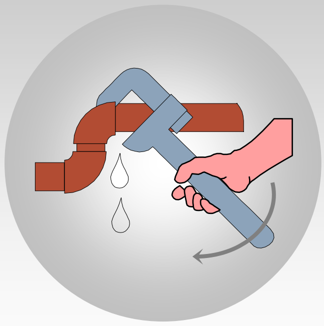


# Rotational Motion

## 7.1 Introduction

Translation is motion along a straight line but rotation is the motion of wheels, gears, motors, planets, the hands of a clock, the rotor of jet engines and the blades of helicopters. First figure shows a skater gliding across the ice in a straight line with constant speed. Her motion is called translation but second figure shows her spinning at a constant rate about a vertical axis. Here motion is called rotation.

Up to now we have studied translatory motion of a point mass. In this chapter we will study the rotatory motion of rigid body about a fixed axis.

(1) Rigid body : A rigid body is a body that can rotate with all the parts locked together and without any change in its shape.

(2) System : A collection of any number of particles interacting with one another and are under consideration during analysis of a situation are said to form a system.

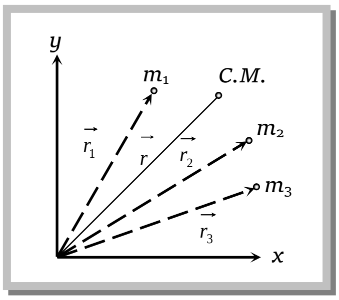
(3) Internal forces : All the forces exerted by various particles of the system on one another are called internal forces. These forces are alone enable the particles to form a well defined system. Internal forces between two particles are mutual (equal and opposite).

(4) External forces : To move or stop an object of finite size, we have to apply a force on the object from outside. This force exerted on a given system is called an external force.

## 7.2 Centre of Mass

Centre of mass of a system (body) is a point that moves as though all the mass were concentrated there and all external forces were applied there.

(1) **Position vector of centre of mass for *n* particle system :** If a system consists of $n$ particles of masses $m_1, m_2, m_3 ...... m_n$, whose positions vectors are $\vec{r}_1, \vec{r}_2, \vec{r}_3 ........ \vec{r}_n$ respectively then position vector of centre of mass




$$\vec{r} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3 + ........... m_n \vec{r}_n}{m_1 + m_2 + m_3 + ........... m_n}$$

Hence the centre of mass of $n$ particles is a weighted average of the position vectors of $n$ particles making up the system.

(2) **Position vector of centre of mass for two particle system :** $\vec{r} = \frac{m_1 \vec{r_1} + m_2 \vec{r_2}}{m_1 + m_2}$

and the centre of mass lies between the particles on the line joining them.

If two masses are equal *i.e.* $m_1 = m_2$, then position vector of centre of mass $\vec{r} = \frac{\vec{r_1} + \vec{r_2}}{2}$

(3) **Important points about centre of mass**

(i) The position of centre of mass is independent of the co-ordinate system chosen.

(ii) The position of centre of mass depends upon the shape of the body and distribution of mass.

*Example* : The centre of mass of a circular disc is within the material of the body while that of a circular ring is outside the material of the body.

(iii) In symmetrical bodies in which the distribution of mass is homogenous, the centre of mass coincides with the geometrical centre or centre of symmetry of the body.

(iv) Position of centre of mass for different bodies

| S. No. | Body | Position of centre of mass |
|---|---|---|
| (a) | Uniform hollow sphere | Centre of sphere |
| (b) | Uniform solid sphere | Centre of sphere |
| (c) | Uniform circular ring | Centre of ring |
| (d) | Uniform circular disc | Centre of disc |
| (e) | Uniform rod | Centre of rod |
| (f) | A plane lamina (Square, Rectangle, Parallelogram) | Point of inter section of diagonals |
| (g) | Triangular plane lamina | Point of inter section of medians |
| (h) | Rectangular or cubical block | Points of inter section of diagonals |
| (i) | Hollow cylinder | Middle point of the axis of cylinder |
| (j) | Solid cylinder | Middle point of the axis of cylinder |
| (k) | Cone or pyramid | On the axis of the cone at point distance $\frac{3h}{4}$ from the vertex where $h$ is the height of cone |

(v) The centre of mass changes its position only under the translatory motion. There is no effect of rotatory motion on centre of mass of the body.

(vi) If the origin is at the centre of mass, then the sum of the moments of the masses of the system about the centre of mass is zero *i.e.* $\Sigma m_i \vec{r_i} = 0$ .

(vii) If a system of particles of masses $m_1, m_2, m_3, ......$ move with velocities $v_1, v_2, v_3, ......$

then the velocity of centre of mass $v_{cm} = \dfrac{\sum m_i v_i}{\sum m_i}$.

(viii) If a system of particles of masses $m_1, m_2, m_3, ......$ move with accelerations $a_1, a_2, a_3, ......$

then the acceleration of centre of mass $A_{cm} = \dfrac{\sum m_i a_i}{\sum m_i}$

(ix) If $\vec{r}$ is a position vector of centre of mass of a system

then velocity of centre of mass $\vec{v}_{cm} = \dfrac{d\vec{r}}{dt} = \dfrac{d}{dt}\left(\dfrac{m_1\vec{r_1} + m_2\vec{r_2} + m_3\vec{r}_3 + ......}{m_1 + m_2 + m_3 + ......}\right)$

(x) Acceleration of centre of mass $\vec{A}_{cm} = \dfrac{d\vec{v}_{cm}}{dt} = \dfrac{d^2\vec{r}}{dt^2} = \dfrac{d^2}{dt^2}\left(\dfrac{m_1\vec{r_1} + m_2\vec{r_2} + .......}{m_1 + m_2 + m_3 + .......}\right)$

(xi) Force on a rigid body $\vec{F} = M\vec{A}_{cm} = M\dfrac{d^2\vec{r}}{dt^2}$

(xii) For an isolated system external force on the body is zero

$$\vec{F} = M\frac{d}{dt}\left(\vec{v}_{cm}\right) = 0 \Rightarrow \vec{v}_{cm} = \text{constant} \ .$$

*i.e.*, centre of mass of an isolated system moves with uniform velocity along a straight-line path.

## Sample problems based on centre of mass

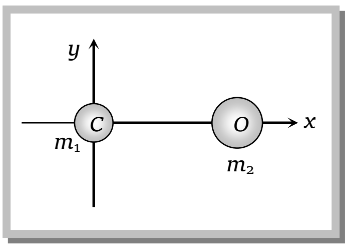
**Problem 1.** The distance between the carbon atom and the oxygen atom in a carbon monoxide molecule is 1.1 Å. Given, mass of carbon atom is 12 *a.m.u.* and mass of oxygen atom is 16 *a.m.u.*, calculate the position of the center of mass of the carbon monoxide molecule

(a) 6.3 Å from the carbon atom (b) 1 Å from the oxygen atom

(c) 0.63 Å from the carbon atom (d) 0.12 Å from the oxygen atom

*Solution* : (c) Let carbon atom is at the origin and the oxygen atom is placed at *x*-axis

$m_1 = 12$ , $m_2 = 16$ , $\vec{r}_1 = 0\hat{i} + 0\hat{j}$ and $\vec{r}_2 = 1.1\hat{i} + 0\hat{j}$



$$\vec{r} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2} = \frac{16 \times 1.1}{28}\hat{i}$$

$\vec{r} = 0.63\,\hat{i}$ *i.e.* 0.63 Å from carbon atom.

***Problem* 2.** The velocities of three particles of masses 20*g*, 30*g* and 50 *g* are $10\vec{i}, 10\vec{j}$, and $10\vec{k}$ respectively. The velocity of the centre of mass of the three particles is **[EAMCET 2001]**

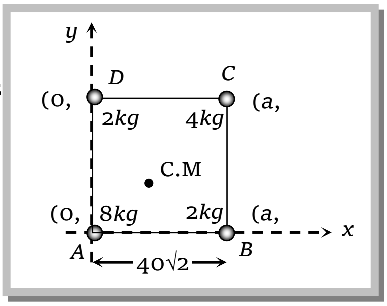
(a) $2\vec{i} + 3\vec{j} + 5\vec{k}$ (b) $10(\vec{i} + \vec{j} + \vec{k})$ (c) $20\vec{i} + 30\vec{j} + 5\vec{k}$ (d) $2\vec{i} + 30\vec{j} + 50\vec{k}$

*Solution* : (a) Velocity of centre of mass $v_{cm} = \frac{m_1 v_1 + m_2 v_2 + m_3 v_3}{m_1 + m_2 + m_3} = \frac{20 \times 10\hat{i} + 30 \times 10\hat{j} + 50 \times 10\hat{k}}{100} = 2\hat{i} + 3\hat{j} + 5\hat{k}$.

***Problem* 3.** Masses $8, 2, 4, 2\,kg$ are placed at the corners $A, B, C, D$ respectively of a square $ABCD$ of diagonal $80\,cm$. The distance of centre of mass from $A$ will be

(a) $20\,cm$ (b) $30\,cm$ (c) $40\,cm$ (d) $60\,cm$

*Solution* : (b) Let corner A of square *ABCD* is at the origin and the mass 8 *kg* is placed at this corner (given in problem) Diagonal of square $d = a\sqrt{2} = 80\,cm \Rightarrow a = 40\sqrt{2}cm$




$m_1 = 8kg,\ m_2 = 2kg,\ m_3 = 4kg,\ m_4 = 2kg$

Let $\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4$ are the position vectors of respective mass

$$\vec{r}_1 = 0\hat{i} + 0\hat{j},\ \vec{r}_2 = a\hat{i} + 0\hat{j},\ \vec{r}_3 = a\hat{i} + a\hat{j},\ \vec{r}_4 = 0\hat{i} + a\hat{j}$$

From the formula of centre of mass

$$\vec{r} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + m_4\vec{r}_4}{m_1 + m_2 + m_3 + m_4} = 15\sqrt{2}i + 15\sqrt{2}\hat{j}$$

$\therefore$ co-ordinates of centre of mass $= (15\sqrt{2}, 15\sqrt{2})$ and co-ordination of the corner $= (0, 0)$

From the formula of distance between two points $(x_1, y_1)$ and $(x_2, y_2)$

distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(15\sqrt{2} - 0)^2 + (15\sqrt{2} - 0)^2} = \sqrt{900} = 30cm$

***Problem* 4.** The coordinates of the positions of particles of mass $7, 4$ and $10\,gm$ are $(1, 5, -3)$, $(2, 5, 7)$ and $(3, 3, -1)cm$ respectively. The position of the centre of mass of the system would be

(a) $\left(-\frac{15}{7}, \frac{85}{17}, \frac{1}{7}\right) cm$ (b) $\left(\frac{15}{7}, -\frac{85}{17}, \frac{1}{7}\right) cm$ (c) $\left(\frac{15}{7}, \frac{85}{21}, -\frac{1}{7}\right) cm$ (d) $\left(\frac{15}{7}, \frac{85}{21}, \frac{7}{3}\right) cm$

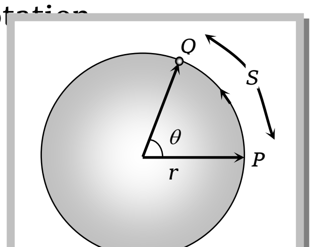
*Solution:* (c) $m_1 = 7gm$, $m_2 = 4gm$, $m_3 = 10gm$ and $\vec{r}_1 = (\hat{i} + 5\hat{j} - 3\hat{k}),\ r_2 = (2i + 5j + 7k),\ r_3 = (3\hat{i} + 3\hat{j} - \hat{k})$

Position vector of center mass $\vec{r} = \frac{7(\hat{i} + 5\hat{j} - 3\hat{k}) + 4(2\hat{i} + 5\hat{j} + 7\hat{k}) + 10(3\hat{i} + 3\hat{j} - \hat{k})}{7 + 4 + 10} = \frac{(45\hat{i} + 85\hat{j} - 3\hat{k})}{21}$

$\Rightarrow \vec{r} = \frac{15}{7}\hat{i} + \frac{85}{21}\hat{j} - \frac{1}{7}\hat{k}$. So coordinates of centre of mass $\left[\frac{15}{7}, \frac{85}{21}, \frac{-1}{7}\right]$.

## 7.3 Angular Displacement

It is the angle described by the position vector $\vec{r}$ about the axis of rotation.



Angular displacement $(\theta) = \dfrac{\text{Linear displacement } (s)}{\text{Radius } (r)}$

(1) Unit : radian

(2) Dimension : $[M^0 L^0 T^0]$

(3) Vector form $\vec{S} = \vec{\theta} \times \vec{r}$

*i.e.*, angular displacement is a vector quantity whose direction is given by right hand rule. It is also known as axial vector. For anti-clockwise sense of rotation direction of $\theta$ is perpendicular to the plane, outward and along the axis of rotation and vice-versa.

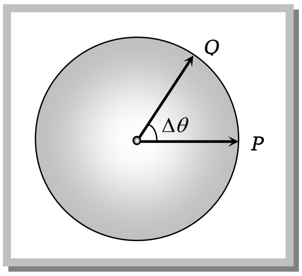
(4) $2\pi$ radian $= 360° = 1$ revolution.

(5) If a body rotates about a fixed axis then all the particles will have same angular displacement (although linear displacement will differ from particle to particle in accordance with the distance of particles from the axis of rotation).

## 7.4 Angular Velocity

The angular displacement per unit time is defined as angular velocity.

If a particle moves from *P* to *Q* in time $\Delta t$, $\omega = \dfrac{\Delta\theta}{\Delta t}$ where $\Delta\theta$ is the angular displacement.




(1) Instantaneous angular velocity $\omega = \lim_{\Delta t \to 0} \dfrac{\Delta\theta}{\Delta t} = \dfrac{d\theta}{dt}$

(2) Average angular velocity $\omega_{av} = \dfrac{\text{total angular displacement}}{\text{total time}} = \dfrac{\theta_2 - \theta_1}{t_2 - t_1}$

(3) Unit : *Radian/sec*

(4) Dimension : $[M^0 L^0 T^{-1}]$ which is same as that of frequency.

(5) Vector form $\vec{v} = \vec{\omega} \times \vec{r}$ [where $\vec{v}$ = linear velocity, $\vec{r}$ = radius vector]

$\vec{\omega}$ is a axial vector, whose direction is normal to the rotational plane and its direction is given by right hand screw rule.

(6) $\omega = \dfrac{2\pi}{T} = 2\pi n$ [where *T* = time period, *n* = frequency]

(7) The magnitude of an angular velocity is called the angular speed which is also represented by $\omega$.

## 7.5 Angular Acceleration

The rate of change of angular velocity is defined as angular acceleration.

If particle has angular velocity $\omega_1$ at time $t_1$ and angular velocity $\omega_2$ at time $t_2$ then,

Angular acceleration $\vec{\alpha} = \dfrac{\vec{\omega}_2 - \vec{\omega}_1}{t_2 - t_1}$

(1) Instantaneous angular acceleration $\vec{\alpha} = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\vec{\omega}}{dt} = \frac{d^2\vec{\theta}}{dt^2}$.

(2) Unit : $rad/sec^2$

(3) Dimension : $[M^0L^0T^{-2}]$.

(4) If $\alpha = 0$, circular or rotational motion is said to be uniform.

(5) Average angular acceleration $\alpha_{av} = \frac{\omega_2 - \omega_1}{t_2 - t_1}$.

(6) Relation between angular acceleration and linear acceleration $\vec{a} = \vec{\alpha} \times \vec{r}$.

(7) It is an axial vector whose direction is along the change in direction of angular velocity *i.e.* normal to the rotational plane, outward or inward along the axis of rotation (depends upon the sense of rotation).

## 7.6 Equations of Linear Motion and Rotational Motion

| | Linear Motion | Rotational Motion |
|---|---|---|
| (1) | If linear acceleration is 0, $u$ = constant and $s = u\,t$. | If angular acceleration is 0, $\omega$ = constant and $\theta = \omega t$ |
| (2) | If linear acceleration $a$ = constant,<br>(i) $s = \frac{(u+v)}{2}t$<br>(ii) $a = \frac{v-u}{t}$<br>(iii) $v = u + at$<br>(iv) $s = ut + \frac{1}{2}at^2$<br>(v) $v^2 = u^2 + 2as$<br>(vi) $s_{nth} = u + \frac{1}{2}a(2n-1)$ | If angular acceleration $\alpha$ = constant then<br>(i) $\theta = \frac{(\omega_1 + \omega_2)}{2}t$<br>(ii) $\alpha = \frac{\omega_2 - \omega_1}{t}$<br>(iii) $\omega_2 = \omega_1 + \alpha t$<br>(iv) $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$<br>(v) $\omega_2^2 = \omega_1^2 + 2\alpha\theta$<br>(vi) $\theta_{nth} = \omega_1 + (2n-1)\frac{\alpha}{2}$ |
| (3) | If acceleration is not constant, the above equation will not be applicable. In this case<br>(i) $v = \frac{dx}{dt}$<br>(ii) $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$<br>(iii) $v\,dv = a\,ds$ | If acceleration is not constant, the above equation will not be applicable. In this case<br>(i) $\omega = \frac{d\theta}{dt}$<br>(ii) $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$<br>(iii) $\omega d\omega = \alpha\, d\theta$ |

## Sample problems based on angular displacement, velocity and acceleration

**Problem 5.** The angular velocity of *seconds* hand of a watch will be

(a) $\frac{\pi}{60} rad/sec$ (b) $\frac{\pi}{30} rad/sec$ (c) $60\pi\ rad/sec$ (d) $30\pi\ rad/sec$

*Solution* : (b) We know that second's hand completes its revolution ($2\pi$) in 60 *sec* $\therefore$ $\omega = \frac{\theta}{t} = \frac{2\pi}{60} = \frac{\pi}{30} rad/sec$

**Problem 6.** The wheel of a car is rotating at the rate of 1200 revolutions per *minute*. On pressing the accelerator for 10 *sec* it starts rotating at 4500 revolutions per *minute*. The angular acceleration of the wheel is **[MP PET 2001]**

(a) 30 *radians/sec*² (b) 1880 *degrees/sec*² (c) 40 *radians/sec*² (d)

*Solution:* (d) Angular acceleration ($\alpha$) = rate of change of angular speed

$$= \frac{2\pi(n_2 - n_1)}{t} = \frac{2\pi\left(\frac{4500-1200}{60}\right)}{10} = \frac{2\pi\frac{3300}{60}}{10} \times \frac{360}{2\pi}\frac{degree}{sec^2} = 1980\ degree/sec^2.$$

**Problem 7.** Angular displacement ($\theta$) of a flywheel varies with time as $\theta = at + bt^2 + ct^3$ then angular acceleration is given by

(a) $a + 2bt - 3ct^2$ (b) $2b - 6t$ (c) $a + 2b - 6t$ (d) $2b + 6ct$

*Solution:* (d) Angular acceleration $\alpha = \frac{d^2\theta}{dt^2} = \frac{d^2}{dt^2}(at + bt^2 + ct^3) = 2b + 6ct$

**Problem 8.** A wheel completes 2000 rotations in covering a distance of $9.5\,km$. The diameter of the wheel is **[RPMT 1999]**

(a) $1.5\,m$ (b) $1.5\,cm$ (c) $7.5\,m$ (d) $7.5\,cm$

*Solution:* (a) Distance covered by wheel in 1 rotation = $2\pi r = \pi D$ (Where $D$= $2r$ = diameter of wheel)

$\therefore$ Distance covered in 2000 rotation = 2000 $\pi D$ = $9.5 \times 10^3 m$ (given)

$\therefore D = 1.5\,meter$

**Problem 9.** A wheel is at rest. Its angular velocity increases uniformly and becomes 60 *rad/sec* after 5 *sec*. The total angular displacement is

(a) $600\ rad$ (b) $75\ rad$ (c) $300\ rad$ (d) $150\ rad$

*Solution:* (d) Angular acceleration $\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{60-0}{5} = 12 rad/sec^2$

Now from $\theta = \omega_1 t + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2}(12)(5)^2 = 150\ rad.$

**Problem 10.** A wheel initially at rest, is rotated with a uniform angular acceleration. The wheel rotates through an angle $\theta_1$ in first one second and through an additional angle $\theta_2$ in the next one second. The ratio $\frac{\theta_2}{\theta_1}$ is

(a) 4 (b) 2 (c) 3 (d) 1

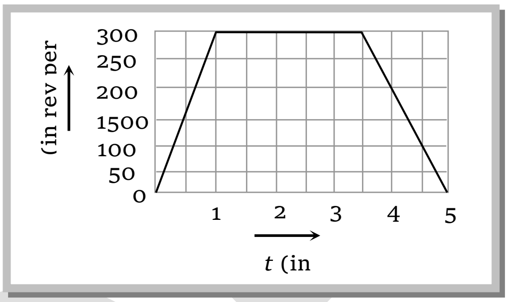
*Solution:* (c) Angular displacement in first one second $\theta_1 = \frac{1}{2}\alpha(1)^2 = \frac{\alpha}{2}$ ......(i) [From $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$ ]

Now again we will consider motion from the rest and angular displacement in total two seconds

$$\theta_1 + \theta_2 = \frac{1}{2}\alpha(2)^2 = 2\alpha \quad \text{......(ii)}$$

Solving (i) and (ii) we get $\theta_1 = \frac{\alpha}{2}$ and $\theta_2 = \frac{3\alpha}{2}$ $\therefore$ $\frac{\theta_2}{\theta_1} = 3$.

***Problem*** **11.** As a part of a maintenance inspection the compressor of a jet engine is made to spin according to the graph as shown. The number of revolutions made by the compressor during the test is

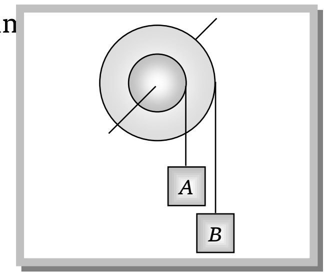



(a) 9000 (b) 16570 (c) 12750 (d) 11250

*Solution:* (d) Number of revolution = Area between the graph and time axis = Area of trapezium

$$= \frac{1}{2}\times(2.5+5)\times 3000 = 11250 \text{ revolution.}$$

***Problem*** **12.** Figure shows a small wheel fixed coaxially on a bigger one of double the radius. The system rotates about the common axis. The strings supporting $A$ and $B$ do not slip on the wheels. If $x$ and $y$ be the distances travelled by $A$ and $B$ in the same tin




(a) $x = 2y$

(b) $x = y$

(c) $y = 2x$

(d) None of these

*Solution:* (c) Linear displacement ($S$) = Radius ($r$) × Angular displacement ($\theta$)

$\therefore S \propto r$ (if $\theta =$ constant)

$$\frac{\text{Distance travelled by mass } A(x)}{\text{Distance travelled by mass } B(y)} = \frac{\text{Radius of pulley concerned with mass } A(r)}{\text{Radius of pulley concerned with mass } B(2r)} = \frac{1}{2} \Rightarrow y = 2x.$$

***Problem*** **13.** If the position vector of a particle is $\vec{r} = (3\hat{i} + 4\hat{j})$ meter and its angular velocity is $\vec{\omega} = (\hat{j} + 2\hat{k})$ *rad/sec* then its linear velocity is (in $m/s$)

(a) $(8\hat{i} - 6\hat{j} + 3\hat{k})$ (b) $(3\hat{i} + 6\hat{j} + 8\hat{k})$ (c) $-(3\hat{i} + 6\hat{j} + 6\hat{k})$ (d) $(6\hat{i} + 8\hat{j} + 3\hat{k})$

*Solution:* (a) $\vec{v} = \vec{\omega} \times \vec{r} = (3\hat{i} + 4\hat{j} + 0\hat{k}) \times (0\hat{i} + \hat{j} + 2\hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ 0 & 1 & 2 \end{vmatrix} = 8\hat{i} - 6\hat{j} + 3\hat{k}$

## 7.7 Moment of Inertia

Moment of inertia plays the same role in rotational motion as mass plays in linear motion. It is the property of a body due to which it opposes any change in its state of rest or of uniform rotation.

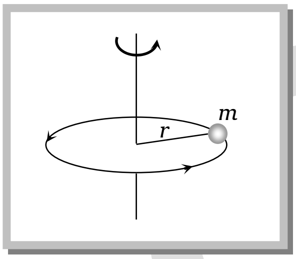
(1) Moment of inertia of a particle $I = mr^2$; where $r$ is the perpendicular distance of particle from rotational axis.

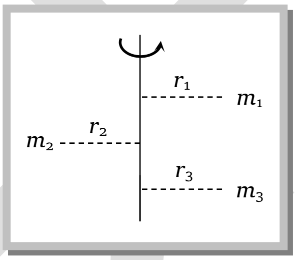
(2) Moment of inertia of a body made up of number of particles (discrete distribution)

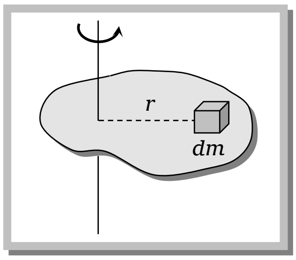
$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + .......$$

(3) Moment of inertia of a continuous distribution of mass, treating the element of mass $dm$ at position $r$ as particle

$$dI = dm\, r^2 \quad i.e., \; I = \int r^2 dm$$








(4) Dimension : $[ML^2T^0]$

(5) S.I. unit : $kgm^2$.

(6) Moment of inertia depends on mass, distribution of mass and on the position of axis of rotation.

(7) Moment of inertia does not depend on angular velocity, angular acceleration, torque, angular momentum and rotational kinetic energy.

(8) It is not a vector as direction (clockwise or anti-clockwise) is not to be specified and also not a scalar as it has different values in different directions. Actually it is a tensor quantity.

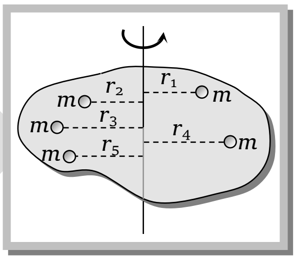
(9) In case of a hollow and solid body of same mass, radius and shape for a given axis, moment of inertia of hollow body is greater than that for the solid body because it depends upon the mass distribution.

## 7.8 Radius of Gyration

Radius of gyration of a body about a given axis is the perpendicular distance of a point from the axis, where if whole mass of the body were concentrated, the body shall have the same moment of inertia as it has with the actual distribution of mass.

When square of radius of gyration is multiplied with the mass of the body gives the moment of inertia of the body about the given axis.

$$I = Mk^2 \text{ or } k = \sqrt{\frac{I}{M}}.$$



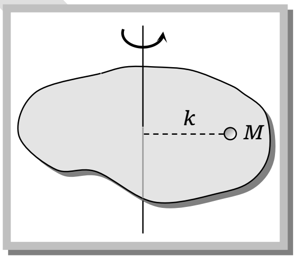

Here $k$ is called radius of gyration.

From the formula of discrete distribution

$$I = mr_1^2 + mr_2^2 + mr_3^2 + ....... + mr_n^2$$

If $m_1 = m_2 = m_3 = ....... = m$ then

$$I = m(r_1^2 + r_2^2 + r_3^2 + .........r_n^2) \qquad ........(i)$$




From the definition of Radius of gyration,

$$I = Mk^2 \qquad ........(ii)$$

By equating (i) and (ii)

$$Mk^2 = m(r_1^2 + r_2^2 + r_3^2 + ........... + r_n^2)$$

$$nmk^2 = m(r_1^2 + r_2^2 + r_3^2 + .......... + r_n^2) \qquad [\text{As } M = nm]$$

$$\therefore \qquad k = \sqrt{\frac{r_1^2 + r_2^2 + r_3^2 + ........... + r_n^2}{n}}$$

Hence radius of gyration of a body about a given axis is equal to root mean square distance of the constituent particles of the body from the given axis.

(1) Radius of gyration $(k)$ depends on shape and size of the body, position and configuration of the axis of rotation, distribution of mass of the body *w.r.t.* the axis of rotation.

(2) Radius of gyration $(k)$ does not depends on the mass of body.

(3) Dimension $[M^0 L^1 T^0]$.

(4) S.I. unit : *Meter*.

(5) Significance of radius of gyration : Through this concept a real body (particularly irregular) is replaced by a point mass for dealing its rotational motion.

*Example* : In case of a disc rotating about an axis through its centre of mass and perpendicular to its plane

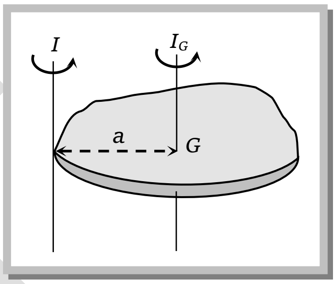
$$k = \sqrt{\frac{I}{M}} = \sqrt{\frac{(1/2)MR^2}{M}} = \frac{R}{\sqrt{2}}$$

So instead of disc we can assume a point mass $M$ at a distance $(R/\sqrt{2})$ from the axis of rotation for dealing the rotational motion of the disc.

Note : ❑ For a given body inertia is constant whereas moment of inertia is variable.

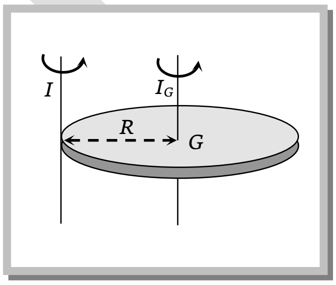
## 7.9 Theorem of Parallel Axes

Moment of inertia of a body about a given axis $I$ is equal to the sum of moment of inertia of the body about an axis parallel to given axis and passing through centre of mass of the body $I_g$ and $Ma^2$ where $M$ is the mass of the body and a is the perpendicular distance between the two axes.




$$I = I_g + Ma^2$$

*Example:* Moment of inertia of a disc about an axis through its centre and perpendicular to the plane is $\frac{1}{2}MR^2$, so moment of inertia about an axis through its tangent and perpendicular to the plane will be




$$I = I_g + Ma^2$$

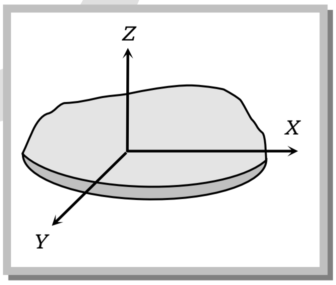
$$I = \frac{1}{2}MR^2 + MR^2$$

$$\therefore \qquad I = \frac{3}{2}MR^2$$

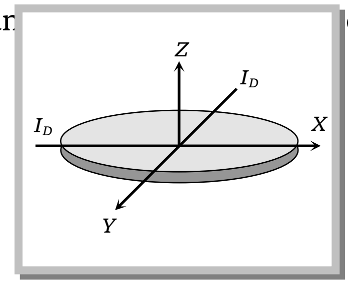
## 7.10 Theorem of Perpendicular Axes

According to this theorem the sum of moment of inertia of a plane lamina about two mutually perpendicular axes lying in its plane is equal to its moment of inertia about an axis perpendicular to the plane of lamina and passing through the point of intersection of first two axes.

$$I_z = I_x + I_y$$




*Example* : Moment of inertia of a disc about an axis through its centre of mass and perpendicular to its plane is $\frac{1}{2}MR^2$, so if the disc is in $x$–$y$ plan of perpendicular axes




*i.e.* $\qquad I_Z = I_X + I_y$

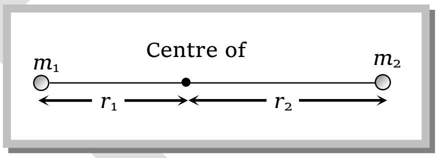
$\Rightarrow \qquad \frac{1}{2}MR^2 = 2I_D$ [As ring is symmetrical body so $I_X = I_y = I_D$]

$\Rightarrow \qquad I_D = \frac{1}{4}MR^2$

*Note* : ❑ In case of symmetrical two-dimensional bodies as moment of inertia for all axes passing through the centre of mass and in the plane of body will be same so the two axes in the plane of body need not be perpendicular to each other.

## 7.11 Moment of Inertia of Two Point Masses About Their Centre of Mass

Let $m_1$ and $m_2$ be two masses distant $r$ from each-other and $r_1$ and $r_2$ be the distances of their centre of mass from $m_1$ and $m_2$ respectively, then




(1) $r_1 + r_2 = r$

(2) $m_1 r_1 = m_2 r_2$

(3) $r_1 = \frac{m_2}{m_1 + m_2} r$ and $r_2 = \frac{m_1}{m_1 + m_2} r$

(4) $I = m_1 r_1^2 + m_2 r_2^2$

(5) $I = \left[\frac{m_1 m_2}{m_1 + m_2}\right] r^2 = \mu r^2$ [where $\mu = \frac{m_1 m_2}{m_1 + m_2}$ is known as reduced mass $\mu < m_1$ and $\mu < m_2$.]

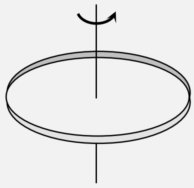
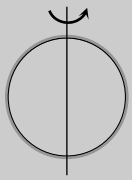
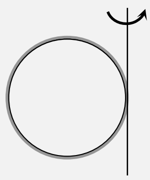
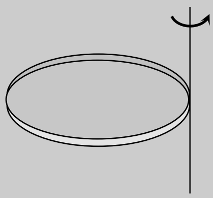
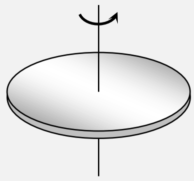
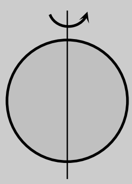
(6) In diatomic molecules like $H_2, HCl$ *etc*. moment of inertia about their centre of mass is derived from above formula.

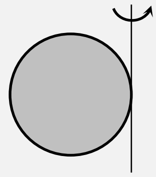
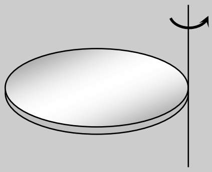
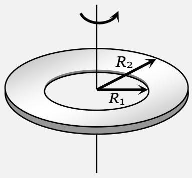
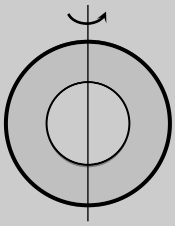
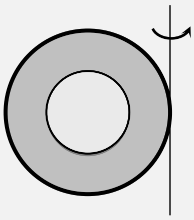
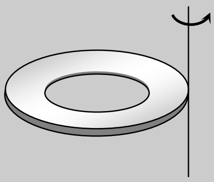
## 7.12 Analogy Between Tranlatory Motion and Rotational Motion

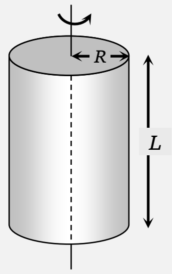
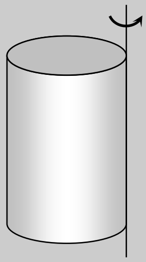
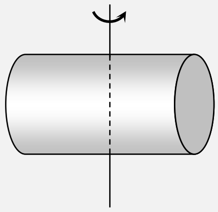
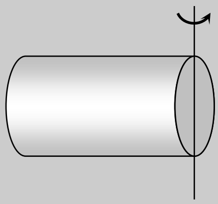
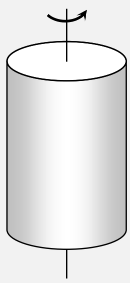
| Translatory motion | | Rotatory motion | |
|---|---|---|---|
| Mass | $(m)$ | Moment of Inertia | $(I)$ |
| Linear momentum | $P = mv$<br>$P = \sqrt{2mE}$ | Angular Momentum | $L = I\omega$<br>$L = \sqrt{2IE}$ |
| Force | $F = ma$ | Torque | $\tau = I\alpha$ |
| Kinetic energy | $E = \frac{1}{2} mv^2$<br>$E = \frac{P^2}{2m}$ | | $E = \frac{1}{2} I\omega^2$<br>$E = \frac{L^2}{2I}$ |

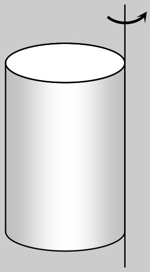
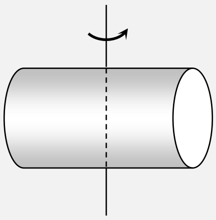
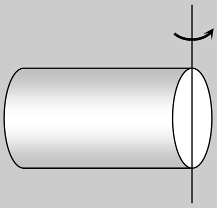
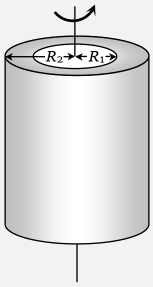
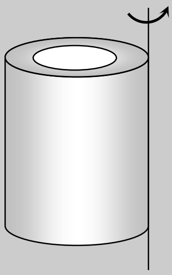
## 7.13 Moment of Inertia of Some Standard Bodies About Different Axes

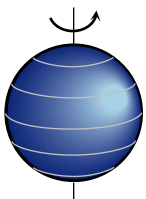
| Body | Axis of Rotation | Figure | Moment of inertia | $k$ | $k^2/R^2$ |
|---|---|---|---|---|---|

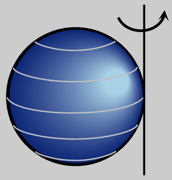
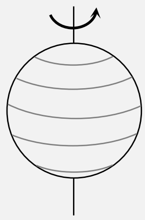
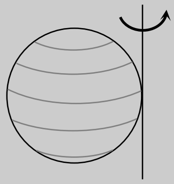
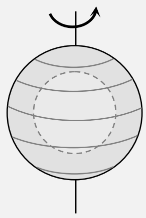
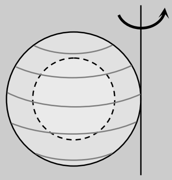
| Body | Axis of Rotation | Figure | Moment of inertia | $k$ | $k^2/R^2$ |
|---|---|---|---|---|---|
| Ring | About an axis passing through C.G. and perpendicular to its plane |  | $MR^2$ | $R$ | 1 |
| Ring | About its diameter |  | $\frac{1}{2}MR^2$ | $\frac{R}{\sqrt{2}}$ | $\frac{1}{2}$ |
| Ring | About a tangential axis in its own plane |  | $\frac{3}{2}MR^2$ | $\sqrt{\frac{3}{2}}R$ | $\frac{3}{2}$ |
| Ring | About a tangential axis perpendicular to its own plane |  | $2MR^2$ | $\sqrt{2}R$ | 2 |
| Disc | About an axis passing through C.G. and perpendicular to its plane |  | $\frac{1}{2}MR^2$ | $\frac{R}{\sqrt{2}}$ | $\frac{1}{2}$ |
| Disc | About its Diameter |  | $\frac{1}{4}MR^2$ | $\frac{R}{2}$ | $\frac{1}{4}$ |

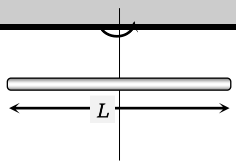
| Body | Axis of Rotation | Figure | Moment of inertia | $k$ | $k^2/R^2$ |
|---|---|---|---|---|---|
| Disc | About a tangential axis in its own plane |  | $\frac{5}{4}MR^2$ | $\frac{\sqrt{5}}{2}R$ | $\frac{5}{4}$ |
| Disc | About a tangential axis perpendicular to its own plane |  | $\frac{3}{2}MR^2$ | $\sqrt{\frac{3}{2}}R$ | $\frac{3}{2}$ |
| Annular disc inner radius = $R_1$ and outer radius = $R_2$ | Passing through the centre and perpendicular to the plane |  | $\frac{M}{2}[R_1^2+R_2^2]$ | – | – |
| Annular disc | Diameter |  | $\frac{M}{4}[R_1^2+R_2^2]$ | – | – |
| Annular disc | Tangential and Parallel to the diameter |  | $\frac{M}{4}[5R_1^2+R_2^2]$ | – | – |
| Annular disc | Tangential and perpendicular to the plane |  | $\frac{M}{2}[3R_1^2+R_2^2]$ | – | – |

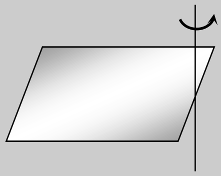
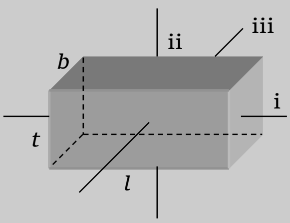
| Body | Axis of Rotation | Figure | Moment of inertia | $k$ | $k^2/R^2$ |
|---|---|---|---|---|---|
| Solid cylinder | About its own axis |  | $\frac{1}{2}MR^2$ | $\frac{R}{\sqrt{2}}$ | $\frac{1}{2}$ |
| Solid cylinder | Tangential (Generator) |  | $\frac{3}{2}MR^2$ | $\sqrt{\frac{3}{2}}R$ | $\frac{3}{2}$ |
| Solid cylinder | About an axis passing through its C.G. and perpendicular to its own axis |  | $M\left[\frac{L^2}{12}+\frac{R^2}{4}\right]$ | $\sqrt{\frac{L^2}{12}+\frac{R^2}{4}}$ | |
| Solid cylinder | About the diameter of one of faces of the cylinder |  | $M\left[\frac{L^2}{3}+\frac{R^2}{4}\right]$ | $\sqrt{\frac{L^2}{3}+\frac{R^2}{4}}$ | |
| Cylindrical shell | About its own axis |  | $MR^2$ | $R$ | 1 |

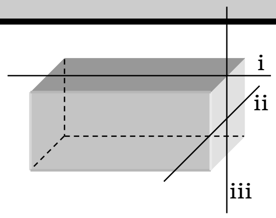
| Body | Axis of Rotation | Figure | Moment of inertia | $k$ | $k^2/R^2$ |
|---|---|---|---|---|---|
| Cylindrical shell | Tangential (Generator) |  | $2MR^2$ | $\sqrt{2}R$ | 2 |
| Cylindrical shell | About an axis passing through its C.G. and perpendicular to its own axis |  | $M\left[\frac{L^2}{12}+\frac{R^2}{2}\right]$ | $\sqrt{\frac{L^2}{12}+\frac{R^2}{2}}$ | |
| Cylindrical shell | About the diameter of one of faces of the cylinder |  | $M\left[\frac{L^2}{3}+\frac{R^2}{2}\right]$ | $\sqrt{\frac{L^2}{3}+\frac{R^2}{2}}$ | |
| Hollow cylinder with inner radius = $R_1$ and outer radius = $R_2$ | Axis of cylinder |   | $\frac{M}{2}(R_1^2+R_2^2)$ | | |
| Hollow cylinder with inner radius = $R_1$ and outer radius = $R_2$ | Tangential |  | $\frac{M}{2}(R_1^2+3R_2^2)$ | | |



| Body | Axis of Rotation | Figure | Moment of inertia | $k$ | $k^2/R^2$ |
|---|---|---|---|---|---|
| Solid Sphere | About its diametric axis | | $\frac{2}{5}MR^2$ | $\sqrt{\frac{2}{5}}R$ | $\frac{2}{5}$ |
| Solid sphere | About a tangential axis |  | $\frac{7}{5}MR^2$ | $\sqrt{\frac{7}{5}}R$ | $\frac{7}{5}$ |
| Spherical shell | About its diametric axis |  | $\frac{2}{3}MR^2$ | $\sqrt{\frac{2}{3}}R$ | $\frac{2}{3}$ |
| Spherical shell | About a tangential axis |  | $\frac{5}{3}MR^2$ | $\sqrt{\frac{5}{3}}R$ | $\frac{5}{3}$ |
| Hollow sphere of inner radius $R_1$ and outer radius $R_2$ | About its diametric axis |  | $\frac{2}{5}M\left[\frac{R_2^5 - R_1^5}{R_2^3 - R_1^3}\right]$ | | |
| Hollow sphere | Tangential |  | $\frac{2M[R_2^5 - R_1^5]}{5(R_2^3 - R_1^3)} + MR_2^2$ | | |




| Body | Axis of Rotation | Figure | Moment of inertia | $k$ | $k^2/R^2$ |
|---|---|---|---|---|---|
| Long thin rod | About on axis passing through its centre of mass and perpendicular to the rod. | | $\frac{ML^2}{12}$ | $\frac{L}{\sqrt{12}}$ | |
| Long thin rod | About an axis passing through its edge and perpendicular to the rod | | $\frac{ML^2}{3}$ | $\frac{L}{\sqrt{3}}$ | |
| Rectangular lamina of length $l$ and breadth $b$ | Passing through the centre of mass and perpendicular to the plane | | $\frac{M}{12}[l^2 + b^2]$ | | |
| Rectangular lamina | Tangential perpendicular to the plane and at the mid-point of breadth |  | $\frac{M}{12}[4l^2 + b^2]$ | | |
| Rectangular lamina | Tangential perpendicular to the plane and at the mid-point of length | | $\frac{M}{12}[l^2 + 4b^2]$ | | |
| Rectangular parallelopiped length $l$, breadth $b$, thickness $t$ | Passing through centre of mass and parallel to (i) Length (x) (ii) breadth (z) (iii) thickness (y) |  | (i) $\frac{M[b^2 + t^2]}{12}$ (ii) $\frac{M[l^2 + t^2]}{12}$ (iii) $\frac{M[b^2 + l^2]}{12}$ | | |



| **Body** | **Axis of Rotation** | **Figure** | **Moment of inertia** | $k$ | $k^2/R^2$ |
|---|---|---|---|---|---|
| Rectangular parallelepiped length $l$, breath $b$, thickness $t$ | Tangential and parallel to (i) length (x) (ii) breadth (y) (iii) thickness(z) | | (i) $\frac{M}{12}[3l^2+b^2+t^2]$ (ii) $\frac{M}{12}[l^2+3b^2+t^2]$ (iii) $\frac{M}{12}[l^2+b^2+3t^2]$ | | |
| Elliptical disc of semimajor axis = $a$ and semiminor axis = $b$ | Passing through CM and perpendicular to the plane | | $\frac{M}{4}[a^2+b^2]$ | | |
| Solid cone of radius R and height h | Axis joining the vertex and centre of the base | | $\frac{3}{10}MR^2$ | | |
| Equilateral triangular lamina with side a | Passing through CM and perpendicular to the plane | | $\frac{Ma^2}{6}$ | | |
| Right angled triangular lamina of sides $a$, $b$, $c$ | Along the edges | | (1) $\frac{Mb^2}{6}$ (2) $\frac{Ma^2}{6}$ (3) $\frac{M}{6}\left[\frac{a^2b^2}{a^2+b^2}\right]$ | | |

***Sample problem based on moment of inertia***

***Problem* 14.** Five particles of mass = 2 *kg* are attached to the rim of a circular disc of radius 0.1 *m* and negligible mass. Moment of inertia of the system about the axis passing through the centre of the disc and perpendicular to its plane is **[BHU 2003]**

(a) 1 $kg\ m^2$ (b) 0.1 $kg\ m^2$ (c) 2 $kg\ m^2$ (d) 0.2 $kg\ m^2$

*Solution:* (b) We will not consider the moment of inertia of disc because it doesn't have any mass so moment of inertia of five particle system $I = 5\,mr^2 = 5\times 2\times (0.1)^2 = 0.1\,kg\text{-}m^2$.

***Problem* 15.** A circular disc $X$ of radius $R$ is made from an iron plate of thickness $t$, and another disc $Y$ of radius $4R$ is made from an iron plate of thickness $\frac{t}{4}$. Then the relation between the moment of inertia $I_X$ and $I_Y$ is **[AIEEE 2003]**

(a) $I_Y = 64 I_X$ (b) $I_Y = 32 I_X$ (c) $I_Y = 16 I_X$ (d) $I_Y = I_X$

*Solution*: (a) Moment of Inertia of disc $I = \frac{1}{2}MR^2 = \frac{1}{2}(\pi R^2 t\rho)R^2 = \frac{1}{2}\pi t\rho R^4$

[As $M = V\times\rho = \pi R^2 t\rho$ where $t =$ thickness, $\rho$ = density]

$\therefore \quad \frac{I_y}{I_x} = \frac{t_y}{t_x}\left(\frac{R_y}{R_x}\right)^4$ [If $\rho$ = constant]

$\Rightarrow \quad \frac{I_y}{I_x} = \frac{1}{4}(4)^4 = 64$ [Given $R_y = 4R_x$, $t_y = \frac{t_x}{4}$ ]

$\Rightarrow \quad I_y = 64 I_x$

***Problem* 16.** Moment of inertia of a uniform circular disc about a diameter is $I$. Its moment of inertia about an axis perpendicular to its plane and passing through a point on its rim will be **[UPSEAT 2002]**

(a) 5 $I$ (b) 6 $I$ (c) 3 $I$ (d) 4 $I$

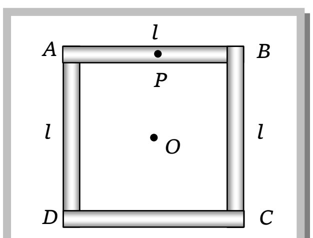
*Solution:* (b) Moment of inertia of disc about a diameter = $\frac{1}{4}MR^2 = I$ (given) $\therefore\ MR^2 = 4I$

Now moment of inertia of disc about an axis perpendicular to its plane and passing through a point on its rim

$= \frac{3}{2}MR^2 = \frac{3}{2}(4I) = 6I$.

***Problem* 17.** Four thin rods of same mass $M$ and same length $l$, form a square as shown in figure. Moment of inertia of this system about an axis through centre $O$ and perpendicular to its plane is **[MP PMT 2002]**

(a) $\frac{4}{3}Ml^2$



(b) $\frac{Ml^2}{3}$

(c) $\frac{Ml^2}{6}$

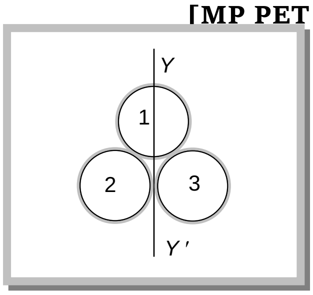
(d) $\frac{2}{3}Ml^2$

*Solution:* (a) Moment of inertia of rod $AB$ about point $P = \frac{1}{12}Ml^2$

M.I. of rod $AB$ about point $O = \frac{Ml^2}{12} + M\left(\frac{l}{2}\right)^2 = \frac{1}{3}Ml^2$ [by the theorem of parallel axis]

and the system consists of 4 rods of similar type so by the symmetry $I_{System} = \frac{4}{3}Ml^2$.

***Problem*** **18.** Three rings each of mass $M$ and radius $R$ are arranged as shown in the figure. The moment of inertia of the system about $YY'$ will be **[MP PET 2000]**




(a) $3\ MR^2$

(b) $\frac{3}{2}MR^2$

(c) $5\ MR^2$

(d) $\frac{7}{2}MR^2$

Solution: (d) M.I of system about $YY'$ $I = I_1 + I_2 + I_3$

where $I_1$ = moment of inertia of ring about diameter, $I_2 = I_3$ = M.I. of inertia of ring about a tangent in a plane

$\therefore$ $I = \frac{1}{2}mR^2 + \frac{3}{2}mR^2 + \frac{3}{2}mR^2 = \frac{7}{2}mR^2$

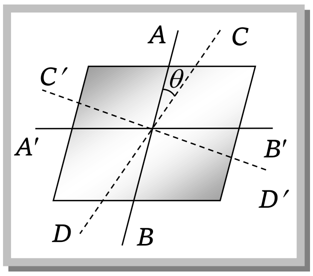
***Problem*** **19.** Let $l$ be the moment of inertia of an uniform square plate about an axis $AB$ that passes through its centre and is parallel to two of its sides. $CD$ is a line in the plane of the plate that passes through the centre of the plate and makes an angle $\theta$ with $AB$. The moment of inertia of the plate about the axis $CD$ is then equal to

**[IIT-JEE 1998]**

(a) $l$ (b) $l\sin^2\theta$ (c) $l\cos^2\theta$ (d) $l\cos^2\frac{\theta}{2}$

*Solution:* (a) Let $I_Z$ is the moment of inertia of square plate about the axis which is passing through the centre and perpendicular to the plane.

$I_Z = I_{AB} + I_{A'B'} = I_{CD} + I_{C'D'}$ [By the theorem of perpendicular axis]



$I_Z = 2I_{AB} = 2I_{A'B'} = 2I_{CD} = 2I_{C'D'}$

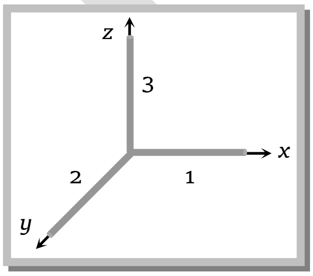
[As $AB$, $A'\ B'$ and $CD$, $C'\ D'$ are symmetric axis]

Hence $I_{CD} = I_{AB} = l$

***Problem* 20.** Three rods each of length $L$ and mass $M$ are placed along $X$, $Y$ and $Z$-axes in such a way that one end of each of the rod is at the origin. The moment of inertia of this system about $Z$ axis is **[AMU 1995]**

(a) $\frac{2ML^2}{3}$ (b) $\frac{4ML^2}{3}$ (c) $\frac{5ML^2}{3}$ (d) $\frac{ML^2}{3}$

*Solution:* (a) Moment of inertia of the system about $z$-axis can be find out by calculating the moment of inertia of individual rod about $z$-axis




$I_1 = I_2 = \frac{ML^2}{3}$ because $z$-axis is the edge of rod 1 and 2

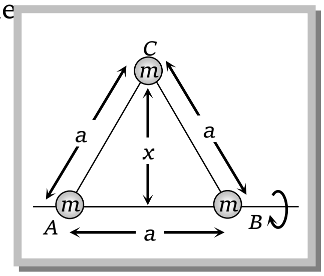
and $I_3 = 0$ because rod in lying on $z$-axis

$\therefore\ I_{\text{system}} = I_1 + I_2 + I_3 = \frac{ML^2}{3} + \frac{ML^2}{3} + 0 = \frac{2ML^2}{3}$.

***Problem* 21.** Three point masses each of mass $m$ are placed at the corners of an equilateral triangle of side $a$. Then the moment of inertia of this system about an axis passing along one side of the triangle is **[AIIMS 1995]**

(a) $ma^2$ (b) $3ma^2$ (c) $\frac{3}{4}ma^2$ (d) $\frac{2}{3}ma^2$

*Solution*: (c) The moment of inertia of system about $AB$ side of triangle

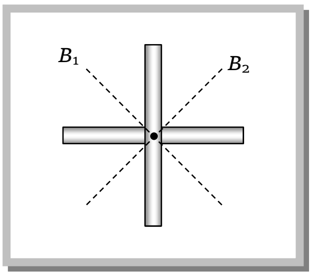



$I = I_A + I_B + I_C$

$= 0 + 0 + mx^2$

$= m\left(\frac{a\sqrt{3}}{2}\right)^2 = \frac{3}{4}ma^2$

***Problem* 22.** Two identical rods each of mass $M$. and length $l$ are joined in crossed position as shown in figure. The moment of inertia of this system about a bisector would be




(a) $\frac{Ml^2}{6}$

(b) $\frac{Ml^2}{12}$

(c) $\frac{Ml^2}{3}$

(d) $\frac{Ml^2}{4}$

*Solution:* (b) Moment of inertia of system about an axes which is perpendicular to plane of rods and passing through the common centre of rods $I_z = \frac{Ml^2}{12} + \frac{Ml^2}{12} = \frac{Ml^2}{6}$

Again from perpendicular axes theorem $I_z = I_{B_1} + I_{B_2} = 2I_{B_1} = 2I_{B_2} = \frac{Ml^2}{6}$ [As $I_{B_1} = I_{B_2}$]

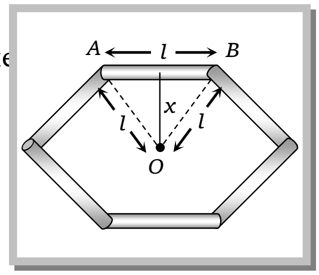
$\therefore \quad I_{B_1} = I_{B_2} = \frac{Ml^2}{12}$.

***Problem* 23.** The moment of inertia of a rod of length $l$ about an axis passing through its centre of mass and perpendicular to rod is $I$. The moment of inertia of hexagonal shape formed by six such rods, about an axis passing through its centre of mass and perpendicular to its plane will be

(a) $16I$ (b) $40\,I$ (c) $60\,I$ (d) $80\,I$

*Solution:* (c) Moment of inertia of rod $AB$ about its centre and perpendicular to the length = $\frac{ml^2}{12} = I \quad \therefore$ $ml^2 = 12I$

Now moment of inertia of the rod about the axis which is passing through $O$ and perpendicular to the plane of hexagon $I_{rod} = \frac{ml^2}{12} + mx^2$ [From the theorem of parallel axe




$$= \frac{ml^2}{12} + m\left(\frac{\sqrt{3}}{2}l\right)^2 = \frac{5ml^2}{6}$$

Now the moment of inertia of system $I_{system} = 6 \times I_{rod} = 6 \times \frac{5ml^2}{6} = 5ml^2$

$I_{system} = 5\,(12\,I) = 60\,I$ [As $ml^2 = 12I$]

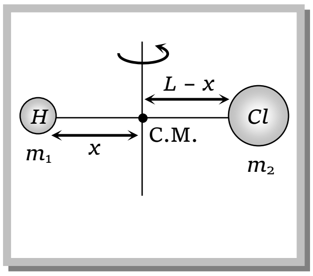
***Problem* 24.** The moment of inertia of $HCl$ molecule about an axis passing through its centre of mass and perpendicular to the line joining the $H^+$ and $Cl^-$ ions will be, if the interatomic distance is 1 Å

(a) $0.61 \times 10^{-47}\, kg.m^2$ (b) $1.61 \times 10^{-47}\, kg.m^2$ (c) $0.061 \times 10^{-47}\, kg.m^2$ (d) 0

*Solution:* (b) If $r_1$ and $r_2$ are the respective distances of particles $m_1$ and $m_2$ from the centre of mass then

$m_1 r_1 = m_2 r_2 \Rightarrow 1 \times x = 35.5 \times (L - x) \Rightarrow x = 35.5(1 - x)$

$\Rightarrow x = 0.973\,Å$ and $L - x = 0.027\,Å$



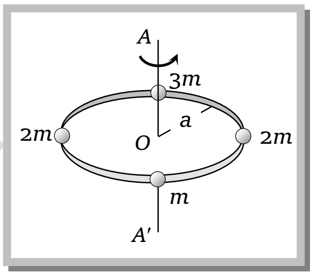

Moment of inertia of the system about centre of mass $I = m_1 x^2 + m_2 (L - x)^2$

$I = 1 amu \times (0.973\,Å)^2 + 35.5\, amu \times (0.027\,Å)^2$

Substituting 1 $a.m.u.$ = $1.67 \times 10^{-27}$ $kg$ and 1 Å = $10^{-10}$ $m$

$I = 1.62 \times 10^{-47}\, kg\, m^2$

***Problem*** **25.** Four masses are joined to a light circular frame as shown in the figure. The radius of gyration of this system about an axis passing through the centre of the circular frame and perpendicular to its plane would be




(a) $a/\sqrt{2}$

(b) $a/2$

(c) $a$

(d) $2a$

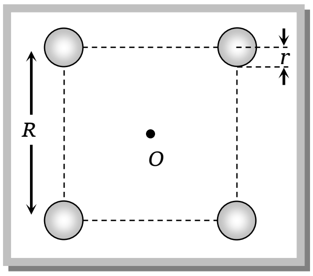
*Solution:* (c) Since the circular frame is massless so we will consider moment of inertia of four masses only.

$I = ma^2 + 2ma^2 + 3ma^2 + 2ma^2 = 8ma^2$ .....(i)

Now from the definition of radius of gyration $I = 8mk^2$ .....(ii)

comparing (i) and (ii) radius of gyration $k = a$.

***Problem*** **26.** Four spheres, each of mass $M$ and radius $r$ are situated at the four corners of square of side $R$. The moment of inertia of the system about an axis perpendicular to the plane of square and passing through its centre will be




(a) $\frac{5}{2}M(4r^2 + 5R^2)$

(b) $\frac{2}{5}M(4r^2 + 5R^2)$

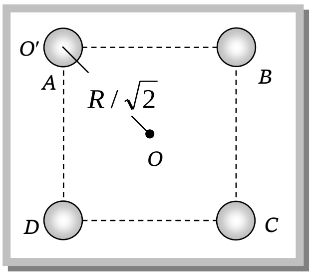
(c) $\frac{2}{5}M(4r^2 + 5r^2)$

(d) $\frac{5}{2}M(4r^2 + 5r^2)$

*Solution:* (b) *M. I.* of sphere *A* about its diameter $I_{O'} = \frac{2}{5}Mr^2$

Now *M.I.* of sphere *A* about an axis perpendicular to the plane of square and passing through its centre will be

$I_O = I_{O'} + M\left(\frac{R}{\sqrt{2}}\right)^2 = \frac{2}{5}Mr^2 + \frac{MR^2}{2}$ [by the theorem of parallel axis]




Moment of inertia of system (i.e. four sphere)= $4I_O = 4\left[\frac{2}{5}Mr^2 + \frac{MR^2}{2}\right]$

$= \frac{2}{5}M\left[4r^2 + 5R^2\right]$

***Problem*** **27.** The moment of inertia of a solid sphere of density $\rho$ and radius *R* about its diameter is

(a) $\frac{105}{176}R^5\rho$ (b) $\frac{105}{176}R^2\rho$ (c) $\frac{176}{105}R^5\rho$ (d) $\frac{176}{105}R^2\rho$

*Solution:* (c) Moment of inertia of sphere about it diameter $I = \frac{2}{5}MR^2 = \frac{2}{5}\left(\frac{4}{3}\pi R^3\rho\right)R^2$ [As $M = V\rho = \frac{4}{3}\pi R^3\rho$]

$I = \frac{8\pi}{15}R^5\rho = \frac{8\times 22}{15\times 7}R^5\rho = \frac{176}{105}R^5\rho$

***Problem* 28.** Two circular discs $A$ and $B$ are of equal masses and thickness but made of metals with densities $d_A$ and $d_B$ $(d_A > d_B)$. If their moments of inertia about an axis passing through centres and normal to the circular faces be $I_A$ and $I_B$, then

(a) $I_A = I_B$ (b) $I_A > I_B$ (c) $I_A < I_B$ (d) $I_A >=< I_B$

*Solution* : (c) Moment of inertia of circular disc about an axis passing through centre and normal to the circular face

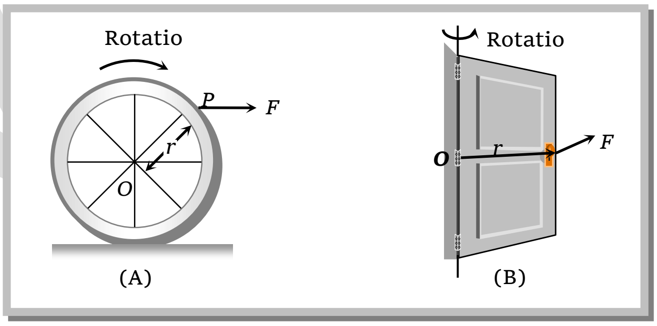
$I = \frac{1}{2}MR^2 = \frac{1}{2}M\left(\frac{M}{\pi t\rho}\right)$ [As $M = V\rho = \pi R^2 t\rho$ $\therefore$ $R^2 = \frac{M}{\pi t\rho}$ ]

$\Rightarrow$ $I = \frac{M^2}{2\pi t\rho}$ or $I \propto \frac{1}{\rho}$ If mass and thickness are constant.

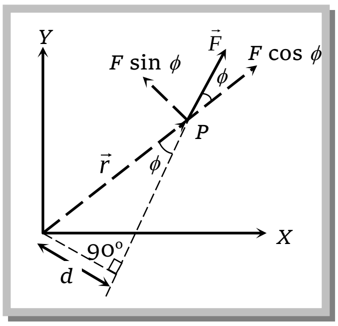
So, in the problem $\frac{I_A}{I_B} = \frac{d_B}{d_A}$ $\therefore I_A < I_B$ [As $d_A > d_B$ ]

### 7.14 Torque.

If a pivoted, hinged or suspended body tends to rotate under the action of a force, it is said to be acted upon by a torque. or The turning effect of a force about the axis of rotation is called moment of force or torque due to the force.




If the particle rotating in $xy$ plane about the origin under the effect of force $\vec{F}$ and at any instant the position vector of the particle is $\vec{r}$ then,



Torque $\vec{\tau} = \vec{r} \times \vec{F}$

$\tau = r F \sin\phi$

[where $\phi$ is the angle between the direction of $\vec{r}$ and $\vec{F}$ ]

(1) Torque is an axial vector. *i.e.,* its direction is always perpendicular to the plane containing vector $\vec{r}$ and $\vec{F}$ in accordance with right hand screw rule. For a given figure the sense of rotation is anti-clockwise so the direction of torque is perpendicular to the plane, outward through the axis of rotation.

(2) Rectangular components of force

$\vec{F}_r = F\cos\phi$ = radial component of force , $\vec{F}_\phi = F\sin\phi$ = transverse component of force

As $\tau = r F \sin\phi$

or $\tau = r F_\phi$ = (position vector) × (transverse component of force)

Thus the magnitude of torque is given by the product of transverse component of force and its perpendicular distance from the axis of rotation *i.e.*, Torque is due to transverse component of force only.

(3) As $\tau = r F \sin\phi$

or $\therefore \tau = F(r\sin\phi) = Fd$ [As $d = r\sin\phi$ from the figure ]

*i.e.* Torque = Force × Perpendicular distance of line of action of force from the axis of rotation.

Torque is also called as moment of force and *d* is called moment or lever arm.

(4) Maximum and minimum torque : As $\vec{\tau} = \vec{r} \times \vec{F}$ or $\tau = r F \sin\phi$

| | | |
|---|---|---|
| $\tau_{maximum} = rF$ | When $\lvert\sin\phi\rvert = \max = 1$ *i.e.,* $\phi = 90°$ | $\vec{F}$ is perpendicular to $\vec{r}$ |
| $\tau_{minimum} = 0$ | When $\lvert\sin\phi\rvert = \min = 0$ *i.e.* $\phi = 0°$ or $180°$ | $\vec{F}$ is collinear to $\vec{r}$ |

(5) For a given force and angle, magnitude of torque depends on *r*. The more is the value of *r*, the more will be the torque and easier to rotate the body.

*Example* : (i) Handles are provided near the free edge of the Planck of the door.

(ii) The handle of screw driver is taken thick.

(iii) In villages handle of flourmill is placed near the circumference.

(iv) The handle of hand-pump is kept long.

(v) The arm of wrench used for opening the tap, is kept long.

(6) Unit : *Newton-metre* (M.K.S.) and *Dyne-cm* (C.G.S.)

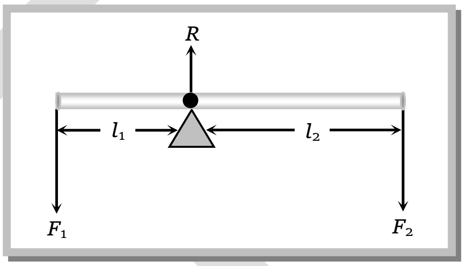
(7) Dimension : $[ML^2T^{-2}]$.

(8) If a body is acted upon by more than one force, the total torque is the vector sum of each torque.

$$\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 + ........$$

(9) A body is said to be in rotational equilibrium if resultant torque acting on it is zero *i.e.* $\Sigma \vec{\tau} = 0$.

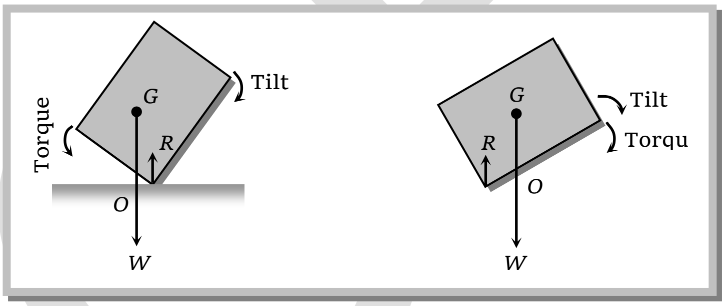
(10) In case of beam balance or see-saw the system will be in rotational equilibrium if,




$\vec{\tau}_1 + \vec{\tau}_2 = 0$ or $F_1l_1 - F_2l_2 = 0$ $\therefore$ $F_1l_1 = F_2l_2$

However if, $\vec{\tau}_1 > \vec{\tau}_2$, L.H.S. will move downwards and if $\vec{\tau}_1 < \vec{\tau}_2$. R.H.S. will move downward. and the system will not be in rotational equilibrium.

(11) On tilting, a body will restore its initial position due to torque of weight about the point *O* till the line of action of weight passes through its base on tilting, a body will topple due to torque of weight about *O*, if the line of action of weight does not pass through the base.



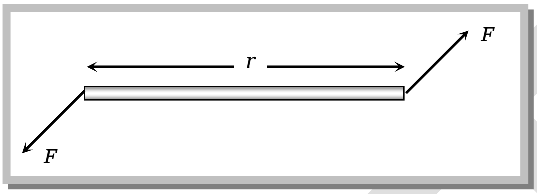

(12) Torque is the cause of rotatory motion and in rotational motion it plays same role as force plays in translatory motion *i.e.,* torque is rotational analogue of force. This all is evident from the following correspondences between rotatory and translatory motion.

| **Rotatory Motion** | **Translatory Motion** |
|---|---|
| $\vec{\tau} = I\vec{\alpha}$ | $\vec{F} = m\vec{a}$ |
| $W = \int \vec{\tau} \cdot d\vec{\theta}$ | $W = \int \vec{F} \cdot d\vec{s}$ |
| $P = \vec{\tau} \cdot \vec{\omega}$ | $P = \vec{F} \cdot \vec{v}$ |
| $\vec{\tau} = \frac{d\vec{L}}{dt}$ | $\vec{F} = \frac{d\vec{P}}{dt}$ |

**7.15 Couple .**

A special combination of forces even when the entire body is free to move can rotate it. This combination of forces is called a couple.

(1) A couple is defined as combination of two equal but oppositely directed force not acting along the same line. The effect of couple is known by its moment of couple or torque by a couple $\vec{\tau} = \vec{r} \times \vec{F}$.

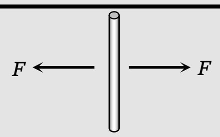
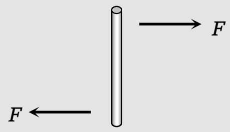
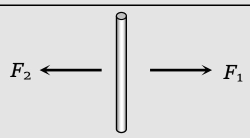
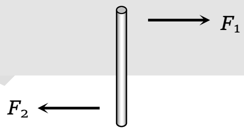



(2) Generally both couple and torque carry equal meaning. The basic difference between torque and couple is the fact that in case of couple both the forces are externally applied while in case of torque one force is externally applied and the other is reactionary.

(3) Work done by torque in twisting the wire $W = \frac{1}{2} C\theta^2$.

Where $\tau = C\theta$; $C$ is known as twisting coefficient or couple per unit twist.

### 7.16 Translatory and Rotatory Equilibrium .

| | | | |
|---|---|---|---|
| Forces are equal and act along the same line. |   | $\sum F = 0$ and $\sum \tau = 0$ | Body will remain stationary if initially it was at rest. |
| Forces are equal and does not act along the same line. |   | $\sum F = 0$ and $\sum \tau \neq 0$ | Rotation *i.e.* spinning. |
| Forces are unequal and act along the same line. |   | $\sum F \neq 0$ and $\sum \tau = 0$ | Translation *i.e.* slipping or skidding. |
| Forces are unequal and does not act along the same line. |   | $\sum F \neq 0$ and $\sum \tau \neq 0$ | Rotation and translation both *i.e.* rolling. |

***Sample problems based on torque and couple***

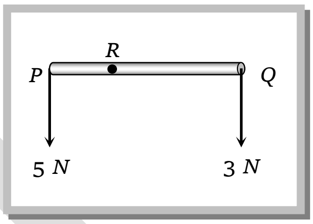
<u>***Problem***</u> **29.** A force of $(2\hat{i} - 4\hat{j} + 2\hat{k})N$ acts at a point $(3\hat{i} + 2\hat{j} - 4\hat{k})$ *metre* from the origin. The magnitude of torque is

(a) Zero (b) 24.4 *N-m* (c) 0.244 *N-m* (d) 2.444 *N-m*

*Solution*: (b) $\vec{F} = (2\hat{i} - 4\hat{j} + 2\hat{k})N$ and $\vec{r} = (3i + 2 - 4\hat{k})$ *meter*

Torque $\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & -4 \\ 2 & -4 & 2 \end{vmatrix}$ $\Rightarrow \vec{\tau} = -12\hat{i} - 14\hat{j} - 16\hat{k}$ and $|\vec{\tau}| = \sqrt{(-12)^2 + (-14)^2 + (-16)^2}$ = 24.4 *N-m*

***Problem* 30.** The resultant of the system in the figure is a force of $8N$ parallel to the given force through $R$. The value of $PR$ equals to




(a) 1/4 $RQ$

(b) 3/8 $RQ$

(c) 3/5 $RQ$

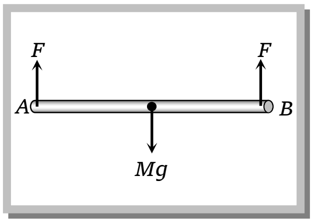
(d) 2/5 $RQ$

*Solution*: (c) By taking moment of forces about point *R*, $5 \times PR - 3 \times RQ = 0 \Rightarrow PR = \frac{3}{5} RQ$.

***Problem* 31.** A horizontal heavy uniform bar of weight *W* is supported at its ends by two men. At the instant, one of the men lets go off his end of the rod, the other feels the force on his hand changed to

(a) $W$ (b) $\frac{W}{2}$ (c) $\frac{3W}{4}$ (d) $\frac{W}{4}$

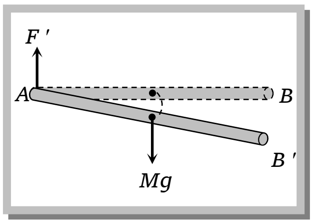
*Solution*: (d) Let the mass of the rod is *M* ∴ Weight (*W*) = *Mg*




Initially for the equilibrium $F + F = Mg$ $\Rightarrow F = Mg/2$

When one man withdraws, the torque on the rod

$\tau = I\alpha = Mg\frac{l}{2}$




$\Rightarrow \frac{Ml^2}{3}\alpha = Mg\frac{l}{2}$ [As $I = Ml^2/3$]

⇒ Angular acceleration $\alpha = \frac{3}{2}\frac{g}{l}$

and linear acceleration $a = \frac{l}{2}\alpha = \frac{3g}{4}$

Now if the new normal force at *A* is $F'$ then $Mg - F' = Ma$

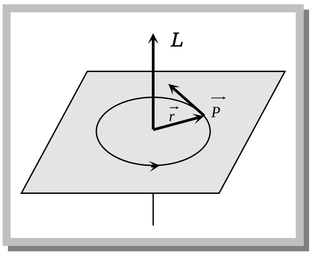
$\Rightarrow F' = Mg - Ma = Mg - \frac{3Mg}{4} = \frac{Mg}{4} = \frac{W}{4}$.

## 7.17 Angular Momentum .

The turning momentum of particle about the axis of rotation is called the angular momentum of the particle.

or

The moment of linear momentum of a body with respect to any axis of rotation is known as angular momentum. If $\vec{P}$ is the linear momentum of particle and $\vec{r}$ its position vector from the point of rotation then angular momentum.




$$\vec{L} = \vec{r} \times \vec{P}$$

$$\vec{L} = r\,P \sin\phi\,\hat{n}$$

Angular momentum is an axial vector *i.e.* always directed perpendicular to the plane of rotation and along the axis of rotation.

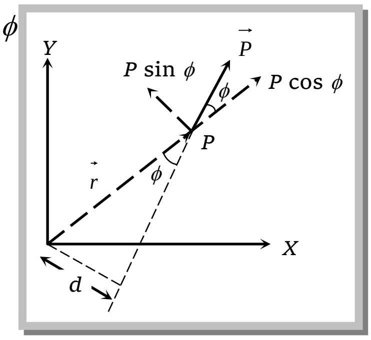
(1) S.I. Unit : $kg\text{-}m^2\text{-}s^{-1}$ or *J-sec*.

(2) Dimension : $[ML^2T^{-1}]$ and it is similar to Planck's constant $(h)$.

(3) In cartesian co-ordinates if $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\vec{P} = P_x\hat{i} + P_y\hat{j} + P_z\hat{k}$

Then $$\vec{L} = \vec{r} \times \vec{P} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ P_x & P_y & P_z \end{vmatrix} = (yP_z - zP_y)\hat{i} - (xP_z - zP_x)\hat{j} + (xP_y - yP_x)\hat{k}$$

(4) As it is clear from the figure radial component of momentum $\vec{P}_r = P\cos\phi$




Transverse component of momentum $\vec{P}_\phi = P\sin\phi$

So magnitude of angular momentum $L = r\,P\sin\phi$

$$L = r\,P_\phi$$

Angular momentum = Position vector × Transverse component of angular momentum

*i.e.*, The radial component of linear momentum has no role to play in angular momentum.

(5) Magnitude of angular momentum $L = P\ (r\sin\phi) = L = Pd$ [As $d = r\sin\phi$ from the figure.]

∴ Angular momentum = (Linear momentum) ×(Perpendicular distance of line of action of force from the axis of rotation)

(6) Maximum and minimum angular momentum : We know $\vec{L} = \vec{r} \times \vec{P}$

∴ $$\vec{L} = m[\vec{r} \times \vec{v}] = m\,v\,r\sin\phi = P\,r\sin\phi$$ [As $\vec{P} = m\,\vec{v}$]

| | | |
|---|---|---|
| $L_{maximum} = mvr$ | When $\lvert\sin\phi\rvert = \max = 1$ *i.e.,* $\phi = 90°$ | $\vec{v}$ is perpendicular to $\vec{r}$ |

| | | |
|---|---|---|
| $L_{\text{minimum}} = 0$ | When $\lvert\sin\phi\rvert = \min = 0$ *i.e.* $\phi = 0°$ or $180°$ | $\vec{v}$ is parallel or anti-parallel to $\vec{r}$ |

(7) A particle in translatory motion always have an angular momentum unless it is a point on the line of motion because $L = mvr\sin\phi$ and $L > 1$ if $\phi \neq 0^o$ or $180^o$

(8) In case of circular motion, $\vec{L} = \vec{r}\times\vec{P} = m(\vec{r}\times\vec{v})$ = $mvr\sin\phi$

$\therefore$ $L = mvr = mr^2\omega$ [As $\vec{r}\perp\vec{v}$ and $v = r\omega$ ]

or $L = I\omega$ [As $mr^2 = I$]

In vector form $\vec{L} = I\vec{\omega}$

(9) From $\vec{L} = I\vec{\omega}$ $\therefore$ $\frac{d\vec{L}}{dt} = I\frac{d\vec{\omega}}{dt}$ = $I\vec{\alpha} = \vec{\tau}$ [As $\frac{d\vec{\omega}}{dt} = \vec{\alpha}$ and $\vec{\tau} = I\vec{\alpha}$ ]

*i.e.* the rate of change of angular momentum is equal to the net torque acting on the particle. [Rotational analogue of Newton's second law]

(10) If a large torque acts on a particle for a small time then 'angular impulse' of torque is given by

$$\vec{J} = \int\vec{\tau}\,dt = \vec{\tau}_{av}\int_{t_1}^{t_2}dt$$

or Angular impulse $\vec{J} = \vec{\tau}_{av}\,\Delta t = \Delta\vec{L}$

$\therefore$ Angular impulse = Change in angular momentum

(11) The angular momentum of a system of particles is equal to the vector sum of angular momentum of each particle *i.e.*, $\vec{L} = \vec{L}_1 + \vec{L}_2 + \vec{L}_3 + ....... + \vec{L}_n$ .

(12) According to Bohr theory angular momentum of an electron in $n^{th}$ orbit of atom can be taken as,

$$L = n\frac{h}{2\pi}$$

[where *n* is an integer used for number of orbit]

**7.18 Law of Conservation of Angular Momentum.**

Newton's second law for rotational motion $\vec{\tau} = \frac{d\vec{L}}{dt}$

So if the net external torque on a particle (or system) is zero then $\frac{d\vec{L}}{dt} = 0$

*i.e.* $\vec{L} = \vec{L}_1 + \vec{L}_2 + \vec{L}_3 + .......$ = constant.

Angular momentum of a system (may be particle or body) remains constant if resultant torque acting on it zero.

As $L = I\omega$ so if $\vec{\tau} = 0$ then $I\omega = \text{constant}$ $\therefore$ $I \propto \frac{1}{\omega}$

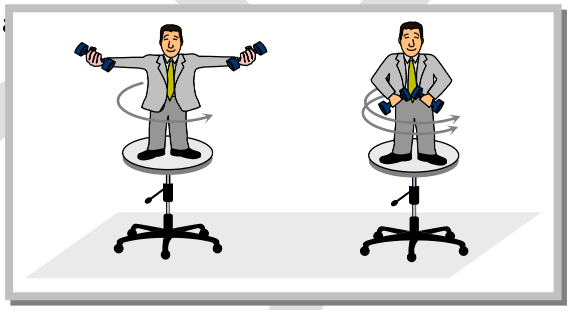
Since angular momentum $I\omega$ remains constant so when $I$ decreases, angular velocity $\omega$ increases and vice-versa.

Examples of law of conservation of angular momentum :

(1) The angular velocity of revolution of a planet around the sun in an elliptical orbit increases when the planet come closer to the sun and vice-versa because when planet comes closer to the sun, it's moment of inertia $I$ decreases there fore $\omega$ increases.

(2) A circus acrobat performs feats involving spin by bringing his arms and legs closer to his body or vice-versa. On bringing the arms and legs closer to body, his moment of inertia $I$ decreases. Hence $\omega$ increases.

(3) A person-carrying heavy weight in his hands and standing on a rotating platform can change the speed of platform. When the person suddenly folds his arms. Its moment of inertia decreases and in accordance the angular speed increa



(4) A diver performs somersaults by Jumping from a high diving board keeping his legs and arms out stretched first and then curling his body.

(5) Effect of change in radius of earth on its time period

Angular momentum of the earth $L = I\omega = \text{constant}$

$$L = \frac{2}{5}MR^2 \times \frac{2\pi}{T} = \text{constant}$$

$\therefore$ $T \propto R^2$ [if $M$ remains constant]

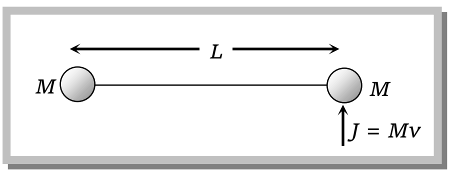
If $R$ becomes half then time period will become one-fourth *i.e.* $\frac{24}{4} = 6\,hrs.$

***Sample problems based on angular momentum***

***Problem*** **32.** Consider a body, shown in figure, consisting of two identical balls, each of mass $M$ connected by a light rigid rod. If an impulse $J = Mv$ is imparted to the body at one of its ends, what would be its angular velocity

**[IIT-JEE (Screening) 2003]**

(a) $v/L$



(b) $2v/L$

(c) $v/3L$

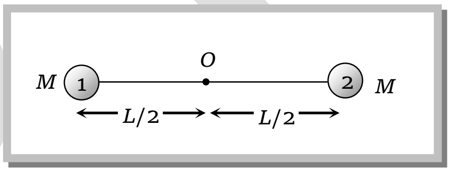
(d) $v/4L$

*Solution*: (a) Initial angular momentum of the system about point $O$

= Linear momentum × Perpendicular distance of linear momentum from the axis of rotation = $Mv\left(\frac{L}{2}\right)$ ....(i)

Final angular momentum of the system about point $O$ $= I_1\omega + I_2\omega = (I_1 + I_2)\omega = \left[M\left(\frac{L}{2}\right)^2 + M\left(\frac{L}{2}\right)^2\right]\omega$ ....(ii)

Applying the law of conservation of angular momentum




$\Rightarrow$ $Mv\left(\frac{L}{2}\right) = 2M\left(\frac{L}{2}\right)^2\omega$

$\Rightarrow$ $\omega = \frac{v}{L}$

***Problem*** **33.** A thin circular ring of mass $M$ and radius $R$ is rotating about its axis with a constant angular velocity $\omega$. Four objects each of mass $m$, are kept gently to the opposite ends of two perpendicular diameters of the ring. The angular velocity of the ring will be

**[CBSE PMT 2003]**

(a) $\frac{M\omega}{M+4m}$ (b) $\frac{(M+4m)\omega}{M}$ (c) $\frac{(M-4m)\omega}{M+4m}$ (d) $\frac{M\omega}{4m}$

*Solution*: (a) Initial angular momentum of ring $= I\omega = MR^2\omega$

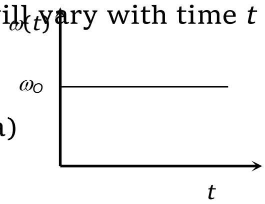
If four object each of mass $m$, and kept gently to the opposite ends of two perpendicular diameters of the ring then final angular momentum = $(MR^2 + 4mR^2)\omega'$

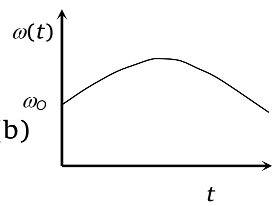
By the conservation of angular momentum

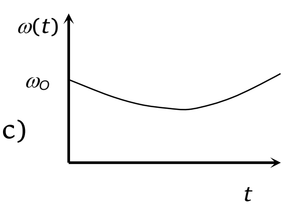
Initial angular momentum = Final angular momentum

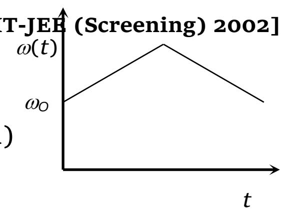
$MR^2\omega = (MR^2 + 4mR^2)\omega'$ $\Rightarrow$ $\omega' = \left(\frac{M}{M+4m}\right)\omega$.

***Problem*** **34.** A circular platform is free to rotate in a horizontal plane about a vertical axis passing through its center. A tortoise is sitting at the edge of the platform. Now, the platform is given an angular velocity $\omega_0$. When the tortoise moves along a chord of the platform with a constant velocity (with respect to the platform), the angular velocity of the platform $\omega(t)$ will vary with time $t$ as **[IIT-JEE (Screening) 2002]**

(a)



(b)



(c)



(d)


*Solution*: (b) The angular momentum ($L$) of the system is conserved *i.e.* $L = I\omega$ = constant

When the tortoise walks along a chord, it first moves closer to the centre and then away from the centre. Hence, M.I. first decreases and then increases. As a result, $\omega$ will first increase and then decrease. Also the change in $\omega$ will be non-linear function of time.

***Problem*** **35.** The position of a particle is given by : $\vec{r} = (\hat{i} + 2\hat{j} - \hat{k})$ and momentum $\vec{P} = (3\hat{i} + 4\hat{j} - 2\hat{k})$. The angular momentum is perpendicular to

(a) $X$-axis

(b) $Y$-axis

(c) $Z$-axis

(d) Line at equal angles to all the three axes

*Solution*: (a) $\vec{L} = \vec{r} \times \vec{p} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 3 & 4 & -2 \end{vmatrix} = 0\hat{i} - \hat{j} - 2\hat{k} = -\hat{j} - 2\hat{k}$ and the $X$- axis is given by $i + 0\hat{j} + 0\hat{k}$

Dot product of these two vectors is zero *i.e.* angular momentum is perpendicular to $X$-axis.

***Problem*** **36.** Two discs of moment of inertia $I_1$ and $I_2$ and angular speeds $\omega_1$ and $\omega_2$ are rotating along collinear axes passing through their centre of mass and perpendicular to their plane. If the two are made to rotate together along the same axis the rotational $KE$ of system will be

(a) $\dfrac{I_1\omega_1 + I_2\omega_2}{2(I_1 + I_2)}$ (b) $\dfrac{(I_1 + I_2)(\omega_1 + \omega_2)^2}{2}$ (c) $\dfrac{(I_1\omega_1 + I_2\omega_2)^2}{2(I_1 + I_2)}$ (d) None of these

*Solution*: (c) By the law of conservation of angular momentum $I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega$

Angular velocity of system $\omega = \dfrac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$

Rotational kinetic energy $= \dfrac{1}{2}(I_1 + I_2)\omega^2 = \dfrac{1}{2}(I_1 + I_2)\left(\dfrac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}\right)^2 = \dfrac{(I_1\omega_1 + I_2\omega_2)^2}{2(I_1 + I_2)}$.

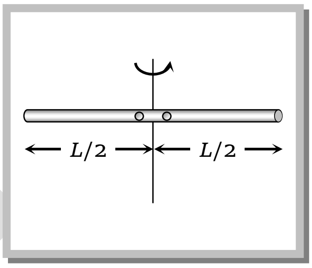
***Problem*** **37.** A smooth uniform rod of length $L$ and mass $M$ has two identical beads of negligible size, each of mass $m$, which can slide freely along the rod. Initially the two beads are at the centre of the rod and the system is rotating with angular velocity $\omega_0$ about an axis perpendicular to the rod and passing through the mid point of the rod (see figure). There are no external forces. When the beads reach the ends of the rod, the angular velocity of the system is **[IIT-JEE 1998]**

(a) $\omega_0$

(b) $\dfrac{M\omega_0}{M+12m}$

(c) $\dfrac{M\omega_0}{M+2m}$

(d) $\dfrac{M\omega_0}{M+6m}$




*Solution*: (d) Since there are no external forces therefore the angular momentum of the system remains constant.

Initially when the beads are at the centre of the rod angular momentum $L_1 = \left(\dfrac{ML^2}{12}\right)\omega_0$ .....(i)

When the beads reach the ends of the rod then angular momentum

$= \left(m\left(\dfrac{L}{2}\right)^2 + m\left(\dfrac{L}{2}\right)^2 + \dfrac{ML^2}{12}\right)\omega'$ .....(ii)

Equating (i) and (ii) $\dfrac{ML^2}{12}\omega_0 = \left(\dfrac{mL^2}{2} + \dfrac{ML^2}{12}\right)\omega' \Rightarrow \omega' = \dfrac{M\omega_o}{M+6m}$.

***Problem*** **38.** Moment of inertia of uniform rod of mass $M$ and length $L$ about an axis through its centre and perpendicular to its length is given by $\dfrac{ML^2}{12}$. Now consider one such rod pivoted at its centre, free to rotate in a vertical plane. The rod is at rest in the vertical position. A bullet of mass $M$ moving horizontally at a speed $v$ strikes and embedded in one end of the rod. The angular velocity of the rod just after the collision will be

(a) $v/L$ (b) $2v/L$ (c) $3v/2L$ (d) $6v/L$

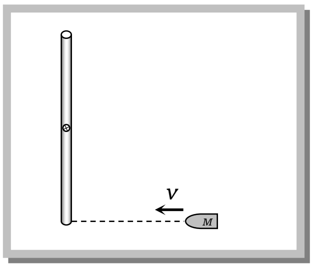
*Solution*: (c) Initial angular momentum of the system = Angular momentum of bullet before collision

$= Mv\left(\dfrac{L}{2}\right)$ .....(i)

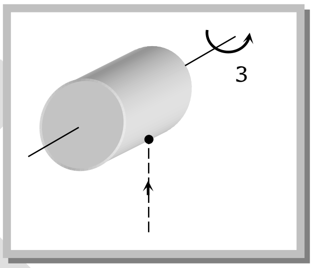
let the rod rotates with angular velocity $\omega$.

Final angular momentum of the system $= \left(\dfrac{ML^2}{12}\right)\omega + M\left(\dfrac{L}{2}\right)^2\omega$ .

By equation (i) and (ii) $Mv\dfrac{L}{2} = \left(\dfrac{ML^2}{12} + \dfrac{ML^2}{4}\right)\omega$ or $\omega = 3v/2L$



***Problem*** **39.** A solid cylinder of mass 2 *kg* and radius $0.2\,m$ is rotating about its own axis without friction with angular velocity $3\,rad/s$. A particle of mass 0.5 *kg* and moving with a velocity 5 *m/s* strikes the cylinder and sticks to it as shown in figure. The angular momentum of the cylinder before collision will be




(a) 0.12 *J-s*

(b) 12 *J-s*

(c) 1.2 *J-s*

(d) 1.12 *J-s*

*Solution*: (a) Angular momentum of the cylinder before collision $L = I\omega = \frac{1}{2}MR^2\omega = \frac{1}{2}(2)(0.2)^2 \times 3$ = 0.12 *J-s*.

***Problem*** **40.** In the above problem the angular velocity of the system after the particle sticks to it will be

(a) 0.3 *rad/s* (b) 5.3 *rad/s* (c) 10.3 *rad/s* (d) 89.3 *rad/s*

*Solution*: (c) Initial angular momentum of bullet + initial angular momentum of cylinder

= Final angular momentum of (bullet + cylinder) system

$$\Rightarrow mvr + I_1\omega = (I_1 + I_2)\omega'$$

$$\Rightarrow mvr + I_1\omega = \left(\frac{1}{2}Mr^2 + mr^2\right)\omega'$$

$$\Rightarrow 0.5 \times 5 \times 0.2 + 0.12 = \left(\frac{1}{2}2(0.2)^2 + (0.5)(0.2)^2\right)\omega'$$

$$\therefore \quad \omega' = 10.3 \ \ rad/sec.$$

## 7.19 Work, Energy and Power for Rotating Body

(1) **Work :** If the body is initially at rest and angular displacement is $d\theta$ due to torque then work done on the body.

$W = \int \tau \, d\theta$ [Analogue to work in translatory motion $W = \int F\, dx$ ]

(2) **Kinetic energy :** The energy, which a body has by virtue of its rotational motion is called rotational kinetic energy. A body rotating about a fixed axis possesses kinetic energy because its constituent particles are in motion, even though the body as a whole remains in place.

| Rotational kinetic energy | Analogue to translatory kinetic energy |
|---|---|
| $K_R = \frac{1}{2}I\omega^2$ | $K_T = \frac{1}{2}mv^2$ |

| | |
|---|---|
| $K_R = \frac{1}{2} L\omega$ | $K_T = \frac{1}{2} Pv$ |
| $K_R = \frac{L^2}{2I}$ | $K_T = \frac{P^2}{2m}$ |

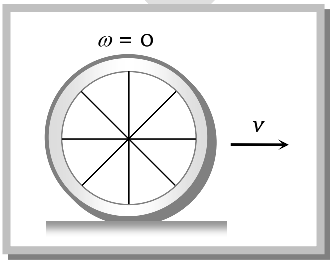
(3) **Power :** Rate of change of kinetic energy is defined as power

$$P = \frac{d}{dt}(K_R) = \frac{d}{dt}\left[\frac{1}{2} I\omega^2\right] = I\omega \frac{d\omega}{dt} = I\omega\alpha = I\alpha\omega = \tau\omega$$

In vector form Power $= \vec{\tau} \cdot \vec{\omega}$ [Analogue to power in translatory motion $P = \vec{F} \cdot \vec{v}$ ]

## 7.20 Slipping, Spinning and Rolling

(1) **Slipping :** When the body slides on a surface without rotation then its motion is called slipping motion.

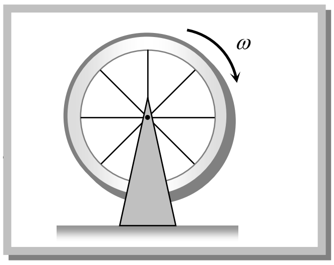



In this condition friction between the body and surface $F = 0$.

Body possess only translatory kinetic energy $K_T = \frac{1}{2} mv^2$.

*Example* : Motion of a ball on a frictionless surface.

(2) **Spinning :** When the body rotates in such a manner that its axis of rotation does not move then its motion is called spinning motion.




In this condition axis of rotation of a body is fixed.

Example : Motion of blades of a fan.

In spinning, body possess only rotatory kinetic energy $K_R = \frac{1}{2} I\omega^2$

or $K_R = \frac{1}{2} mK^2 \frac{v^2}{R^2} = \frac{1}{2} mv^2 \left(\frac{K^2}{R^2}\right)$

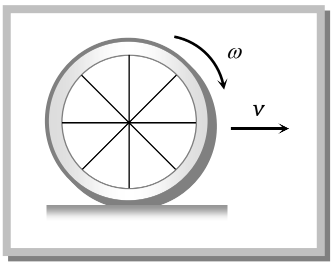
*i.e.*, Rotatory kinetic energy = $\left(\frac{K^2}{R^2}\right)$ times translatory kinetic energy.

Here $\frac{K^2}{R^2}$ is a constant for different bodies. Value of $\frac{K^2}{R^2} = 1$ (ring), $\frac{K^2}{R^2} = \frac{1}{2}$ (disc) and $\frac{K^2}{R^2} = \frac{1}{2}$ (solid sphere)

(3) **Rolling :** If in case of rotational motion of a body about a fixed axis, the axis of rotation also moves, the motion is called combined translatory and rotatory.

*Example* : (i) Motion of a wheel of cycle on a road.

(ii) Motion of football rolling on a surface.



In this condition friction between the body and surface $F \neq 0$.

Body possesses both translational and rotational kinetic energy.

Net kinetic energy = (Translatory + Rotatory) kinetic energy.

$$K_N = K_T + K_R = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2\frac{K^2}{R^2}$$

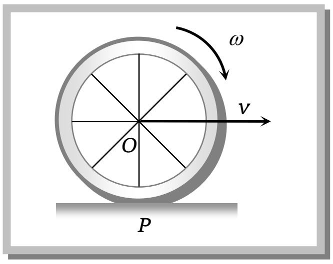
$$\therefore \qquad K_N = \frac{1}{2}mv^2\left(1 + \frac{K^2}{R^2}\right)$$

## 7.21 Rolling Without Slipping

In case of combined translatory and rotatory motion if the object rolls across a surface in such a way that there is no relative motion of object and surface at the point of contact, the motion is called rolling without slipping.

Friction is responsible for this type of motion but work done or dissipation of energy against friction is zero as there is no relative motion between body and surface at the point of contact.

Rolling motion of a body may be treated as a pure rotation about an axis through point of contact with same angular velocity $\omega$.




By the law of conservation of energy

$$K_N = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \qquad [\therefore \text{ As } v = R\omega]$$

$$= \frac{1}{2}mR^2\omega^2 + \frac{1}{2}I\omega^2$$

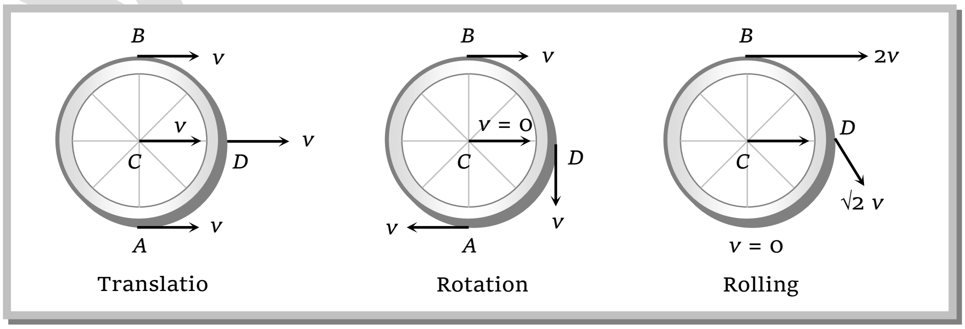
$$= \frac{1}{2}\omega^2[mR^2 + I]$$

$$= \frac{1}{2}\omega^2[I + mR^2] = \frac{1}{2}I_P\omega^2 \qquad [\text{As } I_P = I + mR^2]$$

By theorem of parallel axis, where $I$ = moment of inertia of rolling body about its centre '$O$' and $I_P$ = moment of inertia of rolling body about point of contact '$P$'.

(1) **Linear velocity of different points in rolling :** In case of rolling, all points of a rigid body have same angular speed but different linear speed.

Let $A$, $B$, $C$ and $D$ are four points then their velocities are shown in the following figure.




(2) **Energy distribution table for different rolling bodies :**

| Body | $\frac{K^2}{R^2}$ | Translatory ($K_T$) $\frac{1}{2}mv^2$ | Rotatory ($K_R$) $\frac{1}{2}mv^2\frac{K^2}{R^2}$ | Total ($K_N$) $\frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right)$ | $\frac{K_T}{K_N}$ (%) | $\frac{K_R}{K_N}$ (%) |
|---|---|---|---|---|---|---|
| Ring Cylindrical shell | 1 | $\frac{1}{2}mv^2$ | $\frac{1}{2}mv^2$ | $mv^2$ | $\frac{1}{2}$ (50%) | $\frac{1}{2}$ (50%) |
| Disc solid cylinder | $\frac{1}{2}$ | $\frac{1}{2}mv^2$ | $\frac{1}{4}mv^2$ | $\frac{3}{4}mv^2$ | $\frac{2}{3}$ (66.6%) | $\frac{1}{3}$ (33.3%) |
| Solid sphere | $\frac{2}{5}$ | $\frac{1}{2}mv^2$ | $\frac{1}{5}mv^2$ | $\frac{7}{10}mv^2$ | $\frac{5}{7}$ (71.5%) | $\frac{2}{7}$ (28.5%) |
| Hollow sphere | $\frac{2}{3}$ | $\frac{1}{2}mv^2$ | $\frac{1}{3}mv^2$ | $\frac{5}{6}mv^2$ | $\frac{3}{5}$ (60%) | $\frac{2}{5}$ (40%) |

## Sample problems based on kinetic energy, work and power

***Problem* 41.** A ring of radius 0.5 *m* and mass 10 *kg* is rotating about its diameter with an angular velocity of 20 *rad/s*. Its kinetic energy is

(a) 10 *J* (b) 100 *J* (c) 500 *J* (d) 250 *J*

*Solution*: (d) Rotational kinetic energy $\frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2 = \frac{1}{2}\left(\frac{1}{2}\times 10\times(0.5)^2\right)(20)^2 = 250\ J$

***Problem* 42.** An automobile engine develops 100 *kW* when rotating at a speed of 1800 rev/*min*. What torque does it deliver **[CBSE PMT 2000]**

(a) 350 *N-m* (b) 440 *N-m* (c) 531 *N-m* (d) 628 *N-m*

*Solution*: (c) $P = \tau\omega \Rightarrow \tau = \dfrac{100\times 10^3}{2\pi\frac{1800}{60}} = 531\ N\text{-}m$

***Problem* 43.** A body of moment of inertia of 3 *kg-m*² rotating with an angular velocity of 2 *rad/sec* has the same kinetic energy as a mass of 12 *kg* moving with a velocity of

(a) 8 *m/s* (b) 0.5 *m/s* (c) 2 *m/s* (d) 1 *m/s*

*Solution*: (d) Rotational kinetic energy of the body $= \frac{1}{2}I\omega^2$ and translatory kinetic energy $= \frac{1}{2}mv^2$

According to problem $= \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 \Rightarrow \frac{1}{2}\times 3\times(2)^2 = \frac{1}{2}\times 12\times v^2 \Rightarrow v = 1\,m/s$.

***Problem* 44.** A disc and a ring of same mass are rolling and if their kinetic energies are equal, then the ratio of their velocities will be

(a) $\sqrt{4}:\sqrt{3}$ (b) $\sqrt{3}:\sqrt{4}$ (c) $\sqrt{3}:\sqrt{2}$ (d) $\sqrt{2}:\sqrt{3}$

*Solution*: (a) $K_{disc} = \frac{1}{2}mv_d^2\left(1+\frac{k^2}{R^2}\right) = \frac{3}{4}mv_d^2$ $\left[As\ \frac{k^2}{R^2} = \frac{1}{2}\quad \text{for disc}\right]$

$K_{ring} = \frac{1}{2}mv_r\left(1+\frac{k^2}{R^2}\right) = mv_r^2$ $\left[As\ \frac{k^2}{R^2} = 1\quad \text{for ring}\right]$

According to problem $K_{disc} = K_{ring} \Rightarrow \frac{3}{4}mv_d^2 = mv_r^2 \Rightarrow \frac{v_d}{v_r} = \sqrt{\frac{4}{3}}$.

***Problem* 45.** A wheel is rotating with an angular speed of $20\,rad/sec$. It is stopped to rest by applying a constant torque in $4s$. If the moment of inertia of the wheel about its axis is 0.20 *kg-m*², then the work done by the torque in two seconds will be

(a) 10 *J* (b) 20 *J* (c) 30 *J* (d) 40 *J*

*Solution*: (c) $\omega_1 = 20$ *rad/sec*, $\omega_2 = 0, t = 4sec$. So angular retardation $\alpha = \frac{\omega_1 - \omega_2}{t} = \frac{20}{4} = 5rad/sec^2$

Now angular speed after 2 *sec* $\omega_2 = \omega_1 - \alpha t = 20 - 5 \times 2 = 10$ *rad/sec*

Work done by torque in 2 *sec* = loss in kinetic energy $= \frac{1}{2}I\left(\omega_1^2 - \omega_2^2\right) = \frac{1}{2}(0.20)((20)^2 - (10)^2)$

$= \frac{1}{2} \times 0.2 \times 300 = 30\,J$.

***Problem* 46.** If the angular momentum of a rotating body is increased by 200%, then its kinetic energy of rotation will be increased by

(a) 400% (b) 800% (c) 200% (d) 100%

*Solution*: (b) As $E = \frac{L^2}{2I} \Rightarrow \frac{E_2}{E_1} = \left(\frac{L_2}{L_1}\right)^2 = \left(\frac{3L_1}{L_1}\right)^2$ [As $L_2 = L_1 + 200\%.L_1 = 3L_1$]

$\Rightarrow E_2 = 9E_1 = E_1 + 800\%$ of $E_1$

***Problem* 47.** A ring, a solid sphere and a thin disc of different masses rotate with the same kinetic energy. Equal torques are applied to stop them. Which will make the least number of rotations before coming to rest

(a) Disc (b) Ring

(c) Solid sphere (d) All will make same number of rotations

*Solution*: (d) As $W = \tau\theta$ = Energy $\Rightarrow \theta = \frac{\text{Energy}}{\tau} = 2n\pi$

So, if energy and torque are same then all the bodies will make same number of rotation.

***Problem* 48.** The angular velocity of a body is $\vec{\omega} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and a torque $\vec{\tau} = \hat{i} + 2\hat{j} + 3\hat{k}$ acts on it. The rotational power will be

(a) 20 *W* (b) 15 *W* (c) $\sqrt{17}$ *W* (d) $\sqrt{14}$ *W*

*Solution*: (a) Power $(P) = \vec{\tau}.\vec{\omega} = (i + 2\hat{j} + 3\hat{k}).(2\hat{i} + 3\hat{j} + 4\hat{k}) = 2 + 6 + 12 = 20\,W$

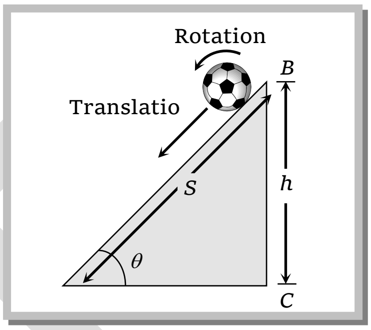
***Problem* 49.** A flywheel of moment of inertia 0.32 *kg-m*² is rotated steadily at $120\,rad/\text{sec}$ by a $50\,W$ electric motor. The kinetic energy of the flywheel is

(a) 4608 *J* (b) 1152 *J* (c) 2304 *J* (d) 6912 *J*

*Solution*: (c) Kinetic energy $K_R = \frac{1}{2}I\omega^2 = \frac{1}{2}(0.32)(120)^2 = 2304\,J$.

## 7.22 Rolling on an Inclined Plane

When a body of mass $m$ and radius $R$ rolls down on inclined plane of height '$h$' and angle of inclination $\theta$, it loses potential energy. However it acquires both linear and angular speeds and hence, gain kinetic energy of translation and that of rotation.




By conservation of mechanical energy $mgh = \frac{1}{2}mv^2\left(1 + \frac{k^2}{R^2}\right)$

(1) **Velocity at the lowest point :** $v = \sqrt{\frac{2gh}{1 + \frac{k^2}{R^2}}}$

(2) **Acceleration in motion :** From equation $v^2 = u^2 + 2aS$

By substituting $u = 0,\ S = \frac{h}{\sin\theta}$ and $v = \sqrt{\frac{2gh}{1 + \frac{k^2}{R^2}}}$ we get

$$a = \frac{g\sin\theta}{1 + \frac{k^2}{R^2}}$$

(3) **Time of descent :** From equation $v = u + at$

By substituting $u = 0$ and value of $v$ and $a$ from above expressions

$$t = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}\left[1 + \frac{k^2}{R^2}\right]}$$

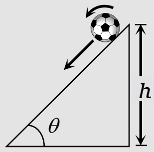
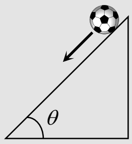
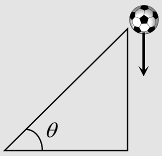
From the above expressions it is clear that, $v \propto \frac{1}{\sqrt{1 + \frac{k^2}{R^2}}}$; $a \propto \frac{1}{1 + \frac{k^2}{R^2}}$; $t \propto \sqrt{1 + \frac{k^2}{R^2}}$

Note :
- Here factor $\left(\frac{k^2}{R^2}\right)$ is a measure of moment of inertia of a body and its value is constant for given shape of the body and it does not depend on the mass and radius of a body.
- Velocity, acceleration and time of descent (for a given inclined plane) all depends on $\frac{k^2}{R^2}$. Lesser the moment of inertia of the rolling body lesser will be the value of $\frac{k^2}{R^2}$. So greater will be its velocity and acceleration and lesser will be the time of descent.
- If a solid and hollow body of same shape are allowed to roll down on inclined plane then as $\left(\frac{k^2}{R^2}\right)_S < \left(\frac{k^2}{R^2}\right)_H$, solid body will reach the bottom first with greater velocity.

- If a ring, cylinder, disc and sphere runs a race by rolling on an inclined plane then as $\left(\frac{k^2}{R^2}\right)_{\text{sphere}} = \text{minimum}$ while $\left(\frac{k^2}{R^2}\right)_{\text{Ring}} = \text{maximum}$, the sphere will reach the bottom first with greatest velocity while ring at last with least velocity.
- Angle of inclination has no effect on velocity, but time of descent and acceleration depends on it.

velocity $\propto \theta^{\circ}$, time of decent $\propto \theta^{-1}$ and acceleration $\propto \theta$.

## 7.23 Rolling Sliding and Falling of a Body

| | | Figure | Velocity | Acceleration | Time |
|---|---|---|---|---|---|
| Rolling | $\frac{k^2}{R^2} \neq 0$ |  | $\sqrt{\frac{2gh}{1+k^2/R^2}}$ | $\frac{g\sin\theta}{1+K^2/R^2}$ | $\frac{1}{\sin\theta}\sqrt{\frac{2h}{g}\left(1+\frac{k^2}{R^2}\right)}$ |
| Sliding | $\frac{k^2}{R^2} = 0$ |  | $\sqrt{2gh}$ | $g\sin\theta$ | $\frac{1}{\sin\theta}\sqrt{\frac{2h}{g}}$ |
| Falling | $\frac{k^2}{R^2} = 0$ $\theta = 90^{\circ}$ |  | $\sqrt{2gh}$ | $g$ | $\sqrt{\frac{2h}{g}}$ |

## 7.24 Velocity, Acceleration and Time for Different Bodies

| Body | $\frac{k^2}{R^2}$ | Velocity $v = \sqrt{\frac{2gh}{1+\frac{k^2}{R^2}}}$ | Acceleration $a = \frac{g\sin\theta}{1+\frac{k^2}{R^2}}$ | Time of descent $t = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}\left(1+\frac{k^2}{R^2}\right)}$ |
|---|---|---|---|---|
| Ring or Hollow cylinder | 1 | $\sqrt{gh}$ | $\frac{1}{2}g\sin\theta$ | $\frac{1}{\sin\theta}\sqrt{\frac{4h}{g}}$ |
| Disc or solid cylinder | $\frac{1}{2}$ or 0.5 | $\sqrt{\frac{4gh}{3}}$ | $\frac{2}{3}g\sin\theta$ | $\frac{1}{\sin\theta}\sqrt{\frac{3h}{g}}$ |
| Solid sphere | $\frac{2}{5}$ or 0.4 | $\sqrt{\frac{10}{7}gh}$ | $\frac{5}{7}g\sin\theta$ | $\frac{1}{\sin\theta}\sqrt{\frac{14}{5}\frac{h}{g}}$ |
| Hollow sphere | $\frac{2}{3}$ or 0.66 | $\sqrt{\frac{6}{5}gh}$ | $\frac{3}{5}g\sin\theta$ | $\frac{1}{\sin\theta}\sqrt{\frac{10}{3}\frac{h}{g}}$ |

***Sample problems based on rolling on an inclined plane***

**Problem 50.** A solid cylinder of mass $M$ and radius $R$ rolls without slipping down an inclined plane of length $L$ and height $h$. What is the speed of its centre of mass when the cylinder reaches its bottom **[CBSE PMT 2003]**

(a) $\sqrt{\frac{3}{4}gh}$ (b) $\sqrt{\frac{4}{3}gh}$ (c) $\sqrt{4\,gh}$ (d) $\sqrt{2\,gh}$

*Solution*: (b) Velocity at the bottom ($v$) $=\sqrt{\frac{2gh}{1+\frac{K^2}{R^2}}}=\sqrt{\frac{2gh}{1+\frac{1}{2}}}=\sqrt{\frac{4}{3}gh}$.

**Problem 51.** A sphere rolls down on an inclined plane of inclination $\theta$. What is the acceleration as the sphere reaches bottom **[Orissa JEE 2003]**

(a) $\frac{5}{7}g\sin\theta$ (b) $\frac{3}{5}g\sin\theta$ (c) $\frac{2}{7}g\sin\theta$ (d) $\frac{2}{5}g\sin\theta$

*Solution*: (a) Acceleration ($a$) $=\frac{g\sin\theta}{1+\frac{K^2}{R^2}}=\frac{g\sin\theta}{1+\frac{2}{5}}=\frac{5}{7}g\sin\theta$.

**Problem 52.** A ring solid sphere and a disc are rolling down from the top of the same height, then the sequence to reach on surface is

(a) Ring, disc, sphere (b) Sphere, disc, ring (c) Disc, ring, sphere (d) Sphere, ring, disc

*Solution*: (b) Time of descent $\propto$ moment of inertia $\propto \frac{k^2}{R^2}$

$$\left(\frac{k^2}{R^2}\right)_{sphere}=0.4,\ \left(\frac{k^2}{R^2}\right)_{disc}=0.5,\ \left(\frac{k^2}{R^2}\right)_{ring}=1 \quad \therefore\ t_{sphere}<t_{disc}<t_{ring}.$$

**Problem 53.** A thin uniform circular ring is rolling down an inclined plane of inclination 30° without slipping. Its linear acceleration along the inclined plane will be

(a) $g/2$ (b) $g/3$ (c) $g/4$ (d) $2g/3$

*Solution*: (c) $a=\frac{g\sin\theta}{1+\frac{k^2}{R^2}}=\frac{g\sin 30^o}{1+1}=\frac{g}{4}$ [As $\frac{k^2}{R^2}=1$ and $\theta=30^o$]

**Problem 54.** A solid sphere and a disc of same mass and radius starts rolling down a rough inclined plane, from the same height the ratio of the time taken in the two cases is

(a) 15 : 14 (b) $\sqrt{15}:\sqrt{14}$ (c) 14 : 15 (d) $\sqrt{14}:\sqrt{15}$

*Solution*: (d) Time of descent $t=\frac{1}{\sin\theta}\sqrt{\frac{2h}{g}\left(1+\frac{k^2}{R^2}\right)}$ $\therefore$ $\frac{t_{\text{shpere}}}{t_{\text{disc}}}=\sqrt{\frac{\left(1+\frac{k^2}{R^2}\right)_{\text{sphere}}}{\left(1+\frac{k^2}{R^2}\right)_{\text{disc}}}}=\sqrt{\frac{1+\frac{2}{5}}{1+\frac{1}{2}}}=\sqrt{\frac{7}{5}\times\frac{2}{3}}=\sqrt{\frac{14}{15}}$

**Problem 55.** A solid sphere of mass $0.1\,kg$ and radius $2\,cm$ rolls down an inclined plane $1.4\,m$ in length (slope 1 in 10). Starting from rest its final velocity will be

(a) $1.4\ m/\sec$ (b) $0.14\ m/\sec$ (c) $14\ m/\sec$ (d) $0.7\ m/\sec$

*Solution*: (a) $v=\sqrt{\frac{2gh}{1+\frac{k^2}{R^2}}}=\sqrt{\frac{2\times 9.8\times l\sin\theta}{1+\frac{2}{5}}}$ [As $\frac{k^2}{R^2}=\frac{2}{5}$, $l=\frac{h}{\sin\theta}$ and $\sin\theta=\frac{1}{10}$ given]

$\Rightarrow v = \sqrt{\dfrac{2 \times 9.8 \times 1.4 \times \frac{1}{10}}{7/5}} = 1.4\, m/s.$

**Problem 56.** A solid sphere rolls down an inclined plane and its velocity at the bottom is $v_1$. Then same sphere slides down the plane (without friction) and let its velocity at the bottom be $v_2$. Which of the following relation is correct

(a) $v_1 = v_2$ (b) $v_1 = \frac{5}{7} v_2$ (c) $v_1 = \frac{7}{5} v_2$ (d) None of these

*Solution*: (d) When solid sphere rolls down an inclined plane the velocity at bottom $v_1 = \sqrt{\frac{10}{7} gh}$

but, if there is no friction then it slides on inclined plane and the velocity at bottom $v_2 = \sqrt{2gh}$

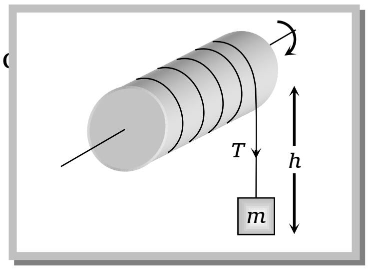
$\therefore \frac{v_1}{v_2} = \sqrt{\frac{5}{7}}$ .

## 7.25 Motion of Connected Mass

A point mass is tied to one end of a string which is wound round the solid body [cylinder, pulley, disc]. When the mass is released, it falls vertically downwards and the solid body rotates unwinding the string

$m$ = mass of point-mass, $M$ = mass of a rigid body

$R$ = radius of a rigid body, $I$ = moment of inertia of rotating bo




(1) **Downwards acceleration of point mass** $a = \dfrac{g}{1 + \dfrac{I}{mR^2}}$

(2) **Tension in string** $T = mg\left[\dfrac{I}{I + mR^2}\right]$

(3) **Velocity of point mass** $v = \sqrt{\dfrac{2gh}{1 + \dfrac{I}{mR^2}}}$

(4) **Angular velocity of rigid body** $\omega = \sqrt{\dfrac{2mgh}{I + mR^2}}$

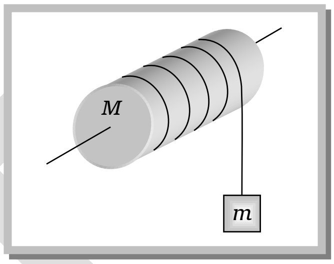
### *Sample problems based on motion of connected mass*

**Problem 57.** A cord is wound round the circumference of wheel of radius *r*. The axis of the wheel is horizontal and moment of inertia about it is *I*. A weight *mg* is attached to the end of the cord and falls from rest. After falling through a distance *h,* the angular velocity of the wheel will be **[MP PMT 1994; DPMT 2001]**

(a) $\sqrt{\dfrac{2gh}{I + mr}}$ (b) $\sqrt{\dfrac{2mgh}{I + mr^2}}$ (c) $\sqrt{\dfrac{2mgh}{I + 2mr^2}}$ (d) $\sqrt{2gh}$

*Solution* : (b) According to law of conservation of energy $mgh = \frac{1}{2}(I + mr^2)\omega^2 \Rightarrow \omega = \sqrt{\dfrac{2mgh}{I + mr^2}}$ .

***Problem*** **58.** In the following figure, a body of mass $m$ is tied at one end of a light string and this string is wrapped around the solid cylinder of mass $M$ and radius $R$. At the moment $t$ = 0 the system starts moving. If the friction is negligible, angular velocity at time $t$ would be




(a) $\frac{mgRt}{(M+m)}$

(b) $\frac{2Mgt}{(M+2m)}$

(c) $\frac{2mgt}{R(M-2m)}$

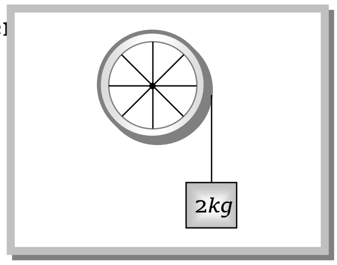
(d) $\frac{2mgt}{R(M+2m)}$

*Solution* : (d) We know the tangential acceleration $a = \frac{g}{1+\frac{I}{mR^2}} = \frac{g}{1+\frac{1/2MR^2}{mR^2}} = \frac{2mg}{2m+M}$ [As $I = \frac{1}{2}MR^2$ for cylinder]

After time $t$, linear velocity of mass $m$, $v = u + at = 0 + \frac{2mgt}{2m+M}$

So angular velocity of the cylinder $\omega = \frac{v}{R} = \frac{2mgt}{R(M+2m)}$.

***Problem*** **59.** A block of mass $2kg$ hangs from the rim of a wheel of radius $0.5m$. On releasing from rest the block falls through $5m$ height in $2s$. The moment of iner




(a) 1 *kg-m*²

(b) 3.2 *kg-m*²

(c) 2.5 *kg-m*²

(d) 1.5 *kg-m*²

*Solution* : (d) On releasing from rest the block falls through 5*m* height in 2 *sec*.

$$5 = 0 + \frac{1}{2}a(2)^2 \qquad \text{[As } S = ut + \frac{1}{2}at^2 \text{]}$$

$$\therefore \qquad a = 2.5\,m/s^2$$

Substituting the value of a in the formula $a = \frac{g}{1+\frac{I}{mR^2}}$ and by solving we get

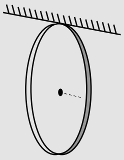
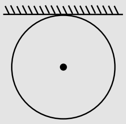
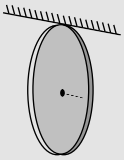
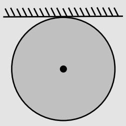
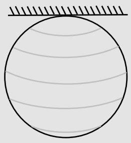
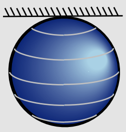
$$\Rightarrow \qquad 2.5 = \frac{10}{1+\frac{I}{2\times(0.5)^2}} \Rightarrow I = 1.5kg - m^2$$

## 7.26 Time Period of Compound Pendulum

Time period of compound pendulum is given by, $T = 2\pi\sqrt{\frac{L}{g}}$ where $L = \frac{l^2+k^2}{l}$

Here $l$ = distance of centre of mass from point of suspension

$k$ = radius of gyration about the parallel axis passing through centre of mass.

| **Body** | **Axis of rotation** | **Figure** | $l$ | $k^2$ | $L = \frac{l^2 + k^2}{l}$ | $T = 2\pi\sqrt{\frac{L}{g}}$ |
|---|---|---|---|---|---|---|
| Ring | Tangent passing through the rim and perpendicular to the plane |  | $R$ | $R^2$ | $2R$ | $T = 2\pi\sqrt{\frac{2R}{g}}$ |
| | Tangent parallel to the plane |  | $R$ | $\frac{R^2}{2}$ | $\frac{3}{2}R$ | $T = 2\pi\sqrt{\frac{3R}{2g}}$ |
| Disc | Tangent, Perpendicular to plane |  | $R$ | $\frac{R^2}{2}$ | $\frac{3}{2}R$ | $T = 2\pi\sqrt{\frac{3R}{2g}}$ |
| | Tangent parallel to the plane |  | $R$ | $\frac{R^2}{4}$ | $\frac{5}{4}R$ | $T = 2\pi\sqrt{\frac{5R}{4g}}$ |
| Spherical shell | Tangent |  | $R$ | $\frac{2}{3}R^2$ | $\frac{5}{3}R$ | $T = 2\pi\sqrt{\frac{5R}{3g}}$ |
| Solid sphere | Tangent |  | $R$ | $\frac{2}{5}R^2$ | $\frac{7}{5}R$ | $T = 2\pi\sqrt{\frac{7R}{5g}}$ |

### Sample problems based on time period of compound pendulum

***Problem* 60.** A ring whose diameter is 1 *meter*, oscillates simple harmonically in a vertical plane about a nail fixed at its circumference. The time period will be

(a) $1/4\ sec$ (b) $1/2\ sec$ (c) $1\ sec$ (d) $2\ sec$

*Solution*: (d) $T = 2\pi\sqrt{\frac{2R}{g}} = 2\pi\sqrt{\frac{1}{g}} = 2\ sec$ [As diameter $2R = 1\ meter$ given]

***Problem* 61.** A number of holes are drilled along a diameter of a disc of radius $R$. To get minimum time period of oscillations the disc should be suspended from a horizontal axis passing through a hole whose distance from the centre should be

(a) $\frac{R}{2}$ (b) $\frac{R}{\sqrt{2}}$ (c) $\frac{R}{2\sqrt{2}}$ (d) Zero

*Solution*: (b) $T = 2\pi\sqrt{\frac{L}{g}}$ where $L = \frac{l^2 + k^2}{l}$

Here $k^2 = \frac{R^2}{2}$ $\therefore$ $L = \frac{l^2 + \frac{R^2}{2}}{l} = l + \frac{R^2}{2l}$

For minimum time period $L$ should be minimum $\frac{dL}{dl} = 0$

$$\Rightarrow \quad \frac{d}{dl}\left(l + \frac{R^2}{2l}\right) = 0$$

$$\Rightarrow \quad 1 + \frac{R^2}{2}\left(\frac{-1}{l^2}\right) = 1 - \frac{R^2}{2l^2} = 0 \Rightarrow l = \frac{R}{\sqrt{2}}.$$



# Practice Problems

## *Problems based on centre of mass*

### ▶ *Basic level*

**1.** Where will be the centre of mass on combining two masses $m$ and $M$ ($M>m$) **[RPET 2003]**

(a) Towards $m$ (b) Towards $M$ (c) Between $m$ and $M$ (d) Anywhere

**2.** Two objects of masses 200 $gm$ and 500$gm$ possess velocities $10\hat{i}$ $m/s$ and $3\hat{i}+5\hat{j}$ $m/s$ respectively. The velocity of their centre of mass in $m/s$ is

(a) $5\hat{i}-25\hat{j}$ (b) $\frac{5}{7}\hat{i}-25\hat{j}$ (c) $5\hat{i}+\frac{25}{7}\hat{j}$ (d) $25\hat{i}-\frac{5}{7}\hat{j}$

**3.** In the $HCl$ molecule, the separation between the nuclei of the two atoms is about 1.27 Å (1 Å = $10^{-10}$ $m$). The approximate location of the centre of mass of the molecule from hydrogen is (assuming the chlorine atom to be about 35.5 times massive as hydrogen)

**[Kerala (Engg.) 2002]**

(a) 1 Å (b) 2.5 Å (c) 1.24 Å (d) 1.5 Å

**4.** Four particle of masses $m$, $2m$, $3m$ and $4m$ are arranged at the corners of a parallelogram with each side equal to $a$ and one of the angle between two adjacent sides is 60°. The parallelogram lies in the $x$-$y$ plane with mass $m$ at the origin and $4m$ on the $x$-axis. The centre of mass of the arrangement will be located at

(a) $\left(\frac{\sqrt{3}}{2}a,\ 0.95a\right)$ (b) $\left(0.95a,\ \frac{\sqrt{3}}{4}a\right)$ (c) $\left(\frac{3a}{4},\ \frac{a}{2}\right)$ (d) $\left(\frac{a}{2},\ \frac{3a}{4}\right)$

**5.** A system consists of 3 particles each of mass $m$ and located at (1, 1) (2, 2) (3, 3). The co-ordinate of the centre of mass are

(a) (6, 6) (b) (3, 3) (c) (2, 2) (d) (1, 1)

**6.** If a bomb is thrown at a certain angle with the horizontal and after exploding on the way the different fragments move in different directions then the centre of mass

(a) Would move along the same parabolic path (b) Would move along a horizontal path

(c) Would move along a vertical line (d) None of these

**7.** Four identical spheres each of mass $m$ are placed at the corners of square of side 2*metre*. Taking the point of intersection of the diagonals as the origin, the co-ordinates of the centre of mass are

(a) (0, 0) (b) (1, 1) (c) (– 1, 1) (d) (1, – 1)

### ▶▶ *Advance level*

**8.** Two particles $A$ and $B$ initially at rest move towards each other under a mutual force of attraction. At the instant when the speed of $A$ is $v$ and the speed of $B$ is $2v$, the speed of centre of mass of the system is

(a) Zero (b) $v$ (c) $1.5v$ (d) $3v$

**9.** A circular plate of uniform thickness has diameter 56 $cm$. A circular part of diameter 42 $cm$ is removed from one edge. What is the position of the centre of mass of the remaining part

(a) 3 $cm$ (b) 6 $cm$ (c) 9 $cm$ (d) 12 $cm$

**10.** Two point masses $m$ and $M$ are separated by a distance $L$. The distance of the centre of mass of the system from $m$ is

(a) $L(m/M)$ (b) $L(M/m)$ (c) $L\left(\frac{M}{m+M}\right)$ (d) $L\left(\frac{m}{m+M}\right)$

**11.** Three identical spheres, each of mass 1 *kg* are placed touching each other with their centres on a straight line. Their centres are marked *K, L* and *M* respectively. The distance of centre of mass of the system from *K* is

(a) $\frac{KL+KM+LM}{3}$ (b) $\frac{KL+KM}{3}$ (c) $\frac{KL+LM}{3}$ (d) $\frac{KM+LM}{3}$

**12.** Two particles of masses 1 *kg* and 3 *kg* move towards each other under their mutual force of attraction. No other force acts on them. When the relative velocity of approach of the two particles is 2*m/s,* their centre of mass has a velocity of 0.5 *m/s.* When the relative velocity of approach becomes 3 *m/s,* the velocity of the centre of mass is

(a) 0.5 *m/s* (b) 0.75 *m/s* (c) 1.25 *m/s* (d) Zero

## Problems based on angular displacement, velocity and acceleration

### Basic level

**13.** In rotational motion of a rigid body, all particle move with

(a) Same linear and angular velocity (b) Same linear and different angular velocity

(c) With different linear velocities and same angular velocities (d) With different linear velocities and different angular velocities

**14.** The angular speed of a fly–wheel making 120 revolution/*minute* is **[Pb. PMT 1999; CPMT 2002]**

(a) $\pi$ *rad/sec* (b) $2\pi$ *rad/sec* (c) $4\pi$ *rad/sec* (d) $4\pi^2$ *rad/sec*

**15.** A flywheel gains a speed of 540 *r.p.m.* in 6 *sec*. Its angular acceleration will be **[KCET (Med.) 2001]**

(a) $3\pi$ *rad/sec*$^2$ (b) $9\pi$ *rad/sec*$^2$ (c) $18\pi$ *rad/sec*$^2$ (d) $54\pi$ *rad/sec*$^2$

**16.** A car is moving at a speed of 72 *km/hr*. the diameter of its wheels is 0.5 *m*. If the wheels are stopped in 20 rotations by applying brakes, then angular retardation produced by the brakes is

(a) – 25.5 *rad/s*$^2$ (b) – 29.5 *rad/s*$^2$ (c) – 33.5 *rad/s*$^2$ (d) – 45.5 *rad/s*$^2$

**17.** A wheel is rotating at 900 *r.p.m.* about its axis. When the power is cut-off, it comes to rest in 1 minute. The angular retardation in *radian/s*$^2$ is

(a) $\pi/2$ (b) $\pi/4$ (c) $\pi/6$ (d) $\pi/8$

### Advance level

**18.** A particle *B* is moving in a circle of radius *a* with a uniform speed *u*. *C* is the centre of the circle and *AB* is diameter. The angular velocity of *B* about *A* and *C* are in the ratio

(a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 4 : 1

**19.** Two particles having mass '*M*' and '*m*' are moving in circular paths having radii *R* and *r*. If their time periods are same then the ratio of their angular velocities will be

(a) $\frac{r}{R}$ (b) $\frac{R}{r}$ (c) 1 (d) $\sqrt{\frac{R}{r}}$

**20.** A body is in pure rotation. The linear speed *v* of a particle, the distance *r* of the particle from the axis and angular velocity $\omega$ of the body are related as $\omega = \frac{v}{r}$, thus

(a) $\omega \propto \frac{1}{r}$ (b) $\omega \propto r$ (c) $\omega = 0$ (d) $\omega$ is independent of *r*

**21.** A strap is passing over a wheel of radius 30 *cm*. During the time the wheel moving with initial constant velocity of 2 rev/sec. comes to rest the strap covers a distance of 25 *m*. The deceleration of the wheel in *rad/s*$^2$ is

(a) 0.94 (b) 1.2 (c) 2.0 (d) 2.5

**22.** A particle starts rotating from rest. Its angular displacement is expressed by the following equation $\theta = 0.025t^2 - 0.1t$ where $\theta$ is in radian and $t$ is in seconds. The angular acceleration of the particle is

(a) 0.5 $rad/sec^2$ at the end of 10 $sec$ (b) 0.3 $rad/sec^2$ at the end of 2 $sec$

(c) 0.05 $rad/sec^2$ at the end of 1 $sec$ (d) Constant 0.05 $rad/sec^2$

**23.** The planes of two rigid discs are perpendicular to each other. They are rotating about their axes. If their angular velocities are 3 $rad/sec$ and 4 $rad/sec$ respectively, then the resultant angular velocity of the system would be

(a) 1 $rad/sec$ (b) 7 $rad/sec$ (c) 5 $rad/sec$ (d) $\sqrt{12}$ $rad/sec$

**24.** A sphere is rotating about a diameter

(a) The particles on the surface of the sphere do not have any linear acceleration

(b) The particles on the diameter mentioned above do not have any linear acceleration

(c) Different particles on the surface have different angular speeds

(d) All the particles on the surface have same linear speed

**25.** A rigid body is rotating with variable angular velocity $(a - bt)$ at any instant of time $t$. The total angle subtended by it before coming to rest will be ($a$ and $b$ are constants)

(a) $\frac{(a-b)a}{2}$ (b) $\frac{a^2}{2b}$ (c) $\frac{a^2 - b^2}{2b}$ (d) $\frac{a^2 - b^2}{2a}$

**26.** When a ceiling fan is switched on, it makes 10 rotations in the first 3 seconds. How many rotations will it make in the next 3 seconds (Assume uniform angular acceleration)

(a) 10 (b) 20 (c) 30 (d) 40

**27.** When a ceiling fan is switched off, its angular velocity falls to half while it makes 36 rotations. How many more rotations will it make before coming to rest (Assume uniform angular retardation)

(a) 36 (b) 24 (c) 18 (d) 12

**28.** Let $\vec{A}$ be a unit vector along the axis of rotation of a purely rotating body and $\vec{B}$ be a unit vector along the velocity of a particle $P$ of the body away from the axis. The value of $\vec{A}.\vec{B}$ is

(a) 1 (b) – 1 (c) 0 (d) None of these

## Problems based on torque, couple

### ▶ Basic level

**29.** Let $F$ be the force acting on a particle having position vector $\vec{r}$ and $\vec{T}$ be the torque of this force about the origin. Then **[AIEEE 2003]**

(a) $\vec{r}.\vec{T} = 0$ and $\vec{F}.\vec{T} = 0$ (b) $\vec{r}.\vec{T} = 0$ and $\vec{F}.\vec{T} \neq 0$

(c) $\vec{r}.\vec{T} \neq 0$ and $\vec{F}.\vec{T} = 0$ (d) $\vec{r}.\vec{T} \neq 0$ and $\vec{F}.\vec{T} \neq 0$

**30.** A couple produces **[CBSE PMT 1997]**

(a) Purely linear motion (b) Purely rotational motion

(c) Linear and rotational motion (d) No motion

**31.** For a system to be in equilibrium, the torques acting on it must balance. This is true only if the torques are taken about

(a) The centre of the system (b) The centre of mass of the system

(c) Any point on the system (d) Any point on the system or outside it

**32.** What is the torque of the force $\vec{F} = (2\hat{i} - 3\hat{j} + 4\hat{k})N$ acting at the pt. $\vec{r} = (3\hat{i} + 2\hat{j} + 3\hat{k})m$ about the origin

(a) $-17\hat{i}+6\hat{j}+13\hat{k}$ (b) $-6\hat{i}+6\hat{j}-12\hat{k}$ (c) $17\hat{i}-6\hat{j}-13\hat{k}$ (d) $6\hat{i}-6\hat{j}+12\hat{k}$

**33.** Two men are carrying a uniform bar of length $L$, on their shoulders. The bar is held horizontally such that younger man gets $(1/4)th$ load. Suppose the younger man is at the end of the bar, what is the distance of the other man from the end

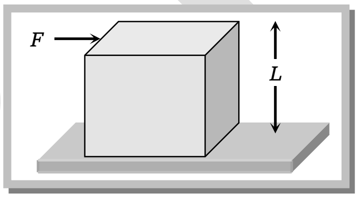
(a) $L/3$ (b) $L/2$ (c) $2L/3$ (d) $3L/4$

**34.** A uniform meter scale balances at the $40\,cm$ mark when weights of $10\,g$ and $20\,g$ are suspended from the $10\,cm$ and $20\,cm$ marks. The weight of the *metre* scale is

(a) $50\,g$ (b) $60\,g$ (c) $70\,g$ (d) $80\,g$

## ▶▶ *Advance level*

**35.** A cubical block of side *L* rests on a rough horizontal surface with coefficient of friction $\mu$. A horizontal force *F* is applied on the block as shown. If the coefficient of friction is sufficiently high so that the block does not slide before toppling, the minimum force required to topple the block is



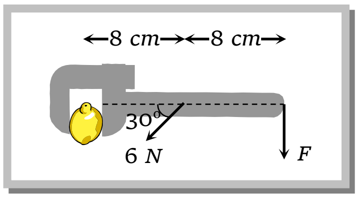

(a) Infinitesimal

(b) *mg*/4

(c) *mg*/2

(d) $mg(1-\mu)$

**36.** When a force of 6.0 *N* is exerted at $30^o$ to a wrench at a distance of 8 *cm* from the nut, it is just able to loosen the nut. What force *F* would be sufficient to loosen it, if it acts perpendicularly to the wrench at 16 *cm* from the nut



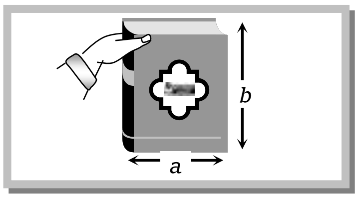

(a) 3 *N*

(b) 6 *N*

(c) 4 *N*

(d) 1.5 *N*

**37.** A person supports a book between his finger and thumb as shown (the point of grip is assumed to be at the corner of the book). If the book has a weight of *W* then the person is producing a torque on the book of




(a) $W\frac{a}{2}$ anticlockwise

(b) $W\frac{b}{2}$ anticlockwise

(c) $Wa$ anticlockwise

(d) $Wa$ clockwise

**38.** Weights of $1g, 2g.....,100\,g$ are suspended from the 1 *cm*, 2 *cm*, ...... 100 *cm*, marks respectively of a light *metre* scale. Where should it be supported for the system to be in equilibrium

(a) 55 *cm* mark (b) 60 *cm* mark (c) 66 *cm* mark (d) 72 *cm* mark

**39.** A uniform cube of side *a* and mass *m* rests on a rough horizontal table. A horizontal force *F* is applied normal to one of the faces at a point that is directly above the centre of the face, at a height $\frac{3a}{4}$ above the base. The minimum value of *F* for which the cube begins to tilt about the edge is (assume that the cube does not slide)

(a) $\frac{mg}{4}$ (b) $\frac{2mg}{3}$ (c) $\frac{3mg}{4}$ (d) $mg$

## *Problems based on moment of inertia*

### ▶ *Basic level*

**40.** A circular disc of radius $R$ and thickness $\frac{R}{6}$ has moment of inertia $I$ about an axis passing through its centre and perpendicular to its plane. It is melted and recasted into a solid sphere. The moment of inertia of the sphere about its diameter as axis of rotation is **[EAMCET 2003]**

(a) $I$ (b) $\frac{2I}{8}$ (c) $\frac{I}{5}$ (d) $\frac{I}{10}$

**41.** The moment of inertia of a meter scale of mass 0.6 *kg* about an axis perpendicular to the scale and located at the 20 *cm* position on the scale in *kg* $m^2$ is (Breadth of the scale is negligible)

(a) 0.074 (b) 0.104 (c) 0.148 (d) 0.208

**42.** Two discs of the same material and thickness have radii 0.2 *m* and 0.6 *m*. Their moments of inertia about their axes will be in the ratio **[MP PET 2003]**

(a) 1 : 81 (b) 1 : 27 (c) 1 : 9 (d) 1 : 3

**43.** A circular disc is to be made by using iron and aluminium, so that it acquires maximum moment of inertia about its geometrical axis. It is possible with

(a) Iron and aluminium layers in alternate order (b) Aluminium at interior and iron surrounding it

(c) Iron at interior and aluminium surrounding it (d) Either (a) or (c)

**44.** The moment of inertia of semicircular ring about its centre is

(a) $MR^2$ (b) $\frac{MR^2}{2}$ (c) $\frac{MR^2}{4}$ (d) None of these

**45.** Moment of inertia of a disc about its own axis is $I$. Its moment of inertia about a tangential axis in its plane is**[MP PMT**

(a) $\frac{5}{2}I$ (b) $3\,I$ (c) $\frac{3}{2}I$ (d) $2\,I$

**46.** A wheel of mass 10 *kg* has a moment of inertia of 160 *kg*–$m^2$ about its own axis, the radius of gyration will be **[Pb. PMT**

(a) 10 *m* (b) 8 *m* (c) 6 *m* (d) 4 *m*

**47.** Four particles each of mass *m* are placed at the corners of a square of side length *l*. The radius of gyration of the system about an axis perpendicular to the square and passing through its centre is

(a) $\frac{l}{\sqrt{2}}$ (b) $\frac{l}{2}$ (c) $l$ (d) $(\sqrt{2})l$

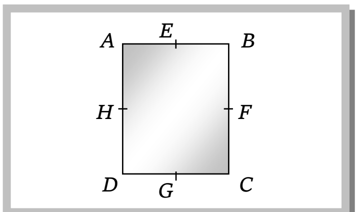
**48.** The moment of inertia of a rod (length *l*, mass *m*) about an axis perpendicular to the length of the rod and passing through a point equidistant from its mid point and one end is

(a) $\frac{ml^2}{12}$ (b) $\frac{7}{48}ml^2$ (c) $\frac{13}{48}ml^2$ (d) $\frac{19}{48}ml^2$

**49.** Three point masses $m_1, m_2, m_3$ are located at the vertices of an equilateral triangle of length '$a$'. The moment of inertia of the system about an axis along the altitude of the triangle passing through $m_1$ is

(a) $(m_2+m_3)\frac{a^2}{4}$ (b) $(m_1+m_2+m_3)a^2$ (c) $(m_1+m_2)\frac{a^2}{2}$ (d) $(m_2+m_3)a^2$

**50.** In a rectangle $ABCD$ $(BC = 2AB)$. The moment of inertia along which axis will be minimum



(a) $BC$

(b) $BD$

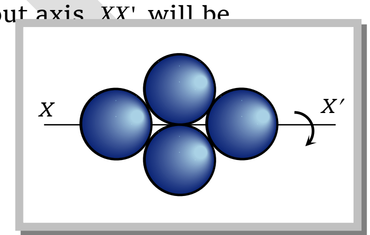
(c) $HF$

(d) $EG$

**51.** Two loops $P$ and $Q$ are made from a uniform wire. The radii of $P$ and $Q$ are $r_1$ and $r_2$ respectively, and their moments of inertia are $I_1$ and $I_2$ respectively. If $I_2/I_1 = 4$ then $\frac{r_2}{r_1}$ equals

(a) $4^{2/3}$ (b) $4^{1/3}$ (c) $4^{-2/3}$ (d) $4^{-1/3}$

**52.** The moment of inertia of a sphere (mass $M$ and radius $R$) about it's diameter is $I$. Four such spheres are arranged as shown in the figure. The moment of inertia of the system about axis $XX'$ will be




(a) $3I$

(b) $5\ I$

(c) $7\ I$

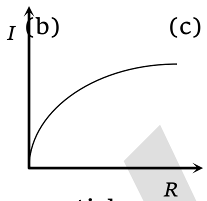
(d) $9\ I$

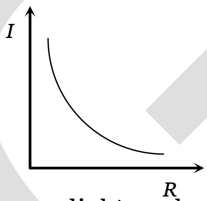
**53.** Three identical thin rods each of length $l$ and mass $M$ are joined together to form a letter $H$. What is the moment of inertia of the system about one of the sides of $H$

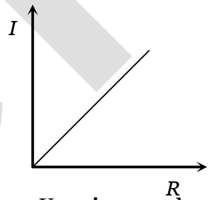
(a) $\frac{Ml^2}{3}$ (b) $\frac{Ml^2}{4}$ (c) $\frac{2Ml^2}{3}$ (d) $\frac{4Ml^2}{3}$

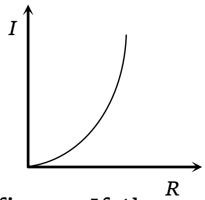
**54.** Moment of inertia of a sphere of mass $M$ and radius $R$ is $I$. Keeping $M$ constant if a graph is plotted between $I$ and $R$, then its form would be

(a) (b) (c) (d)

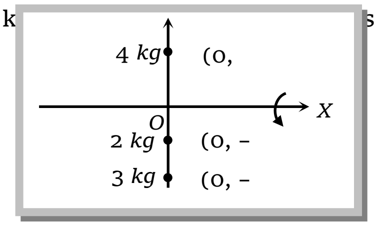



**55.** Three particles are situated on a light and rigid rod along $Y$ axis as shown in the figure. If the system is rotating with an angular velocity of $2\,rad/\sec$ about $X$ axis, then the total k




s

(a) $92\ J$

(b) $184\ J$

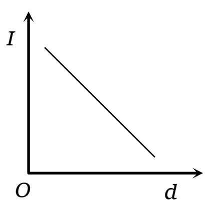
(c) $276\ J$

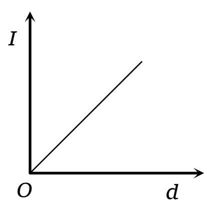
(d) $46\ J$

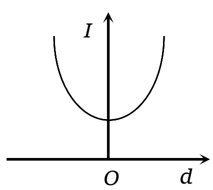
**56.** On account of melting of ice at the north pole the moment of inertia of spinning earth

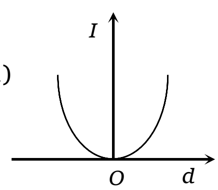
(a) Increases (b) Decreases (c) Remains unchanged (d) Depends on the time

**57.** According to the theorem of parallel axes $I = I_g + Md^2$, the graph between $I$ and $d$ will be

(a)



(b)


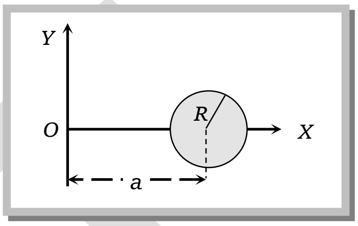

(c)



(d)



**58.** What is the moment of inertia of a square sheet of side $l$ and mass per unit area $\mu$ about an axis passing through the centre and perpendicular to its plane

(a) $\frac{\mu l^2}{12}$ (b) $\frac{\mu l^2}{6}$ (c) $\frac{\mu l^4}{12}$ (d) $\frac{\mu l^4}{6}$

**59.** The adjoining figure shows a disc of mass $M$ and radius $R$ lying in the $X$-$Y$ plane with its centre on $X-$axis at a distance $a$ from the origin. Then the moment of inertia of the disc about the $X$-axis is




(a) $M\left(\frac{R^2}{2}\right)$

(b) $M\left(\frac{R^2}{4}\right)$

(c) $M\left(\frac{R^2}{4}+a^2\right)$

(d) $M\left(\frac{R^2}{2}+a^2\right)$

**60.** We have two spheres, one of which is hollow and the other solid. They have identical masses and moment of inertia about their respective diameters. The ratio of their radius is given by

(a) $5:7$ (b) $3:5$ (c) $\sqrt{3}:\sqrt{5}$ (d) $\sqrt{3}:\sqrt{7}$

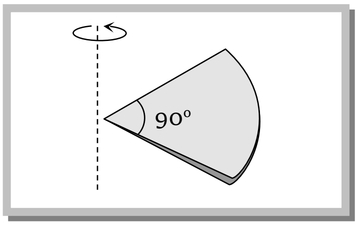
## ▶▶ *Advance level*

**61.** From a uniform wire, two circular loops are made (i) $P$ of radius $r$ and (ii) $Q$ of radius $nr$. If the moment of inertia of $Q$ about an axis passing through its centre and perpendicular to its plane is 8 times that of $P$ about a similar axis, the value of $n$ is (diameter of the wire is very much smaller than $r$ or $nr$)

(a) 8 (b) 6 (c) 4 (d) 2

**62.** One quarter sector is cut from a uniform circular disc of radius $R$. This sector has mass $M$. It is made to rotate about a line perpendicular to its plane and passing through the centre of the original disc. Its moment of inertia about the axis of rotation is

**[IIT-JEE (Screening) 2001]**




(a) $\frac{1}{2}MR^2$

(b) $\frac{1}{4}MR^2$

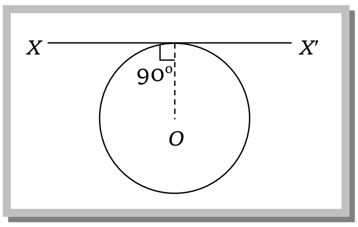
(c) $\frac{1}{8}MR^2$

(d) $\sqrt{2}\,MR^2$

**63.** Two discs of same thickness but of different radii are made of two different materials such that their masses are same. The densities of the materials are in the ratio 1 : 3. The moments of inertia of these discs about the respective axes passing through their centres and perpendicular to their planes will be in the ratio

(a) 1 : 3 (b) 3 : 1 (c) 1 : 9 (d) 9 : 1

**64.** A thin wire of length $L$ and uniform linear mass density $\rho$ is bent into a circular loop with centre at $O$ as shown. The moment of inertia of the loop about the axis $XX'$ is **[IIT-JEE (Screening) 2000]**




(a) $\frac{\rho L^3}{8\pi^2}$

(b) $\frac{\rho L^3}{16\pi^2}$

(c) $\frac{5\rho L^3}{16\pi^2}$

(d) $\frac{3\rho L^3}{8\pi^2}$

**65.** If solid sphere and solid cylinder of same radius and density rotate about their own axis, the moment of inertia will be greater for ($L = R$)

(a) Solid sphere (b) Solid cylinder (c) Both (d) Equal both

**66.** Two point masses of 0.3 *kg* and 0.7 *kg* are fixed at the ends of a rod of length 1.4 *m* and of negligible mass. The rod is set rotating about an axis perpendicular to its length with a uniform angular speed. The point on the rod through which the axis should pass in order that the work required for rotation of the rod is minimum is located at a distance of **[IIT-JEE 1995; AIIMS 2000]**

(a) 0.4 *m* from mass of 0.3 *kg* (b) 0.98 from mass of 0.3 *kg*

(c) 0.70 *m* from mass of 0.7 *kg* (d) 0.98 *m* from mass of 0.7 *kg*

**67.** A circular disc *A* of radius *r* is made from an iron plate of thickness *t* and another circular disc *B* of radius 4*r* is made from an iron plate of thickness *t*/4. The relation between the moments of inertia $I_A$ and $I_B$ is

(a) $I_A > I_B$ (b) $I_A = I_B$

(c) $I_A < I_B$ (d) Depends on the actual values of *t* and *r*

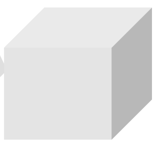
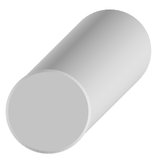
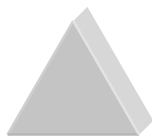
**68.** A thin wire of length *l* and mass *M* is bent in the form of a semi-circle. What is its moment of inertia about an axis passing through the ends of the wire

(a) $\frac{Ml^2}{2}$ (b) $\frac{Ml^2}{\pi^2}$ (c) $\frac{2Ml^2}{\pi^2}$ (d) $\frac{Ml^2}{2\pi^2}$

**69.** If $I_1$ is the moment of inertia of a thin rod about an axis perpendicular to its length and passing through its centre of mass, and $I_2$ is the moment of inertia of the ring formed by bending the rod, then

(a) $I_1 : I_2 = 1 : 1$ (b) $I_1 : I_2 = \pi^2 : 3$ (c) $I_1 : I_2 = \pi : 4$ (d) $I_1 : I_2 = 3 : 5$

**70.** Four solids are shown in cross section. The sections have equal heights and equal maximum widths. They have the same mass. The one which has the largest rotational inertia about a perpendicular through the centre of mass is

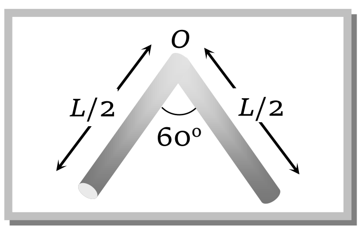
(a)  (b)  (c)  (d) 

**71.** The moment of inertia *I* of a solid sphere having fixed volume depends upon its volume *V* as

(a) $I \propto V$ (b) $I \propto V^{2/3}$ (c) $I \propto V^{5/3}$ (d) $I \propto V^{3/2}$

**72.** A thin rod of length *L* and mass *M* is bent at the middle point *O* at an angle of $60^o$ as shown in figure. The moment of inertia of the rod about an axis passing through *O* and perpendicular to the plane of the rod will be

(a) $\frac{ML^2}{6}$



(b) $\frac{ML^2}{12}$

(c) $\frac{ML^2}{24}$

(d) $\frac{ML^2}{3}$

## *Problems based on angular momentum*

### ▶ *Basic level*

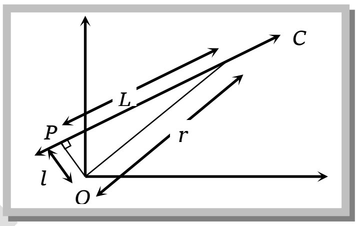
**73.** The motion of planets in the solar system is an example of the conservation of **[AIIMS 2003]**

(a) Mass (b) Linear momentum (c) Angular momentum (d) Energy

**74.** A disc is rotating with an angular speed of $\omega$. If a child sits on it, which of the following is conserved

(a) Kinetic energy (b) Potential energy (c) Linear momentum (d) Angular momentum

**75.** A particle of mass $m$ moves along line $PC$ with velocity $v$ as shown. What is the angular momentum of the particle about $O$ **[AIEEE 2002]**




(a) $mvL$

(b) $mvl$

(c) $mvr$

(d) Zero

**76.** Two rigid bodies $A$ and $B$ rotate with rotational kinetic energies $E_A$ and $E_B$ respectively. The moments of inertia of $A$ and $B$ about the axis of rotation are $I_A$ and $I_B$ respectively. If $I_A = I_B/4$ and $E_A = 100\ E_B$ the ratio of angular momentum ($L_A$) of $A$ to the angular momentum ($L_B$) of $B$ is

(a) 25 (b) 5/4 (c) 5 (d) 1/4

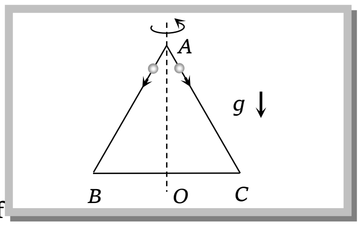
**77.** A uniform heavy disc is rotating at constant angular velocity $\omega$ about a vertical axis through its centre and perpendicular to the plane of the disc. Let $L$ be its angular momentum. A lump of plasticine is dropped vertically on the disc and sticks to it. Which will be constant

**[AMU (Med.) 2001]**

(a) $\omega$ (b) $\omega$ and $L$ both (c) $L$ only (d) Neither $\omega$ nor $L$

**78.** An equilateral triangle $ABC$ formed from a uniform wire has two small identical beads initially located at $A$. The triangle is set rotating about the vertical axis $AO$. Then the beads are released from rest simultaneously and allowed to slide down, one along $AB$ and the other along $AC$ as shown. Neglecting frictional effects, the quantities that are conserved as the beads slide down, are

**[IIT-JEE (Screening) 2000]**




(a) Angular velocity and total energy (kinetic and potential)

(b) Total angular momentum and total energy

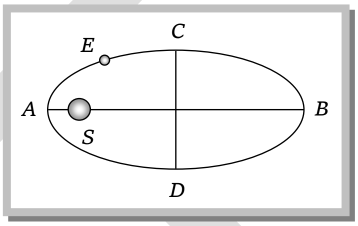
(c) Angular velocity and moment of inertia about the axis of rotation

(d) Total angular momentum and moment of inertia about the axis of

**79.** A thin circular ring of mass $M$ and radius $r$ is rotating about its axis with a constant angular velocity $\omega$. Two objects each of mass $m$ are attached gently to the opposite ends of a diameter of the ring. The ring will now rotate with an angular velocity

(a) $\frac{\omega(M-2m)}{M+2m}$ (b) $\frac{\omega M}{M+2m}$ (c) $\frac{\omega M}{M+m}$ (d) $\frac{\omega(M+2m)}{M}$

**80.** The earth $E$ moves in an elliptical orbit with the sun $S$ at one of the foci as shown in the figure. Its speed of motion will be maximum at the point




(a) $C$

(b) $A$

(c) $B$

(d) $D$

**81.** A rigid spherical body is spinning around an axis without any external torque. Due to change in temperature, the volume increases by 1%. Its angular speed

(a) Will increase approximately by 1% (b) Will decrease approximately by 1%

(c) Will decrease approximately by 0.67% (d) Will decrease approximately by 0.33%

**82.** A uniform disc of mass $M$ and radius $R$ is rotating about a horizontal axis passing through its centre with angular velocity $\omega$. A piece of mass $m$ breaks from the disc and flies off vertically upwards. The angular speed of the disc will be

(a) $\frac{(M-2m)\omega}{(M-m)}$ (b) $\frac{(M+2m)\omega}{(M+m)}$ (c) $\frac{(M-2m)\omega}{(M+m)}$ (d) $\frac{(M+2m)\omega}{(M-m)}$

## ▶▶ *Advance level*

**83.** A particle undergoes uniform circular motion. About which point on the plane of the circle, will the angular momentum of the particle remain conserved

(a) Centre of the circle (b) On the circumference of the circle

(c) Inside the circle (d) Outside the circle

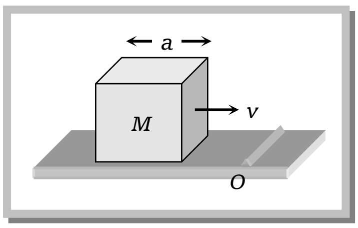
**84.** A thin uniform circular disc of mass $M$ and radius $R$ is rotating in a horizontal plane about an axis passing through its centre and perpendicular to its plane with an angular velocity $\omega$. Another disc of same dimension but of mass $M/4$ is placed gently on the first disc coaxially. The angular velocity of the system now is

(a) $2\omega/5$ (b) $2\omega/\sqrt{5}$ (c) $4\omega/5$ (d) $4\omega/\sqrt{5}$

**85.** A smooth sphere A is moving on a frictionless horizontal plane with angular speed $\omega$ and center of mass with velocity $v$. It collides elastically and head-on with an identical sphere $B$ at rest. Neglect friction everywhere. After the collision, their angular speeds are $\omega_A$ and $\omega_B$ respectively. Then

(a) $\omega_A < \omega_B$ (b) $\omega_A = \omega_B$ (c) $\omega_A = \omega$ (d) $\omega = \omega_B$

**86.** A cubical block of side $a$ is moving with velocity $v$ on a horizontal smooth plane as shown. It hits a ridge at point $O$. The angular speed of the block after it hits $O$ is



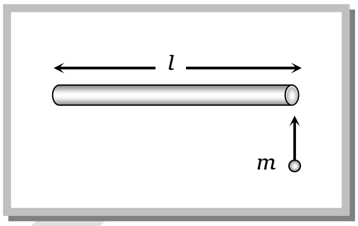

(a) $3v/4a$

(b) $3v/2a$

(c) $\frac{\sqrt{3}v}{\sqrt{2}a}$

(d) Zero

**87.** A stick of length $l$ and mass $M$ lies on a frictionless horizontal surface on which it is free to move in any way. A ball of mass $m$ moving with speed $v$ collides elastically with the stick as shown in the figure. If after the collision ball comes to rest, then what should be the mass of the ball




(a) $m = 2M$

(b) $m = M$

(c) $m = M/2$

(d) $m = M/4$

**88.** In a playground there is a merry-go-round of mass 120 *kg* and radius 4 *m*. The radius of gyration is 3*m*. A child of mass 30 *kg* runs at a speed of 5 *m/sec* tangent to the rim of the merry-go-round when it is at rest and then jumps on it. Neglect friction and find the angular velocity of the merry-go-round and child

(a) 0.2 *rad/sec* (b) 0.1 *rad/sec* (c) 0.4 *rad/sec* (d) 0.8 *rad/sec*

## Problems based on kinetic energy, work and power

### ▶ Basic level

**89.** A ball rolls without slipping. The radius of gyration of the ball about an axis passing through its centre of mass *K*. If radius of the ball be *R*, then the fraction of total energy associated with its rotational energy will be

(a) $\frac{K^2}{R^2}$ (b) $\frac{K^2}{K^2+R^2}$ (c) $\frac{R^2}{K^2+R^2}$ (d) $\frac{K^2+R^2}{R^2}$

**90.** In a bicycle the radius of rear wheel is twice the radius of front wheel. If $r_F$ and $r_r$ are the radii, $v_F$ and $v_r$ are speeds of top most points of wheel, then

(a) $v_r = 2\,v_F$ (b) $v_F = 2\,v_r$ (c) $v_F = v_r$ (d) $v_F > v_r$

**91.** The total kinetic energy of a body of mass 10 *kg* and radius 0.5 *m* moving with a velocity of 2 *m/s* without slipping is 32.8 *joule*. The radius of gyration of the body is

(a) 0.25 *m* (b) 0.2 *m* (c) 0.5 *m* (d) 0.4 *m*

**92.** The moment of inertia of a body about a given axis is 2.4 $kg\text{–}m^2$. To produce a rotational kinetic energy of 750 *J*, an angular acceleration of 5 $rad/s^2$ must be applied about that axis for

(a) 6 *sec* (b) 5 *sec* (c) 4 *sec* (d) 3 *sec*

**93.** A solid sphere of mass 500 *gm* and radius 10 *cm* rolls without slipping with the velocity 20*cm/s*. The total kinetic energy of the sphere will be

(a) 0.014 *J* (b) 0.028 *J* (c) 280 *J* (d) 140 *J*

**94.** The ratio of rotational and translatory kinetic energies of a sphere is **[KCET (Med.) 2001; AFMC 2001]**

(a) $\frac{2}{9}$ (b) $\frac{2}{7}$ (c) $\frac{2}{5}$ (d) $\frac{7}{2}$

**95.** A thin hollow cylinder open at both ends:

(i) Slides without rotating

(ii) Rolls without slipping, with the same speed.

The ratio of kinetic energy in the two cases is **[KCET (Engg./Med.) 2000]**

(a) 1 : 1 (b) 4 : 1 (c) 1 : 2 (d) 2 : 1

**96.** A spherical ball rolls on a table without slipping. Then the fraction of its total energy associated with rotation is

**[MP PMT 1987; BHU 1998]**

(a) $\frac{2}{5}$ (b) $\frac{2}{7}$ (c) $\frac{3}{5}$ (d) $\frac{3}{7}$

**97.** A body is rolling without slipping on a horizontal plane. If the rotational energy of the body is 40% of the total kinetic energy, then the body might be

(a) Cylinder (b) Hollow sphere (c) Solid cylinder (d) Ring

**98.** A body of moment of inertia of $3kg-m^2$ rotating with an angular speed of 2 *rad/sec* has the same kinetic energy as a mass of 12 *kg* moving with a speed of

(a) 1 *m/s* (b) 2 *m/s* (c) 4 *m/s* (d) 8 *m/s*

**99.** Ratio of kinetic energy and rotational energy in the motion of a disc is

(a) 1 : 1 (b) 2 : 7 (c) 1 : 2 (d) 3 : 1

**100.** A solid sphere is moving on a horizontal plane. Ratio of its transitional Kinetic energy and rotational energy is **[CPMT**

(a) 1/5 (b) 5/2 (c) 3/5 (d) 5/7

**101.** The speed of rolling of a ring of mass $M$ changes from $V$ to $3V$. What is the change in its kinetic energy

(a) $3MV^2$ (b) $4MV^2$ (c) $6MV^2$ (d) $8MV^2$

**102.** A disc of radius $1m$ and mass $4kg$ rolls on a horizontal plane without slipping in such a way that its centre of mass moves with a speed of $10\,cm/\sec$. Its rotational kinetic energy is

(a) $0.01$ *erg* (b) $0.02$ *joule* (c) $0.03$ *joule* (d) $0.01$ *joule*

**103.** The ratio of kinetic energies of two spheres rolling with equal centre of mass velocities is 2 : 1. If their radii are in the ratio 2 : 1; then the ratio of their masses will be

(a) 2:1 (b) 1:8 (c) 1:7 (d) $2\sqrt{2}:1$

**104.** A symmetrical body of mass $M$ and radius $R$ is rolling without slipping on a horizontal surface with linear speed *v*. Then its angular speed is

(a) $v/R$ (b) Continuously increasing

(c) Dependent on mass $M$ (d) Independent of radius ($R$)

**105.** A solid sphere of mass 1 *kg* rolls on a table with linear speed 1 *m/s*. Its total kinetic energy is

(a) 1 *J* (b) 0.5 *J* (c) 0.7 *J* (d) 1.4 *J*

**106.** A circular disc has a mass of 1 *kg* and radius 40 *cm*. It is rotating about an axis passing through its centre and perpendicular to its plane with a speed of 10 rev/s. The work done in joules in stopping it would be

(a) 4 (b) 47.5 (c) 79 (d) 158

**107.** Rotational kinetic energy of a given body about an axis is proportional to

(a) Time period (b) $(\text{Time period})^2$ (c) $(\text{Time period})^{-1}$ (d) $(\text{time period})^{-2}$

**108.** If a body completes one revolution in $\pi$ sec then the moment of inertia would be

(a) Equal to rotational kinetic energy (b) Double of rotational kinetic energy

(c) Half of rotational kinetic energy (d) Four times the rotational kinetic energy

**109.** A tangential force $F$ is applied on a disc of radius $R$, due to which it deflects through an angle $\theta$ from its initial position. The work done by this force would be

(a) $FR$ (b) $F\theta$ (c) $\frac{FR}{\theta}$ (d) $FR\theta$

**110.** If the rotational kinetic energy of a body is increased by 300% then the percentage increase in its angular momentum will be

(a) 600% (b) 150% (c) 100% (d) 1500%

**111.** A wheel of moment of inertia 10 $kg\text{-}m^2$ is rotating at 10 rotations per minute. The work done in increasing its speed to 5 times its initial value, will be

(a) 100 *J* (b) 131.4 *J* (c) 13.4 *J* (d) 0.131 *J*

**112.** A flywheel has moment of inertia $4kg-m^2$ and has kinetic energy of 200 *J*. Calculate the number of revolutions it makes before coming to rest if a constant opposing couple of $5N-m$ is applied to the flywheel

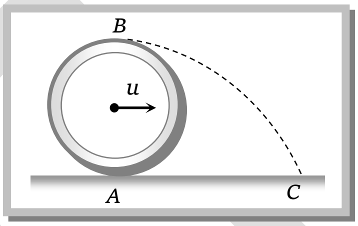
(a) 12.8 *rev* (b) 24 *rev* (c) 6.4 *rev* (d) 16 *rev*

**113.** An engine develops 100 *kW*, when rotating at 1800 rpm. Torque required to deliver the power is

(a) 531 *N-m* (b) 570 *N-m* (c) 520 *N-m* (d) 551 *N-m*

## ▶▶ *Advance level*

**114.** A wheel of radius *r* rolls without slipping with a speed *v* on a horizontal road. When it is at a point *A* on the road, a small jump of mud separates from the wheel at its highest point *B* and drops at point *C* on the road. The distance *AC* will be




(a) $v\sqrt{\frac{r}{g}}$

(b) $2v\sqrt{\frac{r}{g}}$

(c) $4v\sqrt{\frac{r}{g}}$

(d) $\sqrt{\frac{3r}{g}}$

**115.** A fly wheel of moment of inertia *I* is rotating at *n* revolutions per sec. The work needed to double the frequency would be

(a) $2\pi^2 In^2$ (b) $4\pi^2 In^2$ (c) $6\pi^2 In^2$ (d) $8\pi^2 In^2$

**116.** If *L, M* and *P* are the angular momentum, mass and linear momentum of a particle respectively which of the following represents the kinetic energy of the particle when the particle rotates in a circle of radius *R*

(a) $\frac{L^2}{2M}$ (b) $\frac{P^2}{2MR}$ (c) $\frac{L^2}{2MR^2}$ (d) $\frac{MP}{2}$

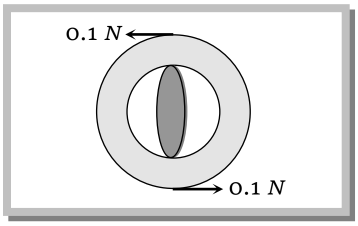
**117.** A uniform thin rod of length *l* is suspended from one of its ends and is rotated at *f* rotations per second. The rotational kinetic energy of the rod will be

(a) $\frac{2}{3}\pi^2 f^2 ml^2$ (b) $\frac{4}{3} f^2 ml^2$ (c) $4\pi^2 f^2 ml^2$ (d) Zero

**118.** A body rotating at 20 *rad/sec* is acted upon by a constant torque providing it a deceleration of 2 $rad/sec^2$. At what time will the body have kinetic energy same as the initial value if the torque continues to act

(a) 20 *secs* (b) 40 *secs* (c) 5 *secs* (d) 10 *secs*

**119.** Part of the tuning arrangement of a radio consists of a wheel which is acted on by two parallel constant forces as shown in the fig. If the wheel rotates just once, the work done will be about (diameter of the wheel = 0.05*m*)




(a) 0.062 *J*

(b) 0.031 *J*

(c) 0.015 *J*

(d) 0.057 *J*

## *Problems based on rolling on incline plane*

## ▶ *Basic level*

**120.** A solid sphere, a hollow sphere and a ring are released from top of an inclined plane (frictionless) so that they slide down the plane. Then maximum acceleration down the plane is for (no rolling)

(a) Solid sphere (b) hollow sphere (c) Ring (d) All same

**121.** A solid sphere (mass 2 *M*) and a thin hollow spherical shell (mass *M*) both of the same size, roll down an inclined plane, then

**[Kerala (Engg.) 2002]**

(a) Solid sphere will reach the bottom first
(b) Hollow spherical shell will reach the bottom first
(c) Both will reach at the same time
(d) None of these

**122.** A hollow cylinder and a solid cylinder having the same mass and same diameter are released from rest simultaneously from the top of an inclined plane. Which will reach the bottom first

(a) The solid cylinder
(b) The hollow cylinder
(c) Both will reach the bottom together
(d) The greater density

**123.** The speed of a homogeneous solid sphere after rolling down an inclined plane of vertical height *h*, from rest without sliding, is

**[CBSE PMT 1992]**

(a) $\sqrt{\frac{10}{7}gh}$
(b) $\sqrt{gh}$
(c) $\sqrt{\frac{6}{5}gh}$
(d) $\sqrt{\frac{4}{3}gh}$

**124.** A solid cylinder rolls down an inclined plane from a height *h*. At any moment the ratio of rotational kinetic energy to the total kinetic energy would be

(a) 1 : 2
(b) 1 : 3
(c) 2 : 3
(d) 1 : 1

**125.** An inclined plane makes an angle of $30^o$ with the horizontal. A solid sphere rolling down this inclined plane from rest without slipping has a linear acceleration equal to

(a) $\frac{g}{3}$
(b) $\frac{2g}{3}$
(c) $\frac{5g}{7}$
(d) $\frac{5g}{14}$

**126.** A solid cylinder of mass *M* and radius *R* rolls down an inclined plane without slipping. The speed of its centre of mass when it reaches the bottom is

(a) $\sqrt{2gh}$
(b) $\sqrt{\frac{4}{3}gh}$
(c) $\sqrt{\frac{3}{4}gh}$
(d) $\sqrt{4\frac{g}{h}}$

**127.** Solid cylinders of radii $r_1, r_2$ and $r_3$ roll down an inclined plane from the same place simultaneously. If $r_1 > r_2 > r_3$, which one would reach the bottom first

(a) Cylinder of radius $r_1$
(b) Cylinder of radius $r_2$
(c) Cylinder of radius $r_3$
(d) All the three cylinders simultaneously

**128.** A solid cylinder of radius *R* and mass *M*, rolls down on inclined plane without slipping and reaches the bottom with a speed *v*. The speed would be less than *v* if we use

(a) A cylinder of same mass but of smaller radius
(b) A cylinder of same mass but of larger radius
(c) A cylinder of same radius but of smaller mass
(d) A hollow cylinder of same mass and same radius

**129.** A body starts rolling down an inclined plane of length *L* and height *h*. This body reaches the bottom of the plane in time *t*. The relation between *L* and *t* is

(a) $t \propto L$
(b) $t \propto 1/L$
(c) $t \propto L^2$
(d) $t \propto \frac{1}{L^2}$

**130.** A hollow cylinder is rolling on an inclined plane, inclined at an angle of $30^o$ to the horizontal. Its speed after travelling a distance of 10 *m* will be

(a) 49 *m/sec*
(b) 0.7 *m/sec*
(c) 7 *m/sec*
(d) Zero

**131.** A solid sphere, a solid cylinder, a disc and a ring are rolling down an inclined plane. Which of these bodies will reach the bottom simultaneously

(a) Solid sphere and solid cylinder
(b) Solid cylinder and disc
(c) Disc and ring
(d) Solid sphere and ring

**132.** A ball of radius 11 *cm* and mass 8 *kg* rolls from rest down a ramp of length 2*m*. The ramp is inclined at $35^o$ to the horizontal. When the ball reaches the bottom, its velocity is (sin 35° = 0.57)

(a) 2 *m/s*
(b) 5 *m/s*
(c) 4 *m/s*
(d) 6 *m/s*

**133.** From an inclined plane a sphere, a disc, a ring and a shell are rolled without slipping. The order of their reaching at the base will be

(a) Ring, shell, disc, sphere (b) Shell, sphere, disc, ring (c) Sphere, disc, shell, ring(d)

**134.** A solid cylinder 30 *cm* in diameter at the top of an inclined plane 2.0 *m* high is released and rolls down the incline without loss of energy due to friction. Its linear speed at the bottom is

(a) 5.29 *m/sec* (b) $4.1 \times 10^3$ *m/sec* (c) 51 *m/sec* (d) 51 *cm/sec*

**135.** A cylinder of mass $M$ and radius $R$ rolls on an inclined plane. The gain in kinetic energy is

(a) $\frac{1}{2}Mv^2$ (b) $\frac{1}{2}I\omega^2$ (c) $\frac{3}{4}Mv^2$ (d) $\frac{3}{4}I\omega^2$

**136.** A disc of radius $R$ is rolling down an inclined plane whose angle of inclination is $\theta$, Its acceleration would be

(a) $\frac{5}{7}g\sin\theta$ (b) $\frac{2}{3}g\sin\theta$ (c) $\frac{1}{2}g\sin\theta$ (d) $\frac{3}{5}g\sin\theta$

**137.** A solid cylinder (i) rolls down (ii) slides down an inclined plane. The ratio of the accelerations in these conditions is

(a) 3 : 2 (b) 2 : 3 (c) $\sqrt{3} : \sqrt{2}$ (d) $\sqrt{2} : \sqrt{3}$

**138.** The acceleration of a body rolling down on an inclined plane does not depend upon

(a) Angle of inclination of the plane (b) Length of plane

(c) Acceleration due to gravity of earth (d) Radius of gyration of body

**139.** A ring, a solid sphere, a disc and a solid cylinder of same radii roll down an inclined plane, which would reach the bottom in the last

(a) Ring (b) Disc (c) Solid sphere (d) Solid cylinder

**140.** A ring is rolling on an inclined plane. The ratio of the linear and rotational kinetic energies will be

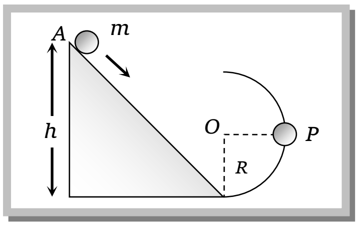
(a) 2 : 1 (b) 1 : 2 (c) 1 : 1 (d) 4 : 1

**141.** The moment of inertia of a solid cylinder about its axis is *I*. It is allowed to roll down an incline without slipping. If its angular velocity at the bottom be $\omega$, then K.E. of the cylinder will be

(a) $I\omega^2$ (b) $\frac{3}{2}I\omega^2$ (c) $2I\omega^2$ (d) $\frac{1}{2}I\omega^2$

## ▶▶ *Advance level*

**142.** A solid ball of mass $m$ and radius $r$ rolls without slipping along the track shown in the fig. The radius of the circular part of the track is $R$. The ball starts rolling down the track from rest from a height of $8R$ from the ground level. When the ball reaches the point P then its velocity will be




(a) $\sqrt{gR}$

(b) $\sqrt{5gR}$

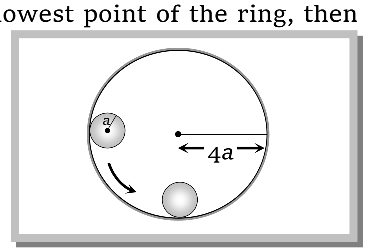
(c) $\sqrt{10gR}$

(d) $\sqrt{3gR}$

**143.** A ring takes time $t_1$ in slipping down an inclined plane of length $L$ and takes time $t_2$ in rolling down the same plane. The ratio $\frac{t_1}{t_2}$ is

(a) $\sqrt{2}:1$ (b) $1:\sqrt{2}$ (c) 1 : 2 (d) 2 : 1

**144.** A ring of radius $4a$ is rigidly fixed in vertical position on a table. A small disc of mass $m$ and radius $a$ is released as shown in the fig. When the disc rolls down, without slipping, to the lowest point of the ring, then its speed will be



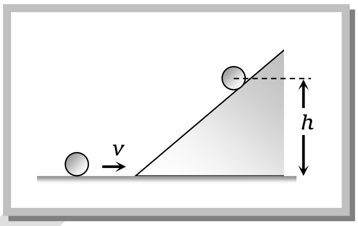

(a) $\sqrt{ga}$

(b) $\sqrt{2ga}$

(c) $\sqrt{3ga}$

(d) $\sqrt{4ga}$

**145.** A disc of mass $M$ and radius $R$ rolls in a horizontal surface and then rolls up an inclined plane as shown in the fig. If the velocity of the disc is $v$, the height to which the disc will rise will be



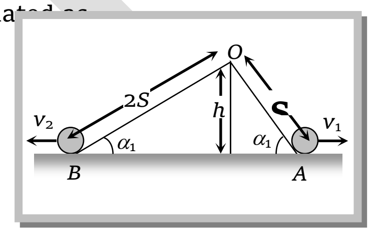

(a) $\frac{3v^2}{2g}$

(b) $\frac{3v^2}{4g}$

(c) $\frac{v^2}{4g}$

(d) $\frac{v^2}{2g}$

**146.** Two uniform similar discs roll down two inclined planes of length $S$ and $2S$ respectively as shown is the fig. The velocities of two discs at the points $A$ and $B$ of the inclined planes are related as




(a) $v_1 = v_2$

(b) $v_1 = 2v_2$

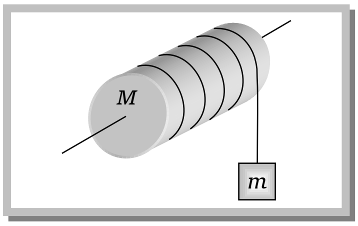
(c) $v_1 = v_1 \frac{v_2}{4}$

(d) $v_1 = \frac{3}{4} v_2$

## Problems based on motion of connected mass

### Basic level

**147.** A mass $M$ is supported by a mass less string would wound a uniform cylinder of mass $M$ and radius $R$. On releasing the mass from rest, it will fall with acceleration



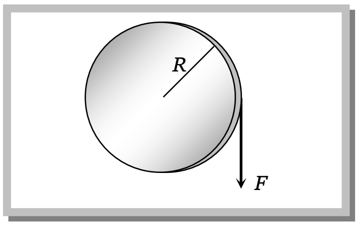

(a) $g$

(b) $\frac{g}{2}$

(c) $\frac{g}{3}$

(d) $\frac{2g}{3}$

**148.** A uniform disc of radius $R$ and mass $M$ can rotate on a smooth axis passing through its centre and perpendicular to its plane. A force $F$ is applied on its rim. See fig. What is the tangential acceleration




(a) $\frac{2F}{M}$

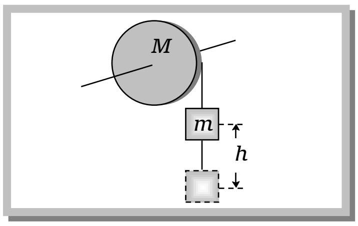
(b) $\frac{F}{M}$

(c) $\frac{F}{2M}$

(d) $\frac{F}{4M}$

**149.** A massless string is wrapped round a disc of mass $M$ and radius $R$. Another end is tied to a mass $m$ which is initially at height $h$ from ground level as shown in the fig. If the mass is released then its velocity while touching the ground level will be

(a) $\sqrt{2gh}$

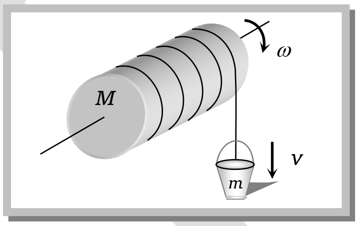


(b) $\sqrt{2gh}\dfrac{M}{m}$

(c) $\sqrt{2ghm / M}$

(d) $\sqrt{4mgh / 2m + M}$

**150.** A cylinder of mass $M$ and radius $r$ is mounted on a frictionless axle over a well. A rope of negligible mass is wrapped around the cylinder and a bucket of mass $m$ is suspended from the rope. The linear acceleration of the bucket will be




(a) $\dfrac{Mg}{M+2m}$

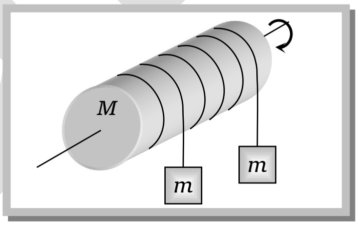
(b) $\dfrac{2Mg}{m+2M}$

(c) $\dfrac{Mg}{2M+m}$

(d) $\dfrac{2mg}{M+2m}$

## ▶▶ *Advance level*

**151.** A uniform solid cylinder of mass $M$ and radius $R$ rotates about a frictionless horizontal axle. Two similar masses suspended with the help two ropes wrapped around the cylinder. If the system is released from rest then the acceleration of each mass will be




(a) $\dfrac{4mg}{M+2m}$

(b) $\dfrac{4mg}{M+4m}$

(c) $\dfrac{2mg}{M+m}$

(d) $\dfrac{2mg}{M+2m}$

**152.** In the above problem the angular velocity of the cylinder, after the masses fall down through distance $h$, will be

(a) $\dfrac{1}{R}\sqrt{8mgh/(M+4m)}$ (b) $\dfrac{1}{R}\sqrt{8mgh/(M+m)}$ (c) $\dfrac{1}{R}\sqrt{mgh/(M+m)}$ (d) $\dfrac{1}{R}\sqrt{8mgh/(M+2m)}$

## *Problems based on compound pendulum*

## ▶ *Basic level*

**153.** A *metre* scale is suspended vertically from a horizontal axis passing through one end of it. Its time period would be

(a) 1.64 *sec* (b) 2 *sec* (c) 2.5 *sec* (d) 3.2 *sec*

**154.** A disc of radius $R$ is made to oscillate about a horizontal axis passing through its periphery. Its time period would be

(a) $2\pi\sqrt{\dfrac{3R}{2g}}$ (b) $2\pi\sqrt{\dfrac{2R}{3g}}$ (c) $2\pi\sqrt{\dfrac{R}{g}}$ (d) $2\pi\sqrt{\dfrac{2R}{g}}$

## Advance level

**155.** A solid cube of side $l$ is made to oscillate about a horizontal axis passing through one of its edges. Its time period will be

(a) $2\pi\sqrt{\frac{2\sqrt{2}}{3}\frac{l}{g}}$ (b) $2\pi\sqrt{\frac{2}{3}\frac{l}{g}}$ (c) $2\pi\sqrt{\frac{\sqrt{3}}{2}\frac{l}{g}}$ (d) $2\pi\sqrt{\frac{2}{\sqrt{3}}\frac{l}{g}}$

**156.** The string of a simple pendulum is replaced by a uniform rod of length $L$ and mass $M$. If the mass of the bob of the pendulum is $m$, then for small oscillations its time period would be (assume radius of bob $r << L$)

(a) $2\pi\sqrt{\frac{2(M+3m)L}{3(M+2m)g}}$ (b) $2\pi\sqrt{\frac{(M+2m)L}{3(M+3m)g}}$ (c) $2\pi\sqrt{\left(\frac{2M}{3m}\right)\frac{L}{g}}$ (d) $2\pi\sqrt{\left(\frac{M+m}{M+3m}\right)\frac{L}{g}}$

# Answer Sheet (Practice problems)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** | **2.** | **3.** | **4.** | **5.** | **6.** | **7.** | **8.** | **9.** | **10.** |
| b | c | c | b | c | a | a | a | c | c |
| **11.** | **12.** | **13.** | **14.** | **15.** | **16.** | **17.** | **18.** | **19.** | **20.** |
| b | a | c | c | a | a | a | b | c | d |
| **21.** | **22.** | **23.** | **24.** | **25.** | **26.** | **27.** | **28.** | **29.** | **30.** |
| a | d | c | b | b | c | d | c | a | b |
| **31.** | **32.** | **33.** | **34.** | **35.** | **36.** | **37.** | **38.** | **39.** | **40.** |
| d | c | c | c | c | d | b | c | b | c |
| **41.** | **42.** | **43.** | **44.** | **45.** | **46.** | **47.** | **48.** | **49.** | **50.** |
| b | a | b | a | a | d | a | b | a | d |
| **51.** | **52.** | **53.** | **54.** | **55.** | **56.** | **57.** | **58.** | **59.** | **60.** |
| b | d | d | d | b | a | c | d | b | c |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **61.** | **62.** | **63.** | **64.** | **65.** | **66.** | **67.** | **68.** | **69.** | **70.** |
| d | a | b | d | a | b | c | d | b | a |
| **71.** | **72.** | **73.** | **74.** | **75.** | **76.** | **77.** | **78.** | **79.** | **80.** |
| c | b | c | d | b | c | c | b | b | b |
| **81.** | **82.** | **83.** | **84.** | **85.** | **86.** | **87.** | **88.** | **89.** | **90.** |
| c | a | a | c | c | a | d | c | b | c |
| **91.** | **92.** | **93.** | **94.** | **95.** | **96.** | **97.** | **98.** | **99.** | **100.** |
| d | b | a | c | c | b | b | a | d | b |
| **101.** | **102.** | **103.** | **104.** | **105.** | **106.** | **107.** | **108.** | **109.** | **110.** |
| d | d | a | a | c | d | d | c | d | c |
| **111.** | **112.** | **113.** | **114.** | **115.** | **116.** | **117.** | **118.** | **119.** | **120.** |
| b | c | a | c | c | c | a | a | b | d |
| **121.** | **122.** | **123.** | **124.** | **125.** | **126.** | **127.** | **128.** | **129.** | **130.** |
| a | a | a | b | d | b | d | d | a | c |
| **131.** | **132.** | **133.** | **134.** | **135.** | **136.** | **137.** | **138.** | **139.** | **140.** |
| b | c | c | a | c | b | b | b | a | c |
| **141.** | **142.** | **143.** | **144.** | **145.** | **146.** | **147.** | **148.** | **149.** | **150.** |
| b | c | b | d | b | a | d | a | d | d |
| **151.** | **152.** | **153.** | **154.** | **155.** | **156.** | | | | |
| b | a | a | a | a | a | | | | |