A2Q7

Consider the function $f(x, y) = 2 \cdot x^2 + 3 \cdot y^2 - x \cdot y - 6$.
We want to visualize the partial derivatives at the point $x = 1$, $y = 1$.

Part (a) First use Maple to compute the partial derivatives $f_x(1, 1)$ and $f_y(1, 1)$. You should get 3 and 5 respectively so both slopes are positive.

```
> restart:
> F := proc(x,y)
      2*x^2 + 3*y^2 - x*y -6
      end;
```

$$F := \mathbf{proc}(x, y)\ 2 * x\verb|^|2 + 3 * y\verb|^|2 - x * y - 6\ \mathbf{end\ proc} \quad (1)$$

```
> f := F(x,y);
```

$$f := 2x^2 - xy + 3y^2 - 6 \quad (2)$$

```
> fx := diff(f,x);
```

$$fx := 4x - y \quad (3)$$

```
> fy := diff(f,y);
```

$$fy := -x + 6y \quad (4)$$

```
> fxm:= eval(fx,{x=1,y=1});
```

$$fxm := 3 \quad (5)$$

```
> fym:= eval(fy,{x=1,y=1});
```

$$fym := 5 \quad (6)$$

Part (b) We want to visualize the partial derivatives at the point $x = 1$, $y = 1$. Generate a 3 dimensional plot of $f(x, y)$ (using the **plot3d** command) and the curves $f(x, 1)$ and $f(1, y)$ in red and blue (using the **spacecurve** command in the plots package) and display all three plots on the same graph (using the **display** command in the plots package).

```
> a,b := 1,1;
```

$$a, b := 1, 1 \quad (7)$$

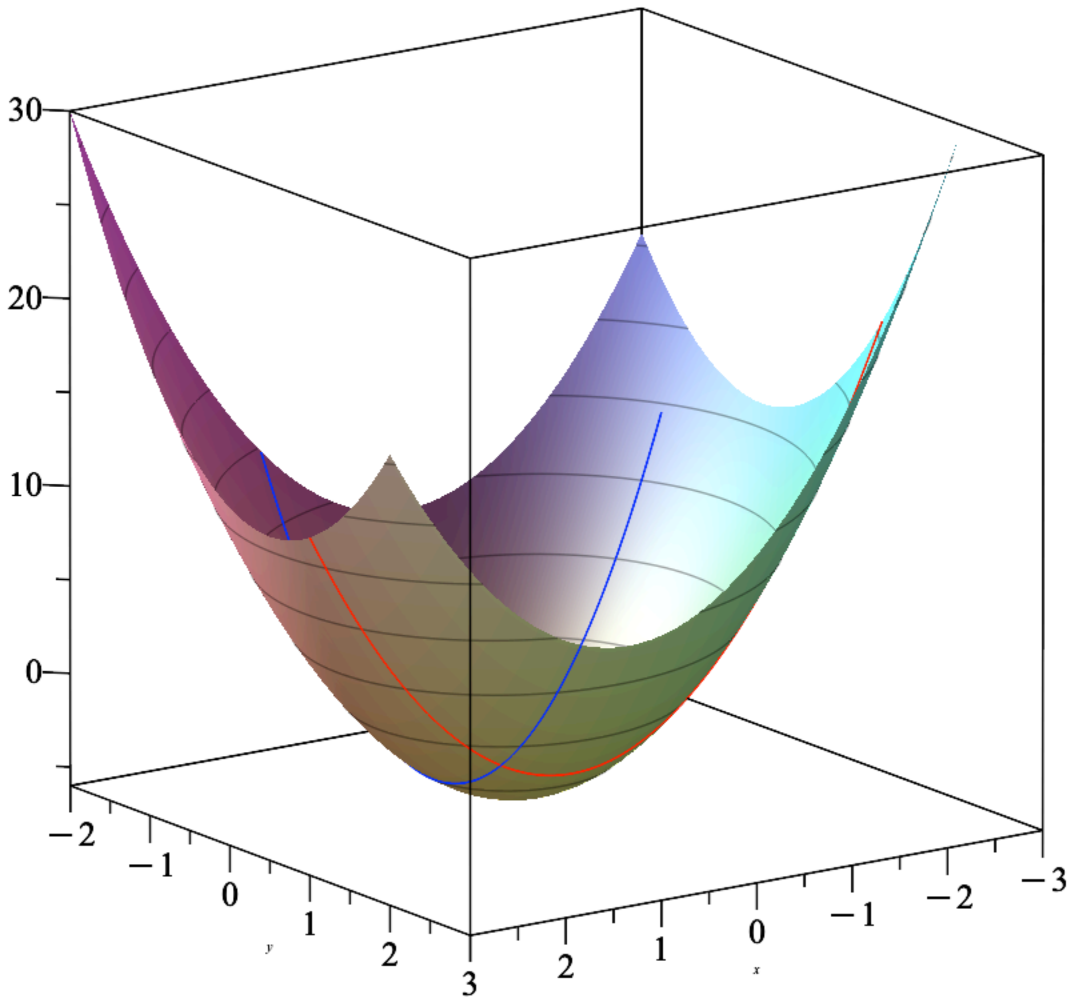
```
> with(plots):
> fplot := plot3d(f,style=patchcontour):
> fx1 := F(x,1);
```

$$fx1 := 2x^2 - x - 3 \quad (8)$$

```
> f1y := F(1,y);
```

$$f1y := 3y^2 - y - 4 \quad (9)$$

```
> fx1plot := spacecurve([x,b,fx1], x=-3..3, color=red):
> f1yplot := spacecurve([a,y,f1y], y=-2..3, color=blue):
> display([fplot,fx1plot,f1yplot]);
```



Part (c) Now we want to visualize the tangent plane at the point $x = 1$, $y = 1$. Construct the formula for the tangent plane and graph it together with $f(x, y)$ on the same plot using suitable options so we can see both surfaces clearly. Include a title for the plot.

The tangent plane is:
T(x,y) = f(a,b) + f_x(a,b) * (x-a) + f_y(a,b) * (y-b)

```
> T := F(a,b) + fxm*(x-a) + fym*(y-b);
```

$$T := -10 + 3x + 5y \quad \textbf{(10)}$$

```
> Tplot := plot3d(T, x=-3..3, y=-2..3):
> display([fplot,Tplot],title="Tangent plane plot");
```

Tangent plane plot