Harvard University
Computer Science 20

Problem Set 9

Due Monday, April 12, 2021 at 11:59pm.

PROBLEM 1

We can use perturbation to generate a big cancellation of terms and a closed form for the series S_n:

$$\begin{aligned} S_n &= 1 + z + z^2 + \cdots z^n \\ zS_n &= z + z^2 + z^3 + \cdots + z^{n+1} \\ S_n - zS_n &= 1 + z^{n+1} \\ S_n &= \frac{1 - z^{n+1}}{1 - z} \end{aligned}$$

Use this same trick to generate a closed form for the series T_n:

$$T_n = 1z + 2z^2 + 3z^3 + \cdots + nz^n$$

PROBLEM 2

Let f and g be monotonically increasing (never decreasing) functions defined on $\mathbb{R}^+$.

(A) Let $f(n) = o(g(n))$. Prove or disprove: $\log f(n) = o(\log g(n))$.
(B) Let $f(n) = O(g(n))$. Prove or disprove: $\log f(n) = O(\log g(n))$.

PROBLEM 3

The game of Hi-Lo works as follows. A computer randomly selects a number between 1 and n (inclusive). The player tries to guess the number. After each guess, the computer tells the player "too low", "too high", or "You got it!"

The optimal strategy for this game is to guess the number in the middle of the range. If the computer says "too high", discard the range above your last guess and recur on the new range which is now only half as large. If the computer says "too low", discard the range below your guess and recur on the new range, again only half as large. If you get all the way to a range of size 1, that must be the right answer.

(A) If $n = 1024$, what is the largest number of guesses it will take (in the worst case) to guess the right answer and get told "You got it!"?
(B) In the recursion tree for Hi-Lo for range n, how many nodes have *depth* 3 in the tree (assuming n is large enough that there are nodes at that depth)?

(C) In terms of n, what is the *height* of the recursion tree (in the worst case)?
(D) Write a recurrence for the algorithm the optimal player is using to play this game in terms of the size of the range n. You may assume that n is a power of 2.
(E) Give a closed form for the recurrence you found for Hi-Lo. You can base your work off your recursion tree analysis or unroll your recurrence.

Problem set by **FILL IN YOUR NAME HERE**

Collaboration Statement: **FILL IN YOUR COLLABORATION STATEMENT HERE (See the syllabus for information)**