T confidence intervals

CLT $\quad Est \pm ZQ \times SE_{Est}$

$\quad Est \pm TQ \times SE_{Est}$

$T \xrightarrow[n \to \infty]{} Z$

William Gosset

Degrees of freedom $\quad \dfrac{\bar{X} - \mu}{S/\sqrt{n}}$ follows t distr with $n-1$ df

Interval is $\bar{X} \pm t_{n-1} S/\sqrt{n}$ where t_{n-1} is the relevant quantile.

T → larger CI

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t interval

It assumes that iid is normal, however, it works well whenever the distribution is roughly symmetrical and mound shaped.

For large degrees of freedom, t quantiles become the same as standard normal quantiles. Therefore this interval converges to the same interval as the CLT yelded

For skewed distributions, the assumptions are violated. In this case the data needs scaling.

evaluated at n-1df

$$mn \pm c(-1,1) \cdot qt(0.975,\ n-1) \cdot s/sqrt(n)$$

mean difference

$\pm$

relevant T quantile

times the SE interval

```
t.test(extra ~ I(relevel(group, 2)), paired = TRUE, data = sleep)
```

outcome is a function of groups

Comparing across independent groups:

S_p pooled sd

$$\bar{Y} - \bar{X} \pm t_{n_x + n_y - 2,\ 1-\alpha/2}\ S_p \left(\frac{1}{n_x} + \frac{1}{n_y}\right)^{1/2}$$

average in one group - average in the other group

times the relative t quantile

standard error of the difference

$$n_x + n_y - 2$$

num observ. in x group

num obs in y group

$$s_p^2 = \{(n_x-1)s_x^2 + (n_y-1)s_y^2\}/(n_x+n_y-2)$$

weighted average of variance

d cast from reshape

Unequal variance.

$$\bar{Y} - \bar{X} \pm t_{df} \times \left(\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}\right)^{1/2}$$

$\uparrow$ Standard error

$\downarrow$

approximation ~~w~~ with t distribution

$$df = \frac{\left(s_x^2/n_x + s_y^2/n_y\right)^2}{\left(\frac{s_x^2}{n_x}\right)^2/(n_x-1) + \left(\frac{s_y^2}{n_y}\right)^2/(n_y-1)}$$

When in doubt, use the unequal variance

Hypothesis testing

H_0 null hypothesis

H_a

$H_0 : \mu = 30$

$H_a : \mu > 30$ μ is mean RPI

$H_0 \to H_0$ Correct null

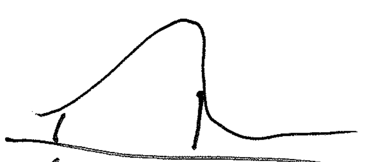
$H_0 \to H_a$ Type I err

$H_a \to H_a$ Correct reject

$H_a \to H_0$ Type II err

Two sided test

$H_a : \mu \neq 30$ split probability in 2 $\to 5/2 = 2.5\%$



qt(.025, 15) qt(.975, 15)

If you failed to reject one-sided test, you also failed to reject ~~two~~-sided test.

Two groups testing

$$\frac{\bar{X} - 30}{s/\sqrt{16}}$$

$H_a : \mu \neq 30$ $\quad 5\%/2 \rightarrow 2{,}5\% \pm$

Exact binomial test

$H_0 : p = 0{,}5$ $\quad H_a : p > 0{,}5$

P values

- Most common value

How unusual is the observed sequence?

pt(2.5, 15, lower.tail = FALSE)

Attained significance level.

$\alpha = 0.05$

The smallest value for α under which you still reject H_0