- Gaussian MSK
  - MSK has three important properties
    - Constant envelope (why?)
    - Relatively narrow bandwidth
    - Coherent detection performance equivalent to that of QPSK
  - However, the PSD of the MSK only drops by $10\log_{10}9 =$ 9.54 dB below its midband value at $fT_b = 0.5$
    - The adjacent channel interference is not low enough
  - We may modify the PSD with the use of a pre-modulation low-pass filter, hereafter referred to as a pulse shaping filter
    - Frequency response with a narrow bandwidth and sharp cutoff
    - Impulse response with a relatively low overshoot
    - Phase values equal to 0 or π at $2nT_b$, or to $\pm\pi/2$ at $(2n+1)T_b$

- These desirable features can be achieved by passing a NRZ data stream through a filter defined by a Gaussian function
- Gaussian-filtered MSK (GMSK)
  - Let W be the 3 dB baseband BW of the pulse shaping filter

$$H(f) = \exp\left(-\frac{\log 2}{2}\left(\frac{f}{W}\right)^2\right)$$

  - The time domain impulse response of this Gaussian filter is

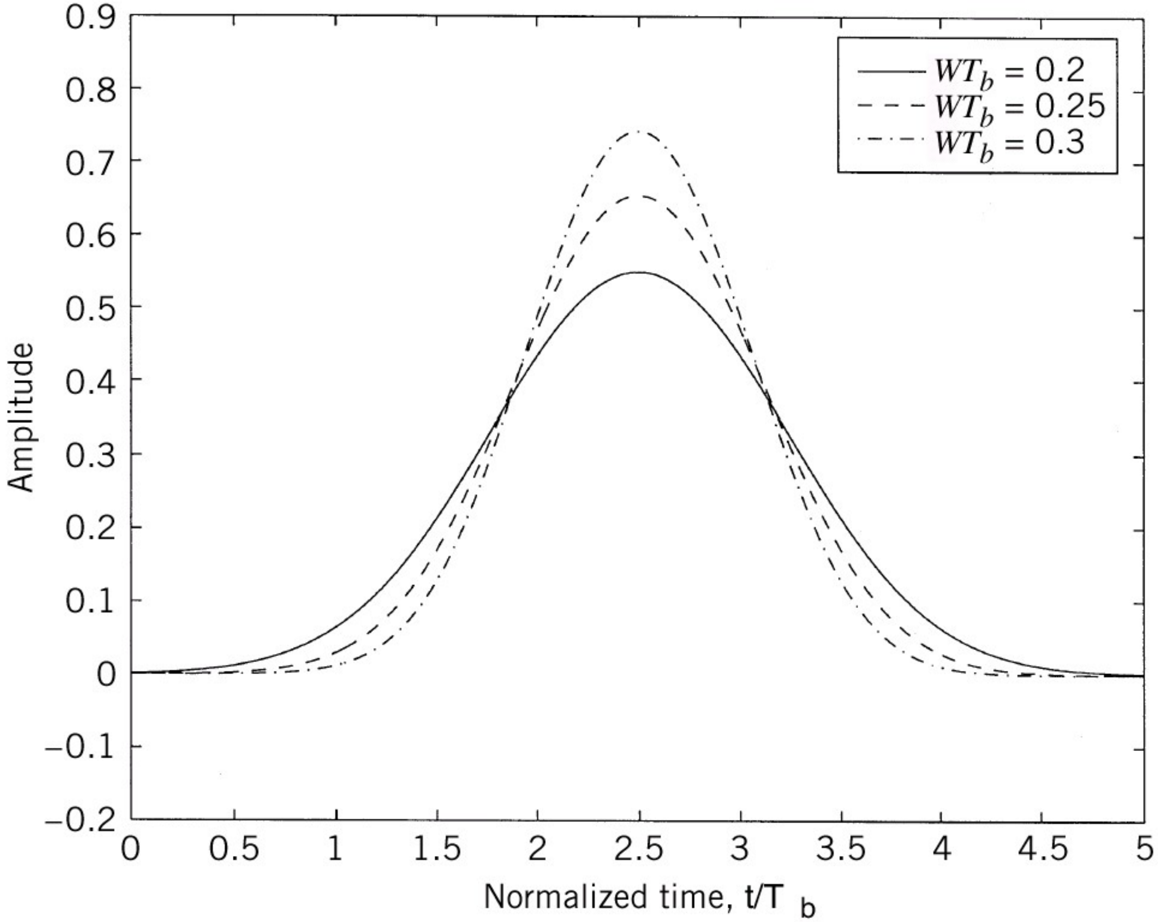
$$h(t) = \sqrt{\frac{2\pi}{\log 2}}W\exp\left(-\frac{2\pi^2}{\log 2}W^2t^2\right)$$

  - The response of $h(t)$ to a rectangular pulse of $U(\frac{-T_b}{2}) - U(\frac{T_b}{2})$ is

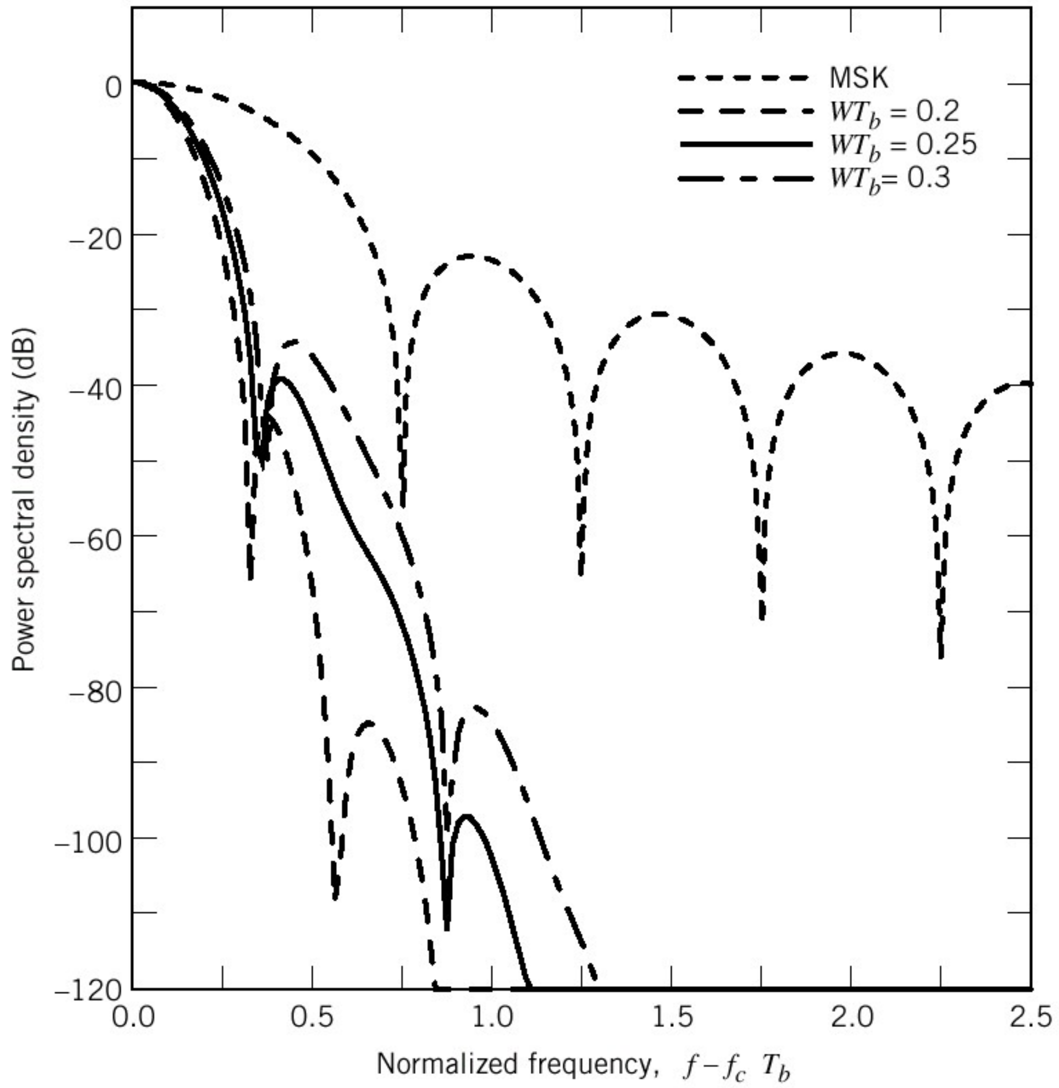
$$g(t) = \sqrt{\frac{2\pi}{\log 2}}W\int_{-T_b/2}^{T_b/2}\exp\left(-\frac{2\pi^2}{\log 2}W^2(t-\tau)^2\right)d\tau$$

- The impulse response to $U(-T_b/2) - U(T_b/2)$ can expressed as
$$g(t) = \frac{1}{2}\left[\operatorname{erfc}\left(\pi\sqrt{\frac{2}{\log 2}}WT_b\left(\frac{t}{T_b}-\frac{1}{2}\right)\right) - \operatorname{erfc}\left(\pi\sqrt{\frac{2}{\log 2}}WT_b\left(\frac{t}{T_b}+\frac{1}{2}\right)\right)\right]$$
- This pulse shaping function $g(t)$ is non-causal in that it is nonzero for $t < -T_b/2$
- For a causal response, $g(t)$ must be truncated and shifted in time
- The time-bandwidth product WT_b is a design parameter
- The case of $WT_b = \infty$ corresponds to the ordinary MSK
- When $WT_b < 1$, increasingly more of the transmit power is concentrated inside the passband of the GMSK signal
- An undesirable feature of GMSK is that the modulated signal of NRZ binary data is no longer confined to a single bit interval as in the ordinary MSK, causing inter-symbol interference (ISI)

- The truncated pulse shaping function $g(t)$ which is shifted in time by 2.5 T_b, and truncated at t = $\pm 2.5T_b$



- The PSD of GMSK

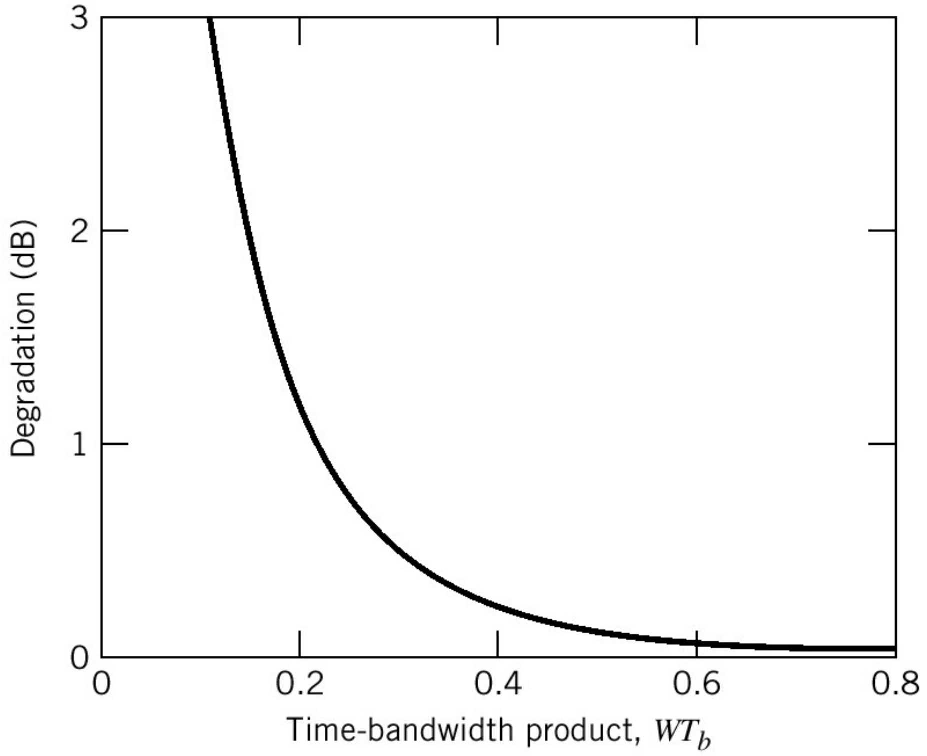


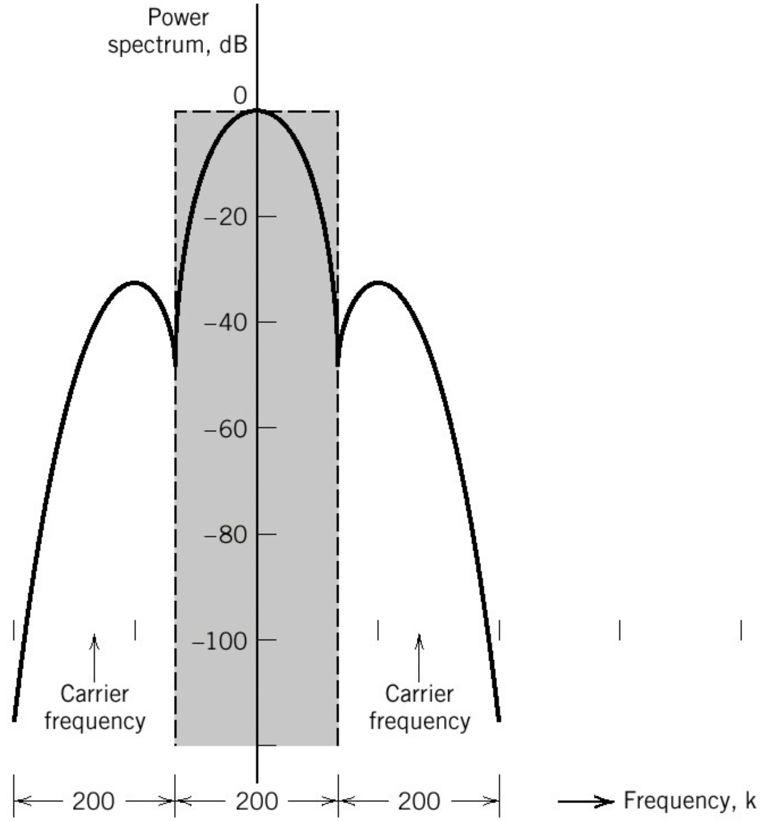
- Therefore, the chose of WT_b offers a tradeoff between spectral compactness and performance loss
- The BER is hard to derive. Let us express it in terms of the BER of MSK, given by

$$P_e = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{\alpha E_b}{2N_0}}\right)$$

- Comparing to the BER of MSK, GMSK has a performance degradation of $10\log_{10}(\alpha/2)$ dB in SNR
- The value of α depends on WT_b
- For MSK, we have $WT_b = \infty$, corresponding to $\alpha = 2$
- For GSM, we have $WT_b = 0.3$, resulting in an SNR degradation of 0.46 dB, corresponding to $\alpha/2 = 0.9$

- The performance degradation $10\log_{10}(\alpha/2)$ v.s. WT_b





- For GSM, data rate is 271kb/s, with BW=200kHz for each channel
- 99% of the power is confined to a BW of 250kHz with $WT_b = 0.3$

M-ary FSK

- We have $s_i(t) = \begin{cases} \sqrt{\frac{2E}{T}} \cos\left(\frac{\pi}{T}(n_c + i)t\right), & 0 \le t \le T \\ 0, & \text{elsewhere} \end{cases}$

  where $i = 1,2,\ldots,M$, and $f_c = n_c /2T$
- Since the individual frequencies are separated by 1/2T Hz, we have

$$\int_0^T s_i(t) s_j(t) dt = 0, \qquad i \neq j$$

- We thus may use the transmitted signals $s_i(t)$ themselves as a complete orthonormal set of basis functions, as shown by

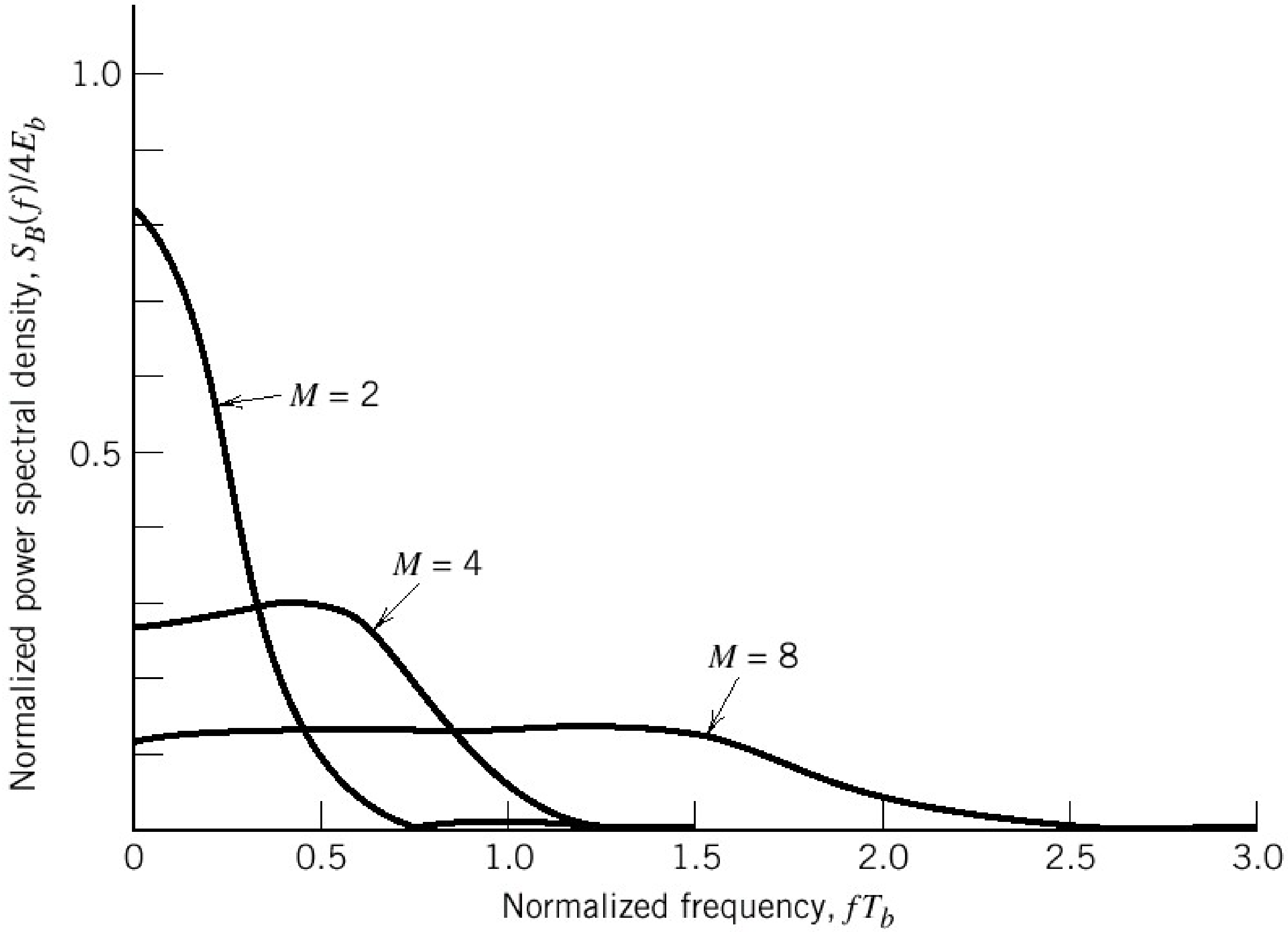
$$\phi_i(t) = \frac{1}{\sqrt{E}} s_i(t), \qquad 0 \le t \le T, i = 1,2,\ldots,M$$

- The optimum receiver consists of a bank of M matched filters

- The receiver makes decisions based on the largest matched filter output in accordance with the ML decision rule
- The exact BER is difficult to derive, while can be bounded from above by $P_e \leq \frac{1}{2}(M-1)\mathrm{erfc}\left(\sqrt{\frac{E}{2N_0}}\right)$ since the minimum distance in M-ary FSK is $\sqrt{2E}$
- For M = 2, i.e. BFSK, the bound becomes an equality

- The PSD of M-ary FSK depends on the frequency assigned to each value of *M*
  - When the spacing is uniform with a deviation *k* = 0.5, that is when frequencies are separated from each other by 1/2T Hz, the PSD is plotted in the next page

- The PSD of M-ary FSK, for $k = 0.5$



- The bandwidth efficiency of M-ary FSK
  - The adjacent frequencies need only be separated from each other by 1/2T to maintain orthogonality
  - We, thus, define the channel bandwidth required to transmit M-ary FSK as $B = M / 2T$
  - Recall that T is equal to $T_b \log_2 M$
  - Let $R_b = \frac{1}{T_b}$, then $\mathrm{B} = \frac{R_b M}{2\log_2 M}$
  - The bandwidth efficiency of an M-ary signal is thus given by

$$\rho = \frac{R_b}{B} = \frac{2\log_2 M}{M}$$

  - Increasing the number of M tends to decrease the bandwidth efficiency of M-ary FSK

- Lab 4
  - Redo Lab3-2 by changing the RC filter with a RRC filter
  - Generate a series of binary random numbers
  - Modulate the binary numbers with MSK, given that the carrier frequency f_c is 1 MHz, and the symbol energy E is 10dB and the symbol frequency is 1KHz
  - Demodulate the transmitted signal using the method on p.p. 100
  - Draw and compare the BERs with the theoretical values
  - Redo the above procedure with GMSK when $WT_b = 0.3$
    - Using the pulse shaping filter on pp. 113
    - Draw the phase trellis of GMSK and compare it with that of MSK

- HW2 (due on 4/21)
  - For FSK: 6.20, 6.21, 6.22, 6.23, 6.26, and 6.28