LGIC 010 & PHIL 005
Problem Set 4
Spring Term, 2021
DUE FRIDAY, APRIL 9 at 11:59 p.m. EDT

1. Let S_1 be the schema $(\forall x)(f(f(f(x))) = x$.

   (a) (15 points) What is the value of $|\mathbf{mod}(S_1, 6)|$?

   (b) (15 points) What is the value of $|\mathbf{iso}(S_1, 6)|$?

2. Let S_2 be the schema

$$(\forall x)\neg Lxx \wedge (\forall x)(\forall y)(Lxy \supset Lyx).$$

(a) (15 points) Suppose $|\mathbf{aut}(A)| = 24$. What is the value of $|\{\mathbb{I}(A) \mid A \in \mathbf{mod}(S_2, 4)\}|$?

(b) (15 points) Suppose $|\mathbf{aut}(A)| = 2$. What is the value of $|\{\mathbb{I}(A) \mid A \in \mathbf{mod}(S_2, 4)\}|$?

3. Let S_3 be a schema such that a structure A satisfies S_3 if and only if A is a 2-regular simple graph.

   (a) (15 points) List all the values of $|\mathbf{aut}(A)|$, for $A \in \mathbf{mod}(S_3, 8)$.

   (b) (15 points) Let T_3 be the conjunction of S_3 and the schema

$$\neg(\exists F)((\exists x)Fx \wedge (\exists y)\neg Fy \wedge (\forall x)(\forall y)((Fx \wedge \neg Fy) \supset \neg Lxy))$$

   What is the value of $|\mathbf{mod}(T_3, 8)|$?

4. (10 points) Let S_4 be the conjunction of the following schemata

$$(\forall x)\neg Lxx \wedge (\forall x)(\forall y)(Lxy \supset Lyx).$$

$$\neg(\exists F)((\exists x)Fx \wedge (\exists y)\neg Fy \wedge (\forall x)(\forall y)((Fx \wedge \neg Fy) \supset \neg Lxy))$$

$$\neg(\exists F)((\exists x)Fx \wedge (\forall x)(Fx \supset (\exists y)(\exists z)(Fx \wedge Fy \wedge y \neq z \wedge (\forall w)(Fw \supset (Lxw \equiv (w = y \vee w = z))))))$$

What are the maximum and minimum values of $|L^A|$ for $A \in \mathbf{mod}(S_4, n)$? Explain your answer.