Homework 5

A portion of the following problems will be graded according to the provided rubric.

1. Let $f_n(x) = \frac{nx}{1+nx^2}$.
    a. Find the pointwise limit of $\{f_n\}$ for all $x \in (0, \infty)$.
    b. Is the convergence uniform on $(0, \infty)$?
    c. Is the convergence uniform on $(0, 1)$?
    d. Is the convergence uniform on $(1, \infty)$?
2. Rudin page 166 problem 5
3. Rudin page 166 problem 6
4. Let $f_n \to f$ pointwise and $f_n' \to g$ uniformly on $[a, b]$. Assume each f_n' is continuous, so that $\int_a^x f_n'\, d\alpha = f_n(x) - f_n(a)$ for all $x \in [a, b]$. Use this to prove $g(x) = f'(x)$.
5. Rudin page 166 problem 7
6. Let $g_n(x) = \frac{nx+x^2}{2n}$ and set $g(x) = \lim_{n\to\infty} g_n(x)$.
    a. Compute $g(x)$ by algebraically taking the limit as $n \to \infty$ and then find $g'(x)$.
    b. Compute $g_n'(x)$ for each $n \in \mathbb{N}$ and show the sequence of derivatives converges uniformly on every interval $[-M, M]$. Conclude $g'(x) = \lim_{n\to\infty} g_n'(x)$.
7. Rudin page 166 problem 8
8. Rudin page 166 problem 9
9. Use the Weierstrass M-Test to prove that if a power series $\sum_{n=0}^{\infty} a_n x^n$ converges absolutely at a point x_0, then it converges uniformly on the closed interval $[-c, c]$ where $c = |x_0|$.