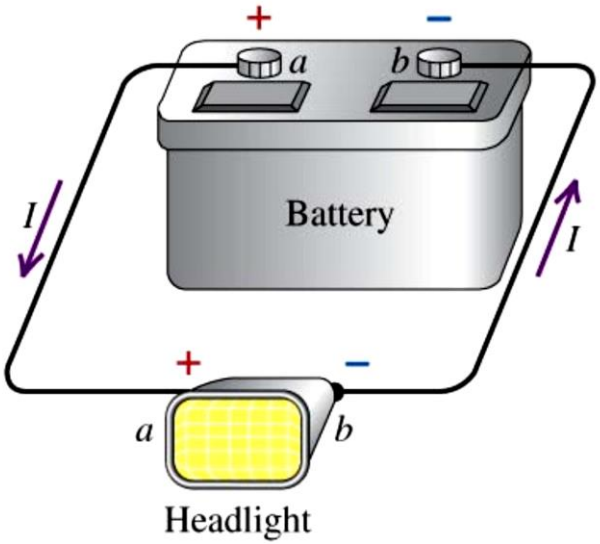
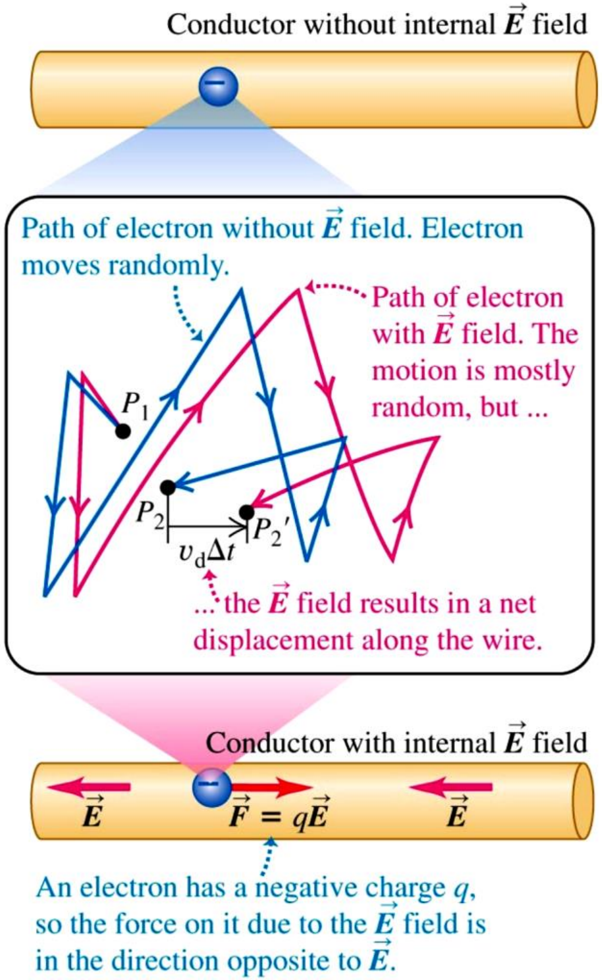
Current, Resistance and Electromotive Force



Current

Electric current

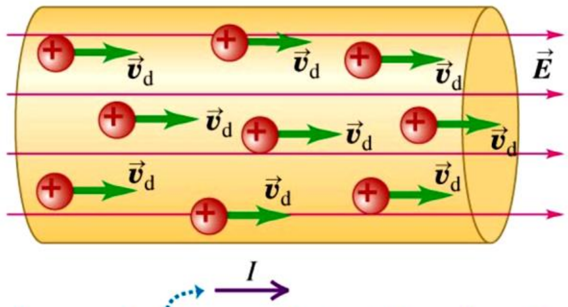
- In the absence of an external electric field, electrons move randomly in a conductor.
- If an electric field $\vec{E}$ exists near the conductor, its force on the electron imposes a drift.
  - $\vec{E}$ does work on moving charges
  - Much of the done by $\vec{E}$ goes into heating the conductor, not into accelerating charges.



Current

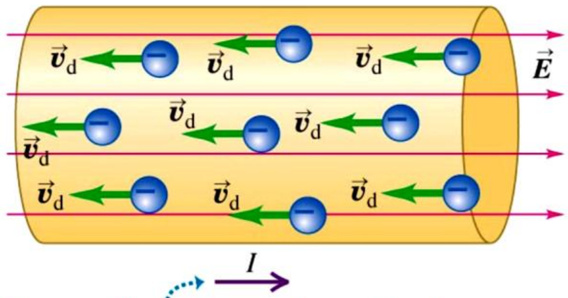
Electric current

- Positive charges would move in the direction of the electric field
- Electrons move in the opposite direction.
- *Conventional current*: direction in which there is a flow of positive charge.
  - This direction is not necessarily the same as the direction in which charged particles are actually moving.



A **conventional current** is treated as a flow of positive charges, regardless of whether the free charges in the conductor are positive, negative, or both.

(b)



In a metallic conductor, the moving charges are electrons — but the *current* still points in the direction positive charges would flow.

Current

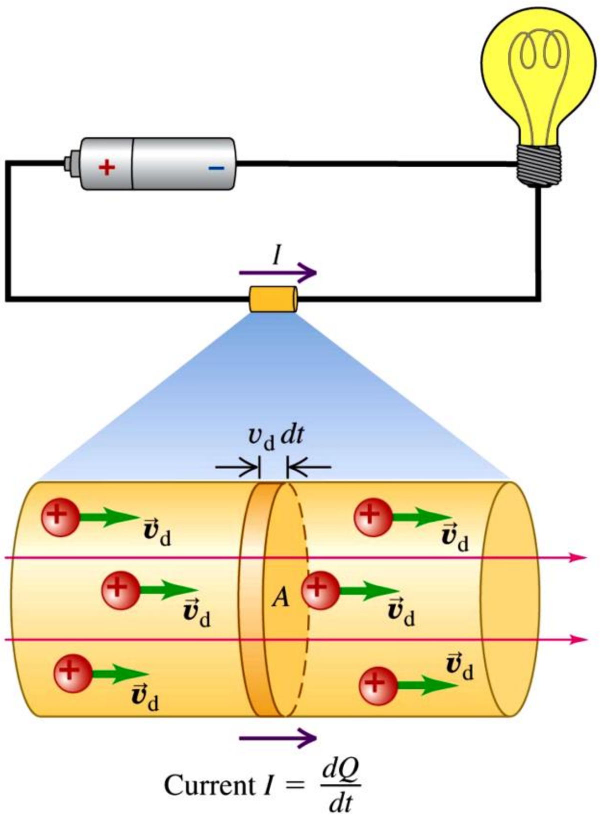
Electric current

- Current I is the time rate of charge transfer through a cross sectional area.

$$I = \frac{dQ}{dt}$$

Units:

$$\frac{\text{Coulombs}}{\text{second}} \equiv \text{Ampere}$$



Current

Drift velocity and current density

- Given

  n = no. of charged particles per unit volume

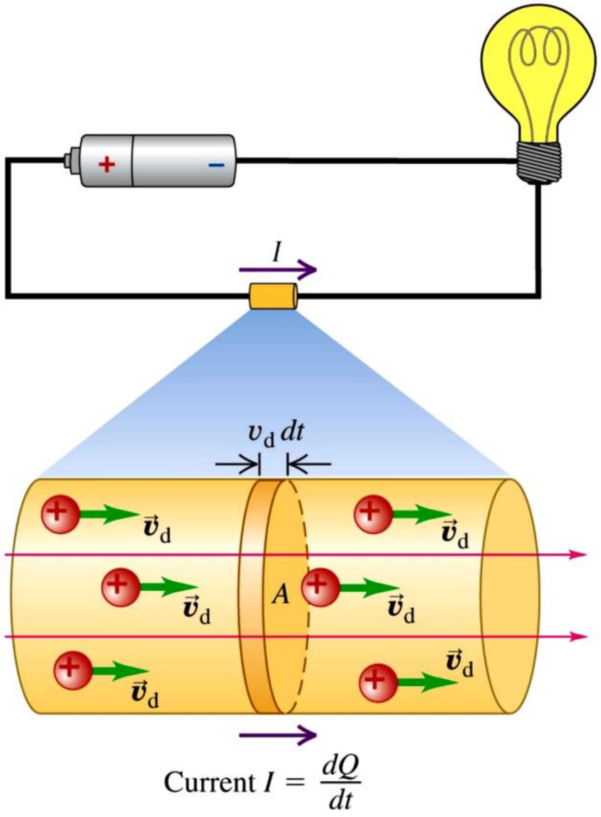
  v_d = drift velocity

- The current I is

$$I = \frac{dQ}{dt} = n\,|q|\,v_d\,A$$

- The current density J is

$$J = \frac{I}{A} = n\,|q|\,v_d$$



Current

Example: Drift velocity and current density in a wire

A copper wire with diameter of 1.02 mm carries a constant current of 1.67 A. The density of free electrons is 8.5×10^{28} electrons/m^3. Find the current density and drift velocity.

Solution:

- Cross-sectional area: $A = \pi R^2 = 8.17 \times 10^{-7}\ \mathrm{m}^2$
- Current density:

$$J = \frac{I}{A} = 2.04 \times 10^{6}\ \mathrm{A/m^2}$$

- Drift velocity:

$$v_d = \frac{J}{n\,|q|} = 1.5 \times 10^{-4}\ \mathrm{m/s}$$

Resistivity

Ohm's law

In a conductor, the current density J is directly proportional to the electric field $\vec{\boldsymbol{E}}$.

- Conductivity:

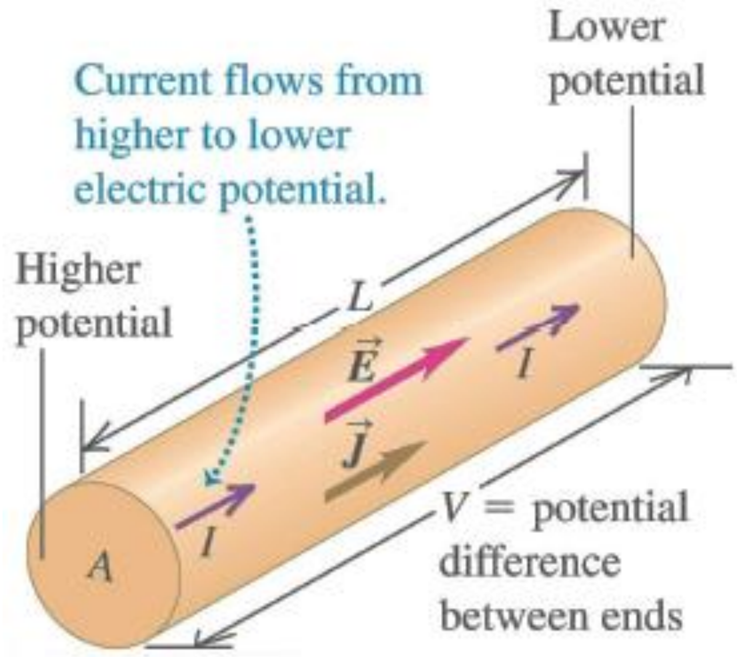
$$J = \sigma\, E$$

$$\sigma = \text{Conductivity}$$

- Resistivity:

$$E = \rho\, J$$

$$\rho = \text{Resistivity}$$



Resistivity

Resistivity of some materials:

Substance		$\rho\,(\Omega \cdot m)$
Conductors		
Metals	Silver	1.47×10^{-8}
	Copper	1.72×10^{-8}
	Gold	2.44×10^{-8}
	Aluminum	2.75×10^{-8}
	Tungsten	5.25×10^{-8}
	Steel	20×10^{-8}
	Lead	22×10^{-8}
	Mercury	95×10^{-8}
Alloys	Manganin (Cu 84%, Mn 12%, Ni 4%)	44×10^{-8}
	Constantan (Cu 60%, Ni 40%)	49×10^{-8}
	Nichrome	100×10^{-8}

Resistivity

Resistivity of some materials:

Substance	$\rho\,(\Omega \cdot m)$
Semiconductors	
Pure carbon (graphite)	3.5×10^{-5}
Pure germanium	0.60
Pure silicon	2300
Insulators	
Amber	5×10^{14}
Glass	10^{10}–10^{14}
Lucite	$>10^{13}$
Mica	10^{11}–10^{15}
Quartz (fused)	75×10^{16}
Sulfur	10^{15}
Teflon	$>10^{13}$
Wood	10^{8}–10^{11}

Resistance

Ohm's law

In a conductor, the current I is directly proportional to the electric potential difference V_{ab}

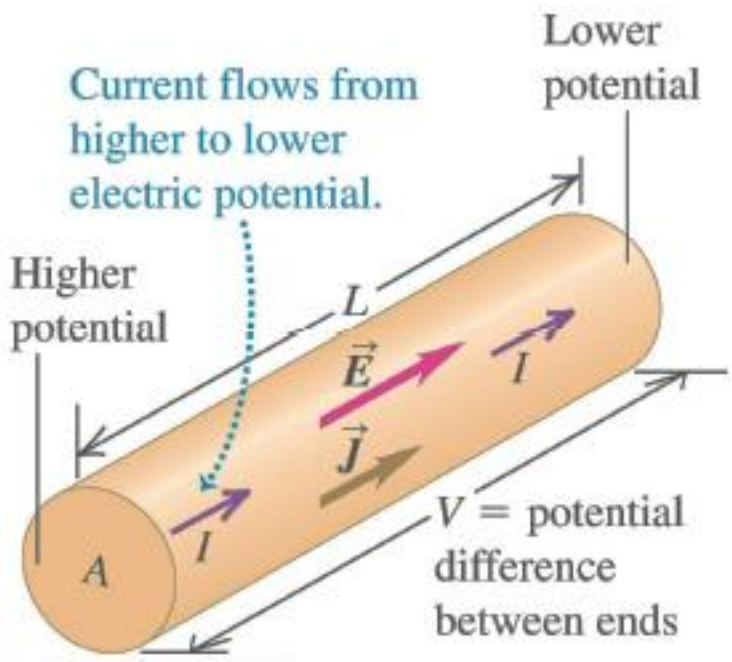
$$V_{ab} = E\,L$$

$$I = J\,A$$

- Resistance:

$$E = \frac{V_{ab}}{L} = \rho\,J = \rho\,\frac{I}{A}$$

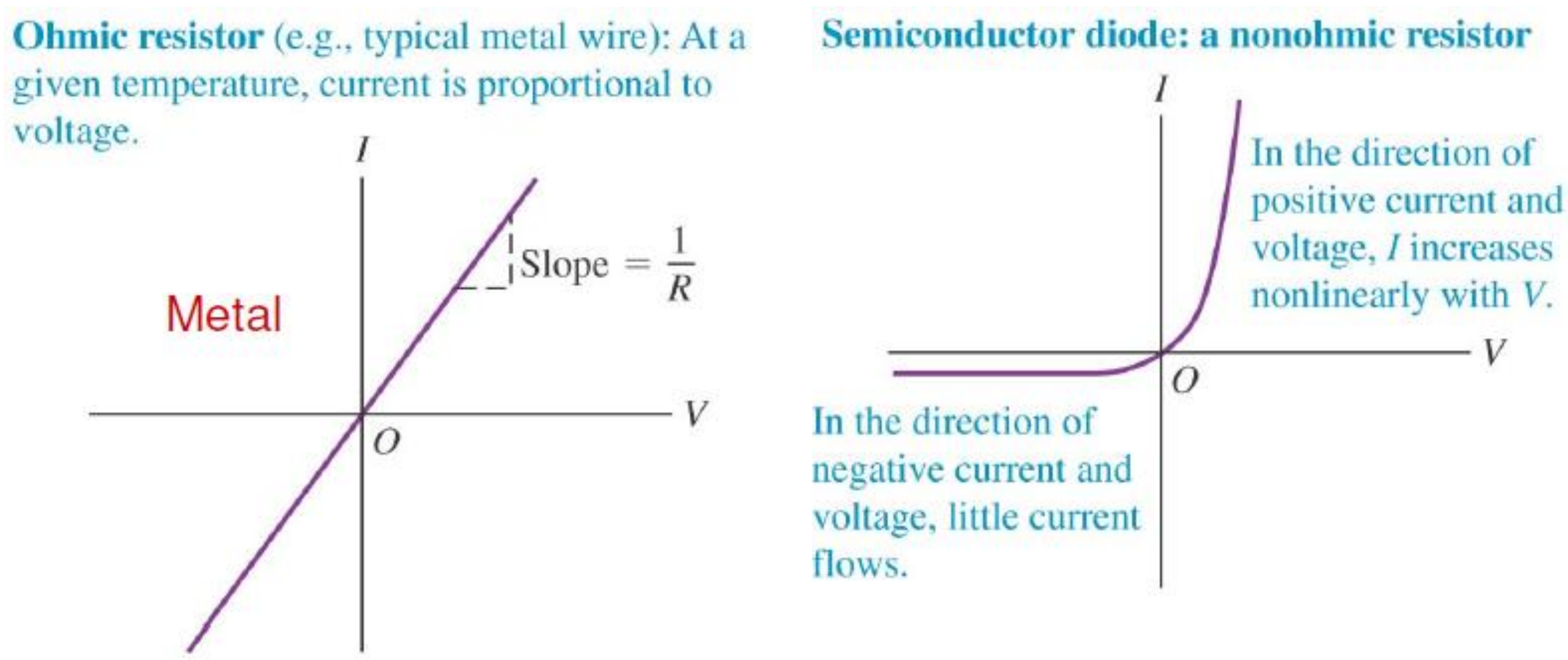
$$V_{ab} = \left(\frac{\rho\,L}{A}\right) I = R\,I$$



Resistance

Ohm’s law

Ohmic and non-ohmic materials.



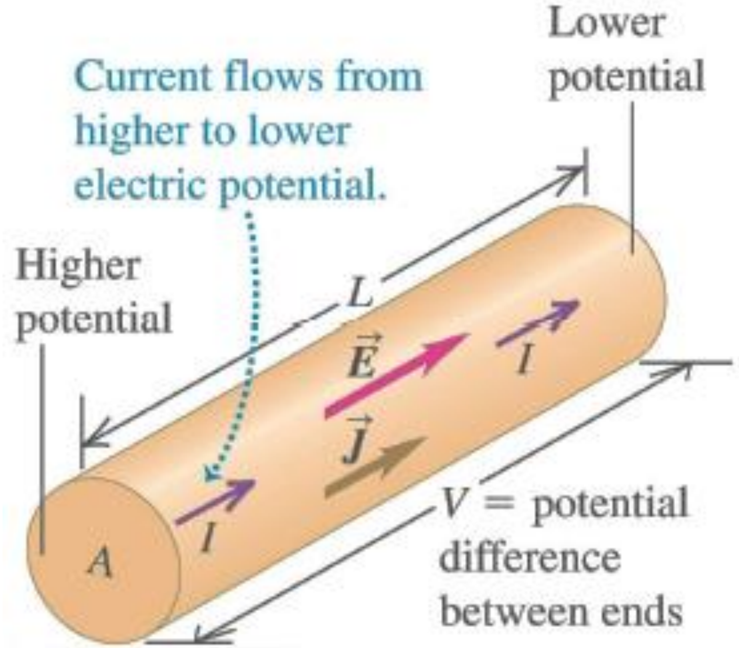
Resistance

Ohm's law

Current direction is from higher *V* end to lower *V* end. Follows direction of $\vec{\boldsymbol{E}}$, independent of sign of moving charges.

$$V_{ab} = \text{"voltage drop"}$$

- As the current flows through a potential difference, electric potential energy is lost.
- This energy is transferred to the ions of conducting material during collisions.

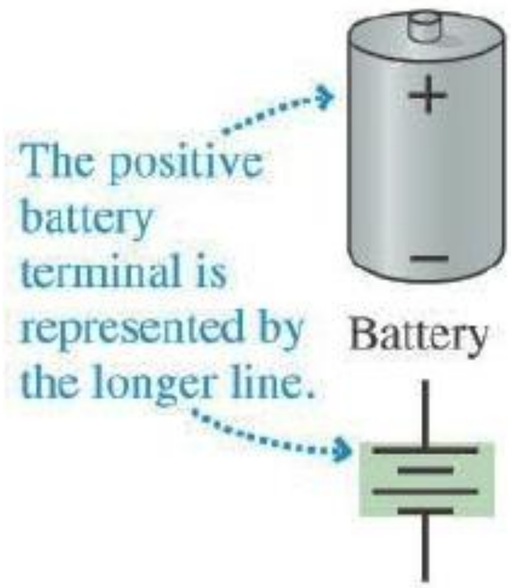


Electromotive Force and Circuits

***Electromotive force* (*emf*)**

Maintaining a current in an electric circuit requires a device that acts like the water pump in a fountain: a source of *emf*.

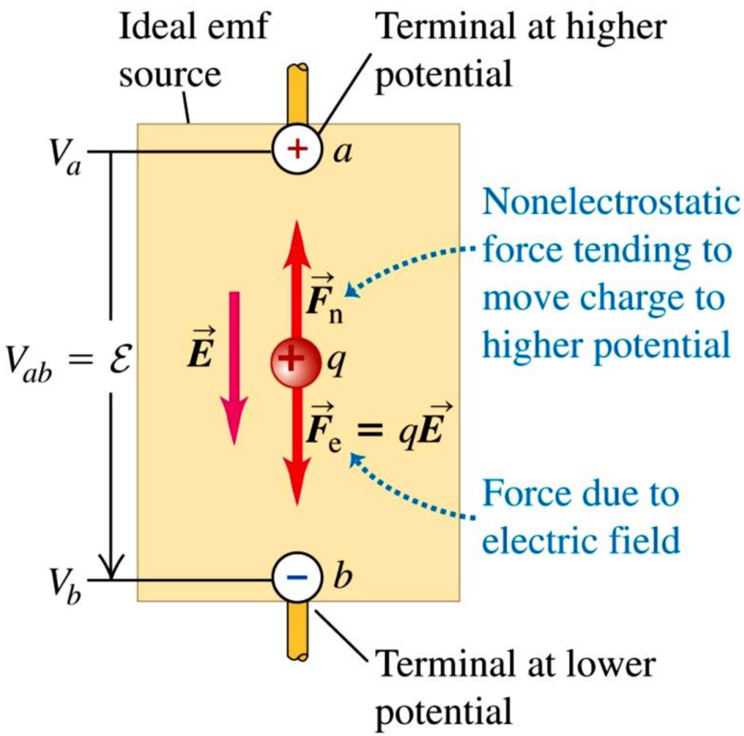
- Device converts energy (mechanical, chemical, thermal) into electric potential energy and transfers it to the circuit.
- In this device, the charge travels “uphill” from lower to higher *V* (opposite to normal conductor) due to the emf force.
- ***emf*** is not a force but energy/unit charge (units: Volt=Joule/Coulomb)



Electromotive Force and Circuits

Electromotive force $\mathcal{E}$

- Ideal emf device maintains a constant potential difference between its terminals, independent of the current *I*.
- Non electrostatic force $\vec{\boldsymbol{F}}_n$ maintains potential difference between terminals.
  - If $\vec{\boldsymbol{F}}_n \rightarrow 0$, charge will flow between terminals until $V_{ab} = 0$.



When the emf source is not part of a closed circuit, $F_n = F_e$ and there is no net motion of charge between the terminals.

Electromotive Force and Circuits

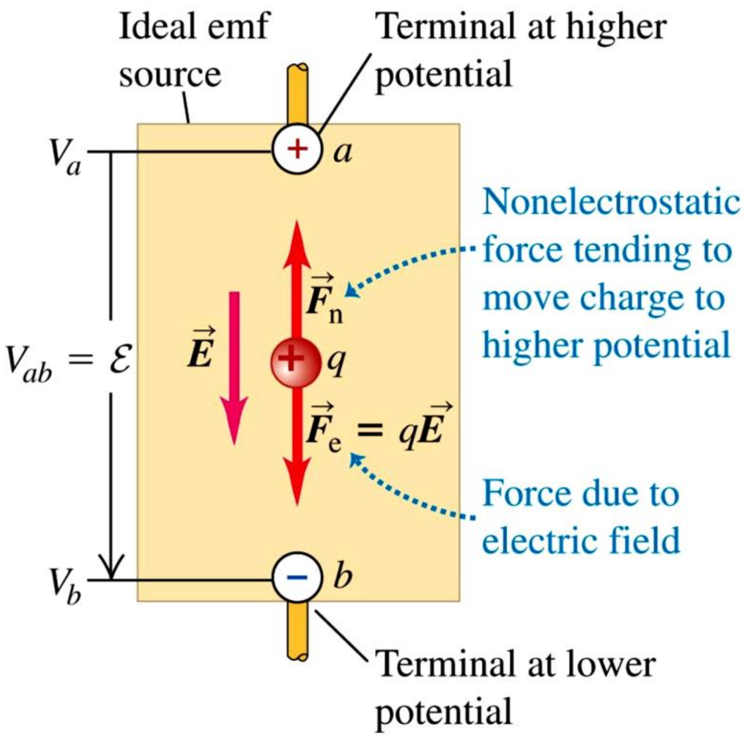
Electromotive force $\mathcal{E}$

- Work done by $\vec{F}_n$:

$$W_{ab} = q(V_a - V_b) = q\,\mathcal{E}$$

  - Potential energy increases by

$$q\,V_{ab} = q\,\mathcal{E}$$



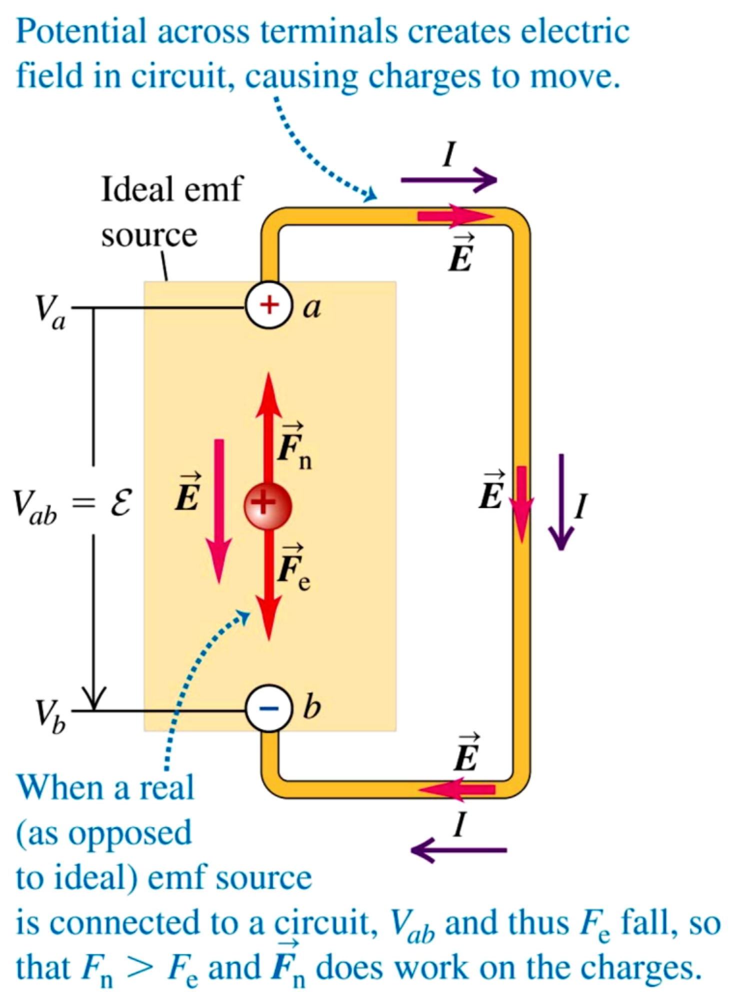
When the emf source is not part of a closed circuit, $F_n = F_e$ and there is no net motion of charge between the terminals.

Electromotive Force and Circuits

Closed circuit

A conducting path connects the terminals of the source of *emf*.

- When a charge q flows around a circuit, the potential rise $\mathcal{E}$ as it passes through the ideal source equal the potential drop V_{ab} as it passes through reminder of circuit.
- The current is same at every point of a circuit—charge is conserved and cannot be accumulated in any part of the circuit.



Electromotive Force and Circuits

A simple circuit

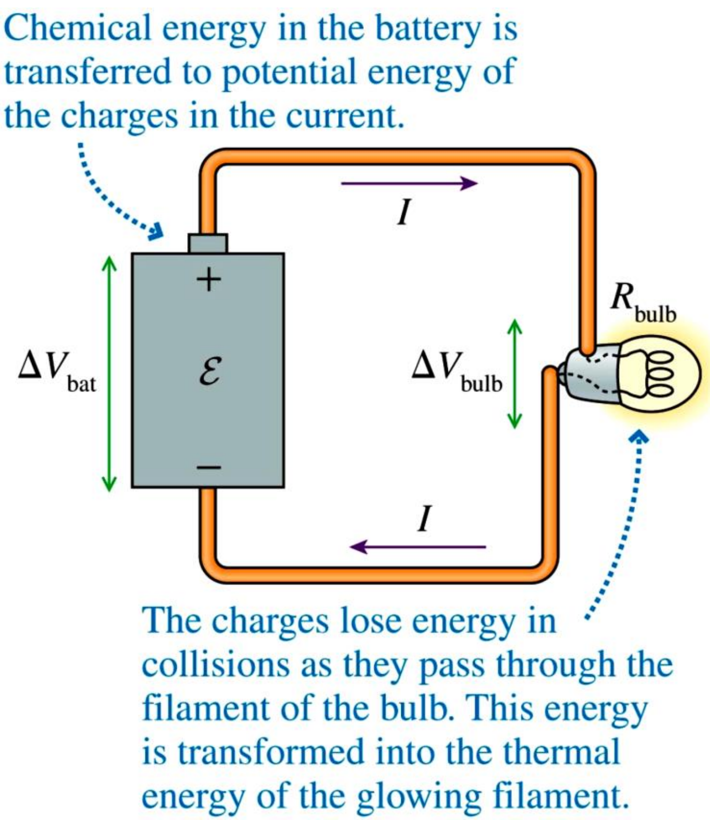
Battery and bulb.

- Net work done around a closed loop is zero.

$$V_{bulb} = \mathcal{E} = R\ I$$

$$I = \frac{\mathcal{E}}{R}$$

- Ideal connecting wires have zero resistance.



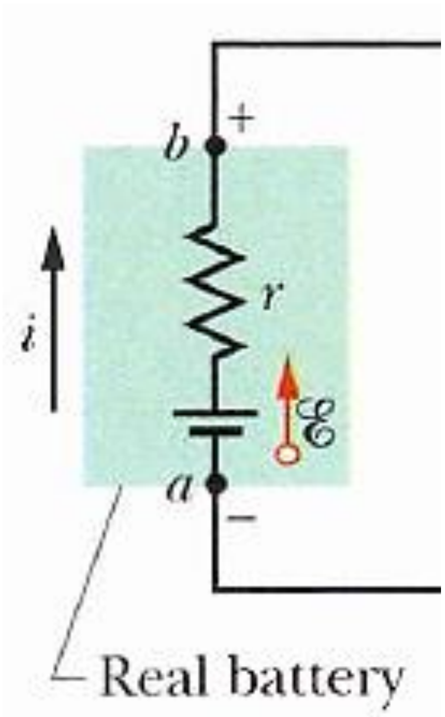
Electromotive Force and Circuits

Internal resistance

The terminal voltage is $V_{ab} = \mathcal{E}$ when the terminals are not connected.

- Connections allows electrons to flow, but internal resistance within battery delivers incrementally less voltage.
- Terminal voltage:

$$V_{ab} = \mathcal{E} - r\,I$$



A simple circuit

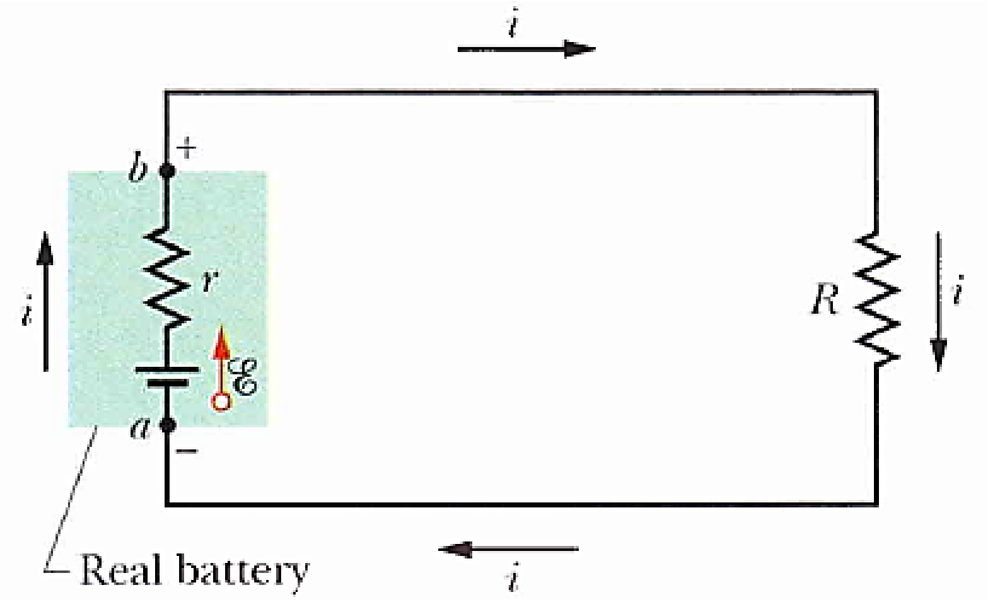
Real battery and bulb.

- Net work done around a closed loop is zero.

$$V_{ab} = \mathcal{E} - r\,I = R\,I$$

$$I = \frac{\mathcal{E}}{R + r}$$

$$V_{ab} = \frac{R}{R + r}\mathcal{E}$$

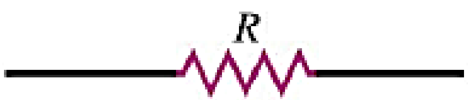


Electromotive Force and Circuits

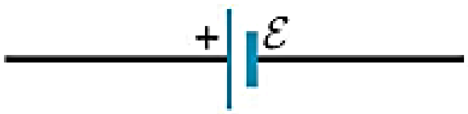
Schematic circuit diagrams

Table 25.4 Symbols for Circuit Diagrams

Conductor with negligible resistance



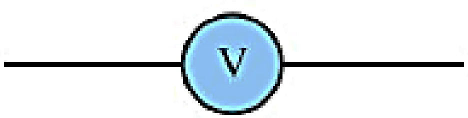
Resistor



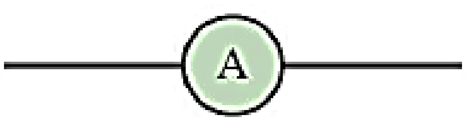
Source of emf (longer vertical line always represents the positive terminal, usually the terminal with higher potential)

or

Source of emf with internal resistance r (r can be placed on either side)



Voltmeter (measures potential difference between its terminals)



Ammeter (measures current through it)

Electromotive Force and Circuits

Example: A simple circuit

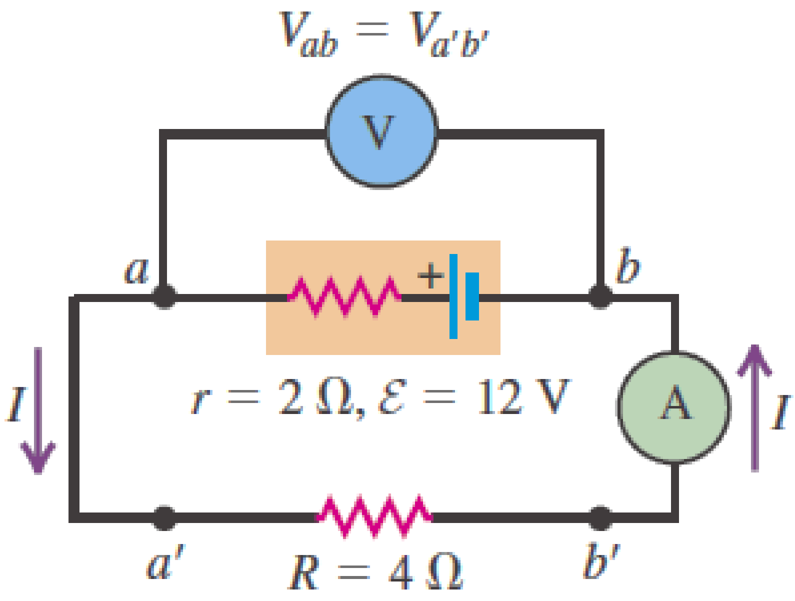
- Current:

$$I = \frac{12\ \text{V}}{4\ \Omega + 2\ \Omega} = 2\ \text{A}$$

- Voltage:

$$V_{ab} = \mathcal{E} - r\,I = R\,I$$

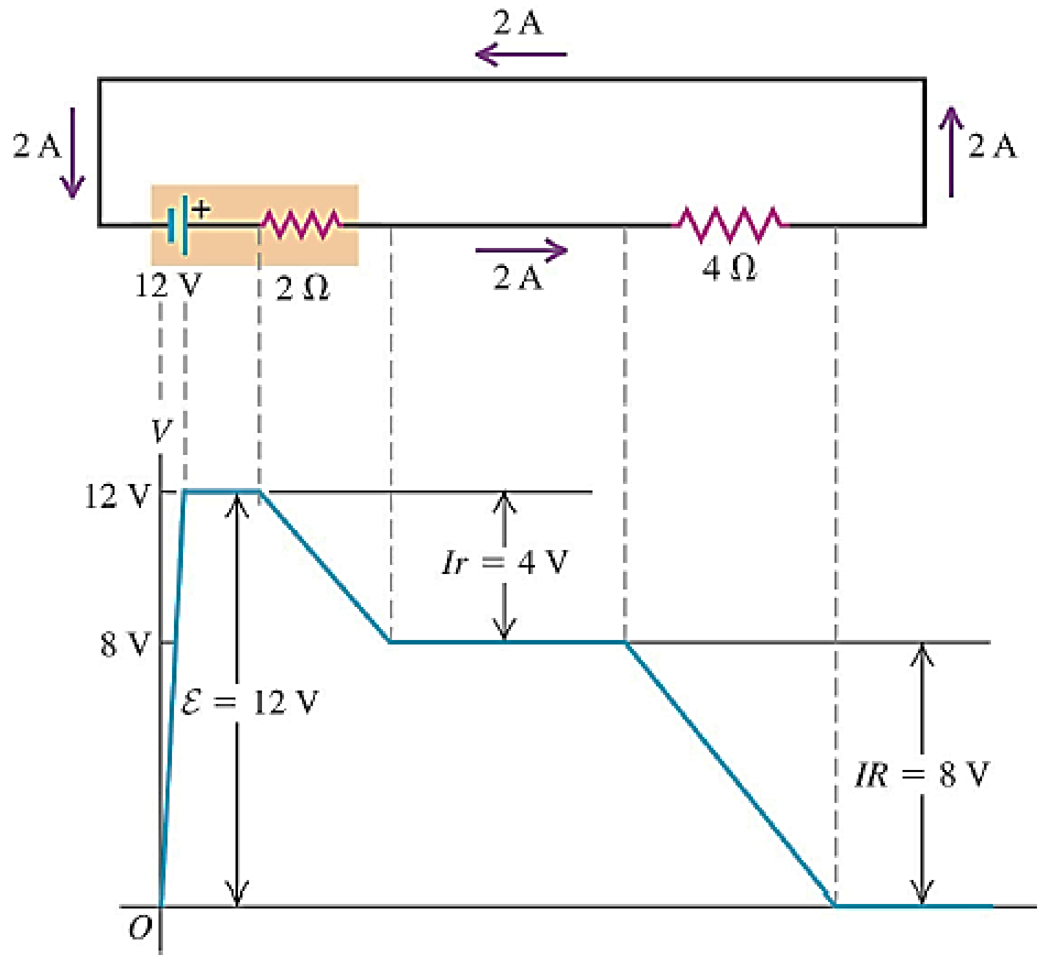
$$= 8\ \text{V}$$



Electromotive Force and Circuits

Potential changes around a circuit

- The net change in potential energy for a charge *q* making a round trip around a complete circuit must be zero.



Energy and Power in Electric Circuits

Potential energy changes

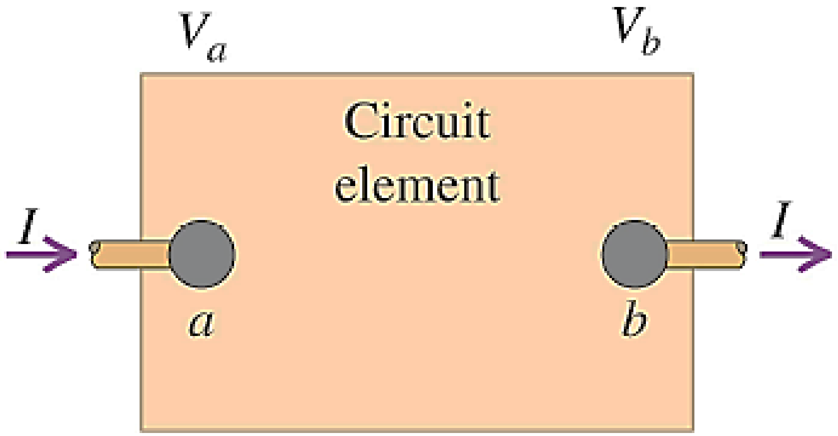
- When amount of charge Δq passes through, potential energy changes by

$$\Delta U = \Delta q\, V_{ab}$$

- The ***power***

$$\frac{\Delta U}{\Delta t} = \left(\frac{\Delta q}{\Delta t}\right) V_{ab} = I \cdot V_{ab}$$

  - Power is the rate at which energy is delivered to or extracted from a circuit element



Energy and Power in Electric Circuits

Power

- In a battery $V_{ab} = \mathcal{E}$

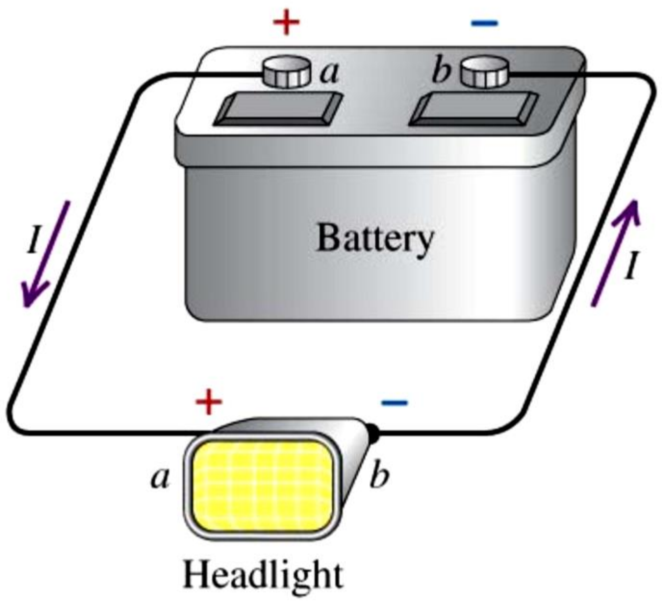
$$P = I \cdot \mathcal{E}$$

  - Rate at which the emf source converts nonelectrical to electrical energy.

- In a resistor $V_{ab} = R \cdot I$

$$P = I^2 \cdot R$$

  - Energy dissipated (as heat) in the resistor



Energy and Power in Electric Circuits

Power in a circuit

- With ideal battery

$$I^2 \cdot R = I \cdot \mathcal{E}$$

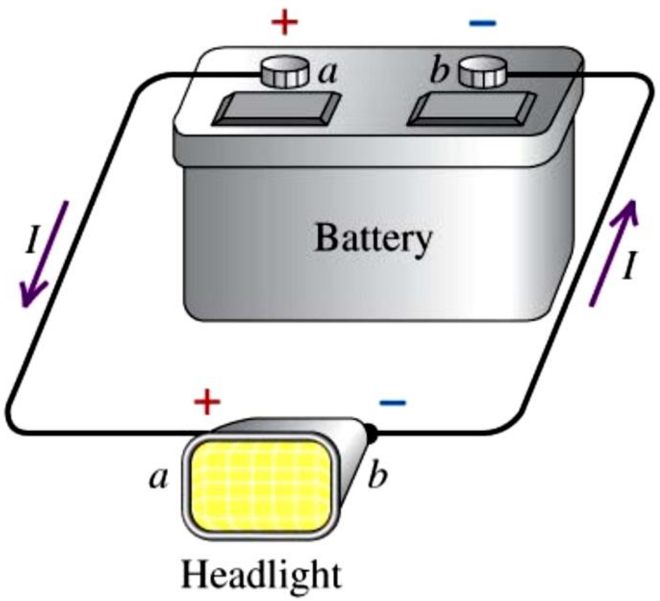
$$\mathcal{E} \cdot I - I^2 \cdot R = 0$$

  - Energy is conserved.

- With real battery

$$I^2 \cdot R = I \cdot (\mathcal{E} - r \cdot I)$$

$$\mathcal{E} \cdot I - I^2 \cdot (R + r) = 0$$



Energy and Power in Electric Circuits

Example: A simple circuit

- Current:

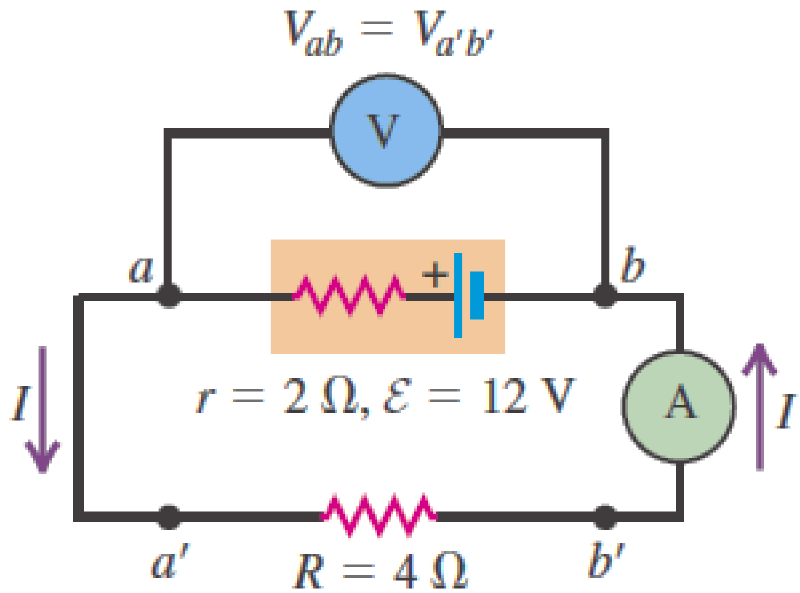
$$I = 2\ \mathrm{A}$$

- Power:

$$\mathcal{E}\, I = 24\ \mathrm{W}$$

$$I^2 r = 8\ \mathrm{W}$$

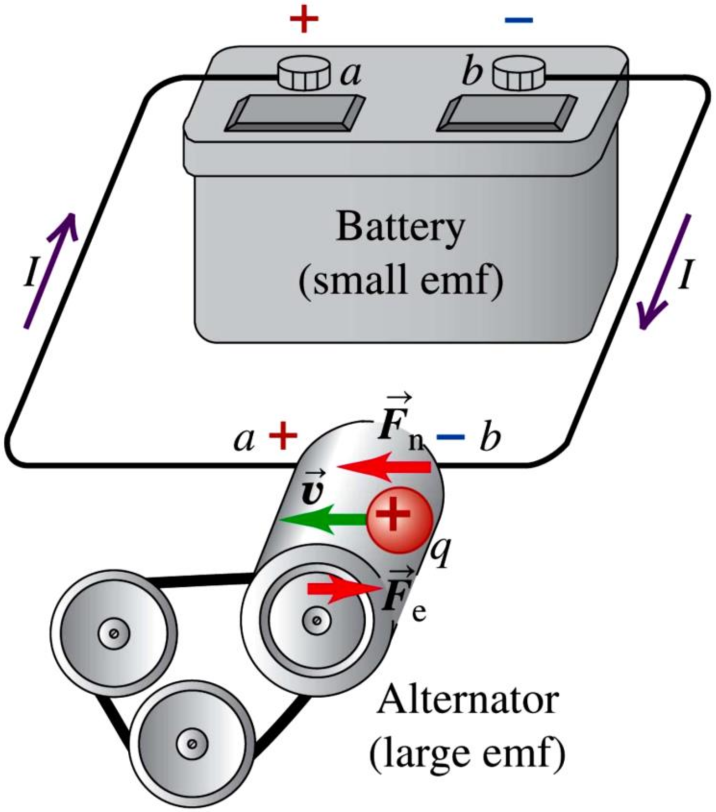
$$I^2 R = 16\ \mathrm{W}$$



Energy and Power in Electric Circuits

Power in a circuit

- Charging a battery
  - Generator pushing current upward through upper source.



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