Harvard University
Computer Science 20

Problem Set 4

Due Tuesday, March 2, 2021 at 11:59pm.

PROBLEM 1

Prove that there exist two distinct natural numbers m and n with $m, n \leq 2049$ such that $19^m - 19^n$ is divisible by 2019.

The truth of this claim does not depend on the exact values 19, 2019, and 2049. So, your solution should give an argument that is easy to adapt to other values for these constants rather than, say, using a computer program to find specific values for m and n.

PROBLEM 2

Determine whether the following sets are finite, countably infinite, or uncountable. Justify your answers.

(A) The set of all total functions from domain $\{0,1\}$ to co-domain $\{0,1\}$

(B) The set of all total functions from domain $\mathbb{N}$ to co-domain $\{0,1\}$

(C) The set of all total functions from domain $\{0,1\}$ to co-domain $\mathbb{N}$

PROBLEM 3

Prove by contradiction that $\sqrt{3} + \sqrt{2}$ is irrational. *Hint:* Consider $(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})$

PROBLEM 4

Recall that we saw that that the set $\mathbb{R}_{(0,1)} = \{r \in \mathbb{R} : 0 < r < 1\}$ is uncountably infinite using Cantor's diagonalization method.

(A) The Schröder-Bernstein Theorem states that for sets S and T, if there exist injective functions $f : S \to T$ and $g : T \to S$, then S and T have the same cardinality. Using the Schröder-Bernstein Theorem show that the cardinality of the set of real numbers in the closed interval $[0,1]$ is the same as the cardinality of the set of all real numbers in the open interval $(0,1)$. To receive full credit on this problem you must formally define two total injective functions $f : \mathbb{R}_{(0,1)} \to \mathbb{R}_{[0,1]}$ and $g : \mathbb{R}_{[0,1]} \to \mathbb{R}_{(0,1)}$.

(B) Using what we have proved about intervals of the real number line, prove that there are at least a countably infinite number of uncountably infinite sets.

Problem set by **FILL IN YOUR NAME HERE**

Collaboration Statement: **FILL IN YOUR COLLABORATION STATEMENT HERE (See the syllabus for information)**