LGIC 010 & PHIL 005
Problem Set 5
Spring Term, 2021
DUE FRIDAY, APRIL 23 at 11:59 p.m. EDT

1. Let S_1 be the conjunction of the following schemata.

$$(\forall x)(\forall y)(\forall v)(\forall w)((Fx \wedge Fy \wedge Fv \wedge Fw) \supset (g(x,y) = g(v,w) \supset (x = v \wedge y = w)))$$

$$(\forall z)(\exists x)(\exists y)(Fx \wedge Fy \wedge g(x,y) = z)$$

   (a) (15 points) What is the value of $|\mathbf{mod}(S_1, 4)|$?

   (b) (10 points) What is $\mathsf{Spec}(S_1)$?

   (c) (10 points) Let T_1 be the schema

$$(\exists x)(\exists y)(x \neq y) \wedge (\forall x)(\forall y)(\forall v)(\forall w)(g(x,y) = g(v,w) \supset (x = v \wedge y = w)).$$

   Is T_1 satisfiable? If so, give an example of a structure that satisfies T_1. If not, explain why.

2. Let S_2 be the schema

$$(\forall x) f(x) \neq x \wedge (\forall x)(\forall y)(f(x) = f(y) \supset x = y).$$

(a) (15 points) List the values of $|\mathbb{I}(A)|$ for those $A \in \mathbf{mod}(S_2, 7)$ with $|\mathbf{def}(A)| = 2$.

(b) (10 points) List the values of $|\mathbb{I}(A)|$ for those $A \in \mathbf{mod}(S_2, 7)$ with $\mathbf{min}(\{|X| \mid X \in \mathbf{def}(A) \text{ and } X \neq \emptyset\}) = 3$.

3. Let S_3 be the schema

$$(\forall x)(Fx \equiv \neg Fg(x)) \wedge (\forall x)(\forall y)(g(x) = g(y) \supset x = y).$$

(a) (15 points) What is the value of $|\mathbf{iso}(S_3, 6)|$?

(b) (15 points) What is $\mathsf{Spec}(S_3)$?

4. (10 points) Let S_4 be the conjunction of SLO and the following schemata.

$$(\exists x)(\forall y)\neg Lyx \wedge (\exists x)(\forall y)\neg Lxy$$

$$(\forall x)((\exists y)Lyx \supset (\exists y)(Lyx \wedge (\forall z)(\neg(Lyz \wedge Lzx))))$$

$$(\forall x)((\exists y)Lxy \supset (\exists y)(Lxy \wedge (\forall z)(\neg(Lxz \wedge Lzy))))$$

Is there a structure A such that $A \models S_4$ and $|\mathbf{aut}(A)| > 1$. If so, give an example of such a structure. If not, explain why?