

## Cascade Control System of Direct Current Motor

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**Abstract:** DC motor speed can be indirectly controlled by controlling armature current. This paper describes the design cascade control system, which has two control loops: the purpose of first control loop determine the set point of armature current according to comparison between actual speed and target speed; the purpose of second control loop secure the armature current set point and so on secure the target speed. The PID controller is used in first control loop and the Generic Model Control (GMC) is used in the second control loop. The Generic Model Control (GMC) algorithm is based on the mathematical model of the Direct Current (DC) motor. The main objective of this paper illustrates how the armature current of the DC motor and so on the speed of DC motor can be controlled using Cascade control system. The performances of cascade are demonstrated using Matlab [Simulink] software under various operation conditions.

**Key words:** Cascade control . PID control . generic model control . DC motor modeling

### INTRODUCTION

DC motors are the important machine in the most control systems such as electrical systems in homes, vehicles, trains and process control [1]. It is well known that the mathematical model is very crucial for a control system design [2]. For a DC motor, there are many models to represent the machine behavior with a good accuracy. However, the parameters of the model are also important because the mathematical model cannot provide a correct behavior without correct parameters in the model [3, 4]. In this paper study the design of Generic Model controller and their application to an industrial DC motor at steps included Structure, characteristic and the mathematical model and simulation of stability response for speed control of DC motor. The design & simulation studies used to demonstrate the basic theoretical feasibility of the system used by Generic Model Control to achieve better response by less noise and less overshoot.

**DC motor model:** DC machines are characterized by their versatility. By means of various combinations of shunt, series and separately-excited field windings they can be designed to display a wide variety of volt ampere or speed-torque characteristics for both dynamic and steady-state operation. Because of the ease with which they can be controlled systems of DC machines have been frequently used in many applications requiring a wide range of motor speeds and a precise output motor control.

In this paper, the DC motor model is chosen according to his good electrical and mechanical performances. The DC motor is driven by applied voltage. Figure 1 show the equivalent circuit of DC motor with separate excitation. The characteristic equations of the DC motor are represented as [5, 6]:

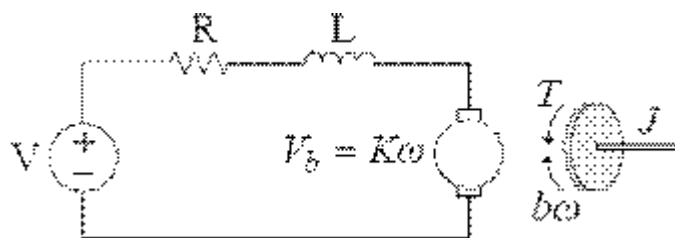


Fig. 1: Equivalent circuit of dc motor

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$$\frac{di_a}{dt} = -\frac{R_a}{L_a}i_a - \frac{K_b}{L_a}\omega_m + \frac{1}{L_a}V_a \quad (1)$$

$$\frac{d\omega_m}{dt} = \frac{K_t}{J}i_a - \frac{B}{J}\omega_m - \frac{1}{J}T_L \quad (2)$$

where  $V_a$  is the voltage applied to the motor,  $L_a$  is the motor inductance,  $i_a$  the current through the motor windings,  $R_a$  the motor winding resistance,  $K_b$  the motor's back electromagnetic force constant,  $\omega_m$  the rotor's angular velocity,  $J$  the rotor's moment of inertia,  $K_t$  the motor's torque constant,  $B$  the motor's viscous friction constant and  $T_L$  the torque applied to the rotor by an external load.

**Design of PID controller:** One of the most common controlling methods in the market is the PID controller. Application of the PID controller involves choosing the KP, KI and KD that provide satisfactory closed-loop performance. These parameters must be selected so that the characteristics: response speed, settling time and proper overshoot rate, all of which guarantee the system stability, would be satisfied. The main method for this purpose is based on trial and error, which is time consuming. There are different processes for different composition of proportional, integral and differential. The duty of control engineering is to adjust the coefficients of gain to attain the error reduction and dynamic responses simultaneously. The transfer function of PID controller is defined as follows [7, 8]:

$$G_{PID} = K_p + \frac{K_i}{s} + K_d s \quad (3)$$

PID control is a linear control methodology with a very simple control structure. In paper this type of controller operates directly on the error signal of motor speed, which is the difference between the desired motor speed and the actual motor speed and generates the desired armature current that drives the second control loop. In the design of PID controller the amount of KI is identified to reach to an intended error in steady state. In PID controller design, KP, KI and KD, related to the closed loop feedback system within the least time is determined and requires a long range of trial and error. As shown in Fig. 2, PID controllers have three basic terms: proportional action, in which the actuation signal is proportional to the error signal, integral action, where the actuation signal is proportional to the time integral of the error signal and derivative action, where the actuation signal is proportional to time derivative of error signal.

To design a particular control loop, the values of the three parameters (KP, KI and KD) have to be adjusted so that the control input provides acceptable performance from the plant. These three parameters have been included in a chromosome as shown in Fig. 2 to be optimized in the optimization procedure. In order to get an acceptable solution, there are several controller design methods that can be applied. For example, classical control methods in the frequency domain or automatic methods like Ziegler-Nichols, known as PID tuning methodology. Although these methods provide a first approximation, the response produced usually needs further manual retuning by the designer before implementation.

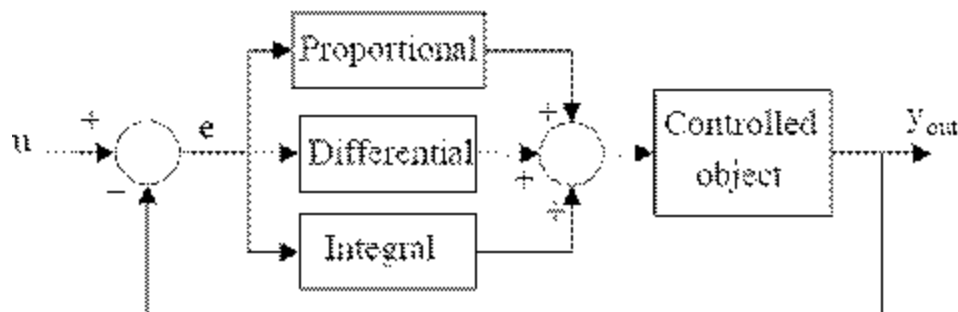


Fig. 2: Closed loop PID controlled system

**Design of generic model controller:** MC is an advanced model-based control strategy which uses linear/nonlinear models of a system to compute the control action. Since the GMC can directly use nonlinear models of a process to determine a control action, the nonlinear models do not need linearization. Generally the GMC controller design is based on the following nonlinear state space [9]:

$$\frac{dx}{dt} = \dot{x} = f(x) + g(x)u \quad (4)$$

$$y = h(x) \quad (5)$$

where  $f(x)$  and  $g(x)$  are vector fields, i.e. they are vector valued functions of a vector and  $h(x)$  is a scalar field, i.e. a scalar valued function of a vector,  $x$ . Generic Model Control (GMC) uses a model of the process in formulating the control law. However, rather than adopting a classical approach of comparing the trajectory of the process output against a desired trajectory, GMC defines the performance objective in terms of the time derivatives of the process output, i.e. minimizing the difference between the desired derivative of the process output and the actual derivative. Good control performance will be given by choose the following desired trajectory.

$$\dot{y}_d = \alpha_1(w - y) + \int \alpha_2(w - y)dt \quad (6)$$

where  $\alpha_1$  and  $\alpha_2$  are design constants and  $w$  is the set point. In order to design a controller so that the system follows the trajectory defined by the above equation as closely as possible, the following performance index is specified:

$$J = \int_0^T e^2 dt = \int (\dot{y}_d - \dot{y})^2 dt \quad (7)$$

i.e. minimize the error squared over a specified time horizon. In order to obtain  $\dot{y}$  from equation (1) the chain rule must be used,

$$\frac{dy}{dt} = \frac{dy}{dx} \frac{dx}{dt} \quad (8)$$

Where

$$\frac{dy}{dx} \quad \text{is a row vector}$$

$$\frac{dx}{dt} \quad \text{is a column vector}$$

Therefore;

$$\frac{dh}{dx} \quad \text{is also a row vector}$$

and

$$\frac{dy}{dt} = \frac{dh}{dx} f(x) + \frac{dh}{dx} g(x)u \quad (9)$$

Using equation (9) the performance index, equation (7), is minimized when  $e = 0$ , i.e.

$$\alpha_1(w - y) + \int \alpha_2(w - y)dt - \left[ \frac{dh}{dx} f(x) + \frac{dh}{dx} g(x)u \right] = 0 \quad (10)$$

**Generic model control design for DC motor:** The choice of a controller based on Generic Model Control for the dynamic model of the DC motor is highly non-linear and the GMC allows including a non-linear model in the control algorithm. The control objective is to regulate motor speed using the voltage input

The dynamic equations (1-2) of the DC motor are evidently highly non-linear due to the inter-relationships of the states variables. The differential equations can be rearranged into the generic vector representation of the process with the following vector and scalar fields:

State Vector:

$$x = \begin{bmatrix} i_a \\ \omega_m \end{bmatrix} \quad (11)$$

Vector fields:

$$f(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \end{bmatrix} \quad (12)$$

$$f_1(x) = -\frac{R_a}{L_a} i_a - \frac{K_b}{L_a} \omega_m \quad (12a)$$

$$f_2(x) = \frac{K_t}{J} i_a - \frac{B}{J} \omega_m - \frac{1}{J} T_L \quad (12b)$$

g vector fields:

$$g(x) = (g_1(x), g_2(x))^T = \begin{bmatrix} 1/L_a \\ 0 \end{bmatrix} \quad (13)$$

The h scalar field describing the output function is simply the state itself,

$$y = h(x) = i_a \quad (14)$$

With  $y = h(x) = i_a$  therefore,

$$\frac{dh(x)}{dx_1} = \frac{dh(x)}{di_a} = 1, \quad \frac{dh(x)}{dx_2} = \frac{dh(x)}{d\omega_m} = 0$$

$$\therefore \frac{dh(x)}{dx} = [1, 0]$$

$$\frac{dy}{dt} = \frac{dh(x)}{dx} f(x) + \frac{dh(x)}{dx} g(x)u$$

$$\frac{dy}{dt} = \dot{y} = f_1(x) + g_1(x)u$$

In other words, the GMC controller is given by:

$$\alpha_1(w - y) + \int \alpha_2(w - y)dt - \left[ \frac{dh}{dx} f(x) + \frac{dh}{dx} g(x)u \right] = 0$$

And solving for the manipulated variable ( $V_a$ ), yields,

$$\alpha_1(i_{a_{set}} - i_m) + \int \alpha_2(i_{a_{set}} - i_m)dt - \left[ \frac{R_a}{L_a} i_a + \frac{K_b}{L_a} \omega_m + \frac{V_a}{L_a} \right] = 0$$

$$\alpha_1(i_{a_{set}} - i_m) + \int \alpha_2(i_{a_{set}} - i_m)dt + \frac{R_a}{L_a} i_a + \frac{K_b}{L_a} \omega_m - \frac{V_a}{L_a} = 0$$

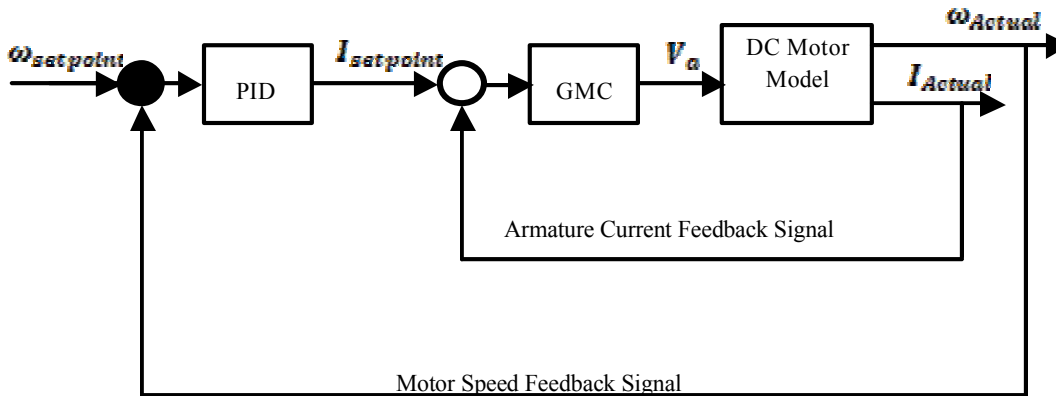


Fig. 3: Matlab/simulink block diagram of DC motor and proposed cascade control system

$$\therefore V_a = L_a \alpha_1 (i_{set} - i_m) + L_a \int \alpha_2 (i_{set} - i_m) dt + R_a i_a + K_b \omega_m \quad (15)$$

**Numerical example:** In order to examine the effectiveness of the developed control strategies, simulation studies were conducted using the model described above for the behavior of the DC motor. The DC motor parameters considered in the simulation are [10]:

$$R_a = 2.3 \text{ ohm}, L_a = 0.03H, J = 0.045 \text{ kg.m}^2,$$

$$B = 0.007 \text{ Nm/(rad/sec)}, K_b = 0.64 \text{ V/(rad/sec)}, K_t = 0.62 \text{ Nm/A}$$

The DC motor model and two control systems are expressed using MATLAB-SIMULINK program. Two examples are presented to evaluate effectiveness of the proposed control strategies. Ziegler-Nichols closed loop method based on ultimate gain and ultimate period is used to calculate the controller parameters. The parameters values of the GMC controller are  $\alpha_1 = 0.5$  and  $\alpha_2 = 0.5$  and the parameters of PID controller are  $K_p = 0.5$ ,  $K_i = 0.05$  and  $K_D = 0.05$ . The control structure in Matlab/Simulink is illustrated in Fig. 3.

## RESULTS AND DISCUSSION

Analysis of DC motor without load ( $T_L = 0 \text{ Nm}$ )

In the state-space model of DC motor, given above, the load torque is taken as  $T_L = 0.0$  (without load), and the set point value of the motor speed is setting at  $\omega_{set} = 330 \text{ rad/sec}$ . PID controller calculates the target value of armature current which in turn; secure through GMC controller by adjusting the armature voltage (manipulated variable). The dynamic responses of armature voltage, armature current, motor speed and motor torque with proposed control system are shown in Fig. 4.

**Analysis of DC motor with load ( $T_L = 30 \text{ Nm}$ ):** In the state-space model of DC motor, the load torque is taken as  $T_L = 30 \text{ Nm}$  in the same time the set point value of motor speed is setting at  $165 \text{ rad/sec}$ . The dynamic responses of armature current, armature voltage, motor speed and torque with control system are shown in Fig. 5.

**Analysis of DC motor with load ( $T_L = 12 \text{ Nm}$ ):** In the state-space model of DC motor, the load torque is taken as  $T_L = 12 \text{ Nm}$  in the same time the set point value of motor speed is setting at  $\omega_{setpoint} = 260 \text{ rad/sec}$ . The dynamic responses of armature current, armature voltage, motor speed and torque with control system are shown in Fig. 6.

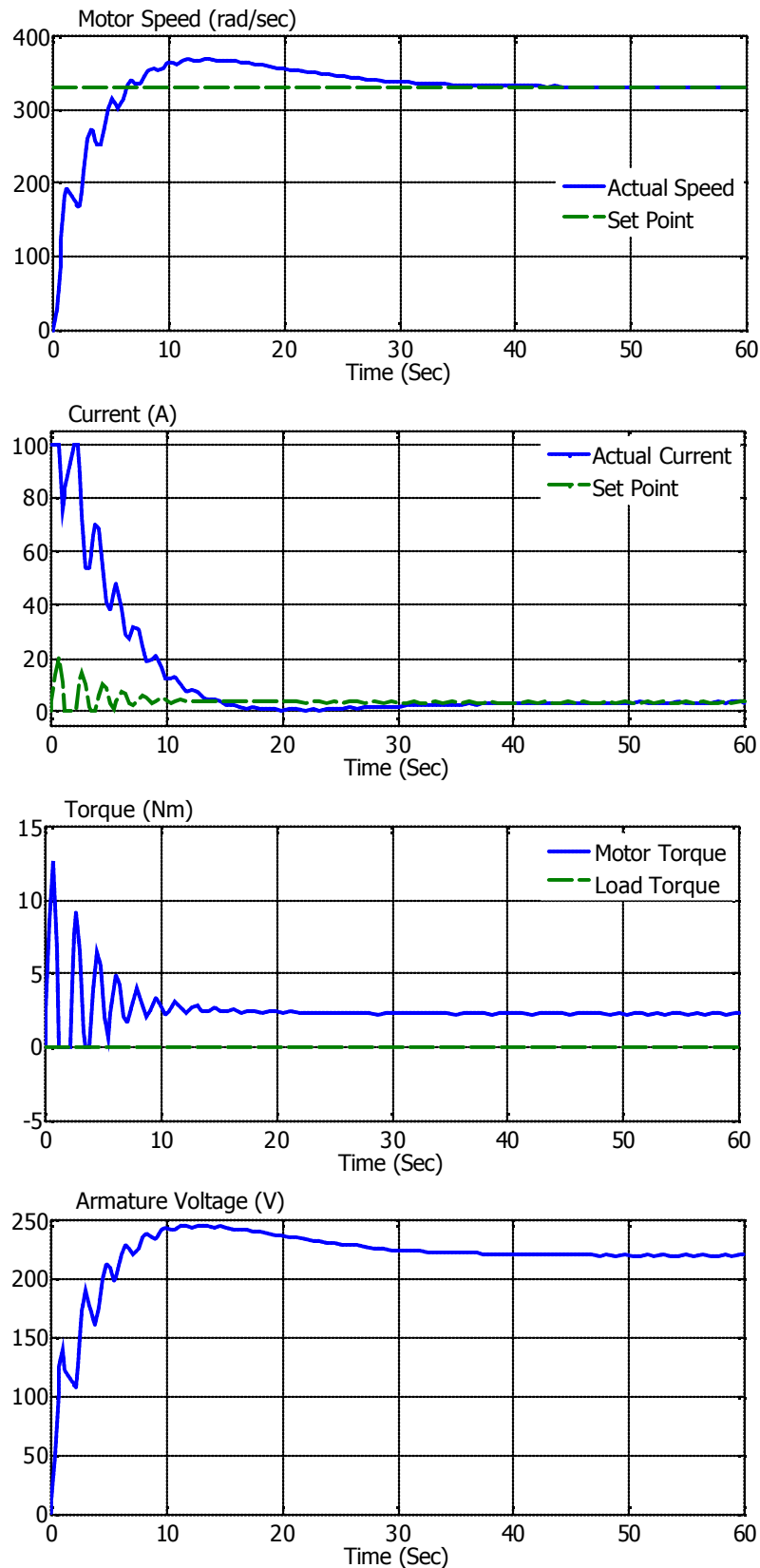


Fig. 4: Dynamic responses of motor speed, armature Current, torque and armature voltage, with the proposed cascade control system ( $T_L = 0$  Nm,  $\omega_{set} = 330$  rad/sec)

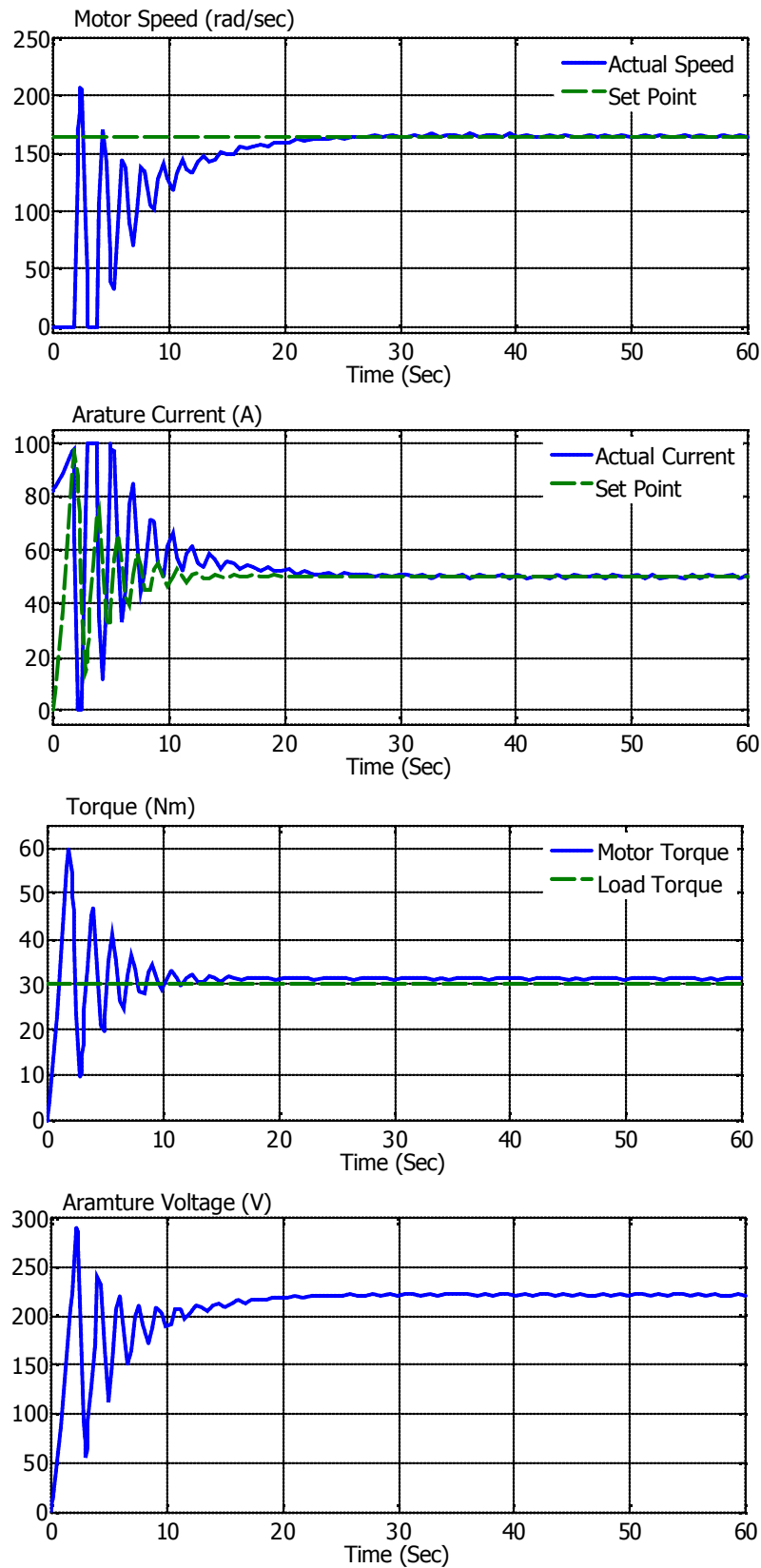


Fig. 5: Dynamic responses of armature current, motor speed, torque and armature voltage, with the proposed cascade control system ( $T_L = 30 \text{ Nm}$ ,  $\omega_{\text{set}} = 165 \text{ rad/sec}$ )

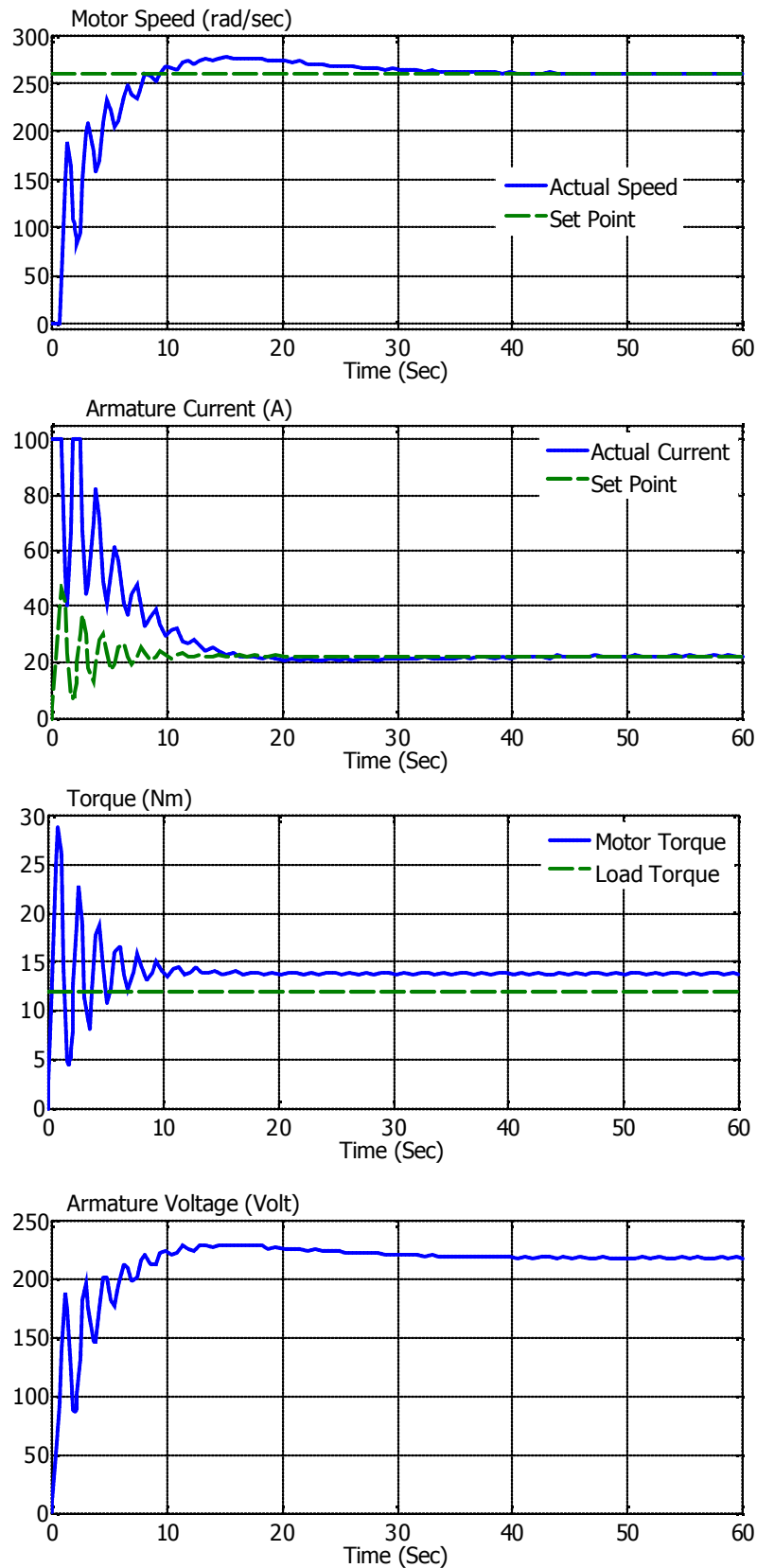


Fig. 6: Dynamic responses of armature current, motor speed, torque and armature voltage, with the proposed cascade control system ( $T_L = 12 \text{ Nm}$ ,  $\omega_{\text{set}} = 260 \text{ rad/sec}$ )



## **CONCLUSION**

The proposed cascade control system is designed and implemented using Matlab/Simulink program to track the motor speed and determine the armature current set point under loading and without load. The simulation results show that the proposed cascade controller has excellent performance. Simulation results indicate that the proposed cascade controller has a better performance in transient, overshoot and steady state response.

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