Homework 7

A portion of the following problems will be graded according to the provided rubric.

1. Prove the following limit statements.
   a. $\lim_{x\to 2}(3x+4) = 10$
   b. $\lim_{x\to 2}(x^2+x-1) = 5$
   c. $\lim_{x\to 3}\frac{1}{x} = \frac{1}{3}$
2. Let $f: X \to \mathbb{R}$ be such that $\lim_{x\to p} f(x) = 0$.
   a. Prove that if g is a bounded function on X, then $\lim_{x\to p} f(x)g(x) = 0$.
   b. Give an example to show that $\lim_{x\to p} f(x)g(x)$ is not always equal to zero.
3. Let $f(x) = \sqrt[3]{x}$.
   a. Prove f is continuous at $p = 0$.
   b. Prove f is continuous at any $p \neq 0$ (and thus is continuous everywhere).
      Hint: $a^3 - b^3 = (a-b)(a^2+ab+b^2)$
4. Let C be the Cantor set. Define $g: \mathbb{R} \to \{0,1\}$ by

$$g(x) = \begin{cases} 1, & \text{if } x \in C \\ 0, & \text{if } x \notin C \end{cases}$$

   a. Prove g is not continuous at any $c \in C$
   b. Prove g is continuous at any $c \notin C$
5. Rudin pg 98 problem 2
6. Let X, Y be metric spaces and $f: X \to Y$. Prove f is continuous if and only if $f^{-1}(C)$ is closed in X for every closed set C in Y.
7. Rudin pg 98 problem 3
8. Rudin pg 98 problem 4