04/29/2024

PHYS31.01: Analytical Physics 1

1. **Equation of Motion of Linear Oscillation**

$$\begin{aligned} a + \omega_f^2 x &= constant \\ a &: \ acceleration \\ \omega_f^2 &: \ proportionality\ constant \\ x &: \ displacement\ of\ object\ about\ equilibrium\ point \end{aligned}$$

2. **Some Formulas**

$$\begin{aligned} &Linear\ Oscillation \\ &a = -\omega_f^2 x \\ &v_{max} = \omega_f A \\ \\ &Rotational\ Oscillation \\ &\alpha = -\omega_f^2 \theta \\ &\omega_{max} = \omega_f^2 A \end{aligned}$$

3. **Equation of Motion of Linear Oscillation**
   Summary of Oscillation Parameters:
   Amplitude: maximum angular displacement
   Period: time to complete one cycle
   Phase: indicates lead or lag of oscillation
   INC

4. **Plots**
   Just some calculus of trig functions lol

5. **Phase Constant**
   When phase constant is *zero*, the position is *max* and velocity is *zero*.
   When phase constant is *negative*, the position
   When phase constant is *positive*, the position
   INC

6. **Problems Involving SHM**
Linear Oscillation (Horizontal)

Recall: Hooke's Law
$F_{spring} = -kx$
How??

$$ma = -kx$$
$$a = -\frac{k}{m}x$$
$$Recall : a = -\omega_f^2 x$$
$$\omega_f^2 = \frac{k}{m}$$
$$\omega_f = \sqrt{\frac{k}{m}}$$

$$f = \frac{\omega_f}{2\pi} = \frac{1}{2\pi}\sqrt{k/m}$$
$$Period(T_{per}) = 2\pi\sqrt{m/k}$$

$$Linear\ Osci$$
$$a + \frac{k}{m}x = 0$$

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Linear Oscillation (Vertical)

$$a + \frac{k}{m}y = -g$$
$$-ky - mg = ma$$

$$\omega_f = \sqrt{k/m}$$

Rotational Oscillation (Simple Pendulum)
The gravitational force does torque on a simple pendulum

Equation of Motion:

$$-mgLsin\theta = I\alpha = mL^2\alpha$$
$$-gsin\theta = L\alpha$$
$$sin\theta \approx \alpha \textit{(for small oscillations)}$$
$$\alpha + \frac{g}{L}\theta = 0$$

Solution:

$$\theta(t) = \theta_0 cos(\omega_f t + \delta)$$

Angular Frequency:

$$\omega_f = \sqrt{\frac{g}{L}}$$

Rotational Oscillation (Physical Pendulum)
The gravitational force does torque on a physical pendulum

Equation of Motion:

??

Solution:

$$\theta(t) = \theta_0 cos(\omega_f t + \delta)$$

Angular Frequency:

$$\omega_f = \sqrt{\frac{MgL}{I}}$$

Math 10: Mathematics in the Modern World

1. **Centrality**
   - a concept in graph theory used to measure the *importance* of certain vertices in a graph. It can be described in a variety of ways.
   - assigns a numerical value to a vertex that helps you compare vertices in your netwrok in terms of importance and criticality.

   (a) **Degree Centrality**
   - the higher the degree, the more important a node is (larger immediate connections).
   Ex: (Social media connections)

   (b) **Closeness Centrality**
   - the *geodesic distance* between two vertices u and v is the number of edges, if any, on a shortest path between u and v. If the vertices are not connected by a path, their geodesic distance is inf.
   - the *closeness centrality* of a vertex is the sum of the geodesic distance between that vertex and all the other vertices in the network, i.e.

   $$C(u) = \sum_{i=1}^{n} d(u, v_i)$$

   - vertices with high closeness centrality are those that can access the rest of the vertices with <u>less work.</u>

   (c) **Betweenness Centrality**
   - measures the importance of a vertex in a network based upon how many times it occurs in the shortest path between all pairs of vertices in a graph.

- It is given as follows:

$$B(u) = \sum_{a,b \in V(G)} \frac{\textit{no. of shortest paths between a and b, passing through u}}{\textit{no. of shortest paths between a and b}}$$

- Vertices with high betweenness centrality are important because their absence or removal in the network would *disrupt* the paths between many pairs of vertices in the network.

2.