


To minimize residual projections:

$$\min \sum \|x_i - (x_i^T u)u\|^2$$

$$\min \sum x_i^T x_i - \sum (x_i^T u)^2 + (x_i^T u)^2$$

$$\min \sum x_i^T x_i - (x_i^T u)^2 \text{ where } \sum x_i^T x_i = 0 \text{ due to centering}$$

$$\min - \sum (x_i^T u)^2 \text{ is same as } \max \sum (x_i^T u)^2$$

Variance of projected point is

$$\text{So solve } \max \sum_{i=1}^n \left[x_i \cdot u - \left(\frac{1}{n} \sum_{i=1}^N x_i \cdot u \right) \right]$$

Since $\frac{1}{N} \sum_{i=1}^N x_i = 0$, the projection is also 0.

So variance simplifies to

$$\max \sum_{i=1}^n (x_i^T u)^2$$
