

A1Q6

Consider the two equations $y^2 = x^3 + x$ and $x^2 + y^2 = 1$ which are the equations of an elliptic curve and a circle. Graph the two curves together using the **implicitplot** command in the **plots** package.

Improve the smoothness of the plot using the **grid** option. The circle will appear distorted. Find out from ?plot, options how to scale the axes so that the circle looks like a circle (so that the plot looks like the figure below).

```
> restart;
```

```
> with(plots):
```

```
> f := y^2 = x^3+x;
```

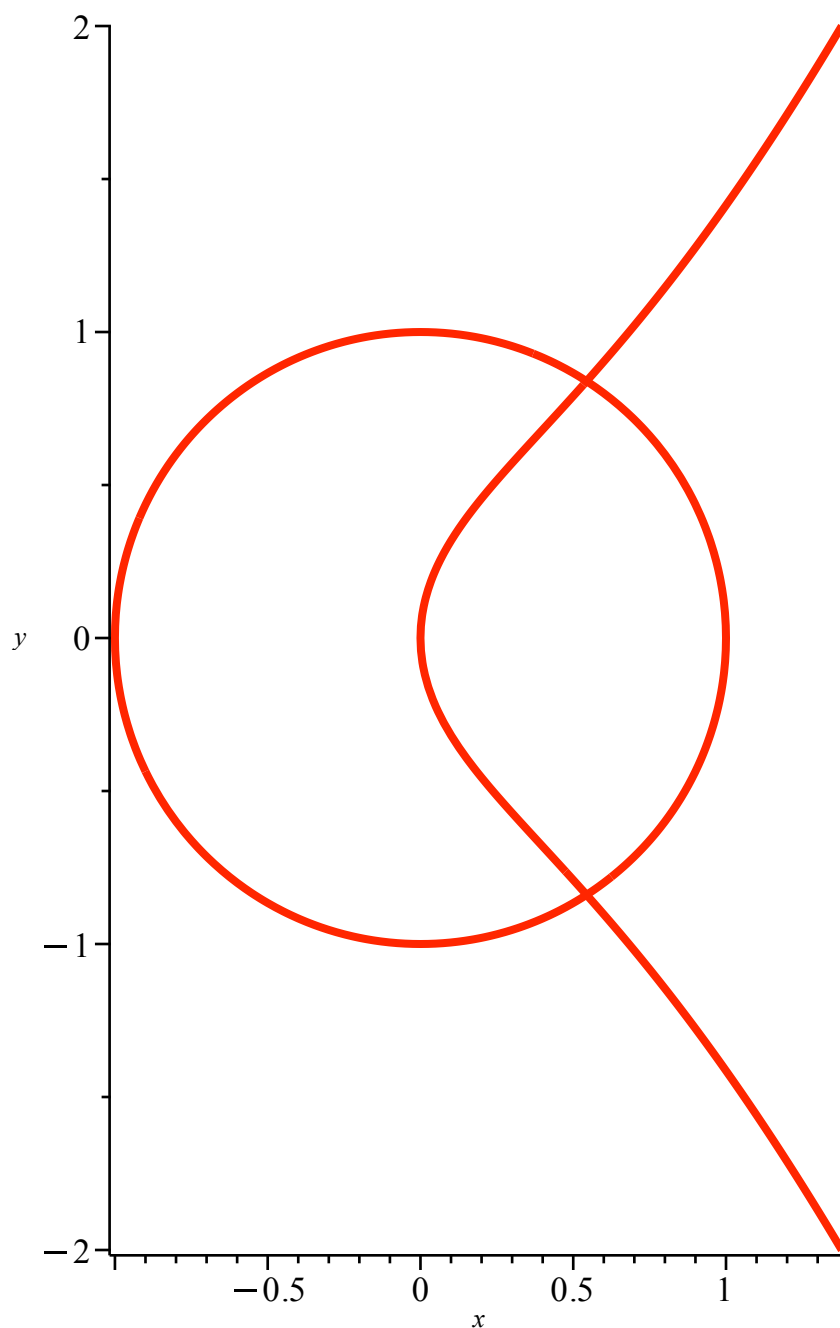
```
g := x^2 + y^2 = 1;
```

$$f := y^2 = x^3 + x$$

$$g := x^2 + y^2 = 1$$

(1)

```
> implicitplot(
[f,g],
x=-1..1.5,
y=-2..2,
color = red,
thickness = 3,
axes = frame,
axis[2] = [tickmarks = [subticks=1]],
axis[1] = [tickmarks = [subticks = 4]],
gridrefine=2,
scaling=constrained) ;
```



You can read off from the plot roughly where the curves intersect.
 Use Maple to calculate the intersection points accurately.
 Do this by solving two simultaneous equations.

```
> sys := {f,g};
```

```
sys := {y^2 = x^3 + x, x^2 + y^2 = 1}
```

(2)

```
> fsolve(sys,{x=-1..1, y=-2..0});
```

```
{x = 0.5436890127, y = -0.8392867552}
```

(3)

```
> fsolve(sys,{x=-1..1, y=0..2});
```

```
{x = 0.5436890127, y = 0.8392867552}
```

(4)

The intersection points are at
 $\{x = 0.5436890127, y = -0.8392867552\}$ and

$$\mathbb{L}_{\{x = 0.5436890127, y = 0.8392867552\}}.$$