CIS 320

Homework Assignment 5 [100 pts]

Due: July 1st, 2:00 PM on Gradescope

Please see Piazza and Canvas for submission logistics and LaTeXtemplate.

1. Suppose you have a flow network G with integer capacities, and an integer maximum flow f . Suppose that, for some edge e, we increase the capacity of e by one. Describe an $O(m)$ time algorithm to find a maximum flow in the modified graph.

2. You are outfitting a spacecraft to perform experiments on Mars and have available n pieces of scientific equipment $s_1, s_2, \ldots, s_n$. For each i, the cost of carrying equipment s_i is c_i and you can choose to take or not take each of the s_i. On Mars, you would like to perform experiments. The j^{th} experiment confers a utility of u_j but in order for this experiment to be performed we must carry a subset S_j of equipment . The goal is to maximize the sum of the utilities of the experiments we are able to perform minus the sum of the costs of the equipment we carry. Note that a piece of equipment can be used for multiple experiments. In other words, it does not get 'used up'. Design a flow network where the min cut provides a solution to this problem and prove your result.

3. You are given an undirected graph $G(V, E)$, with three vertices u, v, and w. Design an $O(m + n)$ time algorithm to determine whether or not G contains a simple path from u to w that passes through v. You do not need to return the path and do not need to give a full proof of the solution. **Hint:** Modify the max flow algorithm to handle capacity constraints at vertices and convert the given question into a question about max flows with vertex and edge capacity constraints. (Feel free to come up with your own solution if you do not want to follow the hint.)

4. We know that the subset sum problem is NP-Complete. Remember that the subset sum problem begins with an array of positive integers A and a targeted sum k. It then asks whether there exists a subset of A that sums up to this target value k. We would like to show that the following problem called Zero sum is also NP-Complete. Given a set of integers (of course, not necessarily all positive) is there a non-empty subset whose sum is zero. Show that the zero sum problem is in NP-Complete.

5. A software company is having a holiday party The CEO would like to invite some of the company's n employees to the party. The company has m projects and each project has a team of employees working on it. (One employee could be part of multiple teams.)

   For $i = 1, 2, \ldots, m$, let S_i be the set of people working on project i.

   The CEO doesn't want to invite all members of any team to the party, since she knows that if she does that the members of the group will just sit and talk about their project. Note that it is totally fine to

invite even all but one member of a group... just not all of them. The CEO wants to know if she can invite at least k people to the party.

Prove that this problem is NP-complete.

If you are having a tough time with the notation, here is an example instance of this problem:

A, B, C, D, E and F are the employees.

A, B, C are working on a project

D, A, B are working on a project

D, E are working on a project

A, F, C are working on a project

Can the CEO invite 4 people?

Yes - A, B, E and F is a safe set of people to invite since no subset of them represents the complete set of people doing a project.

6. We have seen how the independent set (decision) problem is NP complete. Suppose we are given a subroutine that will solve this problem. Show how you can use the subroutine to find a maximum independent set in a graph, making only a polynomial number of calls to the subroutine.