Homework 2

Assigned: February 9; Due: February 27

(1) Compute the improper integral

$$\int_{1/\pi}^{\infty} \frac{1}{x^3} \sin\left(\frac{1}{x}\right) dx.$$

(2) Compute the improper integral

$$\int_0^{\infty} e^{-x} \cos x \, dx.$$

(3) Compute the improper integral

$$\int_0^{\infty} \frac{dx}{x^2 + a^2},$$

and then evaluate

$$\int_0^{\infty} \frac{dx}{(x^2 + a^2)^2}.$$

(4) Compute the improper integral

$$\int_0^{\infty} \frac{e^{-ax} \sin x}{x} \, dx,$$

and then evaluate

$$\int_0^{\infty} \frac{\sin x}{x} \, dx.$$

(5) Provide option prices for European plain vanilla call options struck at 110, 120, 130, and 140 for an asset at 100, so that there is no arbitrage opportunity.

(6) Show that a parallel shift in the zero rate curve generates an identical parallel shift in the instantaneous rate curve, and vice versa.

In other words, denote by $r_1(0,t)$ and $r_2(0,t)$ two zero rate curves with corresponding instantaneous rate curves $r_1(t)$ and $r_2(t)$. Let $\delta r > 0$ denote an arbitrary positive number. Then,

$$r_2(0,t) = r_1(0,t) + \delta r \quad \text{if and only if} \quad r_2(t) = r_1(t) + \delta r.$$

(7) Use Simpson's rule to compute the cumulative distribution of the standard normal variable with 10^{-12} tolerance.

In other words, write a routine that computes $N(t)$ with $tol = 10^{-12}$, where

$$N(t) \;=\; \frac{1}{2} \;+\; \frac{1}{\sqrt{2\pi}} \int_0^t e^{-\frac{x^2}{2}} \, dx.$$

Note that you only need to compute numerical approximations of a definite integral over the finite interval $[0,t]$, if $t > 0$, or $[t,0]$, if $t < 0$.

Compute $N(0.1)$, $N(0.5)$ and $N(1)$ with 12 digits accuracy. Start with $n = 4$ intervals and double the number of intervals until the desired accuracy is achieved. Report the approximate values you obtained for each interval until convergence, for each of the two integrals.

(8) Assume that the continuously compounded zero rate curve is

$$r_c(0,t) \;=\; 0.02 + 0.01\frac{t}{1+t^2}.$$

(i) find the instantaneous interest rate curve;

(ii) compute the corresponding annually compounded zero rate curve;

(iii) compute the corresponding semiannually compounded zero rate curve.

(9) Assume that the continuously compounded instantaneous interest rate curve is

$$r(t) \;=\; 0.04 + 0.01\ln(1+t).$$

Compute the corresponding continuously compounded zero rate curve.

(10) Assume that the continuously compounded zero rate curve has the form

$$r(0,t) \;=\; 0.05 \;+\; 0.005\sqrt{1+t}, \;\; \forall\, t \geq 0.$$

(i) Find the price of a two year semiannual coupon bond with coupon rate 7%.

(ii) Find the price of a semiannual coupon bond with coupon rate 7% and maturity in 19 months.

(11) Assume that the continuously compounded risk–free zero rate curve is given by

$$r(0,t) \;=\; 0.02 + \frac{t}{200-t}, \;\; \forall\, 0 \leq t \leq 5.$$

Find the value of a 19 months bond with coupon rate 4% and face value \$100, if the bond is an annual coupon bond, a semiannual coupon bond, or a quarterly coupon bond.

(12) Find the modified duration and convexity of a 19 months semiannual coupon bond with coupon rate 4% and face value \$100, if the yield of the bond is 2.5%.

(13) The instantaneous rate curve $r(t)$ is given by

$$r(t) \;=\; \frac{0.05}{1 + 2\exp(-(1+t)^2)}.$$

Assume that interest is compounded continuously.

(i) Compute the 6 months, 1 year, and 18 months discount factors with six decimal digits accuracy, and compute the 2 year discount factor with eight decimal digits accuracy, using Simpson's Rule; recall that the discount factor corresponding to time t is

$$\exp\left(-\int_0^t r(\tau)\, d\tau\right).$$

(ii) Find the price of a two year semiannual coupon bond with coupon rate 5% (and face value 100).

(14) (i) The one year zero rate is 5% and the three year zero rate is 5.5%. You are offered a 1×3 forward rate of 5.6%. How do you arbitrage it?

(ii) In a more realistic setting, the bid-ask spread on one year loans/deposits is 5% and 5.05%, and the bid-ask spread on three year loans/deposits is 5.5% and 5.56%. You are offered a bid-ask 1×3 forward rate spread of 5.58 and 5.62. How do you arbitrage it?