

MA 630 - Homework 5 (Module 3 - Section 1)

Solutions must be typeset in \LaTeX and submitted to Canvas as a .pdf file. When applicable, write in complete sentences. Use only results which have been discussed in our class.

1. Let $n \in \mathbb{Z}$. Prove that $\gcd(5n + 2, 12n + 5) = 1$.
2. Let a and b be integers which are not both zero, and let $d = \gcd(a, b)$. Prove that $\gcd\left(\frac{a}{d}, \frac{b}{d}\right) = 1$.
3. Let $a, b, q, r \in \mathbb{Z}$ with $b \neq 0$. Prove that if $a = bq + r$, then $\gcd(a, b) = \gcd(b, r)$.
Hint: Let $d = \gcd(a, b)$. Use Theorem 3.9 to characterize $\gcd(b, r)$, and then show that $d = \gcd(b, r)$.
4. Use the Euclidean algorithm to find an integer x such that $2314x - 1$ is divisible by 3181.
5. Let $p \geq 5$ be a prime.
 - (a) Use the division algorithm to prove that there exists $k \in \mathbb{Z}$ such that either $p = 6k + 1$ or $p = 6k - 1$.
 - (b) Prove that 24 divides $p^2 - 1$. *Hint: The integer k in part (a) is either even or odd.*