Power

Power is the probability of rejecting the null hypothesis when it is false

Type II $\rightarrow$ failing to reject when H_0 is false

The probability of type II err is called β

Note Power $= 1-\beta$

$H_0: \mu = 30$ $\quad H: \mu > 30$

Power

$$P\left(\frac{\bar{X}-30}{s/\sqrt{n}} > t_{1-\alpha, n-1} ; \mu = \mu_\alpha\right) = \alpha$$

Power $(\mu) \rightarrow$ function of difference in means

Calculating Power

We reject if $\dfrac{\bar{X}-30}{\sigma/\sqrt{n}} > Z_{1-\alpha}$

- Equivalently $\bar{X} > 30 + Z_{1-\alpha}\dfrac{\sigma}{\sqrt{n}}$

Under H_0: $\bar{X} \sim N(\mu_0, \sigma^2/n)$

under H_α: $\bar{X} \sim N(\mu_\alpha, \sigma^2/n)$

pnorm (mu0 + z · sigma/sqrt(n), mean = mua, sd = sigma/sqrt(n),
lower.tail = FALSE)

A lot of noise → less power.

When testing

$$1-\beta = P\left(\bar{X} > \mu_0 + z_{1-\alpha}\frac{\sigma}{\sqrt{n}} ; \mu = \mu_\alpha\right)$$

Where $\bar{X} \sim N(\mu_\alpha, \sigma^2/n)$

Unknowns $\mu_\alpha, \sigma, n, \beta$

Knowns μ_0, α

Specify any 3 of the unknowns and you can solve for the remainder.

The calculation for $H_\alpha: \mu < \mu_0$ is similar

For $H_\alpha: \mu \neq \mu_0$ calculate the one sided power using $\alpha/2$ (this is only approx. right, it excludes the probability of getting a large TS in the opposite direction of the truth)

α → larger → power → larger

Power of one sided test is greater than 2 sided

Power goes up as μ_1 gets further away from μ_0.

Power goes up as n goes up.

Power does not depend on these parameters, rather than it depends on the function of these parameters.

$$\frac{\sqrt{n}\,(\mu_a - \mu_0)}{\sigma}$$

$\frac{\mu_a - \mu_0}{\sigma}$ ← effect size, unit-free quantity.

T test power.

$$P\left(\frac{\bar{X} - \mu_0}{S/\sqrt{n}} > t_{1-\alpha,\,n-1} ; \mu = \mu_a\right)$$

power.t.test in r | Calculating this involves a non-central T distro.

```
power.t.test (n=16, delta = 2, sd = 4, type = "one.sample",
              alt = "one.sided") $ power

power.t.test ( power = 0.8, delta = 2/4, sd = 1,
              type = "one.sample", alt = "one.side") $ n
```

Use power.t.test or first attack on the power/size.

Bootstrapping

```
B <- 1k
resamples <- matrix(sample(x, n*B, replace = TRUE), B, n)
medians <- apply(resamples, 1, median)
quantile(medians)
```

BCA interval

Permutation tests.

- Equal distribution
- Consider a dataframe with count and spray
- Permute the spray labels
- Recalculate the statistic
  - Mean difference in counts
  - Geometric means
  - T statistic

Calculate the percentage of simulations where the simulated statistic was more extreme than the observed