MA 630 Quiz 1 - Group 3

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1.

①	②	③	④	⑤	⑥
P	Q	R	$P \vee Q$	④ $\to R$	$\neg$⑤
T	T	T	T	T	F
T	T	F	T	F	T
T	F	T	T	T	F
T	F	F	T	F	T
F	T	T	T	T	F
F	T	F	T	F	T
F	F	T	F	T	F
F	F	F	F	T	F

2.

①	②	③	④	⑤	⑥
P	Q	$P \to Q$	$\neg Q$	$1 \wedge 4$	$3 \wedge 4$
T	T	T	F	F	F
T	F	F	T	T	F
F	T	T	F	F	F
F	F	T	F	F	F

3. (a) $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(y^2 = x)$

   (b) We could simply negate: $\neg[(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(y^2 = x)]$
   However that seems too easy, so perhaps we're meant to push the negation in: $(\exists x \in \mathbb{R})(\forall y \in \mathbb{R})(y^2 \neq x)$.
   There exists a real number x such that for all real numbers y, $y^2 \neq x$.

   (c) The negation, $\neg P$, is the true sentence. It is witnessed by the value $x = -1$ for the existential claim. With this assignment, and for every $y \in \mathbb{R}$, we have $y^2 \neq -1$.

4. Let a be an even integer and b be an odd integer. Then $a = 2m$ and $b = 2n + 1$, for some integers m and n. Thus. $ab = 2m(2n+1) = 4mn+2m = 2(2mn+m)$ Therefore, there is an integer t, namely $t = 2mn + m$, such that $ab = 2t$. Thus, by definition, ab is even.

5. Let n be an integer. Prove that n is even if and only if n^3 is even.

Suppose n is even, then $n = 2k$ for some $k \in \mathbb{Z}$. Then,

$$\begin{aligned} n^3 &= n^3 \\ &= (2k)^3 \\ &= 8k^3 \\ &= 2(4k^3) \end{aligned}$$

Thus, n^3 is even since $4k^3 \in \mathbb{Z}$.

We then need to prove the contrapositive, that if n^3 is odd then n is odd.

Suppose n is odd, then $n = 2k + 1$ for some $k \in \mathbb{Z}$. Then,

$$\begin{aligned} n^3 &= n^3 \\ &= (2k+1)^3 \\ &= (2k+1)(2k+1)(2k+1) \\ &= (4k^2+4k+1)(2k+1) \\ &= 8k^3 + 4k^2 + 8k^2 + 4k + 2k + 1 \\ &= 8k^3 + 12k^2 + 6k + 1 \\ &= 2(4k^3 + 6k^2 + 3k) + 1 \end{aligned}$$

Thus, n^3 is odd since $4k^3 + 6k^2 + 3k \in \mathbb{Z}$.

Because $P \to Q$ and its contrapositive $\neg Q \to \neg P$ are both true, the original proof is true.