MATHEMATICS, Paper – I (A)
(English Version)

Time : 3 Hours ***Max. Marks : 75***

Note : This question paper consists of three sections *A*, *B* and *C*.

SECTION – A

Note : i) Answer **all** questions **$10 \times 2 = 20$**

ii) Each question carries **two** marks

iii) All are very **short answer** type questions

1. Find the domain and range of the function $f(x) = |x| + |1+x|$.
2. If $A : \{-2,-1,0,1,2\}$ and $f : A \to B$ is a surjection defined by $f(x) = x^2 + x + 1$, then find B.
3. If $A = \begin{bmatrix} -2 & 1 \\ 5 & 0 \\ -1 & 4 \end{bmatrix}, B = \begin{bmatrix} -2 & 3 & 1 \\ 4 & 0 & 2 \end{bmatrix}$ then find $2A + B^T$
4. If $A = \begin{bmatrix} 2 & 4 \\ -1 & k \end{bmatrix}$ and $A^2 = O$, find k.
5. $a = 2i - j + k, b = i - 3j - 5k$. Find the vector c such that a, b and c from the sides of a triangle.
6. ABCDE is a pentagon. If the sum of the vectors $\overrightarrow{AB}, \overrightarrow{AE}, \overrightarrow{BC}, \overrightarrow{DC}, \overrightarrow{ED}$ and $\overrightarrow{AC}$ is $\lambda\overrightarrow{AC}$, then find the value of λ.
7. If $\cos\theta = t, 0 < t < 1$ and θ does not lie in the first quadrant, find the values of

   i) $\sin\theta$ ii) $\tan\theta$
8. Prove that $Tanh^{-1}\left(\frac{1}{2}\right) = \frac{1}{2}\log_e 3$
9. Prove that $\frac{\sin 4\theta}{\sin\theta} = 8\cos^3\theta - 4\cos\theta$
10. If $f(x) = \frac{x-1}{x+1}, x \neq \pm 1$, show that $(fof^{-1})(x) = x$.

SECTION – B

II. Note : i) Answer any **five** of the following questions. **5 × 4 = 20**

ii) Each question carries **four** marks.

iii) All are **short answer** type questions.

11. A trust fund has to invert Rs. 30,000 in two different types of bonds. The bond pays 5% interest per year, and the second bond pays 7% interest per year. Using matrix multiplication, determin how to devide Rs.30,000 among the two types of bonds, if the trust fund must obtain an annual total interest of
(a) Rs 1,800 (b) Rs. 2,000

12. If a, b, c are non–coplanar vectors, then show that the four points $6a+2b-c$, $2a-b+3c$, $-a+2b-4c$, $-12a-b-3c$ are coplanar.

13. If $0<A<B<\frac{\pi}{4}, \sin(A+B)=\frac{24}{25}, \cos(A-B)=\frac{4}{5}$, find the value of tan 2A.

14. Solve $6\tan^2 x-2\cos^2 x=\cos 2x$

15. If A is not an integral multiple of π, prove that cos A cos 2A cos 4 A cos 8A = $\frac{\sin(16A)}{16\sin A}$ and hence deduce that $\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{16\pi}{15}=\frac{1}{16}$

16. If $Sin^{-1}x+Sin^{-1}y+Sin^{-1}z=\pi$, prove that $x\sqrt{1-x^2}+y\sqrt{1-y^2}+z\sqrt{1-z^2}=2xyz$

17. In ΔABC, if $C=90^0$ then prove that $\left(\frac{a^2+b^2}{a^2-b^2}\right)\sin(A-B)=1$.

SECTION – C

III. Note : i) Answer any **five** of the following questions. **5 ×7 = 35**

ii) Each question carries **seven** marks.

iii) All are **long answer** type questions.

18. If $f:A\to B$ is a bijection, then $f^{-1}of=I_A$, $fof^{-1}=I_B$.

19. Show that $49^n+16n-1$ is divisible by 64.

20. Examine whether the systems of equation are consistent or inconsistent and if consistent find the complete solutions. $x+y+z=6,\ x-y+z=2,\ 2x-y+3z=9$

21. Show that $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = \begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ac-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} = (a^3+b^3+c^3-3abc)^2$

22. In ΔABC prove that
$$\sin\frac{A}{2}+\sin\frac{B}{2}+\sin\frac{C}{2}=1+4\sin\left(\frac{\pi-A}{4}\right)\sin\left(\frac{\pi-B}{4}\right)\sin\left(\frac{\pi-C}{4}\right)$$

23. $\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)\ldots\left(1+\frac{2n+1}{n^2}\right)=(n+1)^2$.

24. Prove that $\frac{r_1}{bc}+\frac{r_2}{ca}+\frac{r_3}{ab}=\frac{1}{r}-\frac{1}{2R}$

MARCH – 2025
Board of Intermediate Education
Mathematics I(A) 1st Year
SET-1

Time : 3 Hrs | Total Marks : 75 M

SECTION – A

I Answer all the following questions **10x2=20M**

1) Find the domain of definition of the function y(x), given by the equation $2^x+2^y=2$
2) If f: R→R, g:R→R are defined by f(x)=3x-1, g(x)=x^2+1 then find i) fog(2) ii) gof(2a-3)
3) If $A=\begin{bmatrix}0 & 2 & 1\\ -2 & 0 & -2\\ -1 & x & 0\end{bmatrix}$ is a skew symmetric matrix then find x.
4) Find rank of A=$\begin{bmatrix}1 & 2 & 0 & -1\\ 3 & 4 & 1 & 2\\ 2 & 3 & 2 & 5\end{bmatrix}$ using elementary transformation.
5) Find the vector equation of the plane passing through the point $i-2j+5k, -5j-5k$ and $3i+5j$.
6) A= 2i+5j+k and b=4i+mj+nk are collinear vectors then find m and n.
7) If OA-i+j+k, AB=3i-2j+k, BC=i+2j-2k and Cd=2i+j+3k then find vector OD
8) If $\frac{\sin\alpha}{a}=\frac{\cos\alpha}{b}$ then prove that asin2α+bcos2α=b.
9) Draw the graph of y=sin x between -π and π taking four values on X-axis.
10) If Sinhx=3, prove that x=$\log_e(3+\sqrt{10})$.

SECTION – B

II Answer any Five of the following questions **5x4=20M**

11) If 3A=$\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & -2\\ -2 & 2 & -1\end{bmatrix}$ then show that $A^{-1}=A^T$.
12) If $\bar{a},\bar{b},\bar{c}$ are non-coplanar vectors, then prove that the four points $4\bar{b}-3\bar{c}, 3\bar{a}+2\bar{b}-5\bar{c}, -3\bar{a}+8\bar{b}-5\bar{c}$ are coplanar.
13) Prove that the angle in semi circle is right angle.
14) Prove that $\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}=\frac{1+\sin\theta}{\cos\theta}$
15) Solve 4 $\cos^2\theta+\sqrt{3}=2(\sqrt{3}+1)\cos\theta$

16) Prove that $2\sin^{-1}\frac{3}{5} - \cos^{-1}\frac{15}{13} = \cos^{-1}\frac{323}{325}$

17) In ΔABC prove that $\cot A + \cot B + \cot C = \frac{a^2+b^2+c^2}{4}$

III Answer any Five of the following questions 5 x 7 = 35M

18) If $f:A\to B$, $g:B\to C$ are bijective functions then prove that $(gof)^{-1} = f^{-1}og^{-1}$.

19) Using mathematical induction, prove that statement $\forall n \in N$ $\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)\ldots\left(1+\frac{2n+1}{n^2}\right) = (n+1)^2$

20) $\Delta_1 = \begin{vmatrix} a_1^2+b_1+c_1 & a_1a_2+b_2+c_2 & a_1a_3+b_3+c_2 \\ b_1b_2+c_3 & b_2^2+c_2 & b_1b_3+c_1 \\ c_3c_1 & c_3c_2 & c_3^2 \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ then find $\frac{\Delta_1}{\Delta_2}$

21) By gauss Jordan method, solve the system of linear equations 2x-y+3z=9, x+y+z=6 and x-y+z=2.

22) If A=(1,-2,-1), B=(4,0,-3), C=(1,2,-1) and D=(-2,-4,-5) then find the shortest distance between AB and CD lines.

23) If A, B, C are the angles in a triangle, the prove that $\cos\frac{A}{2} + \cos\frac{B}{2} + \cos\frac{C}{2} = 4\cos\left(\frac{\pi-A}{4}\right)\cos\left(\frac{\pi-B}{4}\right)\cos\left(\frac{\pi-C}{4}\right)$.

24) If a=13, b=14, c=15 then show that $R=\frac{65}{8}, r=4, r_1=\frac{21}{2}$, r_2=12 and r_3=14.