

Homework 2

A portion of the following problems will be graded according to the provided rubric.

1. Calculate, without proofs, the suprema and infima for the following sets:
 - a. $A = \left\{ \frac{m}{n} : m, n \in \mathbb{N} \text{ with } m < n \right\}$
 - b. $B = \left\{ \frac{(-1)^m}{n} : m, n \in \mathbb{N} \right\}$
 - c. $C = \left\{ \frac{n}{3n+1} : n \in \mathbb{N} \right\}$
 - d. $D = \left\{ \frac{m}{m+n} : m, n \in \mathbb{N} \right\}$
2. Rudin pg 22 problem 4
3. Rudin pg 22 problem 5
4. Rudin pg 22 problem 9
5. Let $A \subseteq \mathbb{R}$ be nonempty and bounded above. Let $s \in \mathbb{R}$ have the property that for all $n \in \mathbb{N}$, $s + \frac{1}{n}$ is an upper bound for A and $s - \frac{1}{n}$ is not an upper bound for A . Prove $s = \sup(A)$.
6. Prove that if X and Y are countable sets, then $X \times Y = \{(x, y) : x \in X \text{ and } y \in Y\}$ is countable.
7. Give an example of a countable collection of disjoint open intervals. Explain.
8. Prove that the set of odd integers is countable.