1. Let Y_1, Y_2, Y_3 be an iid random sample from an Exponential distribution with density function $f(y) = \dfrac{1}{\theta} e^{-\frac{y}{\theta}}, y > 0$. Find the MSE for each of the following estimators of θ:
   (a) $\hat{\theta} = \frac{2}{3}Y_1 + \frac{1}{3}Y_2$
   (b) $\hat{\theta} = \overline{Y}$
   (c) $\hat{\theta} = Y_1^2$ *You may use a computer for any calculations needed here*
2. A Uniform random variable Y is defined on the interval $(a-2, a)$. Let $Y_1, ..., Y_n$ be a random sample taken from this distribution and $\overline{Y}$ is an estimator of a.
   (a) Compute the bias of $\overline{Y}$.
   (b) Find $MSE(\overline{Y})$.
3. For $X \sim Poisson(\lambda)$, prove that $\overline{X}$ is a consistent estimator for λ, using Markov's inequality.
4. Simulation (problems below the instructions)
   - To begin, create data which are distributed $Binomial(20, 0.15)$ and let $\overline{X}$ be an estimator for μ.
     - The code x = rbinom(500000, 20, 0.15) creates data that are drawn from a binomial experiment with 20 trials and probability of success 0.15.
     - To generate a matrix with 100,000 rows and 5 columns, use the following code: xmat = matrix(data=x, nrow=100000, ncol=5). Each row represents a sample of size 5, and there are 100,000 repetitions.
     - If the desired estimator for $\mu = np$ is $\overline{X}$, use the following code to generate $\overline{X}$ for each sample: xmn = apply(xmat, 1, mean). xmn is a vector of 100,000 sample means.
     - The sample mean of the sample means is found using: m1 = mean(xmn)
     - The sample variance of the sample means is found using: v1 = var(xmn)

   Save the sample mean and variance to answer questions below.

   - Now the same process as above should be followed for estimating μ, using data which are distributed Exponential(2).
     - The code x = rexp(2000000, 2)creates data drawn from an exponential distribution with parameter $\lambda = 2$.
     - To generate a matrix with 100,000 rows and 20 columns, use the following code: xmat = matrix(data=x, nrow=100000, ncol=20). Each row represents a sample of size 20, and there are 100,000 repetitions.
     - The code est = apply(xmat, 1, mean) will construct the mean of each sample
     - The sample mean of the estimators is found using: m2 = mean(est)
     - The sample variance of the estimators is found using: v2 = var(est)

   Save the sample mean and variance to answer questions below.

   (a) Notice that the simulated Bias is the average of all the estimated values minus the true value of the mean, and the simulated variance is the variance of all the estimated values. Report the simulated values for the Bias and MSE for the estimator $\overline{X}$ for μ when $X \sim Bin(20, 0.15)$, using a sample size of 5.

(b) What are the true values for $E[\overline{X}]$ and $V[\overline{X}]$ using the Binomial data? Are your simulated values close?

(c) Report the simulated values for the Bias and MSE for the estimator for μ when $X \sim Exp(2)$, using a sample size of 20.

(d) What are the true values for $E[\overline{X}]$ and $V[\overline{X}]$ using the Exponential data? Are your simulated values close?