03/18/2024

M10: Mathematics in the Modern World

1. Terminologies

   (a) Data - collection of measured information. Can be qualitative or quantitative.

      i. Univariate - single measurement obtained

      ii. multivariate - multiple measurements obtained

   (b) Variable - characteristic of an object or person

      i. Discrete - represented by integers

      ii. Continuous - represented in decimal form

   (c) Data Management - practice of managing data as a valuable resource.

   (d) Population vs Sample Descriptive Measures

      i. Population - described by parameters

      ii. Sample - described by statistics

2. Sample Problems

   (a) Find the mean and median monthly income of 100 families if 95 of them earn 20,000 pesos per month and the rest earn 300,000 pesos per month. What can you infer from the values of the mean and median in this example?

$$samplemean = \bar{x}$$
$$\bar{x} = 9T(20,000) + 5(300,000)/(100)$$
$$\bar{x} = 34,200 \qquad \text{(mean monthly income)}$$

$$samplemedian = \tilde{x}$$
$$\tilde{x} = 20,000 \qquad \text{(median monthly income)}$$

   (b) Find the mean and median age on the table.

$$mean = \bar{x}$$
$$\bar{x} = \frac{7(3) + 8(4) + 9(6) + 10(15) + 11(11) + 12(7) + 13(1)}{3 + 4 + 6 + 15 + 11 + 7 + 1}$$
$$\bar{x} = 10$$

   (c) A teacher grades on 5 tests, a project, and a final exam. Each test counts as 10 percent of the course grade and the final exam counts as 30 percent of the course grade. Sam's test scores (in percent) are

70, 65, 82, 94, and 85. Her project score is 92 percent and her final score is 80 percent. Find Sam's average for the course.

$$
\begin{aligned}
weightedmean &= \bar{x} \\
\bar{x} &= (70 + 65 + 82 + 94 + 85)(0.10) + (92)(0.20) + (85)(0.30) \\
\bar{x} &= 82
\end{aligned}
$$

(d) Two softdrink dispensing machines were designed to dispence 8 ounces of liquid into a cup. The amount of soft drink dispensed by the machine in 5 samples is recorded in the table below. How many ounces of softdrink are being dispensed by each machine? Give a valuable insight based on the data (on the table) and your computation.

$$
\begin{aligned}
Machine1 &= \bar{x_1} \\
\bar{x_1} &= \frac{(9.52)(6.41)(10.07)(5.85)(8.15)}{5} \\
\bar{x_1} &= 8
\end{aligned}
$$

$$
\begin{aligned}
Machine2 &= \bar{x_2} \\
\bar{x_2} &= \frac{(8.01)(7.99)(7.95)(8.03)(8.02)}{5} \\
\bar{x_2} &= 8
\end{aligned}
$$

(e) Using the previous problem, find the range of the two machines.

$$
\begin{aligned}
Machine1 &= R_1 \\
R_1 &= 10.07 - 5.85 \qquad (1) \\
R_1 &= 4.22
\end{aligned}
$$

$$
\begin{aligned}
Machine2 &= R_2 \\
R_2 &= 8.03 - 7.95 \qquad (2) \\
R_2 &= 0.08
\end{aligned}
$$

3. On Culminating Activity

   (a) Think of your groupmates (:heart emoji:)(:heart emoji:)(:heart emoji:)

   (b) Think of a theme for your project similar to what sir will present.

   (c) Think of a topic you can talk about.

      i. Discuss about use of math in your topic.

      ii. Discuss a problem related to your topic.

      iii. Make a proposal to address the problem using data management tools.

M31.2: Mathematical Analysis IB

1. Discussion

   (a) $\int f(sinx, cosx)\,dx = \int u\,du = F(u), u = sinx$
   or
   $\int f(sinx, cosx)\,dx = -\int u\,du = -F(u), u = cosx$

   Example:
   $\int cos^5(x)sin^3(x)\,dx$
   $\int cos^5(x)(1 - cos^2(x))sin(x)\,dx =$
   $\int u^5(1 - u^2)\,du =$
   $\int u^5 - u^7\,du = \frac{u^6}{6} - \frac{u^8}{8}$

   (b) $\int sec^3(x)\,dx$
   Let: $u = secx; du = sectanxdx; dv = sec^2(x)dx; v = tanx$
   $\int sec^3(x)\,dx = secxtanx - \int sec(x)tan^2(x)\,dx$
   $\int sec^3(x)\,dx = secxtanx - \int sec(x)(sec^2(x) - 1)\,dx$
   $\int sec^3(x)\,dx = secxtanx - \int sec^3(x) - sec(x)\,dx$
   $2\int sec^3(x)\,dx = secxtanx + \int sec(x)\,dx$
   $2\int sec^3(x)\,dx = secxtanx + ln|secx + tanx|\,dx$
   $\int sec^3(x)\,dx = \frac{secxtanx+ln|secx+tanx|}{2}\,dx$

   (c) $\int f(x\sqrt[2]{a^2 - x^2})\,dx; x = asin\theta orx = acos\theta$

   Example: $\int \frac{x^2}{\sqrt{1-x^2}}\,dx$
   $\int \frac{sin^2(\theta)}{cos\theta}cos\theta\,d\theta$

   (d) $\int f(x\sqrt[2]{a^2 + x^2})\,dx; x = atan\theta$

   (e) $\int f(x\sqrt[2]{x^2 - a^2})\,dx; x = asec\theta$

   (f) $\int (sin^2(\theta))\,d\theta$

   (g) cos

   (h) $\int sin^2(x)cos^2(x)\,dx$
   $\int (\frac{1+cos2x}{2})(\frac{1-cos2x}{2})\,dx$
   $\int (\frac{1-cos^2(2x)}{4})\,dx$
   $\int \frac{1}{4}\,dx - \int \frac{cos^2(2x)}{4}\,dx$

$\frac{x}{4} - (\frac{1}{4}) \int \frac{1+cos(4x)}{2}$
$\frac{x}{8} - (\frac{1}{8}) \frac{sin(4x)}{4}$

Alternatively...

(i) $\int sin^2(x)cos^2(x)\,dx$
$\int (sinxcosx)^2\,dx$
$\frac{1}{4}\int sin^2(2x)\,dx$
$\frac{1}{4}\int \frac{1-cos(4x)}{2}\,dx$
$\frac{x}{8} - (\frac{1}{8}) \frac{sin(4x)}{4}$

(j) $\int e^{3y}\sqrt{1-e^{2y}}\,dy$
$\int (e^{2y}\sqrt{1-e^{2y}})e^y\,dy$ Let: $u = e^y$
$\int (u^2\sqrt{1-u^2})\,dy$ Let: $u = sin\theta; \theta = sin^{-1}(y)$
$\int sin^2\theta cos^2\theta\,d\theta$
$\frac{\theta}{8} - \frac{sin(4\theta)}{32} + C$
$\frac{sin^{-1}(y)}{8} - \frac{2sin(2\theta)cos(2\theta)}{32} + C$
$\frac{sin^{-1}(y)}{8} - \frac{(2(sin\theta)(cos\theta))(cos^2\theta - sin^2\theta)}{16} + C$
$\frac{sin^{-1}(y)}{8} - (\frac{1}{8})(e^y\sqrt{1-e^{2y}}(1-e^{2y}-e^{2y})) + C$

(k) $\int sin(3x)cos(5x)\,dx$
Let: $u = sin(3x); du = 3cos(3x)dx; dv = cos(5x)dx; v = \frac{sin(5x)}{5}$
$\frac{1}{5}(sin(3x)cos(5x)) - \frac{3}{5}\int sin(5x)cos(3x)\,dx$
Let: $a = cos(3x); da = -3sin(3x)dx; db = sin(5x)dx; b = -\frac{cos(5x)}{5}$
$\frac{1}{5}(sin(3x)cos(5x)) - \frac{3}{5}[-\frac{1}{5}cos(3x)cos(5x) - \frac{3}{5}\int sin(3x)cos(5x)]$
$\frac{34}{5}\int sin(3x)cos(5x)\,dx = \frac{1}{5}(sin(3x)cos(5x)) + \frac{3}{25}cos(3x)cos(5x)$
$\int sin(3x)cos(5x)\,dx = [\frac{1}{5}(sin(3x)cos(5x)) + \frac{3}{25}cos(3x)cos(5x)]\frac{5}{34} + C$

Alternatively... Remember that:
sin(A+B)=sinAcosB+cosAsinB
sin(A-B)=sinAcosB-cosAsinB
sin(A+B)sin(A-B)=2sinAcosB
In the example: A=3x; B=5x
Easier to solve with it:
$\frac{1}{2}\int sin(8x) - sin(2x)\,dx$
$\frac{-cos(8x)}{16} + \frac{cos(2x)}{4}$