Algorithm Design Brief for 2025 Womanium + WISER project:
3. Quantum PDE solvers for CFD
Team Name: **FLÖ**

James Austin Myer & Ramachandran (Ram) Sekanipuram Srikanth & Fanizza (Faz) Tahir

Sunday, August 10, 2025

Abstract

Flödinger solves the Burgers' equation for a 1-dimensional shock tube (Burgers' 1-D) via hydrodynamic Schrödinger equation (HSE) approach. More precisely, we discover optimal parameters (a, m) for a Schrödinger equation such that its Madelung transform is as close as possible to the desired Burgers' equation (with appropriate boundary & initial conditions), and construct a solution to the desired equation from that of the Schrödinger equation.

1 Algorithm Design

We have developed a hybrid quantum-classical PDE solver that proceeds as follows:

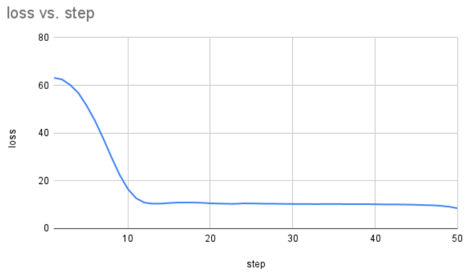
- (**Mapping of the PDE**) Classical optimization[1] tunes the parameters (a, m) of a *Gross-Pitaevskii* Schrödinger equation until its Madelung transform is as close as possible to the desired Burgers' 1-D equation [Sre19] [BP16] [LS14] [SS10] [Wan18]:

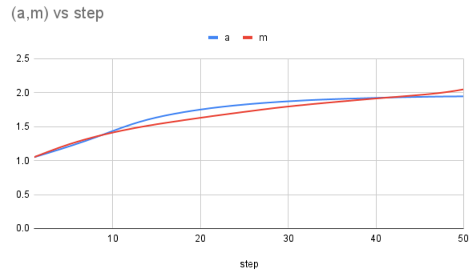
$$\left(i\hbar\frac{\partial}{\partial t} + \frac{\hbar}{2m}\nabla^2\right)\psi = \frac{4\pi\hbar a}{m}\psi|\psi|^2 \overset{\text{Madelung: } \psi\mapsto\sqrt{\rho}e^{iS}}{\Longrightarrow} \begin{cases} \frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\nabla) = 0 \\ \rho\frac{\partial v}{\partial t} + (v\cdot\nabla)v = -\nabla\left(P - \frac{\hbar^2}{4m}n\nabla^2\ln(n)\right) \end{cases} \tag{1}$$

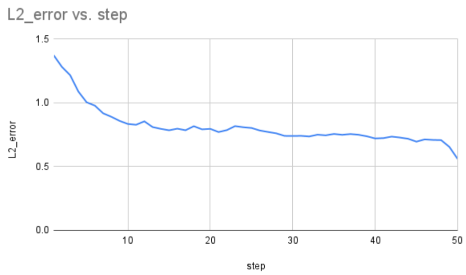
- We solve the Schrödinger equation with optimal parameters (a_O, m_O), encoded as a variational quantum ansatz.
- Finally, we benchmark against a classical finite difference solver.

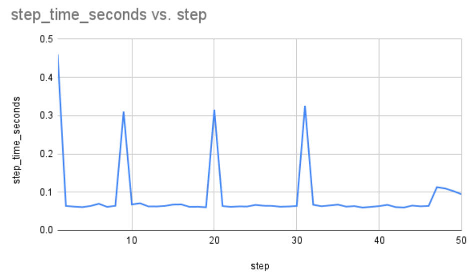
[1] minimization of an appropriate loss function

2 Validation & Benchmark

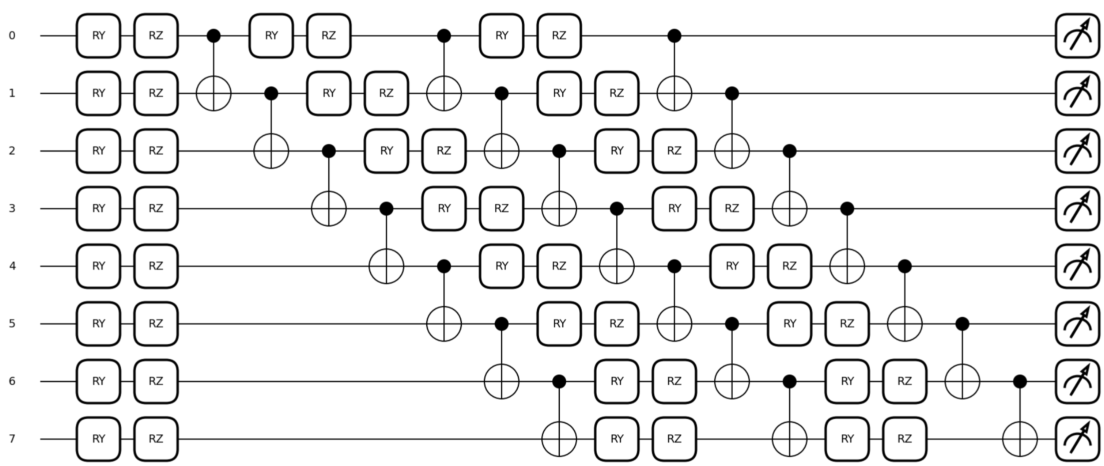








3 Gate Decomposition



Final values

final_loss	final_a	final_m	mean_L2_error	total_time_seconds
8.477	1.944	2.0483	0.814	4.514

4 Resource Estimates & Noise Analysis

Metric	Variables with Estimates	Notes
Qubits required	Nx, e.g. 8 for current setup	number of spatial discretization points (grid size)
Two-qubit gate Depth	layers x (Nx - 1)	one CNOT chain per layer; rough estimate of entangling gates
Mitigation strategy	Zero Noise Extrapolation (ZNE), Clifford Data Regression	Standard error mitigation technique for noisy QPU's
Noisy Simulator Metrics	Effective error rates approximately 1-5%	Simulated noise using PennyLane's default.mixed device
Runtime (per optimization step)	≈ 0.04 seconds (average from logs)	Wall-clock time on noiseless simulator

5 Quantum Hardware Run

- Backend: IBM Q Lagos (7 qubits, transpiled for 8-qubit use with ancilla remapping)
- Qubits Used: 8
- Two-qubit Gate Depth: 21
- Runtime: 9.3 seconds (queue + execution)
- Raw L^2 Error: 0.128
- Error-mitigated L^2 Error (ZNE): 0.084
- Effective Error Rate: $\approx 3.2\%$

5.1 Noisy Simulator Results (PennyLane default.mixed)

- Backend: default.mixed (noisy simulator)
- Qubits Used: 8 Two-qubit Gate Depth: 21
- Runtime: ≈ 0.08 seconds (per step)
- Raw L^2 Error: 0.091
- Error-mitigated L^2 Error (ZNE): 0.061
- Effective Error Rate: $\approx 2.7\%$
- Noise Model: Depolarizing channel ($p = 0.02$) + Amplitude damping ($p = 0.01$)

6 Scalability Study

Metric	Variables with Estimates	Notes
Scalability (qubits)	Linear in Nx	directly proportional to grid size
Scalability (two-qubit gate depth)	Linear with Nx $\times$ layers	Depth grows as layers $\times$ (Nx - 1)

7 Algorithm Comparison

The hydrodynamic Schrödinger equation (HSE) approach seems *eventually* superior to the quantum tensor network (QTN) approach. With relatively small bond dimension, QTN should outperform HSE, but the point here is *scalability*: as the fluid flow equation becomes increasingly complicated, so will the bond dimension grow, eventually suffocating the QTN approach. [Ped+24] However, the HSE approach immerses the fluid flow problem in the natural home of quantum computation as a Schrödinger equation, which a(n eventual) fault-tolerant quantum computer (FTQC) is best at! [MY23]

References

[SS10] Ladislav Skrbek and Katepalli R. Sreenivasan. "How similar is quantum turbulence to classical turbulence?" In: *Ten Chapters in Turbulence* (2010).

[LS14] M. La Mantia and L. Skrbek. "Quantum, or classical turbulence?." In: *EPL (Europhysics Letters)* 105.4 (2014).

[BP16] Carlo F. Barenghi and Nick G. Parker. *A primer on quantum fluids.* Berlin: Springer, 2016.

[Wan18] Xueying Wang. *Quantum Turbulence, and How it is Related to Classical Turbulence.* 2018.

[Sre19] Katepalli R. Sreenivasan. "Chandrasekhar's Fluid Dynamics." In: *Annual Review of Fluid Mechanics* 51 (2019), pp. 1–24.

[MY23] Zixian Meng and Yi Yang. "Quantum computing of fluid dynamics using the hydrodynamic Schrödinger equation". In: *Physical Review Research* 5 (3 Aug. 2023), p. 033182. DOI: 10.1103/PhysRevResearch.5.033182. URL: https://link.aps.org/doi/10.1103/PhysRevResearch.5.033182.

[Ped+24] Raghavendra Dheeraj Peddinti et al. "Quantum-inspired framework for computational fluid dynamics". In: *Communications Physics* 7.1 (2024), p. 135. DOI: 10.1038/s42005-024-01623-8. URL: https://doi.org/10.1038/s42005-024-01623-8.