1. Let A and B be two events with $P(A) = 0.5$ and $P(B) = 0.3$. In each case below find the probability of both events occurring together, $P(A \cap B)$.

   (a) Assume $P(A|B) = 0.2$.

   (b) Assume A, B are independent.

   (c) Assume A, B are disjoint (i.e. mutually exclusive).

2. A box contains three coins. One has two heads, one has two tails, and the other is a fair coin with one head and one tail. A coin is chosen at random, is flipped, and comes up heads.

   (a) What is the probability that the coin chosen is the two-headed coin?

   (b) What is the probability that if it is thrown another time it will come up heads again?

3. Twenty items are submitted for acceptance. It is known that there are 3 defective items in the lot. Assume we choose a sample of size 5 from those twenty items without replacement.

   (a) How many ways can we do so?

   (b) What is the probability of finding exactly 2 defective items in the sample?

   (c) Find the expected number of defective items in the sample.

4. There are n people eligible to vote in a certain election. Voting requires registration. Decisions are made independently. Each of the n people will register with probability p_1. Given that a person registers, he or she will vote with probability p_2. Given that a person votes, he or she will vote for for the 1st candidate with probability p_3. Let X denote the number of votes for the 1st candidate.

   (a) What is the probability mass function of X? Please make sure to specify what values X takes.

   (b) Find $E[X]$.

   (c) Find $\mathrm{SD}(X)$.

5. Suppose that the lifetime X of an electronic component follows an exponential distribution with $\lambda = 0.1$.

   (a) Compute the probability that the lifetime is less than 10.

   (b) Find the expectated value $E[X]$ of the lifetime.

   (c) What is the standard deviation $\mathrm{SD}(X)$ of the lifetime?

6. Consider the following probability mass function (pmf):

k	2	4
$p(k)$	.8	.2

(a) Assume that X comes from the above distribution, i.e. $P(X = k) = p(k)$ for all k. Find the expected value $E[X]$ and standard deviation $\text{SD}(X)$.

(b) Assume now that X_1 and X_2 are independent and both come from the above distribution. Give the sampling distribution of $\bar{X} = \frac{1}{2}(X_1 + X_2)$, that is, list all values $\bar{X}$ can take with corresponding probabilities.

(c) Using the distribution you obtained in (b), find $E[\bar{X}]$. Compare $E[\bar{X}]$ and $E[X]$. Are they equal?

(d) Using the distribution you obtained in (b), find $\text{SD}(\bar{X})$. Compare $\text{SD}(\bar{X})$ and $\text{SD}(X)$. Are they equal? What about $\text{SD}(\bar{X})$ and $\text{SD}(X)/\sqrt{n}$, where $n = 2$?

7. Let R be a continuous random variable with the following probability density function (pdf):

$$f_R(r) = \begin{cases} 12r^2(1-r), & \text{if } 0 < r < 1, \\ 0, & \text{otherwise.} \end{cases}$$

Let S denote the area of the disk of radius R.

(a) Find $P(0 < R < 0.5)$.

(b) Find the probability density function (pdf) of the area S.

(c) Find $P(0 < S < \frac{\pi}{4})$.

8. Suppose that, conditional on N, X has a binomial distribution with N trials and probability p of success, and that N is a Poisson random variable with parameter λ.

   (a) Find the joint probability mass function (joint pmf) $P(X = k, N = n)$ of X and N. Make sure to explicitly specify the support.

   (b) Find the marginal probability mass function (marginal pmf) $P(X = k)$ of X. Please indicate what values X takes.

   (c) Find $E[X]$.