HW2-ML

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1. State the maximum likelihood estimates for the parameters of Gaussian Naive Bayes. Do these estimates change if we use grayscale images instead of RGB images? Explain why or why not.
   - For Gaussian Naive Bayes, the conditional probability of a feature X_i given class $y = c$ is assumed to follow a Gaussian distribution

   $$P(x_i|y = c) = \frac{1}{2\pi\sigma} exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

   Mean: The Maximum Likelihood for the mean μ_c of the Gaussian distribution for feature X_i given class $y = c$ is the average of the feature values in the class c:

   $$\mu_{i,c} = \frac{1}{N_c}\sum_{j=1}^{N_c} X_{i,j}$$

   where N_c is the number of instances with $y = c_i$ and X_i are the values of the feature of those instances
   Variance: The Maximum Likelihood for the variance $\sigma^2_{i,c}$ is:

   $$\sigma^2_{i,c} = \frac{1}{N_c}\sum_{j=1}^{c}(X_{i,j} - \mu_{i,c})^2$$

   where $\sigma^2_c$is the variance of the feature for class c.

   Yes, these estimates change if we use an RGB image instead of a gray-scale image. RGB images have three channels compared to gray-scale images. So, we have to fit three times more features for RGB images. Thus, Gaussian NB have to learn separate means and variances for three channels compared to a gray-scale image, which has to learn only one mean and one variance for each pixel. Moreover, RGB images have three channels: Red, Green, and Blue, which makes increases the dimension of the feature, which lead to different parameter compared to single pixel gray-scale images.

2. The accuracy of QDA using RGB images was lower than that of grayscale images. What assumptions does QDA make that might cause this difference in performance?
   - RGB images have three channels (Red, Green, & Blue), while Gray-scale image has only single channel. The number of feature in RGB image is three times that of gray-scale image. Given the insufficient amount of data, QDA on RGB images might not be able to produce better results than QDA trained on Gray-scale images. Moreover, QDA needs to estimate a separate covariance matrix for each class. With RGB images, the covariance matrix is much larger and more complex than the Grayscale images. Thus, poor estimation of these covariances matrices can degrade the performance of QDA. One more important thing is that the performance of QDA heavily relies on the covariance structure between the features. In case of RGB images, three channels might not be independent, thus impacting the performance of QDA, while the gray-scale images has only single feature, making it more simple. Moreover, the assumption that pixel intensities follow a multi-variate normal distribution might not be true, because of the introduced complexity associated with colored pixels, and thus exhibit non-gaussian pattern.

3. Both LDA and Gaussian Naive Bayes saw reduced test accuracy on grayscale images compared to RGB images. Why might this be the case (is it the data, the model, or something else)?
   RGB images have three channels: Red, Green, and Blue. In comparison to gray-scale images, RGB images have three times more feature than that of gray-scale images. These additional features can provide significant information to help learn the model.
   LDA seeks to find the linear combination of features that best separates the classes; in case of RGB images (feature-rich vector), the images provides more flexibility in finding better hyperplanes to seperate the classes than a single intensity pixel images. In case of Gaussian NB, the gray-scale images may not provide enough features to capture all the variation in the data making it less effective. Thus, three channels RGB images provides more features or contain more information that the model can learn and improve its accuracy.

4. How many parameters are estimated for each model and each image type (RGB and grayscale)?

   (a) Linear Discriminant Analysis

      i. Gray-scale image: 530,040 (10, 240 + 524, 800)
         mean parameters= 10 * 32 * 32 = 10, 240
         co-variance parameters= (32 * 32) *(32 * 32 + 1)/2= 524,800

ii. RGB image: 4,766,976 (92,160 + 4,720,128)
mean parameters = 10 * 3 * 32 * 32 = 92,160
co-variance parameters= (32*32*3) *(32*32*3+1)/2 = 4,720,128

(b) Quadratic Discriminant Analysis
   i. Gray-scale image: 5,258,240
   ii. RGB image: 47,362,560

(c) Gaussian NB
   i. Gray-scale image: 20,480
   ii. RGB image: 61,440