Geometric Partial Matching

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Problem Statement

Geometric Bipartite Matching: Given two equal-sized point sets A, B in the plane, find a *perfect matching* minimizing the sum of lengths of matching edges.
Geometric Partial Matching: Given two point sets A, B in the plane, find a *size-k matching* minimizing the sum of lengths of matching edges.

Background

Let $n = \max(|A|, |B|)$ and $m = |A \times B| = O(n^2)$. Primal-dual algorithms for general graphs can solve the problem in $O(km + k^2 \log n) = O(kn^2 + k^2 \log n)$ time.
However, there are faster algorithms for geometric bipartite matching versus perfect matching in general graphs, using dynamic data structures for **bichromatic closest pair** and **nearest neighbors**. Roughly speaking, we can replace the $O(m)$ with $O(n \operatorname{polylog} n)$ or $O(k \operatorname{polylog} n)$ in many instances. If $O(m)$ is no longer the running time bottleneck, can we design a faster algorithm for partial matching?

Results

Building from existing primal-dual algorithms,

1. An exact algorithm running in time $O((n+k^2) \operatorname{polylog} n)$, using the Hungarian algorithm.
2. A $(1+\varepsilon)$-approximation algorithm running in time $O((n + k\sqrt{k}) \operatorname{polylog} n \log(1/\varepsilon))$, using a **cost-scaling** algorithm for unit-capacity minimum-cost flow by Goldberg, Hed, Kaplan, and Tarjan.

Primal-Dual Augmentation Algorithms

A classical algorithm for min-cost matching and min-cost flow: grow the matching using the least-cost augmenting path. Essentially, solve a **single-source shortest paths** problem (Dijkstra). Improves with geometry — can query the "next shortest edge" as a bichromatic closest pair.

Techniques