MA 638 - Section 7.2 Homework

INSTRUCTIONS: The solutions to all problems here should be typed up in LaTeX using correct mathematical notation and language. Proofs and explanations must be written in complete sentences using correct grammar and punctuation. Use mathematical displays and *implication* or *if and only if* arrows where appropriate to manipulate equations or to validate steps in a proof or argument. Your completed pdf will be uploaded in Canvas. Be sure to save your pdf file using the naming format "first initial last name underscore section number", i.e. Jane Doe would save her this homework as **jdoe_7-2.pdf**.

Throughout assume R is a ring with 1.

1. Define $R[[x]]$ of *formal power series* in the indeterminate x with coefficients from R to be all formal infinite sums
$$\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots .$$
Define addition and multiplication of power series in the same way as for power series with real or complex coefficients i.e., extend polynomial addition and multiplication to power series as though they were "polynomials of infinite degree":
$$\sum_{n=0}^{\infty} a_n x^n \ + \ \sum_{n=0}^{\infty} b_n x^n \ = \ \sum_{n=0}^{\infty} (a_n + b_n) x^n$$
$$\sum_{n=0}^{\infty} a_n x^n \ \times \ \sum_{n=0}^{\infty} b_n x^n \ = \ \sum_{n=0}^{\infty} \left(\sum_{k=0}^{n} a_k b_{n-k} \right) x^n.$$
(The term "formal" is used here to indicate that convergence is not considered, so that formal power series need not represent functions on R.)

   (a) Prove that $R[[x]]$ is a commutative ring with 1.

2. Let S be any ring and let $n \geq 2$ be an integer. Prove that if A is any strictly upper triangular matrix in $M_n(S)$ then $A^n = 0$. (Recall, a strictly upper triangular matrix is one whose entries on and below the main diagonal are all zero.)