NAME: ....

MATH E-156 MATHEMATICAL STATISTICS

ASSIGNMENT 6

1. The joint probability mass function (joint pmf) of two discrete random variables, X and Y, is given in the following table:

	$X = 1$	$X = 2$	$X = 3$
$Y = 1$	.02	.08	.30
$Y = 2$	.04	.32	.04
$Y = 3$	.10	.08	.02

   (a) Find the covariance $\text{Cov}(X, Y)$.

   (b) Find the correlation $\rho_{X,Y}$.

2. Suppose X_1, X_2 are *independent* random variables with means $E(X_1) = \mu_1$, $E(X_2) = \mu_2$ and standard deviations $SD(X_1) = \sigma_1$, $SD(X_2) = \sigma_2$. Find

   (a) $E(4X_1 - 3X_2 + 1)$

   (b) $Var(4X_1 - 3X_2 + 1)$

   (c) $\text{Cov}(X_1 + X_2, X_1 - X_2)$

3. Let $X \sim \text{Beta}(a_1, b_1)$ for some $a_1, b_1 > 0$. Also, let $Y \sim \text{Beta}(a_2, b_2)$ for some $a_2, b_2 > 0$. If the correlation $\rho_{X,Y} = -0.6$, find
   (a) $Cov(X, Y)$

   (b) $E(XY)$

4. Let X and Y be two random variables. Assuming correlations exist, prove that for any constants a, b, c, and d:

$$|\rho_{U,V}| = |\rho_{X,Y}|,$$

   where $U = a + bX$ and $V = c + dY$.

5. A factory has two *independent* production lines. Daily production for line 1 is a random variable X_1 with mean 1500 and standard deviation 100. Daily production for line 2 is a random variable X_2 with mean 2000 and standard deviation 130. What are the mean and standard deviation of the total daily production $T = X_1 + X_2$?

6. A large population has the mean μ and the standard deviation σ. We take a simple random sample X_1, X_2, X_3 of $n = 3$ independent measurements from this population. For the sample mean,

$$\bar{X} \doteq \frac{1}{3}\left(X_1 + X_2 + X_3\right),$$

find

(a) $E(\bar{X})$

(b) $Var(\bar{X})$

(c) $SD(\bar{X})$

7. Assume that a random square has a side length that takes value 1 with probability p and value 2 with probability $1 - p$.

(a) Find the correlation between the side length and the perimeter of the square.

(b) Find the correlation between the side length and the area of the square.

8. Two players play n independent rounds of a game, and the 1st player has probability p of winning each round. Let X and Y denote the numbers of games won by 1st and 2nd player, respectively. Find

   (a) $E[X]$, $\mathrm{SD}(X)$ and $E[Y]$, $\mathrm{SD}(Y)$

   (b) $\mathrm{Cov}(X, Y)$

   (c) $\rho_{X,Y}$