04/25/2024

PHYS31.01: Analytical Physics 1

1. **Kepler's Laws of Planetary Motion**
   1. The planet is moving in an elliptical orbit, with sun is **located at one of the focii** of the ellipse. The shortest distance of planet from its star is **perihelion** while the farthest is **aphelion**.

   2. A line from the sun to a given planet sweeps out **equal areas in equal times**.
   Justification: Angular Momentum Conservation
   Consequences: Perihelion has fastest speeed while aphelion has slowest speed.

   3. The periods of the planets are proportional to the 3/2 poers of the major axis lengths of their orbits.

$$
\begin{aligned}
T_{per} &= \frac{2\pi a^{\frac{3}{2}}}{\sqrt{GM_{cntr}}} \\
T^2 &\propto a^3 \\
where\ a &: \ length\ of\ semi\ major\ axis \\
M_{cntr} &: \ mass\ of\ star
\end{aligned}
$$

$$
\begin{aligned}
&for\ perfectly\ circular\ path... \\
\frac{Gm_s}{R} &= v^2 \\
T^2 &= \frac{4\pi^2}{Gm_s} R^3
\end{aligned}
$$

2. **Periodic Motion**
   Conditions for Oscillation in SHM:
   1. The translational and angular acceleration and angular acceleration is *proportional and of opposite direction* as the translational and angular displacement.
   2. Conditions are valid if the amplitude is small.

$$\alpha + {\omega_f}^2\theta = constant$$
$$\alpha: \ angular\ acceleration$$
$$\omega_f: \ angular\ frequency$$

For object oscillating along ***LINEAR DIRECTION***

$$x(t) = Acos(\omega_f t + \delta)$$
$$A: \ amplitude\ of\ oscillation$$
$$\delta: \ phase\ constant$$
$$\omega_f: \ angular\ frequency\ of\ oscillation$$

MATH 10: Mathematics in the Modern World

1. **Konigsberg's Bridge Problem**
   - In 1736, Euler was tasked to solve the *seven bridge problem.* He found it to be **IMPOSSIBLE** to solve.

2. **Eulerian Graphs**
   - Graphs that can start at a vertex and traverse through **ALL** edges and **RETURN** to the starting vertex.
   Examples: Cycle Graph
   NOT Examples: Path Graph, 7-Bridge Problem

3. **Example:**
   NOT Eulerian

4. **WHY???**
   THEOREM: *A connected graph is Eulerian if and only if each of its vertices has even degree.*

5. **Hamiltonian Graphs**
   - A closed path that starts with one vertex and traverses all other vertices exactly **ONCE**, and **RETURNS** to the starting vertex.
   - The closed path is a hamiltonian cycle.
   - A graph is **NOT** exclusively Hamiltonian or Eulerian. A graph can either be *HAMILTONIAN AND EULERIAN*, *BOTH NOT*, or *either EACH.*

6. **Dirac's Theorem**
   If G is a simple graph with $n \geq 3$ vertices such that the degree of every

vertex in G is at least $\frac{n}{2}$, then G is Hamiltonian.

FALSE *because for some cases, e.g. very large cycles, it can fail.*

7. **Ore's Theorem**
   If G is a simple graph with $n \geq 3$ vertices such that $deg(u) + deg(v) \geq n$ for every pair of nonadjacent vertices u and v in G, then G is Hamiltonian.

8. **Relation of Dirac's and Ore's Theorem**
   Dirac's is a corollary to Ore's because Dirac is a more specific case. Ore's is a more generalized version.

9. **Knight's Tour Problem**
   Is it possible?
   YES.

10. **Travelling Salesman feat. Hamilton Circuit**
    WALA AKONG PIC LOL