The Effects of Informed Bettors on Bookmaker Behavior

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Abstract

Previous literature on the market for sports gambling shows that bookmakers, who effectively set the "price" to gamble, take advantage of biased bettors to earn higher profits. This paper extends an existing model of bookmaker behavior to take into account the effects of smart, "informed" bettors who exploit the bookmaker. Our findings capture the change in bookmaker behavior when informed bettors are introduced, and we present empirical evidence to validate our conclusions. We also discuss the broader significance of this work beyond sports betting to any similarly speculative markets or activities.

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I. ACKNOWLEDGEMENTS

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II. INTRODUCTION

Can smart, well-informed bettors take advantage of a casino that is attempting to profit from public bias towards a particular team? Before introducing the fundamentals of the sports betting market, it is important to define three key terms: bias, the bookmaker, and the bookmaker's primary tool, the point spread:

Point Spread: In sports gambling, the point spread is the handicap that is afforded to the team deemed most likely to lose a game (also known as the "underdog.") If Team A is favored to beat Team B by 5 points, then the point spread is 5 and is represented by the term "Team A -5," or equivalently, "Team B +5." As a result, for the type of betting discussed in the paper, a wager on Team A will win if and only if Team A beats Team B by 5 or more points. In this case, we say that Team A (which we refer to as the "favorite" or the "favored team") "covers the spread."

Bookmaker: Often known as a "bookie," a bookmaker sets a point spread and is responsible for any changes to the point spread. For our purposes, one casino can be thought of as a bookmaker – it posts an opening point spread and is free to change the point spread until the game begins. The bookmaker charges a commission on each dollar wagered, which is almost universally set to 10% (Levitt, 2004); that is, in order to win $100, a bettor must wager $110.

Bias: In this setting, bias describes a situation in which, for a point spread that gives each team an equal probability of covering the spread, bettors are still more likely to bet on one team over the other. For example, suppose there are two evenly matched teams, Team A and Team B, and the bookmaker sets the spread to Team A -0; that is, a bet on either team is a winning bet if that team is victorious. Because they are evenly matched, we would expect an equal probability that a bettor bets for either team. If this is not that case, then we say the public is biased.

Prior to a National Basketball Association (NBA) game, different casinos (bookmakers) publish point spreads, also known as "odds." The first odds they publish are called the "opening lines." Historically, this has been done either the night before the game *or* the morning of the game, depending on the casino. Casinos will move the point spread slightly throughout the night and day. Often, this is for reasons specific to the game; for example, one of the teams may announce that their star player is not playing, and so a casino will adjust the odds so that that team is less favored to win. The final point spread posted by a casino prior to the start of the game is called the "closing line."

Point spreads can also vary significantly among casinos at any given time. That is, casino A might favor a team by 6 points, while casino B might favor the same team by just 4 points. Ultimately, a casino will set the point spread to maximize its expected profit. If, all of a sudden, everyone starts betting on the same team, then the casino may adjust its point spread to attract money to the other side; this is done to minimize its risk. An ideal situation is when a casino's point spread attracts an equal amount of money to both teams, thus guaranteeing a risk-free profit for the casino. This is not always the profit maximizing point spread, but it is nonetheless a viable strategy and is often called the "balanced book method" (Levitt, 2004).

This paper explores the effect of "informed bettors," which we define to be experienced gamblers who can perfectly convert a point spread into the corresponding probability of winning a bet on a team, just as the bookmaker is able to do. More importantly, they can identify situations in which bookmakers attempt to take advantage of public bias, and in turn, they are able to exploit the bookmaker.

The broader significance of this paper extends far beyond the arena of sports gambling. The influences of informed bettors could hypothetically arise in any situation in which people part with their money in a speculative manner in hopes of profiting. If there exists a rational public bias and a firm takes this into account for purposes of pricing at a profit maximizing point, we would expect smart individuals to recognize the firm's biased pricing and exploit it. For instance, consider insider trading of a public company's stock; in this case, we have an "informed" individual who has information to which the public does not have access. Like our bookmaker, however, the public company is fully aware of this information. Thus, this informed investor will make his or her investment decision based on the knowledge that the public is comparatively uninformed. For our purposes, we will explore the expected effect of such bettors on the profit maximizing point spread, and attempt to answer the following question: is there evidence that a subset of "smart" bettors are taking advantage of bookmakers who try to exploit bias?

The results of the model presented in this paper are supported empirically. Data collected during the 2013-2014 NBA season describes opening and closing lines for two types of casinos: ones that posted their first point spread the night before the game (early entrants) and ones that posted their first point spread the morning of the game (late entrants.) The data show that even after controlling for the fact that the early entrants' point spreads were open for a longer period, and thus susceptible to more "noise," (random line movements due to injuries, suspended players, and other reasons) that the early entrants still exhibited significantly larger line changes (the difference between the opening and closing lines) than late entrants. More specifically, when we look at games in which there are no early entrants, the subsequent lines changes are significantly higher than in the case where a casino enters late when there *were* previous *early* entrants.

This discrepancy could be explained by the presence of informed bettors. As a model proposed by Steven Levitt shows, casinos should exploit bettor bias to obtain maximum expected profits (Levitt, 2004). For example, a casino may have the option to set the point spread such that it expects both teams will have a 50% chance of covering the spread; this is referred to as "even odds." However, consider the case when, despite the even odds, more than half the bettors bet on one team (say, Team A.) In this case, the casino can adjust the odds very slightly in such a way that Team A has a slightly less than 50% chance of covering the spread, but still is attracting more than half the bettors. Therefore, the casino can earn strictly higher expected profits than in the case of even odds. However, once they adjust these lines away from even odds, they are susceptible to being exploited by the informed bettors. Therefore, we would expect that the early entrants (who set opening lines to exploit the public bias) must eventually adjust their lines in such a way that that the bias is no longer being exploited, and eventually, the late entrants enter and post opening lines that are closer to even odds

We will first discuss Levitt's model of bookmaker behavior and subsequently extend his model to account for the presence of informed bettors. Finally, we will present the empirical evidence using the data collected during the 2013-2014 NBA season.

III. LITERATURE REVIEW

This paper utilizes and extends a model of bookmaker behavior proposed by Steven Levitt in his 2004 paper “Why Are Gambling Markets Organized So Differently From Financial Markets?” Before extending his model to allow for informed bettors, it is useful to first discuss Levitt’s findings, along with relevant findings of other papers on gambling markets.

By utilizing data from a private betting tournament, Levitt first demonstrates that bettors empirically exhibit a significant amount of bias when placing bets. Based on this, he shows that bookmakers, in the presence of biased bettors, do not set the point spread to balance the number of dollars bet on each team but instead choose a point spread that systematically takes advantage of biased bettors to yield strictly higher expected profits. The theoretical basis for these claims is Levitt’s model of bookmaker behavior, which is supported by empirical findings and is discussed in great detail in the following section. The results have been supported by similar empirical studies, which sought to test a similar hypothesis using a more publicly accessible bookmaker, such as an online bookmaker.

An independent test of Levitt’s model of bookmaker behavior by Rodney Paul and Andrew Weinbach utilized data from Sportsbook.com, an online bookmaker, and yielded results that supported Levitt’s claims (Paul & Weinbach, 2008). In particular, not only did Paul and Weinbach confirm the prevalence of bias in betting markets, they also found that this particular bookmaker “set a price in this market to maximize profits, not to clear the market” – in other words, the bookmaker chose to exploit bias to obtain maximum expected profits rather than attract equal dollar amounts to both teams.

A particularly interesting question is posed at the conclusion of the Paul and Weinbach: “The question of why an informed bettor or group of bettors would not also recognize these biases of the betting public and use the same betting strategy that the sportsbook uses to maximize profits is certainly an interesting and important one to address.” This is precisely the question we will seek to address by extending Levitt’s model to allow for a group of “informed” bettors.

Similarly, a paper by Gandar et al. (1988) yielded results that lead to the conclusion that “informed traders are present and influential in this market,” speaking in particular about the NBA betting market. Their claim that “the brief trading period in this market largely rules out information release,” however, is no longer very applicable with the increasing abundance of online bookmakers and betting resources that were not available 25 years ago.

The term “informed bettor” is often used to describe players in the betting market with *more* information than the bookmaker. Hyun Song Shin discusses the cost of obtaining such information in his 1991 paper “Optimal Betting Odds Against Insider Traders” (Shin, 1991). For our purposes, however, we are more interested in the case where our “informed bettors” have the same information as the bookmaker – and hence, more information than the average bettor. We define our version of informed bettors based on Steven Levitt’s model, independent from other previously defined types of informed bettors such as Shin’s, and examine their theoretical effects alongside the empirical evidence of their existence.

IV. BOOKMAKER BEHAVIOR WITH NO INFORMED BETTORS

We first slightly modify Levitt's model of bookmaker behavior (Levitt, 2004) before introducing the concept of informed bettors in the following section. Levitt defines p to be the probability of winning a bet placed on the favored team, and can be considered a function of the point spread. For a bookmaker, setting p and setting the point spread are equivalent since we assume that the bookmaker can accurately convert a point spread into the corresponding p. Let the probability that a bettor places a bet on the favored team for a particular p be given by $F(p)$, which we assume to be differentiable and strictly increasing. Recall that bettors observe the point spread and not p. As such, as opposed to the bookmaker, we assume that bettors cannot perfectly convert a point spread into a corresponding probability of victory. This is the motivation for the probability function F, the functional form of which the bookmaker knows. The commission v charged by the bookmaker per dollar bet is treated as constant, since it is almost universally set to .1. A bookmaker's payoff function in profit per dollar wagered is given by

$$\pi(p) = [p(1-F(p)) + (1-p)F(p)](1+v) - [pF(p) + (1-p)(1-F(p))] \tag{1}$$

which can be rearranged as

$$\pi(p) = (2+v)[F(p) + p - 2pF(p)] - 1 \tag{2}$$

Since F is strictly increasing, $\frac{\partial F}{\partial p} > 0$. Setting the derivative of π with respect to p equal to zero, the bookmaker will choose p^* such that

$$[1\text{-}2F(p^*)] + [1\text{-}2p^*]\frac{\partial F}{\partial p} = 0 \qquad (3)$$

At even odds (the case where p=.5 and hence a bet on either team has an equal chance to win) it still may be the case that bettors prefer one team to another. We will say that bettors are *collectively biased towards the favorite* if $F(p=.5) > .5$ and conversely are *collectively biased towards the underdog* if $F(p=.5) < .5$. Bettors are said to be *collectively unbiased* if $F(p=.5) = .5$. That is, if the median of F lies below $p = .5$ then bettors are said to be collectively biased towards the favorite, and if the median of the distribution of bettors F lies above $p = .5$ then bettors are said to be collectively biased towards the underdog.

As Levitt demonstrates, in the case where bettors are collectively unbiased, $p = .5$ is the unique solution to (1) and expected profits are $v/2$. If bettors are collectively biased towards the favorite such that $F(p=.5) > .5$, then at $p = .5$, the left hand side of (3) is negative, and so $p = .5$ no longer maximizes π. If we assume that $\frac{\partial^2 \pi}{\partial p^2} < 0$ for all p, then bookmaker must reduce p below .5 to maximize profits (this assumption is expanded upon in the next section.) By attracting more than half of the expected bets on the team which is expected to "cover the spread" less than half the time (since for the new choice of p, $F(p)$ is still greater than .5), the bookmaker can actually take advantage of the biased bettors and obtain expected profits that are strictly higher than in the case of collectively unbiased bettors. Once p has been lowered to the point where $F(p) = .5$, then expected profits are again equal to $v/2$, where they were in the case that $p = .5$. In this case, because the probability that a bettor places a wager on each team is equal to .5, we say that the bookmaker is using a balanced book model.

V. BOOKMAKER BEHAVIOR WITH INFORMED BETTORS

In this section, we seek to quantify the effects of informed bettors on bookmaker behavior and provide an illustrative example of informed bettor entry. We end with a discussion of the bookmaker's ability to predict the fraction of informed bettors over time.

i. Model

We now extend Levitt's model and allow the distribution of bettors to be composed of two types: uninformed bettors and informed bettors. Uninformed bettors can be subject to collective bias, as described previously, and cannot perfectly convert a point spread into a corresponding probability of a winning bet, p. Informed bettors, by contrast, are individually unbiased and all place identical bets. More importantly, we assume that informed bettors can perfectly convert a point spread into probability of victory. That is, if the bookmaker sets a point spread that converts to $p > .5$, then all informed bettors bet on the favorite team and if the bookmaker sets a point spread that converts to $p < .5$, all informed bettors bet on the underdog. If the odds are even such that the point spread converts to $p = .5$ then we assume that the informed bettors bet on the favorite with probability .5. $F(p)$ can be rewritten as a weighted sum of the distributions of uninformed and informed bettors, given by F_U and F_I, respectively, where a is the fraction of informed bettors to total bettors:

$$F(p) = aF_I(p) + (1-a)F_U(p)$$

Then π can be rewritten as to reflect the two types of bettors as

$$\pi(p, a) = \begin{cases} (2+v)\left[(1\text{-}a)\ F_U(p) + p - 2p(1\text{-}a)\ F_U(p)\right]\text{-}1, & p < .5 \\ (2+v)\ \left[\left(1\text{-}a\right)\ F_U(p) + a + p - 2pa - 2p(1\text{-}a)\ F_U(p)\right]\text{-}1, & p > .5 \\ \frac{v}{2}, & p = .5 \end{cases} \qquad (4)$$

F_U is assumed to be strictly increasing and differentiable, so π is differentiable on the intervals (0, .5) and (.5, 1). Note that if the bookmaker sets $p = .5$, then he will obtain the same expected profit regardless of the fraction of informed bettors.

It can be shown that if the betting public is biased, then an increase in the fraction of informed bettors in the betting pool moves the optimal choice of *p* towards .5. Intuitively, this means that as informed bettors enter the bettor pool, the bookmaker will be less able to exploit the biased bettors, and will therefore have to set a new point spread (and therefore a new value of *p*) that is closer to even odds. Before proving two interesting results of the introduction of informed bettors, we present two important lemmas. For the remainder of this section, we assume that the public is collectively biased *towards* the favorite; as we will show in Lemma 1, this means we can restrict our attention to the first expression in (4), since the optimal point spread will be set such that $p^* < .5$. Slightly modified proofs will yield equivalent results in the case of uninformed bettors biased towards the underdog. Lemma 2 presents the necessary condition for a profit function that is concave down with respect to *p*.

Lemma 1. *If uninformed bettors are collectively biased towards the favorite, the profit maximizing* p *cannot lie in the interval (.5, 1]; that is,* p* *must lie in the interval [0, .5].*

Proof. It is sufficient to show that the second expression in (4) is strictly less than $\frac{v}{2}$ for $p > .5$. First note that because bettors are collectively biased towards the favorite, $F_U(p) > .5$. Therefore,

$$(2+v)\,[(1-a)\,F_U(p) + a + p - 2pa - 2p(1-a)\,F_U(p)] - 1$$

$$=(2+v)\,[(1-a)\,F_U(p)[1-2p] + a\,[1-2p]+ p]- 1$$

$$=(2+v)\,[[(1-a)\,F_U(p)+ a][1-2p] + p] - 1$$

$F_U(p) > .5$ and $a \in [0, 1]$,so $[(1-a)\,F_U(p) + a] > .5$, and since $[1-2p] < 0$, we get the relation

$$(2+v)\,[[(1-a)\,F_U(p)+ a][1-2p] + p] - 1 < (2+v)\,[[1-2p]\,F_U(p) + p] - 1$$

$$< (2+v)\,[.5(1-2p) + p] - 1 = \frac{v}{2}$$

■

Lemma 2. *Suppose the fraction of informed bettors if given by* a ϵ *(0, 1). Then the bookmaker's payoff function* π *is concave down with respect to* p *on the interval (0, .5) if and only if the probability distribution of bettor behavior F satisfies the condition*

$$\frac{\dfrac{\partial^2 F_U}{\partial p^2}}{\dfrac{\partial F_U}{\partial p}} < \frac{4}{1 - 2p}$$

Proof. Taking the derivative of π twice with respect to p, we get the following expressions:

$$\frac{\partial \pi(p,a)}{\partial p} = (2+v)[(1-a)[(1-2p)\,\frac{\partial F_U}{\partial p}\,-2F_U(p)] + 1]$$

$$\frac{\partial^2 \pi(p,a)}{\partial p^2} = (2+v)[(1-a)[\frac{\partial^2 F_U}{\partial p^2}[1-2p]-4\,\frac{\partial F_U}{\partial p}]]$$

Since a ≠ 1, the assumption of that π is concave down implies that

$$\frac{\partial^2 F_U}{\partial p^2}[1-2p]-4\,\frac{\partial F_U}{\partial p} < 0$$

Rearranging, we get

$$\frac{\frac{\partial^2 F_U}{\partial p^2}}{\frac{\partial F_U}{\partial p}} < \frac{4}{1 - 2p}$$

■

This condition should not be considered to be very restrictive; note that this condition holds on the relevant interval if *F(p)* is defined to be a truncated normal distribution on [0,1] with non-zero and finite variance; as such, this condition should not be considered to be too restrictive. We now introduce a theorem showing the effect of an increase of informed bettors on *p**.

Theorem 1. *If π is concave down with respect to p on the interval (0, .5) for some* a ϵ *(0, 1) and the bookmaker is maximizing his expected profit, then an increase in the fraction of informed bettors* a *increases the profit-maximizing choice of* p.

Proof. Let *M(p,a)* be the derivative of π with respective to *p*. That is,

$$M(p, a)=\frac{\partial \pi(p,a)}{\partial p} = (1-a)[(1-2p)\ \frac{\partial F_U}{\partial p}\ -2F_U(p)] + 1$$

Under profit-maximizing conditions, *M(p,a) = 0,* so we apply the constraint

$$M(p, a)=(1-a)\left[(1-2p)\frac{\partial F_U}{\partial p}\ -2F_U(p)\right]+ 1=0 \qquad (5)$$

Note that

$$\frac{\partial M(p, a)}{\partial p} =(2+v)(1-a)\left[\frac{\partial^2 F_U}{\partial p^2}(1-2p)\ -4\frac{\partial F_U}{\partial p}\right]$$

$$\frac{\partial M(p, a)}{\partial a} =(2+v)\left[2F_U(p)-\frac{\partial F_U}{\partial p}(1-2p)\right]$$

By the implicit function theorem,

$$\frac{\partial p^*}{\partial a} = -\frac{\frac{\partial M(p^*,a)}{\partial p}}{\frac{\partial M(p^*,a)}{\partial p}} = -\frac{(2+v)\left[2F_U(p^*)-\frac{\partial F_U}{\partial p}(1-2p^*)\right]}{(2+v)(1-a)\left[\frac{\partial^2 F_U}{\partial p^2}(1-2p^*) -4\frac{\partial F_U}{\partial p}\right]}$$

$$= -\frac{\left[2F_U(p^*)-\frac{\partial F_U}{\partial p}(1-2p^*)\right]}{(1-a)\left[\frac{\partial^2 F_U}{\partial p^2}(1-2p^*) -4\frac{\partial F_U}{\partial p}\right]}$$

Since $M(p, a) = 0$, rearrange (5) such that

$$\left[2F_U(p^*)-\frac{\partial F_U}{\partial p}(1-2p^*)\right] = \frac{1}{1-a}$$

Now we can replace the numerator with $\frac{1}{1-a}$ and this yields the expression

$$\frac{\partial p^*}{\partial a} = -\frac{1}{(1-a)^2\left[\frac{\partial^2 F_U}{\partial p^2}(1-2p^*) -4\frac{\partial F_U}{\partial p}\right]}$$

By assumption, π is concave down with respect to p, and so by Lemma 1,

$$\left[\frac{\partial^2 F_U}{\partial p^2}(1-2p^*) -4\frac{\partial F_U}{\partial p}\right] < 0$$

Since $(1-a)^2 > 0$, $\frac{\partial p^*}{\partial a} > 0$ as desired.

■

This result tells us that at some profit maximizing point $p^* < .5$, an increase in the fraction of informed bettors a increases the corresponding p^*. It should be noted, however, that this only holds true on the interval (0, .5). However, if we instead assumed that bettors were collectively biased towards the underdog, then we would restrict our attention to the interval (.5, 1) and a similar proof would should that the profit maximizing value of p decreases with an increase in a. These two results together tell us that, in both cases, the increases in a lead to a convergence of p^* to .5.

Another interesting result can be derived from this concept of convergence to .5. Since increases in the fraction of informed bettors require *p** closer to .5, it is natural to ask whether there is a cut-off for *a*, after which all optimal *p* are equal to .5. As it turns out, there is a well-defined value for *a* that signals the point after which any further increases in *a* will not affect *p**, as it will remain at .5.

Theorem 2. *Suppose the fraction of informed bettors is given by* a ϵ *(0, 1) and let* $M(p, a) = \frac{\partial \pi(p,a)}{\partial p}$*. If uninformed bettors are collectively biased towards the favorite and* $\lim_{p \to .5^-} M(p,a) > 0$ *then the bookmaker will set* $p^* = .5$*. In particular, if*

$$a > 1 - \frac{1}{2F_U(.5)}$$

then the profit maximizing p *is .5.*

Proof. By Lemma 1, since bettors are collectively biased towards the favorite, *p** must lie on the interval [0, .5].

$$\lim_{p \to .5^-} M(p,a) = \lim_{p \to .5^-} (1-a)[(1-2p)\frac{\partial F_U}{\partial p} - 2F_U(p)] + 1 = (1-a)(-2F_U(.5)) + 1$$

If this limit is greater than zero, then due to the concave down nature of π on (0, .5), any *p* slightly less than .5 cannot be optimal and must be adjusted upwards towards the upper limit of the interval [0, .5], $p = .5$. That is, if the following inequality holds, then $p^* = .5$:

$$a > 1 - \frac{1}{2F_U(.5)}$$

■

The implications of these two theorems are best illustrated through an example.

ii. Illustrative Example

To further examine the implications of Theorems 1 and 2, it is useful to look at an illustrative example. Suppose that bettors are distributed according to some truncated normal distribution on the interval [0, 1] with finite and non-zero variance; this yields the probability that bettor will bet for the favored team for each *p*, given by *F(p)*. Figure 1 shows the case where the median of *F(p)* is greater than .5, defined earlier as being collectively biased towards the favorite, and the case where the median of *F(p)* equals .5, defined earlier as being collectively unbiased.

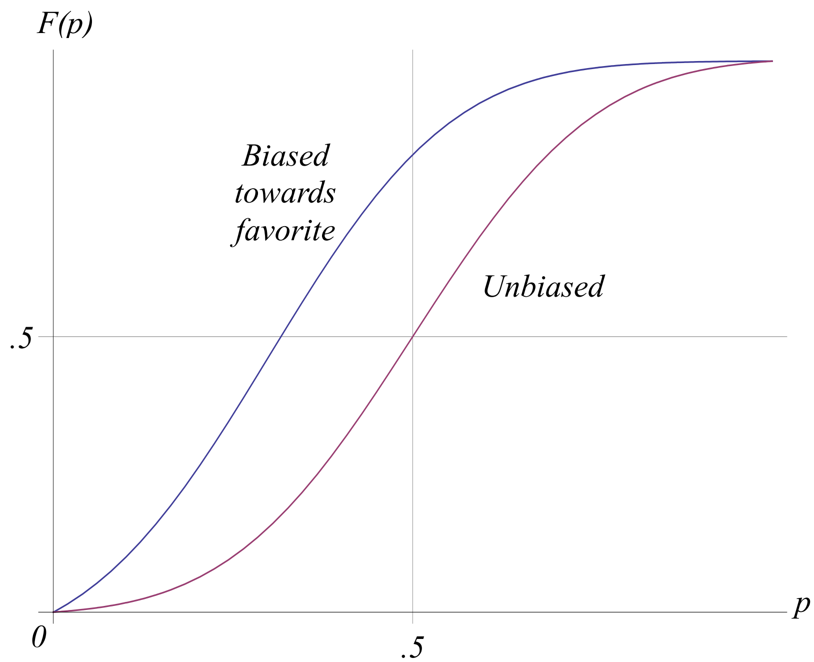

Figure 1: Two distributions of bettor behavior.

As Levitt demonstrates, if bettors are indeed biased towards the favorite, the bookmaker can exploit the bettors to earn higher profits than in the case where bettors are unbiased. As seen in Figure 2, by lowering p to $p^* < .5$, the bookmaker can earn strictly higher profits than setting $p^* = .5$ in the case of an unbiased betting public. As stated earlier, this is one of the main results of Levitt's model.

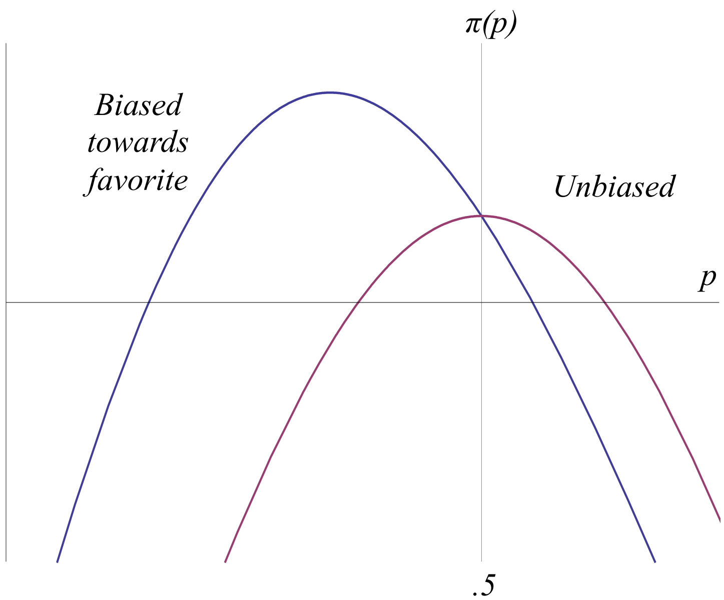

Figure 2: Bookmaker profit with biased and unbiased bettors.

As such, it is worthwhile to ask what the implications would be of setting such point spreads to exploit public bias. In particular, what would happen if an informed bettor takes advantage of these lines? Once we introduce informed bettors (as defined previously) into this model, the profit function for a bookmaker facing a public that is biased towards the favorite is dependent on the fraction of informed bettors, as shown in Figure 3. Specifically, we see that as *a*, the fraction of informed bettors, increases, the maximum attainable profits decrease.

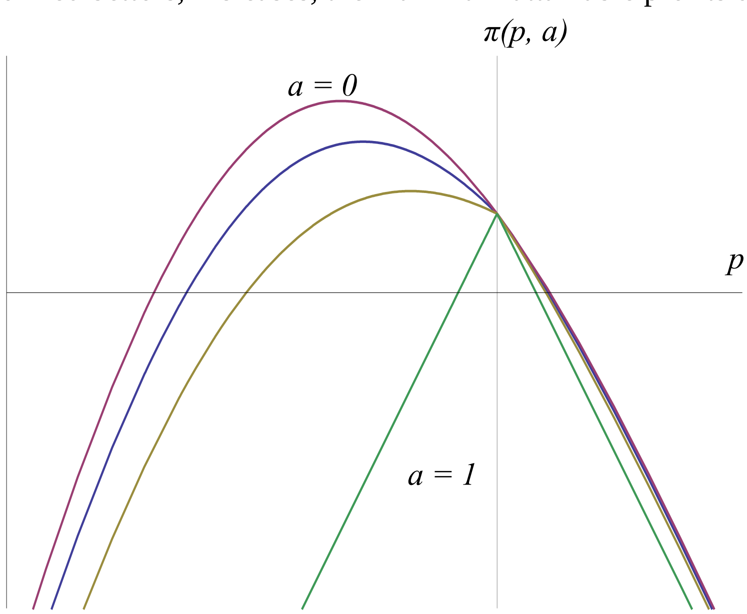

Figure 3: Bookmaker profit with varying degrees of informed bettors.

Figure 4 is an illustrative example of Theorem 1, which states that an increase in the fraction of informed bettors will push the profit maximizing value of *p* towards .5. Suppose the bookmaker has a previous belief of the fraction of informed bettors in the betting market, given by a_1. He then sets p_1* to maximize his profit.

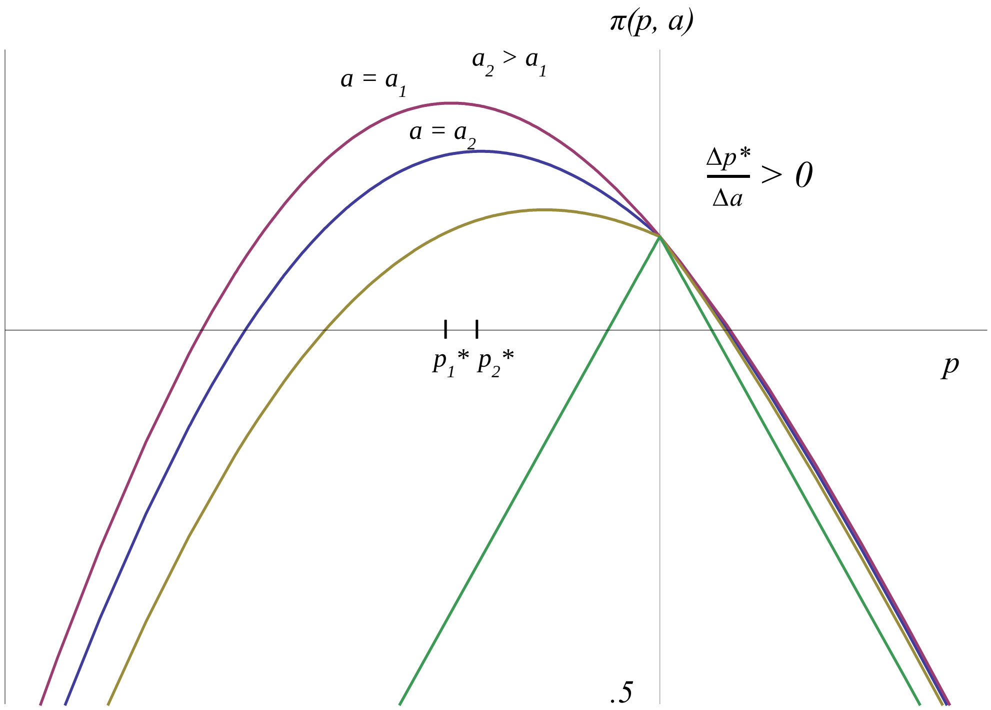

Figure 4: Graphical representation of Theorem 1.

We have assumed that $a \neq 1$ (in other words, there exist some uninformed bettors), so the bookmaker has set p_1* to exploit bias of the uninformed bettors. Subsequently, informed bettors identify that the bookmaker has set a biased *p*, and *a* increases to $a_2 > a_1$; this increase in informed bettors over time is an assumption that is discussed in the next section. The bookmaker's new optimal *p*, given by p_2*, is greater than p_1*, and hence closer to .5.

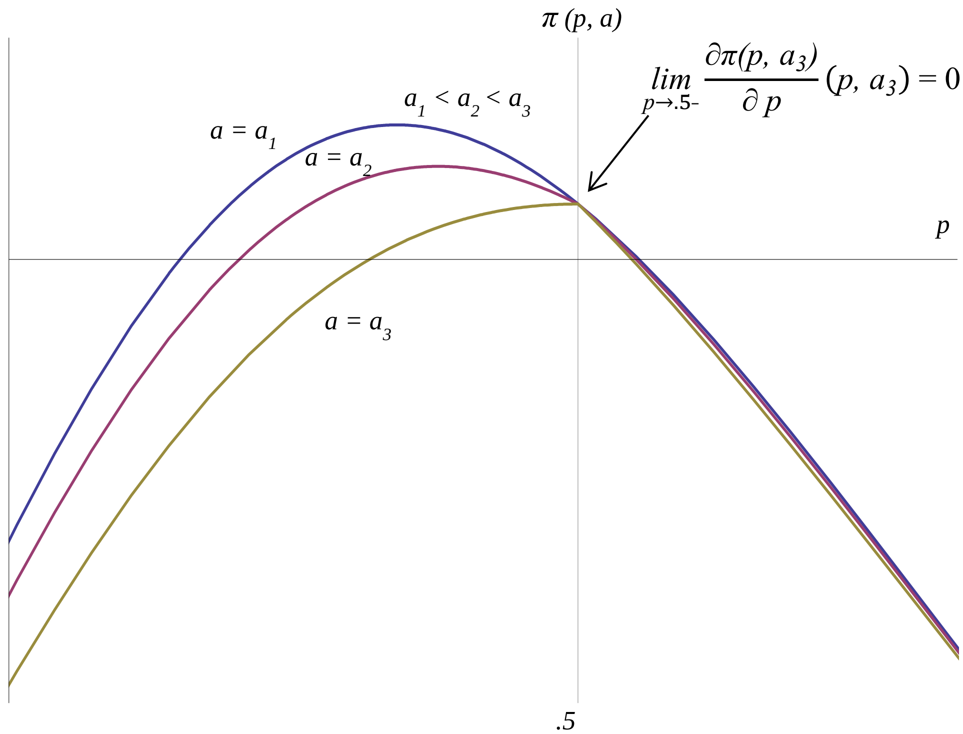

Figure 5: Graphical representation of Theorem 2.

Theorem 2 shows that there is a value of *a* for which this is no longer true, and any subsequent increases in *a* would yield no change in *p**, which would remain at .5.

In our example, shown in Figure 5, this is shown by first assuming the fraction of informed bettors is given by a_3. Increasing *p* towards .5 yields higher profits – in fact, setting $p = .5$ is the best the bookmaker can do in this case. Furthermore, for any higher value of *a*, setting $p = .5$ will be the optimal solution, as this yields strictly higher profits than any other *p*. In this case, we have a “critical mass” of informed bettors, given by a_3, after which the bookmaker can no longer exploit the biased public and must set *p* to .5.

iii. Predicting Changes in the Fraction of Informed Bettors

So far, it has been assumed that the bookmaker can accurately predict the fraction a of informed bettors. In reality, the bookmaker may only have a belief of the true value of a. The effects of an incorrect estimation of a are important in determining how the fraction of informed bettors changes over time. Because informed bettors bet for each team with equal probability when p = .5, their expected profit per dollar wagered can be written as

$$\pi_I(p) = \begin{cases} (1\text{-}p) - p(1+v), & p < .5 \\ p - (1\text{-}p)(1+v), & p > .5 \\ .5 - .5(1+v), & p = .5 \end{cases}$$

It is clear that the informed bettors obtain higher expected profits as p differs more significantly from .5. How might this affect the introduction of more informed bettors? Suppose betting takes place in two stages and that uninformed bettors are collectively biased towards the favorite. In the first stage, the bookmaker will either:

- underestimate the true fraction of informed bettors a_1 and set p equal to p_1 such that $p_1 < p^* < .5$, where p^* is the profit maximizing value of p that would be chosen if the bookmaker had accurately predicted the value of a_1;
- overestimate a_1 and set p equal to p_1 such that $p^* < p_1 < .5$;
- or accurately estimate a_1 and set p equal to p^*.

In this scenario, an underestimation of a_1 and the resulting value of p would result in strictly higher expected profits for the informed bettors than in the case of an overestimation of a_1. This is true simply because for values of p less than .5, profits for the informed bettors increase as p approaches 0.

In the second stage of betting, we would expect an increase in a (assuming no change in the collective biasness of the uninformed bettors). Regardless of the value of p_1, informed bettors fared better in the first round than their uninformed counterparts, and we can expect this information to spread to a certain extent as informed bettors continue to identify biased lines. How much the fraction of informed bettors increases, however, should depend on whether a_1 is under or overestimated. Although profits are not realized until after the outcome of the game has been decided, higher profits that informed bettors *expect* to receive based on their first round bets should attract more informed bettors for the second stage of betting. As such, it is reasonable to assume that there will be a greater increase in the fraction of informed bettors if a_1 is underestimated (higher expected profit for informed bettors) than if a_1 is overestimated (lower expected profit for informed bettors.) An interesting question to explore would be the possibility of the bookmaker setting p^* such that he intentionally overestimates the fraction informed bettors disincentivize other informed bettors from entering. However, this will not be addressed in this paper; instead, we will just keep the simpler claim that the fraction of informed bettors increases with each stage of betting.

The previous finding tells us that the later a bookmaker chooses to enter the betting market, the closer his initial estimate of a should be to 1 (assuming other bookmakers have already entered in an earlier stage), since each stage of betting increases the fraction of informed bettors. As such, a bookmaker that enters the betting market late, in the presence of other bookmakers opening earlier, should be more likely to set his point spread to even odds (that is, $p = .5$). Furthermore, since the late entrant has a continually diminishing ability to exploit bias as betting continues, we would expect a smaller change in opening to closing lines than in the case where the same bookmaker was a late entrant but *all* other bookmakers were also late entrants. In this case, all the bookmakers initially set the point spread to exploit the biased bettors and subsequently are affected by informed bettors, because they have no prior information about the actual fraction of informed bettors provided by early entrants.

VI. EMPIRICAL EVIDENCE FOR INFORMED BETTORS

The data describes relevant metrics from the first 328 games of the 2013-2014 NBA season as published by VegasInsider.com, excluding the first game for each team. Prior to examining the point spread data, the following characteristics of each game were recorded: *Game Date, Home Team, Away Team, Home Points, and Away Points.*

For each game in the sample, six Las Vegas sportsbooks posted opening and closing points spreads, or lines. Any line changes that occurred between the opening and closing lines, however, are not recorded, as they are not relevant to this analysis. Also recorded is whether, for a particular game, a sportsbook posted its opening line the day before the game or the morning of the game. Those that set their opening lines the day before the game are said to be "early entrants" and those that set their opening lines the day of the game are said to be "late entrants." Figure 1 depicts data for one game between the Chicago Bulls and the New York Knicks. An additional metric *All Late* is equal to 1 if all sportsbooks set opening lines the morning of the game and 0 otherwise – in this case, because Atlantis, LVH, SP and William Hill set their opening lines the day prior to the game, *All Late* equals 0 for all sportsbooks.

Game Information						**Spread Information**				
Game Date	*Home*	*Home Points*	*Away*	*Away Points*	*Game ID*	*Book*	*Late Entry*	*Open*	*Close*	*All Late*
10/31/2013	CHI	82	NYK	81	130001	Atlantis	0	-8	-7.5	0
10/31/2013	CHI	82	NYK	81	130001	LVH	0	-8	-8	0
10/31/2013	CHI	82	NYK	81	130001	SP	0	-8	-7.5	0
10/31/2013	CHI	82	NYK	81	130001	WillHill	0	-8	-7.5	0
10/31/2013	CHI	82	NYK	81	130001	MGM	1	-8	-8	0
10/31/2013	CHI	82	NYK	81	130001	Wynn	1	-8	-7.5	0

Figure 7 – Sample inputted data for one NBA game.

From this data, absolute changes in point spreads (|Open – Close|) were extracted. Before proceeding, it is important to note the role of external effects on the movement of point spreads. That is, from the posting of the opening line to the posting of the closing line, there are factors relevant to the expected outcome of the game which may cause the point spread to move; examples of such factors include announcements of injuries or suspension of key players. Whether the sportsbook is an early or late entrant, it can be subject to the variability of some of these factors, which can be thought of as "random noise," as there is no reason to assume a systematic bias towards any teams with regards to injuries or suspensions. However, the longer a sportsbook remains open, the more likely it is to be affected by this noise. Ultimately, we want to show that sportsbooks that enter the betting market early – early entrants – have a greater tendency to move the point spread than late entrants due to the entrance of informed bettors. As such, in the presence of the previously mentioned random noise, it would be difficult to determine whether the point spread of an early entrant moved due to noise or due to some other factor – namely, informed bettors. Figure 8 gives descriptive statistics for early entrants and late entrants. As expected, early entrants exhibited significantly larger line changes than late entrants; this is not surprising, due to the effect of the aforementioned noise. Thus, we must be able to strip the random noise from early entrants before comparing absolute line changes with those of the late entrants.

		|Opening Line - Closing Line|	
All Games	n	*Mean*	*St Dev*
All Bookmakers	1949	0.58	0.63
Opened Line Early	777	0.72	0.70
Opened Line Late	1172	0.48	0.56
t-test: p-value		0.00	

Figure 8: Descriptive Statistics for Early and Late Entrants.

This can be achieved by comparing two special subsets of bookmakers. Empirically, bookmakers opened point spreads in one of two ways for a given game; these two cases are illustrated in Figure 9.

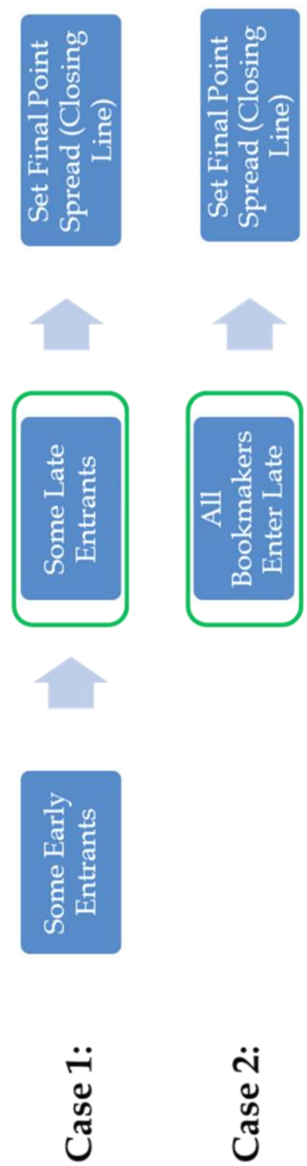

Figure 9: Subsets of Bookmakers

In the first case are games in which there exist both early and late entrants; that is, at least one of the bookmakers set an opening line the day before the game and at least one bookmaker set an opening line the day of the game. In the second case are games in which all the bookmakers set their opening lines the day of the game. By comparing the late entrants in the first case to the bookmakers in second case, we have successfully stripped any effect of noise (because both subgroups have their lines open for the same amount of time).

Without the noise, we are effectively left with the influence of early entrants on the point spreads of late entrants. Since bookmakers have been shown to set their point spreads to exploit bias to maximize profits, rather than minimize risk by attracting equal money to both sides of the bet, any effects captured in this model can be attributed to changes in the bias of the bettor population. Indeed, as shown by Figure 10, bookmakers that set their lines *after* other bookmakers had set their lines the previous day exhibited significantly smaller line changes from opening to closing lines as compared to bookmakers in games for which all bookmakers opened their lines the day of the games.

Games In Which All Bookmakers Entered Late	n	**|Opening Line - Closing Line|** *Mean*	*St Dev*	*t-test: p-value*
Opened Line Late (All Bookmakers)	437	0.58	0.57	
Games with Both Early and Late Entrants	n	**|Opening Line - Closing Line|** *Mean*	*St Dev*	
Opened Line Late	735	0.42	0.54	0.007

Figure 10: Descriptive Statistics for Special Cases.

This result can be interpreted as evidence of informed bettors. In the setting of our extension of Levitt's model, this can be described the following way: In games with early entrants, the early entrants set their opening lines and begin to attract informed bettors to the bettor population. As more informed bettors identify the biased lines, the profit maximizing point spread (p^*) is pushed closer to even odds ($p = .5$), as described by Theorem 1. At some point the day of the game, the late entrants set their opening lines. Eventually, there are enough informed bettors that point spreads are set such that $p=.5$, as described by Theorem 2. Hence, the early entrants' point spreads reflect the entire movement from the first profit maximizing point spread to the final point spread, where $p=.5$. The late entrants' point spreads, however, reflect only the portion of this movement after their opening lines. We effectively use the case in which all bookmakers enter late as a proxy for early entrants in the first case in Figure 9, as these bookmakers would similarly have to bear the entire movement of p from the initial profit maximizing point spread to $p=.5$.

An alternative interpretation of this result not involving informed bettors would be that public bias towards a team is diminished over the course of betting. While this interpretation cannot be fully discredited, it relies on somewhat questionable logic. It is important to note that while the public can be biased towards a team, this is distinctly different from bettors acting irrationally; bettors can simply enjoy betting on their favorite team, for example, and derive a great deal of utility simply from the excitement of betting. If the public's bias towards a team erodes over the course of one game, but the public continues to act rationally, then this implies that their personal preference towards a team has changed over a very short period of time. While this is certainly possible, it is worthwhile to explore alternative explanations, such as the entrance of profit-minded informed bettors.

VII. CONCLUSION

We developed a model to explain the change in behavior exhibited by a profit-maximizing bookmaker when informed bettors are introduced to the betting market. The model is validated by empirical data recorded from the 2013-2014 NBA season. We also considered the broader significance of this model in any kind of speculative market in which inside information is available. While the results presented here are certainly indicative of informed bettors affecting bookmaker behavior, it is impossible to attribute the changes we observed empirically solely to informed bettors; in reality, there may be a number of other unobserved factors that play a role in such changes. At the very least, however, our results show the potential gain and effects that arise from being an informed bettor. It would be particularly interesting to fully develop a model exploring the choice a bookmaker faces when deciding to set his point spread early versus late, and examining the first and second mover advantages in such a case.

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