A2Q9

Let A = $\int_A^B f(x)\,dx$. Recall from assignment 1 that the value of A may be approximated by the Midpoint rule **Mn** on n intervals of width $h = \frac{(b-a)}{n}$ using the formula

$$Mn = h\cdot\left(f\left(a+\frac{h}{2}\right)+f\left(a+\frac{3}{2}h\right)+2\cdot f\left(a+\frac{5}{2}\cdot h\right)+\ldots+f\left(a+\frac{(2\cdot n-1)}{2}\cdot h\right)\right).$$

Write a Maple procedure MidpointRule (f(x), x, a, b, n) that computes Mn and uses 15 digits of precision for arithmetic calculations

i.e., your Maple procedure should begin by initializing Digits := 15;

```
> restart:
> MidpointRule := proc(f,x::name,a::numeric,b::numeric,n::numeric)

  Digits := 15;
  local h,g,w,i,M;

  if n = 0 then
      return 0;
  fi;

  h := (b-a)/n;
  for i from 0 to (n-1) do
      if i = 0 then
          w := a + (i*h) + (h/2);
          g := evalf[Digits](eval(f,x=w));
      else
          w := w + h;
          g := g + evalf[Digits](eval(f,x=w));
      fi;
  od:
  M := evalf[Digits](h*g):
  end:
```

Execute the following:

```
> MidpointRule(sin(x),x,0,1,4);
  MidpointRule(sin(x),x,0,1,8);
  MidpointRule(sin(x),x,0,1,16);
```

0.460897009411942

0.459997112932708

0.459772523245456 **(1)**

Now compute the error in these approximations as follows:

```
> Digits := 15;
```

$Digits := 15$ **(2)**

```
> A := evalf( int(sin(x), x=0..1) );
```

(3)

$$A := 0.459697694131860 \tag{3}$$

```
> for n in [4,8,16,32,64,128] do
      Mn := MidpointRule( sin(x),x,0,1,n);
      e[n] := abs(A-Mn);
      printf("n=%3d error=%.15f\n", n, e[n]);
  od:
n=  4 error=0.001199315280082
n=  8 error=0.000299418800848
n= 16 error=0.000074829113596
n= 32 error=0.000018705679852
n= 64 error=0.000004676320068
n=128 error=0.000001169073773
```

```
> 0.000299418800848 / 0.001199315280082
```

$$0.249658122280847 \tag{4}$$

As we double the number of intervals n, the error is about 25% of the previous error.