Friis Transmission Equation

Introduction

Friis transmission equation is a useful tool for calculating the power relation between two antennas separated by a distance $R > 2D^2/\lambda$ where D is the largest dimension of either antenna.

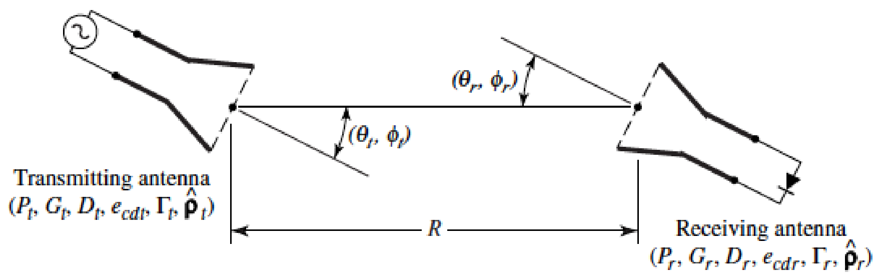

Figure 2.31 Geometrical orientation of transmitting and receiving antennas for Friis transmission equation.

The ratio of two powers

$$\frac{P_r}{P_t} = e_{cdt}e_{cdr}(1 - |\Gamma_t|^2)(1 - |\Gamma_r|^2)\left(\frac{\lambda}{4\pi R}\right)^2 D_t(\theta_t, \phi_t)D_r(\theta_r, \phi_r)|\hat{\rho}_t \cdot \hat{\rho}_r|^2$$

Discussion

Assume that there are two antennas separated by a distance, R. The transmit antenna radiates some power that is isotropic. On the other hand, the receiving antenna will see the power coming from far-field region. Is the amount of power radiated equal to that of the received power?

In practice, the answer is no. There should be accounts for losses such as polarization losses, losses due to effective areas, losses due to reflections in the transmission lines, dielectric losses (what the transmission lines are made of), and etc.

In this context, we also assume that wave propagations are considered as plane waves in the far-field. However, they are not plane wave in real world example.

Now, let's think about the power at the terminals of the transmit antenna (input power, P_t). Since the transmit antenna is assumed to be isotropic power, the governing equation of power density can be described as

$$W_0 = e_t \frac{P_t}{4\pi R^2} \ [W/m^2]$$

(2-113)

where

$$e_t = radiation\ efficiency\ of\ transmit\ antenna\ [dimensionless]$$

Note that this power is a power of plane wave incident on the receiving antenna.

Now, we obtained a general understanding of the power relation between the transmit and receiving side. Let's generalize further: investigate a non-isotropic power case.

Unlike the isotropic power, as you can imagine, a non-isotropic power has some directional account in itself. Fortunately, we have a great tool to describe this directional characteristic, namely the gain.

$$W_t = \frac{P_t G_t(\theta_t, \phi_t)}{4\pi R^2} = e_t \frac{P_t D_t(\theta_t, \phi_t)}{4\pi R^2} \; [W/m^2]$$

(2-114)

Recall that gain and directivity are highly related to one another.

As aforementioned, we need to think about the area that captures the incoming power from the far-field region. The question is that will the receiving antenna fully captures the area of the power of plane wave? (This is another advantage of using plane wave. It's easy to picture it.) In other words, will the effective area of the receiving end be the same as that of the physical area?

$$A_r = e_r D_r(\theta_r, \phi_r)(\frac{\lambda}{4\pi})^2$$

Details about the effective area will be covered later. Interestingly, effective area would be greater than the physical area in general.

So, how much power will be collected at the receiving antenna? The collected power is the product of the effective area of receiving antenna and the power density of plane wave from the far-field region.

$$P_r = A_r W_t$$
$$= e_r D_r(\theta_r, \phi_r)(\frac{\lambda}{4\pi})^2 \cdot e_t \frac{P_t D_t(\theta_t, \phi_t)}{4\pi R^2}$$
$$\frac{P_r}{P_t} = e_r e_t D_r(\theta_r, \phi_r)\, D_t(\theta_t, \phi_t)\, (\frac{\lambda}{4\pi R})^2$$

(2-117)

However, (2-117) still does not capture the whole picture. What we are missing here? Losses due to polarization mismatch and due to reflections. We may use (2-117) when the transmitting and receiving antennas have been matched to their respective lines or loads and when the polarization of receiving antenna is matched to the polarization of impinging wave. To account for these effects, we simply add new terms.

$$\frac{P_r}{P_t} = e_{cdt} e_{cdr} (1 - |\Gamma_t|^2)(1 - |\Gamma_r|^2) \left(\frac{\lambda}{4\pi R}\right)^2 D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r) |\hat{\rho}_t \cdot \hat{\rho}_r|^2$$

(2-118)

Notice that the radiation efficiency does not include the reflection mismatch. The overall antenna efficiency can be described as

$$e_0 = e_r e_c e_d$$

where

$$e_0 = total\ antenna\ efficiency\ [dimensionless]$$
$$e_r = reflection\ mismatch = 1 - |\Gamma|^2 [dimensionless]$$
$$e_c = conduction\ efficiency\ [dimensionless]$$
$$e_d = dielectric\ efficiency\ [dimensionless]$$

where

$$\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

Where

$$Z_{in} = input\ impedance\ of\ antenna$$
$$Z_0 = characteristic\ impedance\ of\ the\ transmission\ line$$

In practice, it is very difficult to compute e_c and e_d. In general, $e_{cd} = e_c e_d$ is used together to account for the radiation efficiency of antenna due to the fact that it is even harder to find each factor separately.

$\left(\frac{\lambda}{4\pi R}\right)^2$ is called the free-space loss factor. It explains the losses due to spherical spreading of the energy by the antenna.

Reference

[1] Antenna Theory: Analysis and Design, 4th Edition by Constantine A. Balanis

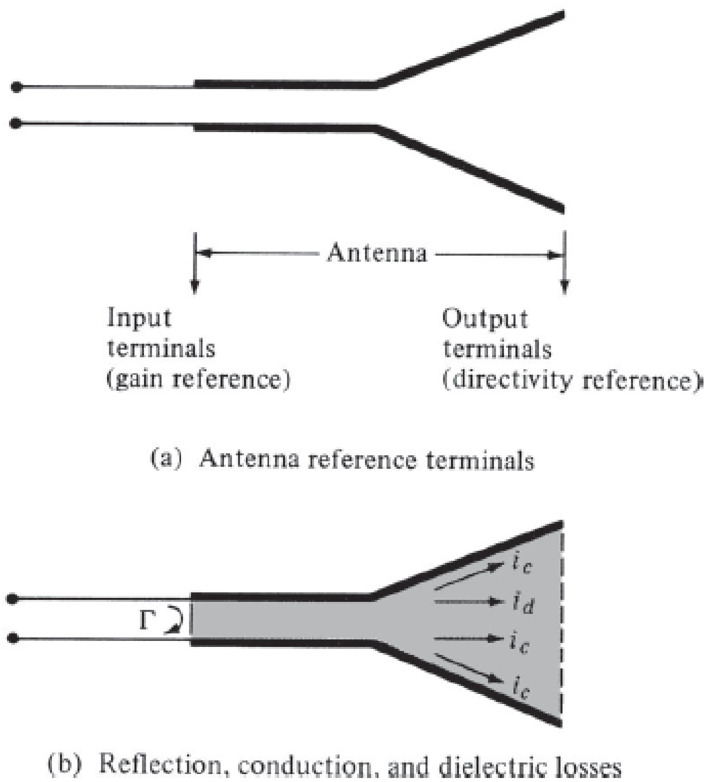

Figure 2.22 Reference terminals and losses of an antenna.