# 3 - Interference and Standing Waves

#phys31-2 #waves #standingwave

## Boundary Conditions

A wave on a string reflects depending on the state of the end of the string:

- If it is a fixed end, inverted reflection occurs.
- If it is a free end, a non-inverted reflection occurs.

## Wave Interference and Superposition

When waves overlap, the resulting final displacement is the algebraic sum of each individual pulse.

### Principle of Superposition

If we have two waves of position $y_1$ and $y_2$, then our final displacement is

$$y(x,t) = y_1(x,t) + y_2(x,t)$$

## Standing Waves on a String

Consider when a sinusoidal wave is reflected on a fixed end.

The resulting wave is called a **standing wave**. This is because the wave "appears" to be non-moving (aka "standing" in place). There are two parts of a standing wave:

- Nodes: points on a wave where the displacement is always 0
- Antinodes: points on a wave where displacement reaches its maximum

For a standing wave to manifest, the length of the string $L$ and the wavelength $\lambda$ should have the following relationship:

$$L = \frac{n}{2}\lambda_n$$

where $n$ is any integer, corresponding to the number of antinodes present in the standing wave.

### Wave Function of a Standing Wave

Given two wave positions $y_1$ and $y_2$,

$$y_1(x,t) = -A\cos(kx+\omega t) y_2(x,t) = A\cos(kx+\omega t)$$

then the final displacement position is

$$\begin{aligned} y(x,t) &= y_1(x,t) + y_2(x,t) \\ &= A(-\cos(kx+\omega t) + A\cos(kx+\omega t)) \\ &= (2A\sin kx)(\sin(\omega t)) && \text{(using trig addition)} \\ &= (A_{\text{sw}}\sin kx)(\sin(\omega t)) && \text{(alternatively)} \end{aligned}$$

To then find the position of the nodes, we can take advantage of the fact that $y(x,t) = 0$ at these points. Because $A_{\text{sw}} > 0$, then $\sin(kx) = 0$. From these we can get the following values:

$$kx = 0, \pi, 2\pi, 3\pi, \dots$$
$$x = 0, \frac{\pi}{k}, \frac{2\pi}{k}, \frac{3\pi}{k}, \dots$$

# Normal Modes of a String

Since we have the relationship

$$L = n\frac{\lambda}{2}$$

then we can have a possible standing wave frequency related to its corresponding wavelength according to:

$$f_n = \frac{v}{\lambda_n} = n\frac{v}{2L}$$

where $f_1$ is the fundamental frequency.

Integer multiples of $f_1$ correspond to $2f, 3f, 4f$, etc. These are known as harmonics or overtones.