MATH 410, FALL 2020: HOMEWORK 3

Due: <u>September 24 before class (1:30 pm US eastern time)</u>

Read Chapters 3 and 4. You can use any results we have covered so far from the book or from class for the following problems, but please state which results you are using when writing your solutions.

Exercise 1. Let $f(z)$ be a complex analytic function over a region D. If there are two constants c_1 and c_2, not both zero, such that $c_1 f(z) + c_2 \overline{f(z)} = 0$ for all $z \in D$, then $f(z)$ must be constant.

Exercise 2. (1) Find the formulas for the real and imaginary parts of the following complex functions in terms of $x = \text{Re } x$ and $y = \text{Im } z$:

$$\cos(z), \quad \sin(z), \quad \cosh(z), \quad \sinh(z)$$

(2) Use the Cauchy-Riemann equations to check that $f(z) = \sin(\bar{z})$ is nowhere complex differentiable.

Exercise 3. Check the following trigonometry identities hold for complex numbers:

(1) $\cos^2 z + \sin^2 z = 1$.
(2) $\cos(z_1 + z_2) = \cos(z_1)\cos(z_2) - \sin(z_1)\sin(z_2)$.
(3) $\sin(z_1 + z_2) = \cos(z_1)\sin(z_2) + \sin(z_1)\cos(z_2)$.

Exercise 4. Sketch the following paths and label their directions:

(1) $\gamma(t) = e^{-2it}, t \in [0, \frac{\pi}{4}]$.
(2) $\gamma(t) = \cosh t + i \sinh t, t \in [-1, 1]$.

Exercise 5. Evaluate $\int_C (z^2 + i)\mathrm{d}z$, where C is a parabola segment:

$$z(t) = t^2 - it, \quad t \in [-1, 1]$$

(1) by finding the anti-derivative of the integrand.
(2) by integrating along the straight line between the end points of C and then using the Closed Curve Theorem.

Exercise 6. Evaluate $\int_C \frac{\mathrm{d}z}{z}$ over the following curves C with given orientations:

(1) $C = \{z \in \mathbb{C} \mid |z| = r\}$ is a circle of radius $r > 0$, clockwise orientation.
(2) $C = \{z \in \mathbb{C} \mid |\text{Re } z| = 1 \text{ or } |\text{Im } z| = 1\}$, counter-clockwise orientation.

$\boxed{\textbf{Exercise 7.}}$ Evaluate

(1) $\displaystyle\int_0^{\pi i} e^z \mathrm{d}z$

(2) $\displaystyle\int_{\frac{\pi}{4}-i}^{\frac{\pi}{4}+i} \cos(2z) \mathrm{d}z$