

# Section 1E. Expectation and Variance

## Statistics for Data Science

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## Random Variables: Expectation

- ▶ Given a discrete r.v.  $X$  with PMF  $p_X$  and a function  $g : \mathbb{R} \rightarrow \mathbb{R}$ , the **expectation** (or **expected value**) of  $g(X)$  is defined as

$$\mathbb{E}[g(X)] = \sum_{x \in \mathcal{X}} g(x) p_X(x)$$

where  $\mathcal{X}$  is the set of possible values  $X$  may take

- ▶ If  $X$  is continuous with PDF  $f_X$ , the **expected value** of  $g(X)$  is

$$\mathbb{E}[g(X)] = \int_{-\infty}^{\infty} g(x) f_X(x) dx$$

When  $g(x) = x$ , we have that  $\mathbb{E}[g(X)] = \mathbb{E}[X]$ , which is known as the **mean** of  $X$ .

- ▶ **Properties** of the expectation:
  - ▶ For any constant  $a \in \mathbb{R}$ ,  $\mathbb{E}[a] = a$  and  $\mathbb{E}[ag(X)] = a\mathbb{E}[g(X)]$
  - ▶  $\mathbb{E}[f(X) + g(X)] = \mathbb{E}[f(X)] + \mathbb{E}[g(X)]$

## Random Variables: Variance

- ▶ The **variance** of a r.v.  $X$  is defined as

$$\text{Var}[X] = \mathbb{E}[(X - \mathbb{E}[X])^2] = \mathbb{E}[X^2] - \mathbb{E}[X]^2$$

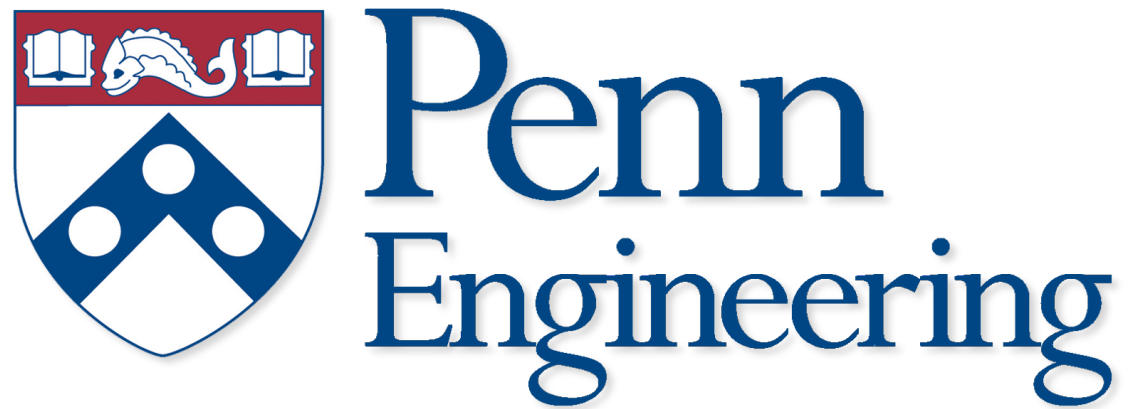
- ▶ **Properties** of the variance:

1. For any constant  $a \in \mathbb{R}$ ,  $\text{Var}[ag(X)] = a^2 \text{Var}[g(X)]$
2. Given two independent r.v.'s  $X$  and  $Y$ , we have that  $\text{Var}[X + Y] = \text{Var}[X] + \text{Var}[Y]$

- ▶ **Example:** Calculate the mean and variance of a r.v. with a uniform PDF, i.e.,  $f_X(x) = 1$  for all  $x \in [0, 1]$ , 0 elsewhere.

$$\mathbb{E}[X] = \int_0^1 x \, dx = 1/2 \text{ and } \mathbb{E}[X^2] = \int_0^1 x^2 \, dx = 1/3$$

$$\text{Hence, } \text{Var}[X] = 1/3 - 1/4 = 1/12$$



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