**Homework 5**

A portion of the following problems will be graded according to the provided rubric.

1. Rudin pg 78 problem 1
2. Let $\{x_n\}_{n=1}^{\infty}$ and $\{y_n\}_{n=1}^{\infty}$ be sequences in $\mathbb{C}^k$. Give an example of the following or explain why such a request is impossible.
    a. Sequences $\{x_n\}$ and $\{y_n\}$ which both diverge, but whose sum $\{x_n + y_n\}$ converges
    b. Sequences $\{x_n\}$ and $\{y_n\}$ where $\{x_n\}$ converges and $\{y_n\}$ diverges, but whose sum $\{x_n + y_n\}$ converges
3. Let $\{x_n\}_{n=1}^{\infty}$ and $\{y_n\}_{n=1}^{\infty}$ be sequences in $\mathbb{R}$. Let $\{x_n\}_{n=1}^{\infty}$ be a bounded (not necessarily convergent) sequence and assume $\lim_{n\to\infty} y_n = 0$. Show that $\lim_{n\to\infty} x_n y_n = 0$.
4. Let $\{x_n\}_{n=1}^{\infty}$ and $\{y_n\}_{n=1}^{\infty}$ be sequences in $\mathbb{R}^k$. Give an example of each of the following or explain why such a request is impossible.
    a. A sequence that has a subsequence that is bounded but contains no subsequence that converges.
    b. A sequence that does not contain 0 or 1 as a term but contains subsequences converging to each of these values.
5. Rudin pg 78 problem 3
6. Prove the Monotone Convergence Theorem for a decreasing sequence.
7. Prove the sequence defined by $x_1 = 1$ and $x_{n+1} = \frac{x_n+1}{4}$ converges and find the limit.
8. Rudin pg 78 problem 5
9. Rudin pg 82 problem 20
10. Let $\{x_n\}_{n=1}^{\infty}$ and $\{y_n\}_{n=1}^{\infty}$ be sequences in $\mathbb{R}^k$. Give an example of each of the following or explain why such a request is impossible.
    a. A Cauchy sequence that is not monotone.
    b. A Cauchy sequence with an unbounded subsequence.
    c. An unbounded sequence containing a subsequence that is Cauchy.
11. Prove that if $\{x_n\}_{n=1}^{\infty}$ and $\{y_n\}_{n=1}^{\infty}$ are Cauchy sequences in $\mathbb{C}^k$, then $\{x_n + y_n\}_{n=1}^{\infty}$ is Cauchy.
12. Prove that if $\{x_n\}_{n=1}^{\infty}$ and $\{y_n\}_{n=1}^{\infty}$ are Cauchy sequences in $\mathbb{R}^k$, then $\{x_n y_n\}_{n=1}^{\infty}$ is Cauchy without using Theorem 3.3.
13. Rudin pg 82 problem 23