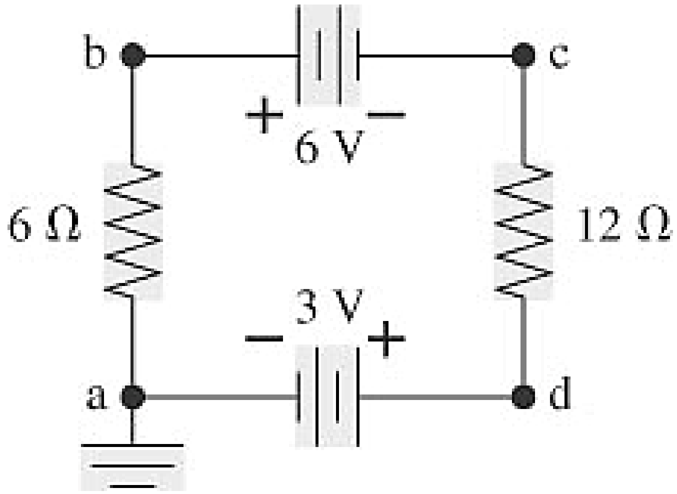
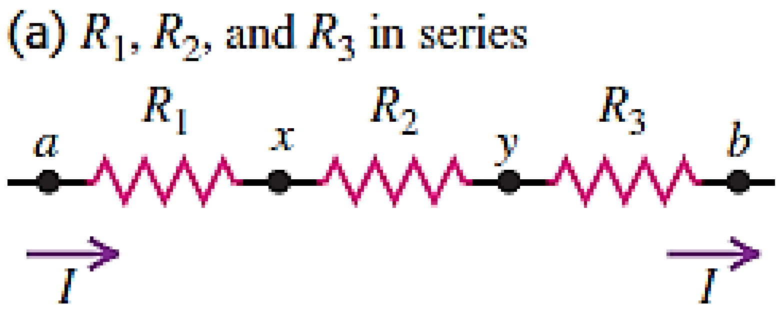
Direct-Current Circuits

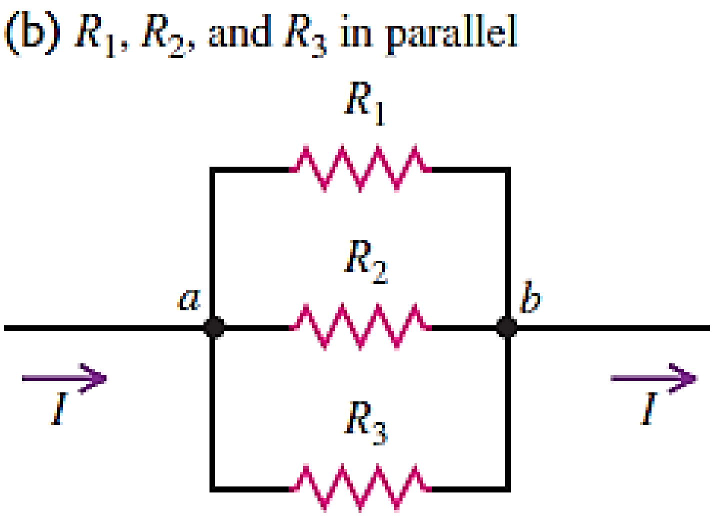


Resistors in Series and Parallel

Combining resistors

- For three resistors R_1, R_2, R_3.



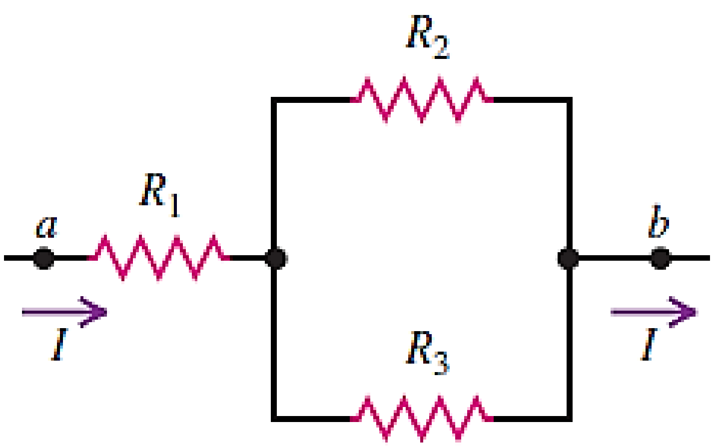


Resistors in Series and Parallel

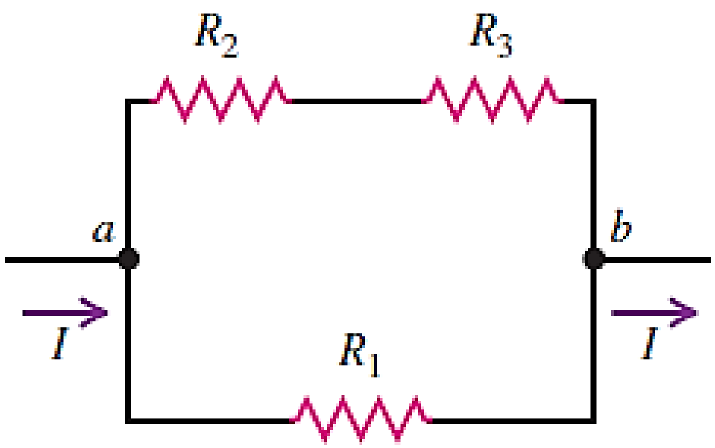
Combining resistors

- For three resistors R_1, R_2, R_3.

(c) R_1 in series with parallel combination of R_2 and R_3



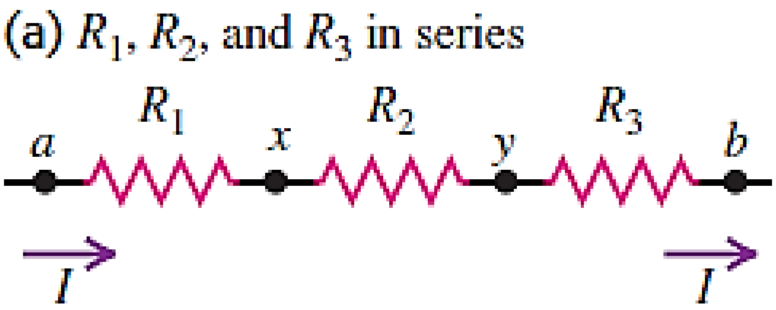
(d) R_1 in parallel with series combination of R_2 and R_3



- For any combination of resistors, we can always find a *single* resistor that could replace the combination and result in the same total current and potential difference.

Resistors in Series and Parallel

Resistors in series



$$V_{ab} = V_{ax} + V_{xy} + V_{yb} = I(R_1 + R_2 + R_3)$$

$$\frac{V_{ab}}{I} = (R_1 + R_2 + R_3)$$

- The equivalent resistance:

$$R_{eq} = (R_1 + R_2 + R_3)$$

Resistors in Series and Parallel

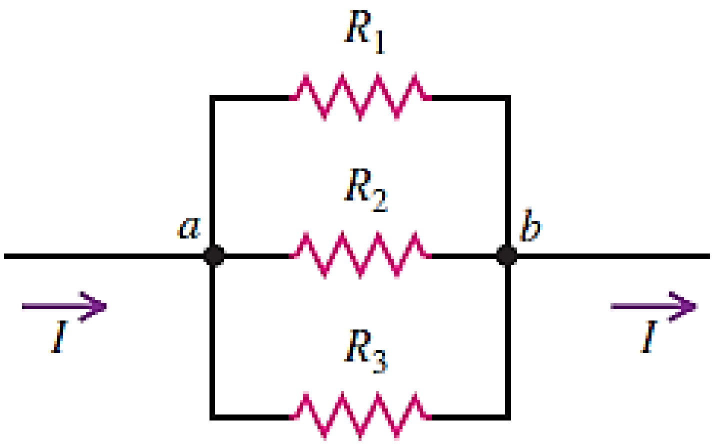
Resistors in parallel

$$I = I_1 + I_2 + I_3$$

$$= \frac{V_{ab}}{R_1} + \frac{V_{ab}}{R_2} + \frac{V_{ab}}{R_3}$$

$$\frac{I}{V_{ab}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

(b) R_1, R_2, and R_3 in parallel



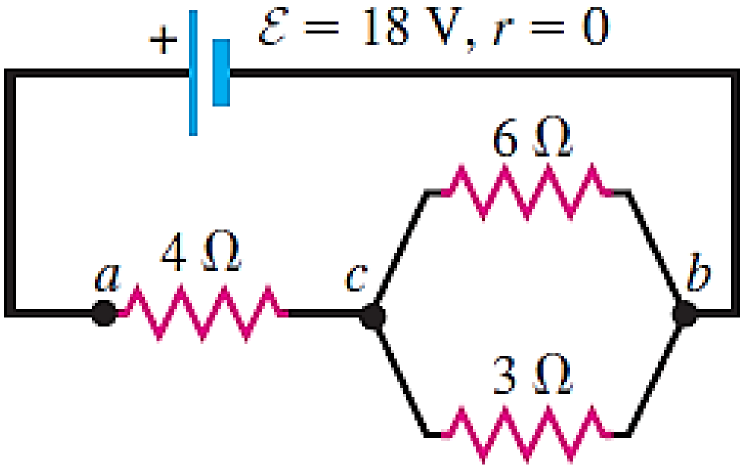
- The equivalent resistance:

$$\frac{1}{R_{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

Resistors in Series and Parallel

Example 1

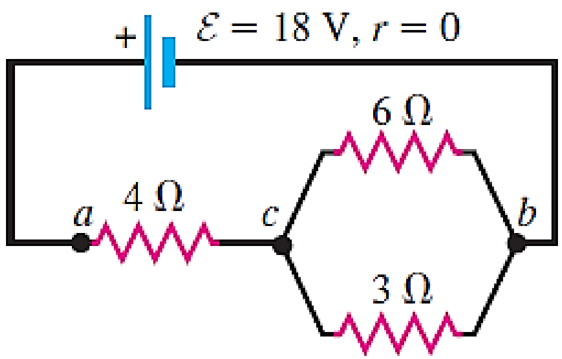
- Find the equivalent resistance of the network shown and find the current in each resistor. The source of emf has negligible internal resistance.



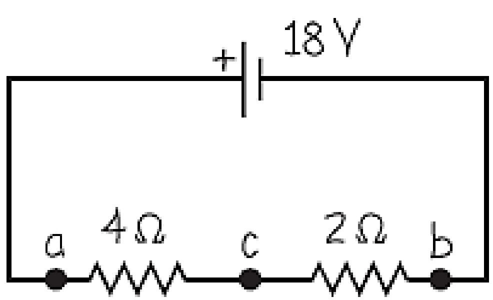
Resistors in Series and Parallel

Example 1

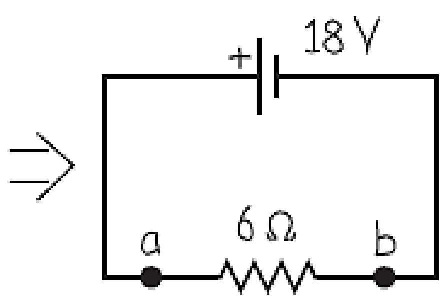
- Solution: The equivalent resistance



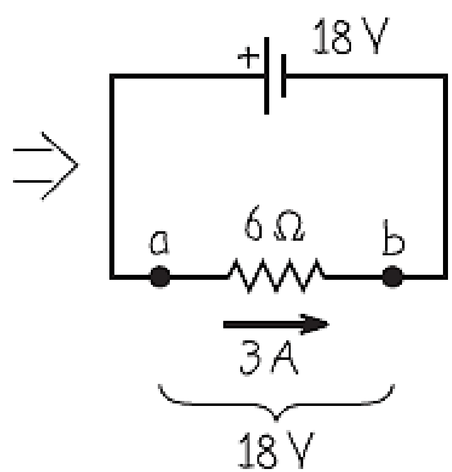
(b)



(c)



(d)

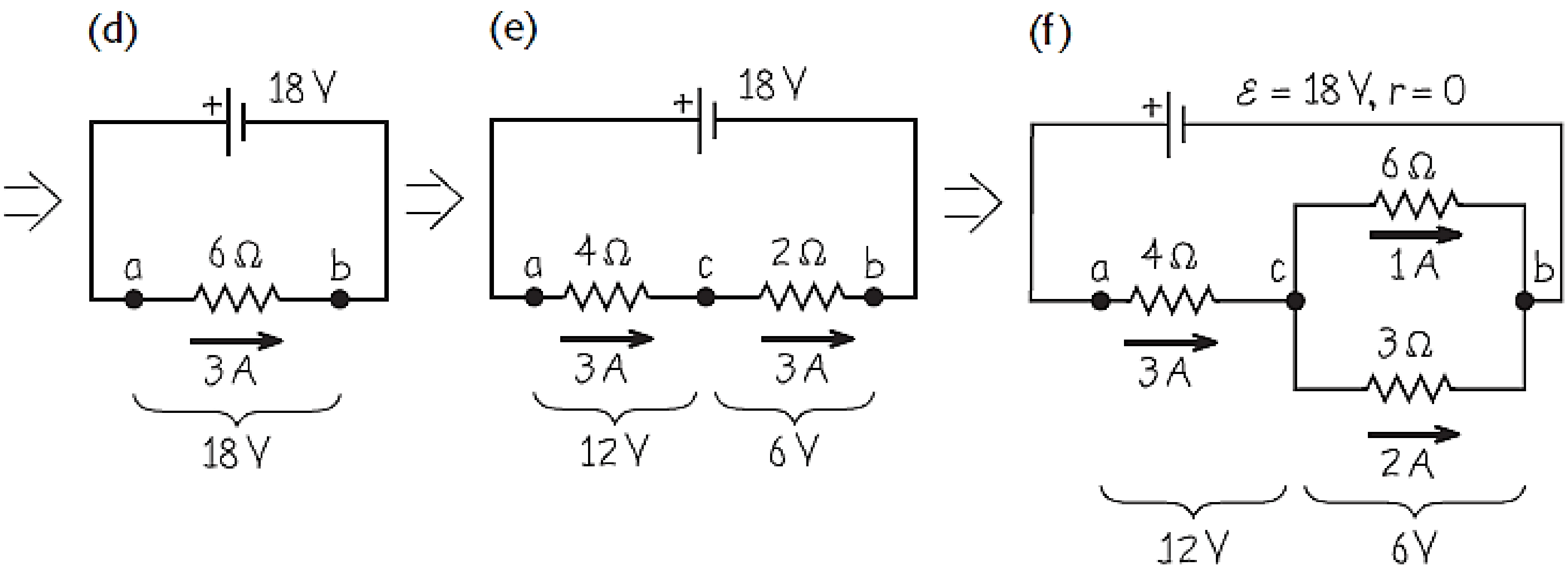


$$R_{eq} = 6\ \Omega$$

Resistors in Series and Parallel

Example 1

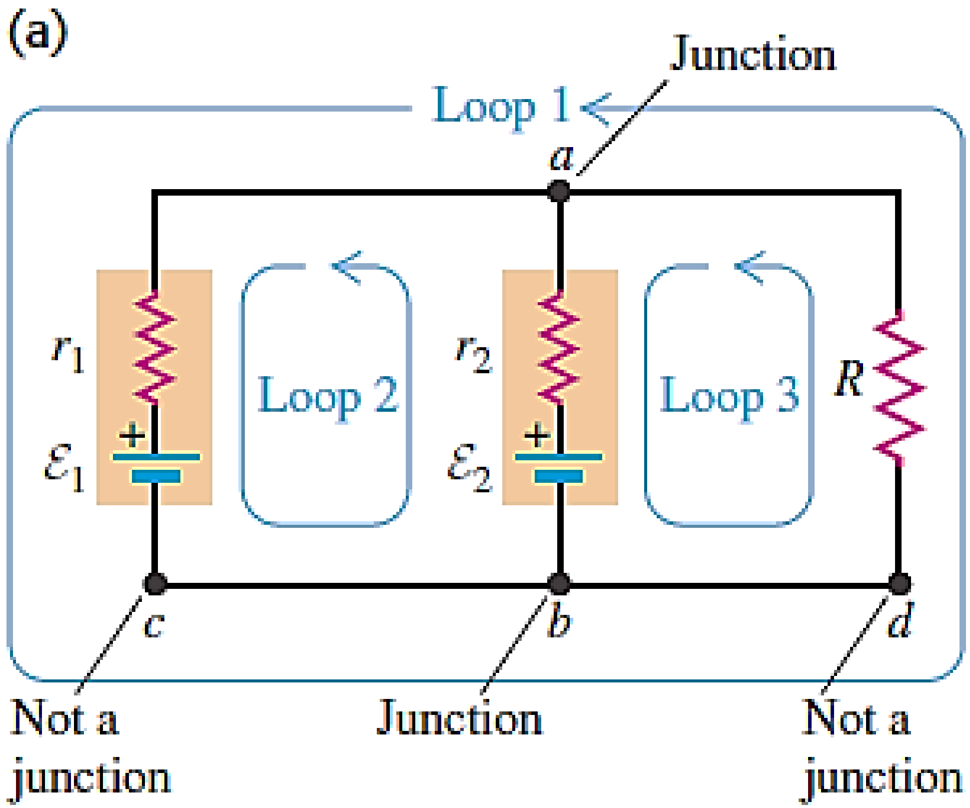
- Solution: Determining the currents



Kichhoff's Rules

Junctions and loops

- Two features of circuits



- ***Junction*** – a point where three or more conductors meet.
- ***Loop*** – is any closed conducting path.

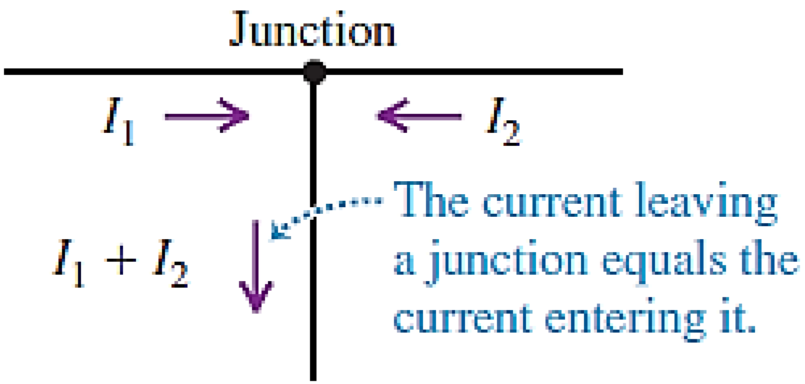
Kichhoff’s Rules

Junctions and loops

Kirchhoff’s junction rule: *The algebraic sum of the currents into any junction is zero.*

$$\sum I = 0$$

- Based on *conservation of electric charge.*



$$\sum I_{in} = \sum I_{out}$$

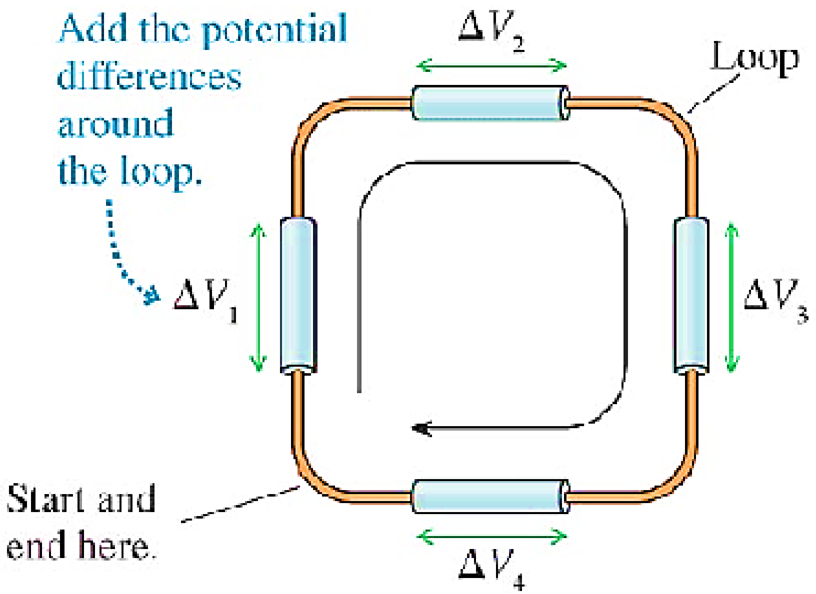
Kichhoff’s Rules

Junctions and loops

Kirchhoff’s loop rule: *The algebraic sum of the potential differences in any loop,* including those associated with emfs and those of resistive elements, *must equal zero.*

$$\sum \Delta V = 0$$

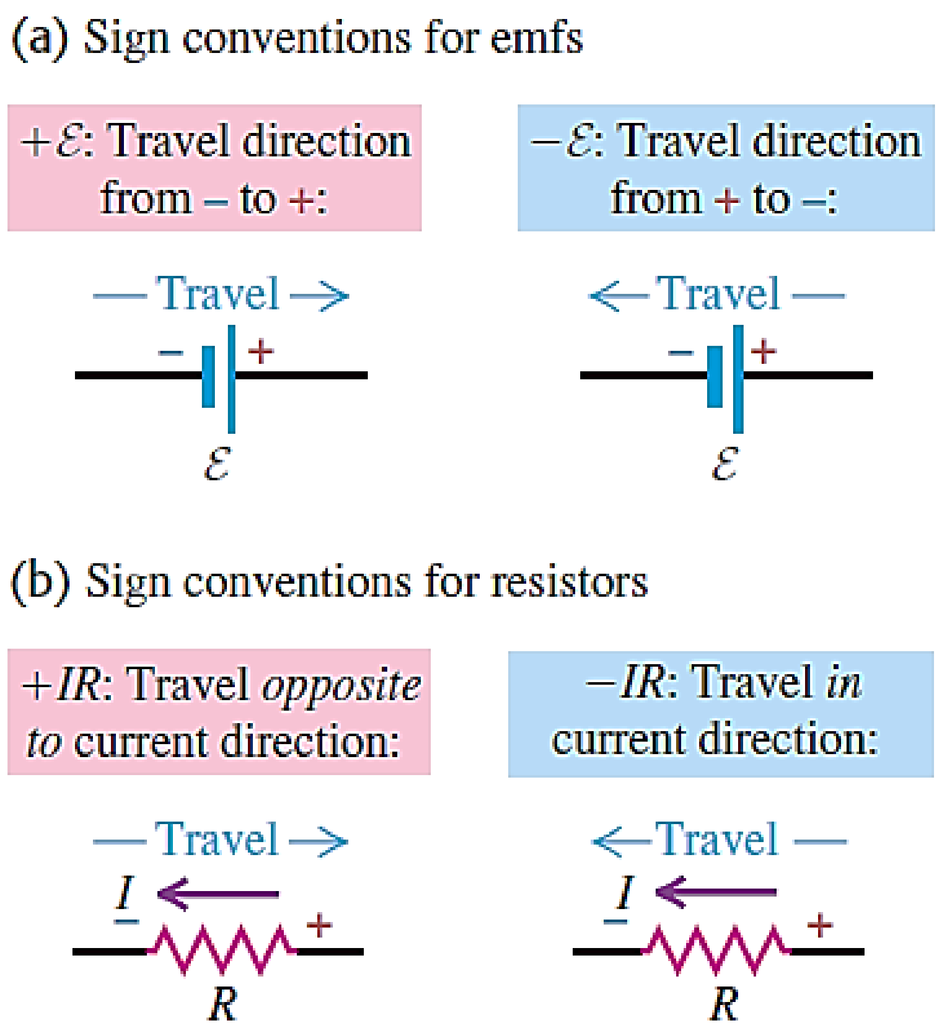
- Based on *conservation of energy.*



$$\sum V_{drop} = \sum V_{rise}$$

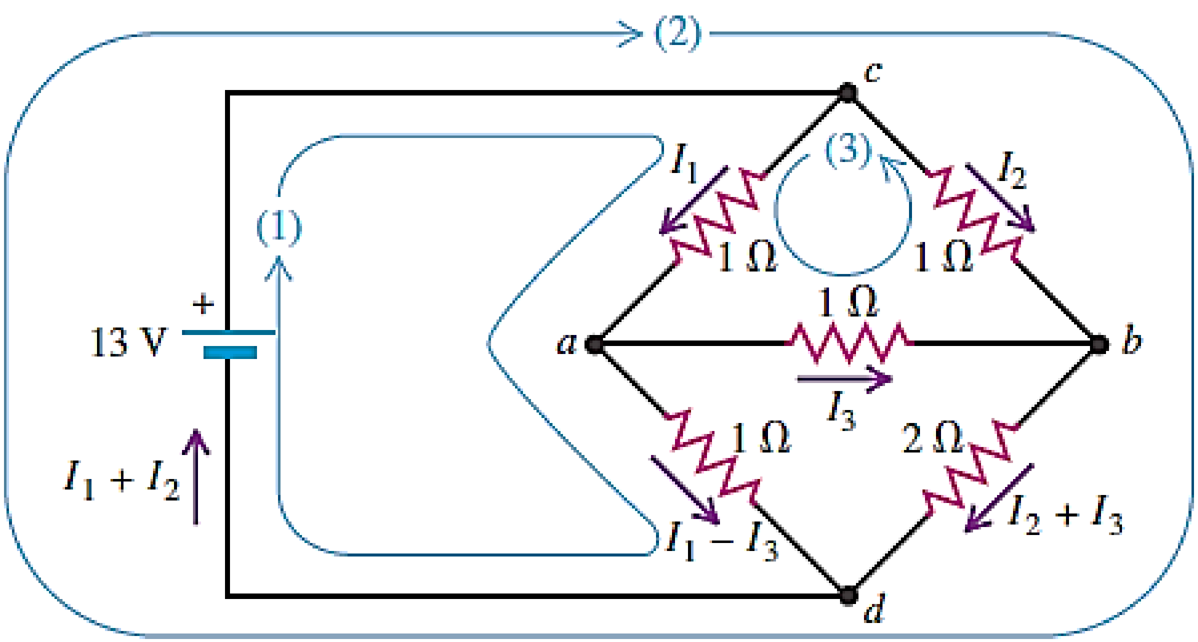
Kichhoff's Rules

Sign convention for the loop rule:



Example 2:

Below is a "bridge" circuit. Find a) the current in each resistor and b) the equivalent resistance of the network of five resistors.



Kichhoff's Rules

Example 2:

Solution:

a) Apply the loop rule to the three loops shown:

$$13\ \text{V} - I_1(1\ \Omega) - (I_1 - I_3)(1\ \Omega) = 0 \qquad (1)$$
$$-I_2(1\ \Omega) - (I_2 + I_3)(2\ \Omega) + 13\ \text{V} = 0 \qquad (2)$$
$$-I_1(1\ \Omega) - I_3(1\ \Omega) + I_2(1\ \Omega) = 0 \qquad (3)$$

- From junction rule: $I_2 = I_1 + I_3$

$$13\ \text{V} = I_1(2\ \Omega) - I_3(1\ \Omega) \qquad (1')$$
$$13\ \text{V} = I_1(3\ \Omega) + I_3(5\ \Omega) \qquad (2')$$

$$I_1 = 6\ \text{A}\ ;\quad I_3 = -1\ \text{A}\ ;\quad I_2 = 5\ \text{A}$$

Kichhoff's Rules

Example 2:

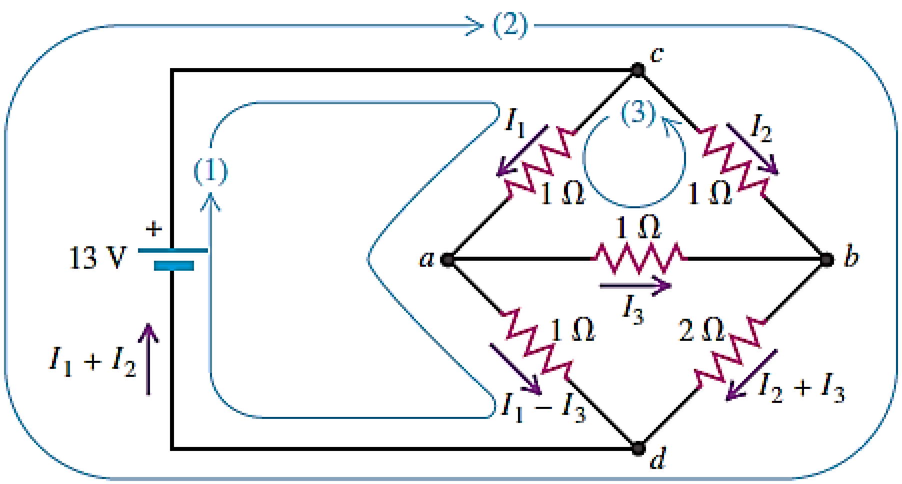
Solution:

b) Equivalent resistance:

$$R_{eq} = \frac{\mathcal{E}}{I_1 + I_2} = \frac{13\ \text{V}}{11\ \text{A}} = 1.2\ \Omega$$

- Potential difference V_{ab}

$$V_{ab} = I_3(1\ \Omega) = -1\ \text{V}$$

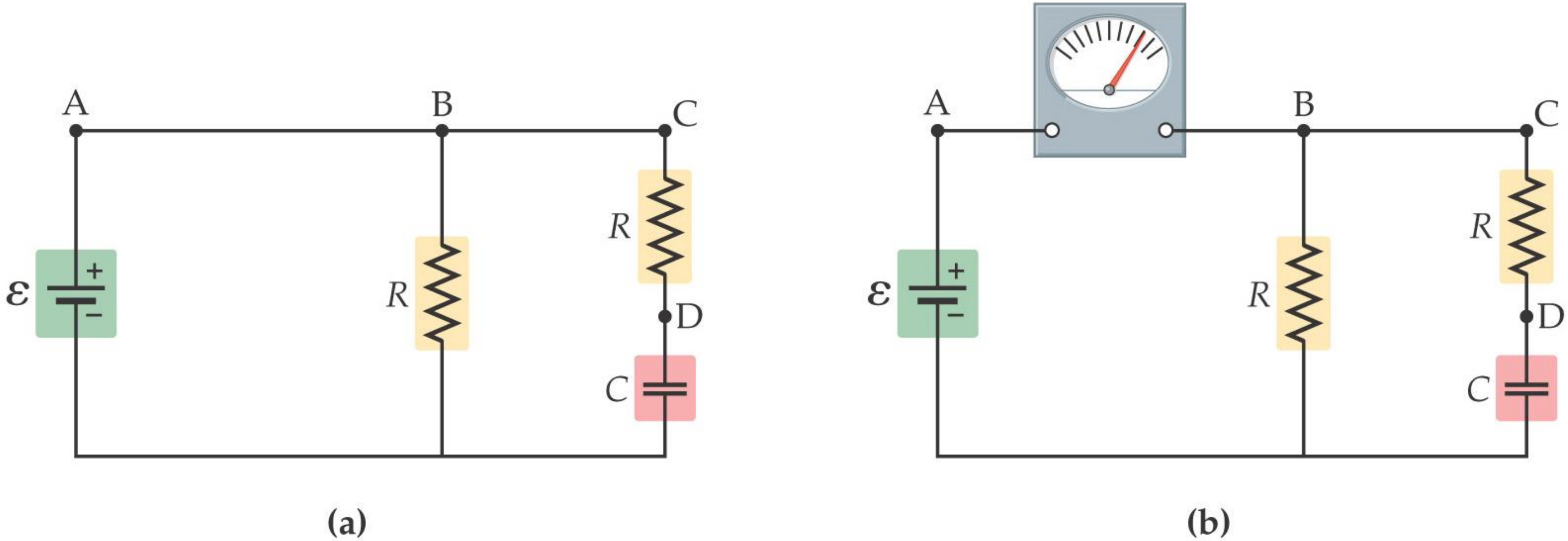


Measuring Instruments

The Ammeter

An *ammeter* is a device for measuring current.

- The current in the circuit must flow through the ammeter; therefore the ammeter should have as low a resistance as possible, for the least disturbance.

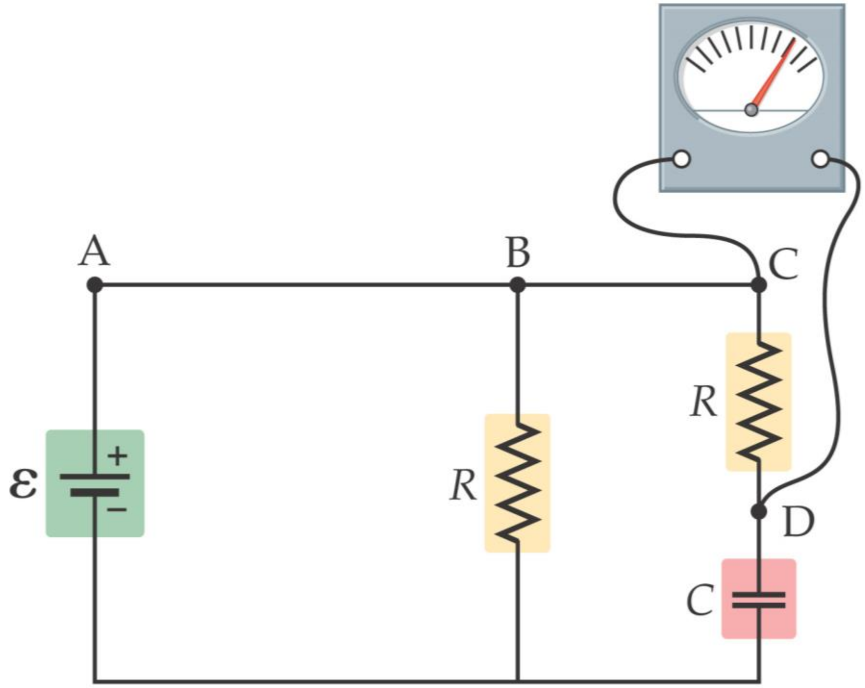


Measuring Instruments

The Voltmeter

A *voltmeter* measures the potential drop between two points in a circuit.

- It therefore is connected in parallel;
- In order to minimize the effect on the circuit, it should have as large a resistance as possible.

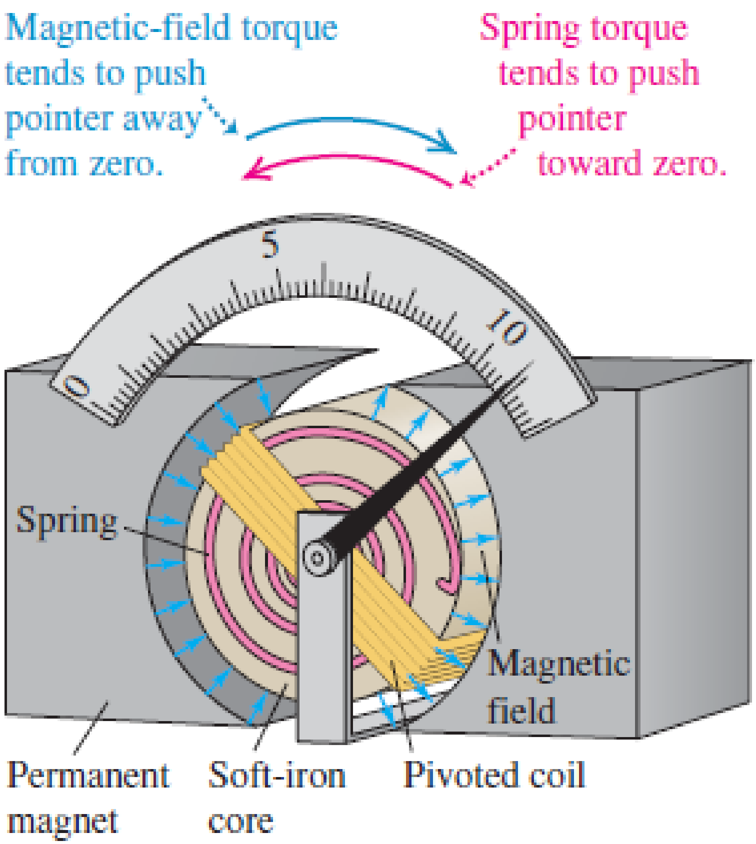


Measuring Instruments

The Galvanometer

Many common devices, measure potential difference (voltage),current, or resistance using a **d’Arsonval galvanometer**

- A pivoted coil of fine wire is placed in the magnetic field of a permanent magnet.
- When there is a current in the coil, the magnetic field exerts a torque on the coil that is proportional to the current.
- The device can be calibrated to measure current or voltage.



Measuring Instruments

The Galvanometer

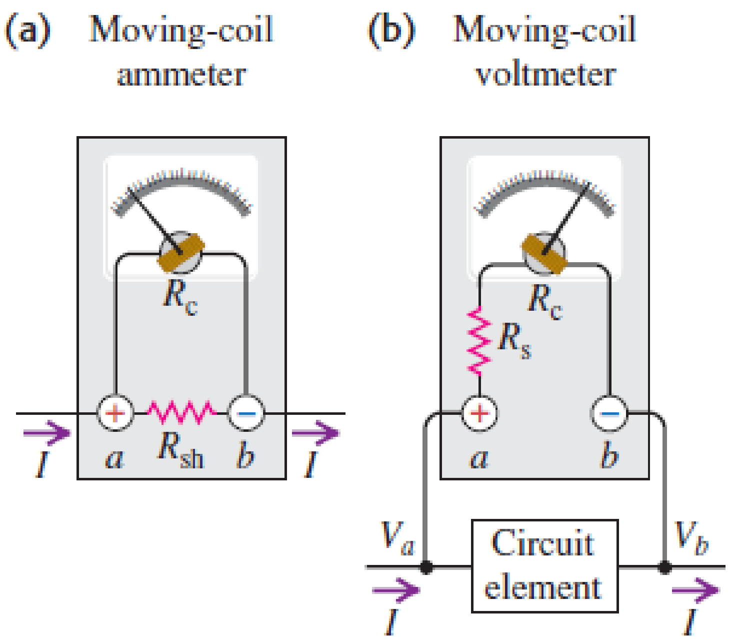
- The essential electrical characteristics of the meter are the current I_{fs} required for *full-scale* deflection (typically on the order of 10 μA to 10 mA) and the resistance R_C of the coil (typically on the order of 10 to 1000 Ω).
- The meter deflection is proportional to the *current* in the coil. If the coil obeys Ohm's law, the current is proportional to the *potential difference* between the terminals of the coil.

$$V_{fs} = I_{fs} R_C$$

Measuring Instruments

The Galvanometer

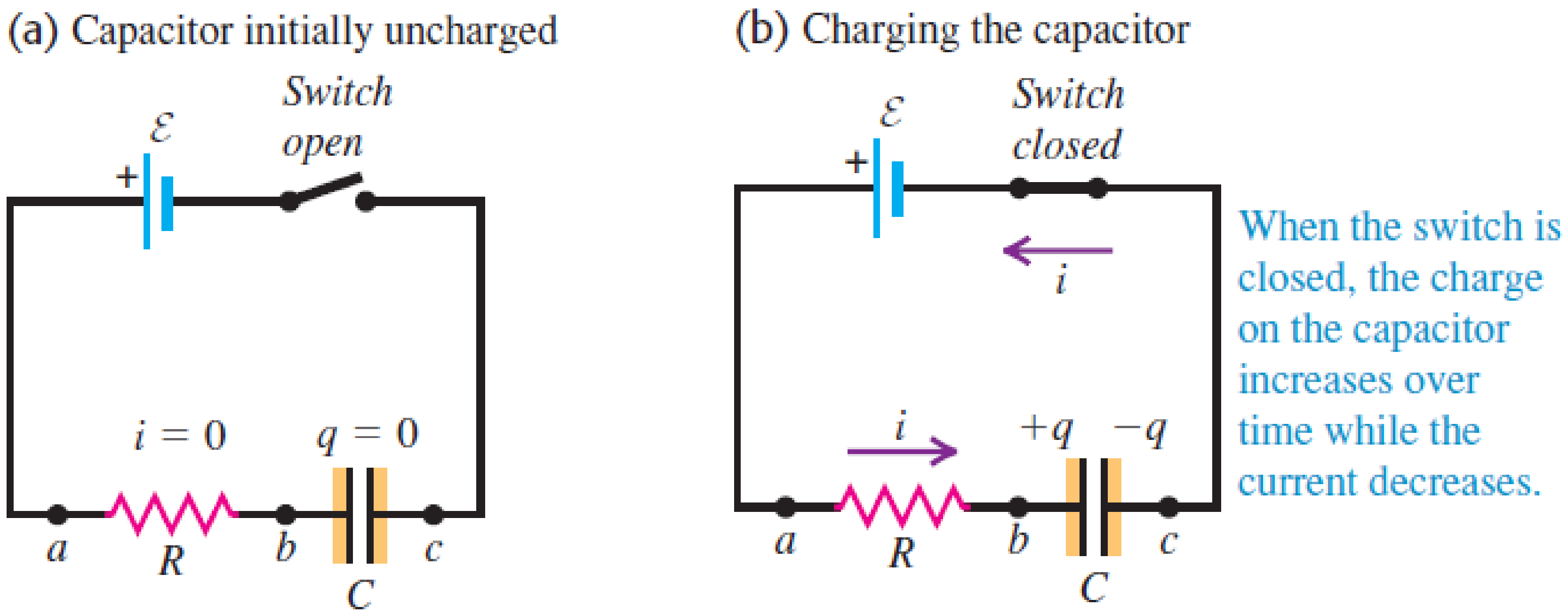
- We can adapt any ammeter to measure currents larger than its full-scale reading by connecting a resistor in parallel with it.
    - The parallel resistor is called a **shunt resistor** or simply a *shunt,* denoted as R_{sh}
- We can extend the range of a voltmeter by connecting a resistor R_S in *series* with the coil.
    - Then only a fraction of the total potential difference appears across the coil itself, and the remainder appears across R_S



R-C Circuits

Charging a capacitor

A ***battery*** is connected to a ***capacitor*** through a ***resistor***.



R-C Circuits

Charging a capacitor

- By Kirchhoff's loop rule:

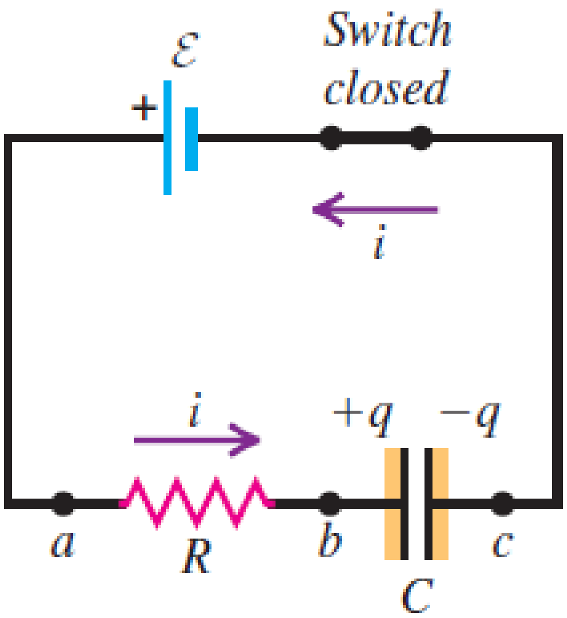
$$\mathcal{E} + v_{ab} + v_{ab} = 0$$

$$\mathcal{E} - Ri - \frac{q}{C} = 0$$

- The current charges the capacitor:

$$R\frac{dq}{dt} + \frac{1}{C}q = \mathcal{E}\,; \quad i = \frac{dq}{dt}$$

- The charge and current are functions of time. [$q = q(t); i = i(t)$]



R-C Circuits

Charging a capacitor

- Initial condition (at $t = 0$):

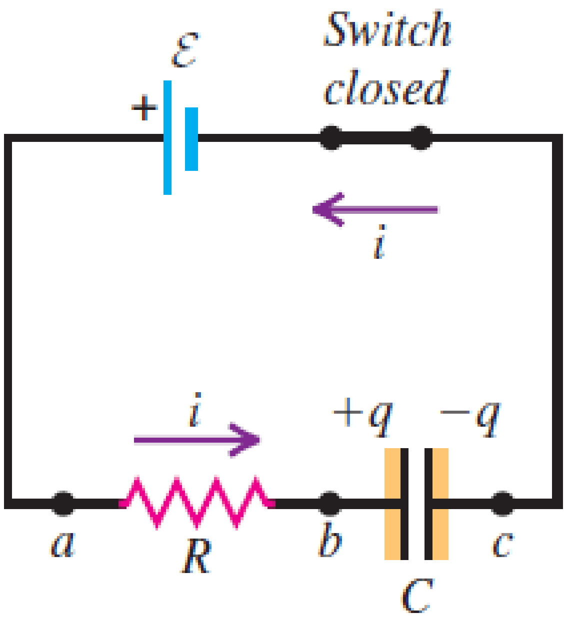
$$q = 0$$

$$i(0) = \frac{\mathcal{E}}{R} = I_0$$

- At steady state ($t \to \infty$):

$$i \to 0$$

$$q(\infty) = C\mathcal{E} = Q_f$$



R-C Circuits

Charging a capacitor

- Solving differential equation:

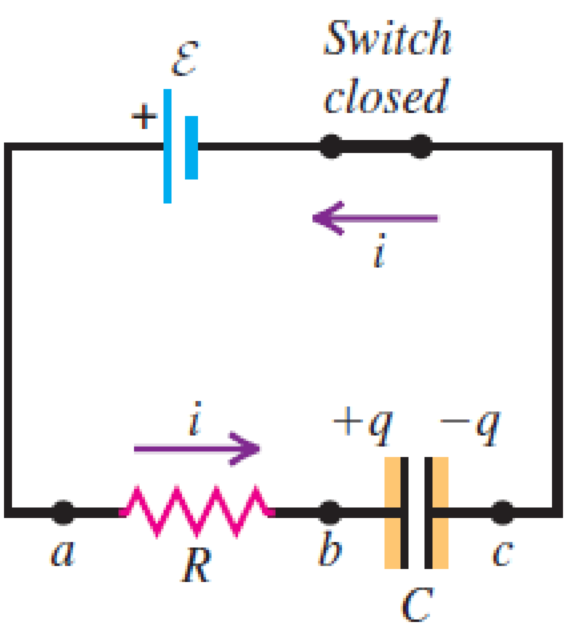
$$\frac{dq}{dt} + \frac{1}{RC} q = \frac{\mathcal{E}}{R}$$

$$q(t) = C\mathcal{E}(1 - e^{-t/RC})$$

$$= Q_f(1 - e^{-t/RC})$$

- The current:

$$i(t) = \frac{Q_f}{RC}\, e^{-t/RC} = I_o\, e^{-t/RC}$$



R-C Circuits

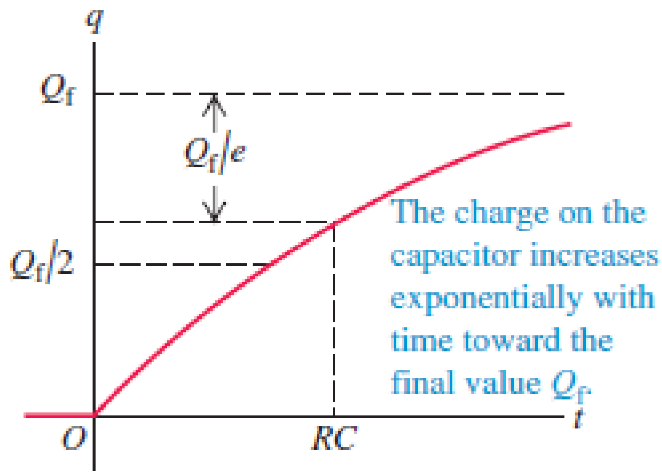
Charging a capacitor

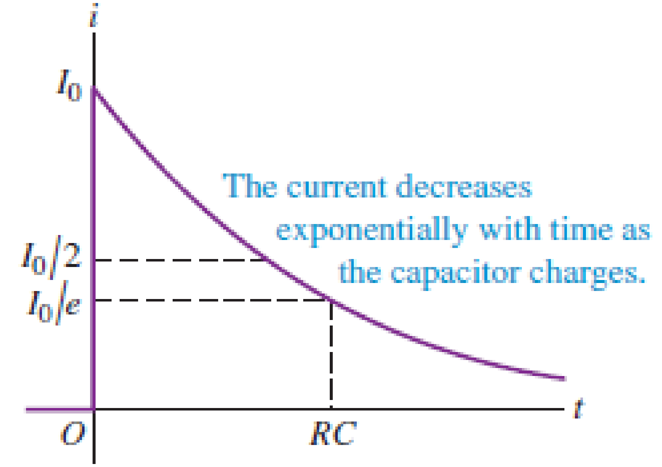
- The charge:

$$q(t) = Q_f(1 - e^{-t/RC})$$

- The current:

$$i(t) = I_o\, e^{-t/RC}$$





R-C Circuits

Charging a capacitor

- The ***time constant***:

$$t = \tau = RC$$

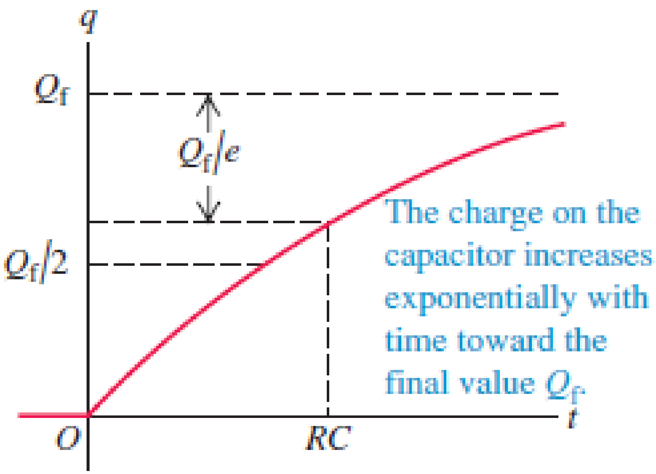
- The charge and current:

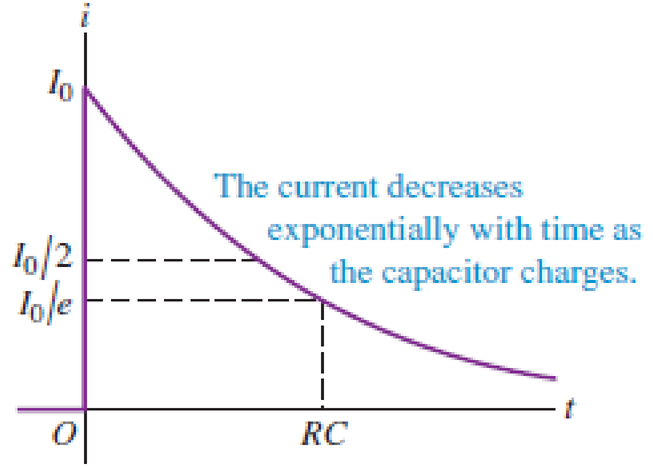
$$q(\tau) = Q_f(1 - e^{-1})$$

$$= 0.63\, Q_f$$

$$i(\tau) = I_o\, e^{-1}$$

$$= 0.37\, I_o$$





R-C Circuits

Charging a capacitor

- ***Energy transfer*** (power):

$$\mathcal{E} = Ri + \frac{q}{C}$$

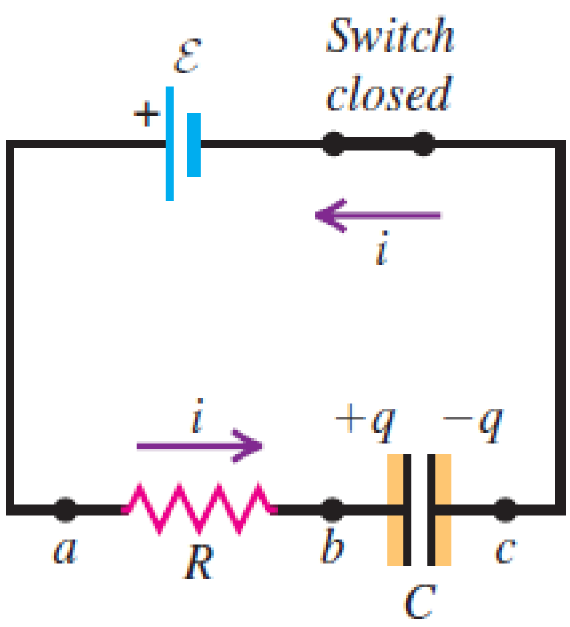
$$\mathcal{E}\, i = R\, i^2 + \frac{q\, i}{C} = P$$

- The total energy transferred:

$$U = \int (\mathcal{E}\, i) dt$$

$$= \mathcal{E} I_o \int_0^{\infty} e^{-t/\tau} dt$$

$$= \mathcal{E}\, Q_f$$



R-C Circuits

Charging a capacitor

- The total energy transferred:

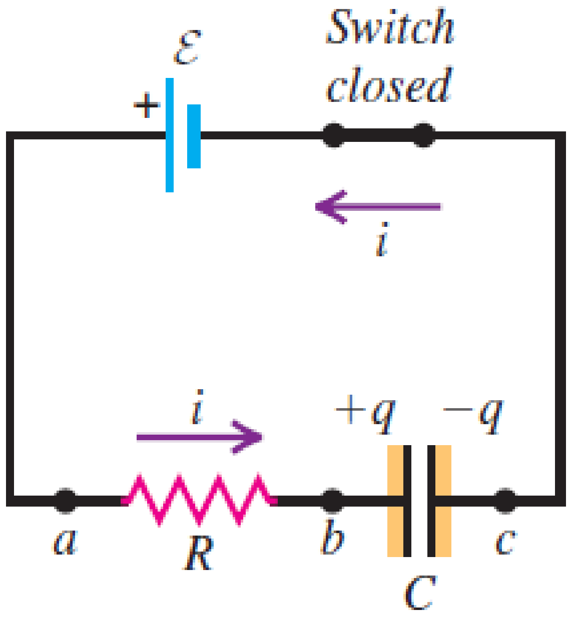
$$U = \mathcal{E}\, Q_f$$

- The ***energy stored*** in capacitor:

$$U_C = \frac{1}{2}\frac{{Q_f}^2}{C} = \frac{1}{2}\mathcal{E}\, Q_f$$

- The ***energy dissipated*** in resistor:

$$U_R = \int (Ri^2)dt = \frac{1}{2}\mathcal{E}\, Q_f$$



R-C Circuits

Example 3: Charging a capacitor

A 10 MΩ resistor is connected in series with a 1.0 μF capacitor and a battery with emf 12.0 V. Before the switch is closed at time *t* = 0 the capacitor is uncharged. (a) What is the time constant? (b) What fraction of the final charge Q_f is on the capacitor at *t* = 46 s? (c) What fraction of the initial current is still flowing at *t* = 46 s?

a) Time constant: $\tau = RC = (10 \times 10^6\ \Omega)(1.0 \times 10^{-6}\ \mathrm{F}) = 10\ s$

Total charge: $Q_f = C\mathcal{E} = 12.0\ \mu\mathrm{C}$

Maximum current: $I_0 = \mathcal{E}/R = 1.2\ \mu\mathrm{A}$

R-C Circuits

Example 3: Charging a capacitor

b) Charge on the capacitor at *t* = 46 s

$$\frac{q}{Q_f} = \left(1 - e^{-t/RC}\right)$$

$$= 1 - e^{-(46)/(10)}$$

$$= 0.99$$

c) Current at *t* = 46 s

$$\frac{i}{I_0} = \left(e^{-t/RC}\right)$$

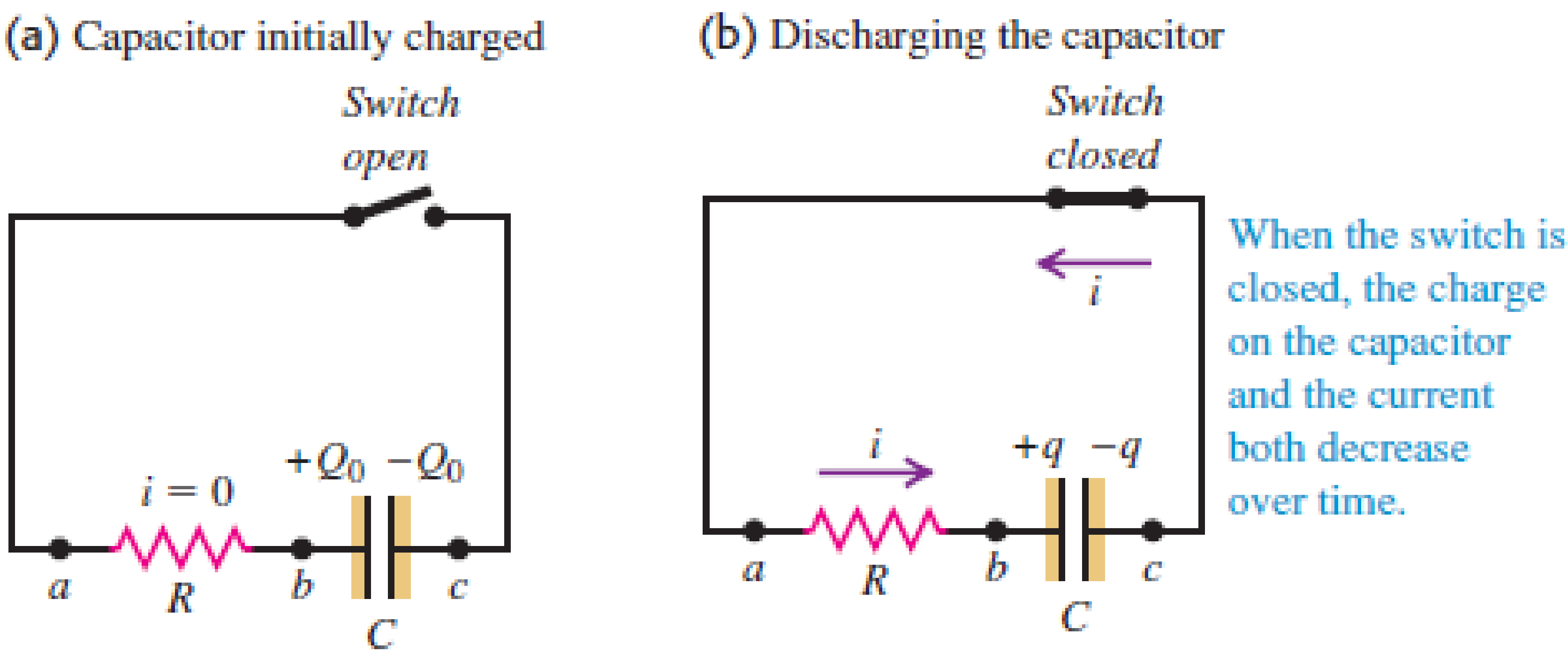
$$= e^{-(46)/(10)}$$

$$= 0.010$$

R-C Circuits

Discharging a capacitor

A ***capacitor*** is discharged through a ***resistor***.



R-C Circuits

Discharging a capacitor

- By Kirchhoff's loop rule:

$$v_{ab} + v_{bc} = 0$$

$$Ri + \frac{q}{C} = 0$$

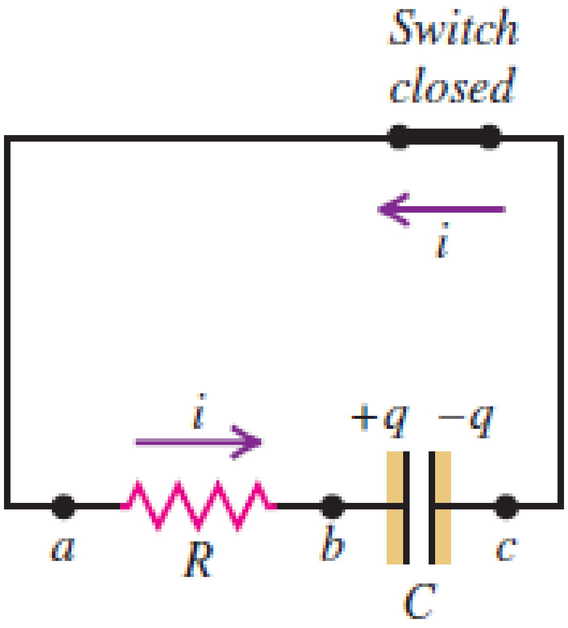
- The discharge current:

$$\frac{dq}{dt} = -\frac{1}{RC}q$$

- Initial conditions (at *t*=0):

$$q(0) = Q_0$$

$$i(0) = -Q_0/RC = I_0$$



R-C Circuits

Discharging a capacitor

- Differential equation:

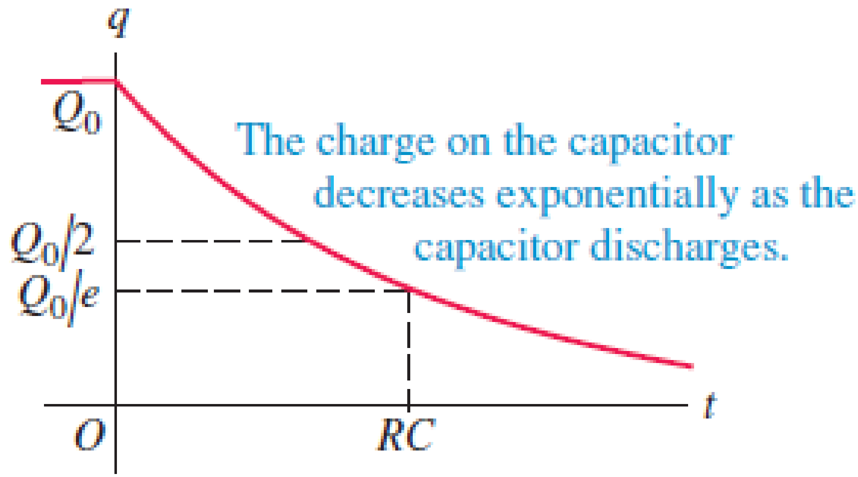
$$\frac{dq}{dt} = -\frac{1}{RC}q$$

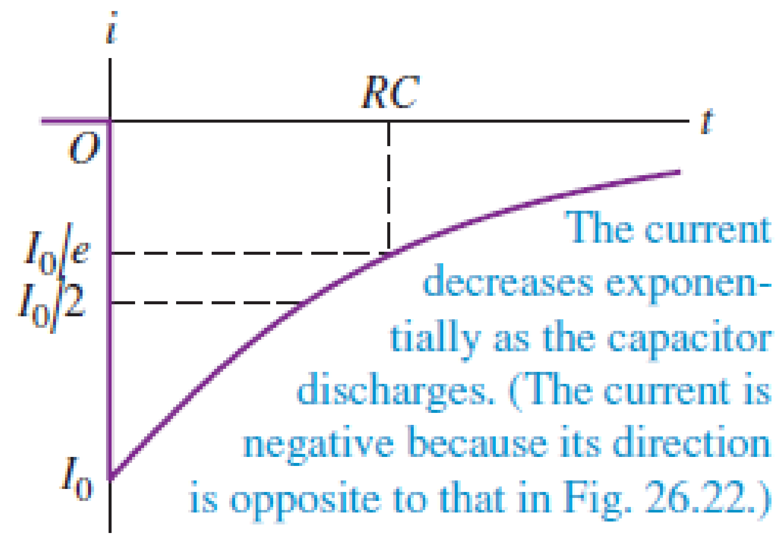
- The charge:

$$q(t) = Q_0\, e^{-t/RC}$$

- The current:

$$i(t) = -I_0\, e^{-t/RC}$$





R-C Circuits

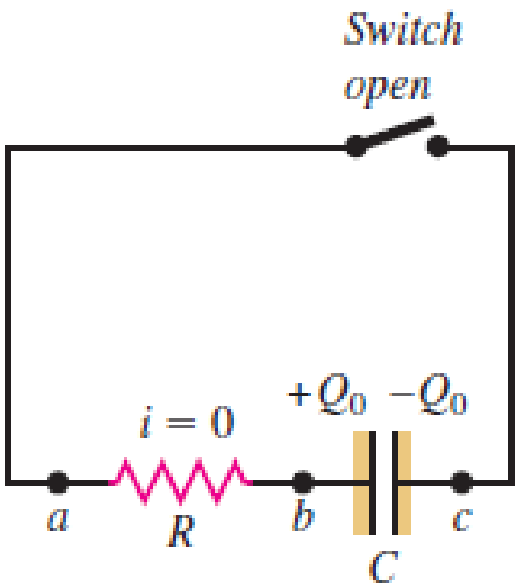
Example 4: Discharging a capacitor

The resistor and capacitor are reconnected as shown. (*R*=10 MΩ; *C*=1.0 μF). The capacitor has an initial charge of Q_0= 5.0 μC and is discharged by closing the switch at *t*=0. (a) At what time will the charge be equal to 0.50 μC. (b) What is the current at this time?

$$\tau = RC = 10\ \mathrm{s}$$

$$Q_0 = 5.0\ \mu\mathrm{C}$$

$$I_0 = Q_0/RC = 0.5\ \mu\mathrm{A}$$



R-C Circuits

Example 4: Discharging a capacitor

a) When the charge on the capacitor = 0.50 μC

$$t = -RC \ln\left(\frac{q}{Q_f}\right) = -(10 \text{ s}) \ln\left(\frac{0.50}{5.0}\right) = 23 \text{ s}$$

b) Current

$$i = I_0\left(e^{-t/RC}\right) = (0.5\ \mu\text{A})e^{-2.3} = 0.05\ \mu\text{A}$$

END