Analysis of a scientific article

Identification of the students:

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Identification of the article:

Title of the article	Available Power Gain, Noise Figure, and Noise Measure of Two-Ports and Their Graphical Representations
Author(s)	H. Fukui
Journal (title, year, volume, edition, pp.)	IEEE Transactions On Circuit Theory, 1966, volume CT-13, No. 2, page 137

Declaration

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Analysis of the content of the article:

1. What is in your own words the main message of the article (≠ abstract)?

It is possible to express the performance metrics of linear two ports as a function of the source impedance, the gain parameters and the noise parameters. This paper focusses on the noise measure and the available gain as import performance metrics.

It is then possible to visualize these different performance metrics of an amplifier on the Smith chart during the design phase of the amplifier, since one can plot those performance metrics in simple geometric shapes, giving a good idea of the optimal source impedance.

2. Which are the basic assumptions on which this article relies? (This *can have different formats: e.g. a model that has been used and of which one assumes it is sufficiently accurate for the purpose, or the assumption that only linear effects are*

important in an active circuit, or the assumption that a new type of circuit should always be better than previous ones,…).

Convenient representation

The author assumes that the geometric places where the performance parameters stay constant, are convenient ways to investigate the influence of the source impedance on the performance. The importance of this paper also depends on the assumption that designers of linear two ports need to visualize the parameters like noise and gain.

Noise figure expression

The author starts from the idea that the noise figure can be expressed in terms of the source impedance using the following expression:

$$F = F_{min} + \frac{R_{ef}}{G_s}\left[\left(G_s - G_{ef}\right)^2 + \left(B_s - B_{ef}\right)^2\right]$$

The author justifies this assumption by referencing the book, *IRE Standards On Electron Tubes Methods of Testing.* In the appendix of part 9 of this book we find a derivation of this formula.

Available Power Gain Expression

The paper assumes the correctness of the expression for the available power gain in function of the two port parameters and the source impedance:

$$G_a = \frac{|y_{21}|^2 G_s}{g_{22}\,|y_{11} + Y_s|^2 - Re\,[y_{12}y_{21}(y_{11} + Y_s)^*]}.$$

This formula can be derived using a procedure described by Linvill and Gibbons in their book *Transistors and Active circuits.*

3. Reformulate in your own words the fundamental reasoning that is made in the article.

This paper searches for more convenient ways to investigate the performance of linear two ports over a wide range of source impedances.
It starts from the basic expression for or definition of the performance parameters, for example the noise measure, defined as:

$$\mathrm{M} = \frac{\mathrm{F} - 1}{1 - \frac{1}{\mathrm{G_a}}}$$

This expression is then rewritten to explicitly contain the source impedance.
Next, the paper rewrites the expression to a formula from which it is more convenient to derive a simple geometric shape as the geometric place; for example, the author transformed the expression for the available power gain from:

$$G_a = \frac{|y_{21}|^2 G_s}{g_{22}\,|y_{11} + Y_s|^2 - Re\,[y_{12}y_{21}(y_{11} + Y_s)^*]}.$$

to:

$$\frac{1}{G_a} = \frac{1}{G_{a_{max}}} + \frac{R_{eg}}{G_s}\left[(G_s - G_{og})^2 + (B_s - B_{og})^2\right]$$

.

The plots of these expression result in families of circles. Finally, the resulting equations are interpreted, with a focus on points like the minimum and their representation on the graph.

The equation are then transformed to the reflection coefficient plane, so that they can be used by designers to create graphical representation of the effect of their source impedance on the noise figure, noise measure and power gain on their circuit.

4. Do you agree with the content and conclusions of the article? Why? (You may check some of the mathematics to come to your decision.).

Expression for the Available Gain

Using some Berkeley Lecture notes (Niknejad, 2005), we can derive the expression for the available power gain as follows:

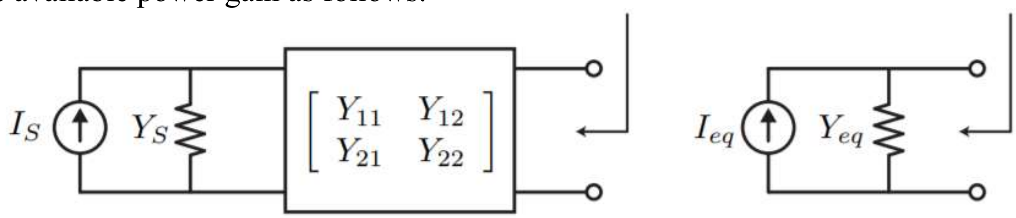


Figure 1: Linear two port and its Norton equivalent

We will use a Norton equivalent as shown in *figure 1:*

$$Power\ at\ the\ source\ = \frac{|I_S|^2}{8 * Re\{Y_S\}}$$

$$Power\ at\ the\ load\ = \frac{|I_{eq}|^2}{8 * Re\{Y_{eq}\}}$$

$$Y_{eq} = Y_{22} - \frac{Y_{21}Y_{12}}{Y_{11} + Y_S}$$

$$I_{eq} = \frac{Y_{21}}{Y_{11} + Y_S} I_S$$

Using this, we can calculate $G_A = \frac{Power\ at\ the\ load}{Power\ at\ the\ source}$

$$G_S = Re\{Y_S\}\ and\ g_{22} = Re\{Y_{22}\}$$

$$G_A = \frac{|Y_{21}|^2 G_S}{|Y_{11} + Y_S|^2 (g_{22} - Re\left\{\frac{Y_{21}Y_{12}}{Y_{11} + Y_S}\right\})}$$

This simplifies to:

$$G_a = \frac{|y_{21}|^2 G_s}{g_{22}|y_{11}+Y_s|^2 - Re[y_{12}y_{21}(y_{11}+Y_s)^*]}.$$

,

since $|Y_{11}+Y_S|^2\, Re\left\{\frac{Y_{21}Y_{12}}{Y_{11}+Y_S}\right\}$ equals $Re\{Y_{21}Y_{12}(Y_{11}-Y_s)*\}$.

Alternate Expression for The Power Gain

$$\frac{1}{G_a} = \frac{1}{G_{a_{max}}} + \frac{R_{eq}}{G_s}[(G_s - G_{eq})^2 + (B_s - B_{eq})^2] \tag{1}$$

With the following parameters:

$$G_{a_{max}} = \left|\frac{y_{21}}{y_{12}}\right| \frac{1}{k+\sqrt{k^2-1}}$$

$$G_{eq} = \frac{|y_{12}y_{21}|}{2g_{22}}\sqrt{k^2-1}$$

$$B_{eq} = -b_{11} + \frac{Im\,(y_{12}y_{21})}{2g_{22}}$$

$$k = \frac{2g_{11}g_{22} - Re\,(y_{12}y_{21})}{|y_{12}y_{21}|}.$$

$$R_{eq} = \frac{g_{22}}{|y_{21}|^2}.$$

If we plug in the values, we get the following:

$$\frac{1}{\mathrm{G_A}} = \left|\frac{\mathrm{y}_{12}}{\mathrm{y}_{21}}\right|\left(k+\sqrt{k^2-1}\right) + \frac{g_{22}}{G_s|y_{21}|^2}\left(G_S^2 + \frac{|y_{12}y_{21}|^2}{4g_{22}^2}(k^2-1) - 2*G_S*\frac{|y_{12}y_{21}|}{2g_{22}}\sqrt{k^2-1} + B_S^2 + \left(b_{11}^2 + \frac{Im\{y_{12}y_{21}\}^2}{4g_{22}^2} - 2b_{11}*\frac{Im\{y_{12}y_{21}\}}{2g_{22}}\right) - 2B_S\left(-b_{11}+\frac{Im\{y_{12}y_{21}\}}{2g_{22}}\right)\right)$$

$$\left|\frac{\mathrm{y}_{12}}{\mathrm{y}_{21}}\right|\left(k+\sqrt{k^2-1}\right) - \frac{g_{22}}{G_s|y_{21}|^2}*2*G_S*\frac{|y_{12}y_{21}|}{2g_{22}}\sqrt{k^2-1} = \left|\frac{\mathrm{y}_{12}}{\mathrm{y}_{21}}\right|k = \frac{2g_{11}g_{22} - [g_{12}g_{21} - b_{12}b_{21}]}{|y_{21}|^2}$$

Now we calculate the factor k^2-1:

$$k^2-1 = \frac{4g_{11}^2g_{22}^2 - 4*g_{11}g_{22}*[g_{12}g_{21}-b_{12}b_{21}] + [g_{12}g_{21}-b_{12}b_{21}]^2 - [g_{12}^2+b_{12}^2][g_{21}^2+b_{21}^2]}{[y_{12}]^2[y_{21}]^2}$$
$$= \frac{4g_{11}^2g_{22}^2 - 4*g_{11}g_{22}*[g_{12}g_{21}-b_{12}b_{21}] - 2g_{12}g_{21}b_{12}b_{21} - [g_{12}^2b_{21}^2+g_{21}^2b_{12}^2]}{|[y_{12}]|^2|[y_{21}]|^2}$$

$$\frac{|y_{12}y_{21}|^2}{4g_{22}^2}(k^2-1) = \frac{4g_{11}^2g_{22}^2 - 4*g_{11}g_{22}*[g_{12}g_{21}-b_{12}b_{21}] - 2g_{12}g_{21}b_{12}b_{21} - [g_{12}^2b_{21}^2+g_{21}^2b_{12}^2]}{4g_{22}^2}$$
$$= g_{11}^2 - \frac{g_{11}}{g_{22}}*Re\{y_{12}y_{21}\} - \frac{[g_{12}b_{21} + g_{21}b_{12}]^2}{4g_{22}^2}$$

Then we can calculate the expression $\frac{1}{\mathrm{G_A}}$:

$$\frac{1}{\mathrm{G_A}} = \frac{(2g_{11}g_{22} - Re\{y_{12}y_{21}\})G_S}{|y_{21}|^2G_S} + \frac{g_{22}}{G_s|y_{21}|^2}\left(G_S^2 + B_S^2 + g_{11}^2 - \frac{g_{11}}{g_{22}}*Re\{y_{12}y_{21}\} - \frac{[g_{12}b_{21} + g_{21}b_{12}]^2}{4g_{22}^2} + \left(b_{11}^2 + \frac{Im\{y_{12}y_{21}\}^2}{4g_{22}^2} - b_{11}*\frac{Im\{y_{12}y_{21}\}}{g_{22}}\right) - 2B_S\left(-b_{11} + \frac{Im\{y_{12}y_{21}\}}{2g_{22}}\right)\right)$$

$$-\frac{[g_{12}b_{21} + g_{21}b_{12}]^2}{4g_{22}^2} + \frac{Im\{y_{12}y_{21}\}^2}{4g_{22}^2} = 0$$

$$\frac{(-Re\{y_{12}y_{21}\})G_S}{|y_{21}|^2G_S} + \frac{g_{22}}{G_s|y_{21}|^2}*\frac{-2B_SIm\{y_{12}y_{21}\}}{2g_{22}} = \frac{g_{22}}{G_s|y_{21}|^2}\left(\frac{-Re\{y_{12}y_{21}\}G_S - B_SIm\{y_{12}y_{21}\}}{g_{22}}\right) = \frac{g_{22}}{G_s|y_{21}|^2}\left(\frac{-Re\{y_{12}y_{21}*Y_S^*\}}{g_{22}}\right)$$

$$-\frac{g_{11}}{g_{22}}*Re\{y_{12}y_{21}\} - b_{11}*\frac{Im\{y_{12}y_{21}\}}{g_{22}} = \frac{-Re\{y_{12}y_{21}*Y_{11}^*\}}{g_{22}}$$

(And of course: Re{a+bj}+Re{c+dj} = a + c = Re{a+bj+c+dj} so we can add the last 2 statements together to get the following:)

$$\frac{1}{\mathrm{G_A}} = \frac{g_{22}}{G_s|y_{21}|^2}\Big(2g_{11}G_S + G_S^2 + B_S^2 + g_{11}^2 + (b_{11}^2) - 2B_S(-b_{11})$$
$$- \frac{Re\{y_{12}y_{21} * (Y_{11} + Y_S)^*\}}{g_{22}}\Big)$$
$$= \frac{g_{22}}{G_s|y_{21}|^2}\left((G_S + g_{11})^2 + (B_S + b_{11})^2 - \frac{Re\{y_{12}y_{21} * (Y_{11} + Y_S)^*\}}{g_{22}}\right) =$$
$$= \frac{g_{22}}{G_s|y_{21}|^2}\left(|Y_{11} + Y_S| - \frac{Re\{y_{12}y_{21} * (Y_{11} + Y_S)^*\}}{g_{22}}\right)$$
$$= \frac{g_{22}|Y_{11} + Y_S| - Re\{y_{12}y_{21} * (Y_{11} + Y_S)^*\}}{G_s|y_{21}|^2}$$

… and if we take the inverse of both sides, we get the expression that we calculate in the section above:

$$G_a = \frac{|y_{21}|^2 G_s}{g_{22}\,|y_{11} + Y_s|^2 - Re\,[y_{12}y_{21}(y_{11} + Y_s)^*]}.$$

We can easily transform this expression into this following shape:

$$(G_s - G_G)^2 + (B_s - B_G)^2 = G_{RG}^2$$

where

$$G_G = G_{og} + \frac{1}{2R_{eg}}\left(\frac{1}{G_a} - \frac{1}{G_{amax}}\right)$$

$$B_G = B_{og}$$

$$G_{RG} = \left[\frac{G_{og}}{R_{eg}}\left(\frac{1}{G_a} - \frac{1}{G_{amax}}\right) + \frac{1}{4R_{eg}^2}\left(\frac{1}{G_a} - \frac{1}{G_{amax}}\right)^2\right]^{1/2}. \quad (2)$$

If we define an X equal to $\frac{1}{G_a} - \frac{1}{G\_amax}$, we can rewrite the alternative expression (1) to the following form: $X * \frac{G_s}{R_{eg}} = \left(G_s - G_{og}\right)^2 + \left(B_s - B_{og}\right)^2$. (3)

Since we have that $Ax = (x - B)^2 + (y - C)^2$ is equivalent to $0 = \left(x - \left[B + \frac{A}{2}\right]\right)^2 + (y - C)^2 - \frac{A^2}{4} - BA$, we can rearrange equation (3) to the desired expression (2).

Because G_a is smaller or equal to G_{amax}, the radius G_{RG} is always positive; hence, the result is a family of real circles.

We can now plot the circles belonging to this family of circles. We plotted three circles with a gain of three, two and one respectively (figure 2). We used the linear two port mentioned in section six: Examples of Graphical Representations of the paper.

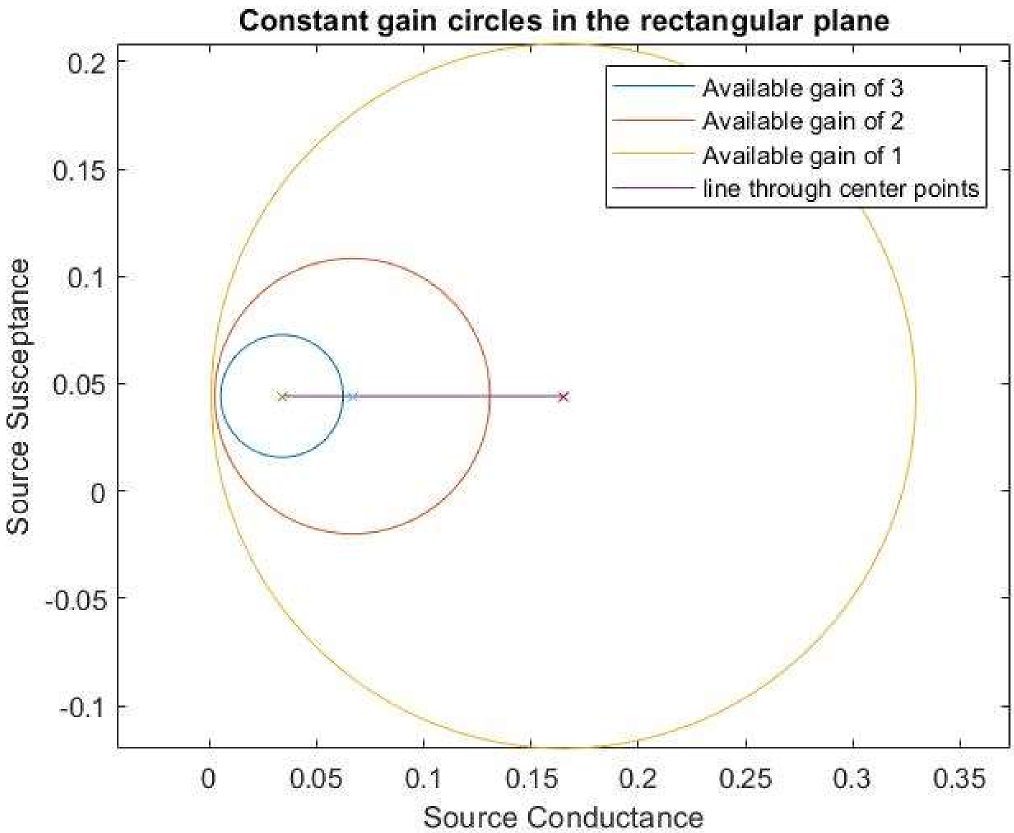


Figure 2: Constant available gain circles and the center points

As the paper mentions the centers of the circles lay on a line parallel to the G_s axis.

Constant Noise Measure Locus

We start from the following expression:

$$M = \frac{F_{\min} - 1 + \frac{R_{ef}}{G_s}\left[(G_s - G_{of})^2 + (B_s - B_{of})^2\right]}{1 - \frac{1}{G_{a\max}} - \frac{R_{eg}}{G_s}\left[(G_s - G_{og})^2 + (B_s - B_{og})^2\right]}. \quad (4)$$

We define the variables C_1 and C_2 in order to reshape this equation:
$C_1 = F_{Min} - 1$ and $C_2 = \frac{1}{G_{amax}} - 1$.
Now, we can rewrite the expression:

$$0 = \frac{G_S}{[R_{ef} + MR_{eg}]}(C_1 + MC_2) + G_S^2 + B_S^2 - 2 * G_S\left[\frac{G_{of}R_{ef} + G_{og}MR_{eg}}{R_{ef} + MR_{eg}}\right] - 2 * B_S\left[\frac{B_{of}R_{ef} + B_{og}MR_{eg}}{R_{ef} + MR_{eg}}\right] + \frac{G_{of}^2\ R_{ef} + G_{og}^2 MR_{eg} + B_{of}^2 R_{ef} + B^2{}_{og}MR_{eg}}{R_{ef} + MR_{eg}} \quad (5)$$

This equation has the following form: $x^2 + y^2 - 2fx - 2gy + c = 0$, (6)
where x and y represent G_s and B_s respectively. We can rearrange that expression (6) to: $(x - f)^2 + (y - g)^2 = f^2 + g^2 - c$. Hence, we can rewrite (5) to the following form:

$$(G_s - G_M)^2 + (B_s - B_m)^2 = G_{RM}^2$$

G_M equals $\left[\frac{G_{of}R_{ef} + G_{og}MR_{eg}}{R_{ef}+MR_{eg}}\right] - \frac{1}{2}\left[\frac{(C_1+MC_2)}{R_{ef}+MR_{eg}}\right] = \frac{2(G_{of}R_{ef} + G_{og}MR_{eg}) - (C_1+MC_2)}{2(R_{ef}+MR_{eg})}$

$$= \frac{M\left(2G_{og}R_{eg} - \frac{1}{\mathrm{G_{Amax}}}+1\right) + 2*G_{of}R_{ef} - F_{MIN} + 1}{2(R_{ef}+MR_{eg})} \quad ,$$

B_M equals $\left[\frac{B_{of}R_{ef} + B_{og}MR_{eg}}{R_{ef}+MR_{eg}}\right]$, and

G_{RM} equals $\sqrt{\left[G_m^2 + B_m^2 - \frac{G_{of}^2\ R_{ef} + G_{og}^2MR_{eg} + B^2_{of}R_{ef} + B^2{}_{og}MR_{eg}}{R_{ef}+MR_{eg}}\right]}$

We have that

$$G_M^2 = \frac{1}{4\left(R_{ef} + MR_{eg}\right)^2} * [\left(\mathrm{M} * \left(\frac{1}{\mathrm{G_{Amax}}} - 1\right) + F_{MIN} - 1\right)^2$$
$$- (2*2)\left(G_{of}R_{ef} + G_{og}MR_{eg}\right) * \left(\mathrm{M} * \left(\frac{1}{\mathrm{G_{Amax}}} - 1\right) + F_{MIN} - 1\right)$$
$$+4 * (G_{of}^2R_{ef}^2 + 2 * G_{of}R_{ef} * G_{og}MR_{eg} + G_{og}^2M^2R_{eg}^2)]$$

and

$$B_M^2 = \frac{1}{4\left(R_{ef}+MR_{eg}\right)^2} * 4 * (B_{of}^2R_{ef}^2 + 2 * B_{of}R_{ef} * B_{og}MR_{eg} + B_{og}^2M^2R_{eg}^2)] \ .$$

Combining those two equations, we get an expression for $G_M^2 + B_M^2$:

$$\frac{1}{4\left(R_{ef} + MR_{eg}\right)^2} * [\left(\mathrm{M} * \left(\frac{1}{\mathrm{G_{Amax}}} - 1\right) + F_{MIN} - 1\right)^2$$

$$- 4\left(G_{of}R_{ef} + G_{og}MR_{eg}\right) * \left(\mathrm{M} * \left(\frac{1}{\mathrm{G_{Amax}}} - 1\right) + F_{MIN} - 1\right)$$
$$+ 4(R_{ef}^2\ \left|Y_{of}\right|^2 + M^2R_{eg}^2\left|Y_{og}\right|^2 + 2R_{ef}\,MR_{eg}(B_{of}B_{og} + G_{of}G_{og}))]$$

The previous equations result in the following expression for G_{RM} :

$$G_{RM}^2 = \frac{1}{4\left(R_{ef} + MR_{eg}\right)^2} * [\left(\mathrm{M} * \left(\frac{1}{\mathrm{G_{Amax}}} - 1\right) + F_{MIN} - 1\right)^2$$

$$- 4\left(G_{of}R_{ef} + G_{og}MR_{eg}\right) * \left(\mathrm{M} * \left(\frac{1}{\mathrm{G_{Amax}}} - 1\right) + F_{MIN} - 1\right)$$
$$+ 4(R_{ef}^2\ \left|Y_{of}\right|^2 + M^2R_{eg}^2\left|Y_{og}\right|^2 + 2R_{ef}\,MR_{eg}(B_{of}B_{og} + G_{of}G_{og}))]$$
$$- \frac{G_{of}^2\ R_{ef} + G_{og}^2MR_{eg} + B^2_{of}R_{ef} + B^2{}_{og}MR_{eg}}{R_{ef} + MR_{eg}}$$
$$= \frac{1}{4\left(R_{ef} + MR_{eg}\right)^2} * [\left(\mathrm{M} * \left(\frac{1}{\mathrm{G_{Amax}}} - 1\right) + F_{MIN} - 1\right)^2$$
$$- 4\left(G_{of}R_{ef} + G_{og}MR_{eg}\right) * \left(\mathrm{M} * \left(\frac{1}{\mathrm{G_{Amax}}} - 1\right) + F_{MIN} - 1\right)$$
$$+ 4(R_{ef}^2\ \left|Y_{of}\right|^2 + M^2R_{eg}^2\left|Y_{og}\right|^2 + 2R_{ef}\,MR_{eg}(B_{of}B_{og} + G_{of}G_{og}))]$$

$$- \frac{R_{ef}\left|Y_{of}\right|^2 + MR_{eg}\left|Y_{og}\right|^2}{R_{ef} + MR_{eg}}$$

$$(7)$$

We make the two denominators the same:

$$\frac{R_{ef}\left|Y_{of}\right|^2+MR_{eg}\left|Y_{og}\right|^2}{R_{ef}+MR_{eg}} = \frac{R_{ef}\left|Y_{of}\right|^2+MR_{eg}\left|Y_{og}\right|^2}{R_{ef}+MR_{eg}} * \frac{4(R_{ef}+MR_{eg})}{4(R_{ef}+MR_{eg})}$$
$$= \frac{4(R_{ef}+MR_{eg})(R_{ef}\left|Y_{of}\right|^2+MR_{eg}\left|Y_{og}\right|^2)}{\left(R_{ef}+MR_{eg}\right)^2}$$
$$= \frac{4(R_{ef}^2\left|Y_{of}\right|^2+M^2R_{eg}^2\left|Y_{og}\right|^2+MR_{eg}\,R_{ef}[\left|Y_{of}\right|^2+\left|Y_{og}\right|^2])}{\left(R_{ef}+MR_{eg}\right)^2}$$

Now, we can simplify expression (7) to:

$$G_{RM}^2 = \frac{1}{4\left(R_{ef} + MR_{eg}\right)^2} * [\left(\mathrm{M} * \left(\frac{1}{\mathrm{G}_{\mathrm{Amax}}} - 1\right) + F_{MIN} - 1\right)^2$$
$$- 4\left(G_{of}R_{ef} + G_{og}MR_{eg}\right) * \left(\mathrm{M} * \left(\frac{1}{\mathrm{G}_{\mathrm{Amax}}} - 1\right) + F_{MIN} - 1\right)$$
$$-4(MR_{eg}\,R_{ef}[\left|Y_{of}\right|^2 + \left|Y_{og}\right|^2 - 2(B_{of}B_{og} + G_{of}G_{og})]$$

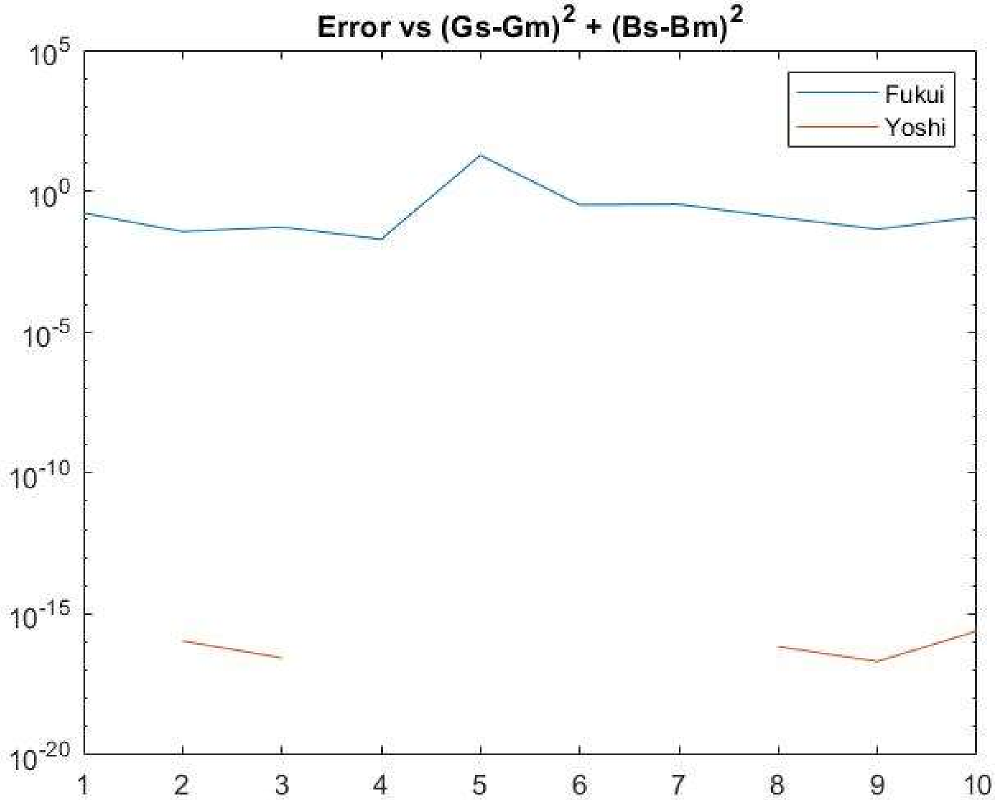
One cannot simplify this last expression in any meaningful way. The careful reader might have noticed that this expression does not equal the expression in the original paper:

$$G_{RM} = \frac{1}{2(MR_{eg} + R_{ef})}\left[\left\{M\left(\frac{1}{G_{a_{max}}} - 1\right) + F_{min} - 1\right\}^2\right.$$
$$- 4(MR_{eg}G_{og} + R_{ef}G_{of})\left\{M\left(\frac{1}{G_{a_{max}}} - 1\right) + F_{min} - 1\right\}$$
$$\left. - 4MR_{eg}R_{ef}(|Y_{og}|^2 + |Y_{of}|^2 - 4B_{og}B_{of})\right]. \qquad (19)$$

Though we suspect that the original paper contains a small error, the overall results are still valid: the constant M loci are circles on the rectangular plane.

We decided to plot the absolute difference of $(G_S - G_M)^2 + (B_S - B_M)^2$ our formula and Fukui's formula (assuming Fukui meant to write G_{RM}^2 when he wrote (19) (this gives a result which aligns much more closely, but not perfectly).

We seeded the MATLAB random number generator 10 times to generate the necessary parameters and got the following result:



We can clearly see that the difference between our estimate of $(G_{RM})^2$ and the value given by $(G_S - G_M)^2 + (B_S - B_M)^2$ is close to machine precision, or even 0, while the value of Fukui's G_{RM}^2 is, while fairly low, not quite the same.

We can now plot different constant noise measure circles as well. We used the same linear two port as the example of section six of the paper (figure 3).

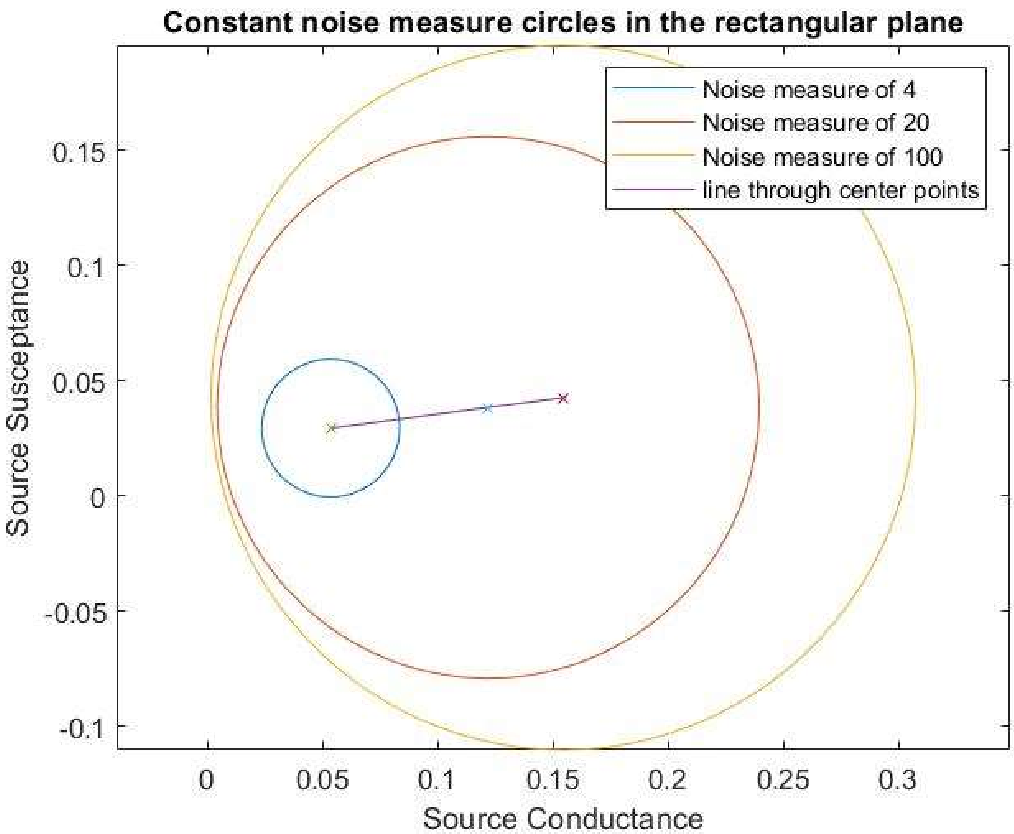


Figure 3 Constant noise measure circles in rectangular plane and the center points

We observe that the radius increases with increasing noise measure and that although the center points still lay on a straight line, the line is no longer parallel to the axis.

Bilinear Transformations

A bilinear transformation is a transformation of the following type:
$w = \frac{ay+b}{cy+d}$, where the product of a and d minus the product of c and b does not equal zero. The transformation used in this paper is $w = \frac{-y+1}{y+1}$.,
and $ad - bc = -1 * 1 - 1 * 1 = 2$. This is indeed a bilinear transformation. As we can read in this chapter [1], such a bilinear transformation maps circles on the complex plane into circles on the reflection coefficient plane.

The reflection coefficient ρ = u + jv can be expressed in terms of the normalized source conductance g_s and the normalized source susceptance b_s.

$$\rho = \frac{1 - y_s}{s + y_s}$$
$$= \frac{1-g_s-j*b_s}{1+g_s+j*b\_s}$$
$$= \frac{1-g_s^2-b_S^2-2b_sj}{(1+g_s)^2+b_S^2}$$

When we plot the constant gain loci of figure two on the reflection coefficient plane, the results are circles as the reader can verify on the following figure (figure 4):

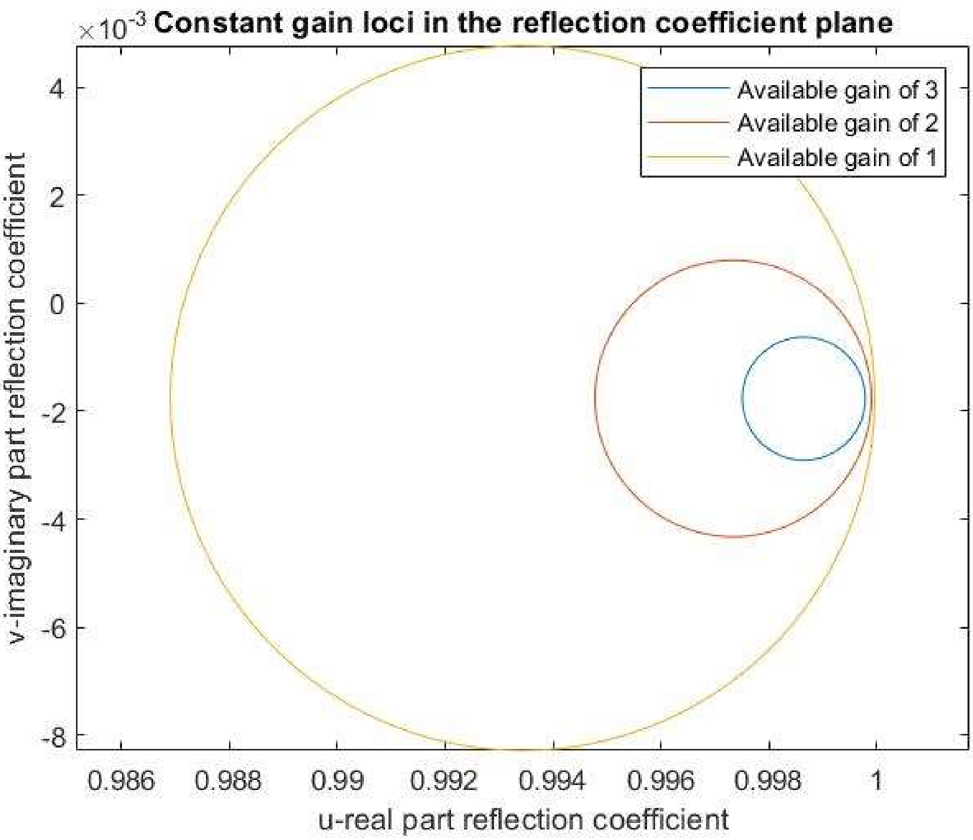


Figure 4 constant gain circles on the reflection coefficient plane

We normalized the admittances with a characteristic admittance of 50Ω.

Smith Chart

We will plot the constant noise figure (figure 5), available gain (figure 6) and noise measure (figure 7) on the Smith chart. We use a characteristic impedance Y_0 of 50Ω and the same linear two port as in the previous plots.

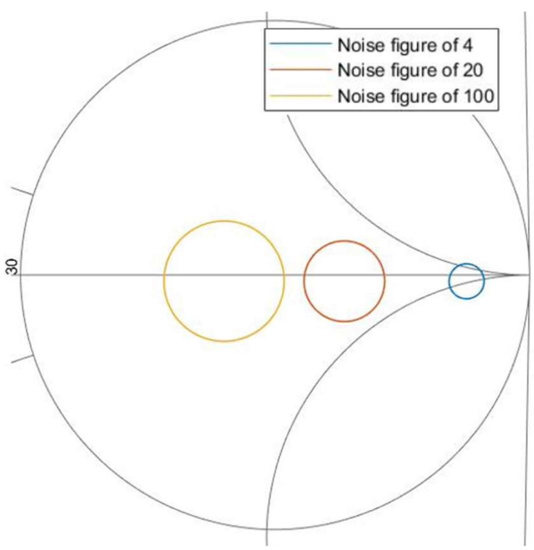


Figure 5 Constant noise figure loci on smith chart

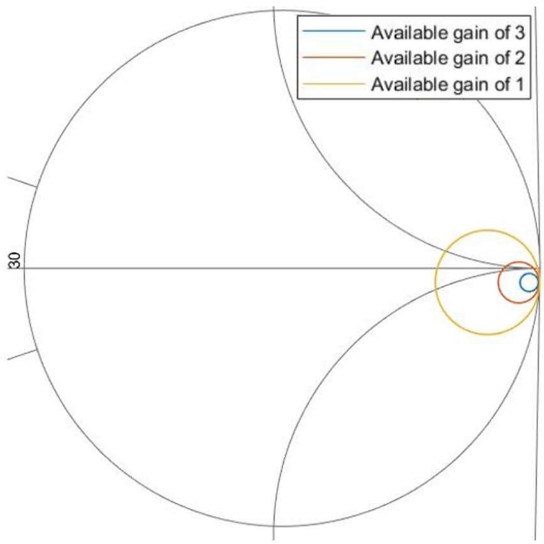


Figure 6 Available gain on the smith chart

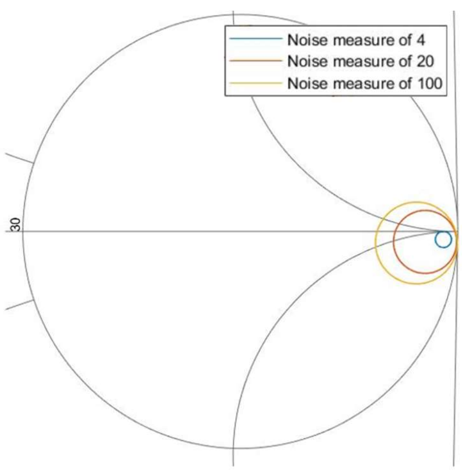


Figure 7 noise measure on smith chart

5. Where and how can the content of this article be applied following you? (You may mention applications indicated in the paper, but also try to come up with your own ideas.)

The usefulness of this article lies in the noise figure, noise measure and available power gain charts (See figure 6, 7 in the article) that can be derived from any two-port using the methods described. These charts are useful to any circuit designer because they tell him how his power gain and noise performance will degrade as he changes his source impedance away from the optimal points.

The charts not only allow him to come up with the ideal source impedance for one of optimizing one of the three performance measures described in the article, but also give insight in how this will impact the other two. Of course, the noise measure is meant to be a performance indicator that gives the best of both worlds, but when for example the power gain is more important than the noise figure (because the design requirement calls for a minimum power gain for example), we can use the chart to quickly find a very good approximation for the best noise figure achieved for the requested minimum power gain.

6. Are there any terms (definitions), or other background information, that you needed to look up in order to understand the article? If yes, explain.

- The paper refers to the original paper of Rothe and Dalke "Theory of Noisy fourpoles", which was found and from which we got the following figure describing the noise figure on a rectangular coordinate system.

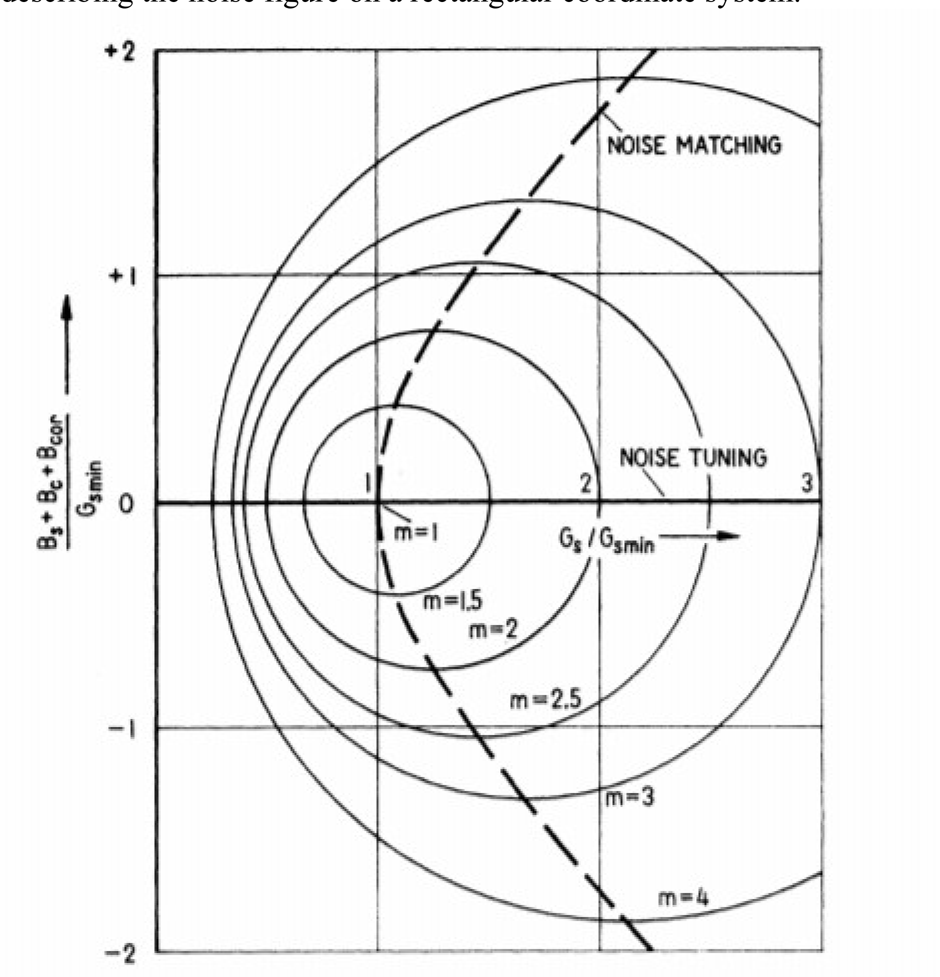


Fig. 6—Noise matching diagram between the signal source and the noise fourpole. The circles are curves for constant noise figure.

- The Berkeley Lecture slides to understand how we could express the amplifier gain as used in the article (the original paper by Linvill and Gibbons could not be found)

- The book *Standards On Electron Tubes Methods of Testing* provided us with a derivation of the noise formula.
- Chapter 9 from Pulak Sahoo gave us the proof that bilinear transformation indeed transform circles into circles. (Sahoo)
- We weren't familiar with the unit millimhos. This unit is equivalent to millisiemens, and is used to express conductivity.

Analysis of the format of the article:

1. Is the article written a sufficiently clear language?

The language of the article is clear, concise and to the point. Each section starts with an initial known formula, which is generally transformed into a form more suited to plotting on a rectangular or reflection coefficient plane. This allows readers to quickly identify the formula of their interest, find the relevant transformation, and use that in their application or research. If they are interested, some of the intermediate steps are given. The final two pages give an example, which considerably helps understanding and shows how the formulas are best employed to find the optimum source impedance for a given two-port.

2. Are the graphs sufficiently clear?

The first 3 figures are quite clear, especially when takes in conjunction with their formulas. Moreover, figure 3 nicely demonstrates that for the noise measure, the center of the locus for the susceptance (B_M) is not constant as M changes.

Figure 4 and 5 are less clear, mainly because the figures are smaller and more (slightly unnecessary) text is added in the form of angle measurements, and a 0.5 magnitude line. They might not allow a reader skimming through the paper to instantly understand the meaning, but the meaning becomes obvious on examination of the bilinear transformations chapter.

Figure 6, 7, and 8 are a great help in understanding the purpose of the article, since they show what can be considered "the final product". A Smith Chart that allows a designer to instantly find in which region he would prefer his source impedance to be to obtain optimal noise and power gain characteristics.

3. Do the captions to the figures and tables contain sufficient information in order to understand them (and possibly reconstruct them)?

The captions only state what is plotted, but this is enough for a reader who has read the abstract. However, they do not contain the formulas which are required to reconstruct them. These can be quickly found by looking at the formulas in their respective sections. Adding the full formulas to the caption would harm the understanding of the article, since they are quite long. Besides, a fair amount of the parameters depend on other parameters previously calculated, so it would not be possible to reconstruct the graph even when the full formula is given. Adding the circle formula to the caption wouldn't help either, since the graph itself clearly demonstrates that a circle is plotted with a given centre and radius.

4. What is the importance / the function of the references to which this article refers?
Which reference is the crucial one (explain)? (<u>Add this reference in pdf to your analysis</u>)

The majority of the references introduces the basic concepts used to derive the results of this paper: the definitions of important concepts as noise measure and the coordinates of the Smith chart, fundamental assumptions like the dependence of the noise figure on the source impedance, etc…

The Rothe and Dahlke paper provides the graphical representation of the noise figure in the source admittance plane. This paper improves upon this result by using the noise measure and the reflection coefficient plane.

As previously mentioned, the book *Standards On Electron Tubes Methods of Testing* provided us with a derivation of the noise formula.

However, since the whole purpose of this paper is to apply a bilinear transformation to these plots, the crucial reference is J.C. Slater's *Microwave Electronics*, more specifically the section on pages 6 and 7 on bilinear transformations. (Slater, 1946)

This reference provides properties of the bilinear transformation and proves that it maps circles from one plane to another (while keeping the circle a circle). This last property is of course the important one, since it allows the use of the formulas previously discovered for the rectangular plane. Transforming is then a matter of changing values, as described in the article.

<u>Bibliography</u>

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