

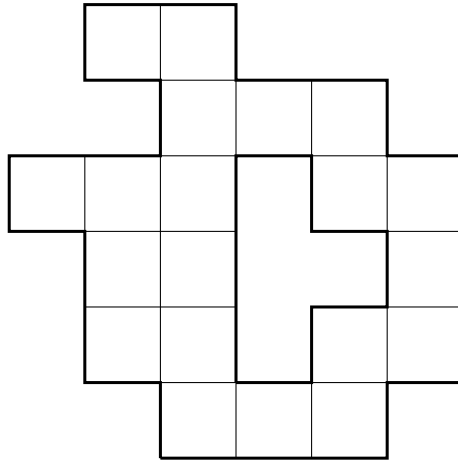
Harvard University  
Computer Science 20

Problem Set 6

Due Wednesday, March 17, 2021 at 11:59pm.

PROBLEM 1

In this problem, a tile is a  $1 \times 1$  square and an  $n$ -tiling is a group of  $n$  tiles placed so that every tile is contiguous along at least one side to another tile. (The tiles cannot be offset. That is, they must be placed into a grid like Scrabble tiles.) The *edge length* of a tiling is the total length of edges that are not touching other tiles. For instance, in the following 20-tiling, the edge length is 36.



Prove that for all  $n$ -tilings where  $n \geq 1$ , the edge length is even.

PROBLEM 2

Prove using strong induction that for *any* positive integer  $n$  and *any*  $x \in \mathbb{R}$  where  $x \neq 0$  that if  $x + \frac{1}{x}$  is an integer then  $x^n + \frac{1}{x^n}$  is also an integer.

PROBLEM 3

Prove that for all nonnegative integers  $n$ . You may use induction or well-ordering (or do it both ways for practice!)

$$\sum_{i=0}^n i^3 = \left( \sum_{i=0}^n i \right)^2$$

*Hint: the following identity may be useful*

$$\sum_{i=0}^n i = \frac{n(n+1)}{2}$$