MA 630 - Homework 2 (Module 1 - Sections 1 and 2)

Solutions must be typeset in LaTeX and submitted to Canvas as a .pdf file. When applicable, write in complete sentences.

1. Let x be a positive real number. Prove that if $x - \dfrac{2}{x} > 1$, then $x > 2$ by
   (a) direct proof.
   (b) contrapositive proof.
   (c) proof by contradiction.
2. Suppose x is an integer. Prove that $5x - 7$ is odd if and only if $9x + 2$ is even. *Hint: In both directions, first prove that x is even.*
3. Prove that if k is an odd integer, then the equation $x^2 + x - k = 0$ has no integral solution.
4. Let m and n be integers. Prove that $(m + 1)n^2$ is even if and only if m is odd or n is even.
5. (a) Let n be an integer. Prove that if n^2 is even, then n^2 is divisible by 4.
   (b) Prove that if k is an odd integer, then $2k$ is not divisible by 4.
   (c) Prove that the sum of the squares of two odd integers can not be equal to the square of an integer.