Section 1I. Multiple Random Variables

Statistics for Data Science

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Two Random Variables: Bayes

Bayes' rule: A useful formula to reverse the order of the conditionals

- For discrete r.v.'s X and Y

$$p_{Y|X}(y|x) = \frac{p_{XY}(x,y)}{p_X(x)} = \frac{p_{X|Y}(x|y)\,p_Y(y)}{\sum_{y\in\mathcal{Y}} p_{X|Y}(x|y)\,p_Y(y)}$$

- For continuous r.v.'s

$$f_{Y|X}(y|x) = \frac{f_{XY}(x,y)}{f_X(x)} = \frac{f_{X|Y}(x|y)\,f_Y(y)}{\int_{-\infty}^{\infty} f_{X|Y}(x|y)\,f_Y(y)\,dy}$$

Multiple Random Variables

Given a collection of random variables $X_1, \ldots, X_n$, we can define the *random vector*

$$\mathbf{X} = \begin{bmatrix} X_1 & X_2 & \cdots & X_n \end{bmatrix}^\mathsf{T} \in \mathbb{R}^n$$

- The *mean* of the random vector is

$$\mathbb{E}[\mathbf{X}] = \mathbb{E}\begin{bmatrix} X_1 & X_2 & \cdots & X_n \end{bmatrix}^\mathsf{T} = \begin{bmatrix} \mathbb{E}[X_1] & \mathbb{E}[X_2] & \cdots & \mathbb{E}[X_n] \end{bmatrix}^\mathsf{T}$$

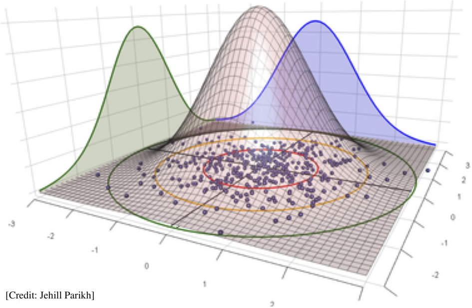
- The *covariance matrix* of the random vector is

$$\Sigma = \begin{bmatrix} \mathrm{Cov}[X_1, X_1] & \cdots & \mathrm{Cov}[X_1, X_n] \\ \vdots & \ddots & \vdots \\ \mathrm{Cov}[X_n, X_1] & \cdots & \mathrm{Cov}[X_n, X_n] \end{bmatrix}$$

Multiple Random Variables (cont.)

An important PDF is the multivariate Gaussian with mean $\mu \in \mathbb{R}^n$ and covariance $\Sigma \in \mathbb{R}^{n \times n}$, denoted as $\mathbf{X} \sim \mathcal{N}(\mu, \Sigma)$, defined as

$$f_{\mathbf{X}}(\mathbf{x}; \mu, \Sigma) = \frac{1}{(2\pi)^{n/2} \det(\Sigma)^{1/2}} \exp\left(-\frac{1}{2} (\mathbf{x} - \mu)^{\mathsf{T}} \Sigma^{-1} (\mathbf{x} - \mu)\right)$$



[Credit: Jehill Parikh]

Reading and next section:

- Reading: "*Review of Probability Theory*" by A. Maleki and T. Do
  Available online at <http://cs229.stanford.edu/section/cs229-prob.pdf>

- Next section: Introduction to Statistical Learning...