Probability

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→ How likely each outcome or event might be?

→ Most experiments show some regularity.

Probability of an event $A \rightarrow Pr(A)$

is the expected relative frequency of event A in a large number of trials.

If an experiment has a total of n_S outcomes in the sample space S, and n_A of these outcomes correspond to the event A will occur is

$$Pr(A) = \frac{n_A}{n_S}$$

Probability Axioms & Theorems

I. For any event A in a sample space S,

$$0 \leq Pr(A) \leq 1$$

impossibility of event A — certainty of an event A

II. For the entire sample space S

$$Pr(S) = \frac{n_S}{n_S} = 1$$

III. For two events A and B with an intersection $A \cap B$

$$n_{A \cup B} = n_A + n_B - n_{A \cap B}$$

needed to remove overcounting of outcomes

$$\frac{n_{A \cup B}}{n_S} = \frac{n_A + n_B - n_{A \cap B}}{n_S}$$

$$Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$$

If $A \cap B = \phi$, then $Pr(A \cap B) = 0$

IV. If $\bar{A}$ is the complement of A then $\bar{A}$ and A are <u>mutually exclusive</u> events

$$1 = Pr(S)$$

$$1 = Pr(A) + Pr(\bar{A})$$

$$Pr(A) = 1 - Pr(\bar{A})$$

Note:

$A \cap \bar{A} = \phi$

$S = A \cup \bar{A}$

$Pr(S) = Pr(A) + Pr(\bar{A})$

V. If $A_1, A_2, \ldots A_n$ are <u>mutually exclusive</u> events, then

$$Pr(A_1 \cup A_2 \cup \ldots \cup A_n) = Pr(A_1) + Pr(A_2) + \ldots + Pr(A_n)$$

$$Pr\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} Pr(A_i)$$

If we further assume that $\underbrace{A_1 \cup A_2 \cup \ldots A_n = S}_{\{A_i\}_{i=1}^{S} \text{ exhaust } S}$

then

$$\sum_{i=i}^{n} Pr(A_i) = 1$$

Example: Calculate the probability of drawing an ace or a spade from a deck of cards.

$A \rightarrow$ event of drawing an ace

$B \rightarrow$ event of drawing a spade

Soln:

$$Pr(A) = \frac{4}{52}$$

$$Pr(B) = \frac{13}{52}$$

$$Pr(A \cap B) = \frac{1}{52}$$

Thus, the probability of drawing an ace or a spade is

$$Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$$

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$$

$$= \frac{16}{52}$$

$$= \frac{4}{13}$$

Exercise:

A card is drawn from a shuffled deck

a) what is the probability that it is black?

b) " " " " " " " a red nine?

c) " " " " " " " a queen of spades?

d) " " " " " " " a king or a red card?

Consider the union of three events

A_1, A_2, and A_3 ← Not mutually exclusive

Define $B = A_2 \cup A_3$, then

$$Pr(A_1 \cup A_2 \cup A_3) = Pr(A_1 \cup B)$$

$$= Pr(A_1) + Pr(B) - Pr(A_1 \cap B)$$

$$Pr(A_1 \cup A_2 \cup A_3) = Pr(A_1) + Pr(A_2) + Pr(A_3) \quad \leftarrow \text{one-fold intersection}$$

$$- Pr(A_2 \cap A_3) - Pr(A_1 \cap A_2) - Pr(A_1 \cap A_3) \quad \leftarrow \text{two-fold intersection}$$

$$+ Pr(A_1 \cap A_2 \cap A_3) \quad \leftarrow \text{three-fold intersection}$$

Q2 Given the union of n general events, prove by induction upon n that

$$Pr(A_1 \cup A_2 \cup \ldots \cup A_n) = \sum_i Pr(A_i) - \sum_{ij} Pr(A_i \cap A_j)$$

$$+ \sum_{i,j,k} Pr(A_i \cap A_j \cap A_k)$$

$$- \ldots + (-1)^{n+1} Pr(A_1 \cap A_2 \cap \ldots A_n)$$

* summation with diff indices are understood not to be equal ($i \neq j$, $i \neq j \neq k \ldots$)