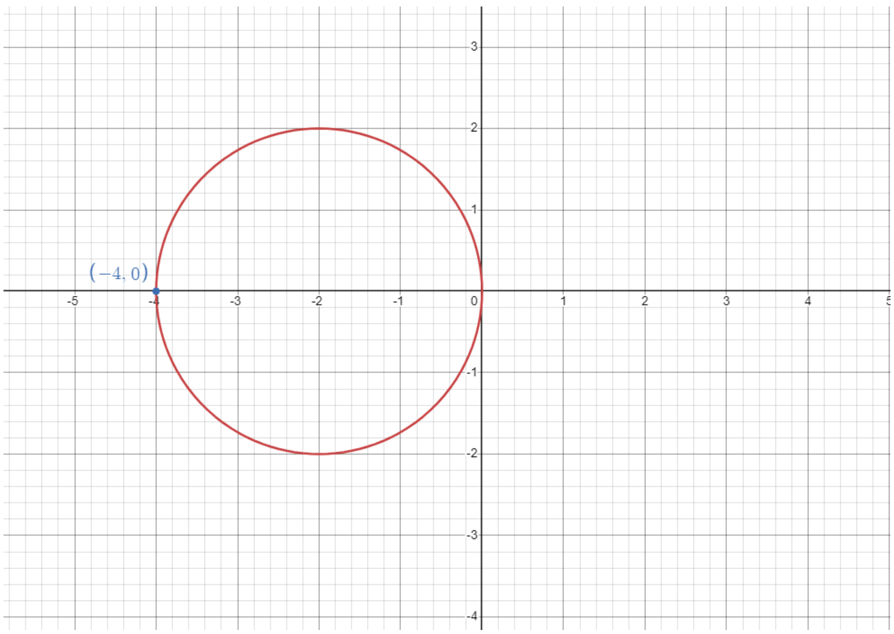
Review (05/07/2024)

1. Sketch the graph of $r = -4\cos\theta$.

Solution:



Alternative way to solve 1:

$$\begin{aligned} r &= -4\cos\theta \\ r^2 &= -4r\cos\theta \\ &= -4x \end{aligned}$$

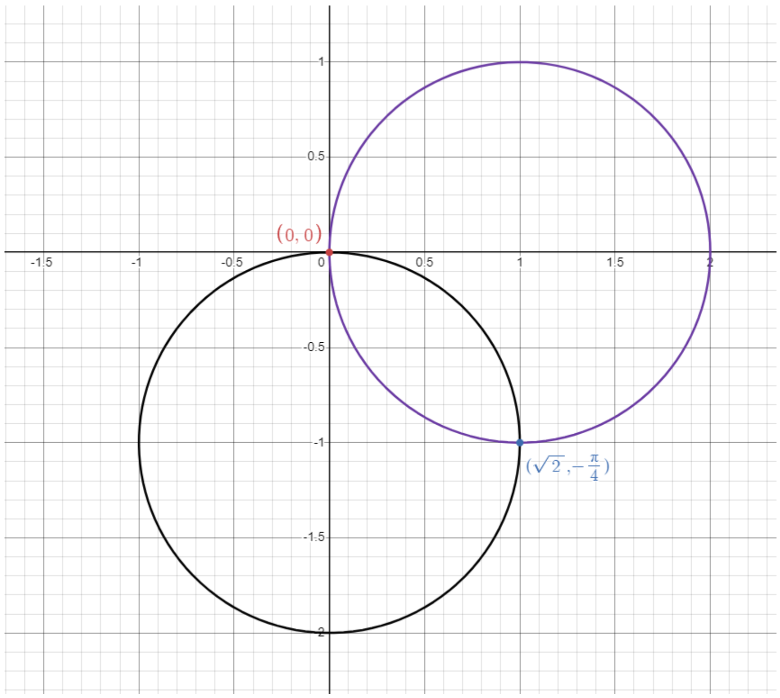
so:

$$\begin{aligned} x^2 + y^2 = r^2 \to x^2 + y^2 &= 4x \\ x^2 + 4x + y^2 &= 0 \\ (x^2 + 4x + 4) + y^2 &= 4 && \text{(completing the square)} \\ (x+2)^2 + y^2 &= 2^2 \end{aligned}$$

essentially, we get an equation of a circle of centre $(-2, 0)$ and radius 2.

2. Sketch the graph of $r = 2\cos\theta$ and $r = -2\sin\theta$ on the same coordinate plane.

Solution:



To find the intersection points:

$$\begin{aligned}2\cos\theta &= -2\sin\theta\\ -\frac{\sin\theta}{\cos\theta} &= 1\\ \tan\theta &= -1\\ \theta &= -\frac{\pi}{4}\end{aligned}$$

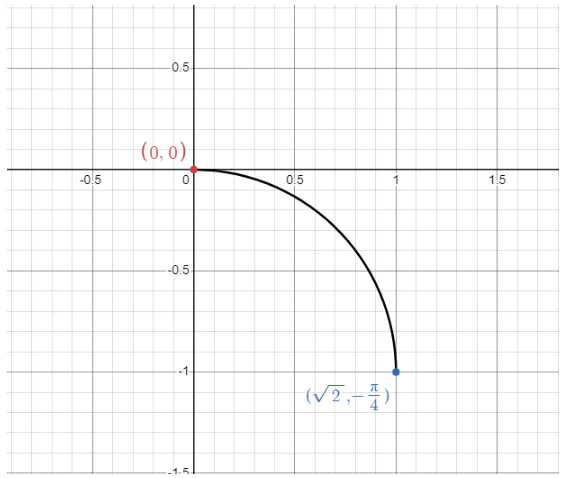
and

$$2\cos(-\frac{\pi}{4}) = \sqrt{2}$$

giving an intersection point of $(\sqrt{2}, -\frac{\pi}{4})$.

3. Find the area of the region common to the curves in 2.

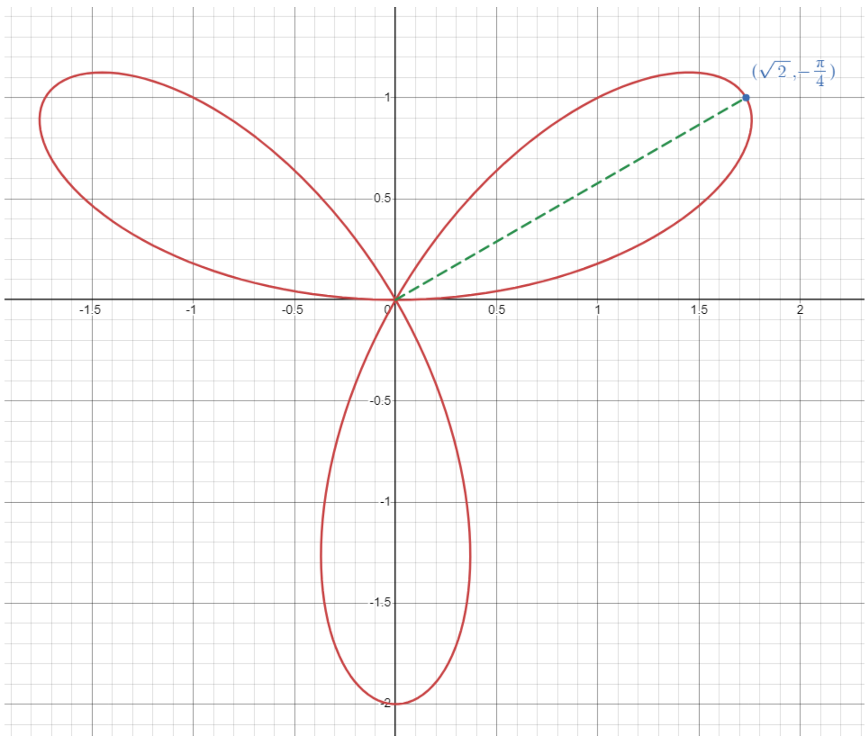
Solution: We can take the area of the sin circle from $-\frac{\pi}{4}$ to 0, and take advantage of the symmetry of the graph.



$$\begin{aligned}
A = 2\left(\frac{1}{2}\int_{-\frac{\pi}{4}}^{0} (-2\sin\theta)^2\, d\theta\right) &= 4\int_{-\frac{\pi}{4}}^{0} \sin^2\theta\, d\theta \\
&= 4\int_{-\frac{\pi}{4}}^{0} \frac{1-\cos 2\theta}{2}\, d\theta \\
&= 4\left(\frac{1}{2}\theta - \frac{\sin 2\theta}{4}\Bigg|_{-\frac{\pi}{4}}^{0}\right) \\
&= 4\left(0 - ((\frac{1}{2})(-\frac{\pi}{4}) - \frac{\sin 2(-\frac{\pi}{4})}{4})\right) \\
&= \frac{\pi}{2} - 1
\end{aligned}$$

4. Sketch a graph of $r = 2\sin 3\theta$, then find the area of one petal.

Solution: $n = 3$, meaning there are three petals.



$$
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