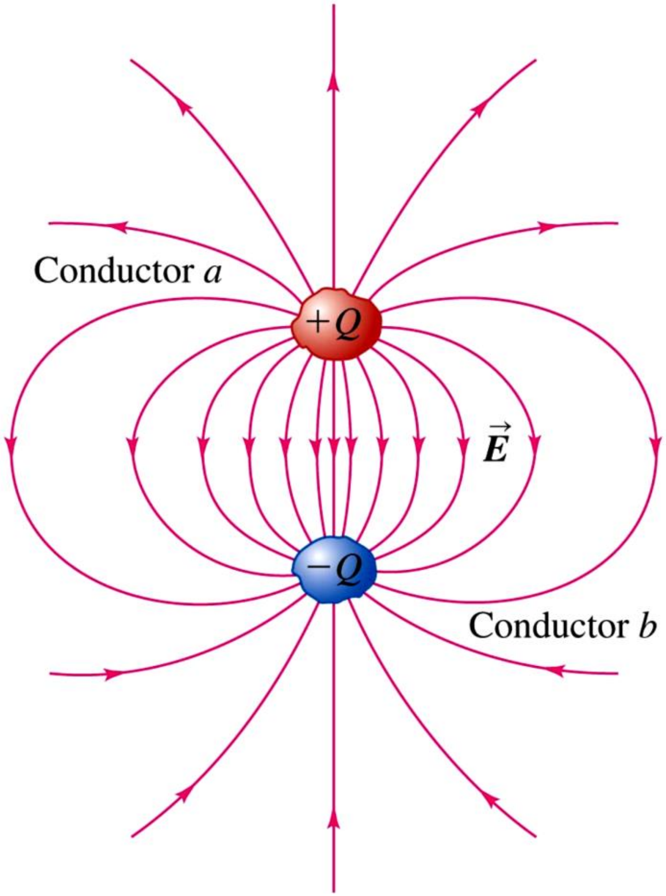
Capacitance and Dielectrics

Capacitors and Capacitance

Capacitors

- A capacitor is a device which stores electric charge.
- Capacitors vary in shape and size, but the basic configuration is two conductors carrying equal but opposite charges



Capacitors and Capacitance

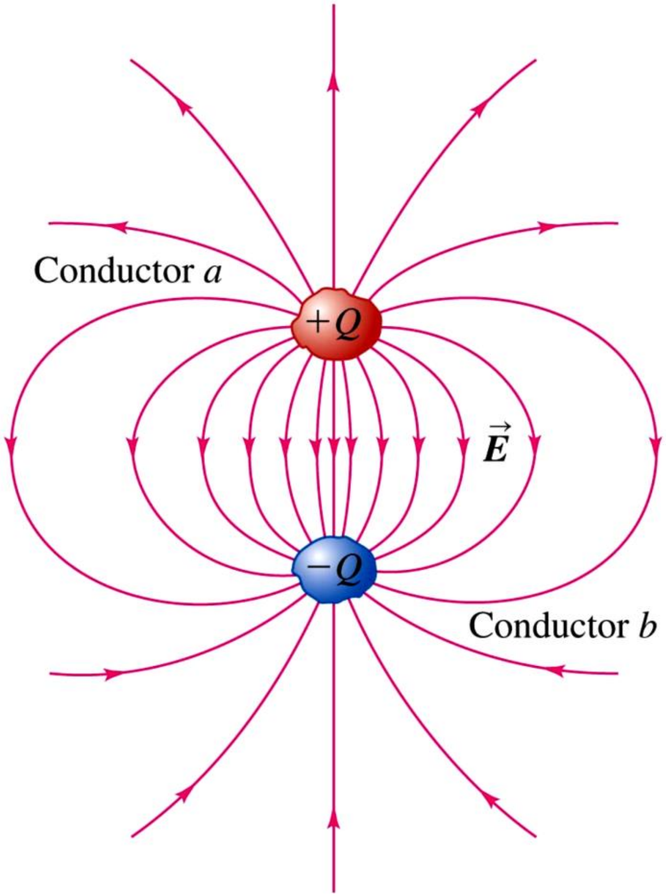
Capacitance

- A fixed potential difference V_{ab} exists between the conductors.
- The electric field $\vec{E}$ is proportional to the charge Q
- The capacitance C:

$$C \equiv \frac{Q}{V_{ab}}$$

- Units:

$$\frac{\text{Coulomb}}{\text{Volt}} \equiv \text{Farad}$$



Capacitors and Capacitance

Parallel-plate capacitor

- The electric field E :

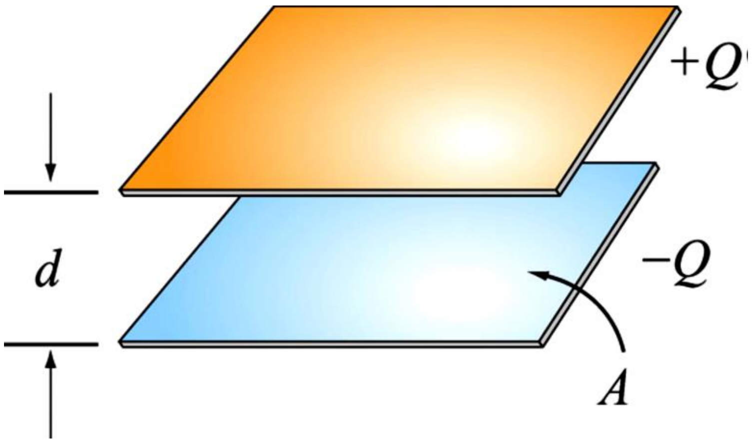
$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

- The potential difference V_{ab}

$$V_{ab} = E \cdot d = \frac{Q \cdot d}{\epsilon_0 A}$$

- The capacitance C:

$$C = \frac{\epsilon_0 A}{d}$$



$$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$$

Capacitors and Capacitance

Example: Size of a 1 F capacitor

A parallel plate capacitor has a capacitance of 1.0 F. If the plates are 1.0 mm apart, what is the area of the plates?

- Solution:

$$A = \frac{C\ d}{\epsilon_0} = \frac{(1.0)(1.0 \times 10^{-3})}{(8.85 \times 10^{-12})}$$

$$= 1.1 \times 10^{8}\ \mathrm{m}^2$$

$$C = \frac{\epsilon_0 A}{d}$$

Capacitors and Capacitance

Example: Parallel-plate capacitor

The plates of a parallel plate capacitor are 5.00 mm apart and 2.00 m^2 in area. A voltage of 10,000 V is applied across the capacitor. Compute the capacitance and the charge on each plate.

- Capacitance:

$$C = \frac{\epsilon_0 A}{d} = \frac{(8.85 \times 10^{-12})(2.00)}{(5.00 \times 10^{-3})} = 3.54 \times 10^{-9}\ \mathrm{F}$$

- Charge:

$$Q = C\,V_{ab} = (3.54 \times 10^{-9})(1.00 \times 10^{4})$$

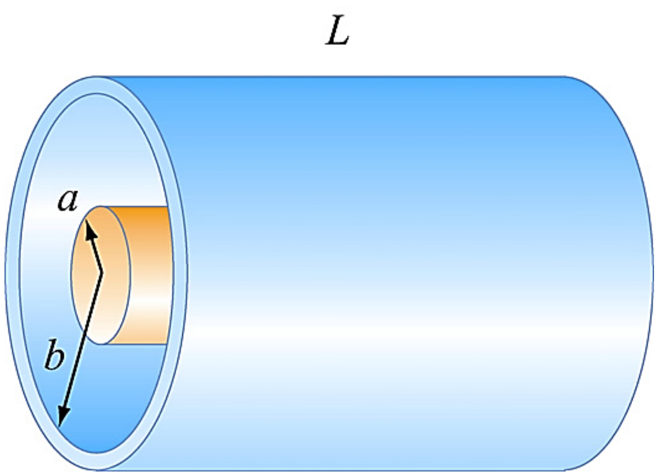
$$= 3.54 \times 10^{-5}\ \mathrm{C}$$

Capacitors and Capacitance

Cylindrical capacitor

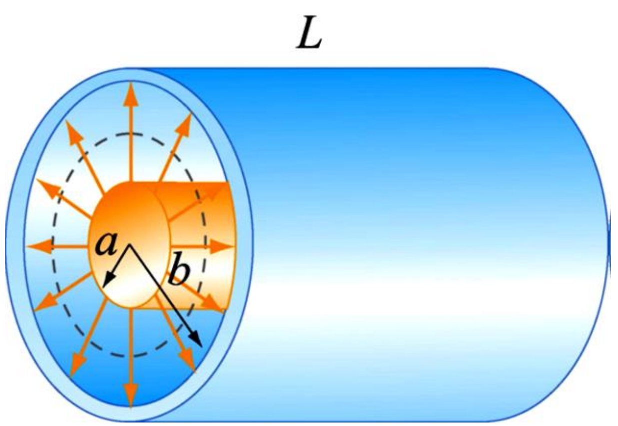
- The electric field E :

$$E = \frac{\lambda}{2\pi\epsilon_0 \, r} \quad \text{where} \;\; \lambda = \left(\frac{Q}{L}\right)$$



- The potential difference V_{ab}:

$$V_{ab} = \int_a^b E \cdot dr = \frac{Q}{2\pi\epsilon_0 L} \ln\left(\frac{b}{a}\right)$$



- The capacitance C:

$$C = \frac{Q}{V_{ab}} = \frac{2\pi\epsilon_0 L}{\ln(b/a)}$$

Capacitors and Capacitance

Spherical capacitor

- The electric field E :

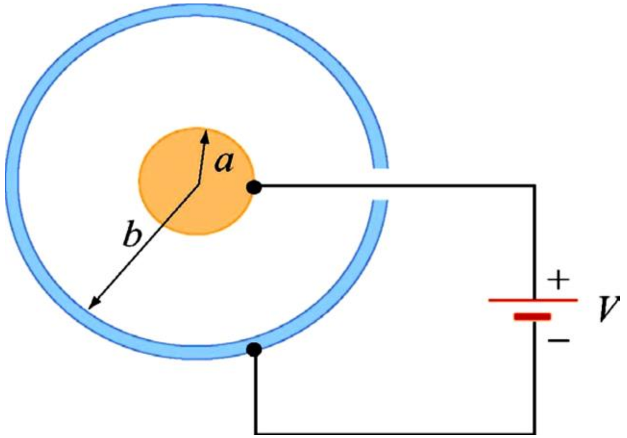
$$E = \frac{Q}{4\pi\epsilon_0 \, r^2}$$

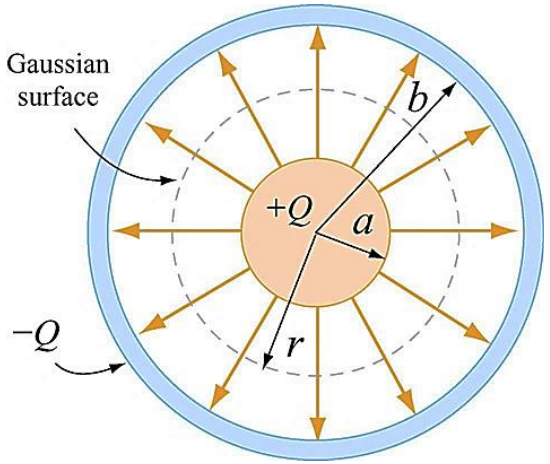
- The potential difference V_{ab}:

$$V_{ab} = \int_a^b E \cdot dr = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

- The capacitance C:

$$C = \frac{Q}{V_{ab}} = 4\pi\epsilon_0\left(\frac{ab}{b-a}\right)$$

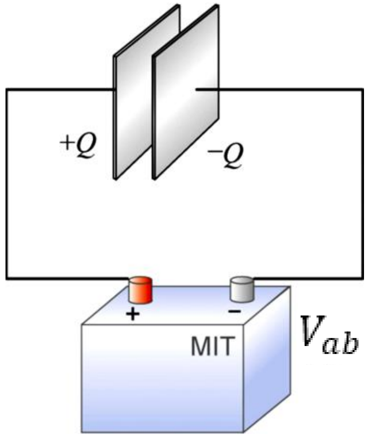




Capacitors and Capacitance

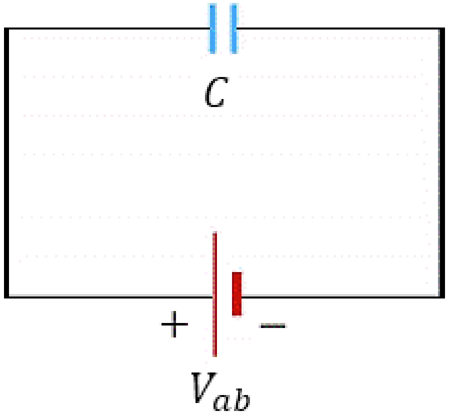
Charging a capacitor

- A capacitor can be charged by connecting the plates to the terminals of a battery, which are maintained at a potential difference called the *terminal voltage*.



- Schematic diagram:

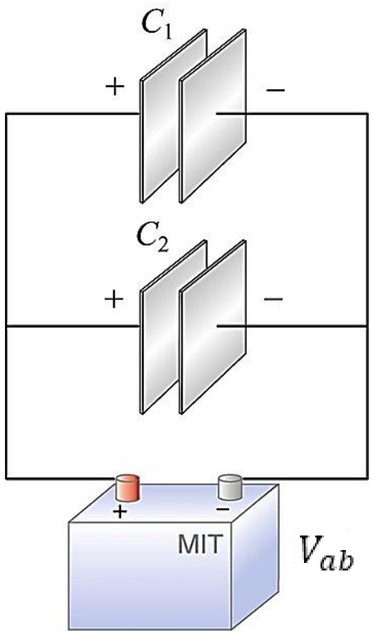
$$Q = CV_{ab}$$

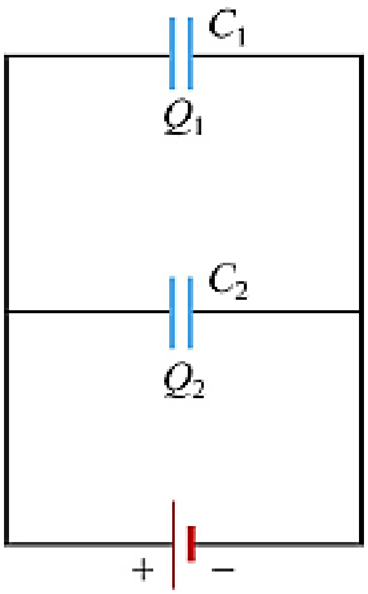


Capacitors in Series and Parallel

Capacitors in parallel

- Two capacitors can be charged by connecting the capacitors to the terminals of a battery at the same time.





Schematic diagram.

Capacitors in Series and Parallel

Capacitors in parallel

- The charge on each capacitor:

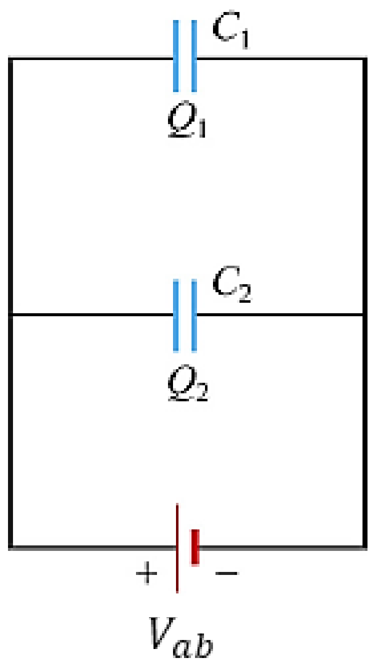
$$Q_1 = C_1 V_{ab} \text{ and } Q_2 = C_2 V_{ab}$$

- The total charge in the system:

$$Q = Q_1 + Q_2 = (C_1 + C_2)V_{ab}$$

- The effective (equivalent) capacitance:

$$\frac{Q}{V_{ab}} = (C_1 + C_2) \equiv C_{eq}$$

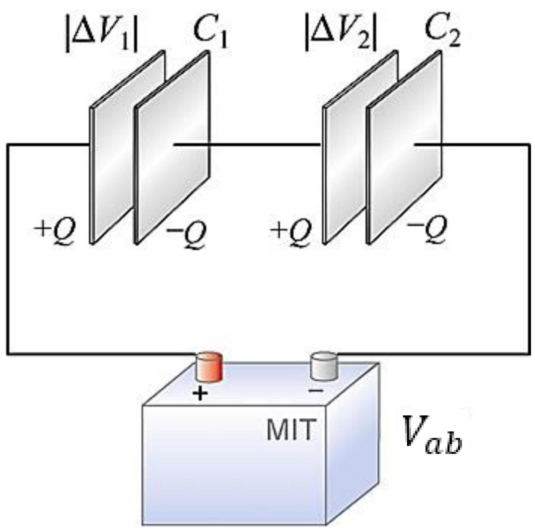


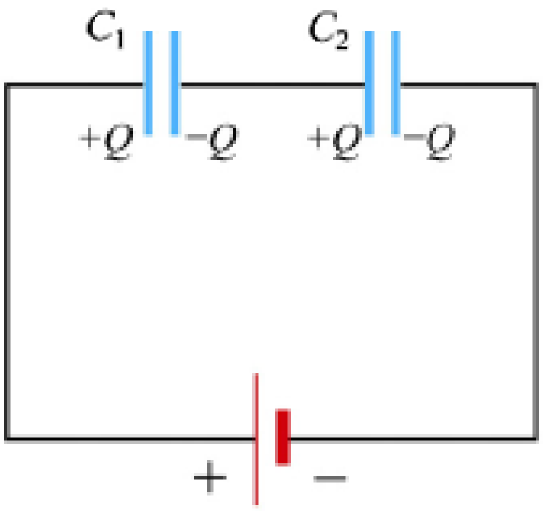
The voltages across capacitors are equal.

Capacitors in Series and Parallel

Capacitors in series

- Two capacitors can be charged by connecting the capacitors in *series* to a battery at the same time.





Schematic diagram.

Capacitors in Series and Parallel

Capacitors in series

- The potential difference across each capacitor:

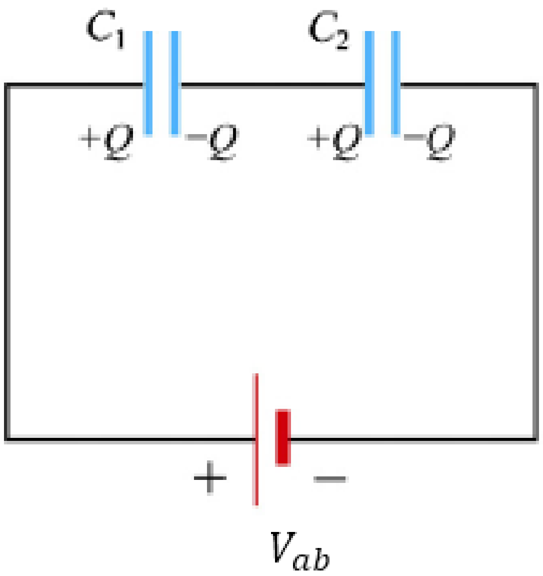
$$V_1 = \frac{Q}{C_1} \quad \text{and} \quad V_2 = \frac{Q}{C_2}$$

- The total potential:

$$V_{ab} = V_1 + V_2 = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

- The effective (equivalent) capacitance:

$$\frac{Q}{V_{ab}} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} \equiv C_{eq}$$

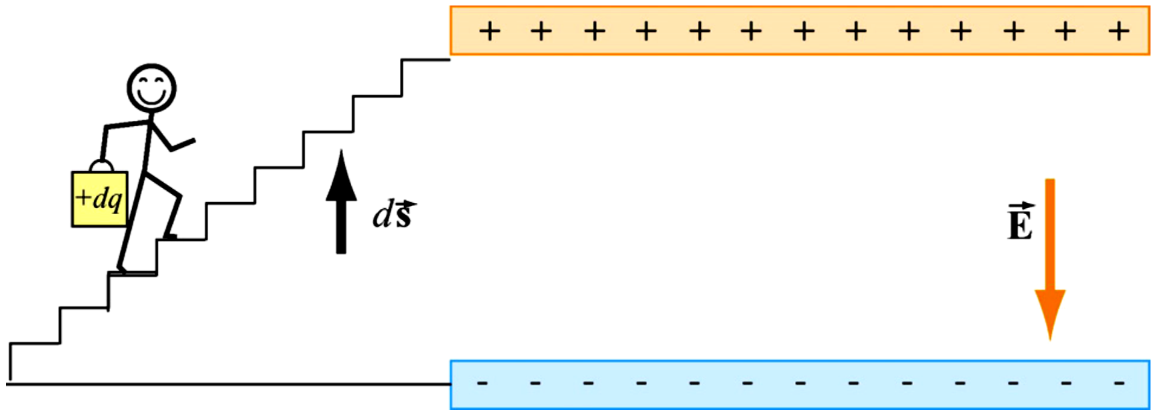


The charges on the capacitors are equal.

Energy Storage in Capacitors

Charging the capacitor

- During the charging process, the battery does work to remove charges from one plate and deposit them onto the other.



Energy Storage in Capacitors

Charging the capacitor

- When the amount of charge on the top plate at some instant is $+q$, and the potential difference between the two plates is $v_{ab} = q/C$.
- To dump an additional amount $+dq$ on the top plate, the amount of work done to overcome electrical repulsion is $dW = v_{ab}dq$
- If at the end of the charging process, the charge on the top plate is $+Q$, then the total amount of work done is

$$W = \int_0^Q v_{ab}dq = \int_0^Q \left(\frac{q}{C}\right) dq$$

Energy Storage in Capacitors

Energy stored in the capacitor

- The total amount of work done is

$$W = \int_0^Q \left(\frac{q}{C}\right) dq = \frac{Q^2}{2C}$$

- This is electric potential energy of the system

$$U_e = \frac{Q^2}{2C} = \frac{1}{2} Q V_{ab} = \frac{1}{2} C {V_{ab}}^2$$

Energy Storage in Capacitors

Energy stored in the capacitor

- In a parallel-plate capacitor

$$C = \frac{\epsilon_0 A}{d} \text{ and } V_{ab} = E \cdot d$$

$$U_e = \frac{1}{2}\left(\frac{\epsilon_0 A}{d}\right)(E \cdot d)^2 = \frac{1}{2}\epsilon_0\, E^2 (Ad)$$

- The electric potential energy density

$$u_e = \frac{U_e}{(Ad)} = \frac{1}{2}\epsilon_0\, E^2$$

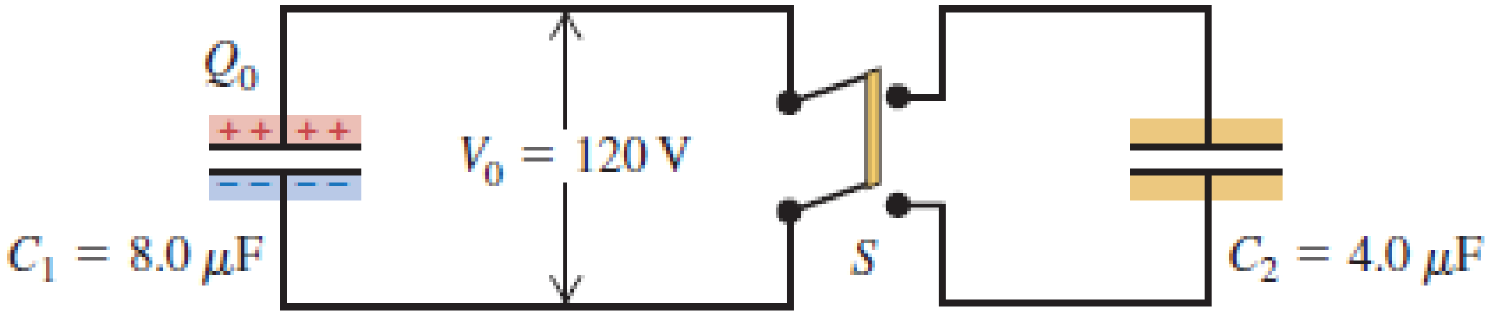
  - One can think of the energy stored in the capacitor as being stored in the electric field itself.

Energy Storage in Capacitors

Example 24.7: Charge and energy between capacitors

We connect a capacitor C_1=8.0 μF to a power supply, charge it to a potential difference V_0=120 V, and disconnect the power supply. Switch is open. (a) What is the charge Q_0 on C_1? (b) What is the energy stored in C_1?

(c) Capacitor C_2=4.0 μF is initially uncharged. We close switch *S*. After charge no longer flows, what is the potential difference across each capacitor, and what is the charge on each capacitor? (d) What is the final energy of the system?



Energy Storage in Capacitors

Example 24.7: Charge and energy between capacitors

a) Initial charge on C_1

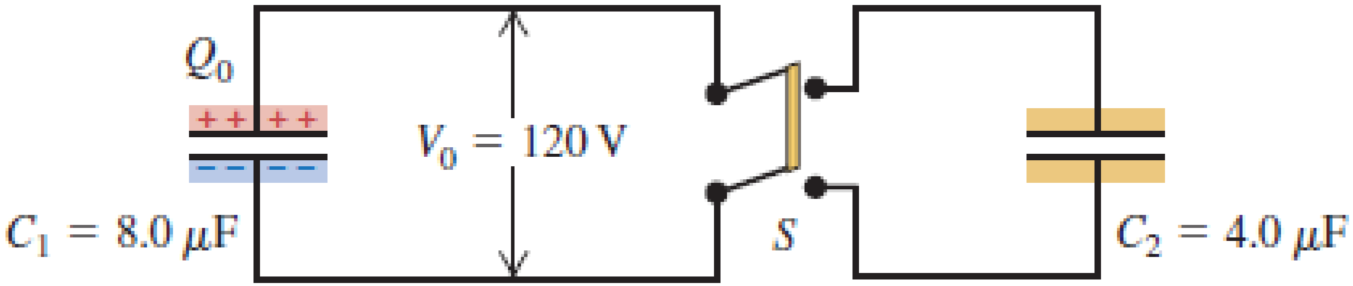
$$Q_0 = C_1 V_0 = (8 \times 10^{-6}\ \mathrm{F})(120\ \mathrm{V})$$

$$= 960 \times 10^{-6}\ \mathrm{C}$$

b) Initial energy stored

$$U_0 = \tfrac{1}{2} Q_0 V_0 = \tfrac{1}{2}(960 \times 10^{-6}\ \mathrm{C})(120\ \mathrm{V})$$

$$= 0.058\ \mathrm{J}$$



Energy Storage in Capacitors

Example 24.7: Charge and energy between capacitors

c) When we close the switch, the charge Q_0 is distributed over the plates of both capacitors, which are now connected in parallel

$$Q_0 = Q_1 + Q_2$$

$$Q_1 = C_1 V_f \qquad\qquad Q_2 = C_2 V_f$$

$$V_f = \frac{Q_0}{C_1 + C_2} = \frac{960 \times 10^{-6}\ \mathrm{C}}{12.0 \times 10^{-6}\ \mathrm{F}} = 80\ \mathrm{V}$$

d) Final energy stored in the system

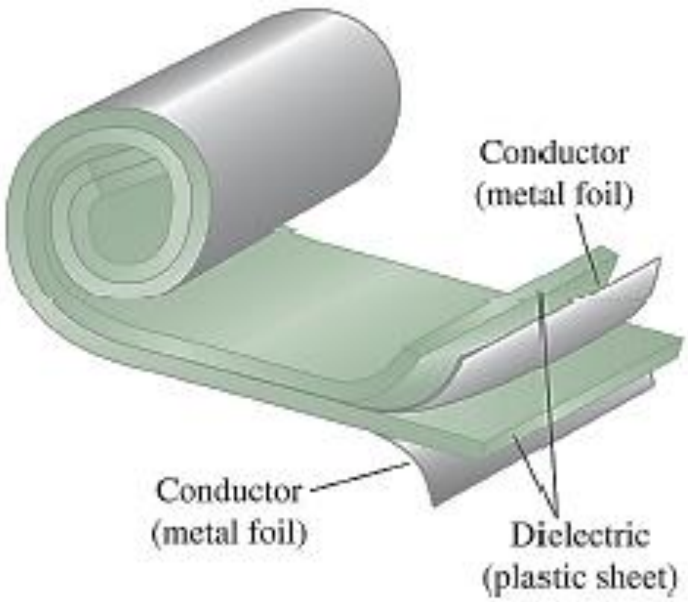
$$U_f = \tfrac{1}{2} Q_1 V_f + \tfrac{1}{2} Q_1 V_f = \tfrac{1}{2} Q_0 V_f$$

$$= \tfrac{1}{2}(960 \times 10^{-6}\ \mathrm{C})(80\ \mathrm{V}) = 0.038\ \mathrm{J}$$

Capacitors and dielectrics

In many capacitors there is an *insulating material* such as paper or plastic between the plates.

- Such material, called a ***dielectric***, can be used to maintain a physical separation of the plates.
- Since dielectrics break down less readily than air, charge leakage can be minimized, especially when high voltage is applied.

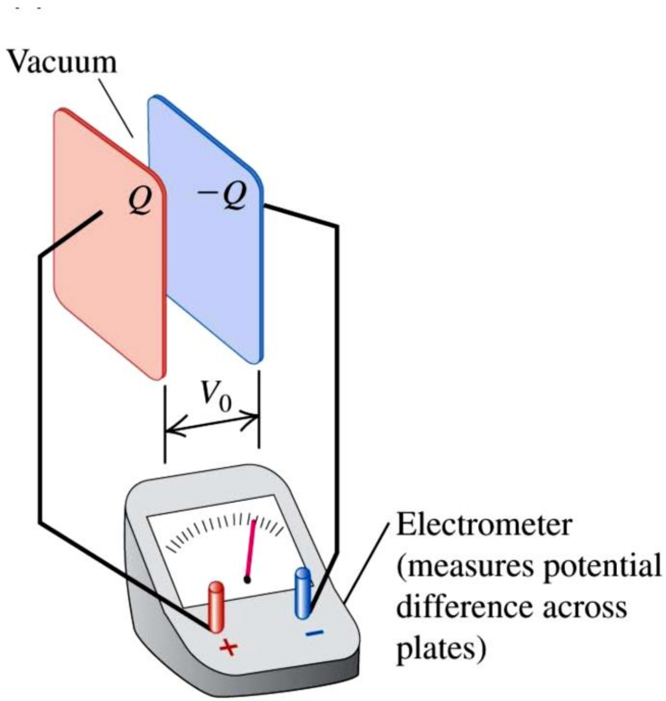


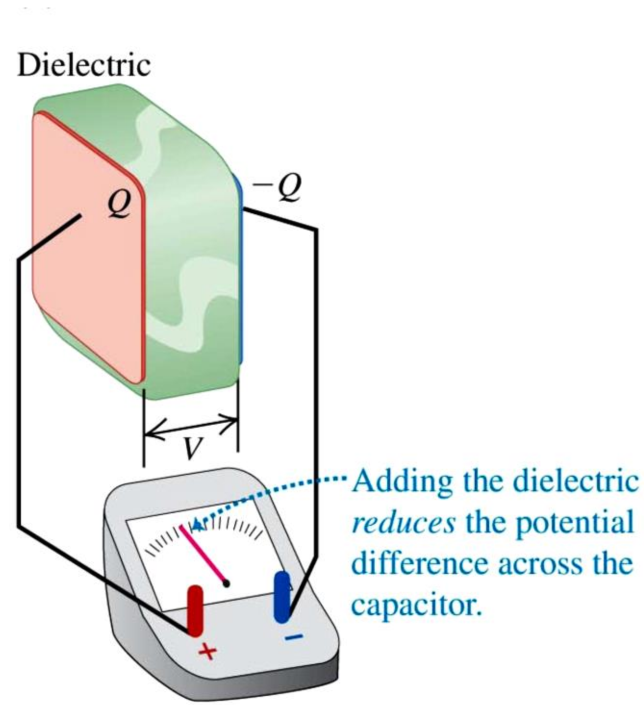
Dielectrics

Capacitors and dielectrics

The capacitance *C* increases when the space between the conductors is filled with dielectrics.

- When a dielectric material is inserted to completely fill the space between the charged plates, the voltage decreases.





Dielectrics

Capacitors and dielectrics

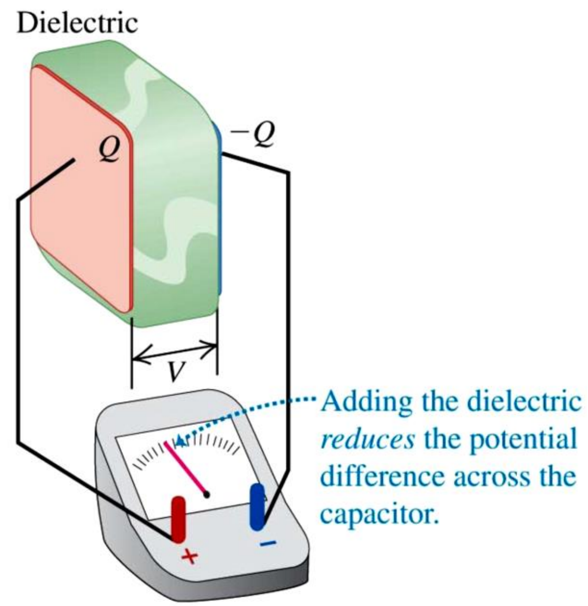
The capacitance C increases when the space between the conductors is filled with dielectrics.

- Before: $C_0 = Q/V_0$
- After: $C = Q/V$ where $V < V_0$

$$\frac{C}{C_0} = \frac{V_0}{V} > 1$$

- Define the dielectric constant

$$\frac{V_0}{V} = K_e; \quad C = K_e C_0$$



Dielectrics

Capacitors and dielectrics

Table 24.1 Values of Dielectric Constant K at 20°C

Material	K	Material	K
Vacuum	1	Polyvinyl chloride	3.18
Air (1 atm)	1.00059	Plexiglas	3.40
Air (100 atm)	1.0548	Glass	5–10
Teflon	2.1	Neoprene	6.70
Polyethylene	2.25	Germanium	16
Benzene	2.28	Glycerin	42.5
Mica	3–6	Water	80.4
Mylar	3.1	Strontium titanate	310

Dielectrics

Dielectric breakdown

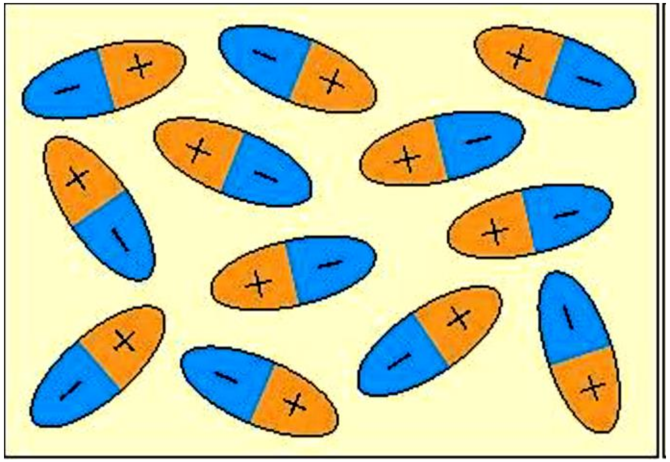
- Partial ionization of an insulating material subjected to a large electric field.

Material	κ_e	Dielectric strength $\left(10^6\,\text{V/m}\right)$
Air	1.00059	3
Paper	3.7	16
Glass	4–6	9
Water	80	–

Molecular Model of Induced Charge

Polar molecules

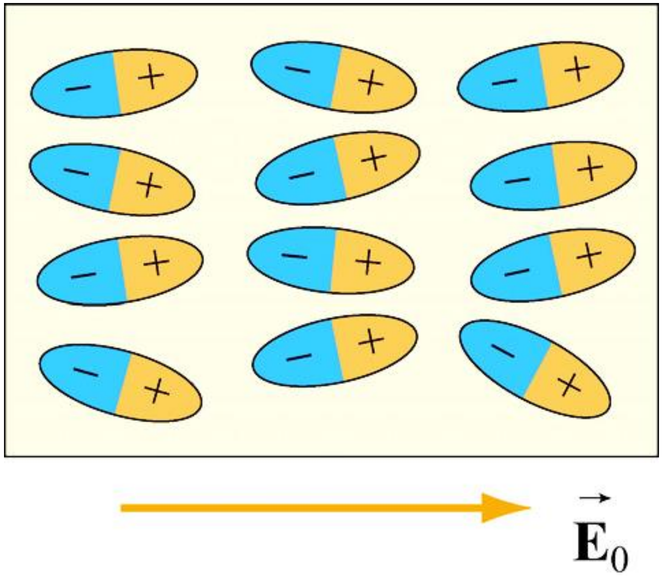
- Polar dielectrics have molecules that have permanent *electric dipole moments*.
- The orientation of polar molecules is random in the absence of an external field.



Molecular Model of Induced Charge

Polar molecules

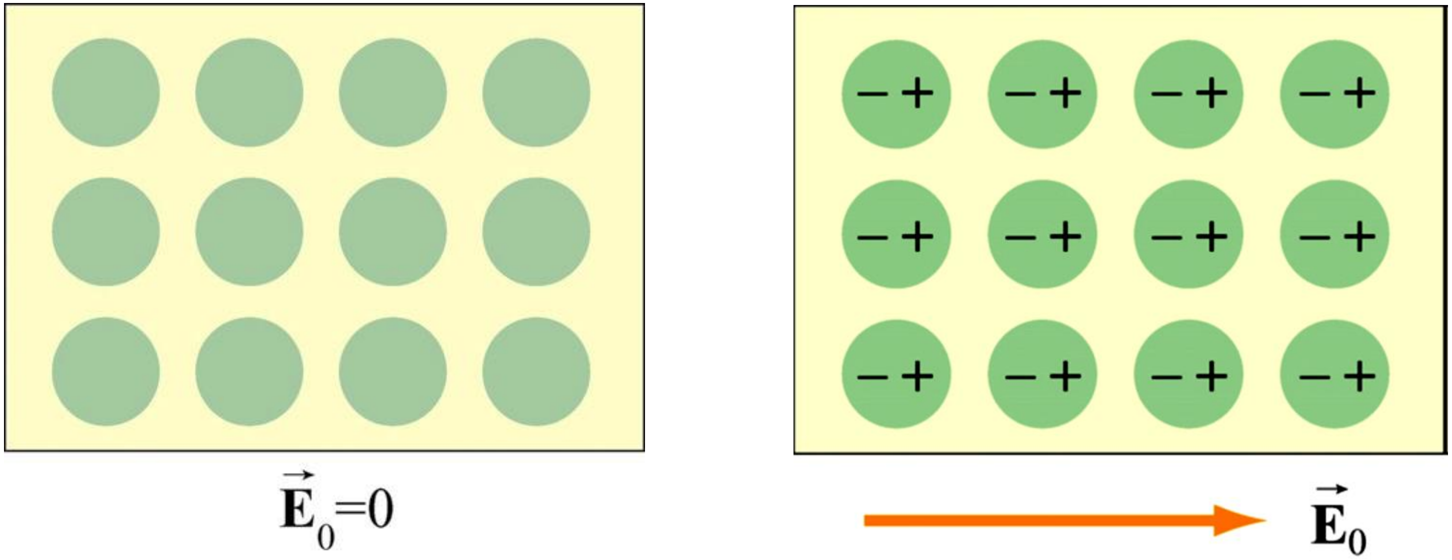
- When an external electric field $\vec{E}_0$ is present, a torque causes the molecules to align with $\vec{E}_0$.
- The aligned molecules then generate an *electric field* that is opposite to the applied field but smaller in magnitude.



Molecular Model of Induced Charge

Non-polar molecules

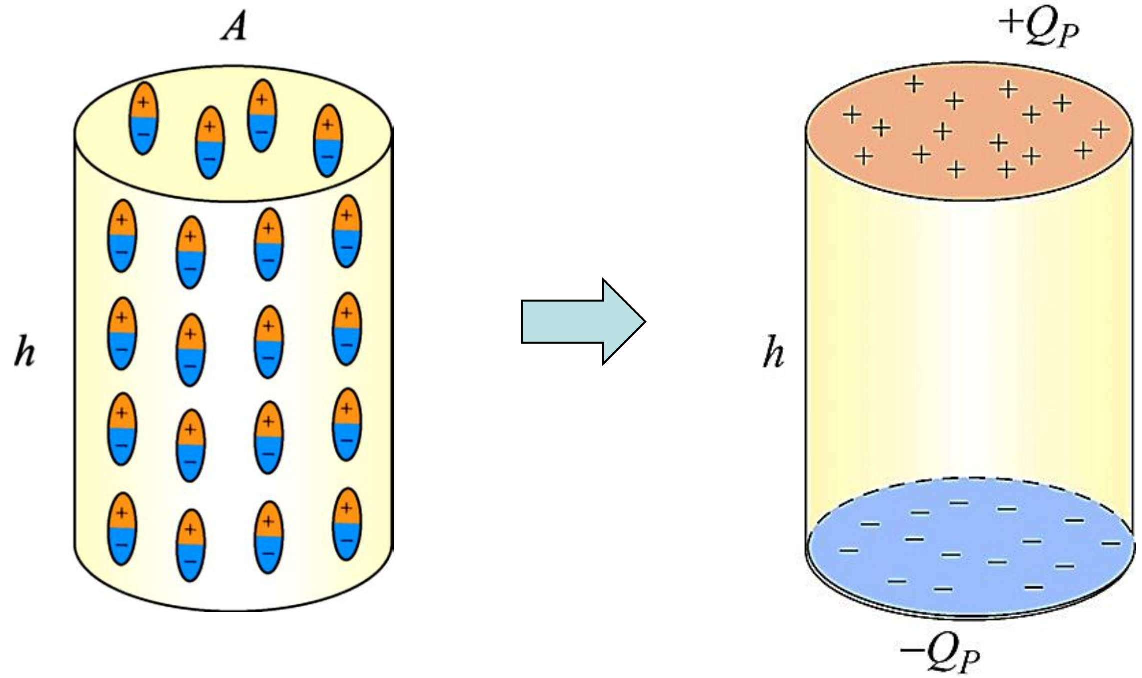
- Dielectrics with molecules that do not possess permanent electric dipole moment.
- Electric dipole moments can be induced by placing the materials in an externally applied electric field.



Molecular Model of Induced Charge

Polarization charges

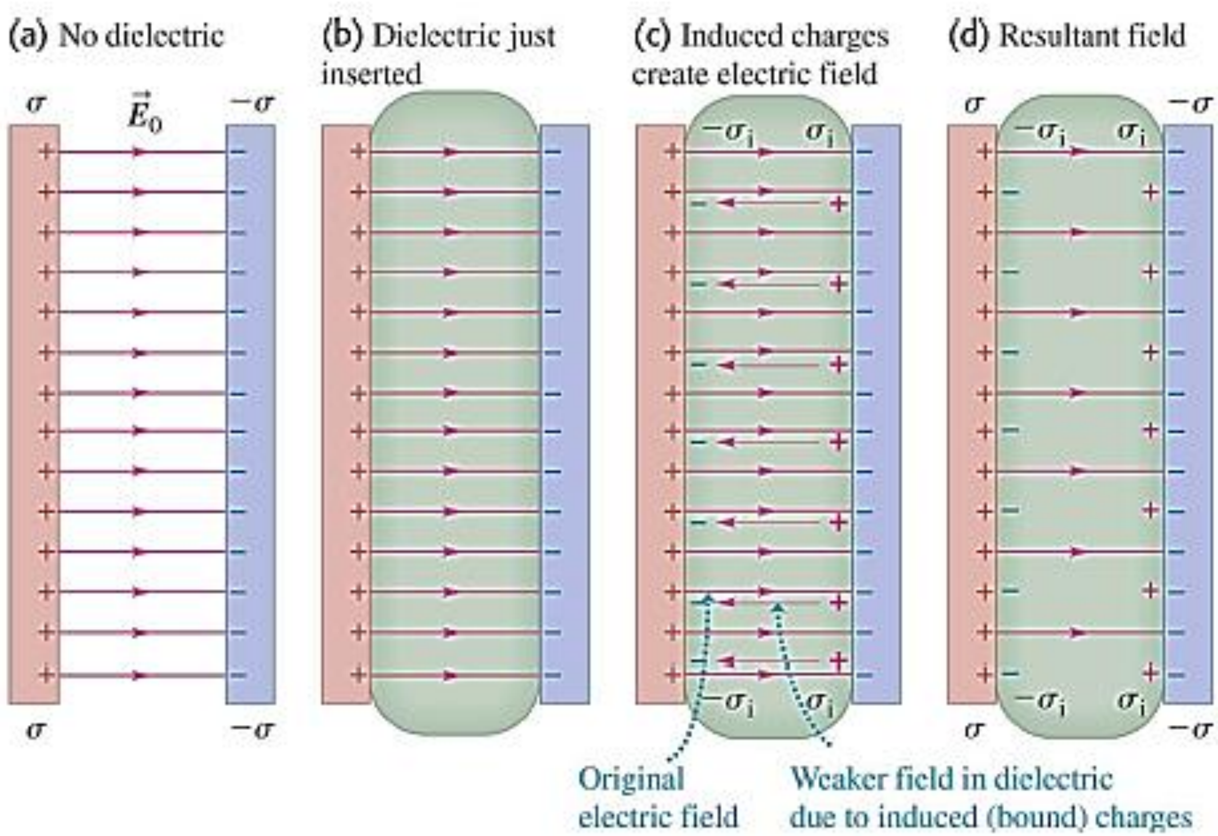
- Polarization of a dielectric in electric field gives rise to induced charges on the surfaces Q_p, creating surface bound charge density σ_i



Molecular Model of Induced Charge

Polarization charges

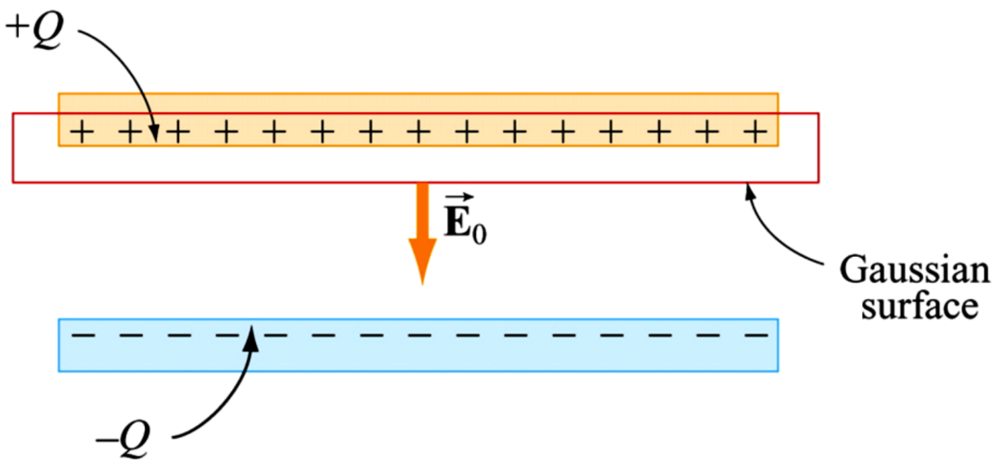
- Introduction of dielectric material reduces the electric field within the capacitor



Gauss' Law in Dielectrics

Gauss' law

- Parallel plate capacitor *without* dielectric



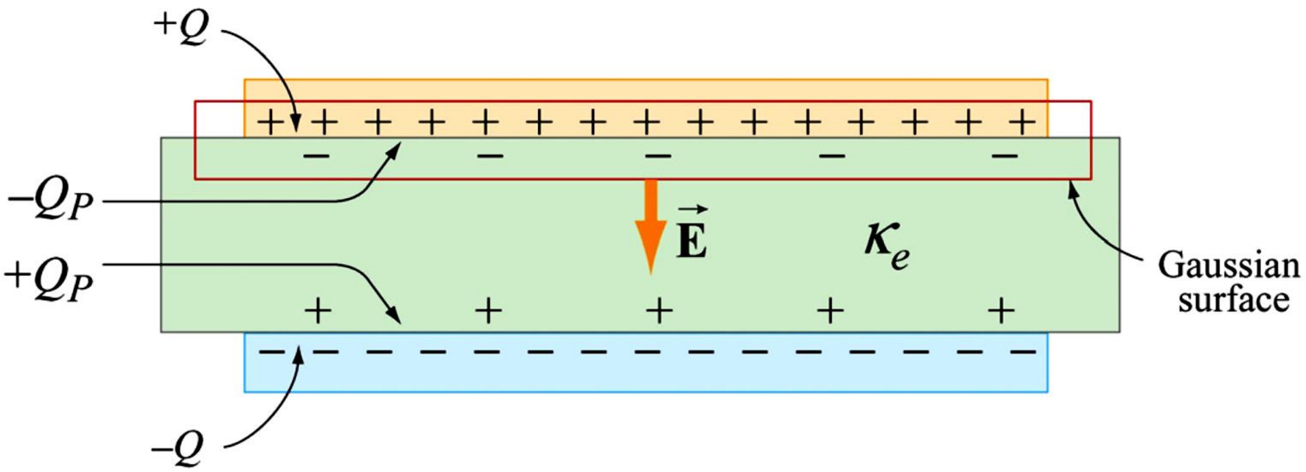
- Applying Gauss' law

$$\oint\!\!\!\oint_S \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}} = E_0 A = \frac{Q}{\epsilon_0} \quad \Longrightarrow \quad E_0 = \frac{\sigma}{\epsilon_0}$$

Gauss' Law in Dielectrics

Gauss' law

- Parallel plate capacitor *with* dielectric



- Applying Gauss' law

$$\oiint_S \vec{E} \cdot d\vec{A} = EA = \frac{Q - Q_p}{\epsilon_0} \quad \Longrightarrow \quad E = \frac{\sigma - \sigma_p}{\epsilon_0}$$

Gauss' Law in Dielectrics

Gauss' law

- Parallel plate capacitor *with* dielectric

$$E = \frac{\sigma - \sigma_i}{\epsilon_0}$$

- Applying Gauss' law: $E = E_0/K_e$

$$\frac{\sigma}{K_e \epsilon_0} = \frac{\sigma - \sigma_i}{\epsilon_0}$$

$$\sigma_i = \sigma\left(1 - \frac{1}{K_e}\right)$$

END