MA 638 - Section 7.5 Homework

INSTRUCTIONS: The solutions to all problems here should be typed up in LaTeX using correct mathematical notation and language. Proofs and explanations must be written in complete sentences using correct grammar and punctuation. Use mathematical displays and *implication* or *if and only if* arrows where appropriate to manipulate equations or to validate steps in a proof or argument. Your completed pdf will be uploaded in Canvas. Be sure to save your pdf file using the naming format "first initial last name underscore sec7_5", i.e. Jane Doe would save her this homework as **jdoe_Sec7_5.pdf**.

Throughout assume R is a ring with identity $1 \neq 0$.

1. Suppose that R, D, and Q are defined as in the proof of Theorem 7.15. Use the equivalence relation along with the defined addition and multiplication operations defined in the proof of Theorem 7.15 to prove the following:

   (a) Prove that Q is an abelian group under addition with additive identity $\dfrac{0}{d}$ for any $d \in D$.

   (b) Prove that multiplication is associative, distributive, and commutative.

   (c) Prove that the map $i : R \to Q$ where $i(r) = \dfrac{rd}{d}$ where $d \in D$ is an injective ring homomorphism. (Remember we already showed that i is well-defined, so you only need to show that i preserves addition and multiplication, and that $\ker(i) = (0)$.)

2. Show by example that $R[x]$ does not have a *field of fractions* if R is not an integral domain.