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Probability Distributions: Discrete

Introduction to Data Science Algorithms

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Poisson distribution

- We showed that the Bernoulli/binomial/categorical/multinomial are all related to each other
- Lastly, we will show something a little different
- The **Poisson** distribution gives the probability that an event will occur a certain number of times within a time interval
- Examples (all real):
  - The number of deaths in Prussian army from horse kicks (`https://goo.gl/iv1ZT1`)
  - Pattern of bombs hitting London in WW-II (`https://goo.gl/4U9XDq`)
  - The number of shark attacks per year (`https://goo.gl/TxGbck`)
- Oddly enough, randomness is clustered.

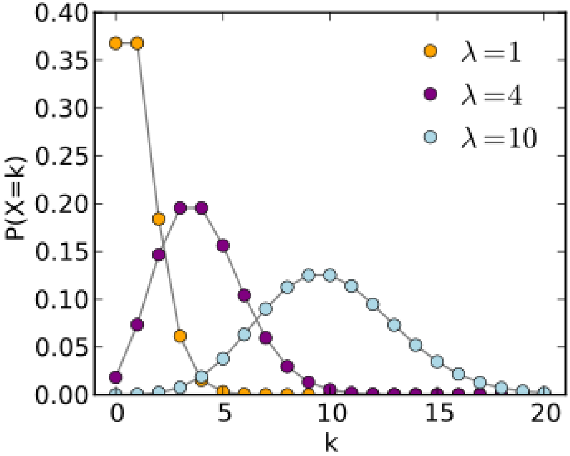
Poisson distribution

- Let the random variable X refer to the count of the number of events over whatever interval we are modeling.
  - X can be any positive integer or zero: $\{0, 1, 2, \dots\}$
- The probability mass function for the Poisson distribution is:

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

- The Poisson distribution has one parameter λ, which is the average number of events in an interval.
  - $\mathbb{E}[X] = \lambda$

Poisson distribution



- Example: Poisson is good model of World Cup match having a certain number of goals
- A World Cup match has an average of 2.5 goals scored: $\lambda = 2.5$
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  - $P(X=0) = \frac{2.5^0 e^{-2.5}}{0!} = \frac{e^{-2.5}}{1} = 0.082$
  - $P(X=1) = \frac{2.5^1 e^{-2.5}}{1!} = \frac{2.5e^{-2.5}}{1} = 0.205$
  - $P(X=2) = \frac{2.5^2 e^{-2.5}}{2!} = \frac{6.25e^{-2.5}}{2} = 0.257$
  - $P(X=3) = \frac{2.5^3 e^{-2.5}}{3!} = \frac{15.625e^{-2.5}}{6} = 0.213$
  - $P(X=4) = \frac{2.5^4 e^{-2.5}}{4!} = \frac{39.0625e^{-2.5}}{24} = 0.133$

    $\ldots$
  - $P(X=10) = \frac{2.5^{10} e^{-2.5}}{10!} = \frac{9536.7432e^{-2.5}}{3628800} = 0.00022$

    $\ldots$