04/18/2024

M31.2: Mathematical Analysis IB

1. **Formulas to Remember:**

$$
\begin{aligned}
x &= rcos\theta \\
y &= rsin\theta
\end{aligned}
$$

$$
\begin{aligned}
\frac{y}{x} &= \frac{rsin\theta}{rcos\theta} \\
\frac{y}{x} &= tan\theta \\
\theta &= tan^{-1}(\frac{y}{x})
\end{aligned}
$$

$$
\begin{aligned}
x^2 + y^2 &= r^2cos^2(\theta) + r^2sin^2(\theta) \\
x^2 + y^2 &= r^2 \\
r &= \sqrt{x^2 + y^2}
\end{aligned}
$$

2. **Examples:**

   (a) $(x, y) = (3, -3)$ give $(r, \theta), 2\pi \leq \theta \leq 4\pi$

$$
\begin{aligned}
r &= \sqrt{3^2 + (-3)^2} \\
&= 3\sqrt{2}
\end{aligned}
$$

$$
\begin{aligned}
\theta &= 2\pi + \frac{7\pi}{4} \\
&= \frac{15\pi}{4}
\end{aligned}
$$

   (b) $(x, y) = (3, -3)$ give $(r, \theta), r < 0, -\pi \leq \theta \leq \pi$

$$
r = -3\sqrt{2}
$$

$$
\theta = \frac{3\pi}{4}
$$

(c) $(x, y) = (-4, -4\sqrt{3})$ give $(r, \theta), r < 0, -8\pi \leq \theta \leq -6\pi$

$$\begin{aligned} r &= \sqrt{(-4)^2 + (-4\sqrt{3})^2} \\ &= 8 \\ &\Rightarrow -8 && (R < 0) \end{aligned}$$

$$\begin{aligned} \theta &= -2\pi + \frac{\pi}{3} \\ &= \frac{-5\pi}{3} \\ &\Rightarrow \frac{-23\pi}{3} && (-8\pi \leq \theta \leq -6\pi) \end{aligned}$$

(d) $(x, y) = (1, 2)$ give $(r, \theta), r < 0, 2\pi \leq \theta \leq 4\pi$

$$\begin{aligned} r &= \sqrt{(1)^2 + (2)^2} \\ &= \sqrt{5} \\ &\Rightarrow -\sqrt{5} && (R < 0) \end{aligned}$$

$$\begin{aligned} \theta &= 1.11 \\ &\Rightarrow 2\pi + 1.11 && (2\pi \leq \theta \leq 4\pi) \\ &\Rightarrow 3\pi + 1.11 && (r < 0) \end{aligned}$$

(e) $(x, y) = (1, -2)$ give $(r, \theta), r < 0, 2\pi \leq \theta \leq 4\pi$

$$\begin{aligned} r &= \sqrt{(-1)^2 + (-2)^2} \\ &\Rightarrow \sqrt{5} && (R > 0) \end{aligned}$$

$$\begin{aligned} \theta &= 1.11 \\ &\Rightarrow 2\pi + 1.11 && (2\pi \leq \theta \leq 4\pi) \end{aligned}$$

(f) **LAHAT NAMAN TAYO AY MAMAMATAY!!!!**

3. **Area of a Circle**
   - it is a known fact that the area of a circle is $[A = \frac{1}{2}r^2\theta]$.

- this can be expressed better using the integral function $[A = \frac{1}{2}\int_{\alpha}^{\beta} f(\theta)^2 d\theta]$, where $r = f(\theta)$.

(a) $r = 2cos\theta$

$$\begin{aligned} r^2 &= 2rcos\theta \\ x^2 + y^2 &= 2x \\ x^2 - 2x + y^2 &= 0 \\ x^2 - 2x + 1 + y^2 &= 1 \\ (x-1)^2 + y^2 &= 1 \end{aligned}$$

$$\begin{aligned} A &= (2)(\frac{1}{2})\int_0^{\pi/2} (2cos\theta)^2 d\theta \\ &= 4\int_0^{\pi/2} cos^2\theta d\theta \\ &= 4\int_0^{\pi/2} \frac{1 + cos^2\theta}{2} d\theta \end{aligned}$$

$$\begin{aligned} r &= 0 \\ 2cos\theta &= 0 \\ \theta &= \frac{\pi}{2} \end{aligned}$$

$$\begin{aligned} &= 2[\theta + \frac{\sin 2\pi}{2}]_0^{\pi/2} \\ &= 2(\frac{\pi}{2} - 0) = \pi \end{aligned}$$

(b) $r = 2sin(2\theta)$

FUN FACT:

$$r = asin(n\theta)$$
$$r = acosn(n\theta)$$

If n is even, rose with 2n petals.

$$r_{max} = 2 \ or \ = -2$$
$$r_{min} = 0$$

$$sin(2\theta) = +-1$$
$$\theta = \frac{\pi}{4}, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$$

$$sin(2\theta) = 0$$
$$\theta = 0, \frac{\pi}{2}, \pi, -\frac{\pi}{2}$$

$$A = (8)(\frac{1}{2}) \int (2sin(2\theta))^2 d\theta$$