Statistics:Expectation and Variance Of The Mean

Dirk Grunwald
University of Colorado, Boulder

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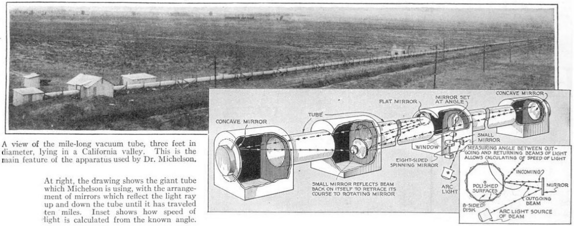
- Running example - Michelson's measurements for speed of light..



A view of the mile-long vacuum tube, three feet in diameter, lying in a California valley. This is the main feature of the apparatus used by Dr. Michelson.

At right, the drawing shows the giant tube which Michelson is using, with the arrangement of mirrors which reflect the light ray up and down the tube until it has traveled ten miles. Inset shows how speed of light is calculated from the known angle.

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- Using linearity of expectation

$$E[\hat{X}_n] = \frac{1}{n} E\left[X_1 + X_2 + \ldots + X_n\right] = \frac{1}{n}(\mu + \mu + \ldots + \mu) = \mu$$

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- Variance:

$$Var(\hat{X}_n) = Var(\frac{1}{n}(X_1 + X_2 + \ldots + X_n)) = \frac{1}{n^2} Var(\sigma^2 + \ldots + \sigma^2) = \frac{\sigma^2}{n}$$

Expectation and Variance of An Average

Definition: Expectation and Variance of An Average

If $\hat{X}_n$ is the average of n independent variables with the same expectation μ and variance σ^2, then

$$E[\hat{X}_n] = \mu \qquad \text{and} \qquad Var(\hat{X}_n) = \frac{\sigma^2}{n}$$

The standard deviation of the average is a factor $\sqrt{n}$ less than single measurement.

Example

Mean and variance of the Binomial distribution.
Let S_n be the sum of n binomial samples with parameter p.

Sample Mean:

$$E[\frac{S_n}{n}] = p$$

Sample Variance:

$$Var[\frac{S_n}{n}] = \frac{np(1-p)}{n^2} = \frac{p(1-p)}{n}$$

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Mean and Variance of number of events

$$E[S_n] = np \qquad Var[S_n] = np(1-p)$$