1-5 Polar Graphs

Similar to regular cartesian coordinates, complex numbers may be represented in polar form.

Given $z = x + iy$, we can then use the following properties:

$$\cos\theta = \frac{x}{r} \rightarrow x = r\cos\theta$$
$$\sin\theta = \frac{y}{r} \rightarrow y = r\sin\theta$$

which then means that

$$\begin{aligned} z &= r\cos\theta + ir\sin\theta \\ &= r(\cos\theta + i\sin\theta) \end{aligned}$$

This is the polar form of a complex number.

So given a complex number $1 + i$, its polar form would be:

$$\begin{aligned} r &= \sqrt{1^2 + 1^2} = \sqrt{2} \\ \theta &= \tan^{-1}(\frac{1}{1}) = 45^\circ \\ 1 + i &= r(\cos\theta + i\sin\theta) \\ &= \sqrt{2}(\cos 45^\circ + \sin 45^\circ) \end{aligned}$$

We can use Euler's formula $e^{i\theta} = \cos\theta + i\sin\theta$ (see proof here) to determine the exponential polar form, which would be

$$z = re^{i\theta}.$$

Additional Info

This form makes it easier to do exponential equations. For example,

$$(\sqrt{2}e^{i45^\circ})^6$$

is much easier to evaluate than

$$(1 + i)^6.$$

You can also find the roots of complex numbers using this form.