LGIC 010 & PHIL 005
Problem Set 2
Spring Term, 2021
DUE FRIDAY, MARCH 5

For the purposes of this problem set, we restrict attention to pure monadic quantificational schemata all of whose predicate letters are among F and G, and to structures which interpret exactly these predicate letters.

We employ the following terminology in the problems on the following page.

- A pure monadic schema S implies a pure monadic schema T if and only if for every structure A, if $A \models S$, then $A \models T$. S and T are equivalent if and only if each implies the other.
- A list of pure monadic schemata is *succinct* if and only if no two schemata on the list are equivalent.
- If X is a finite set, we write $|X|$ for the number of members of X.
- We write $\mathcal{M}_n$ for the set of structures A such that $U^A = \{1, \ldots, n\}$.
- If S is a schema, we write $\mathsf{mod}(S, n)$ for the set of structures $A \in \mathcal{M}_n$ such that $A \models S$.
- Structures A and B are *monadically equivalent* ($A \approx_M B$) if and only if for every pure monadic schema S
$$A \models S \text{ if and only if } B \models S.$$
- A pure monadic schema S is *complete* if and only if S is satisfiable, and for all structures A and B,
$$\text{if } A \models S \text{ and } B \models S, \text{ then } A \approx_M B.$$
- Let A and B be structures. A *is isomorphic to* B ($A \cong B$) if and only if there is a bijection $h : U^A \mapsto U^B$ such that for every $a \in U^A$,
$$a \in F^A \text{ if and only if } h(a) \in F^B,$$
and
$$a \in G^A \text{ if and only if } h(a) \in G^B.$$
- For S a schema,
$$\mathsf{prob}(S, n) = \frac{|\mathsf{mod}(S, n)|}{|\mathcal{M}_n|}.$$
- For A a structure
$$\mathbb{E}(A, n) = \{B \in \mathcal{M}_n \mid B \approx_M A\},$$
and
$$\mathbb{I}(A, n) = \{B \in \mathcal{M}_n \mid B \cong A\}.$$

PROBLEMS

1. (10 points) Let S be the following schema.

$$(\forall x)(Fx \supset Gx) \wedge (\exists x)(Fx \oplus Gx)$$

   What is the value of $\mathsf{mod}(S, 6)$?

2. (15 points) What is the length of a longest succinct list of complete pure monadic schemata?

3. (15 points) Suppose that $\mathsf{mod}(S, 5) \leq 64$. What is the maximum possible number of pairwise inequivalent complete schemata that imply S.

4. Suppose $A \in \mathcal{M}_5$.

   (a) (15 points) What is the maximum possible value of $|\mathbb{E}(A, n)|$?

   (b) (15 points) What is the maximum possible value of $|(\mathbb{I}(A, n)|$?

5. (15 points) Is there a complete pure monadic schema S such that $\mathsf{prob}(S, 5) > .5$? If so, write down such a schema and explain why it has this property. If not, explain why none such exists.

6. (15 points) Is there a complete pure monadic schema S such that $\mathsf{prob}(S, 10) > .5$? If so, write down such a schema and explain why it has this property. If not, explain why none such exists.