Assigned: February 17, 2021. Due: February 24, 2021 at 6:00pm.
Late homework will not be accepted.

(1) (2.7.21) Let (X, Y) be a random vector with probability mass function $p_{X,Y}(i, j) = \frac{1}{10}$, for $1 \le i \le j \le 4$.
  (a) Show that this is indeed a probability mass function.
  (b) Compute the marginal distributions of X and Y.
  (c) Are X and Y independent?
  (d) Compute $E(XY)$.

(2) (2.7.22) Compute $E(X|Y = y)$ and $E(Y|X = x)$ in the previous exercise.

(3) (2.7.23) Suppose that 15 percent of families in a certain country have no children, 20 percent have 1, 35 percent have 2, and 30 percent have 3. Suppose further that each child is equally likely to be a boy or a girl, independent of the other children. A family is chosen at random, and we write B for the number of boys in this family, and G for the number of girls. Write down the joint probability mass function of (B, G).

(4) (2.7.24) We roll two fair dice. Find the joint probability mass function of X and Y when:
  (a) X is the largest value obtained, and Y is the sum of the values;
  (b) X is the value on the first die, and Y is the largest value;
  (c) X is the smallest value, and Y is the largest.

(5) (2.7.25) Compute $E(Y|X = x)$ for all random variables in the previous exercise.

(6) (2.7.26) Let (X, Y) have joint mass function $p(k, n) = \frac{C \cdot 2^{-k}}{n}$, for $k = 1, 2, \ldots$ and $n = 1, 2, \ldots, k$, and suitable constant C. Compute $E(X|Y = y)$.

(7) (2.7.32) Let X and Y be independent and geometrically distributed with the same parameter p. Compute the probability mass function of $X - Y$. Can you compute $P(X = Y)$ now?

(8) (2.7.33) Let X and Y be as in the previous exercise. Compute $E(X|X + Y = k)$ for all $k = 2, 3, \ldots$.

(9) (2.7.37) Let X and Y be independent binomial random variables with parameters n and p. Denote their sum by Z. Show that

$$P(X = k|Z = m) = \frac{\binom{n}{k}\binom{n}{m-k}}{\binom{2n}{m}},$$

for $k = 1, \ldots, m$.

(10) (2.7.41) Suppose we throw a coin 5 times. Let X be the number of heads, and Y be the number of tails. We assume that the coin is biased, and that the probability of head is equal to $\frac{1}{3}$. We are interested in $Z = X - Y$.

(a) Express Z as a function of X.

(b) Compute $E(Z)$ without computing the probability mass function of Z.

(c) Compute the probability mass function of Z, and use this to compute $E(Z)$ once again.