MA 630 - Homework 1 (Module 1 - Section 1)

Solutions must be typeset in LaTeX and submitted to Canvas as a .pdf file. When applicable, write in complete sentences.

1. Let x and y be real numbers. Prove that if $2x^2 - 6x = 2y^2 - 6y$ and $x \neq y$, then $x + y = 3$. (Be sure to comment or note why it is important that $x \neq y$.)
2. Let a and b be integers. Prove that ab is odd if and only if a and b are both odd.
3. Let a and b be integers.
   (a) Prove that if ab is odd, then $a + b$ is even. As always, feel free to reference a previous homework problem.
   (b) Is the converse true? Either prove it or give a counterexample.
4. Let m and n be integers which are greater than or equal to 2. Prove that $mn + 1$ is not divisible by m.
5. Let n be an integer such that n^2 is even. Prove that n^2 is divisible by 4.
6. Prove that for any natural number n, either n is a prime or a perfect square, or n divides $(n - 1)!$.