A1Q9
Consider the odd primes 3, 5, 7, 11, 13, 17, 19, etc.
Let S be the odd primes congruent to 1 mod 4 and T be the odd primes congruent to 3 mod 4.
So S = 5, 13, 17, 29, etc. and T = 3, 7, 11, 19, 23, etc.

Suppose we go through the odd primes in order counting the number of primes in S and T.
So at the first step S = {} and T = {3} so T has more primes than S.
At the second step S = {5} and T = {3} so S and T have the same number of primes.
At the third step S = {5} and T = {3,7} so T has more primes than S again.

Notice that |T| >= |S| for the first 10 primes. Does it ever happen that |S| >= T? Yes, it does. When? How often?

Using the builtin nextprime command write a loop that counts |S| and |T| for primes up to 10^6 and prints out the first time |S| > |T| and, at the end, how often |S| > |T|. NB: You don't need to construct the sets S and T!! You only need to know |S| and |T|.

```
> restart:
> s := 0; t := 0; i := 0;
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$$s := 0$$
$$t := 0$$
$$i := 0 \tag{1}$$

```
> L := {};
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$$L := \varnothing \tag{2}$$

```
> x := 3; #the first odd prime
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$$x := 3 \tag{3}$$

```
> while x < 10^6 do
      if modp(x,4) = 1 then
          s := s + 1;
      else #all odd primes are either 1mod4 or 3mod4
          t := t + 1;
      fi;

      if s > t then
          L := L union {x};
      fi;

      x := nextprime(x):
  od:
  L[1]; #the first time |S| > |T|
  nops(L); #the number of times |S| > |T|
```

$$26861$$
$$239 \tag{4}$$

|S| >= |T| at every x stored in L, which happens 239 times.
The first time |S| > |T| is at 26861, and |S| > |T| 239 times while x < 10^6.