_n$

$e^{s\Delta T} = z$

$\dfrac{1}{\Delta T}\ln(z)$

```
theta =

    0.0594
   -0.0015
   -1.7701
    0.9097


k =

    2.4092


m =

    0.1627


d =

    0.1539


Go =

               1
  ----------------------------
  0.1627 s^2 + 0.1539 s + 2.409

Continuous-time transfer function.
Model Properties
>>
```




Linear Simulation Results
TF with mdk
measured y
Amplitude
Time (seconds)
8
6
4
2
0
-2
-4
-6
-8
0
20
40
60
80
100
120
140
160
180
200