Homework 3

A portion of the following problems will be graded according to the provided rubric.

1. Let $f(x) = 2x + 1$ over the interval $[1,3]$ and let P be the partition consisting of the points $\left\{1, \frac{3}{2}, 2, 3\right\}$.
    a. Compute $L(P, f)$, $U(P, f)$, and $U(P, f) - L(P, f)$.
    b. What happens to $U(P, f) - L(P, f)$ when the point $\frac{5}{2}$ is added to the partition?
    c. Find a partition P' of $[1,3]$ for which $U(P', f) - L(P', f) < 2$.
2. Show that the function $f(x) = k$ where $k \in \mathbb{R}$ is integrable and find $\int_a^b f \, d\alpha$.
3. Rudin page 138 problem 1
4. Rudin page 138 problem 2
5. Let $f \in \mathcal{R}(\alpha, [a, b])$ and $c \in \mathbb{R}$. Prove if $a < c < b$, then $f \in \mathcal{R}(\alpha, [a, c]) \cap \mathcal{R}(\alpha, [c, b])$ and $\int_a^b f \, d\alpha = \int_a^c f \, d\alpha + \int_c^b f \, d\alpha$.
6. Prove if $f \in \mathcal{R}(\alpha_1, [a, b]) \cap \mathcal{R}(\alpha_2, [a, b])$, then $f \in \mathcal{R}(\alpha_1 + \alpha_2, [a, b])$ and $\int_a^b f \, d(\alpha_1 + \alpha_2) = \int_a^b f \, d\alpha_1 + \int_a^b f \, d\alpha_2$
7. Let $f \in \mathcal{R}(\alpha, [a, b])$ and $c \in \mathbb{R}$ be positive. Prove $\int_a^b f \, d(c\alpha) = c \int_a^b f \, d\alpha$
8. Rudin page 138 problem 3
9. Rudin page 138 problem 5
10. Rudin page 138 problem 8
11. Prove the Cauchy-Schwarz inequality: If $f, g \in \mathcal{R}(\alpha, [a, b])$ and $f, g \geq 0$, then $\int fg \, d\alpha \leq (\int f^2 \, d\alpha)^{1/2} (\int g^2 \, d\alpha)^{1/2}$.
12. Rudin page 139 problem 11
13. Rudin page 139 problem 10abc
14. Rudin page 139 problem 12