**LGIC 010 & PHIL 005**
**Problem Set 3**
**Spring Term, 2021**
**DUE FRIDAY, MARCH 26 at 11:59 p.m. EDT**

1. Let $S_1$ be the schema

$$(\forall x)\neg Lxx \wedge (\forall x)(\forall y)(Lxy \supset Lyx) \wedge (\forall x)(\forall y)(Lxy \supset (Fx \oplus Fy))$$

(a) (5 points) Specify a structure $A_1$ which is a member of $\mathsf{mod}(S_2, 5)$.

$U^{A_1} =$

$L^{A_1} =$

$F^{A_1} =$

(b) (10 points) What is the value of $|\mathsf{mod}(S_1, 5)|$?

(c) (10 points) Let $T_1$ be the conjunction of $S_1$ and the schema

$$(\forall x)(\exists y)(\forall z)(Lxz \equiv z = y)$$

What is the value of $|\mathsf{mod}(T_1, 5)|$?

2. Let $S_2$ be the schema

$$(\forall x)(\exists y)(\forall z)(Lxz \equiv z = y) \wedge (\forall x)(\forall y)(Lxy \supset Lyy).$$

(a) (5 points) Specify a structure $A_2$ which is a member of $\mathsf{mod}(S_2, 6)$.

$U^{A_2} =$

$L^{A_2} =$

(b) (10 points) What is the value of $|\mathsf{mod}(S_2, 6)|$?

(c) (10 points) Let $T_2$ be the conjunction of $S_2$ and the schema.

$$(\forall y)(\exists x)Lxy$$

What is the value of $|\mathsf{mod}(T_2, 6)|$?

3. Let $S_3$ be the conjunction of the following schemata

$$(\forall x)(\forall y)(Lxy \supset \neg Lyx) \wedge (\forall x)(\forall y)(x \neq y \supset (Lxy \vee Lyx))$$

$$(\exists x)(\exists y)(\exists z)(Lxy \wedge Lyz \wedge \neg Lxz)$$

(a) (5 points) Specify a structure $A_3$ which is a member of $\mathsf{mod}(S_3, 6)$.

$U^{A_3} =$

$L^{A_3} =$

(b) (10 points) What is the value of $|\mathsf{mod}(S_3, 6)|$?

(c) (10 points) Let $T_3$ be the conjunction of $S_3$ and the schema

$$(\exists x)(\forall y)(y \neq x \supset Lyx)$$

What is the value of $|\mathsf{mod}(T_3, 6)|$?

4. Let $S_4$ be the conjunction of the following schemata

$$(\forall x)(\forall y)(Lxy \supset \neg Lyx) \wedge (\forall x)(\forall y)(\forall z)((Lxz \wedge Lyz) \supset x = y) \wedge (\exists x)(\forall y)((\forall z)\neg Lzy \equiv y = x)$$

$$(\forall x)((\forall y)\neg Lxy \vee (\exists v)(\exists w)(w \neq v \wedge (\forall z)(Lxz \equiv (z = v \vee z = w))))$$

(a) (5 points) Specify a structure $A_4$ which is a member of $\mathsf{mod}(S_4, 7)$.

$U^{A_4} =$

$L^{A_4} =$

(b) (10 points) What is the value of $|\mathsf{mod}(S_4, 7)|$?

(c) (10 points) Let $T_4(x)$ be the schema

$$(\forall y)\neg Lxy$$

What is the maximum value of $|T_4[A]|$, for $A \in \mathsf{mod}(S_4, 7)$?