# Geometric Series

## I. Introduction

Geometric series can be shown in many fields of engineering and physics to approximate a function, etc. Laurent series is in the form of geometric series to approximate a function even at the singularity point of domain where Taylor series can't be defined.

## II. General form

$$\sum_{k=0}^{n-1} ar^k = a + ar + ar^2 + ar^3 + \cdots + ar^{n-1}$$

where $r$ is the common ratio. [1]

Notice that there are total n terms in the equation above.

## III. Derivation

Let's assume that the result of the summation converges to some number, $s$.

Then,

$$s = a + ar + ar^2 + ar^3 + \cdots + ar^{n-1} \quad (1)$$

And we multiply $s$ by $r$ which results,

$$rs = ar + ar^2 + ar^3 + \cdots + ar^n \quad (2)$$

Now subtract (1) from (2):

$$rs - s = (ar + ar^2 + ar^3 + \cdots + ar^n) - (a + ar + ar^2 + ar^3 + \cdots + ar^{n-1})$$

$$s(r-1) = ar^n - a = a(r^n - 1)$$

$$\therefore s = \frac{(r^n - 1)}{(r - 1)}$$

## IV. Example

Investigate the maximum value of IEEE754 single precision.

The maximum value should have the maximum values both in mantissa and exponent field which leads to:

$$1.111\ 1111\ 1111\ 1111\ 1111\ 1111 \times 2^{127}$$

$$= 1 + \sum_{k=1}^{23} 1 \cdot \frac{1}{2^k} \cdot 2^{127}$$

Notice that $a = 1$ and $r = 2^{-1} = 0.5$ and find the sum of the geometric series first.

$$s = \frac{1 - \frac{1}{2^{23}}}{1 - \frac{1}{2}} = 1.999\ 9997\ 62$$

Putting them together would yield:

$$\approx 3.402\ 8232\ 64 \times 10^{38}$$

The maximum of the value in IEEE754 single precision format is in power of 38.

# V. Proof of Convergence

In progress

# VI. Infinite Geometric Series Sum

In progress

# VII. Reference

[1] Wikipedia: https://en.wikipedia.org/wiki/Geometric_series