Homework 4

A portion of the following problems will be graded according to the provided rubric.

1. If g is continuous on $[a,b]$, show there exists a point $c \in (a,b)$ where $g(c) = \frac{1}{b-a}\int_a^b g\, d\alpha$.
2. Let $f(x) = \int_x^{x+1} \sin(t^2)\, dt$.
    a. Prove that $f(x) = \frac{\cos(x^2)}{2x} - \frac{\cos((x+1)^2)}{2(x+1)} - \int_{x^2}^{(x+1)^2} \frac{\cos(u)}{4u^{3/2}} du$
    b. Use the result from part a to show that the improper integral $\int_0^\infty \sin(t^2)\, dt$ converges.
3. Rudin page 141 problem 15
4. Rudin page 141 problem 19
5. Define curves $\gamma_1, \gamma_2 : [0,1] \to \mathbb{R}^2$ by $\gamma_1(t) = \begin{cases} (0,0), & if\ t = 0 \\ \left(t, t sin\left(\frac{1}{t}\right)\right), & if\ t \neq 0 \end{cases}$ and

$$\gamma_2(t) = \begin{cases} (0,0), & if\ t = 0 \\ \left(t, t^3 sin\left(\frac{1}{t}\right)\right), & if\ t \neq 0 \end{cases}.$$

    a. Show that γ_1 is not rectifiable.
    b. Show that γ_2 is rectifiable.