1a. Uniform : $\frac{1}{b-a}$

$$\left.\begin{aligned} P(x|c_1) &= \tfrac{1}{2} \text{ for } -1 \le x \le 1 \\ p(x|c_2) &= \tfrac{1}{4} \text{ for } -2 \le x \le -2 \\ p(x|c_3) &= \tfrac{1}{8} \text{ for } -4 \le x \le 4 \end{aligned}\right\} \rightarrow 0 \text{ otherwise}$$

b. Optimal boundary at $P(C_i|X) = P(C_j|X)$

and classified as c_i if $i = \operatorname{argmax}\{P(C_j|x)\}$

$P(C_1|x) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ for $-1 \le x \le 1 \rightarrow$ MAP for $-1 \le x \le 1$

$P(C_2|x) = \frac{1}{4} \cdot \frac{1}{3} = \frac{1}{12}$ for $-2 \le x \le 2 \rightarrow$ MAP for $-2 \le x < 1$ and $1 < x \le 2$

$P(C_3|x) = \frac{1}{8} \cdot \frac{1}{6} = \frac{1}{48}$ for $-4 \le x \le 4 \rightarrow$ MAP for $-4 \le x < -2$, $2 < x \le 4$

Note: Boundaries at $x = \pm 1, \pm 2$, and ± 4

c. $\underbrace{\int_{-1}^{1} \frac{1}{48} dx}_{\text{pred } c_1 \text{ but actual } c_3} + \underbrace{\int_{-1}^{1} \frac{1}{12} dx}_{\text{Pred } c_1 \text{ but actual } c_2} + \underbrace{\int_{-2}^{1} \frac{1}{48} dx}_{\text{Pred } c_2 \text{ but } c_3} + \underbrace{\int_{1}^{2} \frac{1}{48} dx}_{\text{Pred } c_2 \text{ but } c_3}$

$$\frac{1}{24} + \frac{1}{6} + \frac{3}{48} + \frac{3}{48} = \frac{1}{3}$$

2a. $L(w \mid X, Y) = \prod_{n=1}^{N} \left(\frac{\exp(w_{y_n}^T x_n)}{1 + \sum_{i=1}^{K-1} \exp(w_i^T x_n)} \right) \left(\frac{1}{1 + \sum_{i=1}^{K-1} \exp(w_i^T x_n)} \right)$

for $y_n < k$ for $y_n = k$

b. $\log L(w \mid X, Y) = \sum_{n=1}^{N} \sum_{k=1}^{K} \left(w_k^T x_n - \log\left(1 + \sum_{i=1}^{k-1} \exp(w_i^T x_n)\right)\right)$

$\frac{\partial \log L(w \mid X, Y)}{\partial w_i}$ is gradient or ∇L

I couldnt figure out the calculus

$w_i^{new} = w_i^{old} + \eta \nabla L$