Homework 4

A portion of the following problems will be graded according to the provided rubric.

1. Rudin pg 44 problem 12
2. Rudin pg 44 problem 14
3. Rudin pg 44 problem 15
4. Let $K \subset \mathbb{R}$ be nonempty and compact. Show that $\inf(K) \in K$.
5. Let $K \subset \mathbb{R}^k$ be compact and $F \subset \mathbb{R}^k$ be closed. Determine if the following sets are always compact. If yes, prove it. If no, provide a counterexample.
   a. $\overline{F^C \cup K^C}$
   b. $K \backslash F$
   c. $\overline{K \cap F^C}$
6. Let $P \subset \mathbb{R}^k$ be perfect and $K \subset \mathbb{R}^k$ be compact.
   a. Is $P \cap K$ always compact? If yes, prove it. If no, provide a counterexample.
   b. Is $P \cap K$ always prefect? If yes, prove it. If no, provide a counterexample.
7. Does there exist a perfect set consisting of only rational numbers? Why or why not?
8. Rudin pg 44 problem 19
9. Give an example to show that if A is open and B is closed and A and B are disjoint sets, then they are not always separated.
10. Find an example of a disconnected set whose closure is connected.