2 - Wave Velocity, Energy, Power, and Intensity

#phys31-2 #waves

Velocity of Waves on a String

Speed of a transverse wave on a string:

$$v = \sqrt{\frac{F}{\mu}}$$

where F is the tension of the string and μ is the linear mass density of the string.
In other words, this can be expressed as

$$\text{velocity} = \sqrt{\frac{\text{Restoring force returning to equil.}}{\text{Inertia resisting return}}}$$

Energy & Power in Wave Motion

Energy

$\frac{F_y}{F}$ is equal to the slope of the string. Then:

$$F_y(x,t) = -F\frac{\delta y(x,t)}{\delta x}$$

Power

Since we know that $P = Fv$, then

$$P(x,t) = F_y(x,t)v_y(x,t) = -F\frac{\delta y(x,t)}{\delta x}\frac{\delta y(x,t)}{\delta t}$$

This is valid for any point on the string.

For a sinusoidal wave, this then becomes

$$P(x,t) = Fk\omega A^2 \sin^2(kx - \omega t)$$

Then with $\omega = vk$ and $v^2 = \frac{F}{\mu}$,

$$P(x,t) = \sqrt{\mu F}\omega^2 A^2 \sin^2(kx - \omega t)$$

which also implies that the maximum power at any point on the wave is

$$P_{\text{max}} = \sqrt{\mu F}\omega^2 A^2$$

Intensity

Intensity (aka flux) is used to measure a radiant energy.

Consider waves where energy is transported omnidirectionally (e.g. sound in air).
Now consider two points of the wave with values r_i (radius) and I_i (intensity). Say that we get two points $i = 1$ and $i = 2$, where $i = 2$ is farther from the wave source. Then we can assume the following relationships:

$$I_2 < I_1 \qquad r_1 < r_2$$

We can also get the relationship between r and I:

$$I \propto \frac{1}{r^2}; \qquad I = \frac{P}{4\pi r^2}; \qquad \frac{I_1}{I_2} = \frac{(r_2)^2}{(r_1)^2}$$

The third equation is due to the inverse square law.

Summary:

1. (Propagation) Speed of a Transverse Wave on a String: $v = \sqrt{\frac{F}{\mu}}$
2. Power of a wave: $P(x,t) = \sqrt{\mu F}\omega^2 A^2 \sin^2(kx - \omega t)$