Basic Statistical Models

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Basic Statistical Models

- What do we mean by “model”
- Random samples
- Statistical model for repeated measurements
- Example: disk drive failures
- Distribution features and statistical models
  - Sample expectation
  - Sample variance
- Linear regression models

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- And, we'll need to determine how <u>confident</u> we are in our estimate of those parameters.

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Example: Monitor a data center full of disk drives, count the number of failures per month.

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A dataset consisting of values $x_1, x_2, \ldots, x_n$ of repeated measurements of the same quantity is modeled as the realization of a random sample $X_1, X_2, \ldots, X_n$. The model may include a partial specification of the probability distribution function for each X_i.

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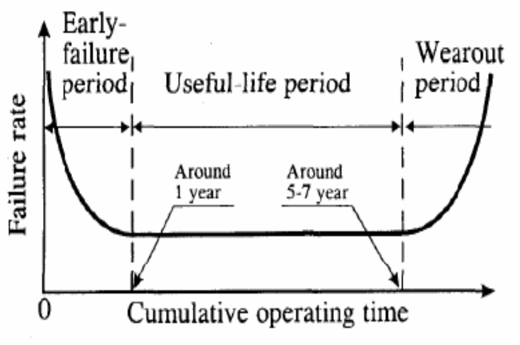
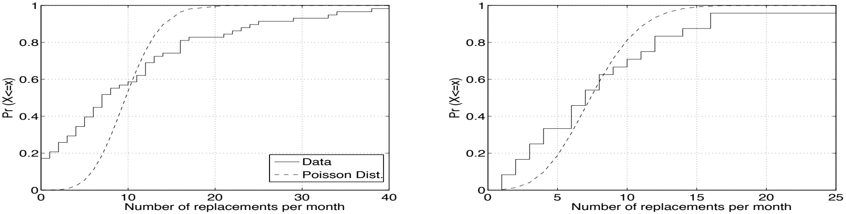
Example: Manufactures rate disk drives using an exponential mean-time-to-failure (MTTF) model. Accurate?

Disk failures in the real world: What does an MTTF of 1,000,000 hours mean to you?, Bianca Schroeder, Garth A. Gibson (CMU), File systems and Storage Technology, 2007.



Example: disk drive failures II

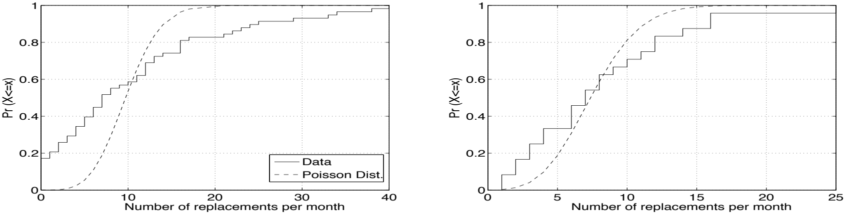


All years — Years 2-3

Figure 5: *CDF of number of disk replacements per month in HPC1*

- The statistical model includes three parts; birth, mid-life and death.

Example: disk drive failures II



All years   Years 2-3

Figure 5: *CDF of number of disk replacements per month in HPC1*

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- OK agreement with exponential assumption in mid-life phase.

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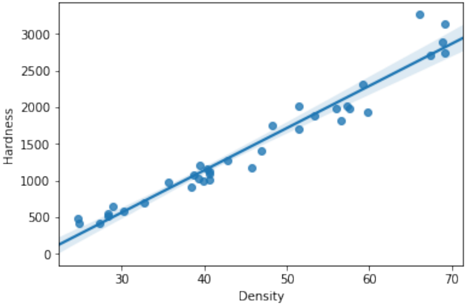
  - Why $n-1$? We'll discuss unbiased estimators later.

Linear Regression Models



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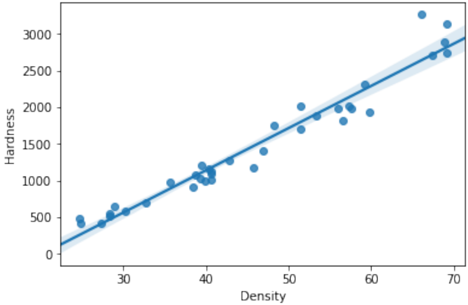
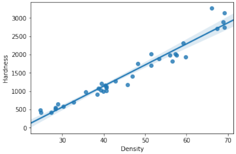
Linear Regression Models



- Our model is hardness $\sim$ density of timber
- A regression model would be
  hardness $\sim$ density of timber + noise

Linear Regression Models



Simple Linear Regression Model

A simple linear regression model for $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ assumes the x_i are non-random and the y_i are realizations of random variables Y_i satisfying

$$Y_i = \alpha + \beta x_i + R_i$$

where R_i are independent random variables with $E[U_i] = 0$ and $Var[U_i] = \sigma^2$.

True distribution sometimes not linear

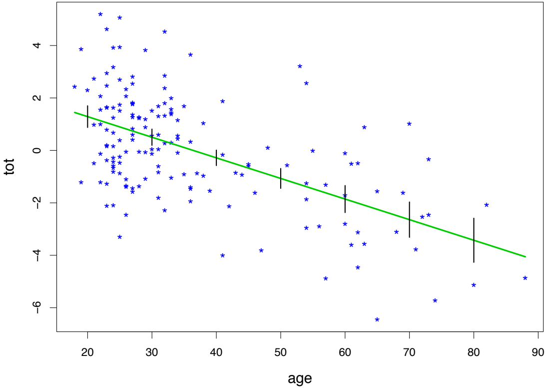


Figure 1.1 Kidney fitness `tot` vs `age` for 157 volunteers. The line is a linear regression fit, showing ± 2 standard errors at selected values of `age`.

Beyond simple linear regression

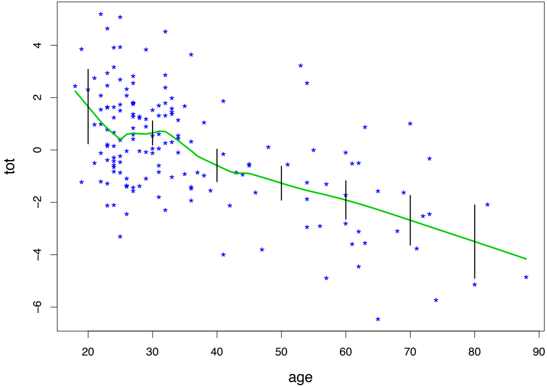


Figure 1.2 Local polynomial `lowess(x,y,1/3)` fit to the kidney-fitness data, with ± 2 bootstrap standard deviations.

Bootstrapped Models

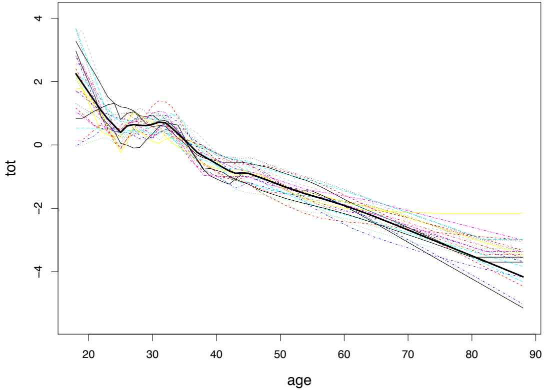


Figure 1.3 25 bootstrap replications of `lowess(x,y,1/3)`.