A1Q10

Let A = $\int_a^b f(x)\, dx$. Recall that the value of A may be aproximated by the **Midpoint rule** on n intervals of width $h = \frac{(b-a)}{n}$ using the formula

$$M_n = h \cdot \left(f\left(a + \frac{h}{2}\right) + f\left(a + h + \frac{h}{2}\right) + \ldots + f\left(a + (n-1) \cdot h + \frac{h}{2}\right) \right).$$

For $f(x) = x^2 \sin(2 \cdot x)$ and $a = 0$, $b = 1$, calculate A using Maple to 10 decimal places by using Maple's integration command. Now using a Maple loop, calculate M_8, M_{16} and M_{32} using 10 digit arithmetic (the default). Note, since M_n is a numerical approximation to an area we want to do the computation using decimal arithmetic, not exact arithmetic.

```
> restart:
> f := x^2*sin(2*x);
  a := 0;
  b := 1;
  A := int(f,x=a..b);
```

$$f := x^2 \sin(2\, x)$$

$$a := 0$$

$$b := 1$$

$$A := -\frac{1}{4} - \frac{\cos(2)}{4} + \frac{\sin(2)}{2} \quad \textbf{(1)}$$

```
> evalf(A);
```

$$0.3086854225 \quad \textbf{(2)}$$

```
> for n in [8,16,32] do
      h := evalf((b-a)/n);
      Mn := 0.0;
      for i from 0 to n-1 do
          Mn := Mn + h*eval(f,x=a+i*h+h/2);
      od;
      print(n,Mn);
  od:
```

8, 0.3080327263

16, 0.3085242331

32, 0.3086452486 **(3)**