Microwave Filters

I. Introduction

Filter is a two-port network for frequency response.

Image Parameter method is useful for low-frequency filters. Most microwave filter design is based on the Insertion loss method. Both image parameter method and insertion loss method lead to lumped element circuits. Richard transformation and Kuroda’s Identities will lead to appropriate transmission line equivalent.

What is a perfect filter? A perfect filter exhibits zero insertion loss in the passband, infinite attenuation in the stopband, and a linear phase response in the passband.

Insertion loss method provides many variables to control the filter specification.

II. Periodic Structure

Periodic structure supports slow-wave propagation (slower than the phase velocity of unloaded line <<I’m not sure what it supposes to mean>>).

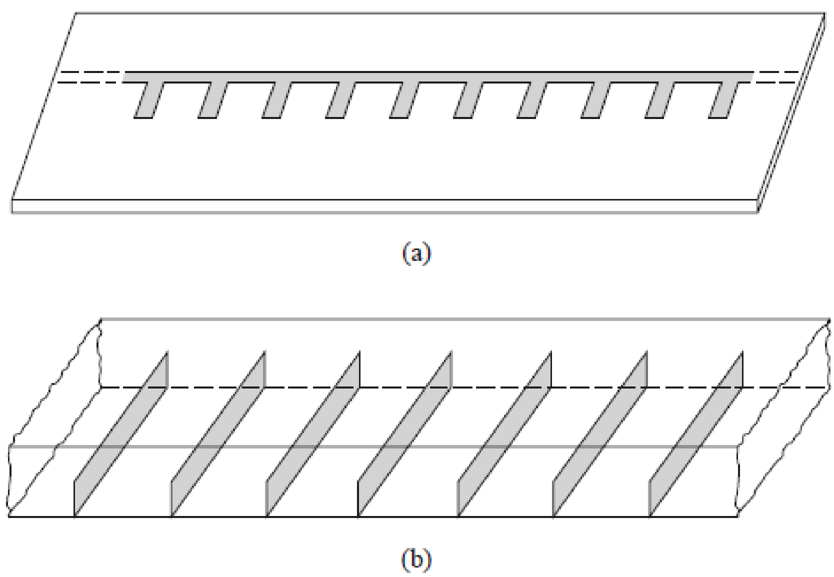

FIGURE 8.1 Examples of periodic structures. (a) Periodic stubs on a microstrip line. (b) Periodic diaphragms in a waveguide.

a. Analysis of Infinite Periodic Structures

III. The Image Parameter Method

It consists of a simpler cascade two-port filter sections to provide cutoff. The procedure is relatively simple but it requires many iterations to get the desired result.

IV. The Insertion Loss Method

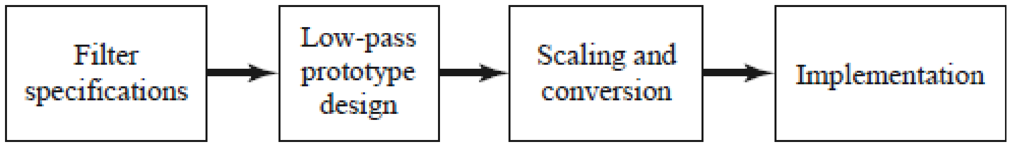

FIGURE 8.23 The process of filter design by the insertion loss method.

Network synthesis techniques. It allows completely specified frequency response. It utilizes the prototype of low-pass filter section with normalized impedance and frequency. Transformation taken in place to convert into desired impedance and frequency range.

Insertion loss method provides a high degree of control over the passband and stopband magnitude and phase characteristics. There are trade-offs but it will help meeting the specified requirements by evaluating them, which cannot be done in the image parameter method.

	Advantage	Disadvantage
Binomial Expansion	Minimum Insertion loss	
Chebyshev	The sharpest cutoff	Ripple in passband
Linear Phase Filter Design	Better Phase Response	Attenuation Rate

Insertion loss improves the filter performance ***at the expense of a higher order filter***. Chebyshev polynomial is a great tool to approximate the response in this sense because it overcomes the Runge’s phenomenon.

Elliptic function filters often provide better cutoff rate by specifying minimum stopband attenuation as well as the maximum passband attenuation (rather than infinite). See Figure. 8.22.

Linear phase filters are important, especially in multiplexing circuit. It is very important to have linear phase in the passband to avoid signal distortion. Note that sharp-cutoff responses are usually incompatible with linear phase. Hence, sharp-cutoff response circuit should be combined with phase response of a filter to overcome the issue; however, leading to a poor attenuation characteristic.

a. Characterization by Power Loss Ratio

$$\text{Insertion loss} = \text{Power loss ratio} = P_{LR}$$

Which is defined by:

$$P_{LR} = \frac{\text{Power available from source}}{\text{Power delivered to load}} = \frac{P_{inc}}{P_{load}} = \frac{1}{1 - |\Gamma(\omega)|^2}$$

P_{LR} is reciprocal of $\lvert S_{12}\rvert^2$ if ***both*** load and source is matched. Note that this power loss ratio is dimensionless quantity.

$$IL = 10 \log_{10}(P_{LR}) \; [dB]$$

$\lvert\Gamma(\omega)\rvert^2$ is an even function. Check the section 4.1 in the textbook. Hence, it can be expressed as polynomials in ω^2. <<I should come back for this point later. Just accept for now.>>

$$|\Gamma(\omega)|^2 = \frac{M(\omega^2)}{M(\omega^2) + N(\omega^2)}$$

Which can be expressed as power loss ratio:

$$P_{LR} = 1 + \frac{M(\omega^2)}{N(\omega^2)}$$

Power loss ratio must take the above form to design a realizable circuit. In addition, it constraints the reflection coefficient, $\lvert\Gamma(\omega)\rvert$.

Now, let's look at what are practical filter responses.

Maximally Flat Response (Butterworth)
Equal Ripple Response

b.Maximally Flat Response

Maximally flat response is also known as binomial or Butterworth response. This type of response provides the flattest passband response (i.e. almost no ripple in the passband). The pioneer of this filter, the British engineer Stephen Butterworth, said something and I quote here [2]:

"An ideal electrical filter should not only completely reject the unwanted frequencies but should also have ***uniform sensitivity for the wanted frequencies***".

For the low-pass filter,

$$P_{LR} = 1 + k^2 \left(\frac{\omega}{\omega_c}\right)^{2N}$$

where

$$N = \text{order of filter}$$

$$\omega_c = \text{cutoff frequency}$$

As we see the passband is defined in the following region:

$$0 \leq \omega \leq \omega_c$$

The corresponding Power loss ratio would be:

$$1 \leq P_{LR} \leq 1 + k^2$$

As always, the cutoff frequency should be defined such that the ratio quantity would have:

$$P_{LR}|_{\omega_c} = 1 + k^2 = -3dB$$

$$k = 1$$

I was stuck at this part. The power loss ratio is defined as something like $\frac{P_{in}}{P_{out}}$. But I think the text wanted to mean something like:

$$\frac{1}{1+k^2} = -3dB$$

Which indeed lead to

$$k = 1$$

Note that dB conversion for power ratio:

$$dB \to 10\log\left(\frac{P_{out}}{P_{in}}\right)$$

Whereas for the voltage gain:

$$dB \to 20\log\left(\frac{V_{out}}{V_{in}}\right)$$

Now, let's look at the stopband characteristic. The maximally flat response would decrease at the rate of $20N\ {}^{dB}/_{Dec}$.

c. Equal Ripple Response

A Chebyshev polynomial to specify the insertion loss:

$$P_{LR} = 1 + k^2 T_N^2\left(\frac{\omega}{\omega_c}\right)$$

Although there's some ripple in the passband, this type of filter will provide the sharpest transition between the passband and the stopband. The ripple is coming from the Chebyshev polynomial coefficients as shown in the equation in the power loss ratio.

A quick note on the characteristics of Chebyshev polynomial [3][4]?

The idea of Chebyshev polynomial is very important in a sense that it minimizes the Runge's phenomenon.
The Chebyshev polynomial coefficients (T_N) are bound in $[-1,1]$.

Chebyshev function (as well as the Butterworth response) is also an elliptic function (????), which behaves as

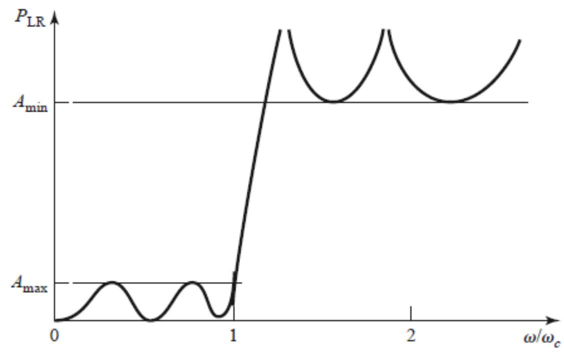

FIGURE 8.22 Elliptic function low-pass filter response.

Runge's phenomenon [4]?
It states that going to higher order degree to approximate a function using polynomials does not always lead to an accurate result.

Gibb's phenomenon? <<From my head, it may not be accurate.>>
It occurs when a finite number of the Fourier series terms to approximate a (periodic) function. The error is recognized as "overshoot". However, the overshoot will be overcome by considering the infinite sum (or very higher order).

d.Linear Phase Response

Linear phase can be achieved by:

$$\phi(\omega) = A\omega\left[1 + p\left(\frac{\omega}{\omega_c}\right)^{2N}\right]$$

where

$$\phi(\omega) = \text{phase of voltage transfer function}$$

$$p = \text{some constant}$$

Phase response is related to the group delay function:

$$\tau_d = \frac{d\phi(\omega)}{d\omega} = A\left[1 + p2N\left(\frac{\omega}{\omega_c}\right)^{2N-1}\right]$$

$$= A\left[1 + p(2N+1)\left(\frac{\omega}{\omega_c}\right)^{2N}\right]$$

Which is in the form of ***Maximally flat response***. In conclusion, linear phase response has maximally flat time delay response.

e. Maximally Flat Low Pass Filter Prototype

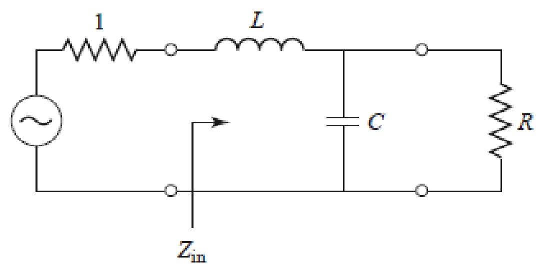

FIGURE 8.24 Low-pass filter prototype, $N = 2$.

where

$$Z_s = 1\ ohm$$

$$\omega_c = 1\ {}^{rad}\!/_{sec}$$

The maximally flat response for $N = 2$ case,

$$P_{LR} = 1 + (\omega)^4$$

I think the power loss ratio should have been

$$P_{LR} = 1 + k^2(\omega)^4$$

I don't think $k^2 = 1$ is true.

The input impedance of Figure. 8.24:

$$Z_{in} = j\omega L + {R\left(\frac{1}{j\omega C}\right)}\Big/{R + \frac{1}{j\omega C}}$$

$$Z_{in} = j\omega L + \frac{R(1 - j\omega RC)}{1 + \omega^2 R^2 C^2}$$

The reflection coefficient is defined as:

$$\Gamma = \frac{Z_{in} - Z_s}{Z_{in} + Z_s} = \frac{Z_{in} - 1}{Z_{in} + 1}$$

Using the fact that the power loss ratio is related to the magnitude of reflection coefficient:

$$P_{LR} = \frac{1}{1 - |\Gamma|^2} = \frac{|Z_{in} + 1|^2}{2(Z_{in} + Z_{in}^*)}$$

After some extensive amount of math, we will observe that:

$$L = C = \sqrt{2}$$

This procedure can be used for larger N; however, clearly not efficient or difficult if N becomes too large.

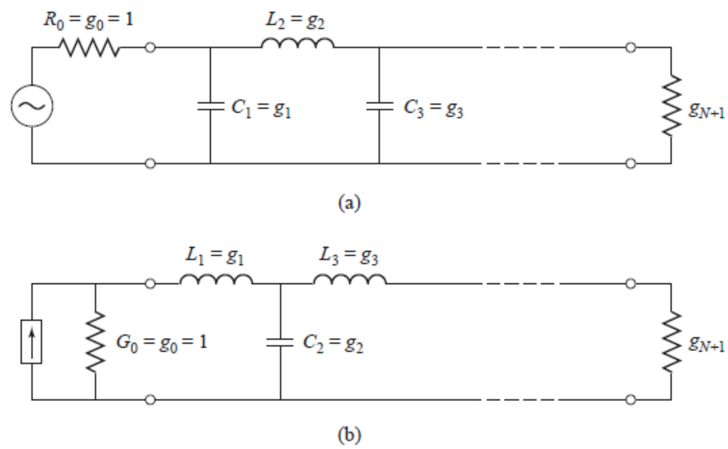

FIGURE 8.25 Ladder circuits for low-pass filter prototypes and their element definitions. (a) Prototype beginning with a shunt element. (b) Prototype beginning with a series element.

Luckily, we have a Table for the element values for the ladder circuits for low-pass filter prototype.

TABLE 8.3 Element Values for Maximally Flat Low-Pass Filter Prototypes ($g_0 = 1$, $\omega_c = 1$, $N = 1$ to 10)

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.0000

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Where the elements can be identified as:

$$g_0 = \begin{cases} \text{generator resistance (network of Figure 8.25a)} \\ \text{generator conductance (network of Figure 8.25b)} \end{cases}$$

$$\begin{matrix} g_k \\ (k = 1 \text{ to } N) \end{matrix} = \begin{cases} \text{inductance for series inductors} \\ \text{capacitance for shunt capacitors} \end{cases}$$

$$g_{N+1} = \begin{cases} \text{load resistance if } g_N \text{ is a shunt capacitor} \\ \text{load conductance if } g_N \text{ is a series inductor} \end{cases}$$

Figure. [1]

How can we select the order, or size N?

The size of filter can be determined by the specification on the insertion loss (at some frequency) in the stopband. If $N > 10$ is required, then consider ***cascading two designs of lower orders***. To examine this point, the following figure will be helpful:

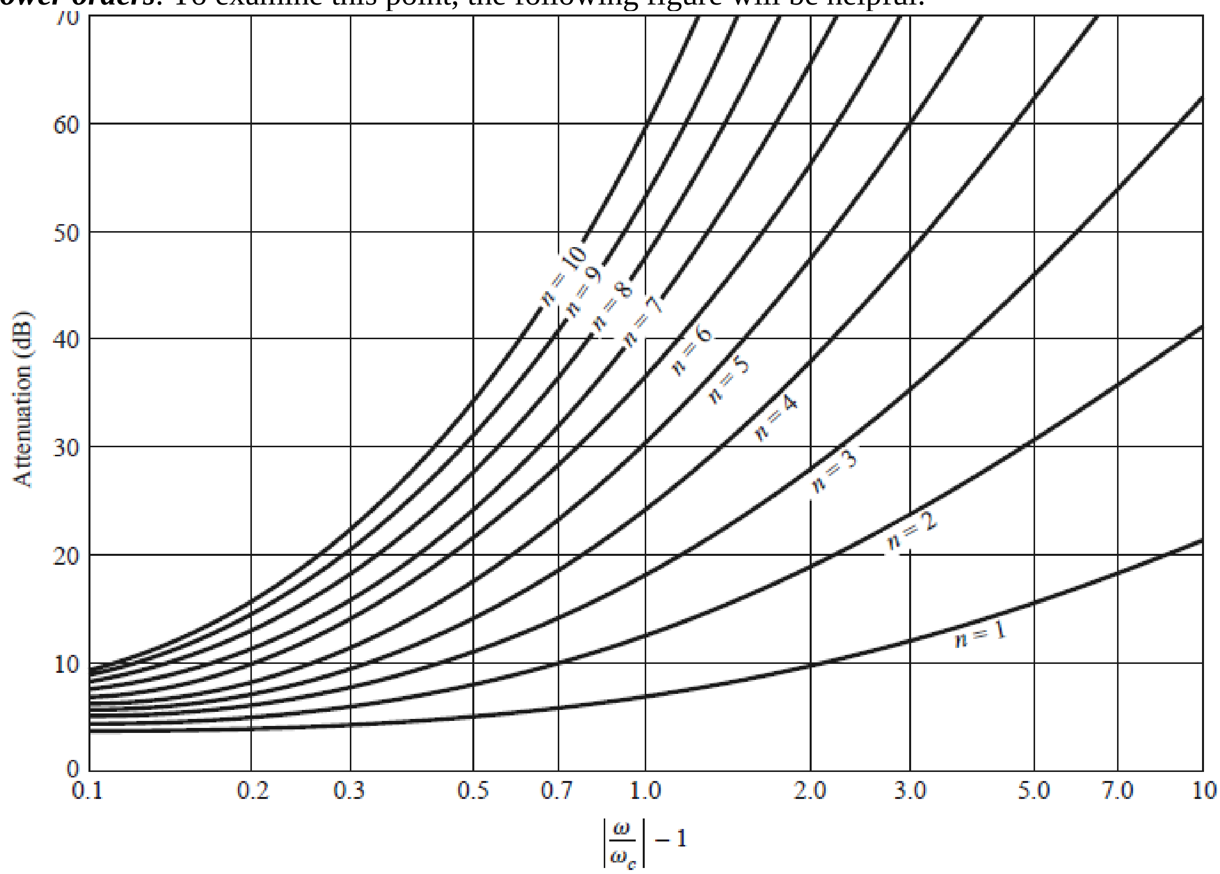


FIGURE 8.26 Attenuation versus normalized frequency for maximally flat filter prototypes.

Adapted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*, Artech House, Dedham, Mass., 1980, with permission.

f. Equal Ripple Low Pass Filter Prototype

By examining $N = 2$, we realized that there's an extensive amount of math waiting for us. Hence, let us move onto the Table and Figure.

For $0.5dB$ Ripple, the attenuation versus normalized frequency plot can be shown as:

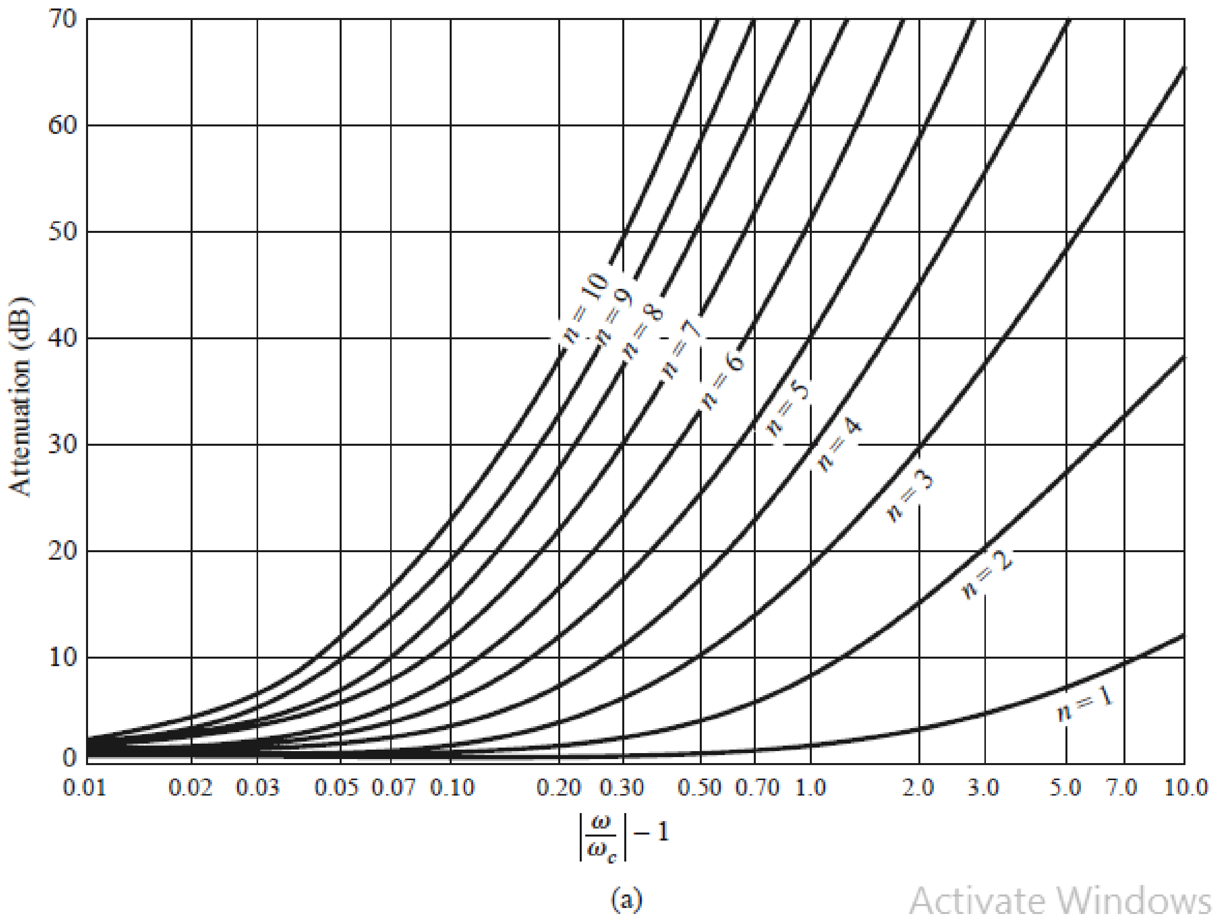
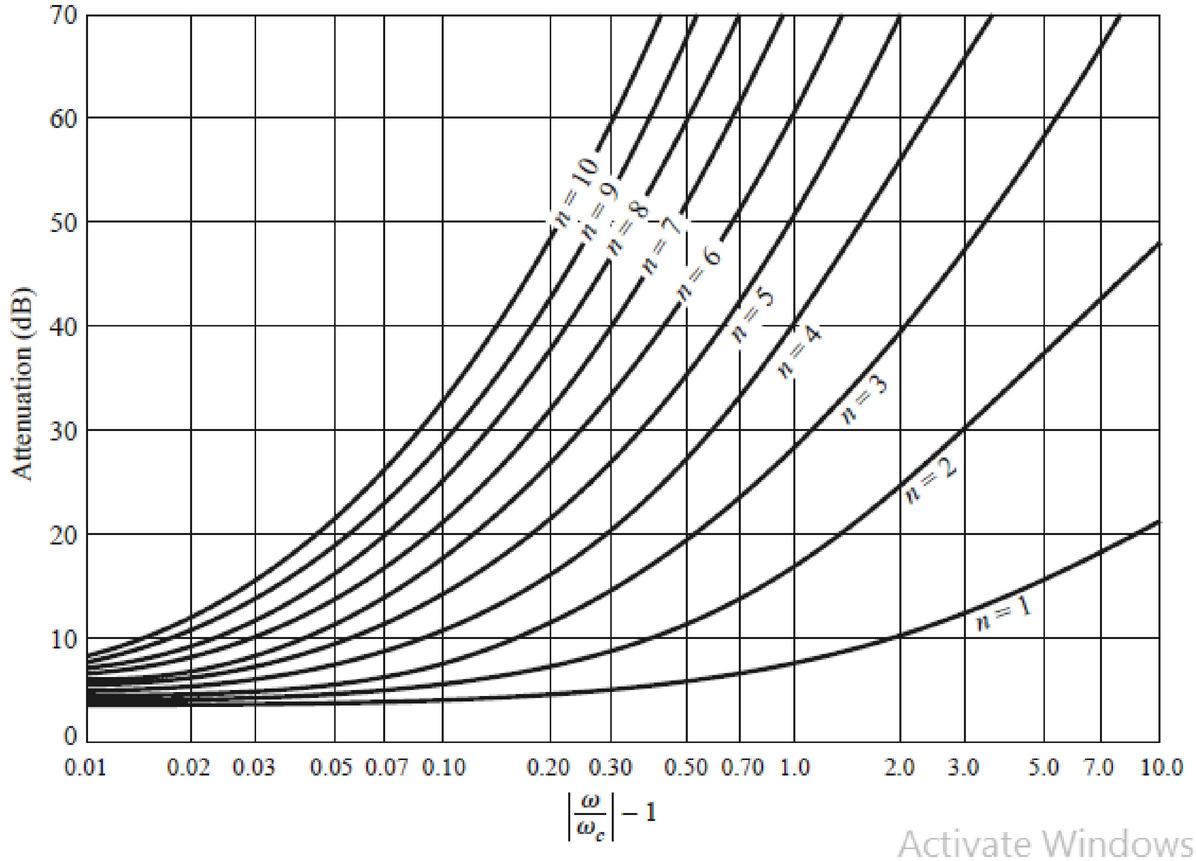


Figure. Attenuation for $0.5dB$

TABLE 8.4 Element Values for Equal-Ripple Low-Pass Filter Prototypes ($g_0 = 1, \omega_c = 1$, $N = 1$ to 10, 0.5 dB and 3.0 dB ripple)

					0.5 dB Ripple						
N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	0.6986	1.0000									
2	1.4029	0.7071	1.9841								
3	1.5963	1.0967	1.5963	1.0000							
4	1.6703	1.1926	2.3661	0.8419	1.9841						
5	1.7058	1.2296	2.5408	1.2296	1.7058	1.0000					
6	1.7254	1.2479	2.6064	1.3137	2.4758	0.8696	1.9841				
7	1.7372	1.2583	2.6381	1.3444	2.6381	1.2583	1.7372	1.0000			
8	1.7451	1.2647	2.6564	1.3590	2.6964	1.3389	2.5093	0.8796	1.9841		
9	1.7504	1.2690	2.6678	1.3673	2.7239	1.3673	2.6678	1.2690	1.7504	1.0000	
10	1.7543	1.2721	2.6754	1.3725	2.7392	1.3806	2.7231	1.3485	2.5239	0.8842	1.9841

For $3dB$ Ripple, the attenuation versus normalized frequency plot can be shown as:



(b)

Figure. Attenuation for $3dB$

3.0 dB Ripple

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	1.9953	1.0000									
2	3.1013	0.5339	5.8095								
3	3.3487	0.7117	3.3487	1.0000							
4	3.4389	0.7483	4.3471	0.5920	5.8095						
5	3.4817	0.7618	4.5381	0.7618	3.4817	1.0000					
6	3.5045	0.7685	4.6061	0.7929	4.4641	0.6033	5.8095				
7	3.5182	0.7723	4.6386	0.8039	4.6386	0.7723	3.5182	1.0000			
8	3.5277	0.7745	4.6575	0.8089	4.6990	0.8018	4.4990	0.6073	5.8095		
9	3.5340	0.7760	4.6692	0.8118	4.7272	0.8118	4.6692	0.7760	3.5340	1.0000	
10	3.5384	0.7771	4.6768	0.8136	4.7425	0.8164	4.7260	0.8051	4.5142	0.6091	5.8095

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Note that the Tables are already calculated for given attenuations. Important note can be made from the Tables:

"***The last element, a resistor, is the unity if the order is <u>an odd number</u>***."

This fact can lead to a simpler design consideration overall. If the order were an even number then the last element, a resistor, wouldn't be the unity; therefore, extra matching network would be needed to overcome the mismatch.

The ladder circuit introduced in maximally flat low pass filter prototype can also be used for the Equal Ripple Low Pass Filter Prototype.

g. Linear Phase Low-Pass Filter Prototype

Recall that linear phase responses exhibit maximally flat time delay response. Design will be done in the same way as before for other type of responses; however, linear phase response would be more complicated because the voltage transfer function should take account not only the magnitude but also the phase response.

TABLE 8.5 Element Values for Maximally Flat Time Delay Low-Pass Filter Prototypes ($g_0 = 1, \omega_c = 1, N = 1$ to 10)

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.5774	0.4226	1.0000								
3	1.2550	0.5528	0.1922	1.0000							
4	1.0598	0.5116	0.3181	0.1104	1.0000						
5	0.9303	0.4577	0.3312	0.2090	0.0718	1.0000					
6	0.8377	0.4116	0.3158	0.2364	0.1480	0.0505	1.0000				
7	0.7677	0.3744	0.2944	0.2378	0.1778	0.1104	0.0375	1.0000			
8	0.7125	0.3446	0.2735	0.2297	0.1867	0.1387	0.0855	0.0289	1.0000		
9	0.6678	0.3203	0.2547	0.2184	0.1859	0.1506	0.1111	0.0682	0.0230	1.0000	
10	0.6305	0.3002	0.2384	0.2066	0.1808	0.1539	0.1240	0.0911	0.0557	0.0187	1.0000

Source: Reprinted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*, Artech House, Dedham, Mass., 1980, with permission.

The ladder circuit can also be used for linear phase response for $\omega_c' = 1\, {}^{rad}/_{sec}$, which is in Table 8.5. The resulting normalized group delay in the passband will be

$$\tau_d = {}^{1}/_{\omega_c'}$$

h.Filter Transformations

After selecting the element values for the ladder circuit appropriately, we can find the actual values for inductance and capacitance that made of the ladder circuit. There are two types of scaling: impedance scaling and frequency scaling. Frequency scaling will lead to the type of filter: low-pass, high-pass, bandpass, or band-reject. The prime quantities represent scaled quantities.

Impedance Scaling:

$$L' = R_0 L$$

$$C' = \frac{C}{R_0}$$

$$R_s' = R_0$$

$$R_L' = R_0 R_L$$

Keep in mind that $R_s = 1\ ohm$ and $\omega_c = 1\ {}^{rad}/_{sec}$.
$L, C, and\ R_L$ are the values from prototype Table.

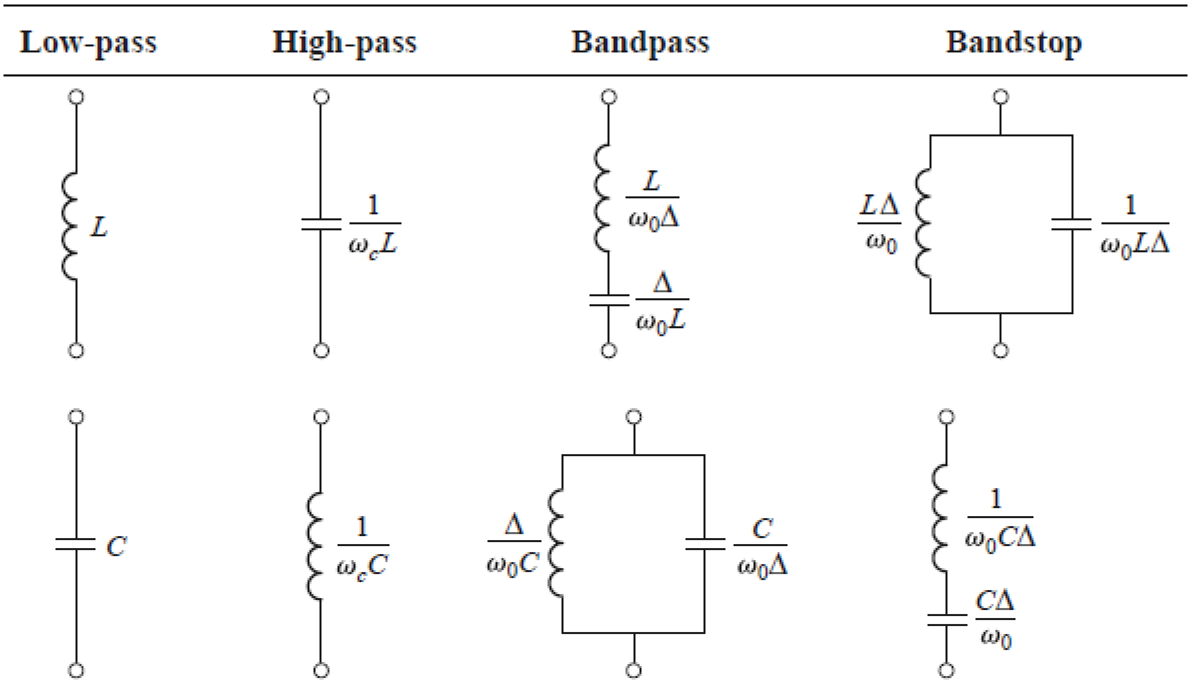
Frequency Scaling: <<The derivation process is somewhat interesting. Please make sure understand them.>>

	L_k'	C_k'
Low Pass	$\frac{R_0 L_k}{\omega_c}$	$\frac{C_k}{R_0 \omega_c}$
High Pass	$\frac{R_0}{\omega_c C_k}$	$\frac{1}{R_0 \omega_c L_k}$
Band-pass	$\frac{\Delta}{\omega_0 C_k}$	$\frac{C_k}{\omega_0 \Delta}$
Band-reject	$\frac{L_k \Delta}{\omega_0}$	$\frac{1}{\omega_0 L_k \Delta}$

$$\Delta = \text{fractional bandwidth of passband} = \frac{\omega_2 - \omega_1}{\omega_0}$$

$$\omega_0 = \text{center frequency} = \sqrt{\omega_1 \omega_2}$$

TABLE 8.6 Summary of Prototype Filter Transformations $\left(\Delta = \frac{\omega_2 - \omega_1}{\omega_0}\right)$



i. Examples

i. Butterworth Low pass filter Design (Example 8.3)

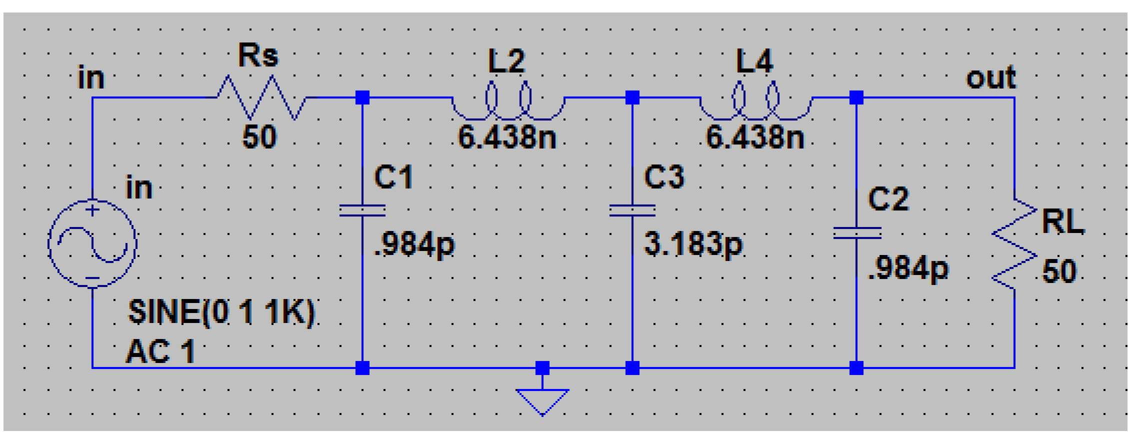


Figure. Low Pass Filter Example

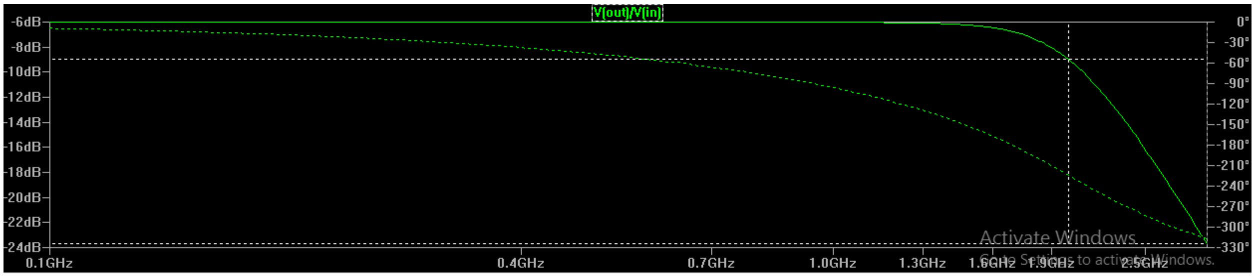


Figure. Frequency Response

Observed Results:

	Magnitude	Phase
$2GHz$	$-8.99dB$	$-224.52°$
$3GHz$	$-23.70dB$	$-317.03°$

Note that the passband magnitude of this filter is already:

$$\left.\frac{V_{out}}{V_{in}}\right|_{passband} = -6.0dB$$

Phase difference between $1GHz \leq \measuredangle H \leq 2GHz$:

$$-130.87°$$

Phase difference between $2GHz \leq \measuredangle H \leq 3GHz$:

$$-90°$$

One might want to recover the shifted phase by considering phase shifting unit if required.

In addition, we observe that

$$23.7\ dB - 9\ dB = 14.7\ dB$$

Which should have been around $16{\sim}17\ dB$ for $N = 5$ as shown in the attenuation versus normalized frequency plot.

ii. Butterworth Low Pass Filter Design

Let's design Butterworth Filter with order $N = 4$. $f_c = 700MHz$. (In practice, the way the order of circuit can be determined by specifying the attenuation rate between the cutoff frequency and one particular frequency that yield a certain attenuation (relative to the cutoff one) and see what order can do so. For our example, let's say

$$\frac{f}{f_c} = 1.5$$

And we want about 10 dB more attenuation from the cutoff at that frequency. Any order that is greater than $N = 3$ could do so. Hence, we select $N = 4$.)

From the Table,

$$g_1 = 0.7654$$

$$g_2 = 1.8478$$

$$g_3 = 1.8478$$

$$g_4 = 0.7654$$

$$g_5 = 1.0000$$

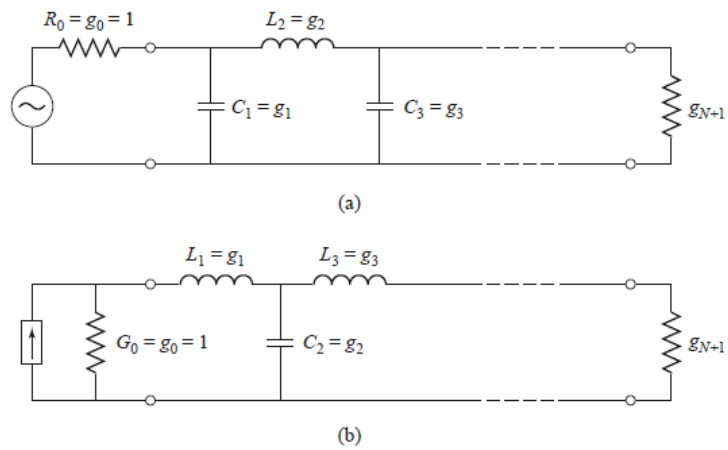


FIGURE 8.25 Ladder circuits for low-pass filter prototypes and their element definitions. (a) Prototype beginning with a shunt element. (b) Prototype beginning with a series element.

Let's first look at the values of source and load resistors since they are independent of frequency.

Considering mismatch by cascading other circuit, 50 ohm should be ideal for source and load resistors.

$$R_s' = R_0 = 50$$

$$R_L' = R_0 R_L = 50 = g_5'$$

Using the following relationship:

$$L_k' = \frac{R_0 L_k}{\omega_c}$$

$$C_k' = \frac{C_k}{R_0 \omega_c}$$

$$C_1' = \frac{0.7654}{50(2\pi)700MHz} = 3.48pF$$

$$L_2' = \frac{50(1.8478)}{(2\pi)700MHz} = 21nH$$

$$C_3' = \frac{1.8478}{50(2\pi)700MHz} = 8.4pF$$

$$L_4' = \frac{50(0.7654)}{(2\pi)700MHz} = 8.7nH$$

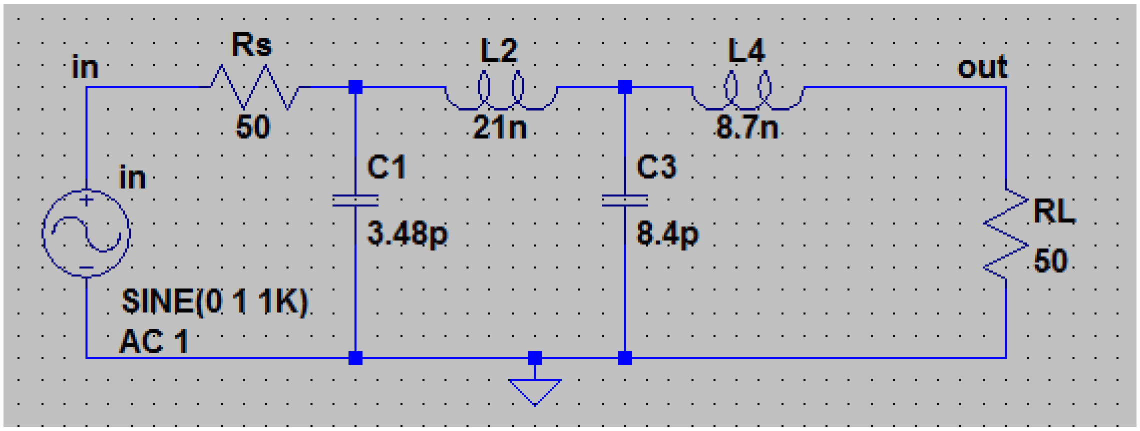


Figure. Butterworth LPF Designed Circuit

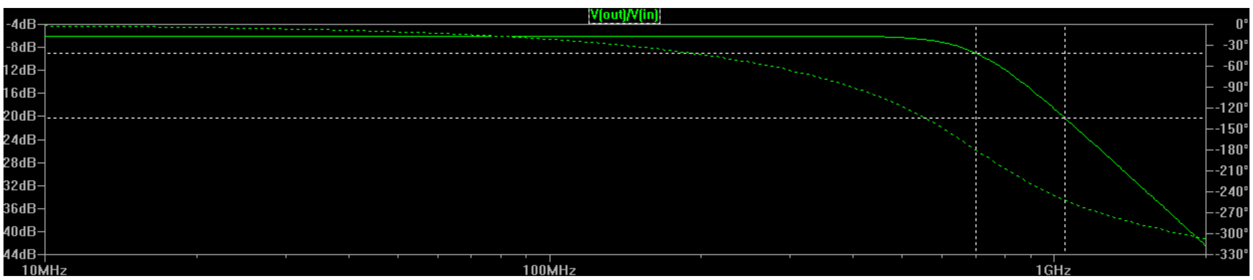


Figure. Frequency Response

	Magnitude	Phase
$700.14\ MHz$	$-9.031\ dB$	$-179.98°$
$1051\ MHz$	$-20.29\ dB$	$-251.79°$

In conclusion:

Theory and actual simulation results show a good accordance. However, I don't think I can control the phase of output response, which showed almost random phase difference between input and output signal as shown in the transfer function phase result.

For $N = 4$, the attenuation versus normalized frequency plot suggests:

$$\approx 14dB$$

From the simulation result,

$$20.29\ dB - 9.031\ dB = 11.26\ dB$$

Hence, this point also follows the theoretical value. Calculated error percentage (for attenuation):

$$\approx 8.3\ \%$$

iii. Butterworth Bandpass Filter (Example 8.4)

The response type is 0.5 dB Equal-Ripple. 10% Bandwidth. 1GHz center frequency. 50 ohm impedance for source and load. Use $N = 3$.

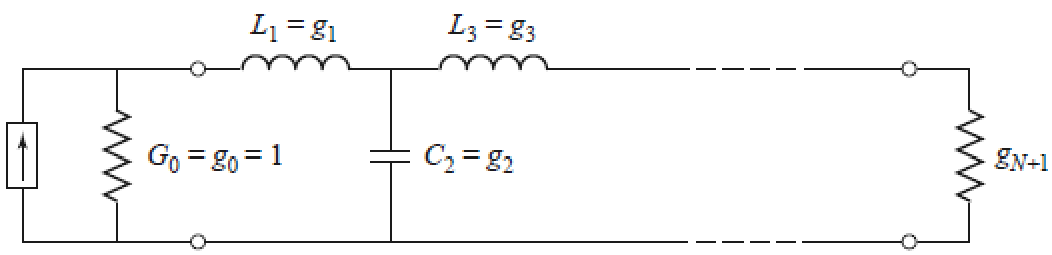
From the Table 8.4:

$$g_1 = 1.5963 \rightarrow L_1$$

$$g_2 = 1.0967 \rightarrow C_2$$

$$g_3 = 1.5963 \rightarrow L_3$$

$$g_4 = 1.0000 \rightarrow R_L$$



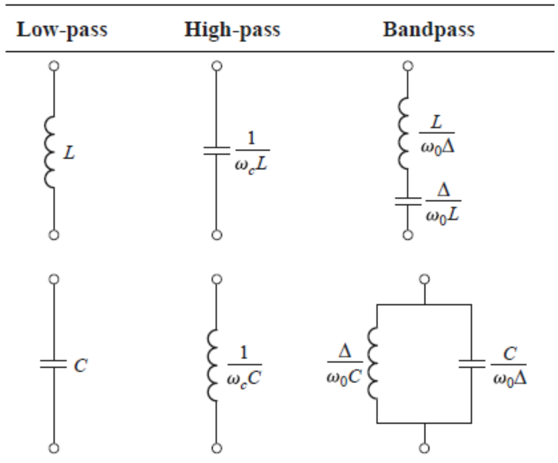
(b)

Figure. 8.25 (b)

Now, let's transform into scaled values:

$$L_1' = \frac{L_1 R_0}{\omega_0 \Delta} = 127.0 \text{ nH},$$
$$C_1' = \frac{\Delta}{\omega_0 L_1 R_0} = 0.199 \text{ pF},$$
$$L_2' = \frac{\Delta R_0}{\omega_0 C_2} = 0.726 \text{ nH},$$
$$C_2' = \frac{C_2}{\omega_0 \Delta R_0} = 34.91 \text{ pF},$$
$$L_3' = \frac{L_3 R_0}{\omega_0 \Delta} = 127.0 \text{ nH},$$
$$C_3' = \frac{\Delta}{\omega_0 L_3 R_0} = 0.199 \text{ pF}.$$

For the bandpass filter, the circuit can be transformed with the following relation:



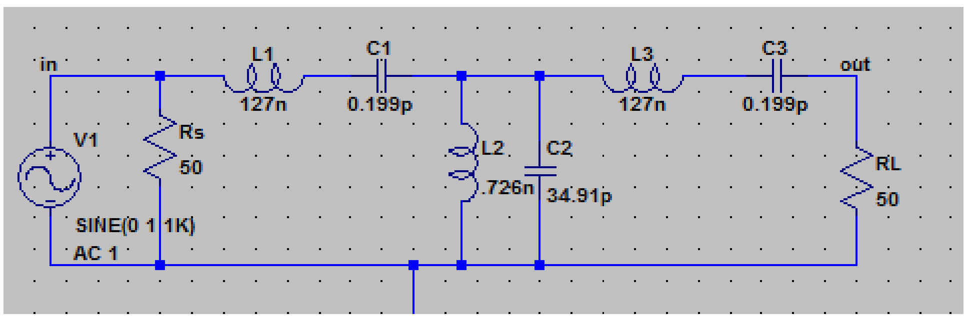


Figure. Butterworth BPF $1GHz$ 10% Bandwidth

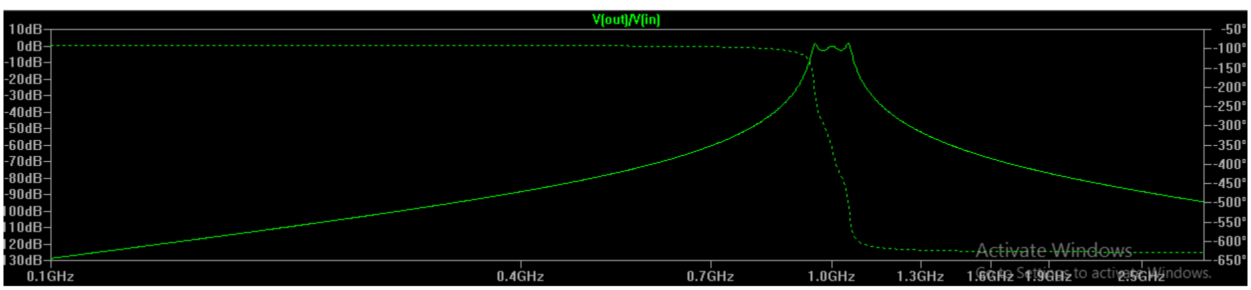


Figure. Frequency Response

$$\Delta = \frac{\omega_2 - \omega_1}{\omega_0} = \frac{\omega_2 - \omega_1}{2\pi(1GHz)} = 10\%$$

$$\omega_0 = \sqrt{\omega_1 \omega_2} = 2\pi(1GHz)$$

$$\omega_1 = \frac{\left(2\pi(1GHz)\right)^2}{\omega_2}$$

After some math, we note that

$$f_1 = 0.951\ GHz$$

$$f_2 = 1.05\ GHz$$

	Magnitude	Phase
$1.00\ GHz$	$0.024\ dB$	$-356.23°$
$0.9502\ GHz$	$0.884\ dB$	$-195.76°$
$1.050\ GHz$	$1.361\ dB$	$-501.92°$

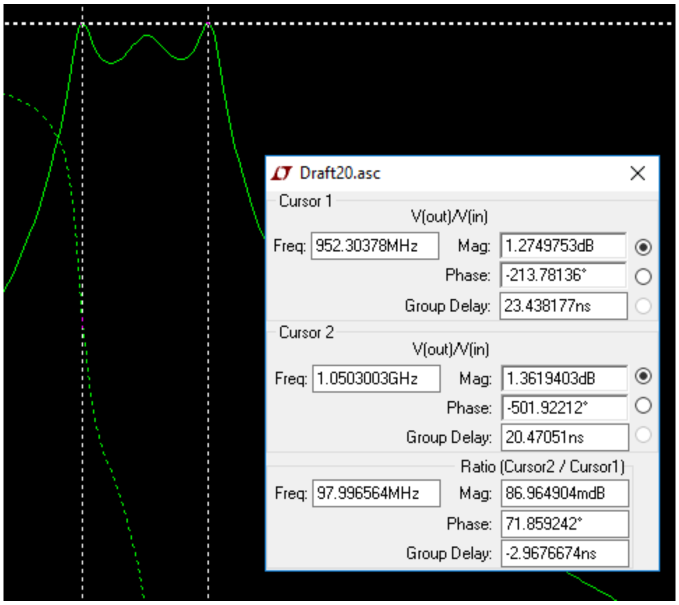


Figure. Fractional Bandwidth

It seems results are in the way we expected.

V. Filter Implementation

Lumped element filters are easy to design and we observe that everything works out great. However, there's a practical problem why lumped element circuits are not desirable at microwave or higher frequency ranges. The problem is that, as in the previous examples, all the inductances and capacitances found to meet the specification may not be available in the market. In other word, we might not be able to find the set of appropriate capacitors and inductors.

The solution to this problem can be solved by considering distributed elements such as short-circuited or open-circuited transmission-line stubs.

Richards' transformation can be used to convert lumped elements to transmission-line sections.

Kuroda's identities can then be used to physically separate filter elements by transmission line sections. Such additional transmission line sections do not affect the filter response; therefore, this type of design is called redundant filter synthesis.

a. Richards' Transformation

To transform lumped element model into transmission line model, we need to convert frequency variable to transmission line length variable. Let's first identify the transformation between two planes: ω-plane to Ω-plane.

$$\Omega = tan(\beta l) = \tan\left(\frac{\omega l}{v_p}\right)$$

The characteristic of Ω-plane is that images in codomain repeat with a period of ${}^{\omega l}/_{v_p} = 2\pi$. <<I want to say the transformation is a surjective function.>> Then, we can re-define the impedance for capacitor (susceptance) and inductor (reactance) in the following way:

$$j\omega L = jX_L = j\Omega L = jLtan(\beta l)$$

$$jB_C = j\Omega C = jCtan(\beta l)$$

By investigating above results, we conclude that

	Characteristic impedance	Type of stub with length βl
Inductor	L	Short-circuit
Capacitor	${}^{1}/_{C}$	Open-circuit

<<I'm little confused with the capacitor case. Let me try this.>>

In the low pass prototype unit, the cutoff frequency occurs at the unity; therefore, to obtain the consistent result, we equate:

$$\Omega = 1 = tan(\beta l)$$

Thus, we note that

$$l = {}^{\lambda}/_{8}$$

Where λ is the wavelength for the cutoff-frequency.

b.Kuroda's Identities

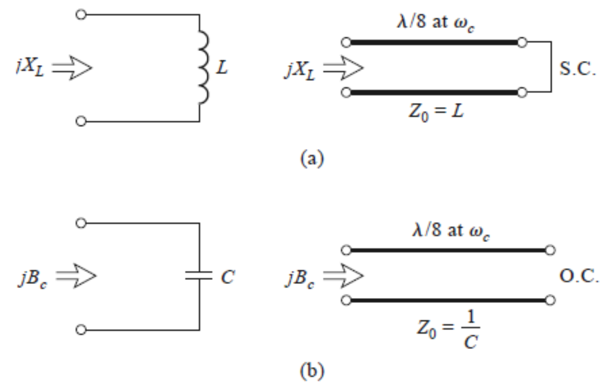


FIGURE 8.34 Richards' transformation. (a) For an inductor to a short-circuited stub. (b) For a capacitor to an open-circuited stub.

Unit element?

It is the additional transmission line section that does not affect the frequency response. The length is $^{\lambda}/_{8}$ at ω_c, which is why it is commensurate with the stubs (short circuit or open circuit) to replace the inductors and capacitors, respectively. ***Each box in the Figure below represents the unit element***.

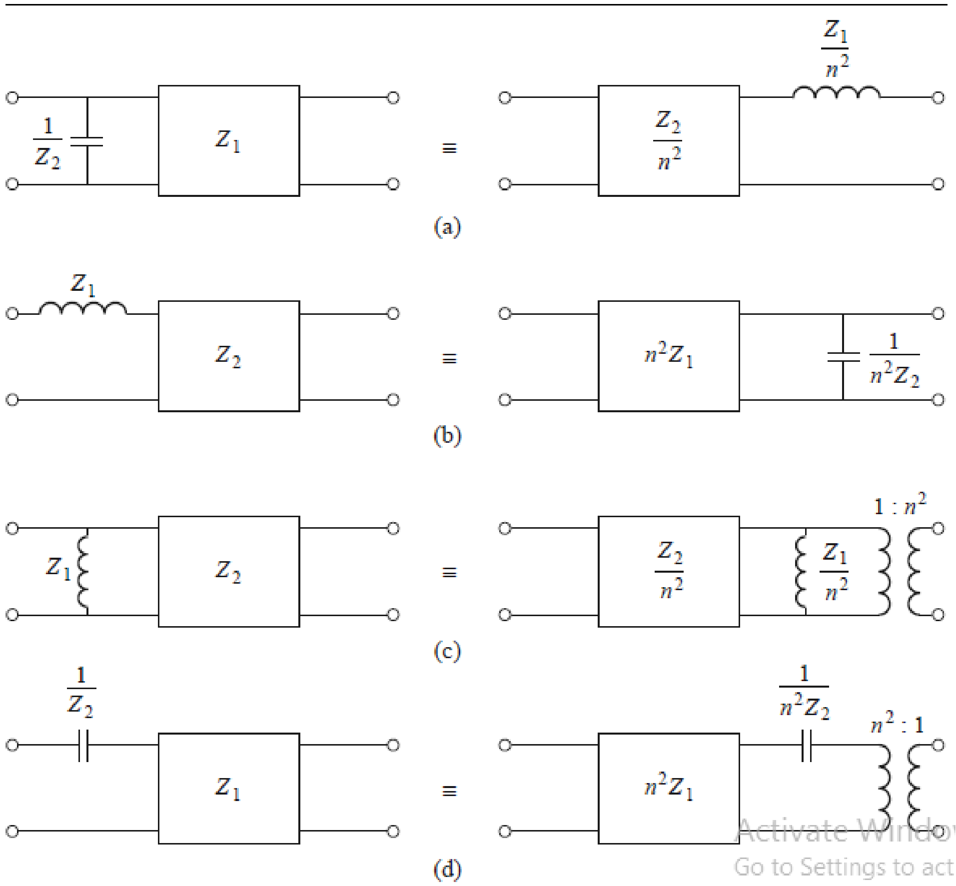


Figure. [1] Table 8.7 $n^2 = 1 + {}^{Z_2}/_{Z_1}$

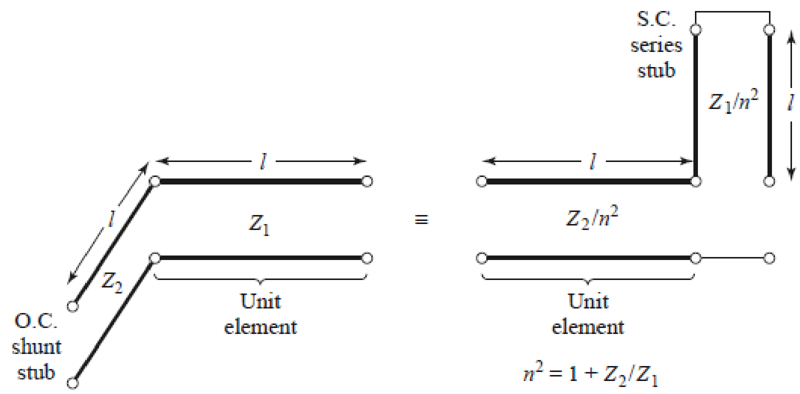


FIGURE 8.35 Equivalent circuits illustrating Kuroda identity (a) in Table 8.7.

So, how can we show that the open circuit stub (on the left in Figure.8.35) is equivalent to the short stub (on the right in Figure.8.35)? This can be shown by investigating their $ABCD$ matrices and examining if they are equal. Showing their equality might be a good practice.

c. Example

i. Example 8.5

4GHz, 50-ohm, and 3rd order 3dB equal-ripple response LPF Design.

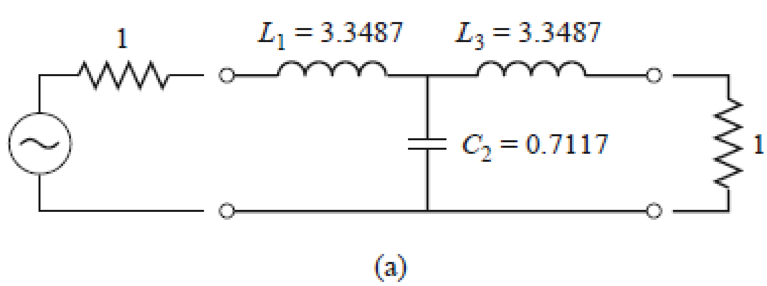


Figure. Normalized low-pass prototype element

Now, we apply Richard's transformation to convert the passive elements into series and shunt stubs.

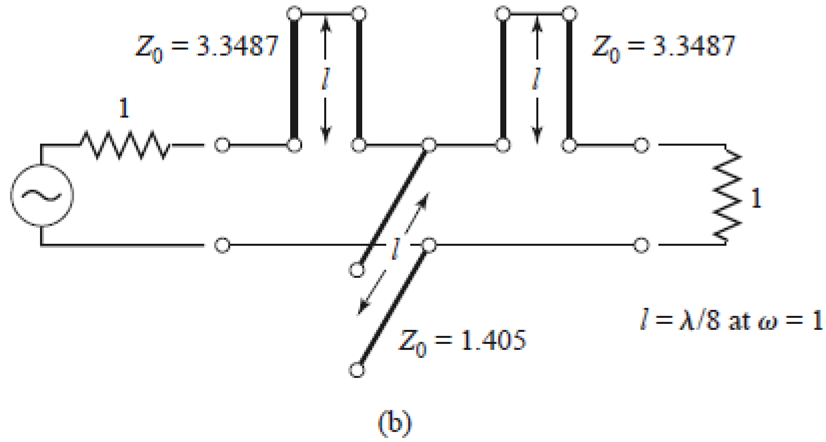


Figure. After Richard's transformation

Now, we add unit elements to utilize the Kuroda's identities.

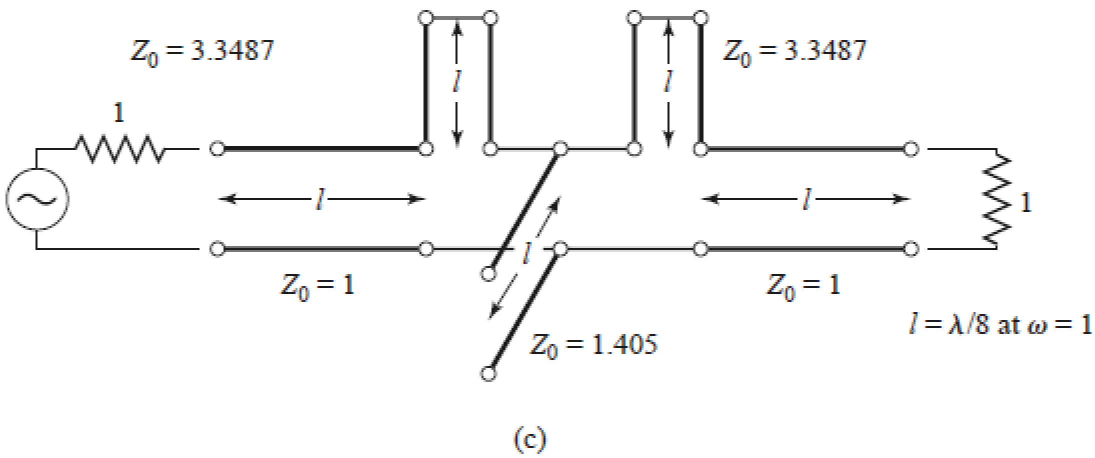


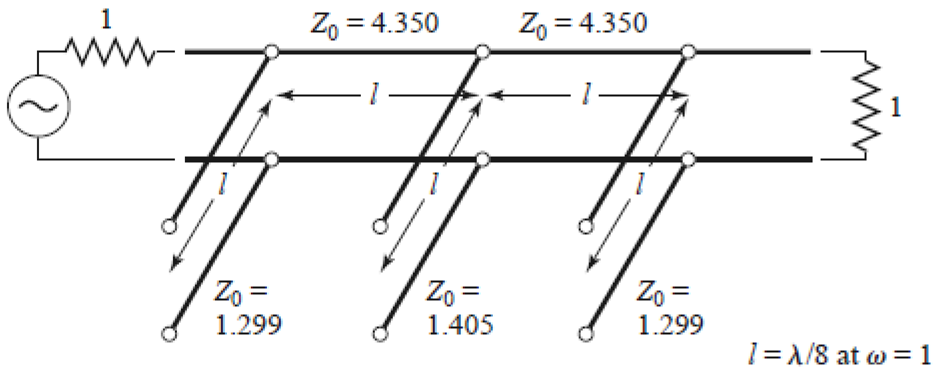
Figure.

To be honest, the following three steps require no brain. However, the rest of procedure might get complicated or time-consumed, depending on the number of the order of circuit. At this point, we simply apply the second Kuroda's identity where important parameters are nailed as:

$$n^2 = 1.2986$$

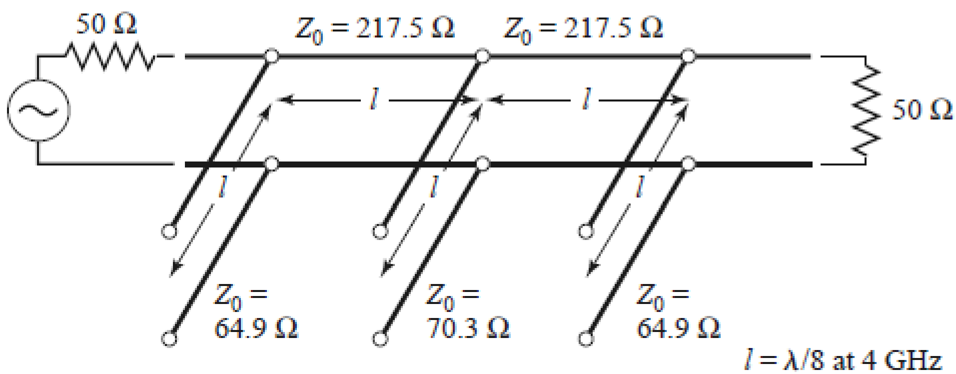
$$Z_1 = n^2 L_1 = n^2 L_3 = 4.3487$$

$$Z_2 = n^2 = 1.2986$$



(d)

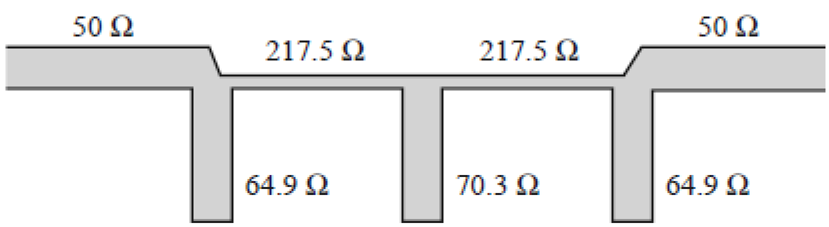
Now, simply scale up using $Z_0 = 50$-ohm



(e)

Figure. Scaling

Therefore, the equivalent transmission-line version (Microstrip line) circuit should look like:



(f)

Figure. Final filter on Microstrip line

Finally, let’s point out an interesting point about this technique by considering the following figure.

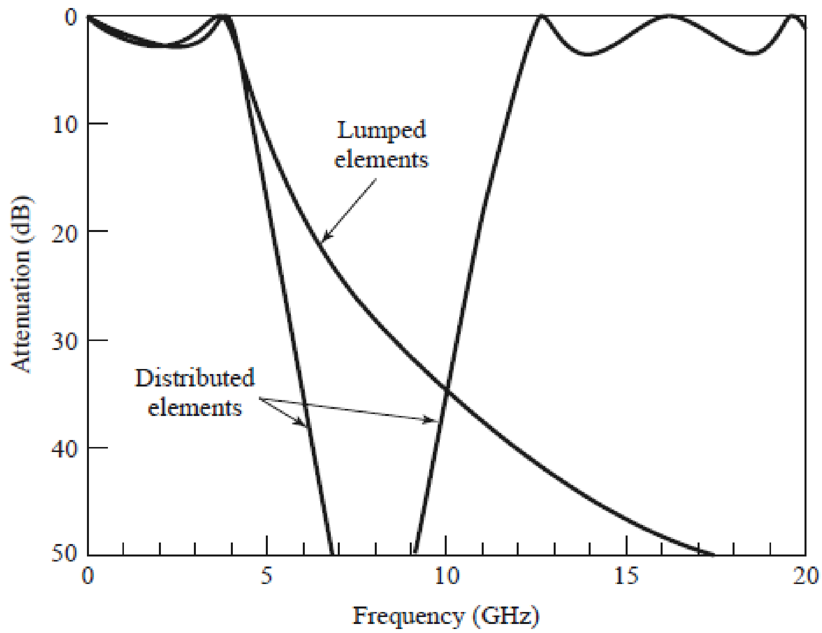


FIGURE 8.37 Amplitude responses of lumped-element and distributed-element low-pass filter of Example 8.5.

We note that the distributed-element low-pass filter exhibits repetitive behavior at every 16 GHz due to the periodic nature of Richard's transformation.

For substrate, let's use $\varepsilon_r = 4.1$, $t = 1.5748\ mm$, and $\tan\delta = 0.01$

The length of each stub can be found from

$$f = 4\ GHz$$
$$\lambda = 75\ mm$$
$$L = 9.375\ mm$$

Width of microstrip line can be easily computed using [5].

	50-ohm	217.5-ohm	64.9-ohm	70.3-ohm
Width	3.088 mm	0.032 mm	1.958 mm	1.675 mm
Length ($\lambda/8$)	5.23 mm	5.69 mm	5.34 mm	5.37 mm

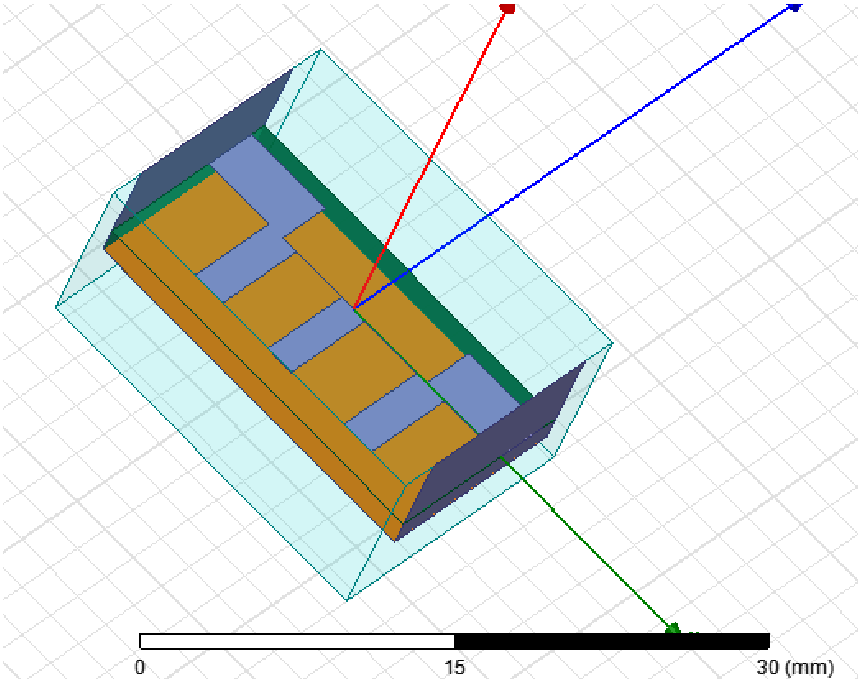


Figure. HFSS Design

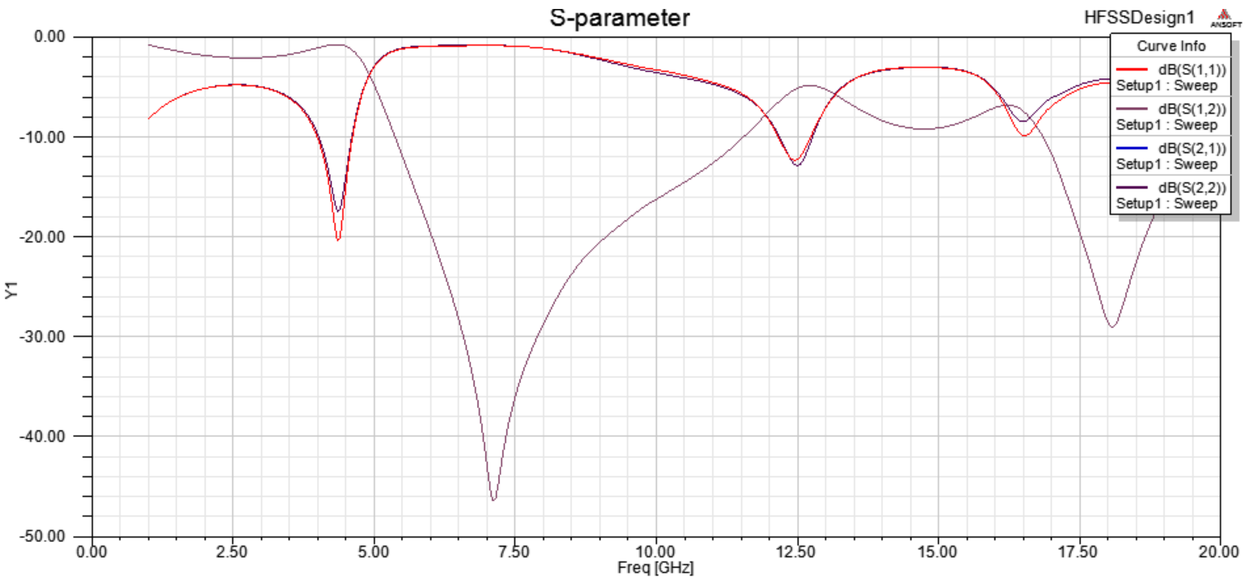


Figure. S-parameters

$$S_{21} = Attenuation = -1.1432\ dB$$

$$S_{21}|_{-3.0737dB} \rightarrow f = 4.85\ GHz$$

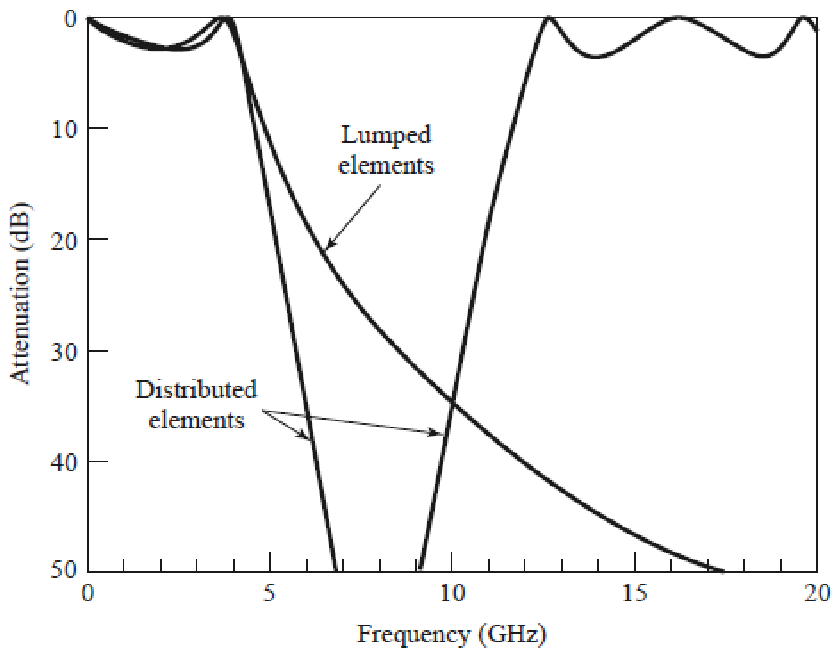


FIGURE 8.37 Amplitude responses of lumped-element and distributed-element low-pass filter of Example 8.5.

One can compare the result from HFSS and the one from the textbook author.

As you may note, the width of 217-ohm (32 μm) transmission line on microstrip line may become an obstacle to overcome in real life fabrication.

ii. Example Butterworth $N = 5$

Minimum attenuation at 3.25 GHz

Center Frequency at 2.5 GHz

Dielectric Constant of substrate 4.1

Thickness 1.5748 mm

Loss tangent of substrate 0.01

$$\left|\frac{\omega_{10dB}}{\omega_c}\right| - 1 = 0.3$$

The Butterworth frequency response suggests any order greater than $N = 4$ would yield what I want, here we use $N = 5$, therefore.

Maximally-flat (Butterworth) Response with $N = 5$.

$$g_1 = C_1 = 0.6180$$

$$g_2 = L_2 = 1.6180$$

$$g_3 = C_3 = 2.0000$$

$$g_4 = L_4 = 1.6180$$

$$g_5 = C_5 = 0.6180$$

$$g_6 = r_L = 1.0000$$

If we apply the Richard's transformation:

$z_1 = {}^{1}/_{g_1}$	$z_2 = g_2$	$z_3 = {}^{1}/_{g_3}$	$z_4 = g_4$	$z_5 = {}^{1}/_{g_5}$
O.C.	S.C	O.C.	S.C.	O.C.
1.6181	1.6180	0.5000	1.6180	1.6181

Since we are observing a symmetry along z_3, we only need to calculate the first two elements for transformation and for Kuroda's identities. Now, we need to insert the unit element such that the magnitudes don't get changed (the phase might be changed due to the shift in the reference frame) and it converts into more realizable circuit implementation.

Series stubs may be physically impossible or difficult to fabricate in practice.

The idea here is that we are going to change the S.C. (short-circuit) stubs into O.C. (open-circuit) stubs. To realize such change, we consider the Kuroda's identities. As $N = 4$, the process might be complicated due to multi-steps for conversion (the Kuroda's identities). One must understand the important fact about *matrix multiplications that they are **not** usually **commutative*** and recall that we write $ABCD$ matrices for Kuroda's identities.

Hence, it is obvious that we need to add the unit element at the source and the load ends and perform the Kuroda's identities.

	O.C.	U.E.	O.C.	U.E.	O.C.	U.E.	O.C.	U.E.	O.C.
	$z_1{}'$		$z_2{}'$		$z_3{}'$		$z_4{}'$		$z_5{}'$
Normalized	3.618	1.382	0.854	2.236	0.5	2.236	0.854	1.382	3.618
Unnormalized	180.9	69.1	42.7	111.8	25	111.8	42.7	69.1	180.9

Since we are going to implement the above low-pass filter using microstrip line, you may simply refer to [5] to calculate length and width of microstrip line.

	O.C.	U.E.	O.C.	U.E.	O.C.	U.E.	O.C.	U.E.	O.C.
	$z_1{}'$		$z_2{}'$		$z_3{}'$		$z_4{}'$		$z_5{}'$
Normalized	3.618	1.382	0.854	2.236	0.5	2.236	0.854	1.382	3.618
Unnormalized	180.9	69.1	42.7	111.8	25	111.8	42.7	69.1	180.9
Length [mm]	9.075	8.635	8.345	8.89	8.04	8.89	8.345	8.635	9.075
Width [mm]	0.086	1.756	4.006	0.544	8.398	0.544	4.006	1.756	0.086

Recall that all lengths were $\lambda/8$;however, they won't be the same due to different characteristic impedance of line. In addition, we need to calculate the 50-ohm lines (e.g. source and load) too.

	50-ohm
Normalized	1
Length [mm]	8.44
Width [mm]	3.13

Designing the above filter might take some time with some effort (I think the previous $N = 3$ filter took about $10 \sim 15$ minutes so please understand). I will add HFSS once I overcome the laziness but you're welcomed to use the above data for design.

VI. References

[1] Microwave Engineering, David Pozar, 4th Edition

[2] https://en.wikipedia.org/wiki/Butterworth_filter

[3] https://en.wikipedia.org/wiki/Chebyshev_polynomials

[4] https://en.wikipedia.org/wiki/Runge%27s_phenomenon

[5] http://janielectronics.com/szamitasok/Transmission%20Line/Microstrip%20Line%20Calculator/janilab.php