Neuroprosthetics - Exercise 2

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24. November 2019

1 Plot Slope Fields and Isoclines

Figures 1 and 2 show the slope fields and isoclines for the ordinary differential equations (ODE) 1 and 2 respectively. An isocline is a line at which the slope (here $\frac{dV}{dt}$) equals to a constant k. The isoclines for the set of constants $K = \{-2, -1, 0, 1, 2\}$ are plotted.

$$\frac{dV}{dt} = 1 - V - t \tag{1}$$

$$\frac{dV}{dt} = sin(t) - \frac{1}{1.5}V \tag{2}$$

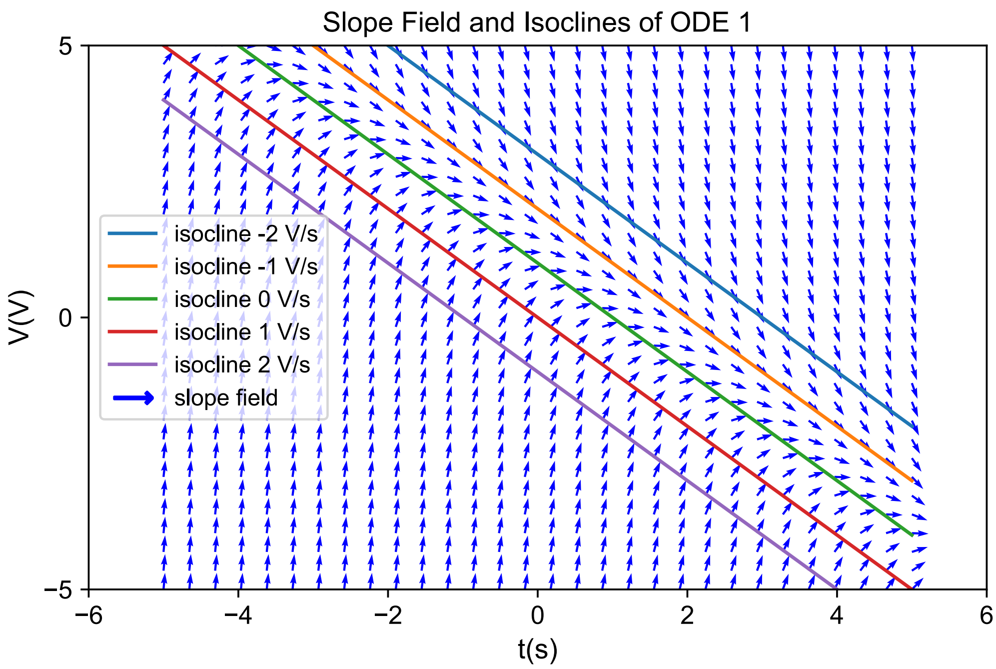

Figure 1: Slope field and isoclines of ODE 1

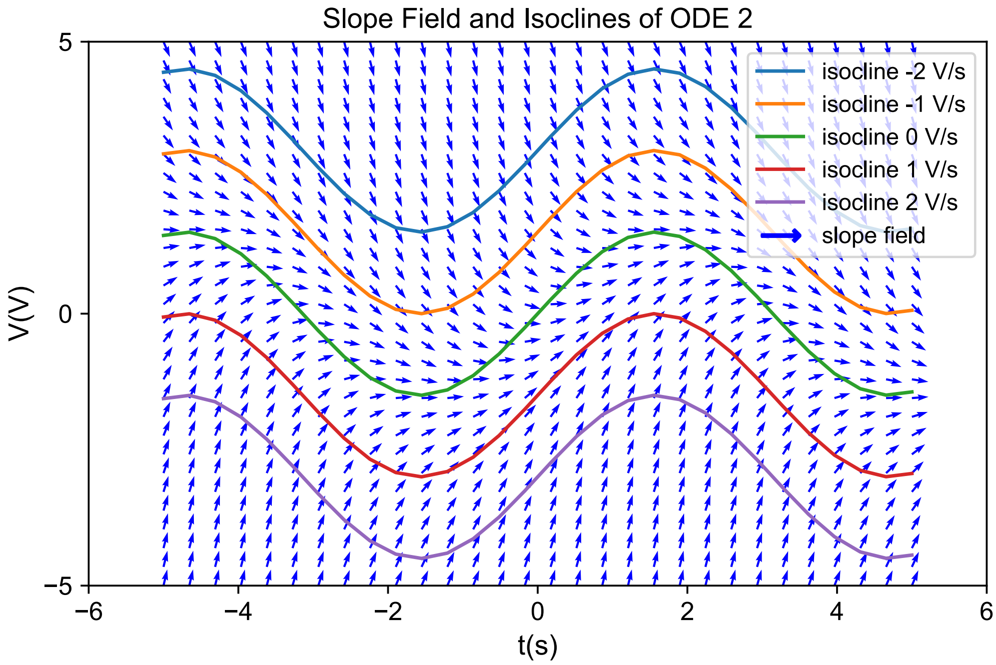

Figure 2: Slope field and isoclines of ODE 2

2 Differential Equations of a Simple Cell Model

The differential equation of the leaky integrate and fire neuron model can be derived from Kirchhoffs law as shown in equation 3. The external current is known and defined by equation 4. The expression for the current through the resistor I_r in equation 5 can simply be derived from Ohm's law $R = \frac{V}{I}$. Furthermore, the capacitor's displacement current in equation 6 can be derived from the relations $I = \frac{dQ}{dt}$ and $Q = C \cdot V$.

$$I_{ex}(t) = I_r(t) + I_c(t) \tag{3}$$

$$I_{ex}(t) = I_{max} \cdot sin(t) \tag{4}$$

$$I_r(t) = \frac{V_r(t)}{R_l} \tag{5}$$

$$I_c(t) = C_m \cdot \frac{dV}{dt} \tag{6}$$

By plugging the results for the components' currents in the original formula 3 we get equation 7 and by rearranging we obtain the final governing differential equation 8. The exercise asks to include another constant term D to the current part of the differential equation, which is done in equation 9.

$$I_{max} \cdot sin(t) = \frac{V_r(t)}{R_l} + C_m \cdot \frac{dV}{dt} \tag{7}$$

$$\frac{dV}{dt} = \frac{1}{C_m}\left(I_{max} \cdot sin(t) - \frac{V_r(t)}{R_l}\right) \tag{8}$$

$$\frac{dV}{dt} = \frac{1}{C_m}\left(I_{max} \cdot sin(t) + D - \frac{V_r(t)}{R_l}\right) \tag{9}$$

Finally, plots of this ordinary differential equation are presented. There are four plots with the following parameters.

R	C_m	I_{max}	D	Plot
1.3Ω	0.8F	0A	0A	A
1.3Ω	0.8F	1A	0A	B
1.3Ω	0.8F	0A	2A	C
1.3Ω	0.8F	1A	2A	D

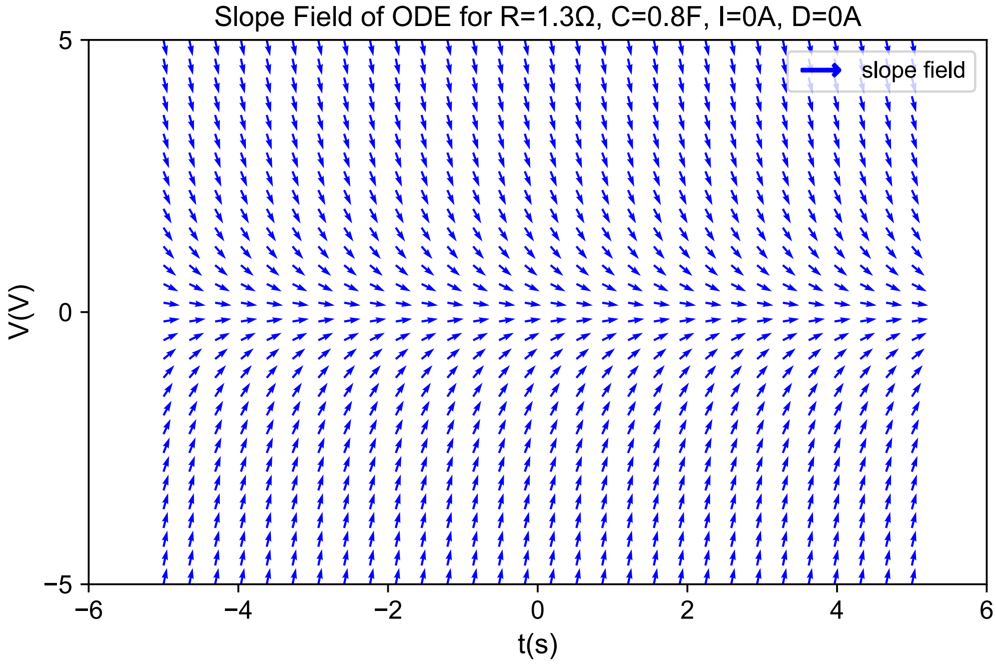

Figure 3: Plot A

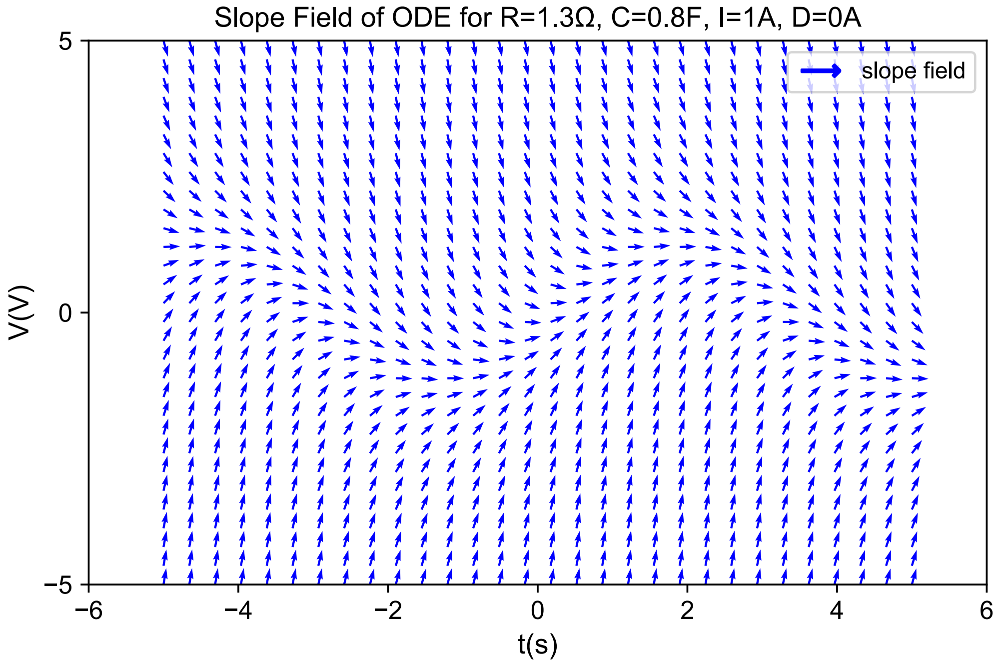

Figure 4: Plot B

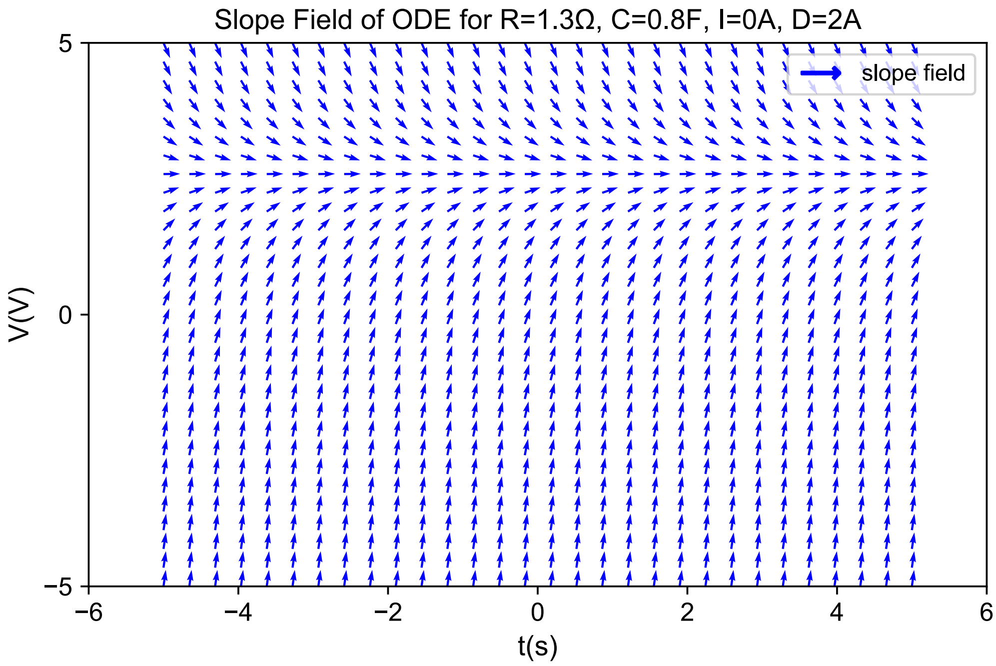

Figure 5: Plot C

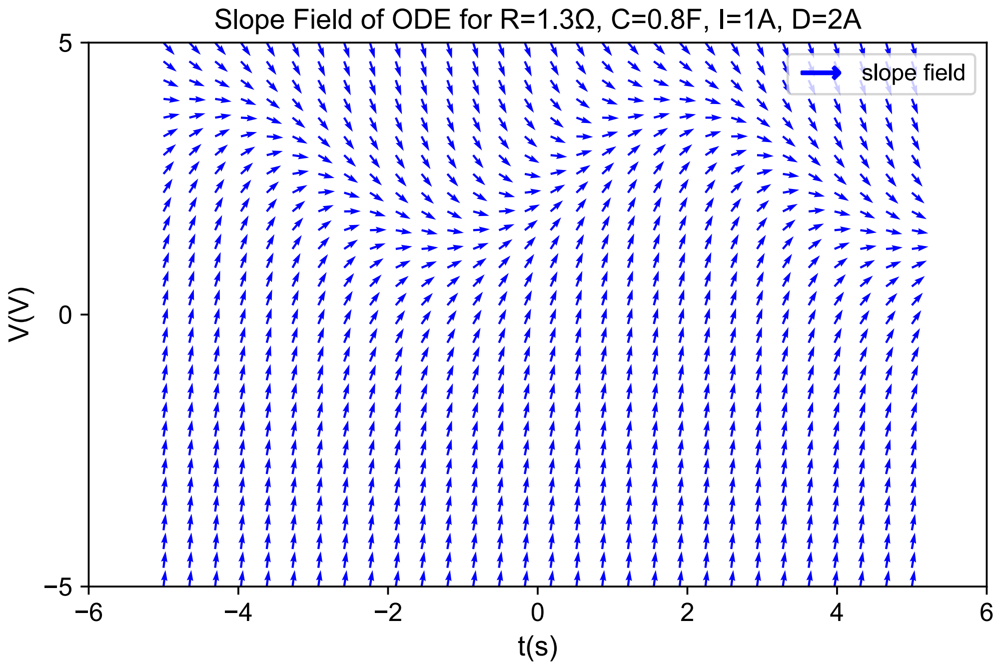

Figure 6: Plot D