Section 3A. Linear Algebra Review

Statistics for Data Science

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Basic Concepts and Notation

- We denote by $A \in \mathbb{R}^{m \times n}$ a *matrix* with m rows, n columns, where the entries are real numbers.
  - We typically use capital letters for matrices.
  - The element of A in the i-th row and j-th column is denoted by either $[A]_{ij}$ or A_{ij}
  - The *transpose* of a matrix A is denoted by $A^\intercal$, i.e., $[A^\intercal]_{ij} = [A]_{ji}$. In other words, the transpose results from converting rows of A into columns of $A^\intercal$. The transpose operation satisfies: $(A^\intercal)^\intercal = A$, $(AB)^\intercal = B^\intercal A^\intercal$, $(A + B)^\intercal = A^\intercal + B^\intercal$.

Matrix Algebra

- Given two matrices $A \in \mathbb{R}^{m \times n}$ and $B \in \mathbb{R}^{n \times p}$, the product $C = A \cdot B \in \mathbb{R}^{m \times p}$ is defined entry-wise as
$$C_{ij} = \sum_{k=1}^{n} A_{ik} B_{kj}$$
- Matrix multiplication is associative (i.e., $(AB)\,C = A\,(BC)$), distributive (i.e., $A\,(B + C) = AB + AC$), but (in general) not commutative (i.e., $AB \neq BA$)
- The identity matrix, denoted by $I_n \in \mathbb{R}^{n \times n}$, is a square matrix with ones on the diagonal and zeros everywhere else. This matrix satisfies
$$AI_n = A = I_n A$$