Name: ....
MATH E-156 Mathematical Statistics
Assignment 1

1. Draw Venn diagrams to illustrate $(A^c \cap B^c)^c = A \cup B$.

2. Prove that $P(A \cap B) \geq P(A) + P(B) - 1$ for all events $A, B \subseteq \Omega$.

3. Prove that $P(A|B)P(B) = P(B|A)P(A)$ assuming all probabilities exist.

4. Show that if $P(A|E) \geq P(B|E)$ and $P(A|E^c) \geq P(B|E^c)$, then $P(A) \geq P(B)$.

5. Director of a research lab has the following information: The probability that the equipment needed for a project will be delivered on time is 0.90 and the probability that the equipment will be delivered on time and the project will be completed on time is 0.70. What is the probability that the project will be completed on time, given that all of the necessary equipment was delivered on time?

6. Prove that $\varnothing$ is independent of A for any event A.

7. Show that if A and B are independent, then A and B^c are independent.

8. Consider an election with two candidates. Every voter is invited to participate in an exit poll, where they are asked whom they voted for; some accept and some refuse. For a randomly selected voter, let A be the event that they voted for the 1st candidate, and W be the event that they are willing to participate in the exit poll. Suppose that $P(W|A) = 0.7$ but $P(W|A^c) = 0.2$. In the exit poll, 60% of the respondents say they voted for the 1st candidate (assume that they are all honest), suggesting a comfortable victory for the 1st candidate. Find $P(A)$, the true proportion of people who voted for the 1st candidate.