Directional Couplers

I. Introduction

What does a directional coupler do?

It’s used for “sampling a signal”. Sampling is described as “the process of a reduction of a continuous-time signal to a discrete-time signal.”

From reference [2], we know that:

Low frequencies	Lumped elements possible
Microwave	Transmission line
Higher band	Waveguide couplers are preferred

Couplers “couple” or combine two input signals at the output port. Key terminologies:

Three ports	T-junctions
	Power Dividers
Four ports	Hybrids
	Directional Couplers

Power Dividers do the opposite: they divide the input signal’s power at the output port. Hence, divider and coupler share basic properties and S-matrix that describes their behaviors. Let’s first go over three-junctions and then four-port networks.

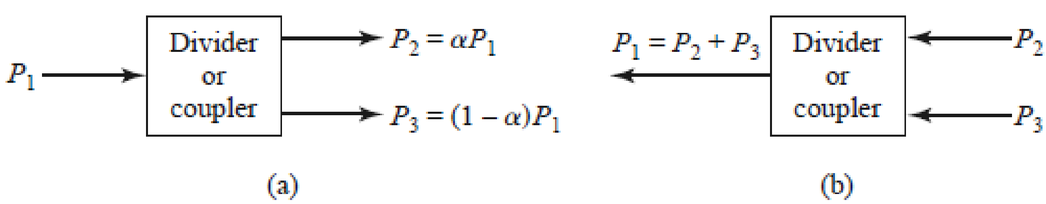


FIGURE 7.1 Power division and combining. (a) Power division. (b) Power combining.

Like usual, the main concern is the power loss at the output port. This point will be discussed shortly.

II. Basic concept of three port networks

To come up with S-matrix equation, we need to know the basic relation between the system characteristics and the elements in the S-matrix which is covered in the other document. <<Insert the name of document.>>

Three port network takes 3×3 matrix for analysis. We write the general S-parameter:

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

At this point, we will assume/apply a few matrix characteristics so that the matrix can be simplified. First, we assume the system is reciprocal which allows:

$$S_{ij} = S_{ji}$$

Hence, our S matrix will become:

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix}$$

<<Detailed derivation will be covered in another document.>>

We further assume that the system is matched at all ports ($S_{ii} = 0$). Ideally, it would have been really nice if we could design three-ports network that is lossless and matched at all ports as well as reciprocal; however, such system can't exist. So, just assume that the system is lossless and matched at all ports at this moment for which the S matrix is given:

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix}$$

Since we ***assumed the system is lossless, its scattering matrix follows the unitary matrix property***, that is:

$$[S][S^*] = I$$

$$[S^*] = \begin{bmatrix} 0 & S_{21}^* & S_{31}^* \\ S_{12}^* & 0 & S_{32}^* \\ S_{13}^* & S_{23}^* & 0 \end{bmatrix}$$

Note that "***Hermitian Conjugate***" [5] has been applied to the S matrix because of the unitary property.

$$[S][S^*] = \begin{bmatrix} S_{12}S_{12}^* + S_{13}S_{13}^* & S_{13}S_{23}^* & S_{12}S_{32}^* \\ S_{23}S_{13}^* & S_{21}S_{21}^* + S_{23}S_{23}^* & S_{21}S_{31}^* \\ S_{32}S_{12}^* & S_{31}S_{21}^* & S_{31}S_{31}^* + S_{32}S_{32}^* \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Or equivalently,

$$\sum_{k=1}^{N} S_{ki}S_{ki}^* = 1 \, for \; (i = j)$$

$$\sum_{k=1}^{N} S_{ki} S_{kj}^{*} = 0\ for\ (i \neq j)$$

Hence,

$$S_{12}S_{12}^{*} + S_{13}S_{13}^{*} = |S_{12}|^2 + |S_{13}|^2 = 1$$

$$S_{21}S_{21}^{*} + S_{23}S_{23}^{*} = |S_{21}|^2 + |S_{23}|^2 = 1$$

$$S_{31}S_{31}^{*} + S_{32}S_{32}^{*} = |S_{31}|^2 + |S_{32}|^2 = 1$$

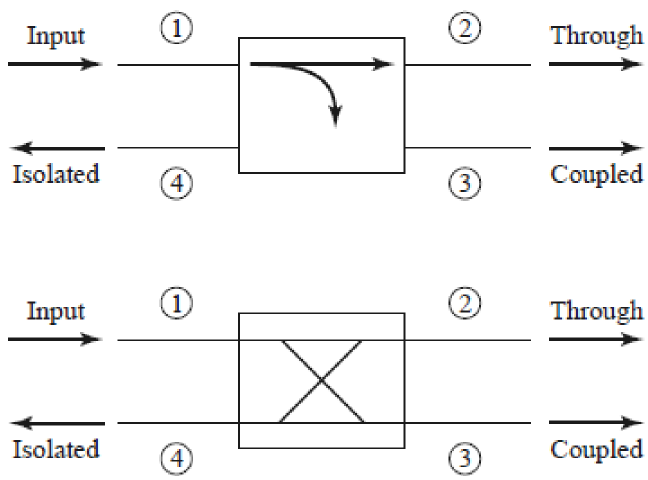


FIGURE 7.4 Two commonly used symbols for directional couplers, and power flow conventions.

III. Basics to Four Port Networks

Three assumptions are made before starting analysis:

(1) Matched at all ports
(2) Lossless (related to the Unitary matrix)
(3) Reciprocal

The according scattering matrix can be shown:

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{21} & 0 & S_{23} & S_{24} \\ S_{31} & S_{32} & 0 & S_{34} \\ S_{41} & S_{42} & S_{43} & 0 \end{bmatrix}$$

From the Unitary property and the energy conservation, 10 different equations can be generated.

$$S_{14} = S_{23} = 0$$

At the end of the day, we end up with choosing:

$$S_{12} = S_{34} = \alpha$$

$$S_{13} = \beta e^{j\theta}$$

$$S_{24} = \beta e^{j\varphi}$$

α and β are both real; θ and φ are the two phases to be determined (one of which we are still able to choose.)

The relation between the phases:

$$\theta + \varphi = \pi \pm 2n\pi$$

From this relation, we can choose the type of coupler:

(1) Symmetric coupler ($\theta = \varphi = \frac{\pi}{2}$)

$$[S] = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

(2) Anti-symmetric coupler ($\theta = 0 \; and \; \varphi = \pi$)

$$[S] = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

What makes them different from each other?
Merely initial selection of the reference plane.

Are the two amplitudes independent to each other?
No. They are related by:

$$\alpha^2 + \beta^2 = 1$$

Conclusion?
Any reciprocal, matched at all ports, and lossless four port networks are directional couplers.

Port relations?

(1) Port 1 and port 3 are (***the coupled port***) where port 1 takes input power. The coupling factor: $|S_{13}|^2 = \beta^2$

(2) Port 2 takes the remained of input power. (***the Through port***) The coefficient for this remainder power: $|S_{12}|^2 = \alpha^2 = 1 - \beta^2$

(3) Port 4 is isolated from the input power. (***the Isolated port***) An ideal coupler exhibits no power is delivered to port 4.

4 Parameters that characterize a directional coupler:

(1) Coupling:

$$C = 10log\frac{P_1}{P_3} = -20log\beta\ [dB]$$

(2) Directivity:

$$D = 10log\frac{P_3}{P_4} = 20log\frac{\beta}{|S_{14}|}\ [dB]$$

(3) Isolation:

$$I = 10log\frac{P_1}{P_4} = -20log|S_{14}|\ [dB]$$

(4) Insertion Loss: (Power delivered to the Through port)

$$L = 10log\frac{P_1}{P_2} = -20log|S_{12}|\ [dB]$$

IV. Hybrid Coupler

Hybrid coupler is a special case where $\alpha = \beta = \frac{1}{\sqrt{2}}\ and\ C = 3\ [dB]$. Hybrid couplers should be considered when power must be equally split between two ports. <<a little vague here, equally split between two ports? Which are? I think port 1 power should be equally distributed to port 2 and port 3.>>

There are two types of Hybrid coupler:

(1) Quadrature Hybrid (phase difference between port 2 and 3 is 90°)

$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & j & 0 \\ 1 & 0 & 0 & j \\ j & 0 & 0 & 1 \\ 0 & j & 1 & 0 \end{bmatrix}$$

Symmetric Coupler

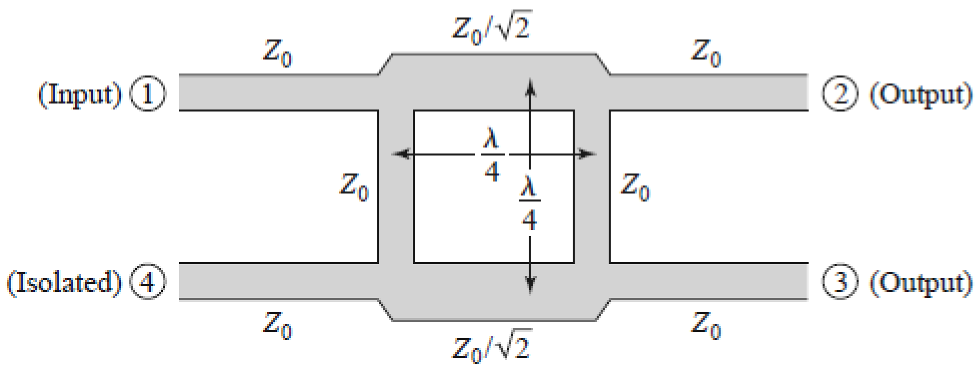


FIGURE 7.21 Geometry of a branch-line coupler.

Figure. Quadrature 90 Coupler

(2) Magic-T and Rat-Race (phase difference port 2 and 3 is 180°)

$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

Anti-symmetric Coupler

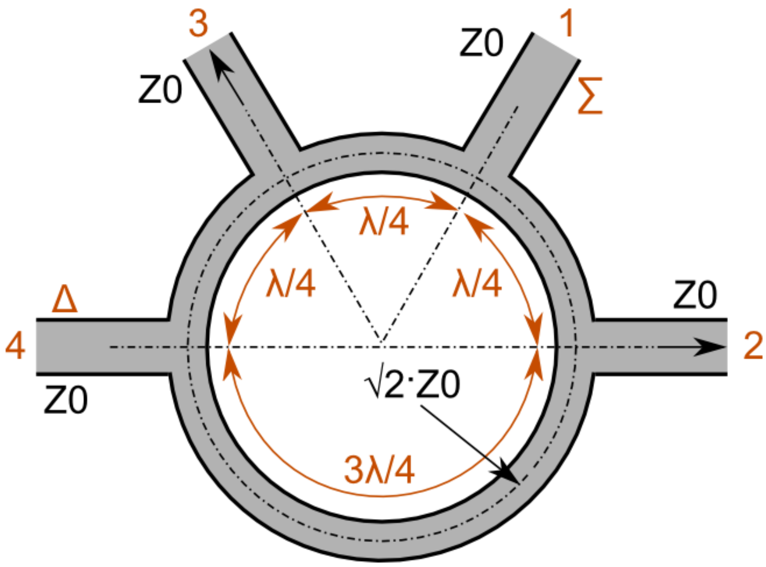


Figure. Rat-Race Coupler [4]

V. Wilkinson Power Divider

a. Even Mode Analysis

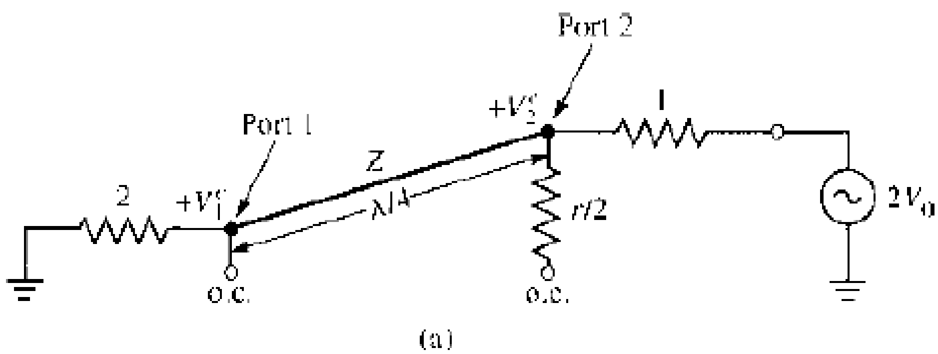


Figure. Equivalent Circuit for Even Mode

The input impedance seen from port 2 can be written (the transmission line's length is quarter wave length):

$$Z_{in}^{e} = Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)} = \frac{Z^2}{2}$$

Since our source impedance is unity, input impedance matching occurs if

$$Z = \sqrt{2}$$

Which is the normalized characteristic impedance of the transmission line (i.e. $Z_0 = 1$).

Now, let’s consider the voltage expression of the transmission line. In phasor domain, the standing wave of transmission line can be written in the following form:

$$\tilde{V}(x) = V_0\left(e^{-j\beta x} + \Gamma e^{j\beta x}\right)$$

V_2^e	V_0
V_1^e	$jV_0\left(\frac{\Gamma+1}{\Gamma-1}\right)$

We investigate this port voltages because they are related to the scattering matrix. Note that the following relation is valid:

$$\frac{V_1}{V_2} = S_{12}$$

Where the V_1 and V_2 are the two voltages at port 1 and port 2, respectively, which were derived from the superposition of even and odd mode analysis. This relation can be checked from the two-port network analysis. Hence, we can complete the scattering matrix for the Wilkinson power divider.

The completed scattering matrix for Wilkinson power divider:

$$[S] = \begin{bmatrix} 0 & -\frac{j}{\sqrt{2}} & -\frac{j}{\sqrt{2}} \\ -\frac{j}{\sqrt{2}} & 0 & 0 \\ -\frac{j}{\sqrt{2}} & 0 & 0 \end{bmatrix}$$

This is a singular matrix, meaning there’s no inverse to it.

VI. Waveguide Directional Coupler

I will not cover the Bethe Hole Coupler in this document since it is a waveguide type. The analysis for waveguide is little complicated but it allows higher frequency band design.

I might cover the waveguide version later if I feel like I need to.

VII. The Quadrature Coupler

It’s a hybrid couple which yields 3dB equal power splits and 90 degree of phase shift between output ports (the through and the coupled arms, or port 2 and port 3). If the quadrature coupler was made on microstrip or strip line, then it would be considered as “***Branch-line coupler***”. For analysis, even and odd mode will be used just as we did in Wilkinson power divider analysis.

$$[S] = -\frac{1}{\sqrt{2}}\begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}$$

The Branch-line coupler exhibits high degree of symmetry (meaning any of the four ports can be used as "input") and all ports are matched.

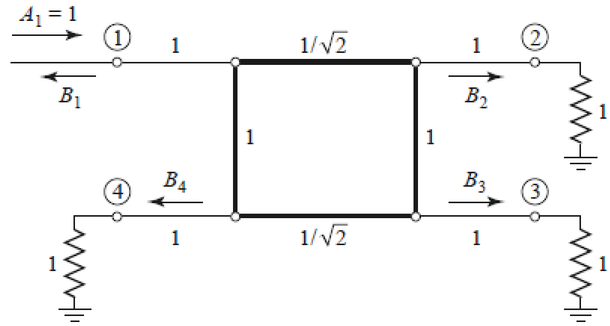


FIGURE 7.22 Circuit of the branch-line hybrid coupler in normalized form.

a. Even-Odd Mode Analysis

Note that the term "normalized" refers to the fact that the circuit parameters are normalized to the characteristic impedance (Z_0) of the line.

Let's look at two sets of excitations: even and odd.

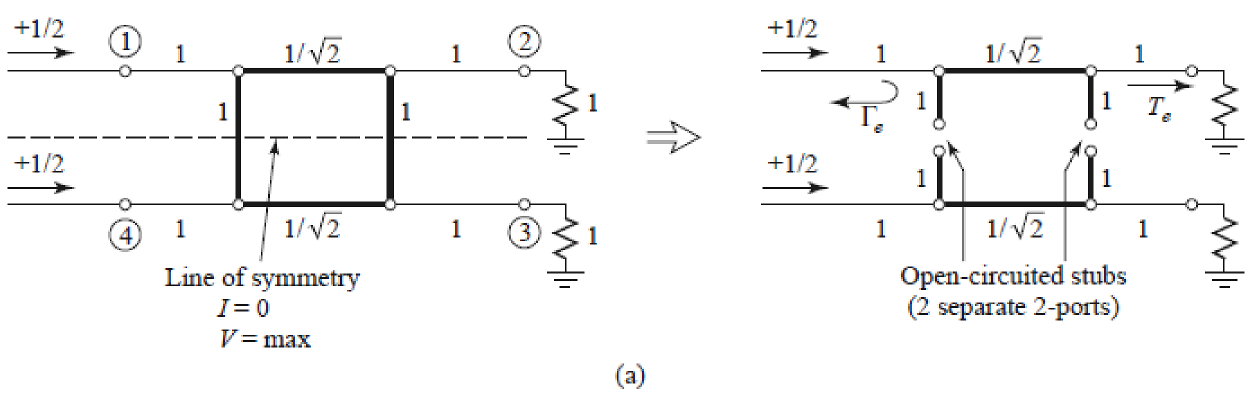
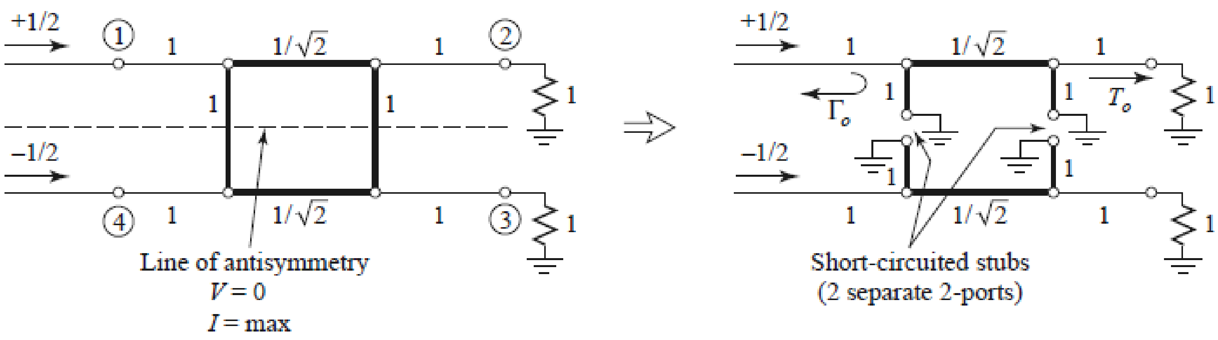


Figure. Even Mode Excitation



(b)

Figure. Odd Mode Excitation

In addition, if we utilize the symmetry and anti-symmetry properties of each mode, the analysis becomes even simpler. In short, we may relate the parameters in Figure.7.22 to those in Figure.7.23 in the following way:

$$B_1 = \frac{1}{2}\Gamma_e + \frac{1}{2}\Gamma_o$$

$$B_2 = \frac{1}{2}T_e + \frac{1}{2}T_o$$

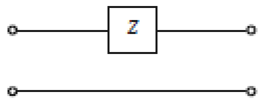
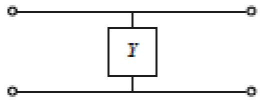
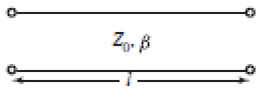
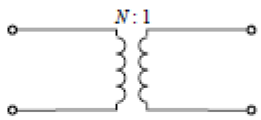
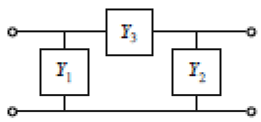
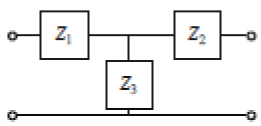
$$B_3 = \frac{1}{2}T_e - \frac{1}{2}T_o$$

$$B_4 = \frac{1}{2}\Gamma_e - \frac{1}{2}\Gamma_o$$

Note that $B's$ refer to the amplitudes of waves incident in the system for each port number indicated. To find each reflection and transmission coefficients for each mode excitation, we need to come up with the S matrix that explains each system.

$ABCD$ matrix is a great tool to calculate the S matrix when there are multiple circuit components in transmission line. The following Table will help setting up the matrix.

TABLE 4.1 ***ABCD*** **Parameters of Some Useful Two-Port Circuits**

Circuit	*ABCD* Parameters	
	$A = 1$ $C = 0$	$B = Z$ $D = 1$
	$A = 1$ $C = Y$	$B = 0$ $D = 1$
	$A = \cos\beta\ell$ $C = jY_0 \sin\beta\ell$	$B = jZ_0 \sin\beta\ell$ $D = \cos\beta\ell$
	$A = N$ $C = 0$	$B = 0$ $D = \frac{1}{N}$
	$A = 1 + \frac{Y_2}{Y_3}$ $C = Y_1 + Y_2 + \frac{Y_1 Y_2}{Y_3}$	$B = \frac{1}{Y_3}$ $D = 1 + \frac{Y_1}{Y_3}$
	$A = 1 + \frac{Z_1}{Z_3}$ $C = \frac{1}{Z_3}$	$B = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3}$ $D = 1 + \frac{Z_2}{Z_3}$

$ABCD$ matrix for the even mode excitation:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_e = \begin{bmatrix} 1 & 0 \\ j & 1 \end{bmatrix} \begin{bmatrix} 0 & \frac{j}{\sqrt{2}} \\ j\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ j & 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 & j \\ j & -1 \end{bmatrix}$$

In the above matrix form, one may realize that

[Shunt $Y = j$][$\lambda/_4$ Transmission Line][Shunt $Y = j$]

"The shunt is j because the shunt open circuit stub for $l = \lambda/_8$ is equal to $jtan\left(\beta\, \lambda/_8\right) = j$"

Now, to convert this $ABCB$ matrix to the S matrix, we can simply use the Table for conversion. They are related as follow:

$$\Gamma_e = \frac{A + B - C - D}{A + B + C + D} = 0$$

$$T_e = \frac{2}{A + B + C + D} = -\frac{1}{\sqrt{2}}(1 + j)$$

Likewise, the $ABCD$ matrix for the odd mode excitation:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_o = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix}$$

$$\Gamma_o = \frac{A+B-C-D}{A+B+C+D} = 0$$

$$T_o = \frac{2}{A+B+C+D} = \frac{1}{\sqrt{2}}(1-j)$$

Now, it's time to superimpose the results from the even and odd mode analysis:

$$B_1 = \frac{1}{2}\Gamma_e + \frac{1}{2}\Gamma_o = 0\ (port1\ is\ matched)$$

$$B_2 = \frac{1}{2}T_e + \frac{1}{2}T_o = -\frac{j}{\sqrt{2}}\ (half-power, from\ port1\ to\ port2\ with\ 90°\ phase\ shift)$$

$$B_3 = \frac{1}{2}T_e - \frac{1}{2}T_o = -\frac{1}{\sqrt{2}}\ (half-power, from\ port1\ to\ port3\ with\ 180°\ phase\ shift)$$

$$B_4 = \frac{1}{2}\Gamma_e - \frac{1}{2}\Gamma_o = 0\ (no\ power\ from\ port1\ to\ port4)$$

Reminder!

All these B's are ***the responses*** when the unity amplitude input was applied to port 1. For $B_1 = 0$, which implies that no power has been bounced back, leading to $S_{11} = 0$. This occurs only when port 1 is matched. So, it makes sense. For $B_2 = -\frac{j}{\sqrt{2}}, \frac{1}{\sqrt{2}} = -3dB$ below than the input level should be injected into port 2. In addition, $-j = e^{-\frac{j\pi}{2}}$ led us to conclude that the port 2 will receive a signal that is 90 degrees out of phase than the port 1 signal.

This just sums up the analysis and characterizes what the Quadrature coupler does.

Bandwidth of Branch-line Coupler?
In practice, the bandwidth is only about 10~20%; however, it can be increased by considering multi-section line coupler.

How does the quadrature coupler behave when port 1 and port 4 are excited at the same time with different sources?

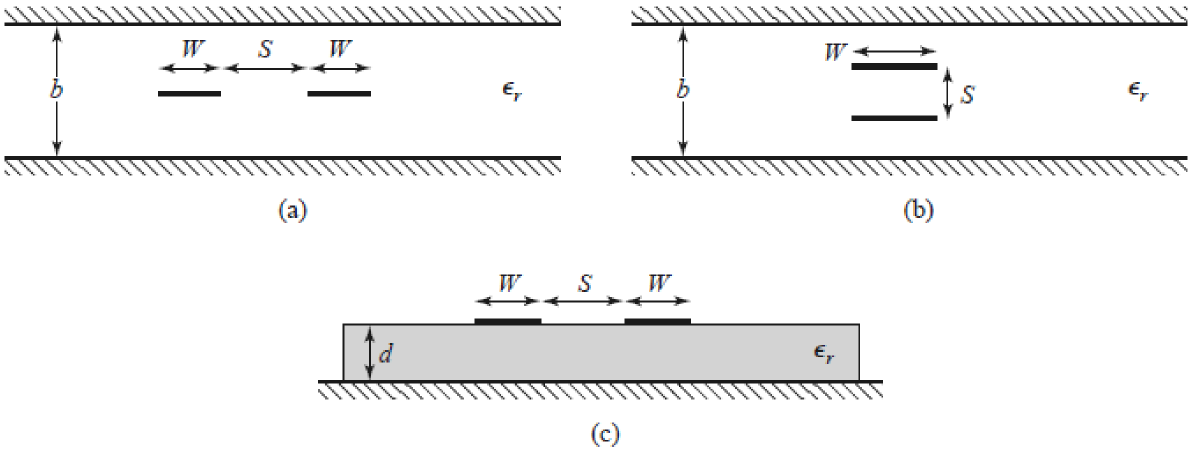


FIGURE 7.26 Various coupled transmission line geometries. (a) Coupled stripline (planar, or edge-coupled). (b) Coupled stripline (stacked, or broadside-coupled). (c) Coupled microstrip lines.

Figure.7.26 helps visualizing how microstrip line Quadrature coupler should look like.

b.Design the Quadrature

The givens:

$$f_0 = 2.5\ GHz$$
$$Thickness\ (t) = 1.5748\ mm$$
$$\epsilon_r = 4.1$$
$$Z_0 = 50\ ohm$$
$$tan\delta = 0.01$$

Calculated	Z_0	$Z_0/\sqrt{2}$
Width	$3.13\ mm$	$5.274\ mm$
$\lambda/4$	$16.88\ mm$	$16.47\ mm$

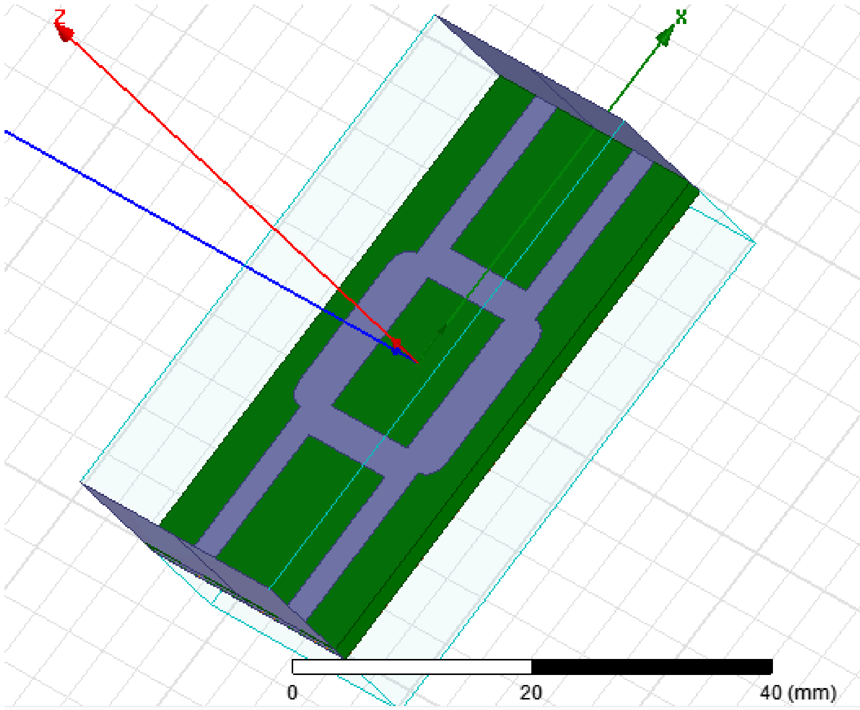


Figure. HFSS design of the quadrature coupler

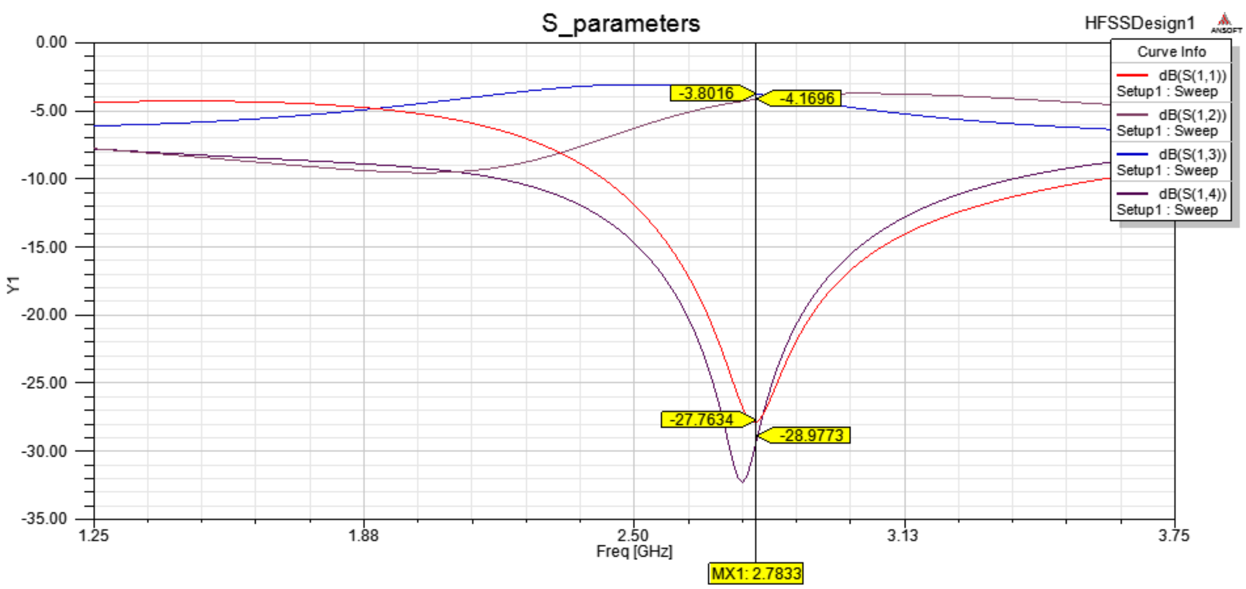


Figure. Response at port 1 from S-parameters

<<The center frequency of the coupler has been shifted about 12% which made me baffled. However, this opens up a new path for learning. I will research a little to find out what’s gone wrong.>>

VIII. The 180 Hybrid Coupler

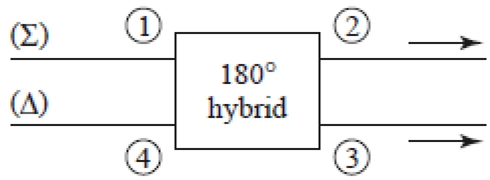


FIGURE 7.41 Symbol for a 180° hybrid junction.

In practice, both 180 and 360 phase shifts are possible between the output ports.

Basic characteristics of the 180 Hybrid coupler?

Input can be applied to either port 1 or port 4 in which case would yield different phase shift between output ports (port 2 and 3). (1) If input is applied to port 1 and then two in-phase components will go to port 2 and 3. The port 4 is isolated. (2) If input is applied to port 4, then the port 2 and 3 will receive two components that are 180 out of phase each other. The port 1 is isolated.

Let's see the S matrix for the ideal 3dB 180 Hybrid coupler:

$$[S] = -\frac{j}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

This matrix is (1) unitary and (2) symmetric. (Verifying them might be a good exercise!)

There are three types of 180 Hybrid but the third one is waveguide type; therefore, I will skip it for now. The other two are microstrip/strip line type.

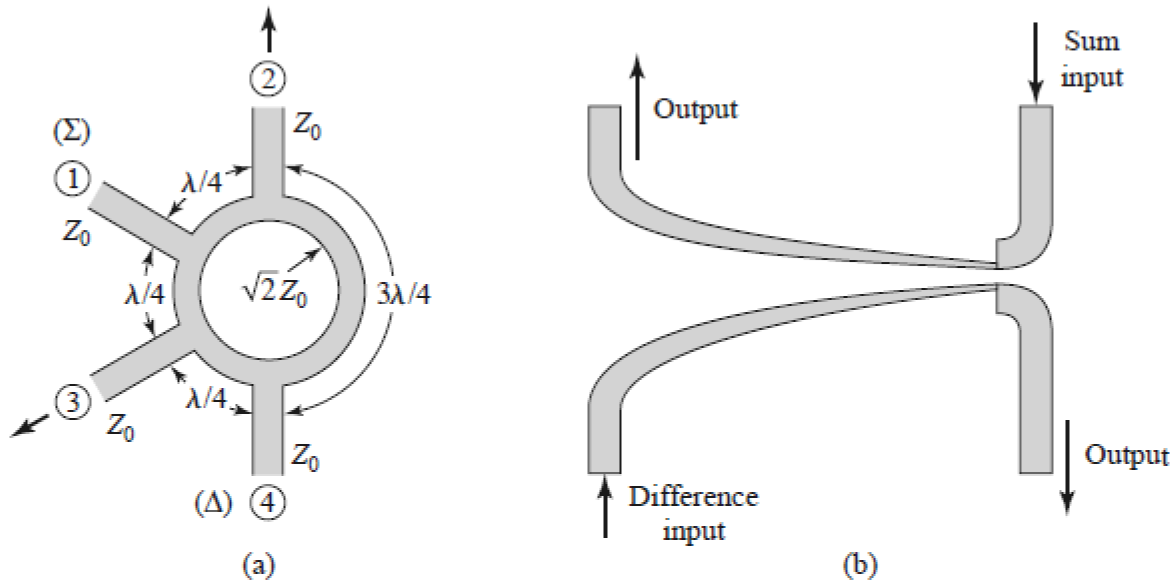


Figure. (a) Ring Hybrid, or Rat-Race (b) A tapered coupled Hybrid

For analysis of Ring Hybrid, even-odd mode analysis will be considered. How useful idea is it? (***Even-odd mode analysis can be applied to linear system, right? Check this point!***)

a. Even Mode Analysis

Even-odd mode analysis begins with the assumption that unit amplitude input has been applied to port 1.

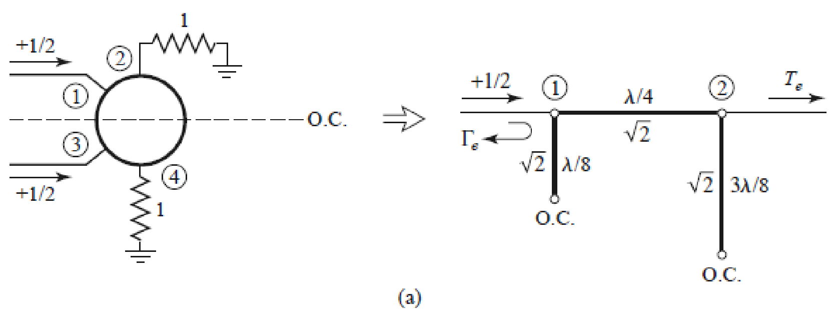


Figure. Even Mode Circuit for 180 Hybrid

$ABCD$ matrix can be expressed:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_e = \begin{bmatrix} 1 & j\sqrt{2} \\ j\sqrt{2} & -1 \end{bmatrix}$$

Hence, we find the reflection and transmission coefficients for the even mode:

$$\Gamma_e = -\frac{j}{\sqrt{2}}$$

$$T_e = -\frac{j}{\sqrt{2}}$$

b.Odd Mode Analysis

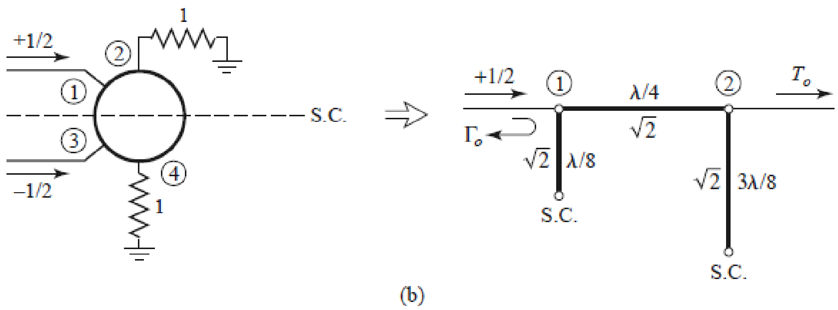


Figure. Odd Mode Circuit for 180 Hybrid

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_o = \begin{bmatrix} -1 & j\sqrt{2} \\ j\sqrt{2} & 1 \end{bmatrix}$$

$$\Gamma_o = \frac{j}{\sqrt{2}}$$

$$T_o = -\frac{j}{\sqrt{2}}$$

Now, we find the four amplitudes (four responses) when the input is applied to port 1 with unity amplitude.

$$B_1 = \frac{1}{2}\Gamma_e + \frac{1}{2}\Gamma_o = 0\ (matched\ at\ port\ 1)$$

$$B_2 = \frac{1}{2}T_e + \frac{1}{2}T_o = -\frac{j}{\sqrt{2}}\ (3dB\ power\ split\ with\ 90\ phase\ shift)$$

$$B_3 = \frac{1}{2}\Gamma_e - \frac{1}{2}\Gamma_o = -\frac{j}{\sqrt{2}}\ (3dB\ power\ split\ with\ 90\ phase\ shift)$$

$$B_4 = \frac{1}{2}T_e - \frac{1}{2}T_o = 0\ (no\ power\ flows\ into\ port\ 4)$$

Bandwidth of Ring Hybrid?

Usually, limited to 20~30%. This can be improved by considering additional sections, or a symmetric ring circuit. However, I was not able to access the reference introduced in the textbook.

IX. Tapered Coupled Line Hybrid

I think this type will provide a general understanding of “coupled” lines since the ABCD matrix is derived from the transformers that somewhat explain the idea of coupled line.

X. References

[1] Microwave Engineering, David Pozar, 4th Edition

[2] https://en.wikipedia.org/wiki/Power_dividers_and_directional_couplers

[3] https://www.comsol.com/blogs/modeling-branch-line-coupler/

[4] http://www.antennamagus.com/database/antennas/antenna_page.php?id=293

[5] https://en.wikipedia.org/wiki/Hermitian_adjoint