STATISTICAL INFERENCE
Autumn 2016
Quiz 1

1. Please **print** your name and student ID number in the upper right corner of this page.
2. This is a closed book, closed-notes examination. You can refer to one two-sided page of notes.
3. Please write the answers in the space provided. If you do not have enough space, use the back of a nearby page or ask for additional blank paper. Make sure you sign any loose pages.
4. In order to receive full credit for a problem, you should show all of your work and explain your reasoning. Good work can receive substantial partial credit even if the final answer is incorrect.

Question	Total Points	Credit
1	20	
2	20	
3	20	
total	60	

1. Suppose that X is a discrete random variable with $P(X = 1) = \theta$ and $P(X = 2) = 1 - \theta$. Three independent observations of X are made: $x_1 = 1$, $x_2 = 2$, $x_3 = 2$.

   (a) Find the method of moments estimate of θ.

   (b) What is the maximum likelihood estimator of θ?

(c) If Θ has a uniform prior on $[0, 1]$, what is the Bayes solution under the squared error loss?

2. Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables from the distribution with density $f(x) = \frac{1}{2}\exp(-\lambda|x|)$, $x \in \mathbb{R}$.

   (a) Find the best test of the hypothesis $H_0 : \lambda = \lambda_0$ versus $H_A : \lambda = \lambda_1$, where $\lambda_1 > \lambda_0$ with significance level α.

   (b) Is this the UMP test for $H_A : \lambda > \lambda_0$? Justify your answer.

(c) Compute and plot the power function for $\lambda > \lambda_0$.

3. Suppose that $X_1, \ldots, X_n$ are a random sample from the normal distribution with an unknown mean μ and known variance σ^2. How large a random sample must be taken in order that there will be a confidence interval for μ with confidence level 0.95 and length less than 0.01σ?