Second Law of Cosine

I. Introduction

Simple derivation of the second law of cosine and the Pythagorean theorem will be covered in this page. The second law of cosine is very useful in linear algebra since it allows to define if two vectors are orthogonal or not.

II. Derivation

To derive the second law of cosine, we need a vector (defined in n-dimensional space but this feature does not matter).

For an arbitrary vector $\vec{c}$, if we consider the dot product of itself:

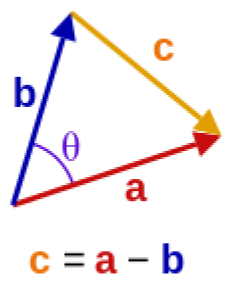

Figure. [1]

$$\vec{c} \cdot \vec{c} = \left(\vec{a} - \vec{b}\right) \cdot \left(\vec{a} - \vec{b}\right)$$

$$= \vec{a} \cdot \vec{a} - 2\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{b}$$

$$= \|a\|^2 - 2\vec{a} \cdot \vec{b} + \|b\|^2$$

Where the dot product is defined as:

$$\vec{a} \cdot \vec{b} = \|a\|\|b\| \cos(\theta)$$

Which complete the derivation,

$$\|c\|^2 = \|a\|^2 - 2\|a\|\|b\| \cos(\theta) + \|b\|^2$$

Since any value in bracket represents the magnitude of a vector,

$$c^2 = a^2 + b^2 - 2abcos(\theta)$$

III. References

[1] https://en.wikipedia.org/wiki/Dot_product