1

If X and Y are independent exponential random variables with parameters1 and 2, evaluate $E[(1+X+Y)^2]$

2

Alice and Bob are tossing a fair coin. For each head Alice gets a dollar. For each tail Bob gets a dollar. (After each toss somebody gets a dollar and nobody looses money in this game). Before the start of the game Alice had 1 dollar and Bob had $0.

(a) What is the probability that after 200 tosses Alice has exactly102 dollars?

(b) What is the probability that after each of 200 tosses Alice has more money than Bob, and after the last of the 200 tosses Alice has exactly 102 dollars?

3

A point is chosen uniformly at random from the rectangle with vertices (0, 0), (23, 0), (23, 18), (0, 8).

If the y coordinate of the point is smaller than the x coordinate, then the player earns $ 20.

Otherwise the player wins $10. Let M be the random variable that represents the money that the player wins in this game.

Determine the cumulative distribution function of the random variable M and draw its graph.

4

Let W be a standard Brownian motion. Evaluate $E[(W_5 e^{w9} | W_6 = 1)]$

5

Let n be a positive integer and $(a_1, a_2, ..., a_n)$ a permutation of the set $\{1, 2, ... , n\}$. An element a_i is called boring if it is smaller than each of the terms that appears after it In, other words, if $i < n$, then a_i is boring if $a_i < a_i+1$, $ai < a_i+2$, ... , and $a_i < a_n$. The number a_n is considered boring because there is nothing after it.

For example, if n = 7 and the permutation is (6, 4, 1, 2, 7, 3, 5), then the numbers 1, 2, 3, and 5 are boring. Every number is boring in the permutation (1, 2, 3, 4, 5, 6, 7), and only the number 1 is boring in (7, 6, 5, 4, 3, 2, 1).

Calculate the expected number of boring elements in a random permutation of the set $\{1, 2, ..., n\}$.