Name: ....
MATH E-156 Mathematical Statistics
Assignment 8

1. Carefully state and then prove the Law of Large Numbers (LLN).

2. Let X_k be random variables with $E[X_k] = \mu$ and $\mathrm{Var}(X_k) = \sigma^2$ for $k = 1, 2, 3, \ldots$ If $Y_k = \frac{X_k - \mu}{\sigma}$ for all k, find
   (a) $E[Y_k]$

   (b) $Var(Y_k)$

   (c) $E[Y_k^2]$

3. In <u>each</u> case below, specify a random variable (i.e. distribution) to which the sequence of random variables $W_1, W_2, W_3, \ldots$ converges in distribution. Justify your answer.

   (a) $W_n \sim \text{Unif}[0, 1 + \frac{1}{n}]$

   (b) $W_n \sim \text{Normal}\,(0, (1/\sqrt{n})^2)$

4. The Central Limit Theorem (CLT) can be used to analyze round-off error. Suppose that the round-off error is represented as a uniform random variable on $[-1/2, 1/2]$. If 100 numbers are averaged, approximate the probability that the round-off error exceeds 0.05 in absolute value. Please use the cumulative Normal distribution table or software to evaluate $\Phi(x)$.

5. Let $X_1 \sim \text{Gamma}(\alpha_1, \lambda)$ and $X_2 \sim \text{Gamma}(\alpha_2, \lambda)$ be independent. Use moment generating functions to show that $X_1 + X_2 \sim \text{Gamma}(\alpha_1 + \alpha_2, \lambda)$.

6. Prove that $X \sim F_{m,n}$ implies $X^{-1} \sim F_{n,m}$.

7. Show that the Cauchy distribution and t_1 (i.e. t distribution with 1 degree of freedom) are the same.

8. Prove that $T \sim t_n$ implies $T^2 \sim F_{1,n}$.