

**Harvard University
Computer Science 20**

Problem Set 3

Due Monday, February 22, 2021 at 11:59pm.

PROBLEM 1

Let A , B , and C be sets. Prove that $(A - B) \cup (B - C) \subseteq (A \cup B) - (A \cap B \cap C)$.

PROBLEM 2

For any integers s and t , we'll define the set $L(s, t)$ as follows: $L(s, t) = \{sx + ty \mid x, y \in \mathbb{Z}\}$.

Prove the following claim. You may use the theorem proved in class that $a \equiv b \pmod{n}$ if and only if $n \mid (a - b)$.

Claim: For any integers a , r , m , where m is positive, if $a \equiv r \pmod{m}$, then $L(a, m) \subseteq L(r, m)$.

PROBLEM 3

Let $g : \mathbb{Z} \rightarrow \mathbb{Z}$ be an injective function. Define $f : \mathbb{Z}^2 \rightarrow \mathbb{Z}^2$ such that $f(x, y) = (g(x) + g(y), g(x) - g(y))$. Show that f is also injective.