Problem Set 1
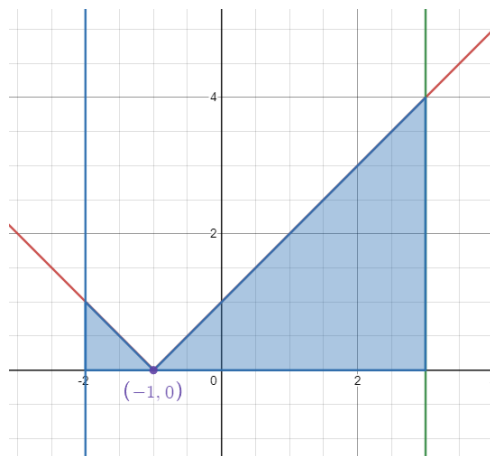
1. Find the area of region bounded by the x-axis, $x = -2$, $x = 3$, and $y = |x+1|$.

Solution:

$$A = \int_{-2}^{-1} -(x+1)\,dx + \int_{-1}^{3} x+1\,dx$$

$$= -\int_{-2}^{-1} x+1\,dx + \int_{-1}^{3} x+1\,dx$$

$$= \left[\frac{x^2}{2} + x\right]_{-2}^{-1} + \left[\frac{x^2}{2} + x\right]_{-1}^{3}$$

$$= \left(\frac{1}{2}\right) + \left(\frac{15}{2} + \frac{1}{2}\right)$$

$$= \frac{17}{2}$$

2. If $\frac{df}{dx} = e^x - 2x$ and $f(0) = 2$, obtain $f(x)$.

Solution:

$$\int \frac{df}{dx}\,dx = f(x) + C$$

$$\int e^x - 2x\,dx = f(x) + C$$

$$e^x - x^2 = f(x) + C$$

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If f(0) = 2, then:

$$e^0 - x^0 = 2 + C$$
$$1 + C = 2$$
$$C = 1$$

Thus:

$$f(x) = e^x - x^2 + 1$$

5. Write the following limit of a Riemann sum as a definite integral and obtain its value:

$$\lim_{n \to \infty} \frac{1^2 + 2^2 + ... + n^2}{n^3}$$

Solution:
Rewrite: