1 Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

2 Integration of Trigonometric Functions

Trigonometric Substitution

$$\begin{array}{ll}
\int f(\sqrt{1-x^2})\,dx & x = \sin\theta \text{ or } \cos\theta \\
\int f(\sqrt{a^2+x^2})\,dx & x = a\tan\theta \\
\int f(\sqrt{a^2-x^2})\,dx & x = a\sin\theta \text{ or } a\cos\theta \\
\int f(x\sqrt{x^2-a^2})\,dx & x = a\sec\theta
\end{array}$$

Useful properties/identities

If you see $\int \sin(A)\cos(B)dx$, remember that

$$\sin(A+B) = \sin A\cos B + \sin A\cos B$$
$$\sin(A-B) = \sin A\cos B - \sin A\cos B$$

That means that

$$\sin(A+B) + sin(A-B) = 2\sin A\cos B$$

$$\int f(\sin x, \cos x)dx$$
$$= \int f(\sin x)\cos x dx = \int f(u)du, u = \sin x$$

$$\int f(\sec x, \tan x)dx$$
$$= \int f(\cos x)\sin x dx = -\int f(u)du, u = \cos x$$

$$\int f(\csc x, \cot x)dx$$
$$= \int \cos^5 x \sin^3 x dx = \int \cos^5 x \sin^2 x \sin x dx$$
$$= \int \cos^5 x (1 - \cos^2 x)\sin x dx$$
$$= -\int u^5 (1 - u^2) du, u = \cos x$$
$$= -\frac{u^6}{6} + \frac{u^8}{8} + c$$

$$\int f(\tan x)\sec^2 x dx = \int f(u)du, u = \tan x$$

$$\int f(\sec x)\sec x \tan x dx = \int f(u)du, u = \sec x$$

$$\int \sec^4 x dx = \int \sec^2 x \sec^2 x dx$$
$$= \int (1 + \tan^2 x)\sec^2 x dx$$
$$= \int (1 + u^2)du = u + \frac{u^3}{3} + c, u = \tan x$$

$$\int \sec x dx = \ln \sec x + \tan x + c$$

$$I = \int \sec^3 x dx = uv - \int v du$$
Through IBP: $u = \sec x, du = \sec x \tan x dx, v = \tan x, dv = \sec^2 x dx$
$$= \int \sec x \tan x - \int \sec x \tan^2 x dx$$
$$= \int \sec x \tan x - \int \sec x (\sec^2 x - 1)dx$$
$$= \int \sec x \tan x - \int \sec^3 x dx - \int \sec x dx$$
$$= \int \sec x \tan x + \int \sec x dx - I$$