Name: ....
MATH E-156 Mathematical Statistics
Assignment 2

1. Consider a discrete random variable X with the following probability mass function (pmf):

k	5	6	7	13
$P(X = k)$	.1	.3	.4	.2

   (a) Find $P(X > 6)$, $P(X \geq 6)$, and $P(\{X < 6\}^c)$.

   (b) Sketch a graph of the cumulative distribution function, $F_X(x)$. Please draw a large graph and show lots of details.

2. Let X be an integer-valued random variable. Show that the probability mass function (pmf) is related to the cumulative distribution function (cdf) as follows:

$$P(X = k) = F_X(k) - F_X(k-1) \text{ for any integer } k.$$

3. Prove that the cumulative distribution function (cdf) of a random variable X is non-decreasing, that is, $F_X(x_1) \leq F_X(x_2)$ whenever $x_1 < x_2$.
   Hint: Recall that $P(A) \leq P(B)$ whenever $A \subseteq B$.

4. Fix some $p \in (0, 1]$. Let X be a discrete random variable that takes values $1, 2, 3, \ldots$ with

$$P(X = k) = (1-p)^{k-1}p \quad \text{for all} \quad k \in \{1, 2, 3, \ldots\}.$$

   Show that $P(X = k)$ is indeed a probability distribution, that is,

   (a) $P(X = k) \geq 0$ for all k

   (b) $\sum_{k=1}^{\infty} P(X = k) = 1$

5. Fix some $\lambda > 0$. Let X be a continuous random variable with the following probability density function (pdf):

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & \text{if } x \geq 0, \\ 0, & \text{if } x < 0. \end{cases}$$

   Show that

   (a) $f_X(x) \geq 0$ for all $x \in \mathbb{R}$

   (b) $\int_{-\infty}^{+\infty} f_X(x)dx = 1$

6. Let $X \sim \text{Unif}[0,1]$. For the random variable $\sqrt{X}$,

(a) find the cumulative distribution function (cdf), $F_{\sqrt{X}}(s)$;

(b) find the probability density function (pdf), $f_{\sqrt{X}}(s)$;

(c) verify that $\int_{-\infty}^{+\infty} f_{\sqrt{X}}(s)ds = 1$.

7. How many ways can

(a) 6 different objects be lined up?

(b) 6 books be chosen out of 13 books if the order of chosen books matters?

(c) 6 books be chosen out of 13 books if the order does not matter?

8. For integers $n \geq k \geq 0$, we define $\binom{n}{k} \doteq \frac{n!}{k!(n-k)!}$, which represents the number of ways k objects can be chosen out of n objects without replacement, assuming the order of chosen objects does not matter. Prove the following identity algebraically:

$$\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1} \quad \text{for integers} \quad n \geq k \geq 1.$$