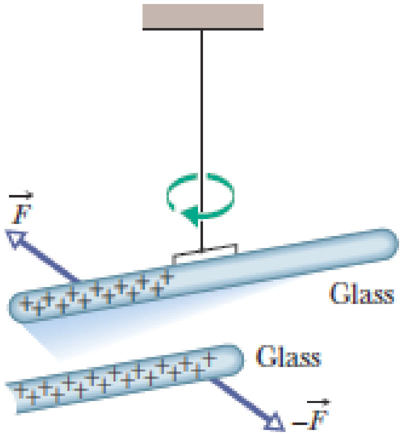
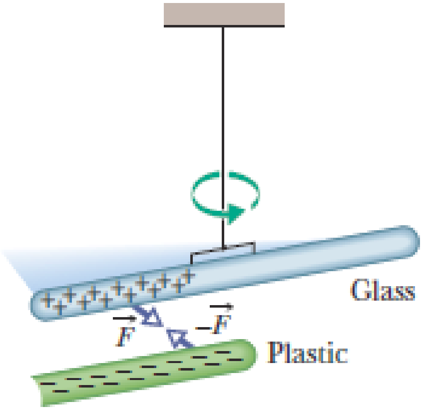
Chapter 21

Electric Charge and Electric Field



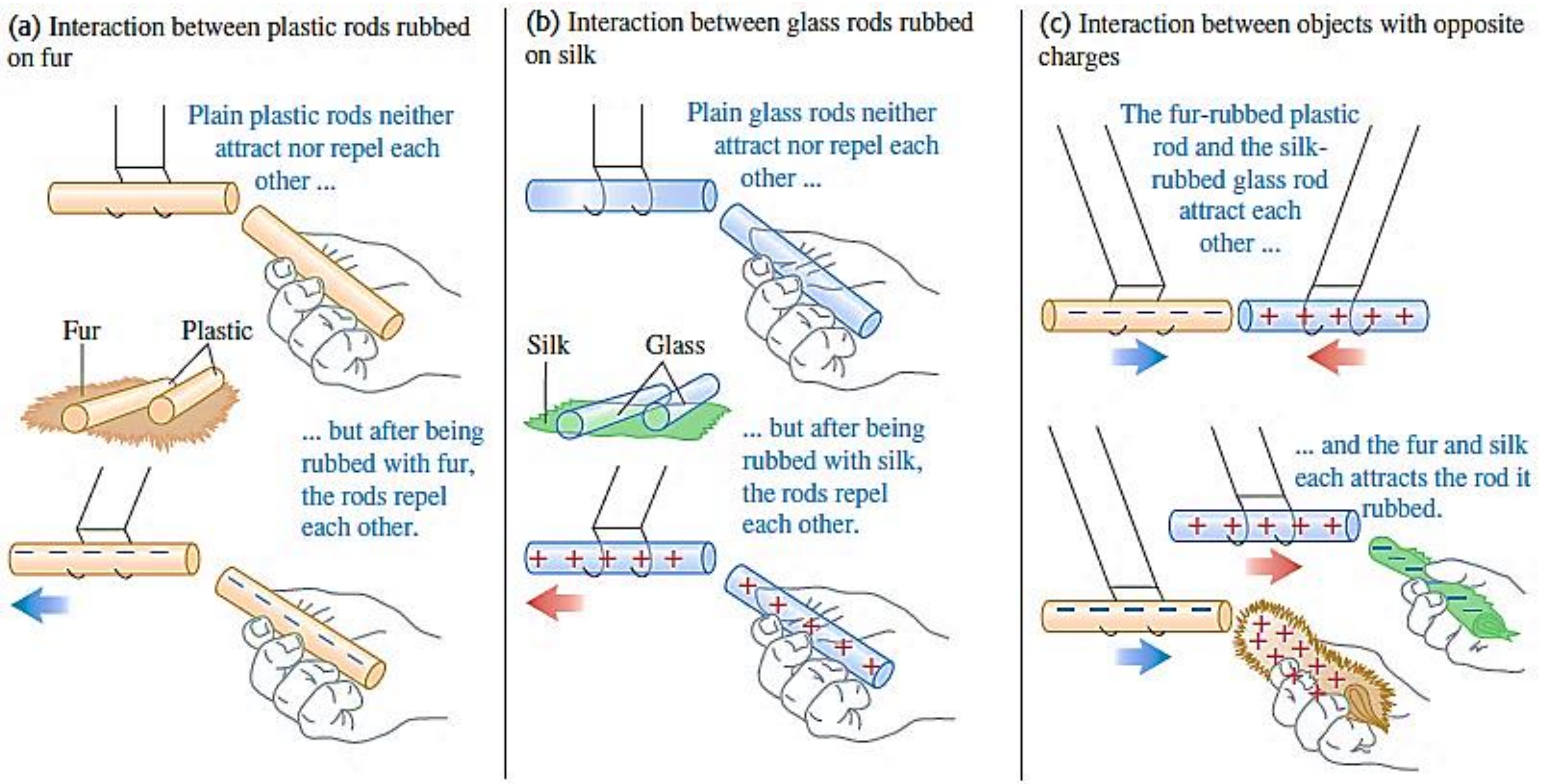
(a)



(b)

Electric Charge

Electric Interactions



Electric Charge

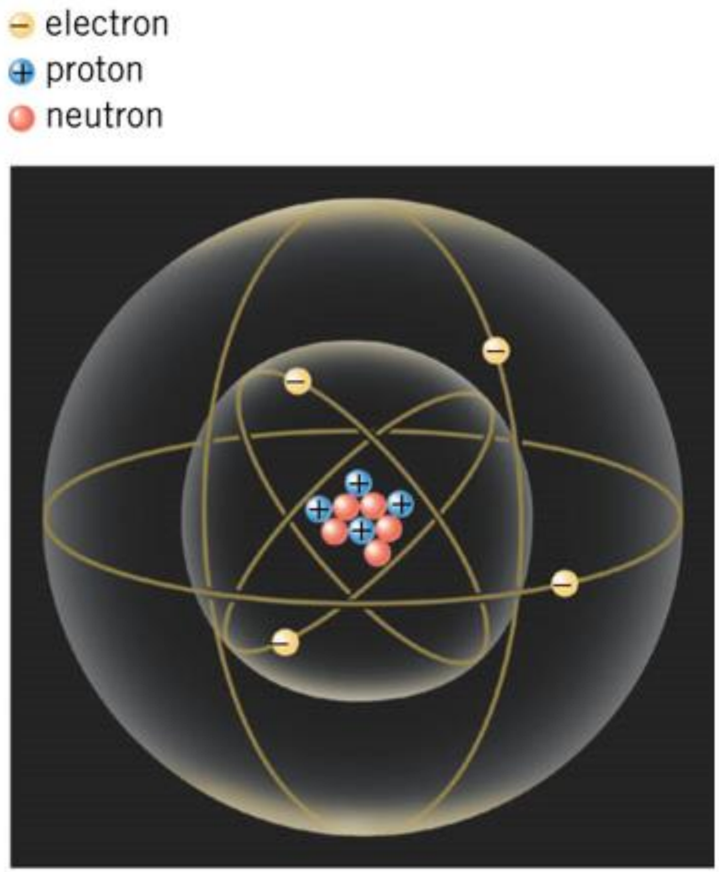
Electrical Charge and the Structure of Matter

The electrical nature of matter is inherent in the atomic structure.

- The sub-atomic particles.

$$m_p = 1.673 \times 10^{-27}\,\text{kg}$$
$$m_n = 1.675 \times 10^{-27}\,\text{kg}$$
$$m_e = 9.11 \times 10^{-31}\,\text{kg}$$

- These particles also possess ***electrical charge***.



Electric Charge

Electrical Charge and the Structure of Matter

There are two kinds of electric charges: ***positive*** and ***negative***

- *Negative* charges are the type possessed by electrons.
- *Positive* charges are the type possessed by protons.

Charges of the same sign ***repel*** one another and charges with opposite signs ***attract*** one another.

Electric Charge

Electrical Charge and the Structure of Matter

The electric charge, q, is said to be ***quantized***.

- Electric charge exists as discrete packets.
- $q = Ne$ where N is an integer
  - e is the fundamental unit of charge
  - $e = 1.6 \times 10^{-19}\ \mathrm{C}$ (Coulombs)
  - Electron: $q = -e$
  - Proton: $q = +e$

Electric Charge

Electrical Charge and the Structure of Matter

- In nature, atoms are normally found with *equal* numbers of protons and electrons, so they are electrically neutral.
- By adding or removing electrons from matter it will acquire a net electric charge with magnitude equal to e times the number of electrons added or removed, N.

$$q = Ne$$

Example: How many electrons are there in one coulomb of negative charge?

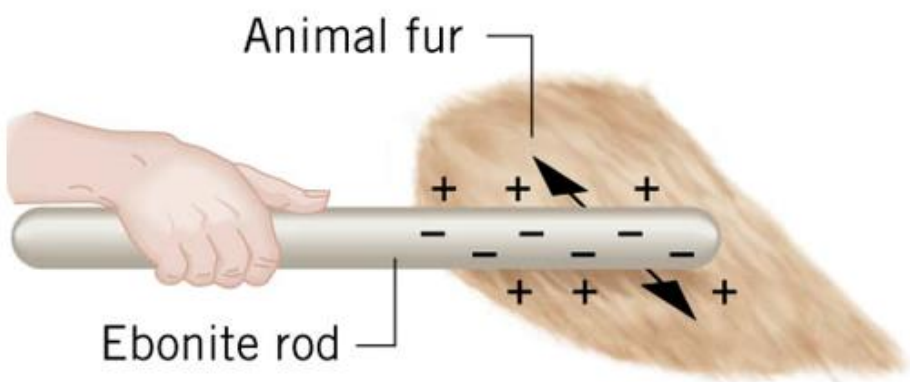
$$N = \frac{q}{e} = \frac{1.00\ \mathrm{C}}{1.60 \times 10^{-19}\ \mathrm{C}} = 6.25 \times 10^{18}$$

- Typical charges can be in the $\mu\mathrm{C}$ $(1 \times 10^{-6}\ \mathrm{C})$ range.

Electric Charge

Conservation of electric charge

- It is possible to transfer electric charge from one object to another.

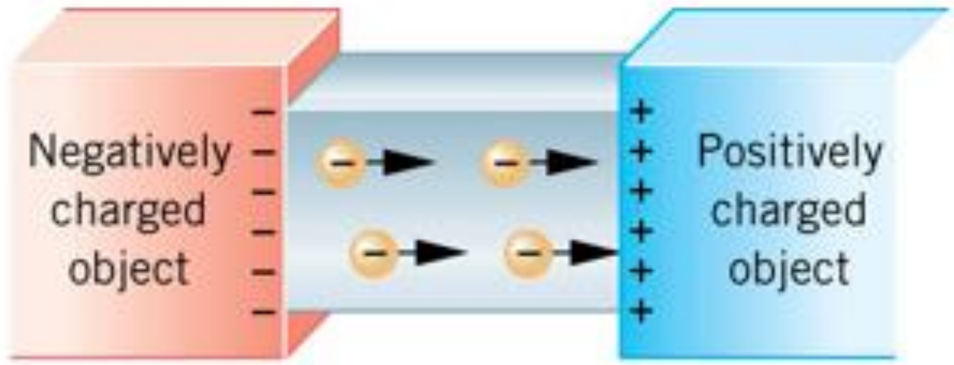


- The body that loses electrons has an excess of positive charge, while the body that gains electrons has an excess of negative charge.
- During any process, the net electric charge of an isolated system remains constant (*is conserved*).

Conductors, Insulators and Induced Charge

Conduction and Insulation

- Not only can electric charge exist *on an object,* but it can also move *through an object*.



- Substances that readily conduct electric charge are called ***electrical conductors***.
- Materials that conduct electric charge poorly are called ***electrical insulators.***

Conductors, Insulators and Induced Charge

Conductors

Electrical **conductors** are materials in which some of the electrons are free electrons.

- Free electrons are loosely bound to the atoms.
- These electrons can move relatively freely through the material.
- Examples of good conductors include copper, aluminum and silver.
- When a good conductor is charged in a small region, the charge readily distributes itself over the entire surface of the material.

Conductors, Insulators and Induced Charge

Insulators

Electrical **insulators** are materials in which all of the electrons are bound to atoms.

- These electrons can not move relatively freely through the material.
- Examples of good insulators include glass, rubber and wood.
- When a good insulator is charged in a small region, the charge is unable to move to other regions of the material.

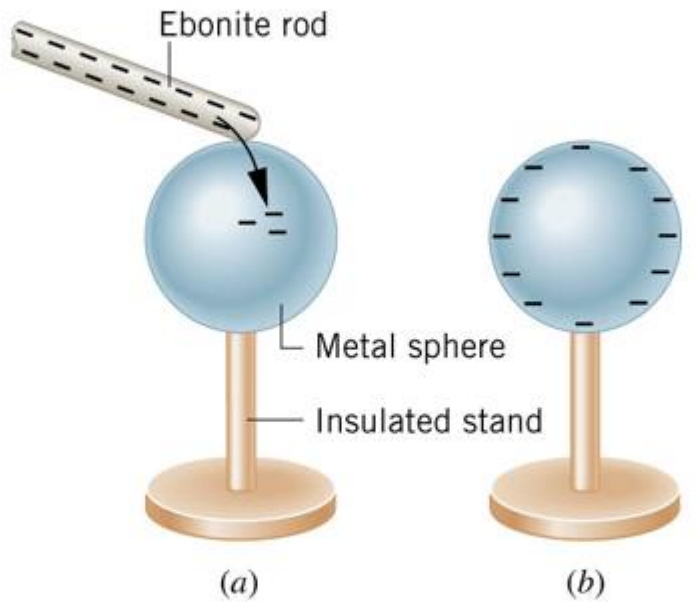
Semiconductors

The electrical properties of **semiconductors** are somewhere between those of insulators and conductors.

- Examples of semiconductor materials include silicon and germanium.

Conductors, Insulators and Induced Charge

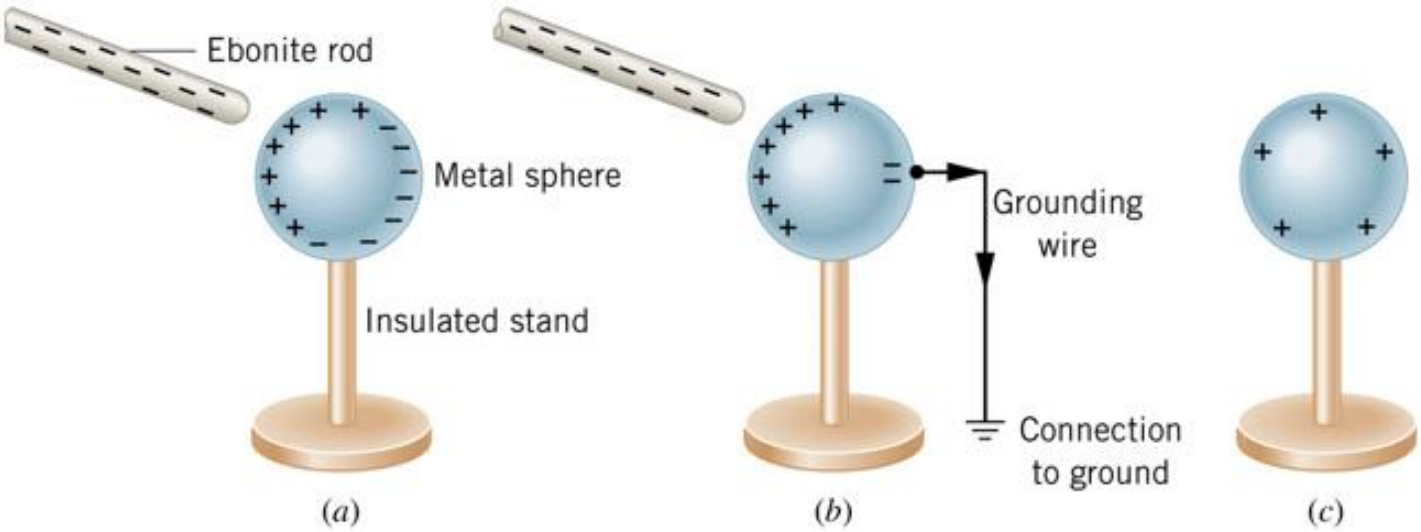
Charging by contact



- When a negatively charged ebonite rod is rubbed on a metal object, some of the excess electrons from the rod are transferred to the metal.
- Once the rod is removed, the electrons on the metal sphere (where they can move readily) repel one another and spread out over the sphere's surface.

Conductors, Insulators and Induced Charge

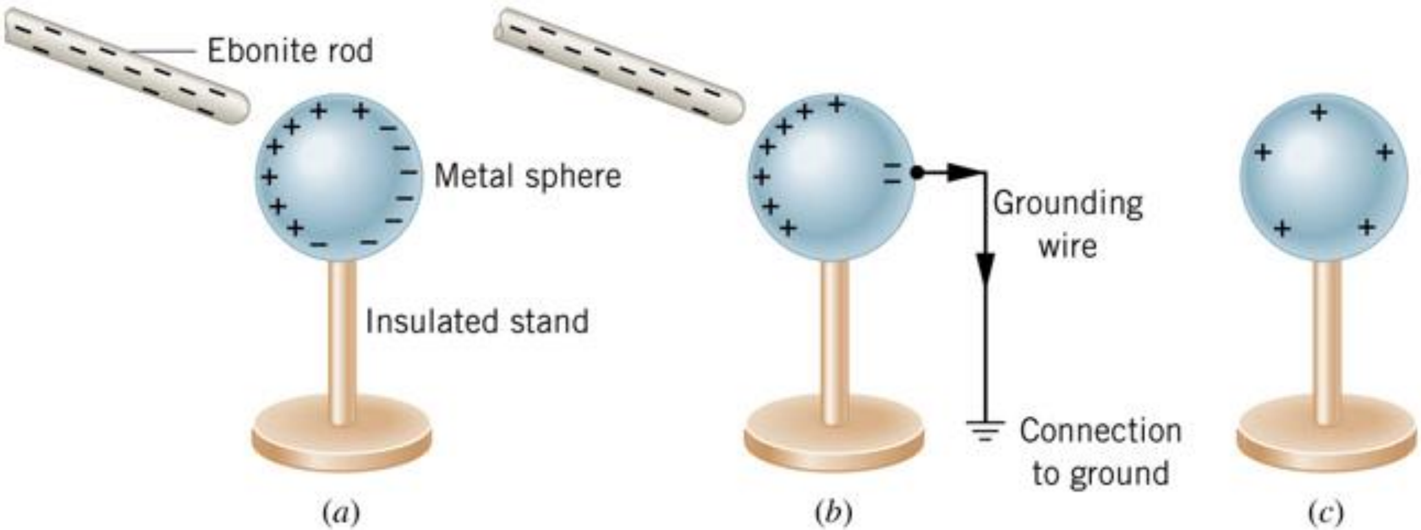
Charging by induction



- A negatively charged rod is brought close to, *but does not touch,* a metal sphere.
- The positively and negatively charged regions have been “induced” or “persuaded” to form because of the repulsive force between the negative rod and the free electrons in the sphere.

Conductors, Insulators and Induced Charge

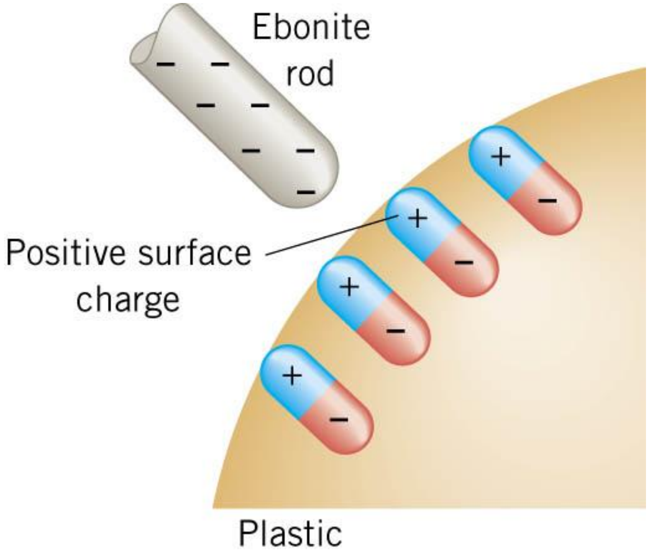
Charging by induction



- When a metal wire is attached between the sphere and the ground, some of the free electrons leave the sphere and distribute themselves over the much larger earth.
- If the grounding wire is then removed, followed by the ebonite rod, the sphere is left with a positive net charge.

Conductors, Insulators and Induced Charge

Electrical force on uncharged object

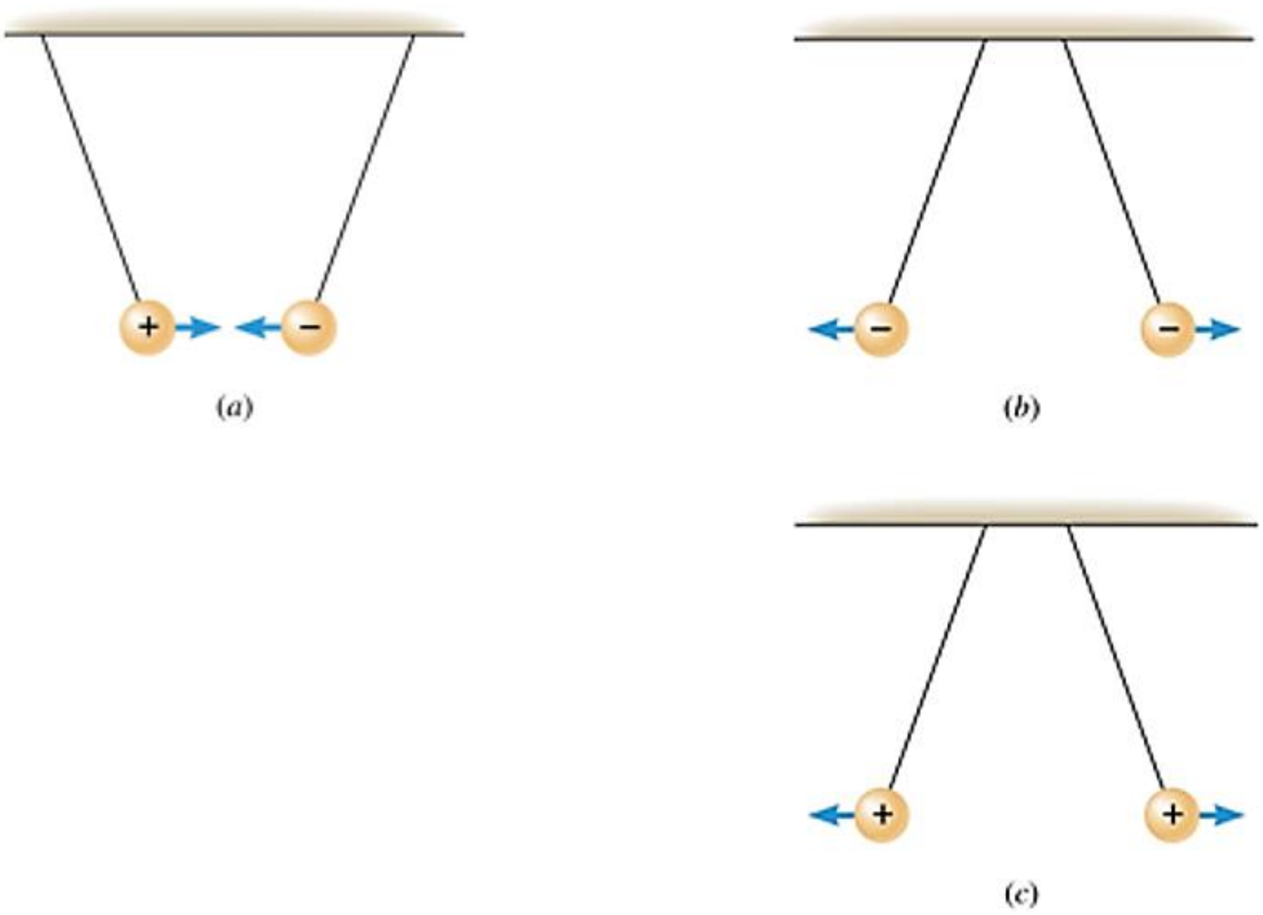


- The negatively charged plastic comb causes a slight shifting of charge within the molecules of the neutral insulator, an effect called *polarization.*
- A charged object of *either* sign exerts an attractive force on an uncharged insulator.

Coulomb's Law

Interaction between charges

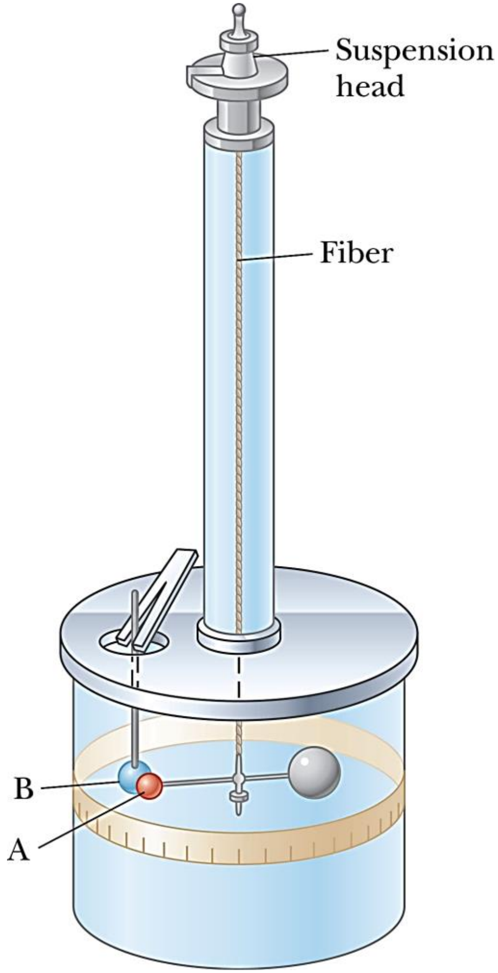
Unlike charges attract and like charges repel each other.



Coulomb's Law

Charles Coulomb measured the magnitudes of electric forces between two small charged spheres.

- The force is inversely proportional to the *square of the separation* r between the charges and directed along the line joining them.
- The force is proportional to the *product of the charges*, q_1 and q_2, on the two particles.
- The electrical force between two stationary point charges is given by ***Coulomb's Law***.

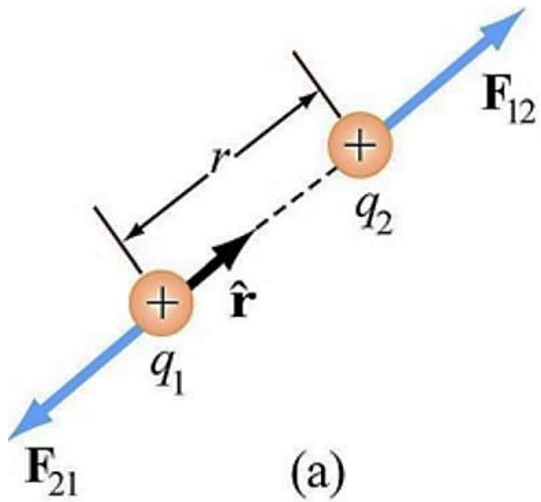


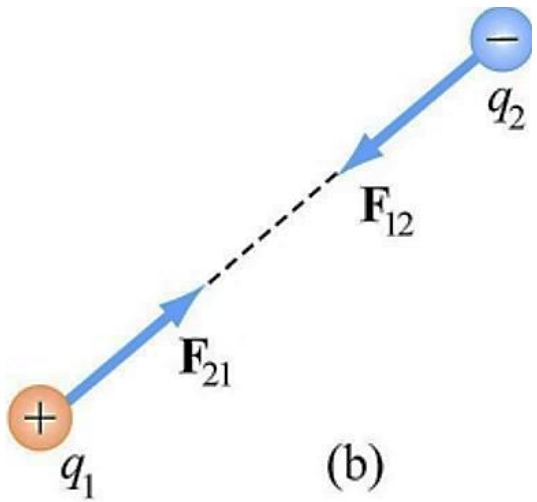
Coulomb's Law

Coulomb's force

a) Repulsion

b) Attraction





Coulomb's Law

Coulomb's force

Force on q_2 by q_1:

$$\vec{\mathbf{F}}_{12} = k\frac{q_1 q_2}{r^2}\hat{\mathbf{r}}$$

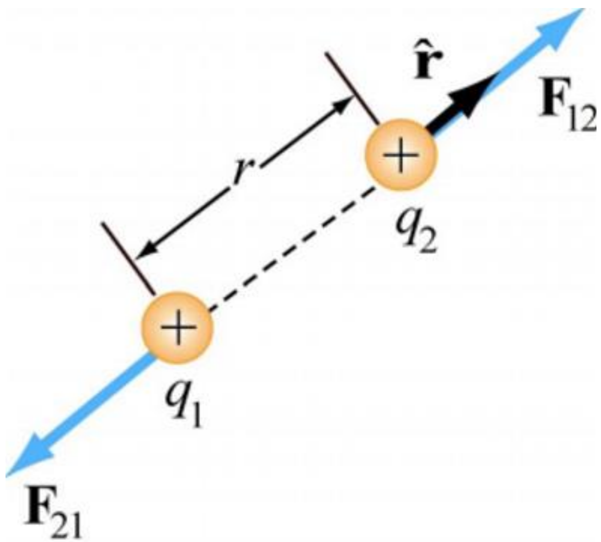
$\hat{\mathbf{r}} =$ unit vector from q_2 to q_1

$$k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \ \mathrm{N \cdot m^2/C^2}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \ \mathrm{C^2/N \cdot m^2}$$

(Permittivity of vacuum)

$$k \approx 9 \times 10^9 \ \mathrm{N \cdot m^2/C^2}$$



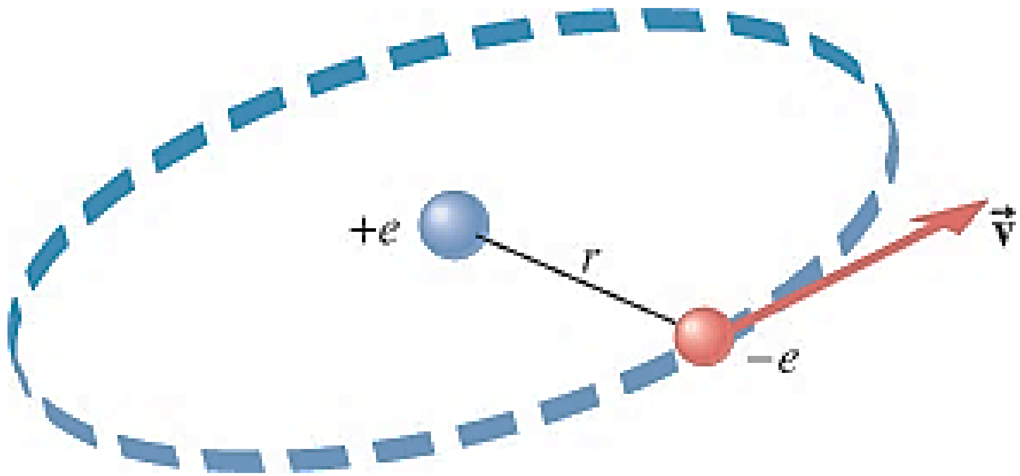
By Newton's 3rd law:

$$\vec{\mathbf{F}}_{21} = -k\frac{q_1 q_2}{r^2}\hat{\mathbf{r}} = -\vec{\mathbf{F}}_{12}$$

Coulomb's Law

***Example**:* The Hydrogen Atom

In the Bohr model of the hydrogen atom, the electron is in orbit about the nuclear proton at a radius of 5.29×10^{-11} m. Determine the speed of the electron, assuming the orbit to be circular.



Coulomb's Law

***Example**:* The Hydrogen Atom

Solution:

$$F = k\frac{|q_1||q_2|}{r^2} = (9 \times 10^9)\frac{(1.60 \times 10^{-19}\ \mathrm{C})^2}{(5.29 \times 10^{-11}\ \mathrm{m})^2}$$

$$= 8.22 \times 10^{-8}\ \mathrm{N}$$

- For circular motion.

$$F = ma_c = m\frac{v^2}{r}$$

$$v = \sqrt{\frac{Fr}{m}} = \sqrt{\frac{(8.22 \times 10^{-8}\ \mathrm{N})(5.29 \times 10^{-11}\ \mathrm{m})}{(9.11 \times 10^{-31}\ \mathrm{kg})}} = 2.18 \times 10^6\ \mathrm{m/s}$$

Coulomb's Law

Superposition of forces

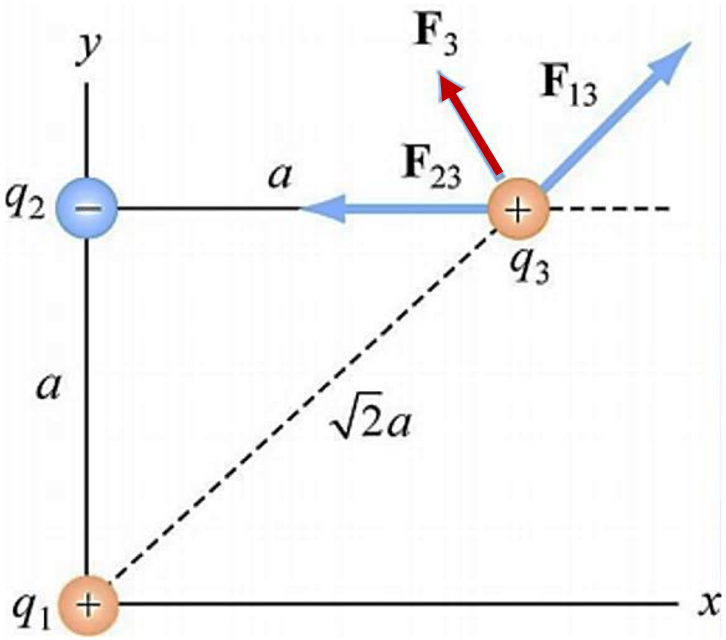
For three or more point charges.

- Net force on q_3

$$\vec{\mathbf{F}}_3 = \vec{\mathbf{F}}_{13} + \vec{\mathbf{F}}_{23}$$

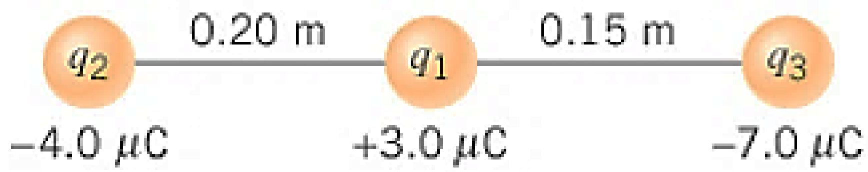
- In general, for *N* point charges, the net force on q_i

$$\vec{\mathbf{F}}_i = \sum_{j=1, j\neq i}^{N} \vec{\mathbf{F}}_{ij}$$



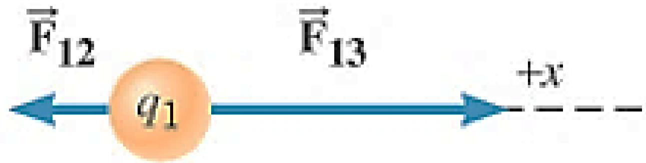
Coulomb's Law

Example: Three point charges in a line.



(a)

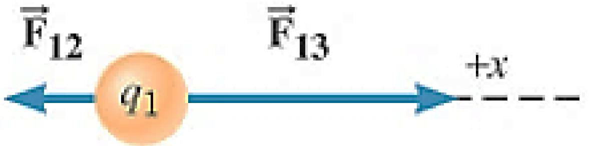
- The net force on q_1



(b) Free-body diagram for q_1

Coulomb's Law

Example: Three point charges in a line.



(b) Free-body diagram for q_1

- The net force on q_1 :

$$F_{12} = k\frac{q_1 q_2}{r^2} = \frac{(9 \times 10^9)(3.0 \times 10^{-6}\ \text{C})(4.0 \times 10^{-6}\ \text{C})}{(0.20\ \text{m})^2} = -2.7\ \text{N}$$

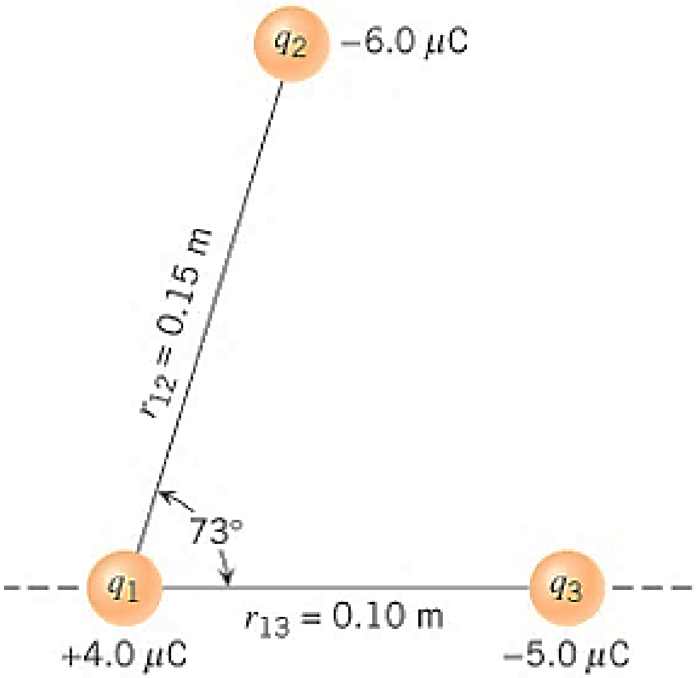
$$F_{13} = k\frac{q_1 q_3}{r^2} = \frac{(9 \times 10^9)(3.0 \times 10^{-6}\ \text{C})(7.0 \times 10^{-6}\ \text{C})}{(0.15\ \text{m})^2} = +8.4\ \text{N}$$

$$F_1 = F_{12} + F_{13} = -2.7\ \text{N} + 8.4\ \text{N} = +5.7\ \text{N}$$

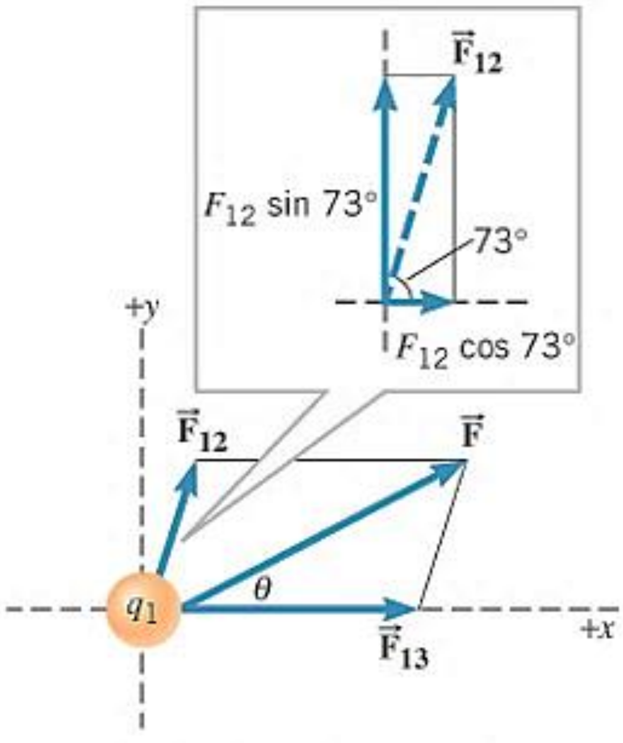
Coulomb's Law

Example: Three point charges in a plane.

- The net force on q_1:



(a)



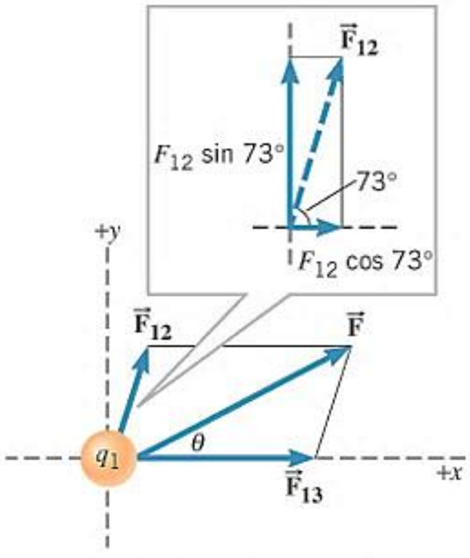
(b) Free-body diagram for q_1

Coulomb's Law

Example: Three point charges in a plane.

- Net force on q_1:

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_{12} + \vec{\mathbf{F}}_{13}$$



(b) Free-body diagram for q_1

$$F_{12} = k\frac{q_1 q_2}{r^2} = \frac{(9 \times 10^9)(4.0 \times 10^{-6}\ \mathrm{C})(6.0 \times 10^{-6}\ \mathrm{C})}{(0.15\ \mathrm{m})^2} = 9.6\ \mathrm{N}$$

$$F_{13} = k\frac{q_1 q_3}{r^2} = \frac{(9 \times 10^9)(4.0 \times 10^{-6}\ \mathrm{C})(5.0 \times 10^{-6}\ \mathrm{C})}{(0.10\ \mathrm{m})^2} = 18\ \mathrm{N}$$

Coulomb's Law

Example: Three point charges in a plane.

- Net force on q_1:

$$F_{12} = 9.6\ \text{N} \qquad F_{13} = 18\ \text{N}$$

$$F_{12x} = (9.6)\ \cos 73° = +2.8\ \text{N} \qquad F_{12x} = (9.6)\ \sin 73° = +9.2\ \text{N}$$

$$F_{13x} = +18\ \text{N} \qquad F_{13y} = 0\ \text{N}$$

$$F_x = +20.8\ \text{N} \qquad F_y = +9.2\ \text{N}$$

$$F = \sqrt{{F_x}^2 + {F_y}^2} = \sqrt{(20.8)^2 + (9.2)^2} = 22.7\ \text{N}$$

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{9.2}{20.8}\right) = 24°$$

Electric Field and Electric Forces

Electric field

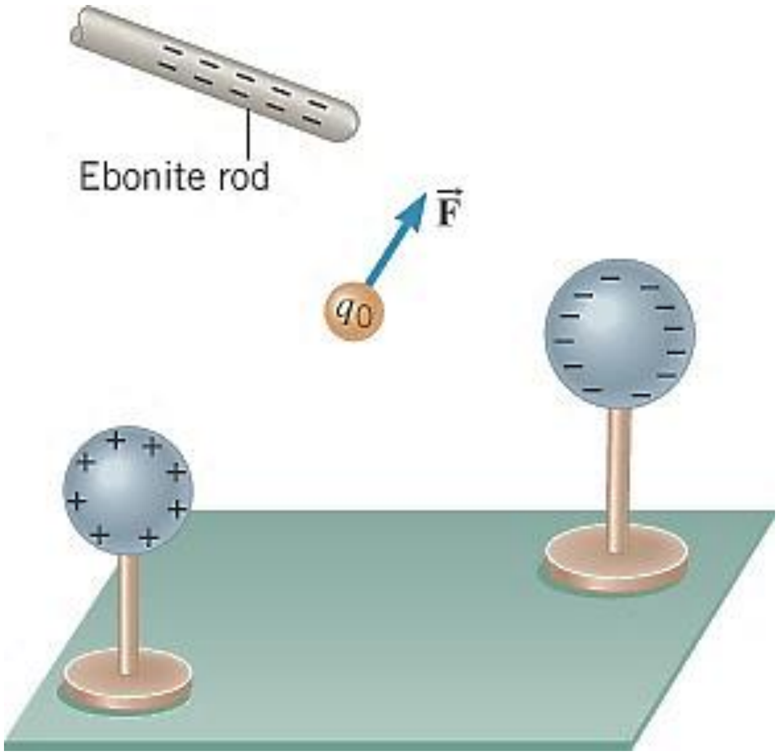
- The electric force is a *field force*.
  - Field forces can act through space.
  - The effect is produced even with no physical contact between objects.

- An ***electric field*** is said to exist in the region of space around a charged object.
  - Field forces can act through space.
  - This charged object is the **source charge.**
  - When another charged object enters this electric field, an electric force acts on it.

Electric Field and Electric Forces

Electric field

- A positive ***test charge*** $\boldsymbol{q_0}$ serves as the detector of the field.
  - The *test charge* experiences a force which is the vector sum of the forces exerted by the charged objects around it.
  - This *test charge* should have a small magnitude so it doesn't affect the other charges significantly.



Electric Field and Electric Forces

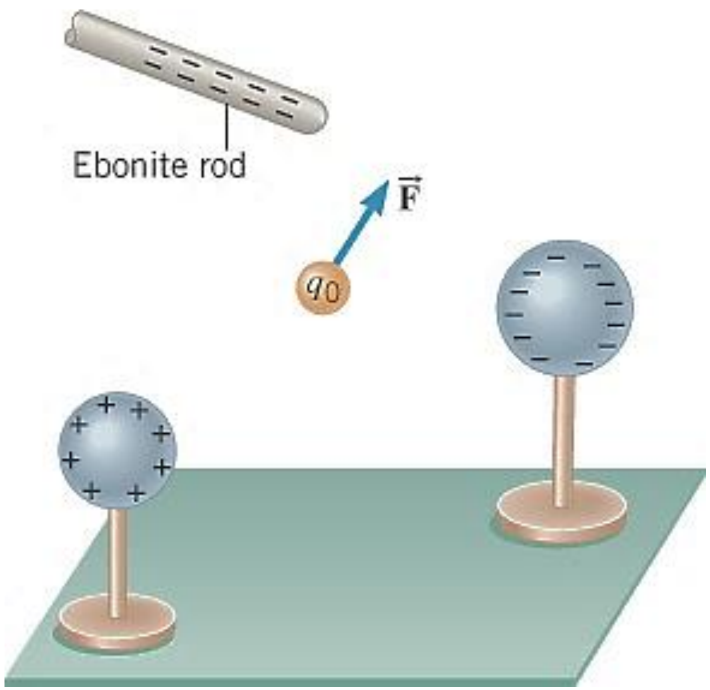
Electric field

- The electric field $\vec{\mathbf{E}}$ is defined as the *electric force on the test charge per unit charge*.

$$\vec{\mathbf{E}} \equiv \frac{\vec{\mathbf{F}}}{q_0}$$

  - *SI Units:* Newton per Coulomb (N/C)

- The ***direction*** of $\vec{\mathbf{E}}$ is that of the force on the positive test charge.



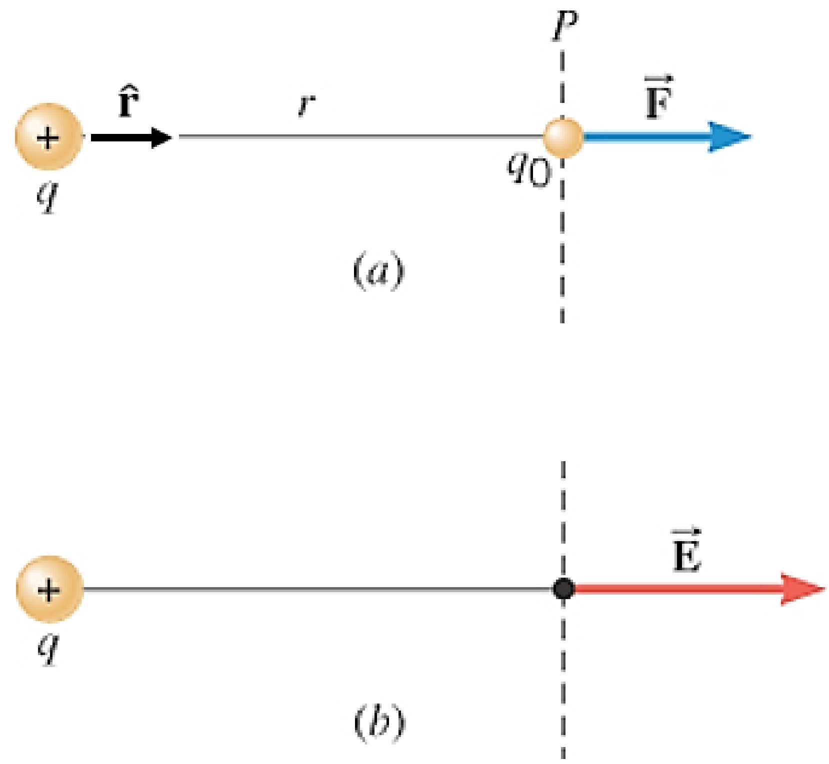
Electric Field and Electric Forces

Electric field of a point charge

(*a*)

$$\vec{\mathbf{F}} = k\frac{|q||q_0|}{r^2}\hat{\mathbf{r}}$$

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{|q_0|} = k\frac{|q|}{r^2}\hat{\mathbf{r}}$$



(*b*)

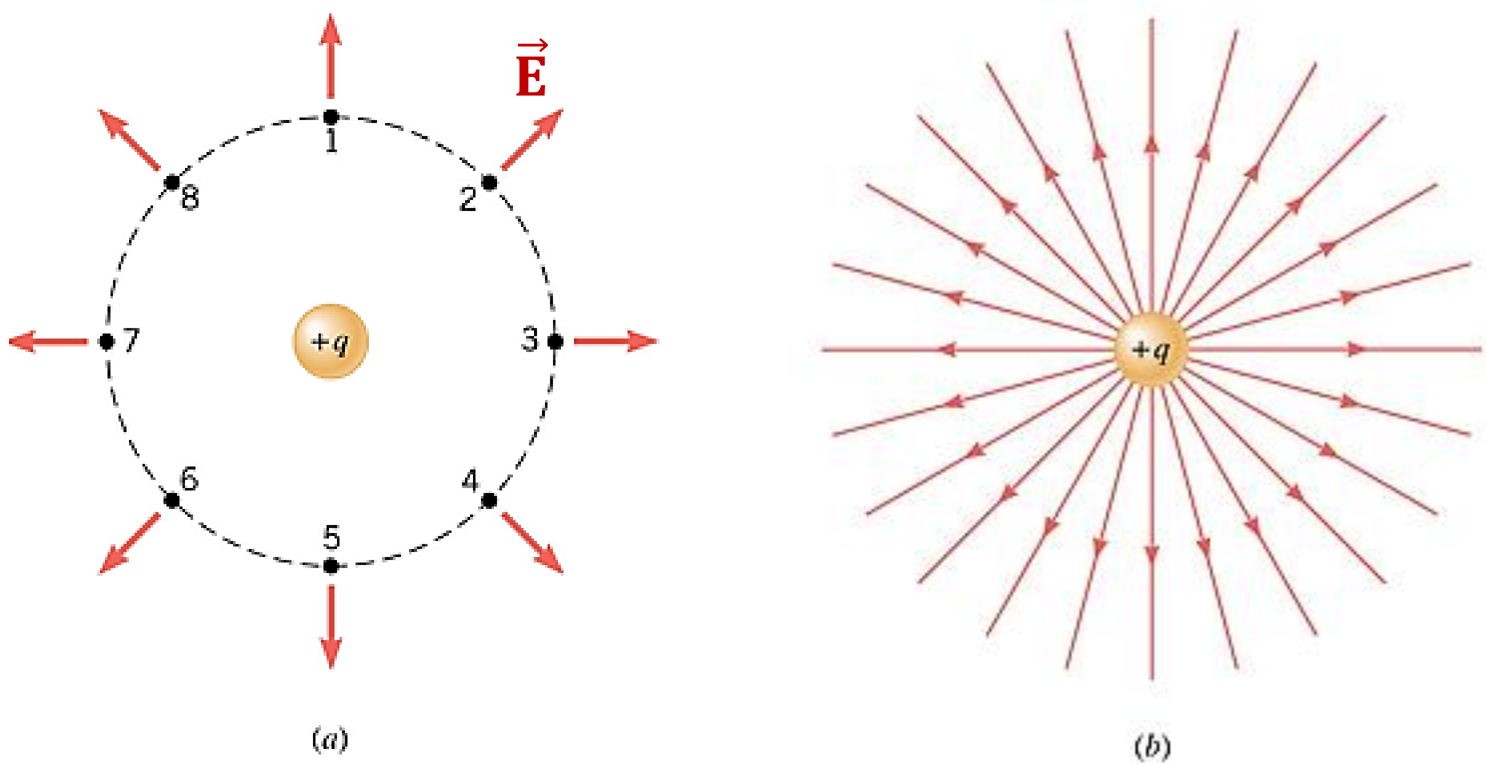
$$\vec{\mathbf{E}} = k\frac{q}{r^2}\hat{\mathbf{r}}$$

The electric field does not depend on the test charge.

Electric Field and Electric Forces

Electric field of a point charge

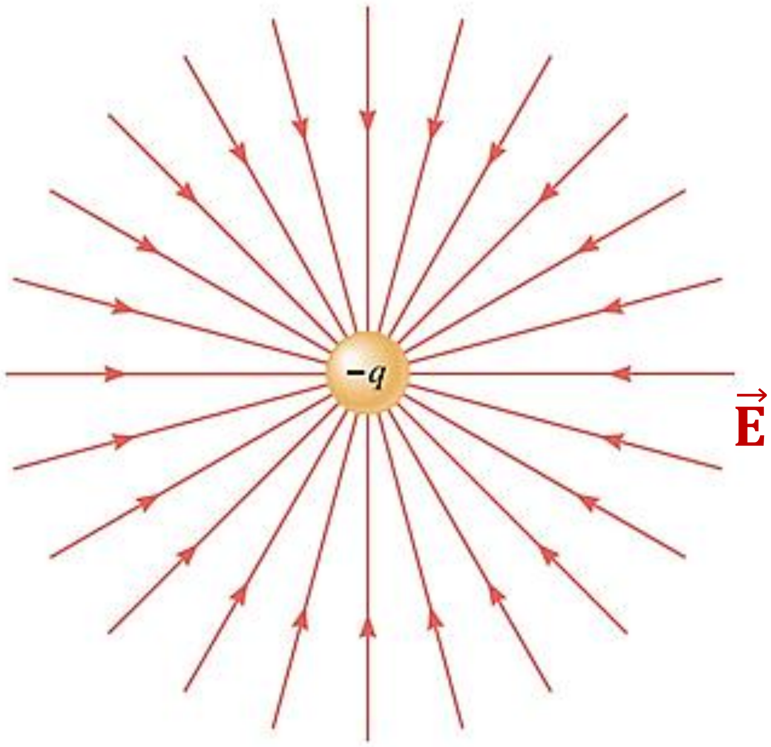
$$\vec{\mathbf{E}} = k\frac{q}{r^2}\hat{\mathbf{r}}$$

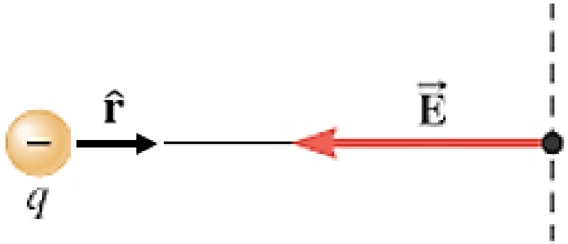


Electric Field and Electric Forces

Electric field of a point charge

(c) *Electric field of negative charge.*



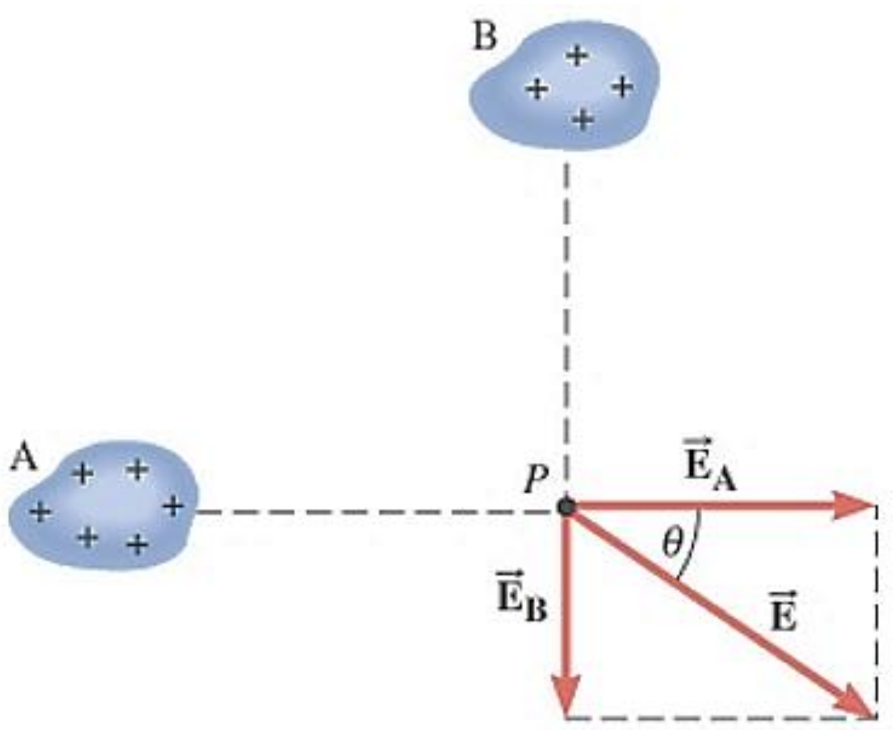


The electric field line is the ***line of force****.*

Electric Field Calculations

Superposition of fields

- Electric fields from different sources add as vectors.



Electric Field Calculations

Superposition of fields: Multiple point charges

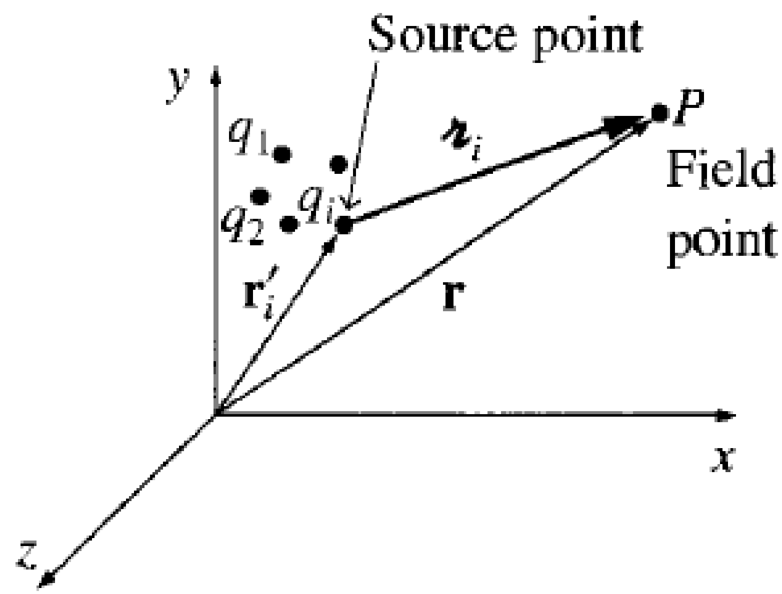
- At any point *P*, the total electric field due to *N* point charges equals the vector sum of the electric fields of all the charges.

- The *net electric field*.

$$\vec{\mathbf{E}}_{net} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 + \vec{\mathbf{E}}_3 + \cdots$$

- or

$$\vec{\mathbf{E}}_{net} = \sum_{j=1}^{N} \vec{\mathbf{E}}_j = k \sum_{j=1}^{N} \frac{q_j}{{r_j}^2} \hat{\mathbf{r}}_j$$



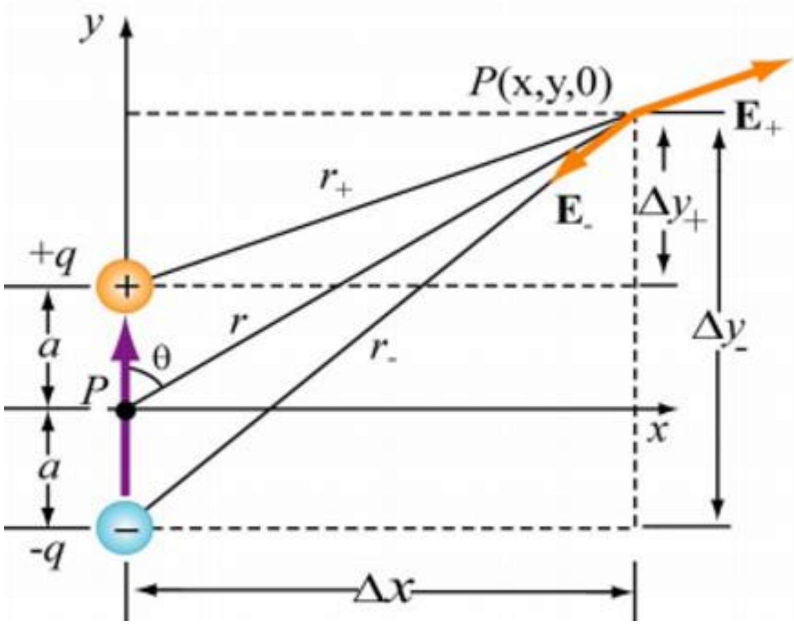
Electric Field Calculations

***Example**:* The Electric Dipole

Two equal but oppositely charged point charges separated by a fixed distance $2a$.

$$\vec{\mathbf{E}}_{+} = k\frac{q}{r_{+}^{2}}\hat{\mathbf{r}}_{+} \qquad \vec{\mathbf{E}}_{-} = k\frac{q}{r_{-}^{2}}\hat{\mathbf{r}}_{-}$$

$$\frac{\hat{\mathbf{r}}}{r^2} = \frac{\vec{\mathbf{r}}}{r^3} = \frac{\Delta x}{r^3}\,\hat{\mathbf{i}} + \frac{\Delta y}{r^3}\,\hat{\mathbf{j}}$$



- Net electric field:

$$\vec{\mathbf{E}}_{net} = E_x\,\hat{\mathbf{i}} + E_y\,\hat{\mathbf{j}}$$

Electric Field Calculations

***Example**:* The Electric Dipole

- Net electric field:

$$\vec{\mathbf{E}}_{net} = E_x\,\hat{\mathbf{i}} + E_y\,\hat{\mathbf{j}}$$

$$E_x = kq\left(\frac{\Delta x}{r_+^3} - \frac{\Delta x}{r_-^3}\right) = kq\left(\frac{x}{[x^2 + (y-a)^2]^{3/2}} - \frac{x}{[x^2 + (y+a)^2]^{3/2}}\right)$$

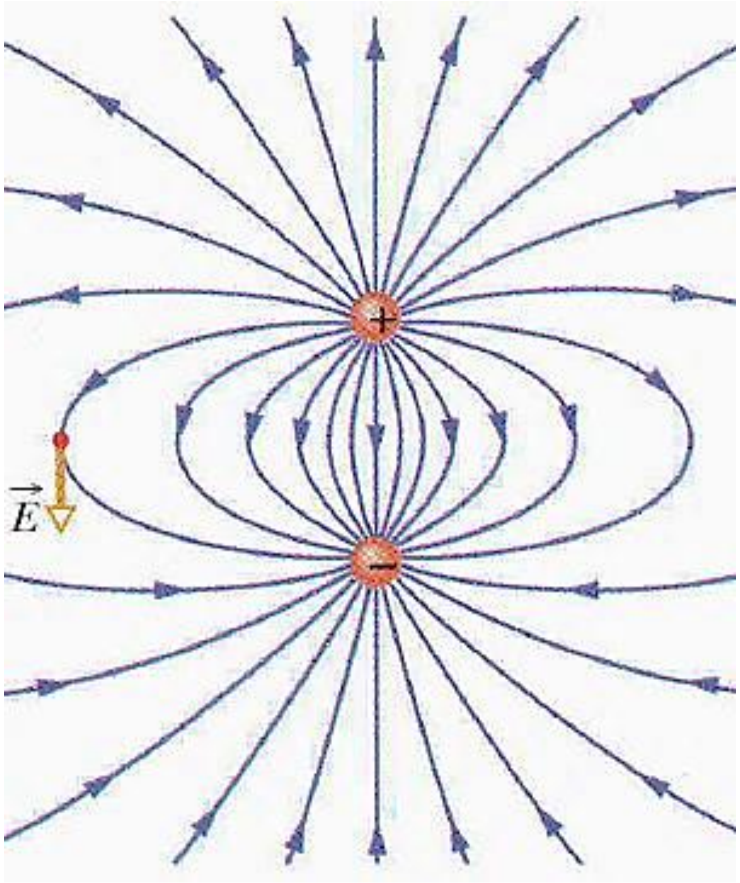
$$E_y = kq\left(\frac{\Delta y_+}{r_+^3} - \frac{\Delta y_-}{r_-^3}\right) = kq\left(\frac{y-a}{[x^2 + (y-a)^2]^{3/2}} - \frac{y+a}{[x^2 + (y+a)^2]^{3/2}}\right)$$

Electric Field Calculations

***Example**:* The Electric Dipole

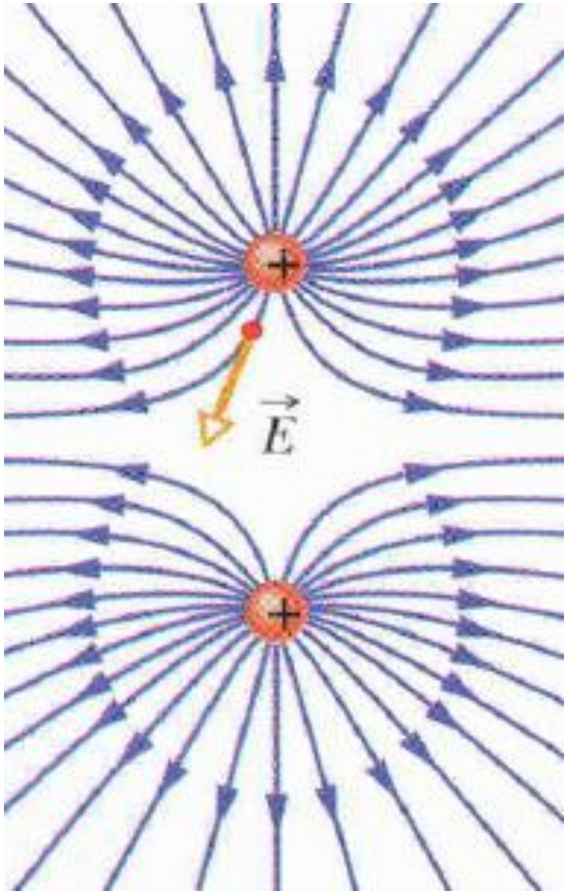
$$E_x = kq\left(\frac{x}{[x^2+(y-a)^2]^{3/2}} - \frac{x}{[x^2+(y+a)^2]^{3/2}}\right)$$

$$E_y = kq\left(\frac{y-a}{[x^2+(y-a)^2]^{3/2}} - \frac{y+a}{[x^2+(y+a)^2]^{3/2}}\right)$$



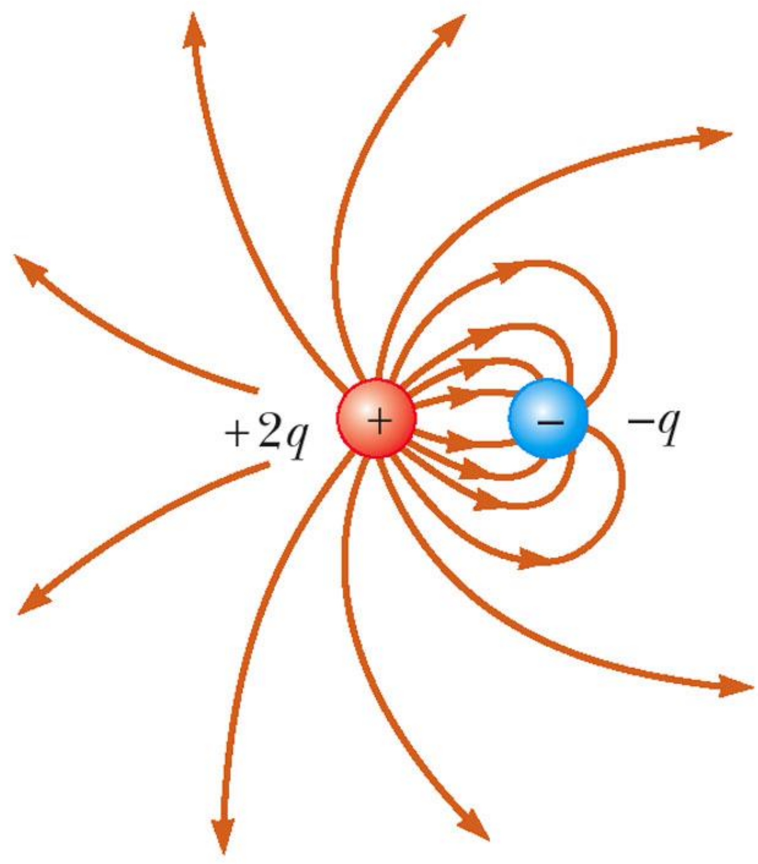
Electric Field Calculations

Example: Equal like charges



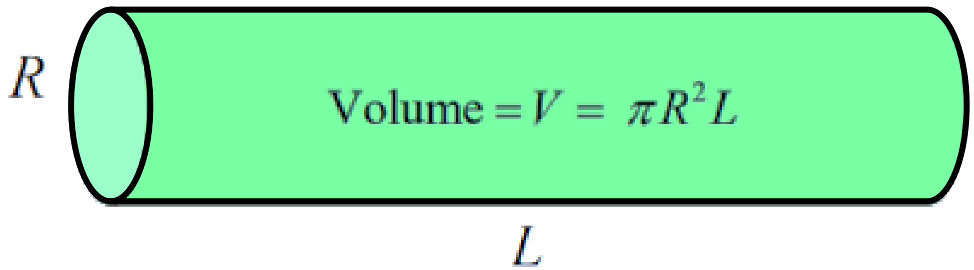
Electric Field Calculations

***Example**:* Unequal charges



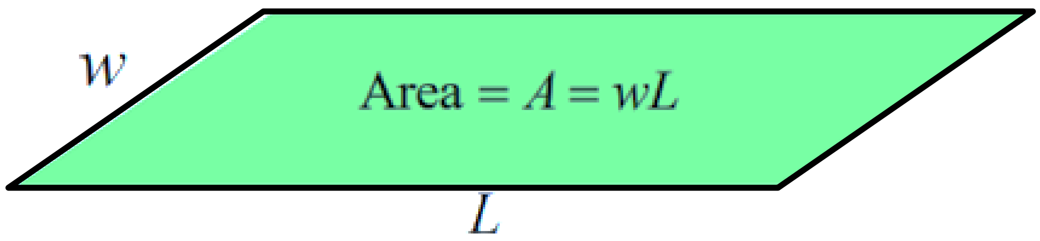
Continuous charge distributions

- Charge density :



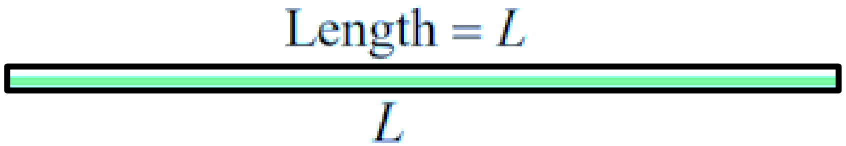
Volume density:

$$\boldsymbol{\rho} = Q/V$$



Surface density:

$$\boldsymbol{\sigma} = Q/A$$



Linear density:

$$\boldsymbol{\lambda} = Q/L$$

Electric Field Calculations

Continuous charge distributions

To calculate $\vec{\mathbf{E}}$:

- Divide distribution into small elements Δq:

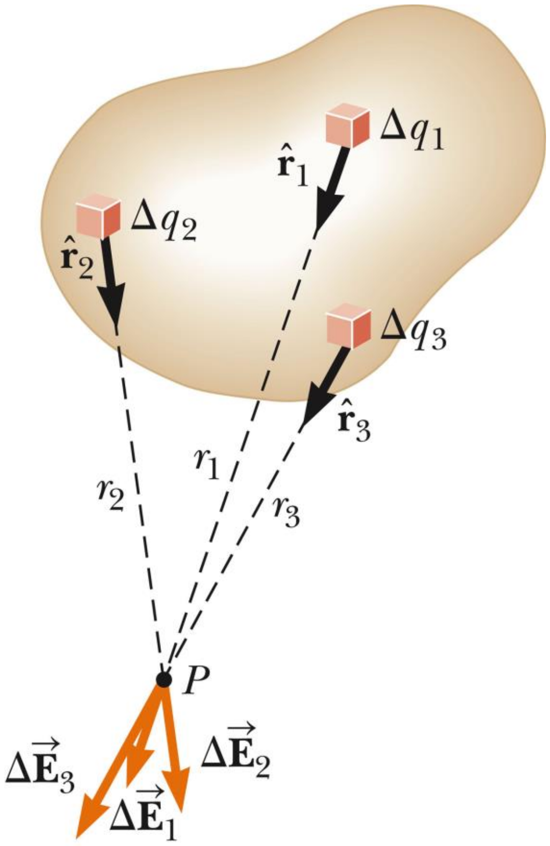
$$Q = \sum \Delta q \longrightarrow \int dq$$

- Electric field at P due to Δq:

$$\Delta\vec{\mathbf{E}} = k\frac{\Delta q}{r^2}\hat{\boldsymbol{r}} \longrightarrow d\vec{\mathbf{E}} = k\frac{dq}{r^2}\hat{\boldsymbol{r}}$$

- By superposition:

$$\vec{\mathbf{E}} = \sum \Delta\vec{\mathbf{E}} \longrightarrow \int d\vec{\mathbf{E}} = k\int\frac{dq}{r^2}\hat{\boldsymbol{r}}$$

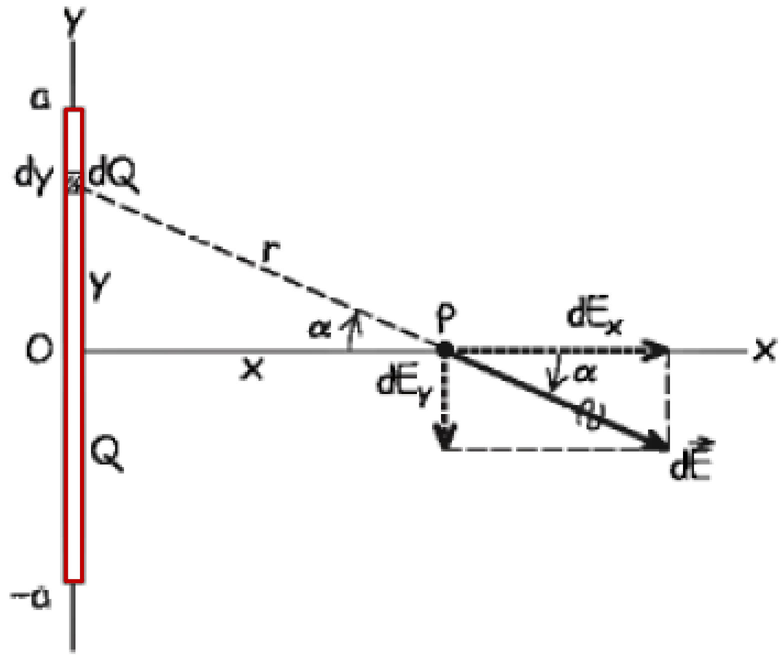


Electric Field Calculations

Line charge of length $2a$ and total charge Q

- Divide infinitesimal segments Δy:

$$\lambda = \frac{Q}{2a} \longrightarrow dQ = \lambda dy$$



- Electric field at P due to dQ :

$$dE = kQ \frac{dy}{2a(x^2 + y^2)}$$

- The electric field components:

$$dE_x = kQ \frac{xdy}{2a(x^2 + y^2)^{3/2}} \qquad dE_y = kQ \frac{ydy}{2a(x^2 + y^2)^{3/2}}$$

Electric Field Calculations

Line charge of length $2a$ *and total charge* Q

- Integrate:

$$E_x = \frac{kQ}{2a}\int_{-a}^{a}\frac{xdy}{(x^2+y^2)^{3/2}}$$

$$= kQ\frac{1}{x(x^2+a^2)^{1/2}}$$

$$E_y = -\frac{kQ}{2a}\int_{-a}^{a}\frac{ydy}{(x^2+y^2)^{3/2}} = 0$$

$$\vec{\mathbf{E}} = kQ\frac{1}{x\sqrt{x^2+a^2}}\,\hat{\boldsymbol{\imath}} = 2k\frac{\lambda a}{x\sqrt{x^2+a^2}}\,\hat{\boldsymbol{\imath}}$$

Electric Field Calculations

Line charge of length $2a$ *and total charge* Q

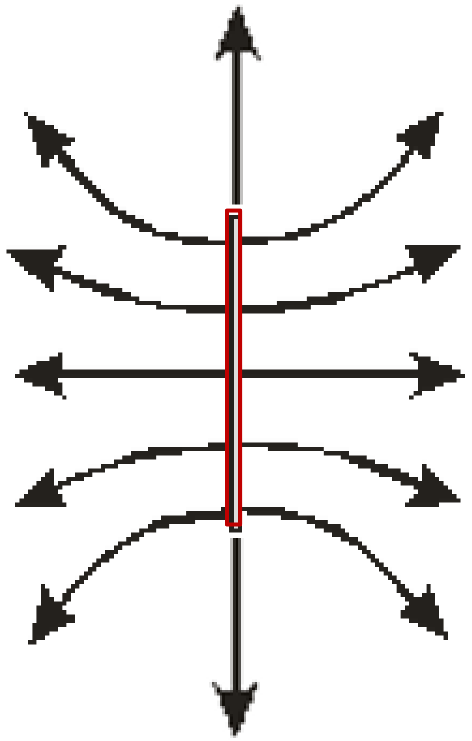
$$\vec{\mathbf{E}} = 2k\frac{\lambda a}{x\sqrt{x^2 + a^2}}\,\hat{\boldsymbol{\imath}}$$

- Special case 1 : $x \gg a$

$$\vec{\mathbf{E}} = \frac{k(2\lambda a)}{x^2}\,\hat{\boldsymbol{\imath}}$$

- Special case 2 : $a \gg x$

$$\vec{\mathbf{E}} = \frac{2k\lambda}{x}\,\hat{\boldsymbol{\imath}}$$

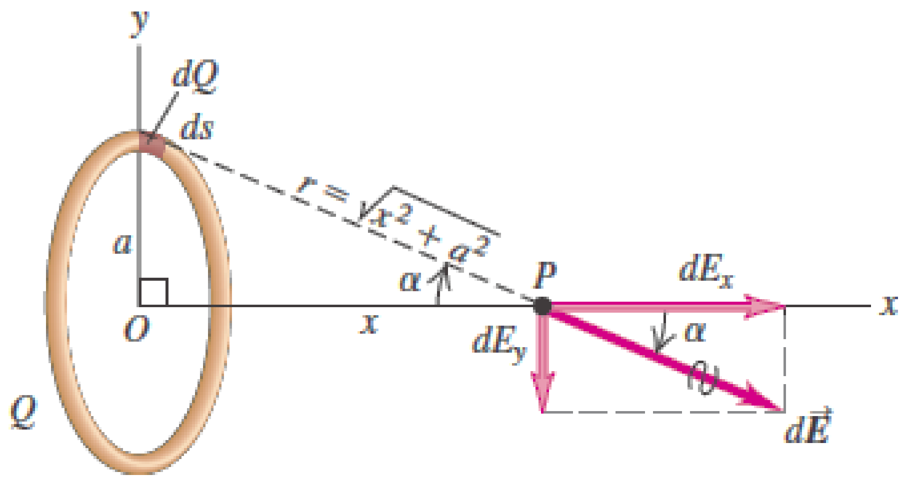


Electric Field Calculations

Charged ring of radius a and total charge Q

- Divide into infinitesimal arcs ds :

$$\lambda = \frac{Q}{2\pi a} \longrightarrow dQ = \lambda ds$$



- Electric field at P due to dQ :

$$dE = k\frac{\lambda ds}{(x^2 + a^2)}$$

- The electric field components:

$$dE_x = k\lambda\frac{x\ ds}{(x^2 + y^2)^{3/2}} \qquad dE_y = k\lambda\frac{a\ ds}{(x^2 + y^2)^{3/2}}$$

Electric Field Calculations

Charged ring of radius a and total charge Q

- Integrate:

$$E_x = k\lambda x \int_0^{2\pi a} \frac{ds}{(x^2 + y^2)^{3/2}}$$

$$= kQ \frac{x}{(x^2 + a^2)^{3/2}}$$

- By symmetry:

$$E_y = 0$$

$$\vec{\mathbf{E}} = kQ \frac{x}{(x^2 + a^2)^{3/2}} \hat{\boldsymbol{\imath}} = 2\pi k \frac{\lambda\, x}{(x^2 + a^2)^{3/2}} \hat{\boldsymbol{\imath}}$$

(*Field on the axis only*)

Electric Field Calculations

Charged ring of radius a *and total charge* Q

$$\vec{\mathbf{E}} = 2\pi k \frac{\lambda\, x}{(x^2 + a^2)^{3/2}}\, \hat{\boldsymbol{\imath}}$$

- Special case 1 : $x \gg a$

$$\vec{\mathbf{E}} = \frac{kQ}{x^2}\, \hat{\boldsymbol{\imath}}$$

- Special case 2 : $x = 0$

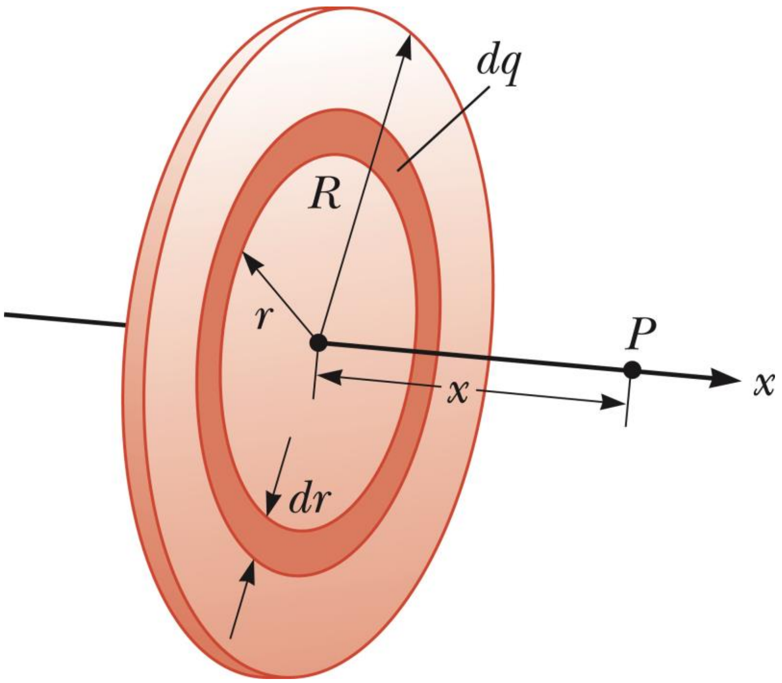
$$\vec{\mathbf{E}} = 0$$

Electric Field Calculations

Uniformly charged disk of radius R and total charge Q

- Divide into thin rings dQ :

$$\sigma = \frac{Q}{\pi R^2} \longrightarrow dQ = \sigma(2\pi r dr)$$



- Electric field at P due to dQ :

$$dE_x = k\,dQ\,\frac{x}{(x^2 + r^2)^{3/2}}$$

$$= k\,(2\pi\sigma x)\,\frac{r\,dr}{(x^2 + r^2)^{3/2}}$$

Electric Field Calculations

Uniformly charged disk of radius R and total charge Q

- Integrate:

$$E_x = 2\pi k\sigma x \int_0^R \frac{r\,dr}{(x^2 + r^2)^{3/2}}$$

$$= 2\pi k\sigma \left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right]$$

(*Field on the axis only*)

- Special case: $R \gg x$

$$E_x = 2\pi k\sigma = \frac{\sigma}{2\epsilon_0}$$

(*Field of infinite plane*)

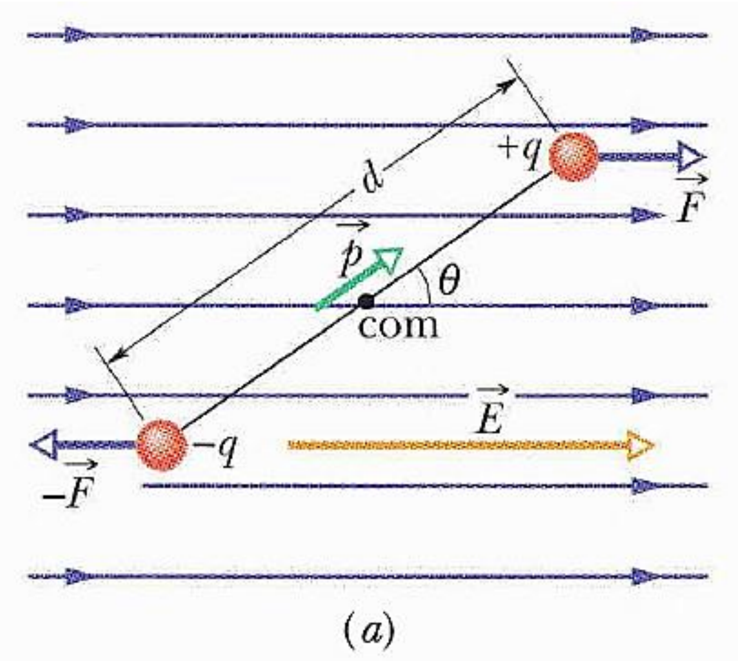
Electric Dipoles

Force and torque on an electric dipole

Electric dipole is placed in an external uniform electric field $\vec{\boldsymbol{E}}$ **:**

- Net force:

$$\vec{\boldsymbol{F}}_{net} = \vec{\boldsymbol{F}}_+ - \vec{\boldsymbol{F}}_- = 0$$



- Torque:

$$\tau = (F_+)(d\ \sin\theta) = (qd)E\ \sin\theta$$

- Define the ***dipole moment vector*** :

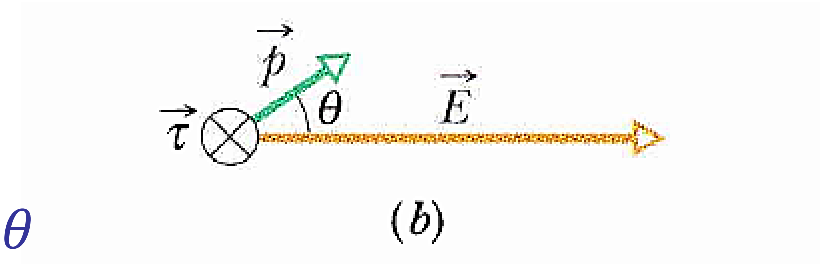
$$\vec{\boldsymbol{p}} = q\vec{\boldsymbol{d}}$$

$$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{p}} \times \vec{\boldsymbol{E}}$$

Electric Dipoles

Force and torque on an electric dipole

- $\vec{\boldsymbol{p}}$ tends to align with the electric field



- Work done in rotating the dipole:

$$W = \int_{\theta_1}^{\theta_2} \tau \, d\theta = \int_{\theta_1}^{\theta_2} pE \sin\theta \, d\theta$$

$$= pE(\cos\theta_2 - \cos\theta_1)$$

- Potential energy:

$$U = -W = -pE\cos\theta$$

$$U(\theta) = -\vec{\boldsymbol{p}} \cdot \vec{\boldsymbol{E}}$$

END