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A *hydrodynamic Schrödinger equation* approach to solve fluid flöw problems

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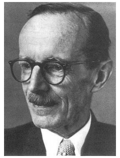
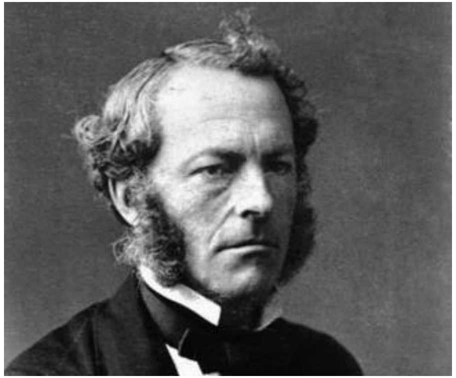
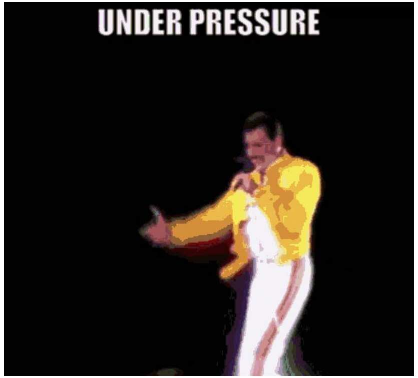
What are we doing, and why should you care?

We solve classically intractable fluid flow problems, e.g. Burgers' & Navier-Stokes

via a quantum analogue à la Schrödinger  & Madelung 

 =  − 

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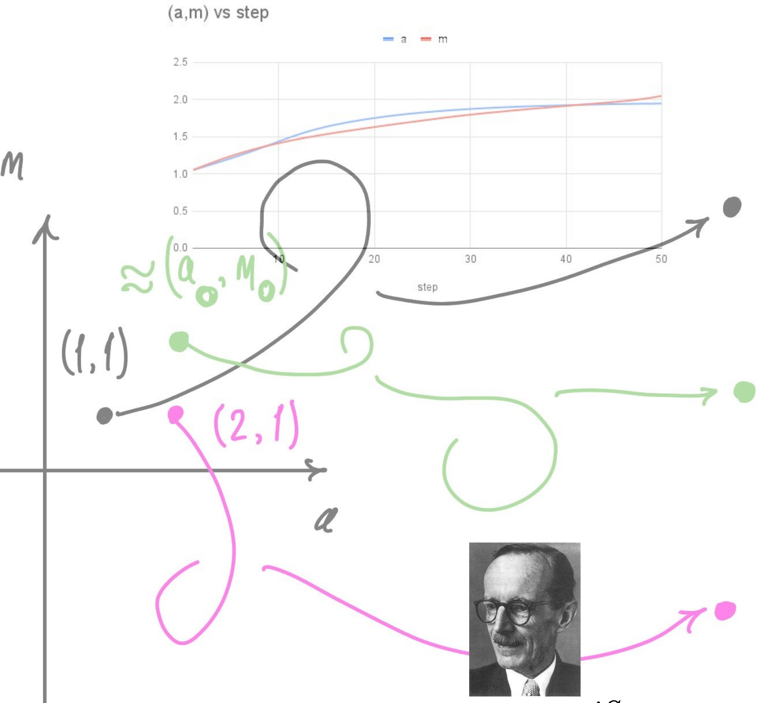
Flödinger approach? We like *Burger(s)*, not (just any) equations!

$$\left(i\hbar\frac{\partial}{\partial t}+\frac{\hbar}{2m}\nabla^2\right)\psi=\frac{4\pi\hbar a}{m}\psi|\psi|^2$$



$$\begin{cases}\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\nabla)=0\\ \rho\frac{\partial v}{\partial t}+(v\cdot\nabla)v=-\nabla\left(P-\frac{\hbar^2}{4m}n\nabla^2\ln(n)\right)\end{cases}$$

$$\psi\mapsto\sqrt{\rho}e^{iS}$$

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Quantum advantage?

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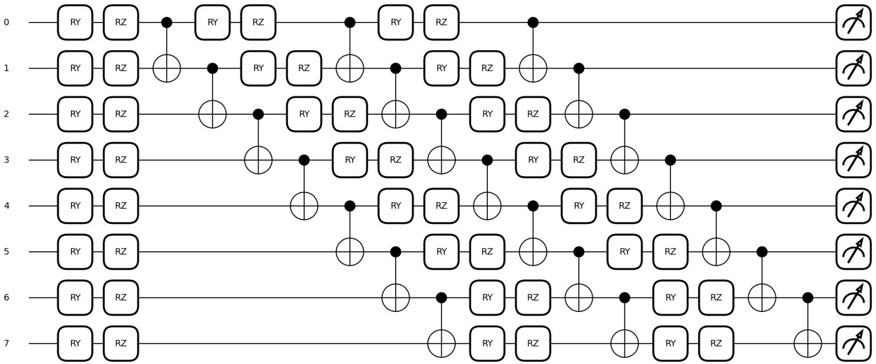


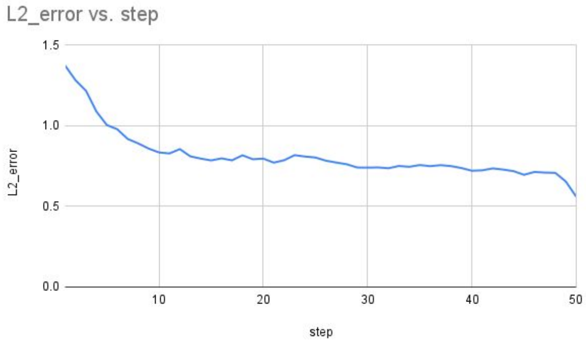
5 Quantum Hardware Run

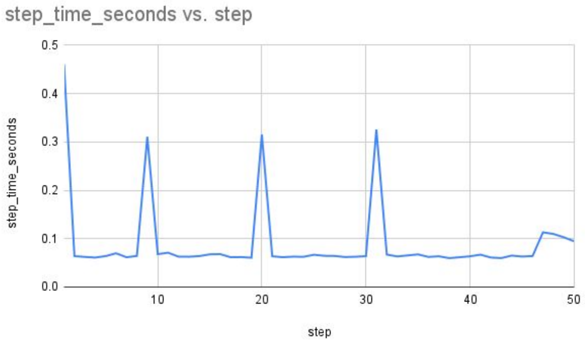
- Backend: IBM Q Lagos (7 qubits, transpiled for 8-qubit use with ancilla remapping)
- Qubits Used: 8
- Two-qubit Gate Depth: 21
- Runtime: 9.3 seconds (queue + execution)
- Raw L^2 Error: 0.128
- Error-mitigated L^2 Error (ZNE): 0.084
- Effective Error Rate: $\approx$ 3.2%

5.1 Noisy Simulator Results (PennyLane default.mixed)

- Backend: default.mixed (noisy simulator)
- Qubits Used: 8 Two-qubit Gate Depth: 21
- Runtime: $\approx$ 0.08 seconds (per step)
- Raw L^2 Error: 0.091
- Error-mitigated L^2 Error (ZNE): 0.061
- Effective Error Rate: $\approx$ 2.7%
- Noise Model: Depolarizing channel ($p = 0.02$) + Amplitude damping ($p = 0.01$)

What did we achieve?





Metric	Variables with Estimates	Notes
Qubits required	Nx, e.g. 8 for current setup	number of spatial discretization points (grid size)
Two-qubit gate Depth	layers x (Nx - 1)	one CNOT chain per layer; rough estimate of entangling gates
Mitigation strategy	Zero Noise Extrapolation (ZNE), Clifford Data Regression	Standard error mitigation technique for noisy QPU's
Noisy Simulator Metrics	Effective error rates approximately 1-5%	Simulated noise using PennyLane's default.mixed device
Runtime (per optimization step)	$\approx$ 0.04 seconds (average from logs)	Wall-clock time on noiseless simulator
Scalability (qubits)	Linear in Nx	directly proportional to grid size
Scalability (two-qubit gate depth)	Linear with Nx $\times$ layers	Depth grows as layers $\times$ (Nx - 1)

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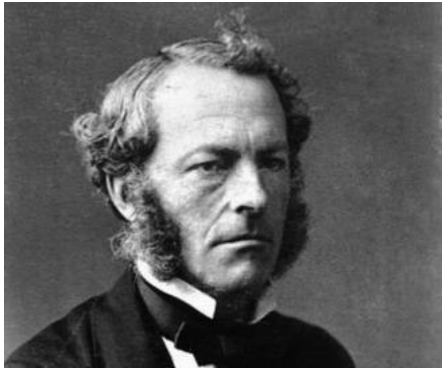
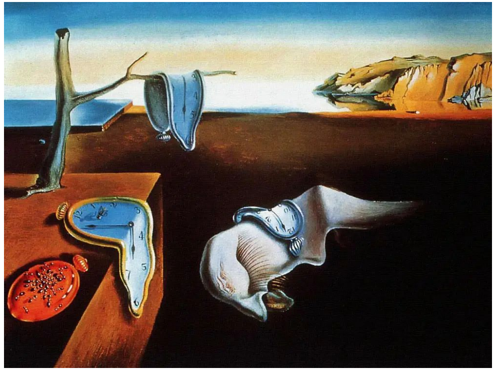
Who's next?





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F(löd)in(ger).

Thank you for listening! Questions?

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Citations.

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