A1Q3

Consider the polynomials $f := x^4 - 1$ and $g := x^4 - 4x^3 + 8x - 4$ and $h := x^4 - 4x^3 + 4x^2 - 8x + 4$.

Graph each separately on a suitable domain for x so that we can see all real roots.

Now factor the polyomials using the **factor** command.

Why does the **factor** command not factor them into linear factors?

Now, using **solve**, solve for the roots. Using Maple, verify that the 4 roots of the polynomial g are correct.

Now, using **fsolve**, solve for numerical approximations of the roots, including the complex roots.

Read the help page for **?fsolve** to find out how to get the complex roots.

```
> restart:
> f := x^4-1;
```

$$f := x^4 - 1 \tag{1}$$

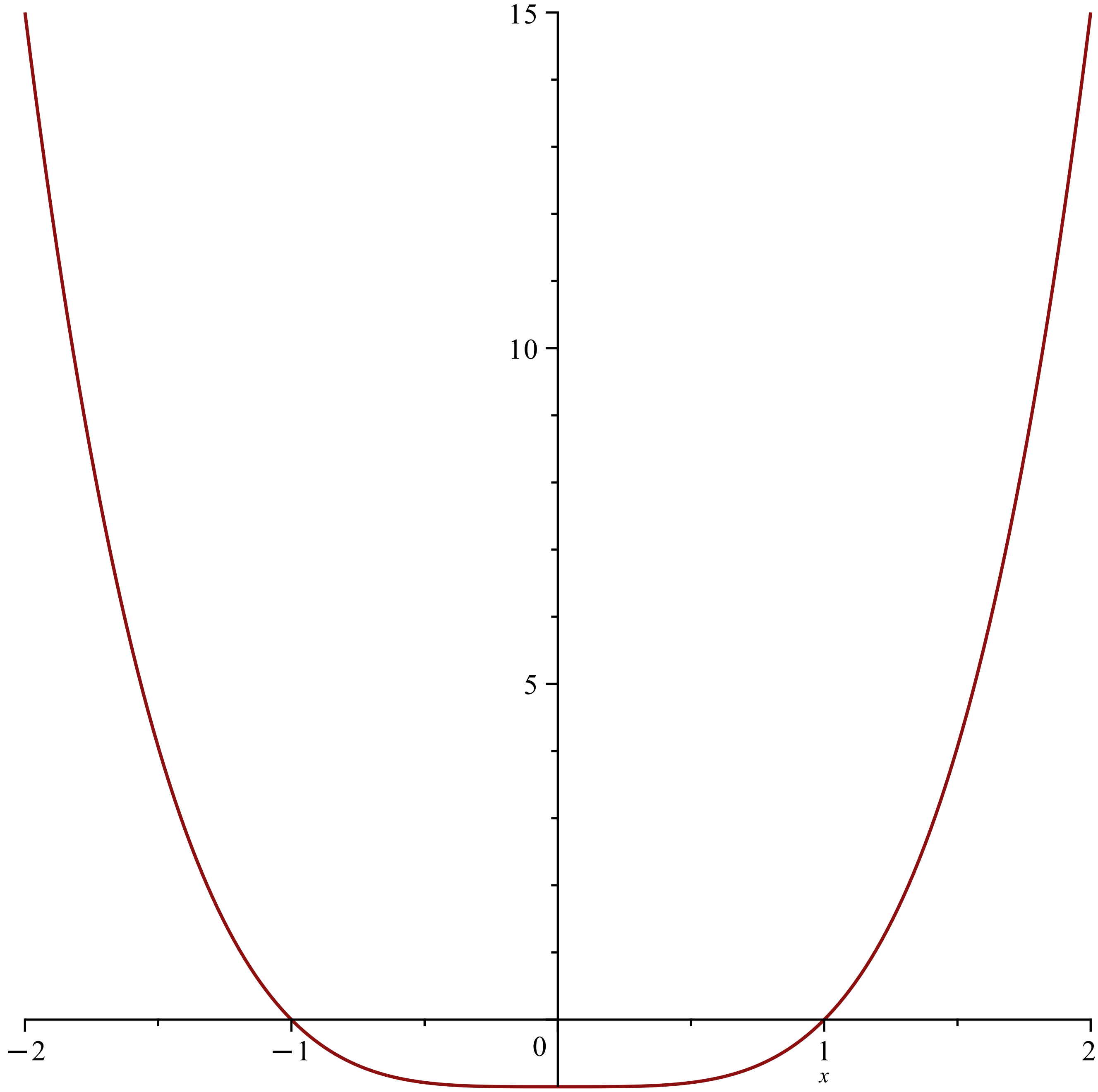
```
> g := x^4 - 4*x^3 + 8*x - 4;
```

$$g := x^4 - 4x^3 + 8x - 4 \tag{2}$$

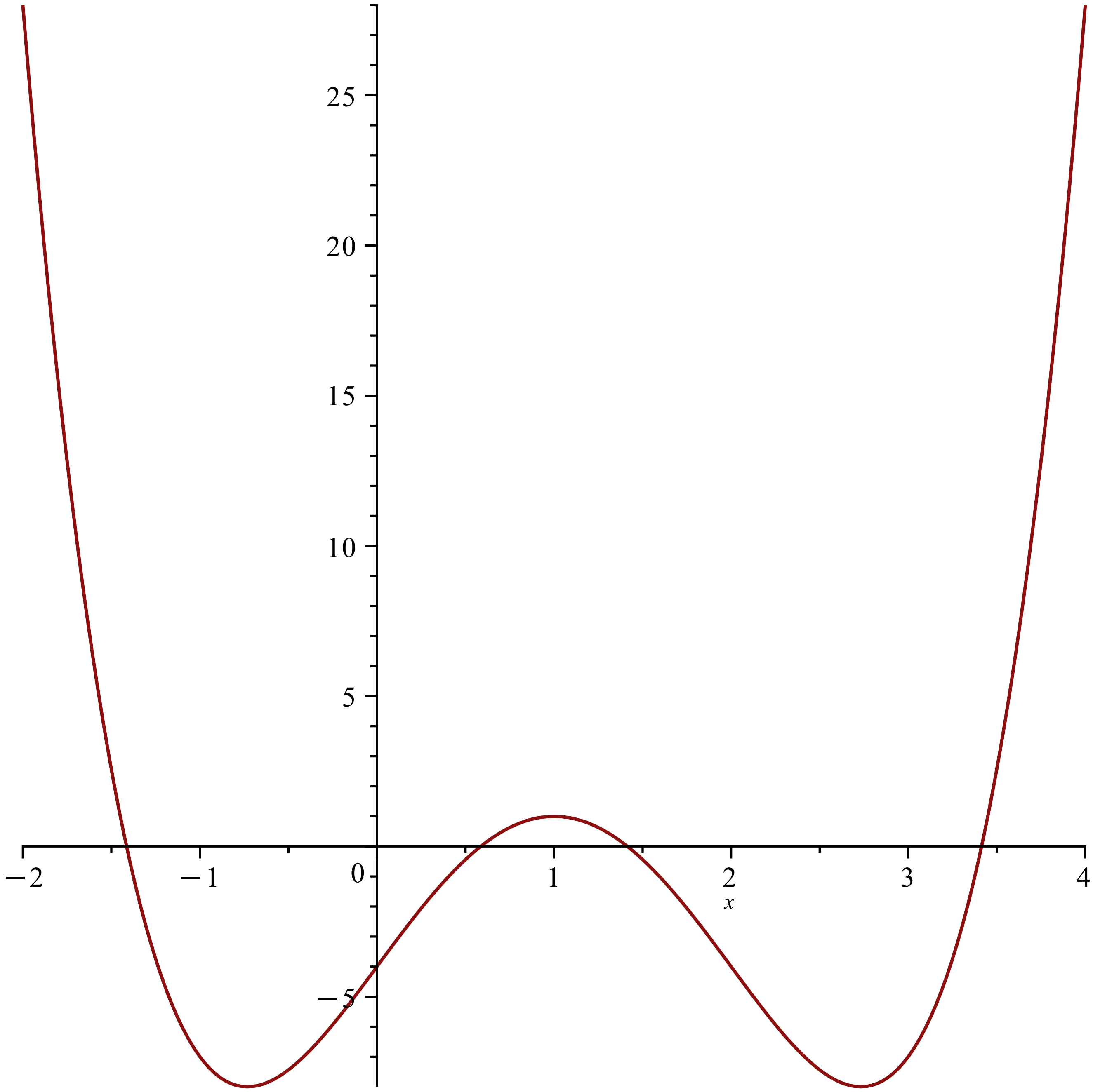
```
> h := x^4 - 4*x^3 + 4*x^2 - 8*x + 4;
```

$$h := x^4 - 4x^3 + 4x^2 - 8x + 4 \tag{3}$$

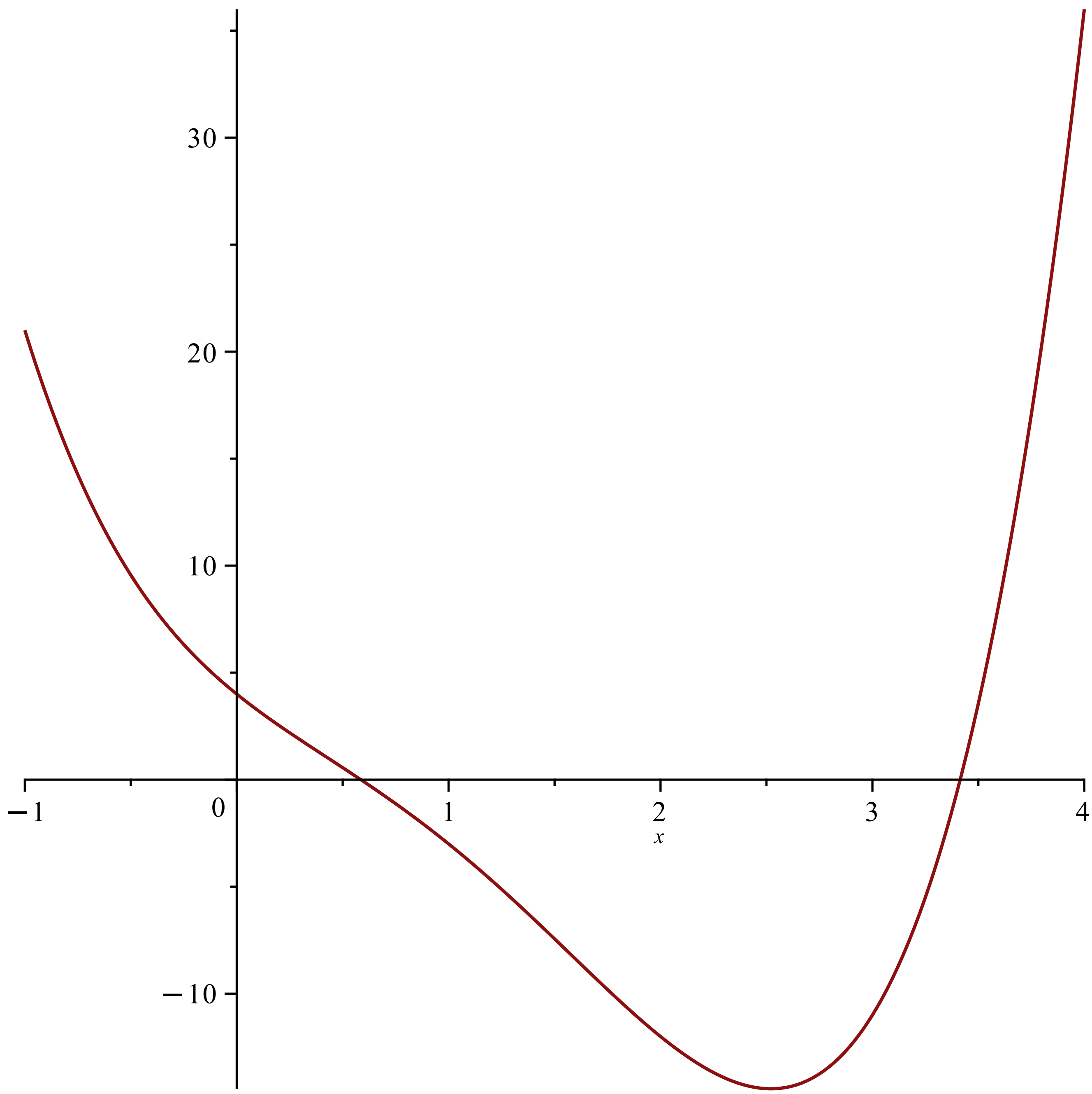
```
> plot(f, x=-2..2);
```




```
> plot(g, x=-2..4);
```




```
> plot(h,x=-1..4);
```



Factor the polynomials using the **factor** command.

```
> factor(f);
```

$$(x-1)\,(x+1)\,(x^2+1) \quad (4)$$

```
> factor(g);
```

$$(x^2-4\,x+2)\,(x^2-2) \quad (5)$$

```
> factor(h);
```

$$(x^2+2)\,(x^2-4\,x+2) \quad (6)$$

If there is no second field K, then factor factors the polynomial implied by the coefficients. In the cases for f,g, and h, the coefficients are all integers so factor computes up to the irreducible factors with integer coefficients. So the factors are not all linear.

Using **solve**, solve for the roots.

```
> solve(f=0,x);
```

$$1, -1, I, -I \tag{7}$$

```
> solve(g=0,x);
```

$$2-\sqrt{2}, 2+\sqrt{2}, \sqrt{2}, -\sqrt{2} \tag{8}$$

```
> solve(h=0,x);
```

$$I\sqrt{2}, -I\sqrt{2}, 2-\sqrt{2}, 2+\sqrt{2} \tag{9}$$

Verify that the 4 roots of g are correct

```
> evalf(eval(g,x = 2 - sqrt(2)));
```

$$0. \tag{10}$$

```
> evalf(eval(g,x = 2 + sqrt(2)));
```

$$0. \tag{11}$$

```
> eval(g, x = sqrt(2));
```

$$0 \tag{12}$$

```
> eval(g, x = -sqrt(2));
```

$$0 \tag{13}$$

Using **fsolve**, solve for numerical approximations of the roots, including the complex roots.

```
> fsolve(f,complex);
```

$$-1.00000000000000, -I, I, 1.00000000000000 \tag{14}$$

```
> fsolve(g); # does not have complex roots
```

$$-1.414213562, 0.5857864376, 1.414213562, 3.414213562 \tag{15}$$

```
> fsolve(h,complex);
```

$$6.657757553 \times 10^{-18} - 1.414213562\,I, 6.657757553 \times 10^{-18} + 1.414213562\,I, 0.5857864376, 3.414213562 \tag{16}$$