Trapezoid Rule

I. Introduction

It is one of the approximation techniques in mathematics. The motivation of this document arouses from reviewing the method of moment for Pocklington's integral equation where the impedance matrix is expressed from the point matching technique using the Dirac-delta function for the weighing function.

II. Theory

It's a linear approximation for a definite integral. First, consider the following integral:

$$\int_a^b f(x)dx$$

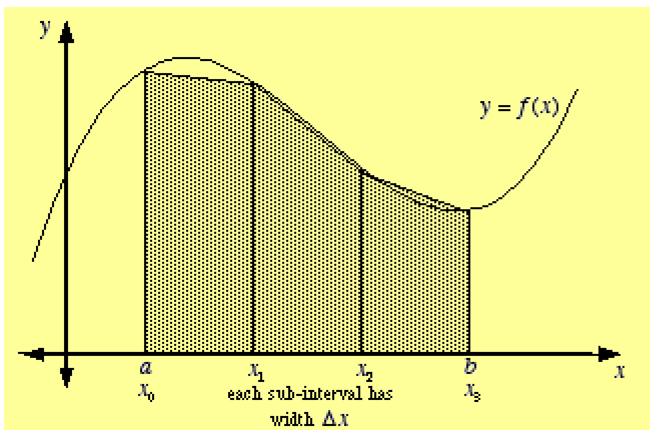

Figure. [1]

Recall the Riemann sum for the definition of integral, one can sense that we need to divide the interval into n subintervals to form the Summation.

$$\Delta x = \frac{b-a}{n}$$

Now that we set up the sub-interval, let's consider the area of a trapezoid. [2]

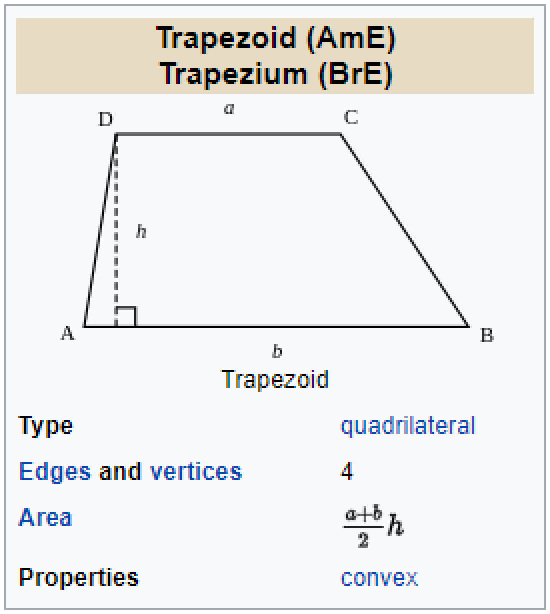

Figure. [2]

$$A = \frac{a+b}{2}h$$

The sum of all trapezoids under the curve (the function $f(x)$) can be approximated to yield the integral:

$$\int_a^b f(x)dx \approx A_0 + A_1 + A_2 + \cdots + A_n$$

$$= \frac{f(x_0) + f(x_1)}{2}\Delta x + \frac{f(x_1) + f(x_2)}{2}\Delta x + \frac{f(x_2) + f(x_3)}{2}\Delta x + \cdots + \frac{f(x_{n-1}) + f(x_n)}{2}\Delta x$$

$$= \frac{\Delta x}{2}\left(f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + f(x_n)\right)$$

Obviously, as n approaches to infinity, the difference between the integral and the Trapezoid summation becomes negligible.

III. References

[1] http://www.mathwords.com/t/trapezoid_rule.htm

[2] https://en.wikipedia.org/wiki/Trapezoid