

# Efficient Algorithms for Geometric Partial Matching

Pankaj K. Agarwal   Hsien-Chih Chang   Allen Xiao

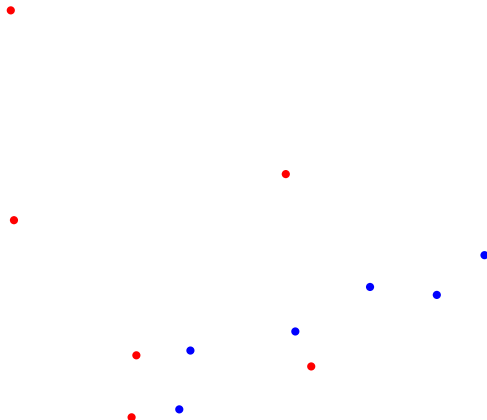
Department of Computer Science, Duke University

June 2019

# Geometric (bipartite) matching



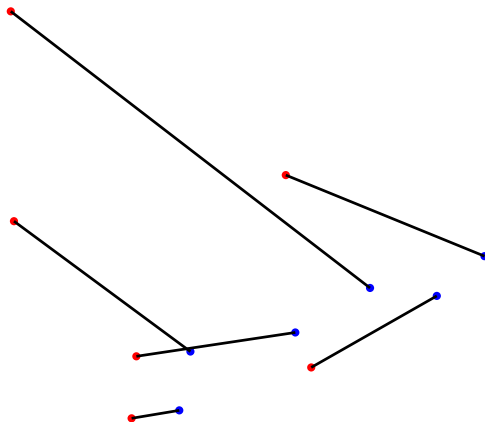
$A, B$



# Geometric (bipartite) matching



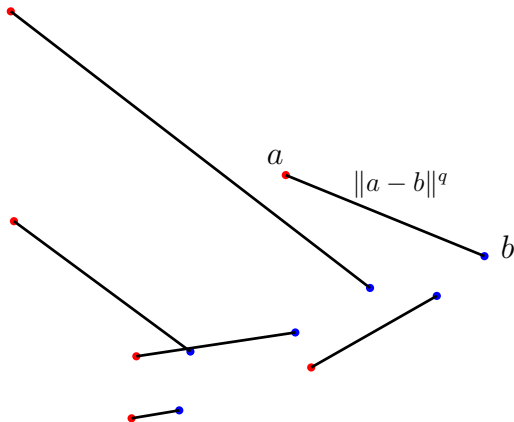
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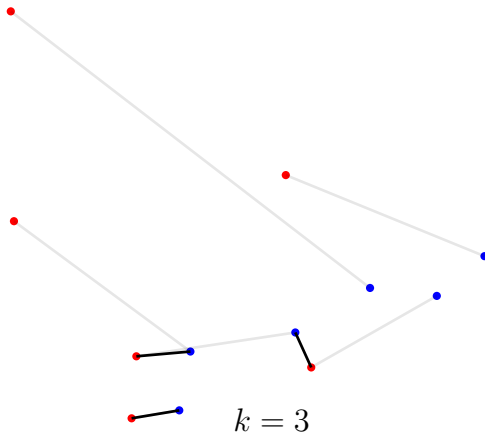
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# Geometric (bipartite) partial matching



$A, B$

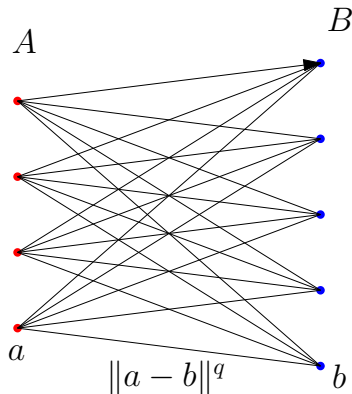




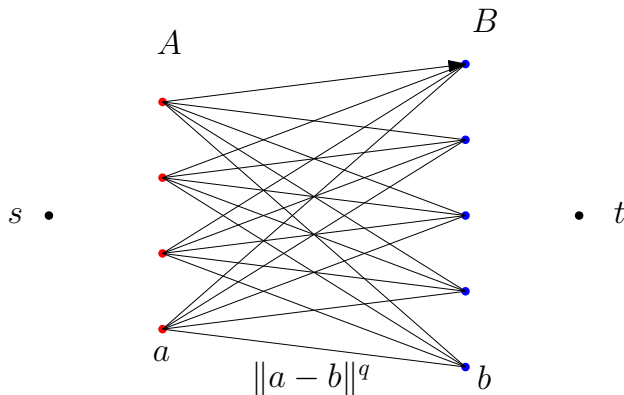
	<b>approx.</b>	<b>time</b>	<b>valid <math>q</math></b>
Hungarian algorithm (Kuhn)	exact	$O(km + k^2 \log n)$	$q \geq 1$
Ramshaw, Tarjan 2012	exact <sup>1</sup>	$O(kn \text{ polylog } n)$	$q \geq 1$
		$O(m\sqrt{k} \log(kC))$	
	$(1 + \varepsilon)$	$O(n\sqrt{k} \text{ polylog } n \log(1/\varepsilon))$	
Sharathkumar, Agarwal 2012	$(1 + \varepsilon)$	$O(n \text{ poly}(\log n, 1/\varepsilon))$	$q = 1$
new (Hungarian)	1	$O((n + k^2) \text{ polylog } n)$	$q \geq 1$
new (cost-scaling)	$(1 + \varepsilon)$	$O((n + k\sqrt{k}) \text{ polylog } n \log(1/\varepsilon))$	$q \geq 1$

<sup>1</sup>Assuming integer costs  $\leq C$ .

# Unit-capacity min-cost flow formulation

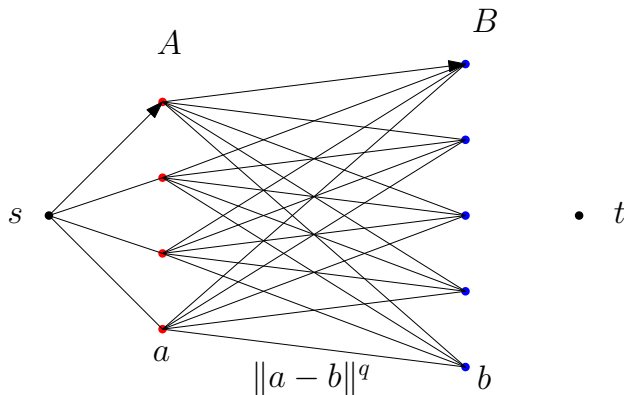


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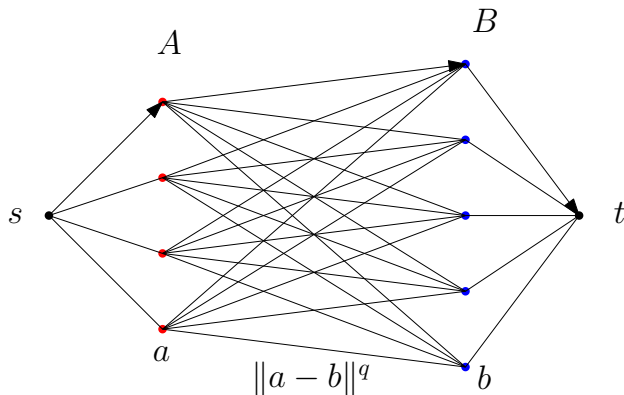




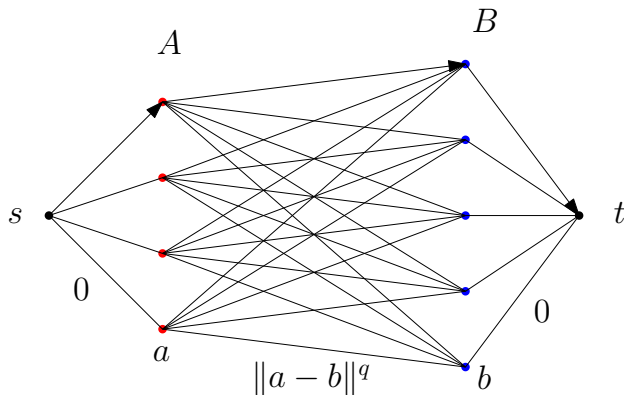
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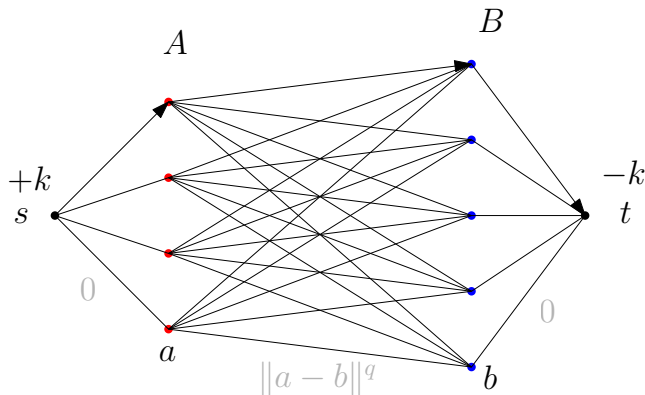
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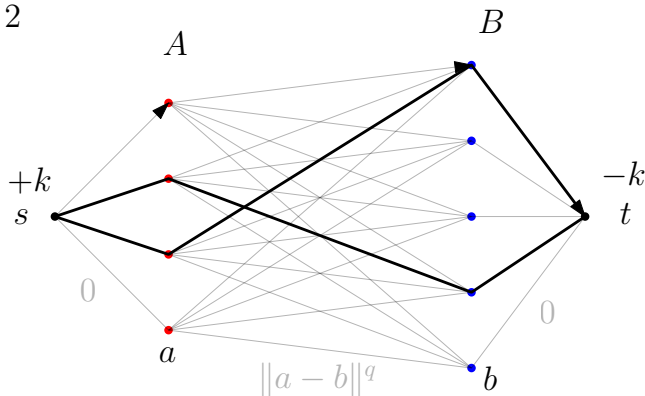
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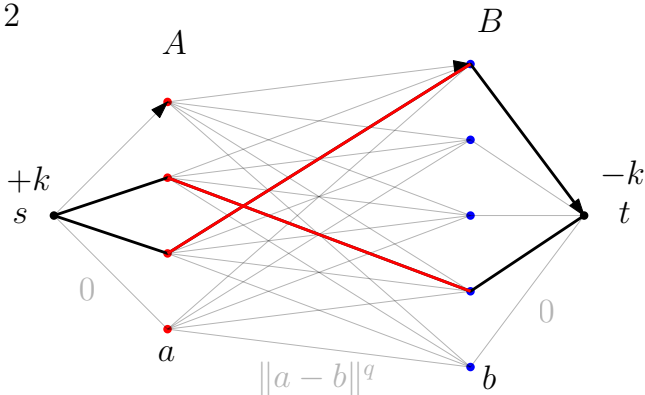
$$k = 2$$

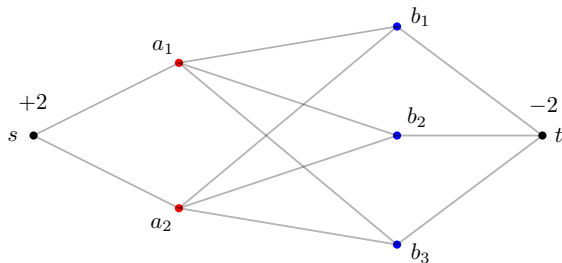


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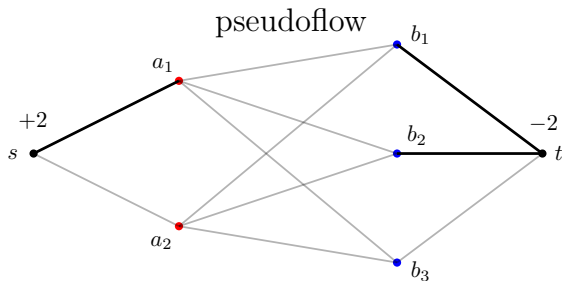


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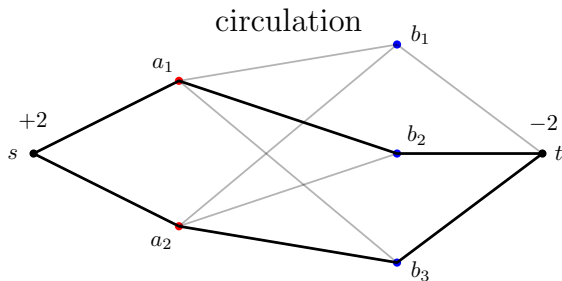


- ▶ Reduced cost:  $c_\pi(v \rightarrow w) := c(v \rightarrow w) - \pi(v) + \pi(w)$
- ▶  $\theta$ -optimality:  $c_\pi(v \rightarrow w) \geq -\theta$  on all residual arcs
- ▶ admissible residual arcs:  $c_\pi(v \rightarrow w) \leq 0$

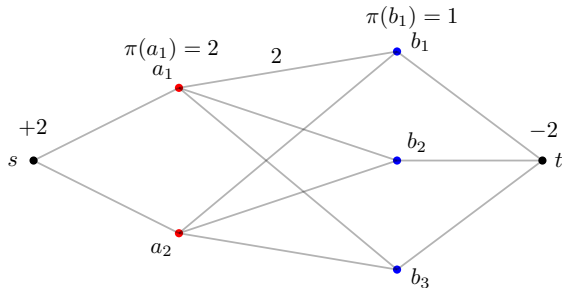


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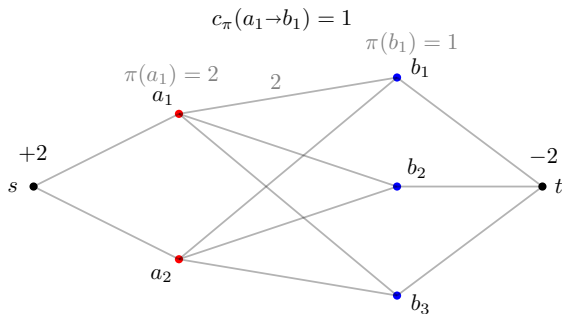




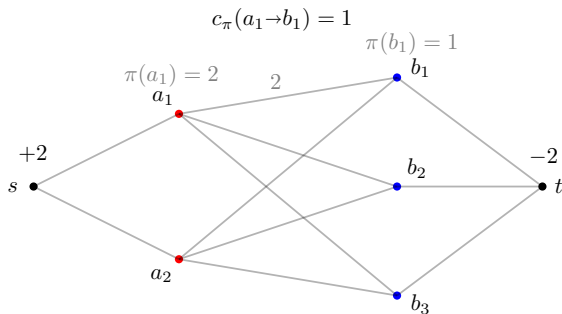
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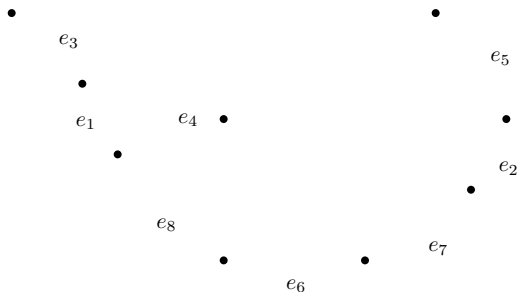
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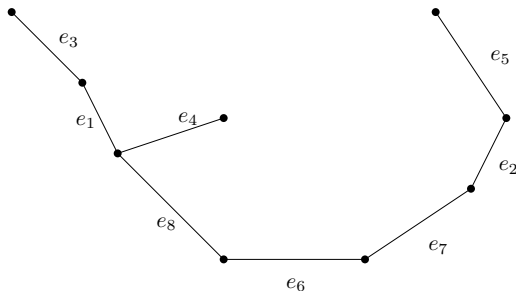
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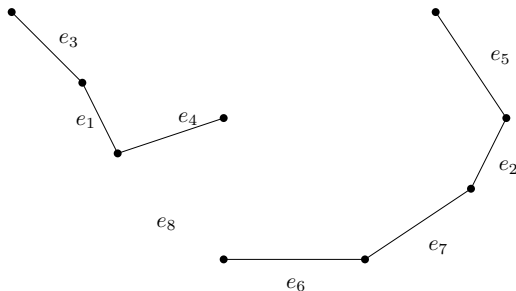
- ▶  **$\theta$ -optimality**:  $c_\pi(v \rightarrow w) \geq -\theta$  on all residual arcs
- ▶  $\theta$ -optimal circulation is  $+n\theta$  approx. in general ( $+6k\theta$  in our graph).
- ▶ Find  $\theta$ -optimal circulations for geometrically decreasing values of  $\theta$ :
  1. Reduce  $\theta \leftarrow \theta/2$ , while creating  $O(k)$  excess.
  2. **Refine** this pseudoflow into a circulation, while preserving  $\theta$ -optimality



- Compute  $\alpha \geq 0$  satisfying:
  1. there exists a  $k$ -matching whose longest edge has cost  $\leq n^q \cdot \alpha$
  2. a  $(\varepsilon\alpha/6k)$ -optimal circulation is  $(1 + \varepsilon)$ -approx.
- $(1 + \varepsilon)$ -approx. geometric partial matching by executing  $O(\log(n^q/\varepsilon))$  cost scales.

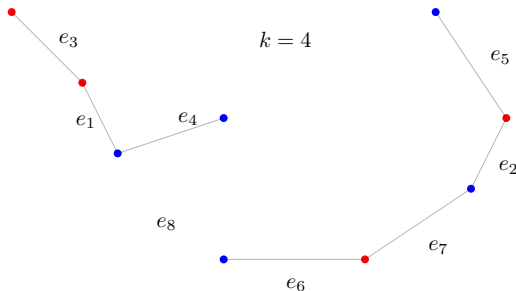


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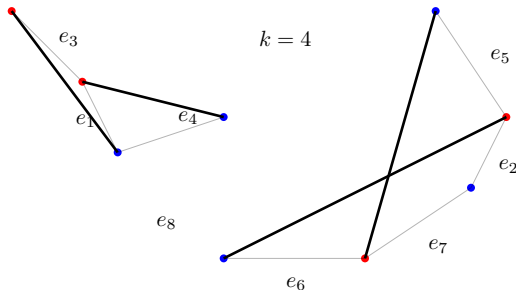


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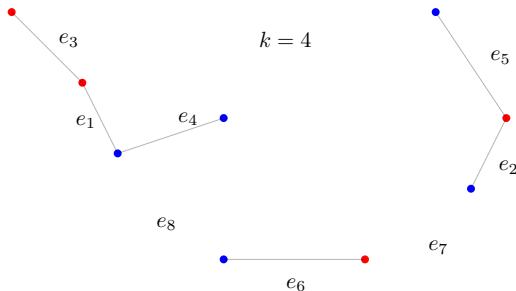


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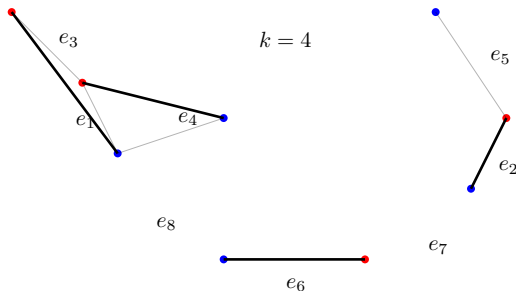


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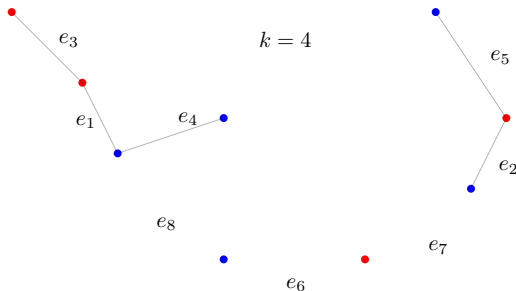
# Cost-scaling for geometric partial matching



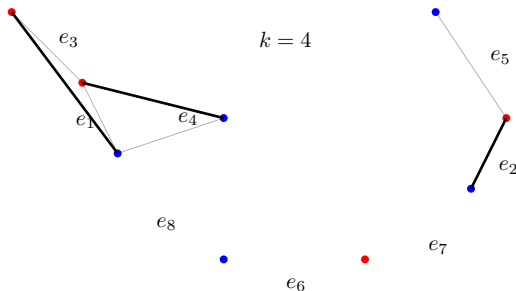
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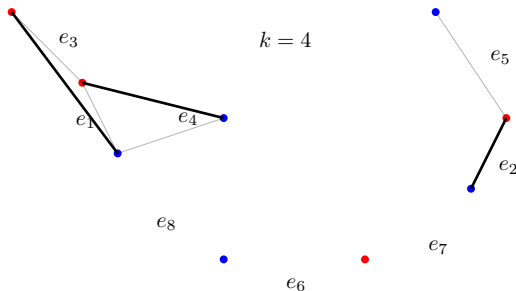
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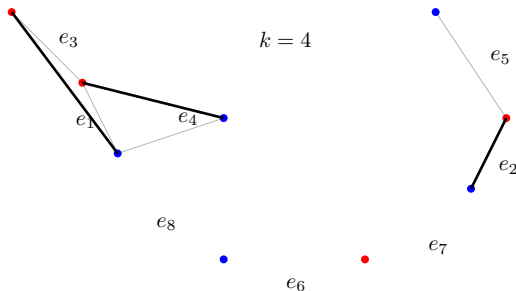
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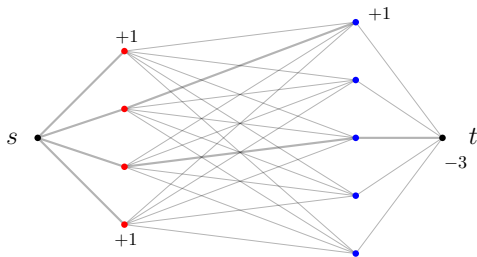
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## ► Inside Refine:

1. Hungarian search: raise potentials until an excess-deficit admissible path exists.
2. Augment by an admissible **blocking flow**.

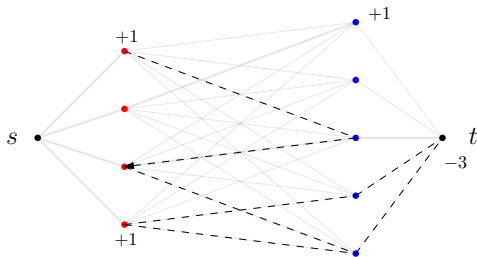


►  $O(\sqrt{k})$  blocking flows before  $f$  becomes a circulation.



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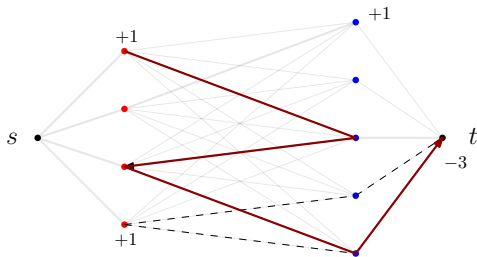


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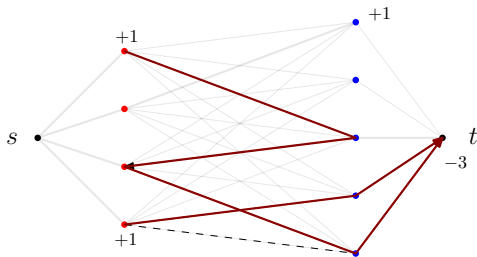


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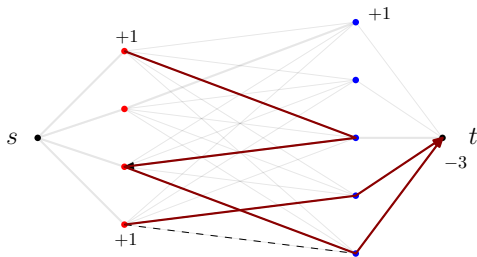


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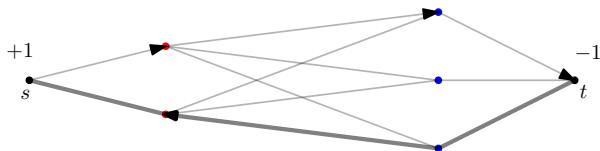
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- After  $O(n \text{ polylog } n)$ -time preprocessing, perform Hungarian search and find each blocking flow in  $O(k \text{ polylog } n)$  time.



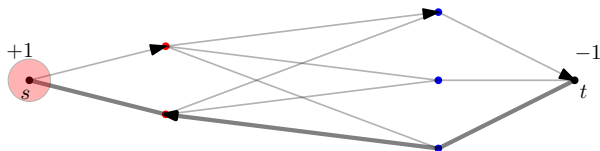
$X$ : admissible reachable from an excess node



- ▶ Dynamic 2D BCP with  $O(\text{polylog } n)$  update time,  $O(\log^2 n)$  query time (Kaplan *et al.* SODA'17)
- ▶ Possible to batch potential updates (Vaidya) — only need to bound number of relaxations.



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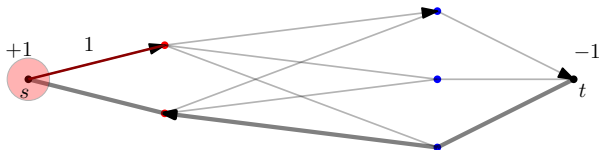


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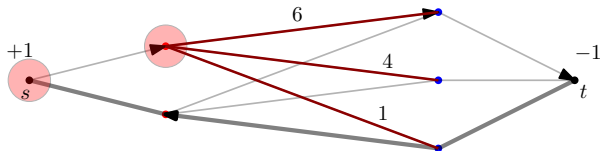
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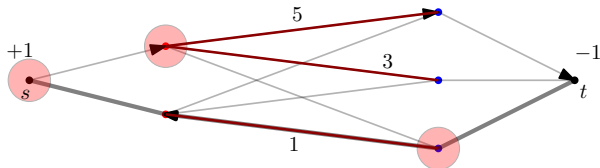
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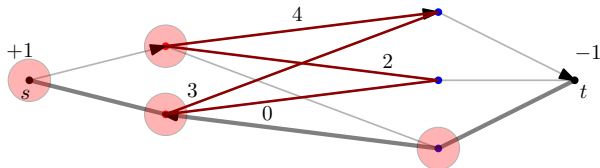
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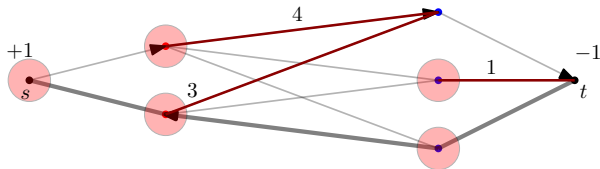
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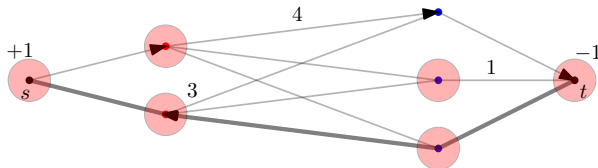
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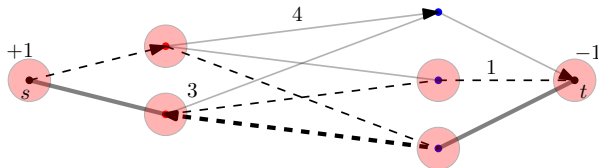
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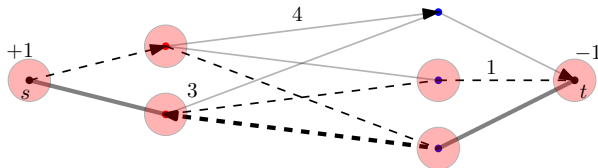
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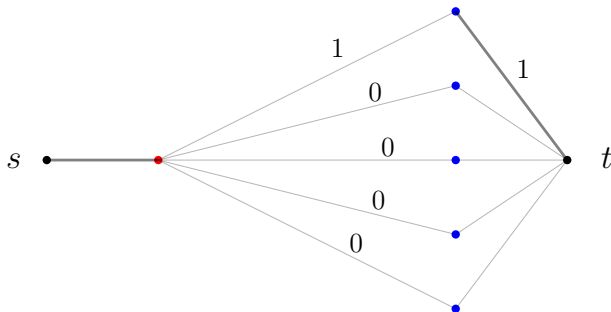
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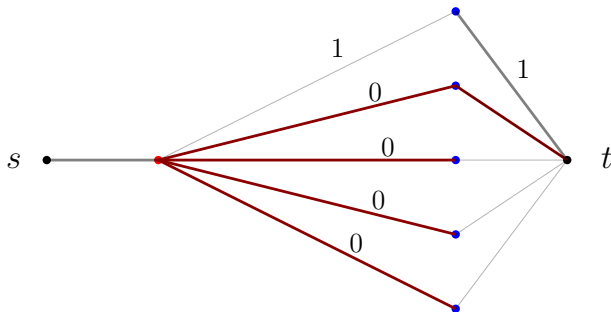
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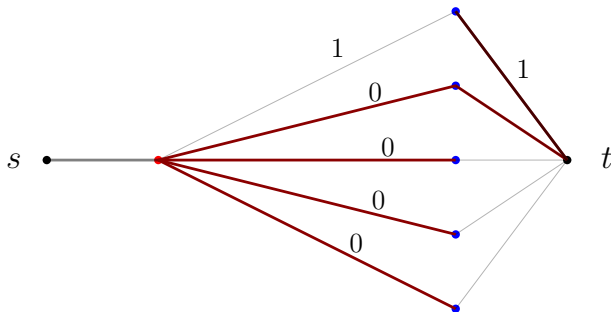
# Problem: Hungarian search looks at too many nodes



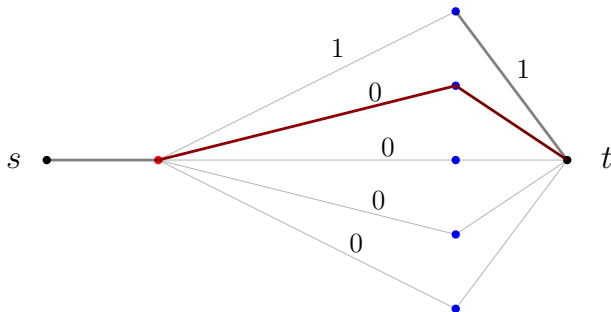
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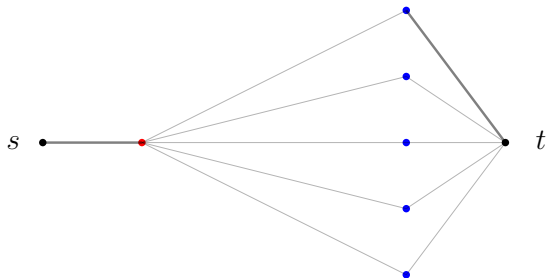


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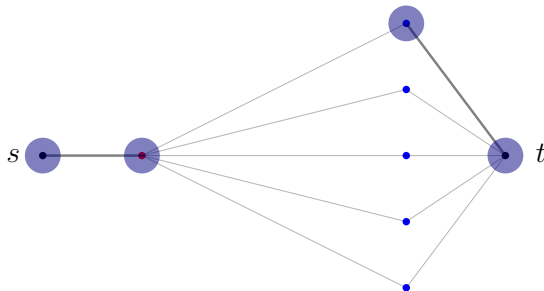
- ▶ **Alive nodes**: nonzero excess/deficit, or adjoining flow support arcs.
- ▶ **Dead nodes**: ones which aren't alive.



- ▶ **Alive path**: residual path between two alive nodes with no other alive nodes in between.



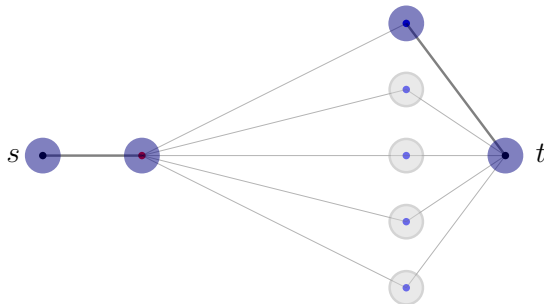
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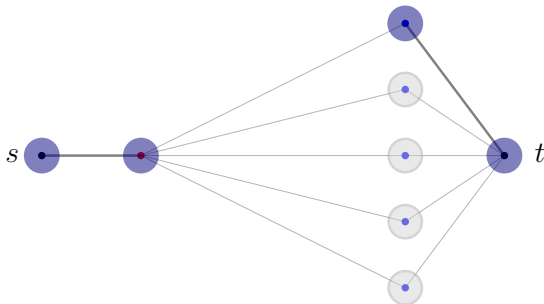
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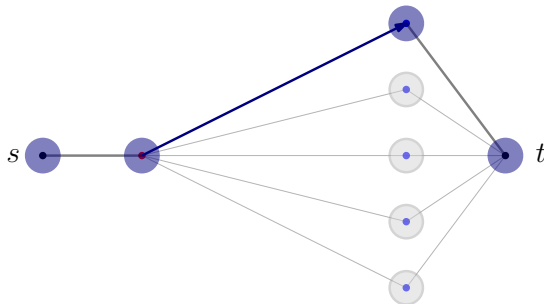


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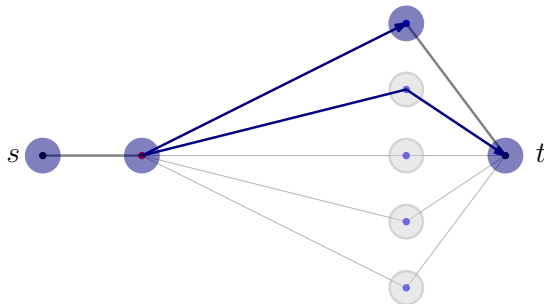
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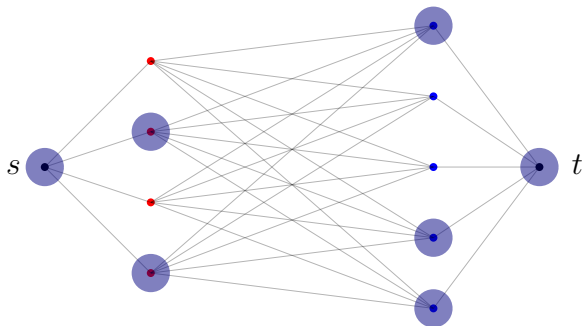


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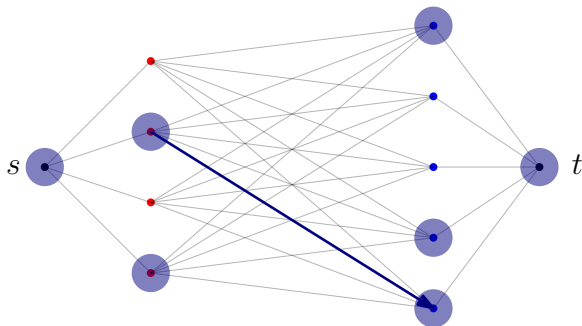
- ▶ Alive paths have length 1, 2, or 3.



- ▶ Telescoping:  $c_\pi(s \rightarrow a \rightarrow b) = c(a \rightarrow b) - \pi(s) + \pi(b)$  (use BCP)
- ▶ Only  $O(k)$  relaxations per Hungarian search.
- ▶ Also: find a blocking flow in  $O(k)$  relaxations (DFS).



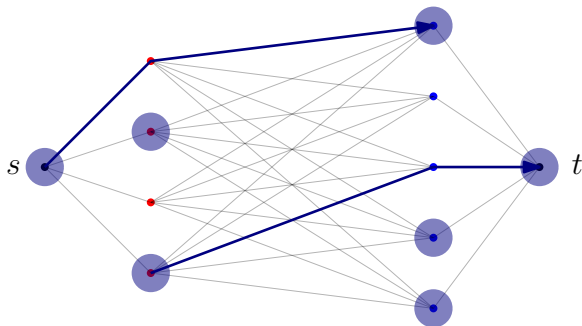
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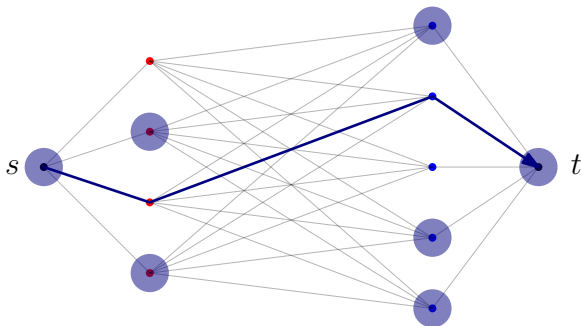


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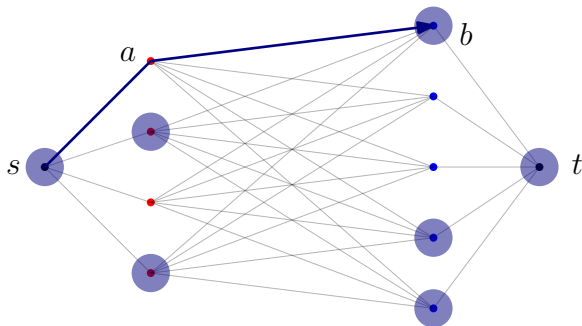
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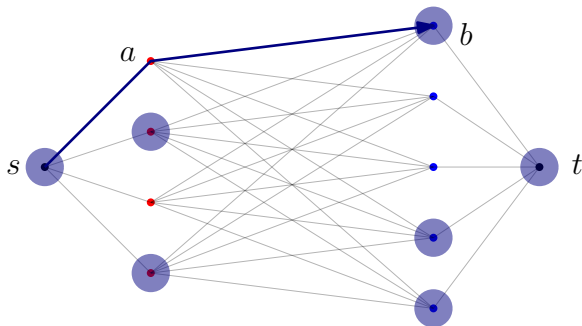
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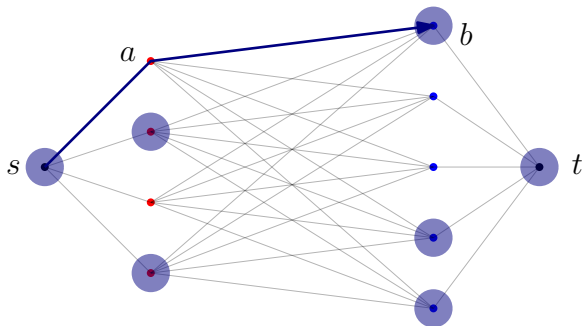
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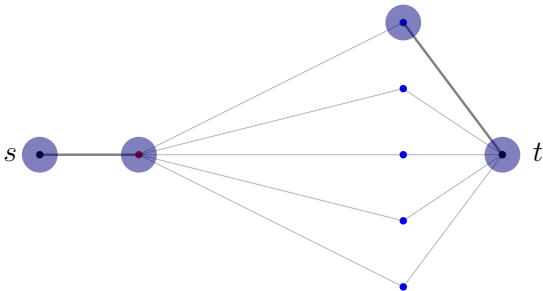
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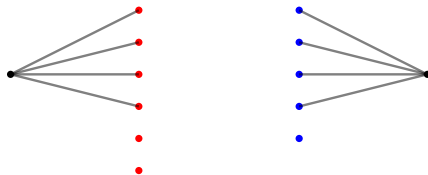
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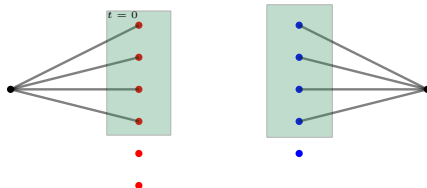
- Dynamic 2D BCP with  $O(\text{polylog } n)$  update time,  $O(\log^2 n)$  query time (Kaplan *et al.* SODA'17)



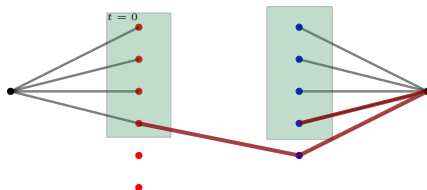
- Some BCP may begin a Hungarian search with  $\Theta(n)$  vertices.
- Can't afford to construct from scratch for every Hungarian search.



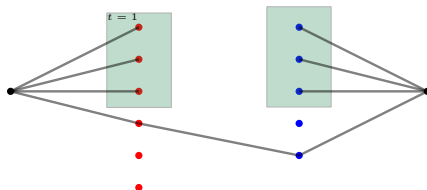
- ▶ Let  $X_t$  be the set  $X$  at the start of the  $t$ -th Hungarian search.
- ▶  $X_t$  and  $X_{t+1}$  differ by only a few nodes (newly matched, dead/alive changes).
- ▶ To generate  $X_{t+1}$ :
  1. **Rewind** the BCP updates of the last Hungarian search to obtain  $X_t$ .
  2. Apply the few changes (newly matched, dead/alive).
- ▶ Persistence?



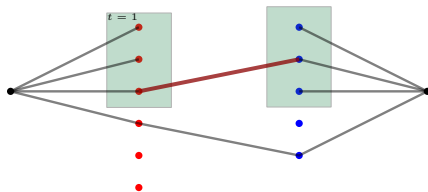
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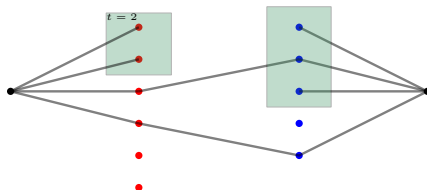
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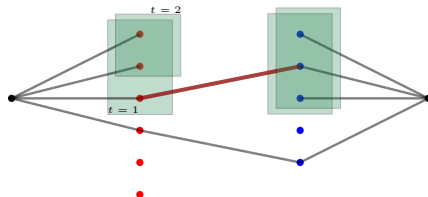


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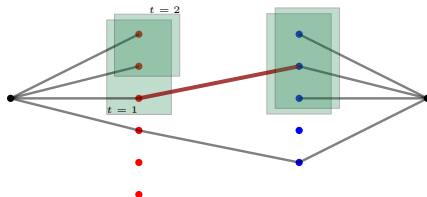


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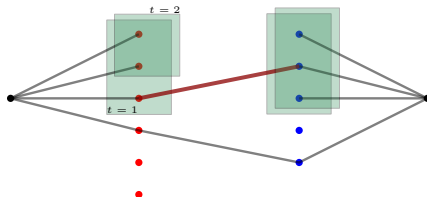




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- ▶ Each blocking flow's Hungarian search uses  $O(k)$  relaxations (**alive paths**).
- ▶ Each blocking flow's Hungarian search BCP can be initialized using the previous one in  $O(k \text{ polylog } n)$  time (**rewinding**). Need to spend  $O(n \text{ polylog } n)$  once per scale to build data structures for  $t = 0$ .
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