**Exercise 4.1**

1. Using Matlab we calculate the eigenvectors and the corresponding eigenvalues

$$e_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \qquad e_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \qquad e_3 = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}.$$
$$\lambda_1 = 1, \qquad \lambda_2 = \tfrac{1}{2}, \qquad \lambda_3 = \tfrac{1}{4}$$

2. We can diagonalize L via $L = SL_DS^{-1}$ (with S columns beeing the eigenvectors), and thus $L^n = SL_D^nS^{-1}$ , so using

$$L_D^n \overset{n\to\infty}{\to} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\Rightarrow p^\infty = \frac{1}{6} \begin{pmatrix} 1 & 4 & 1 \end{pmatrix} \begin{pmatrix} p_-^0 \\ p^0 \\ p_+^0 \end{pmatrix}.$$

3. Letting $p^{j+1} := \frac{3}{4}p^j + \frac{1}{4}p_-^j$ and $p_-^{j+1} := \frac{3}{4}p_-^j + \frac{1}{4}p^j$ we easily conclude

$$L_{Chaikin} = \frac{1}{4} \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$$

using the first row for $p_+$ and the second for $p$. As before we compute the diagonal form

$$L_{Chaikin,D} = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

to the eigenvectors $\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \end{pmatrix}$, apply the same argument as above and observe

$$p^\infty = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} p_+^0 \\ p^0 \end{pmatrix}.$$