Name: ....
MATH E-156 Mathematical Statistics
Assignment 4

1. A basket contains 5 muffins and 8 scones.
   (a) Find the number of ways to choose two muffins and four scones.
   (b) If we randomly pick 6 pastries without replacement, what is the probability that exactly two of them are muffins?
2. Let $X \sim \text{Poisson}(\lambda)$ with some $\lambda > 0$. Show that $E(X) = \lambda$.
3. Entergy receives 70 calls per min (on average) on its toll free number for any given minute. Assuming independence, find
   (a) probability that there will be exactly 100 calls between 10:00 and 10:01 AM.
   (b) expected number of calls from 10:00 to 10:02 AM.
   (c) probability that there will be exactly 100 calls between 10:00 and 10:02 AM.

4. Assume that radius R of a sphere is an exponential random variable with parameter $\lambda = 1$, that is, it has the following probability density function (pdf):

$$f_R(r) = \begin{cases} e^{-r}, & \text{if } r \geq 0, \\ 0, & \text{if } r < 0. \end{cases}$$

Find the probability density function (pdf) of the volume.

5. The joint probability mass function (joint pmf) of two discrete random variables, X and Y, is given in the following table:

	$X = 1$	$X = 2$	$X = 3$	$X = 4$	$X = 5$
$Y = 0$	$8/20$	$2/20$	0	0	0
$Y = 1$	$1/20$	$1/20$	$1/20$	$1/20$	$6/20$

(a) Find the marginal probability mass function (marginal pmf) of Y. Present the answer in the form of a table.

(b) What is the conditional probability mass function (conditional pmf) of Y given X. Present the answer in the form of a table.

6. Two players play 10 independent rounds of a game, and each player has probability $\frac{1}{2}$ of winning each round. Find the joint probability mass function (joint pmf) of the numbers of games won by each of the two players. Make sure to explicitly state when the joint pmf is zero.

7. Let (X, Y) be uniformly distributed over the region $[0,1] \times [0,2]$, i.e. the joint probability density function (joint pdf) of X and Y is

$$f_{X,Y}(x,y) = \begin{cases} c, & \text{if } (x,y) \in [0,1] \times [0,2], \\ 0, & \text{otherwise.} \end{cases}$$

   (a) Find constant c.

   (b) Find the marginal probability density function (marginal pdf) of X, $f_X(x)$.

   (c) Find the marginal probability density function (marginal pdf) of Y, $f_Y(y)$.

   (d) Show that X and Y are independent, i.e. $f_{X,Y}(x,y) = f_X(x) f_Y(y)$ for all $x, y \in \mathbb{R}$.

8. A point (X, Y) is chosen randomly in the disk

$$D = \{(x, y) : \; x^2 + y^2 \leq a^2\}$$

of some fixed radius $a > 0$.

(a) What is the joint probability density function (joint pdf), $f_{X,Y}(x, y)$, of X and Y? Make sure to specify all possible cases.

(b) Find the marginal probability density function (marginal pdf), $f_Y(y)$, of Y.

(c) Compute the conditional probability density function (conditional pdf), $f_{X|Y}(x|y)$, of $X = x$ given $Y = y$. Please specify all possible cases.