In this document, we revise Theorem 9 on the sensitivity of $\log_{10}(P\text{-values})$ in the Cochran-Armitage trend test. In Section S1.3 of Supplementary Material, we stated that the test has 2 degrees of freedom, which is incorrect. The Cochran-Armitage trend test has 1 degree of freedom. Therefore, the P-value corresponding to a value x of the χ^2-statistic under the null χ^2-distribution is

$$P = \frac{1}{\sqrt{2\pi}} \int_x^\infty x^{-\frac{1}{2}} \cdot \mathrm{e}^{-\frac{x}{2}} \, \mathrm{d}x.$$

Theorem 9. *(Revised ver.) The sensitivity of* $\log_{10}(p\text{-values})$ *obtained from the* χ^2*-statistic of the Cochran-Armitage trend test based on a* 3×2 *contingency table, in which the margins are positive and the number of the case and the control are both* $N/2$*, is less than* 4.2*.*

Proof. (The flow of this proof is the same as the proof of Theorem 4.)
We let

$$f(x) = \log_{10}\left(\int_x^\infty x^{-\frac{1}{2}} \cdot \mathrm{e}^{-\frac{x}{2}} \, \mathrm{d}x \right).$$

Because the sensitivity of the χ^2-statistics from the Cochran-Armitage trend test is $\frac{16N(N^2+6N+4)}{(N+18)(N^2+8N-4)} < 16$ [$\because$ Theorem 8, $N \geq 1$], that of $\log_{10}(P\text{-values})$ is less than the maximum value of $f(x) - f(x+16)$. Here,

$$f(x) - f(x+16) = \log_{10}\left(\frac{\int_x^\infty x^{-\frac{1}{2}} \cdot \mathrm{e}^{-\frac{x}{2}} \, \mathrm{d}x}{\int_{x+16}^\infty x^{-\frac{1}{2}} \cdot \mathrm{e}^{-\frac{x}{2}} \, \mathrm{d}x} \right),$$

and we let

$$g(x) = \left(\frac{\int_x^\infty x^{-\frac{1}{2}} \cdot \mathrm{e}^{-\frac{x}{2}} \, \mathrm{d}x}{\int_{x+16}^\infty x^{-\frac{1}{2}} \cdot \mathrm{e}^{-\frac{x}{2}} \, \mathrm{d}x} \right).$$

Because $g'(x) < 0$ (see the proof of Theorem S4 in Supplementary Material), $g(x)$ and $f(x) - f(x+16)$ take the maximum values when $x = 0$. Because $f(0) - f(16) < 4.2$, the sensitivity of $\log_{10}(P\text{-values})$ is less than 4.2. $\square$

From this theorem, we should add Laplace noise with scale 4.2 when publishing ϵ-differentially private $\log_{10}(P\text{-values})$ in the Cochran-Armitage trend test. However, referring to Figure 6 in the published paper and Figure S10 in Supplementary Material, this approach is not expected to provide sufficiently high output accuracy. In the future, we plan to utilize the concept of *smooth sensitivity* and investigate noise distributions other than Laplace distribution to develop more accurate methods for publishing GWAS statistics.