Question 1

4 pts

Def. We say a sequence of random variables $W_1, W_2, W_3, \ldots$ *converges in probability* to $\alpha \in \mathbb{R}$ if for every $\varepsilon > 0$ we have

$\lim_{n\to\infty} P(|W_n - \alpha| > \varepsilon) = 0.$

Problem: Let $X_1, X_2, X_3, \ldots$ be a sequence of independent random variables with $E(X_k) = \mu$ and $\mathrm{Var}(X_k) = \sigma^2$ for $k = 1, 2, 3, \ldots$

Also, let $\bar{X}_n \doteq \frac{1}{n}\sum_{k=1}^{n} X_k$.

Then

$\bar{X}_1, \bar{X}_2, \bar{X}_3, \ldots$ converges in probability to

(A) 0

(B) μ

(C) $\mu + \varepsilon$

(D) None of (A), (B), (C)

Please select:

- ○ A
- ○ B
- ○ C
- ○ D

Question 2

4 pts

Let $X_1, X_2, X_3, \ldots$ be a sequence of independent random variables with $E(X_k) = \mu$ and $\mathrm{Var}(X_k) = \sigma^2$ for $k = 1, 2, 3, \ldots$

If $\mu = 4$ and $\sigma = 1$, find $\lim_{n\to\infty} P(\bar{X}_n > 4.000001)$,

where $\bar{X}_n \doteq \frac{1}{n}\sum_{k=1}^{n} X_k$.

Question 3

4 pts

Let $X_1, X_2, X_3, \ldots$ be a sequence of independent random variables with moment generating functions $M_{X_1}(t), M_{X_2}(t), M_{X_3}(t), \ldots$, respectively.

Let random variable X have the moment generating function $M_X(t)$.

If there exists $\varepsilon > 0$ such that for every $t \in (-\varepsilon, \varepsilon)$, $M_{X_n}(t) \to M_X(t)$ as $n \to \infty$, then X_n converges in distribution to X.

- ○ True
- ○ False

Question 4

4 pts

Let X be a continuous random variable with the following probability density function (pdf):

$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$ for all $x \in \mathbb{R}$.

Then the moment generating function (mgf) of X is

(A) $M_X(t) = \frac{1}{1-t}$ for $t < 1$

(B) $M_X(t) = \left(1 - \frac{t}{\lambda}\right)^{-\alpha}$ for $t < \lambda$.

(C) $M_X(t) = e^{\frac{t^2}{2}}$ for all $t \in \mathbb{R}$

(D) $M_X(t) = (1 - 2t)^{-n/2}$ for $t < \frac{1}{2}$.

- ○ A
- ○ B
- ○ C
- ○ D

Question 5

4 pts

Let $Z \sim N(0,1)$ and $U \sim \chi_n^2$ be independent. Then the random variable $\frac{Z}{\sqrt{U/n}}$ is distributed according to

(A) Normal

(B) Chi-square

(C) t_n

(D) $F_{m,n}$

(E) None of (A), (B), (C), (D)

Please select:

- ○ A
- ○ B
- ○ C
- ○ D
- ○ E