

CSE 202 Homework 1

Winter, 2014

All parts are worth 20 points. Due

Order Say that $f(n) \in O(g(n))$ are functions from non-negative integers to non-negative integers. Then which of the following follows:

1. $f(n) + g(n) \in O(f(n))$
2. $f(n) + g(n) \in O(g(n))$.
3. $2^{f(n)} \in O(2^{g(n)})$
4. $f(n)g(n) \in O(g^2(n))$.
5. $f(n^2) \in O(g(n^2))$.

For each, either give a short proof that the conclusion holds for all such functions or give counter-example functions for which it is false.

Next larger integer Consider the problem of, given an array of integers $A[1..n]$ (not sorted), finding for each I , the least $J > I$ so that $A[J] > A[I]$ (or “NIL” if no such J exists). Find and analyze an efficient algorithm for this problem.

Graph theory A *Hamiltonian path* in an undirected graph is a simple path that visits every node exactly once. Show that if every node in a graph on n nodes has degree greater than $2/3n$, then the graph has a Hamiltonian path.

Triangles A triangle in a graph are 3 nodes any two of which are adjacent. Present two $O(|E||V|)$ time algorithms for determining whether a graph has a triangle, one if the graph is given as an adjacency matrix and the other if it is given in adjacency list format.

Implementation (20 points) Implement the algorithm you gave for the triangles problem above, when inputs are in adjacency matrix format.

Test it on randomly constructed graphs where each edge is in the graph with probability $1/2$, for $|V|$ as many different powers of 2 as you can get data for. Plot the time taken on a log-log scale. Then perform the same experiment with random sparse graphs, with edge density $4/|V|$. Finally, perform the same experiment on random bipartite graphs where $|L| = |R| = |V|/2$, and edges are in the graph with probability $1/2$. Give an explanation for any differences in performance on these distributions.