Dipole Antenna

I. Introduction

In this document, the characteristics of dipole antenna will be introduced. It often helps understanding more complicated structure antenna. HFSS simulation will be provided for reference.

In theory section, the following items will be covered:

(1) Infinitesimal dipole
(2) Small dipole
(3) Finite length dipole

Negligible diameters of all types of dipole antenna are assumed throughout this document for reducing heavy computations. However, this analysis still provides a good approximation. Finite radii dipole antenna analysis should be considered in another document.

Throughout this document, the primed coordinate will represent the source. The unprimed coordinate is the coordinate that forms the basis for any mathematical operation. The primed coordinate can be thought as the actual point where antenna is located.

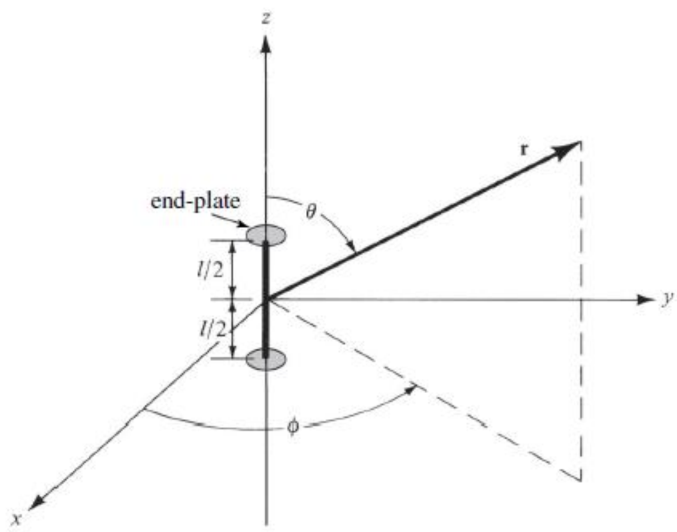
II. Theory

a. Infinitesimal Dipole

i. Fields

When the length of the wire of dipole is much shorter than the wavelength, it is said to be infinitesimal dipole antenna. Infinitesimal dipole antenna is not practical; however, it provides a good insight into the idea of dipole antenna and other antenna parameters. Note that infinitesimal dipole represents capacitor-plate antenna (i.e. current distribution over dipole).

Current distribution is very important in terms of analyzing antenna. For infinitesimal dipole case, the current is uniform due to the capacitive loading at the ends. In addition to this, an assumption about the small radius of dipole (very thin) is made. In conclusion, current can be treated as uniform.



(a) Infinitesimal dipole

Figure.

$$\vec{I}(z') = \hat{a}_z I_0$$

To find the radiated field by the infinitesimal dipole, we need to find the vector potential derived from the current function. Then, we can induce the electric field out of the vector potential. We can assume that the magnetic vector potential is zero since there's no magnetic current in infinitesimal dipole. The electric vector potential is given in terms of electric current:

$$\vec{A}(x,y,z) = \frac{\mu}{4\pi}\int_C \vec{I}_e(x',y',z')\frac{e^{-jkR}}{R}dl'$$

$$= \frac{\mu}{4\pi}\int_C \hat{a}_z\, I_0 \frac{e^{-jkR}}{R}dl'$$

$$= \frac{\mu}{4\pi}\int_{-l/_2}^{l/_2} I_0 \frac{e^{-jkr}}{r}dz'$$

Where $R = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2} = \sqrt{x^2+y^2+z^2} = r$ and $dl' = dz'$

$$\because x' = y' = z' = 0$$

<<The source is located at the origin of the unprimed coordinate.>>

$$\vec{A} = \hat{a}_z \frac{\mu I_0 l e^{-jkr}}{4\pi r}$$

We have convenient coordinate transformation, which is given as:

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\varphi \end{bmatrix} = \begin{bmatrix} sin\theta cos\varphi & sin\theta sin\varphi & cos\theta \\ cos\theta cos\varphi & cos\theta sin\varphi & -sin\theta \\ -sin\varphi & cos\varphi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

<<Vector component transformation can be found in another document named "Coordinate Transformation". >>

Note that there's only z-component in the vector.

$$A_z = \frac{\mu I_0 l e^{-jkr}}{4\pi r}$$

$$A_r = A_z cos\theta = \frac{\mu I_0 l e^{-jkr}}{4\pi r} cos\theta$$

$$A_\theta = -A_z sin\theta = -\frac{\mu I_0 l e^{-jkr}}{4\pi r} sin\theta$$

$$A_\varphi = 0$$

Now that we successfully found all the vector components of the Vector potential in the Spherical coordinate, we can find the magnetic field with the following relation:

$$\vec{H}_A = \frac{1}{\mu} \nabla \times \vec{A}$$

The curl of a vector in the Spherical coordinate system: [3]

$$\nabla \times \vec{A} = \frac{1}{rsin\theta}\left(\frac{\partial}{\partial\theta}(A_\varphi sin\theta) - \frac{\partial A_\theta}{\partial\varphi}\right)\hat{e}_r + \frac{1}{r}\left(\frac{1}{sin\theta}\frac{\partial A_r}{\partial\varphi} - \frac{\partial}{\partial r}(rA_\varphi)\right)\hat{e}_\theta + \frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right)\hat{e}_\varphi$$

Fortunately, if you look at the vector potential components, there is no φ variation and $A_\varphi = 0$. So, we can simplify further:

$$\frac{1}{\mu}\nabla \times \vec{A} = \frac{1}{\mu}\frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right)\hat{e}_\varphi$$

$$= \frac{1}{\mu r}\left[\frac{-\mu I_0 l}{4\pi}(-jk)e^{-jkr} sin\theta - \left(-\frac{\mu I_0 l e^{-jkr}}{4\pi r} sin\theta\right)\right]$$

$$= \frac{I_0 l}{4\pi r} e^{-jkr} sin\theta \left(jk + \frac{1}{r}\right)$$

$$H_\varphi = j\frac{kI_0 l}{4\pi r} e^{-jkr} sin\theta \left(1 + \frac{1}{jkr}\right)$$

To find the electric field, use the Maxwell's equations.

$$\nabla \times \vec{H}_A = \vec{J} + j\omega\varepsilon\vec{E}_A$$

but here $\vec{J} = 0$.

However, since we have the vector potential, we could use it too. The electric field is given as:

$$\vec{E}_A = -\nabla\phi_e - j\omega\vec{A} = -j\omega\vec{A} - j\frac{1}{\omega\mu\varepsilon}\nabla(\nabla \cdot \vec{A})$$

where the *Lorentz gauge* was used in the above equation. Refer to another document to help understanding.

I will use the Maxwell's equation to find the electric field; however, it might be a good exercise to use the vector potential as we check the beauty of the Lorentz gauge.

The Maxwell's equation under the condition: $H_r = H_\theta = 0$ and no φ variation still. Find the electric field:

$$\vec{E}_A = \frac{\nabla \times \vec{H}_A}{j\omega\varepsilon} = \frac{1}{j\omega\varepsilon}\frac{1}{rsin\theta}\left(\frac{\partial}{\partial\theta}\left(H_\varphi sin\theta\right) - \frac{\partial H_\theta}{\partial\varphi}\right)\hat{e}_r + \frac{1}{r}\left(\frac{1}{sin\theta}\frac{\partial H_r}{\partial\varphi} - \frac{\partial}{\partial r}\left(rH_\varphi\right)\right)\hat{e}_\theta$$
$$+\frac{1}{r}\left(\frac{\partial}{\partial r}(rH_\theta) - \frac{\partial H_r}{\partial\theta}\right)\hat{e}_\varphi$$
$$= \frac{1}{j\omega\varepsilon}\left[\frac{1}{rsin\theta}\left(\frac{\partial}{\partial\theta}\left(H_\varphi sin\theta\right)\right)\hat{e}_r + \frac{1}{r}\left(-\frac{\partial}{\partial r}\left(rH_\varphi\right)\right)\hat{e}_\theta\right]$$
$$= \frac{1}{j\omega\varepsilon}\left[\frac{1}{rsin\theta}\left(j\frac{2kI_0 l}{4\pi r}e^{-jkr}sin\theta cos\theta\left(1 + \frac{1}{jkr}\right)\right)\hat{e}_r\right.$$
$$\left. - \frac{1}{r}\left(j\frac{(-jk)kI_0 l}{4\pi}e^{-jkr}sin\theta\left(1 + \frac{1}{jkr}\right) + j\frac{kI_0 l}{4\pi r}e^{-jkr}sin\theta\left(\frac{-1}{jkr^2}\right)\right)\hat{e}_\theta\right]$$

$$\boxed{\begin{gathered} E_r = \mu\frac{I_0 lcos\theta}{2\pi r^2}e^{-jkr}\left(1 + \frac{1}{jkr}\right) \\ E_\theta = j\mu\frac{kI_0 lsin\theta}{4\pi r}e^{-jkr}\left(1 + \frac{1}{jkr} - \frac{1}{(kr)^2}\right) \\ E_\varphi = 0 \end{gathered}}$$

These fields components are supposed to be valid in all regions (near-field, far-field, etc.) However, it's not the case for real-world antenna. Here comes the beauty of fields expressions for infinitesimal dipole: it can play as a building block of wire-type antenna and simplify the analysis.

1. Radial Distance

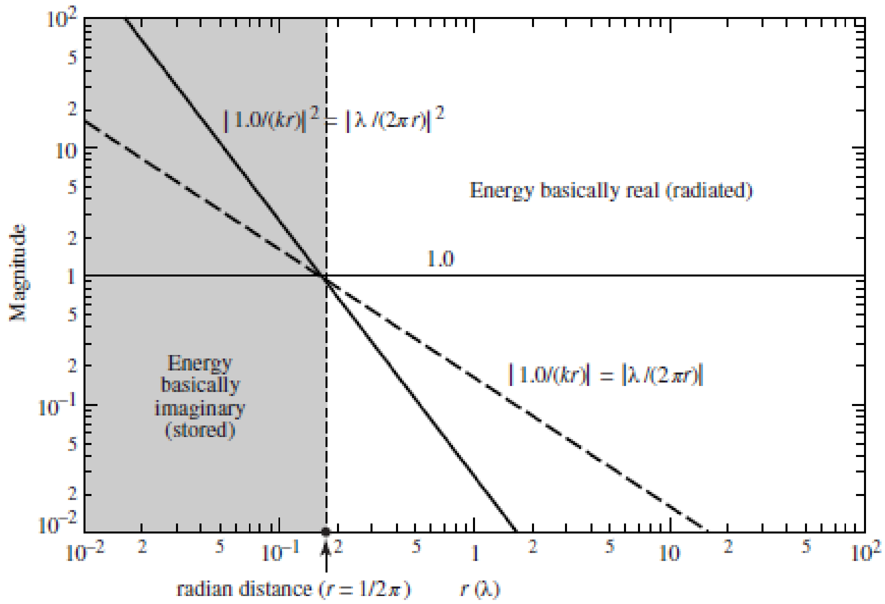


Figure 4.2 Magnitude variation, as a function of the radial distance, of the field terms radiated by an infinitesimal dipole.

As shown in the figure, power density *at the radian distance* is called ***the radian sphere***, which defines the transition between near-field (reactive power density is greater than radiated power) and far-field (radiated power density is greater than reactive power density). You may find the radian sphere as a figure of merit for other antennas.

Let's define the three regions in the following way:

(1) Near-field:

$$r < \frac{\lambda}{2\pi}$$

$$kr < 1$$

(2) Intermediate-field:

$$r > \frac{\lambda}{2\pi}$$

$$kr > 1$$

(3) Far-field:

$$r \gg \frac{\lambda}{2\pi}$$

$$kr \gg 1$$

In far-field, energy is basically real. If one investigates fields in this region:

$$E_r = \mu \frac{I_0 l cos\theta}{2\pi r^2} e^{-jkr} \left(1 + \frac{1}{jkr}\right)$$

$$E_\theta = j\mu \frac{kI_0 l sin\theta}{4\pi r} e^{-jkr} \left(1 + \frac{1}{jkr} - \frac{1}{(kr)^2}\right)$$

Note that $\left(1 + \frac{1}{jkr} - \frac{1}{(kr)^2}\right)$ in E_θ component:

as $kr \gg 1$, the following relation holds:

$$1 > \frac{1}{jkr} > \frac{1}{(kr)^2}$$

2. Far-field Region

The figure of merit for far-field region occurs as the factor kr approaches to infinite (or far greater than unity).

The condition for far-field region? $$kr \gg 1$$

One can approximate the field in the far-field region of infinitesimal dipole:

Far-field region for infinitesimal dipole

$$E_r \approx 0$$
$$E_\theta = j\mu \frac{kI_0 l sin\theta}{4\pi r} e^{-jkr}$$
$$E_\varphi = 0 = H_r = H_\theta$$
$$H_\varphi = j \frac{kI_0 l}{4\pi r} e^{-jkr} sin\theta$$

Note that the reason why E_r component is approximated as zero is because $1/_{r^2}$ converges to zero at the rate faster than that of $1/_{r}$.

In conclusion, for far-field, the electric field and magnetic field are transverse which leads to the fact that the wave propagates in space in far-field region as TEM mode. In addition, ***all antennas will exhibit this feature in far-field region***.

<<I might write about this point later though.>>

ii. Power Density and Radiation Resistance

Radiation resistance is an equivalent circuit model for accounting for the radiation mechanism. To find the radiation resistance, one must calculate the power density. The power density can be derived from the electric and magnetic field of dipole.

To achieve the power density expression, one might want to consider the Poynting theorem to connect the field and the power density. Poynting theorem is like work-energy conservation theorem in classical Mechanics. It states the conservation of energy for the electromagnetics. [4] By virtue of this, one can design antenna with directed gain in certain direction with angles such as HPBW and etc.

Dipole is a good starting point to understand how antennas behave. For example, dipole may exhibit radiation pattern in 3-D as the follow:

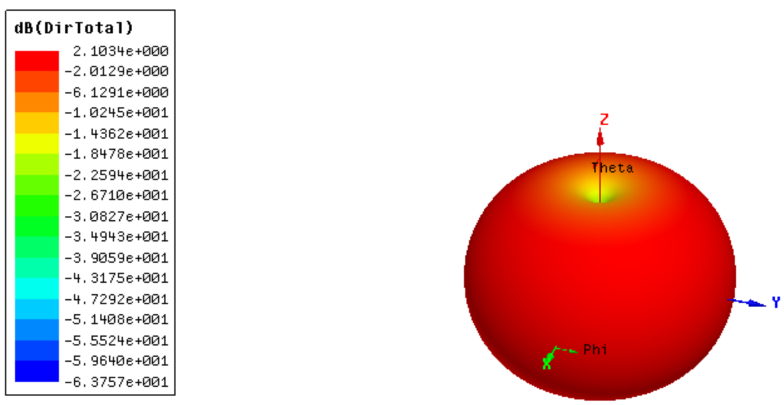


Figure. Dipole Antenna Radiation Pattern (Directivity & Gain)

By changing some design parameters (geometric shape or electric length, etc.), one might achieve something like

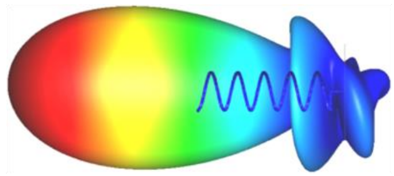


Figure. Helical Antenna Radiation Pattern

It might be confusing why all of sudden a helical antenna pops out to be compared with the radiation pattern of dipole. The point is here, that we can think of the radiation pattern as a balloon that we can "squeeze" in the middle of balloon and note how the shape would change. Other than the middle part (i.e. upper and lower parts of balloon) would gain more volume to compensate the reduction of that in the middle. The Poynting theorem is important in this sense.

Back to our discussion, the Poynting vector can be found by:

$$\vec{W} = \frac{1}{2}\left[\vec{E} \times \vec{H}^*\right]$$

Keep in mind that all these quantities are in Phasor.

For an infinitesimal dipole, recall that

$E_r = \mu \frac{I_0 l cos\theta}{2\pi r^2} e^{-jkr}\left(1 + \frac{1}{jkr}\right)$ $E_\theta = j\mu \frac{kI_0 l sin\theta}{4\pi r} e^{-jkr}\left(1 + \frac{1}{jkr} - \frac{1}{(kr)^2}\right)$ $E_\varphi = 0$	$H_\varphi = j\frac{kI_0 l}{4\pi r} e^{-jkr} sin\theta \left(1 + \frac{1}{jkr}\right)$ $H_r = H_\theta = 0$

Hence, we can simplify the Poynting vector further:

$$\vec{W} = \frac{1}{2}\left[(E_r\hat{e}_r + E_\theta\hat{e}_\theta) \times \left(H_\varphi^*\hat{e}_\varphi\right)\right]$$

Applying the right-hand rule,

$$= \frac{1}{2}\left[-E_r H_\varphi^* \hat{e}_\theta + E_\theta H_\varphi^* \hat{e}_r\right]$$

After some math,

$$W_r = \frac{1}{2}E_\theta H_\varphi^* = \frac{\eta}{8}\left|\frac{I_0 l}{\lambda}\right|^2 \frac{\sin^2\theta}{r^2}\left(1 - j\frac{1}{(kr)^3}\right)$$

$$W_\theta = -\frac{1}{2}E_r H_\varphi^* = j\eta\frac{k|I_0 l|^2 \cos\theta\sin\theta}{16\pi^2 r^3}\left(1 + \frac{1}{(kr)^2}\right)$$

To find the complex power from the Poynting vector, we need surface integral as implied on their units. It would be easier if you could think of the surface area of a sphere for this (Gaussian) integral. Therefore, we can investigate the power radiated by the antenna as the independent variable remains in the radial component. (That is, the magnitude of Power will be different on the radial direction but the same on each surface.) <<Shouldn't be the concept of equi-potential surface similar to this?>>

$$P = \oiint \vec{W}\cdot d\vec{s} = \int_0^{2\pi}\int_0^{\pi}(W_r\hat{e}_r + W_\theta\hat{e}_\theta)\cdot(d\vec{s})$$

For this surface integral, the radial component will not contribute. Hence,

$$\begin{aligned} d\vec{s} &= d\vec{s}_1 = V du^2 du^3 \vec{a}_1 \\ &= d\vec{s}_r = \left(\vec{e}_r\cdot\left(e_\theta \times \vec{e}_\varphi\right)\right) d\theta d\varphi \vec{e}_r \\ &= r^2 \sin\theta \, d\theta d\varphi \vec{e}_r \end{aligned}$$

One can try the following relation to find the factor:

$$V = \sqrt{\mathcal{G}} = \left|g_{ij}\right|$$

which would yield the same that we just found. $\left|g_{ij}\right|$ is the determinant of tensor. See the document named "Tensor".

In addition, the surface integral

$$\int_0^{2\pi}\int_0^{\pi} ds_r = \int_0^{2\pi}\int_0^{\pi} r^2\sin\theta\, d\theta d\varphi = 4\pi r^2$$

which is the surface area of a sphere formula.

$$P = \int_0^{2\pi}\int_0^{\pi}(W_r\hat{e}_r + W_\theta\hat{e}_\theta)\cdot(r^2\sin\theta\, d\theta d\varphi\vec{e}_r)$$

$$= \int_0^{2\pi}\int_0^{\pi}\left(\frac{\eta}{8}\left|\frac{I_0 l}{\lambda}\right|^2\frac{\sin^2\theta}{r^2}\left(1 - j\frac{1}{(kr)^3}\right)\right) r^2\sin\theta\, d\theta d\varphi$$

$$= \frac{\eta}{8}\left|\frac{I_0 l}{\lambda}\right|^2\left(1 - j\frac{1}{(kr)^3}\right)(2\pi)\frac{4}{3}$$

$$P = \frac{\pi\eta}{3}\left|\frac{I_0 l}{\lambda}\right|^2\left(1 - j\frac{1}{(kr)^3}\right)$$

Note that

$$P = \frac{\pi\eta}{3}\left|\frac{I_0 l}{\lambda}\right|^2 \left(1 - j\frac{1}{(kr)^3}\right)$$

does ***not*** represent the total complex power radiated by the antenna for the transverse component W_θ (which is purely imaginary) didn't contribute to the integral.

Let's look at the power closely. We see the real part (radiated) and the imaginary part (reactive).

$$P = P_{rad} + jP_{reactive}$$

$$P = P_{rad} + j2\omega(\widetilde{W}_m - \widetilde{W}_e)$$

$$P = Power$$
$$P_{rad} = Radiated\ real\ part\ power$$
$$\widetilde{W}_m = Time - average\ Magnetic\ Energy\ Density$$
$$\widetilde{W}_e = Time - average\ Electric\ Energy\ Density$$
$$2\omega\left(\widetilde{W}_m - \widetilde{W}_e\right) = Time - average\ Reactive\ Power$$

Note that all these quantities are in *Radial direction*.

Recall the basic circuit theory, the real power can be modeled from an equivalent resistance. Hence, radiation resistance can be found from the circuit view:

$$P_{rad} = \frac{\pi\eta}{3}\left|\frac{I_0 l}{\lambda}\right|^2 = \frac{1}{2}|I_0|^2 R_r$$

Thus, the radiation resistance can be shown as:

$$R_r = \pi\eta\frac{2}{3}\left|\frac{l}{\lambda}\right|^2 \approx 80\pi^2\left|\frac{l}{\lambda}\right|^2$$

where the intrinsic impedance in isotropic material $\eta \approx \eta_0 \approx 120\pi$ [5]

<<One can examine radiation resistance at the location of the maximum current occurs. Correct me if I'm wrong.>>

For an infinitesimal dipole antenna, the overall length of dipole should be usually

$$l \leq \frac{\lambda}{50}$$

Investigate the reactive part of an infinitesimal dipole. Is it capacitive or inductive? This point can be easily shown from examining the input impedance of infinitesimal dipole with the transmission line view:

$$Z_{in}(\beta l) = \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}$$

Since the infinitesimal dipole antenna is treated as open-circuited transmission line (the end points of dipole) and the difference between the input and the one end-point of dipole is $l/_2$:

$$Z_{in} = -jZ_0 \cot(\beta l)$$

Hence, it is always capacitive for $l \ll \lambda$.

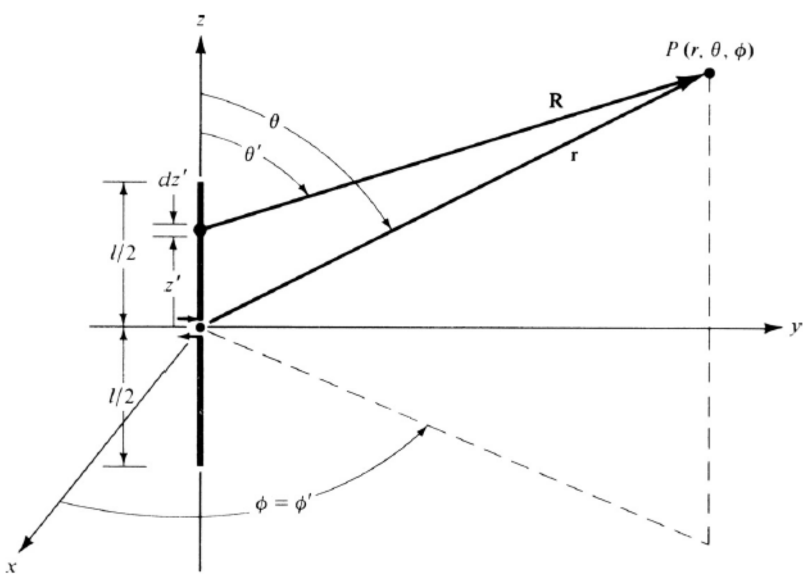
b.Small Dipole

For the analysis of an infinitesimal dipole antenna, we assumed the current distribution is uniform and constant which is not realizable in real life.

In later of this document, the Method of Moment will be shown to approximate the current distribution of dipole antenna where we use thin wire antenna model to explain the current distribution. The Moment method will first calculate the impedance matrix of the wire antenna and examine the current distribution for voltage excitation. I'm excited to show the beauty of this method in this document.

i. Vector Potential of Small Dipole

For now, again, we assume the current distribution which is given



(a) Dipole and geometry

Figure. Small Dipole Model

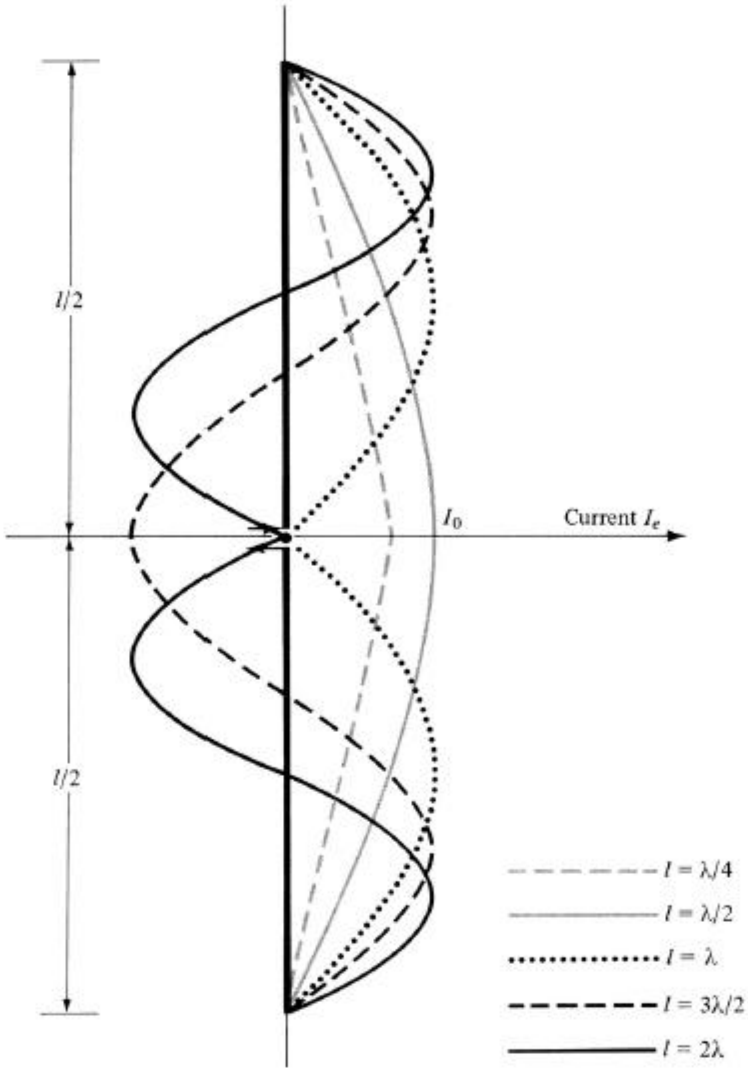


Figure 4.8 Current distributions along the length of a linear wire antenna.

At the two ends, the current goes zero for all length of linear wire antenna type.

$$\vec{I}_e(x', y', z') = \begin{cases} \hat{e}_z I_0 \left(1 - \frac{2}{l} z'\right) \text{ for } 0 \le z \le {}^{l}/_{2} \\ \hat{e}_z I_0 \left(1 + \frac{2}{l} z'\right) \text{ for } - {}^{l}/_{2} \le z \le 0 \end{cases}$$

where $I_0 = \text{constant}$

Then, we can find the vector potential due to this current distribution:

$$\vec{A}(x, y, z) = \frac{\mu}{4\pi} \int_C \vec{I}_e(x', y', z') \frac{e^{-jkR}}{R} dl'$$

$$= \hat{e}_z \frac{\mu}{4\pi} \int_0^{l/2} I_0 \left(1 - \frac{2}{l} z'\right) \frac{e^{-jkr}}{r} dz' + \frac{\mu}{4\pi} \int_{-l/2}^{0} I_0 \left(1 + \frac{2}{l} z'\right) \frac{e^{-jkr}}{r} dz'$$

$$= \hat{e}_z \frac{\mu}{4\pi} I_0 \left(-\frac{l}{4}\right) \frac{e^{-jkr}}{r} - \frac{\mu}{4\pi} I_0 \left(+\frac{l}{4}\right) \frac{e^{-jkr}}{r}$$

$$\vec{A}(z) = \hat{e}_z \frac{\mu}{8\pi} I_0 l \frac{e^{-jkr}}{r}$$

Now, we can find the magnetic field and electric field from the vector potential. Before, jumping on the fields calculation, let's compare it to the infinitesimal case.

Infinitesimal Dipole	Small Dipole
$\vec{A}(z) = \hat{e}_z \frac{\mu I_0 l e^{-jkr}}{4\pi r}$	$\vec{A}(z) = \hat{e}_z \frac{1}{2}\left[\frac{\mu I_0 l e^{-jkr}}{4\pi r}\right]$

The vector potential of a small dipole antenna is half of that of the infinitesimal dipole! In addition, if the current distribution were triangular distribution, then the potential function of triangular is half of that of the uniform distribution. Hence, we can sense that the distribution of current also affect the magnitude of the radiated field!

Note that the vector potential function for small dipole becomes more accurate as $kr \to \infty$.

$kr \to \infty$ simply implies that one would like to consider the "far-field" region of the field of antenna.

ii. Power and Radiation Resistance

Intuitively, since small dipole exhibits half of the field of infinitesimal dipole (with other parameters remain the same), it's not difficult to predict that the power would be one-fourth of the that of infinitesimal dipole. If one can't convince oneself, then try it and see the result.

Recall from the infinitesimal dipole, the radiated power can be found as

$$P_{rad}^{small} = \frac{1}{2} |I_0|^2 R_r = \frac{1}{4} \left(P_{rad}^{inf}\right)$$

where

P_{rad}^{small} = radiated power for small dipole

P_{rad}^{inf} = radiated power for infinitesimal dipole

Conveniently, $P_{rad} = P_{rad}^{small}$ for this sub-section since we are dealing with small dipole.

Then,

$$R_r = \frac{2P_{rad}}{|I_0|^2} \approx 20\pi^2 \left|\frac{l}{\lambda}\right|^2$$

Obviously, the radiation resistance of small dipole is one-fourth of that of infinitesimal dipole.

<<See a document named “Laurent Series”.>>

c. Input Resistance

Input impedance is one of the important parameters in antenna analysis if one wants to integrate antenna with other circuit elements.

For the sake of reducing complexity, if we consider an antenna to be lossless, then we may drop the reactance part of input impedance. For lossless antenna, input impedance is therefore reduced to input resistance.

From transmission line view, a lossless circuit may refer to those with load impedance is zero, open-circuited. Luckily, we already verified that the input impedance at the two ends of dipole to be capacitive.

What does it mean by lossless transmission line in terms of power transmission? Shouldn’t the power at the input be the same at which the current maximum occurs? Mathematically, this leads to:

$$P_{in} = P_{rad}$$

Please recall that P_{rad} refers to the power at which the maximum current occurs. Thus,

$$\frac{1}{2}|I_{in}|^2 R_{in} = \frac{1}{2}|I_0|^2 R_r$$

$$\boxed{R_{in} = \frac{|I_0|^2}{|I_{in}|^2} R_r}$$

However, what if current at input terminal equals the maximum current? This won’t happen because current at input terminal does not equal to the maximum current.

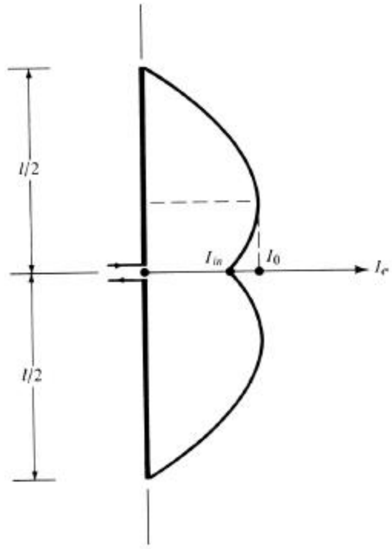


Figure 4.10 Current distribution of a linear wire antenna when current maximum does not occur at the input terminals.

If input current can be expressed as sinusoidal function,

$$I_{in} = I_0 \sin\left(\frac{kl}{2}\right)$$

Then, we can re-write the input resistance equation in the following way:

$$R_{in} = \frac{|I_0|^2}{\left|I_0 \sin\left(\frac{kl}{2}\right)\right|^2} R_r = \frac{R_r}{\sin^2\left(\frac{kl}{2}\right)}$$

However, the radiation resistance we just investigated was derived from an assumption of the ideal current distribution, which does not occur in practice. (radiation resistance at input terminals happens to be infinite!) For more rigorous analysis, one must account for finite radius of wire and finite gap at the feed point of wire antenna.

$$R = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}$$

d.Region Separation

i. Far-field Approximation

Before proceeding finite length dipole analysis, we need to develop a general expression for the distance between ***any point on the source*** and coordinate system that helps mathematical operations. Since we are mostly interested in the fields in the far-field region, let's develop a general expression for

$$R = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}$$

Luckily, we have tools to reduce the equation further: very thin dipole and symmetrically position along the origin. Hence,

$$x' = y' = 0$$

$$R = \sqrt{(x)^2 + (y)^2 + (z - z')^2} = \sqrt{r^2 - 2rz'cos\theta + z'^2}$$

where

$$r^2 = x^2 + y^2 + z^2$$

$$z = rcos\theta$$

This is $R(z')$ function. Now, we consider the Taylor series for approximation. To derive such series, we need the derivatives of R function:

$$R'(z') = \frac{1}{2}(r^2 - 2rz'cos\theta + z'^2)^{-\frac{1}{2}}(2z' - 2rcos\theta)$$

$$R''(z') = \frac{-1}{4}(r^2 - 2rz'cos\theta + z'^2)^{-\frac{3}{2}}(2z' - 2rcos\theta)^2 + \frac{1}{2}(r^2 - 2rz'cos\theta + z'^2)^{-\frac{1}{2}}(2)$$

$$\vdots$$

along this, we may find:

$$R(0) = r$$

$$R'(0) = -\frac{rcos\theta}{r}$$

$$R''(0) = -\frac{r^2\cos^2\theta}{r^2 r} + \frac{1}{r} = \frac{\sin^2\theta}{r}$$

Hence,

$$R(z') = \sum_{n=0}^{\infty} \frac{R^{(n)}(0)}{n!}(z')^n$$

$$= r - z'cos\theta + \frac{\sin^2\theta}{2r}(z')^2 + \cdots$$

$$\boxed{R(z') = r - z'cos\theta + \frac{\sin^2\theta}{2r}(z')^2 + \cdots}$$

For far-field analysis, one may use the approximation

$$R \approx r - z'cos\theta$$

for the fact that $1/_{r^n}$ factors wouldn't contribute to R too much as $r \to \infty$.

e. Finite Length Dipole Antenna

In this section, let's develop some useful formulae for finite size dipole antenna and move onto half-wavelength dipole antenna case. First, let's find out the current distribution for finite length dipole. From a good approximation, it is given as:

$$\vec{I}_e(x' = 0, y' = 0, z') = \begin{cases} \hat{e}_z I_0 \sin\left[k\left(\frac{l}{2} - z'\right)\right], & 0 \le z' \le \frac{l}{2} \\ \hat{e}_z I_0 \sin\left[k\left(\frac{l}{2} + z'\right)\right], & -\frac{l}{2} \le z' \le 0 \end{cases}$$

The characteristics of this current is that it is center-fed and vanishes at the two end points $\left(\pm {l}/_{2}\right)$.

What we are going to do next is that we take the infinitesimal dipole as a building block of this finite length dipole. To do so, consider the differential quantity of infinitesimal dipole:

$$dE_\theta \approx\ j\mu\frac{kI_e(x', y', z')e^{-jkR}}{4\pi R}sin\theta dz'$$

$$dH_\varphi \approx j\frac{kI_e(x', y', z')e^{-jkR}}{4\pi R}sin\theta dz'$$

Note that in differential quantity, we need the following substitutions

$$l \to dz'$$

Also,

$$r \to R$$

since finite antenna has *finite dimension* in the Cartesian coordinate. Recall that

$$R = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}$$

Accounting for the far-field approximation:

$$R \approx r - z'cos\theta$$

$$dE_\theta \approx j\mu\frac{kI_e(x', y', z')e^{-jk(r - z'cos\theta)}}{4\pi(r - z'cos\theta)}sin\theta dz'$$

Clean up a little and further simplify as

$$dE_\theta \approx j\mu \frac{kI_e(x',y',z')e^{-jkr}}{4\pi r} sin\theta e^{jkz\prime cos\theta} dz'$$

Note that $4\pi(r - z'cos\theta)$ gets further simplified by $4\pi r$

To find the total E_θ component, we need to perform integral over dE_θ along the total length of the wire.

$$E_\theta = \int_{-l/2}^{l/2} dE_\theta = \frac{j\mu ksin\theta e^{-jkr}}{4\pi r}\left[\int_{-l/2}^{l/2} I_e(x',y',z')e^{jkz'cos\theta}dz'\right]$$

> Note that all the parameters that do not vary along with the spatial variable are called the
> ***"element factor"***
> whereas those that do vary along with the spatial variable are called the
> ***"space factor"***
> which leads to an important conclusion:
>
> ***"the total field of an antenna is the product of the element factor and space factor"***
>
> which is also known as ***"the pattern multiplication"***.
> Further note that the idea of this continuous pattern multiplication is analogous to that of discrete pattern multiplication, or "***Arrays of antenna***".

f. Half-wavelength Dipole Antenna

i. Radiation Resistance

One noticeable characteristic of half-wavelength dipole is that its input resistance is around 73-ohm. In turn, one can easily match it to 50-ohm or 75-ohm coaxial cable in market.

> If you plug in all the given variables to calculate the input resistance from the equation we found from the ideal current distribution:
>
> $$R_{in} \approx 49.35\ ohm$$

ii. Fields

III. Design

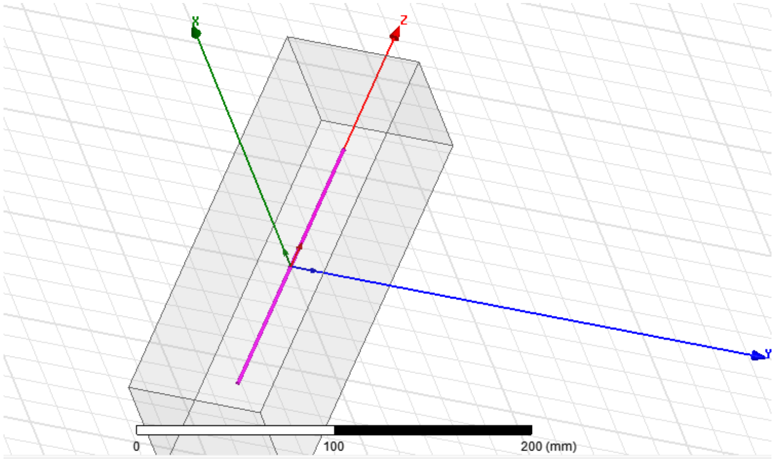


Figure. Dipole Antenna in the air box

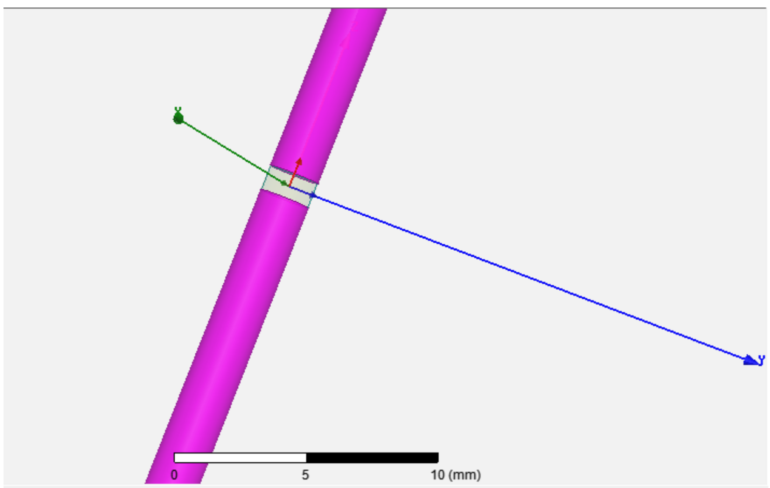


Figure. YZ cut plane view

a. HFSS Simulation Results

i. Impedance

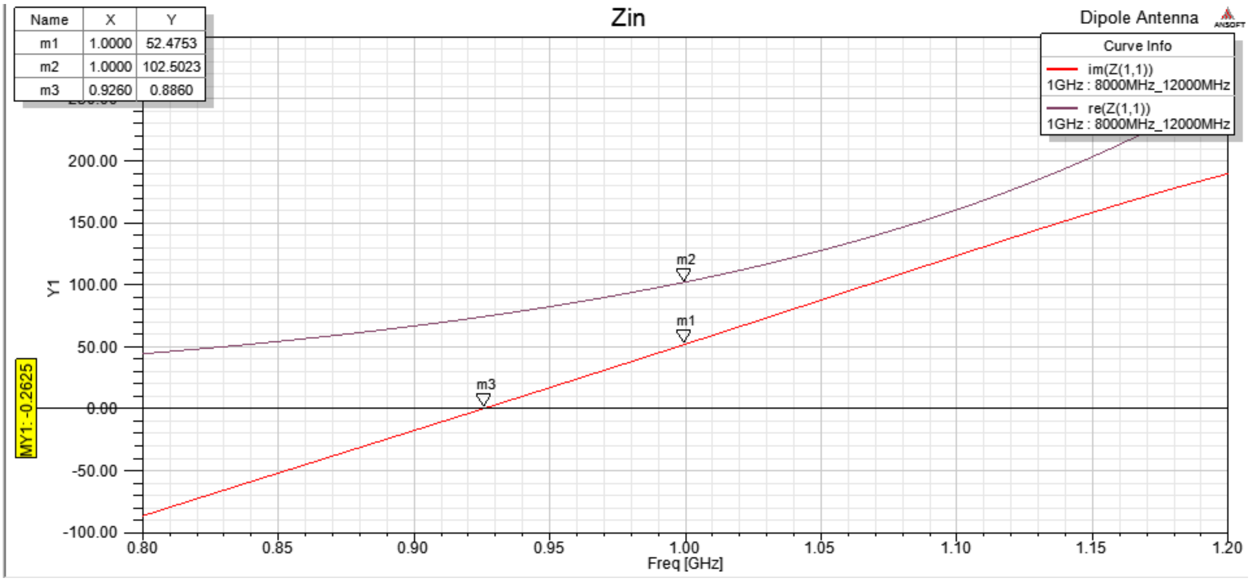


Figure. Input Impedance in ohm

The marker m_3 suggests that the operating point occurs around (Maximum power transfer occurs when the reactance part of input impedance is zero. Without the impedance matching network inserted, the intrinsic operating point should be around this frequency.)

$$f_0 = 926MHz$$

Where the original design attempt was at $1GHz$.

The reactance shown at $f = 1GHz$:

$$X_{in} = 52.4753\ ohm$$

ii. S parameters

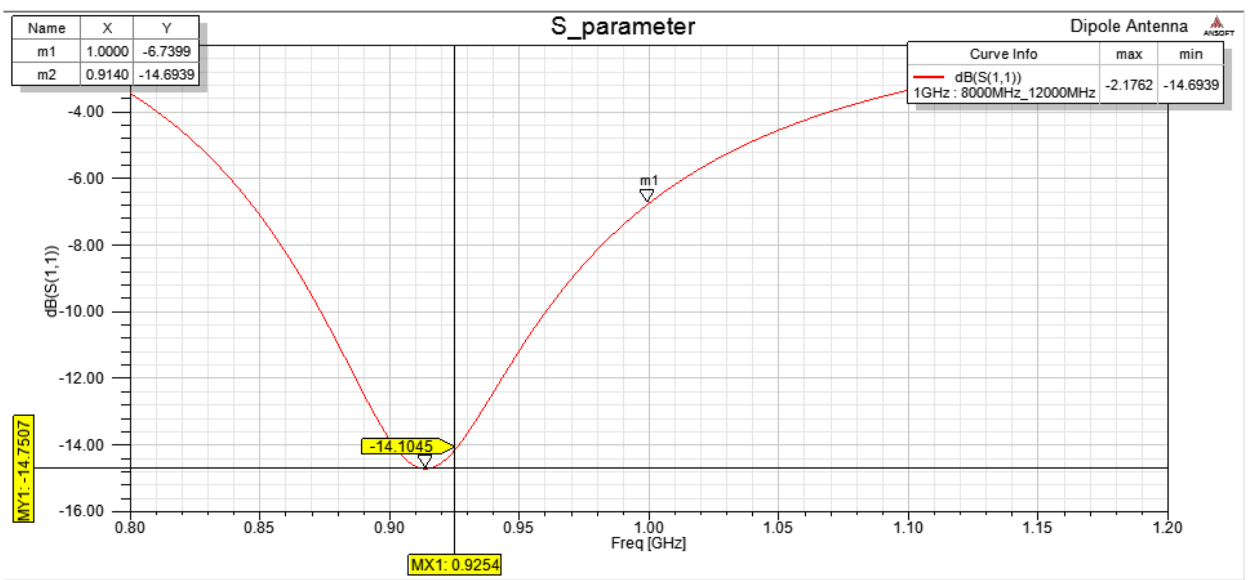


Figure. S parameter in dB

Around the operating frequency,

$$S_{11} = -14.1045\ dB = 0.197$$

Which suggests that the feedline is well matched.

iii. VSWR

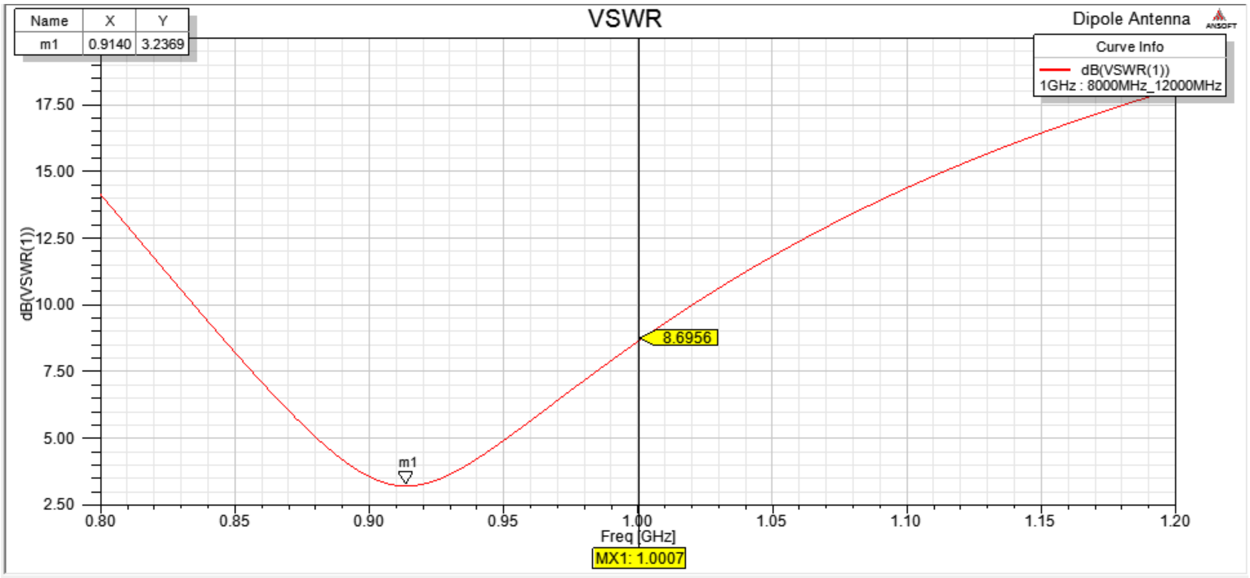


Figure. VSWR

iv. Radiation Pattern

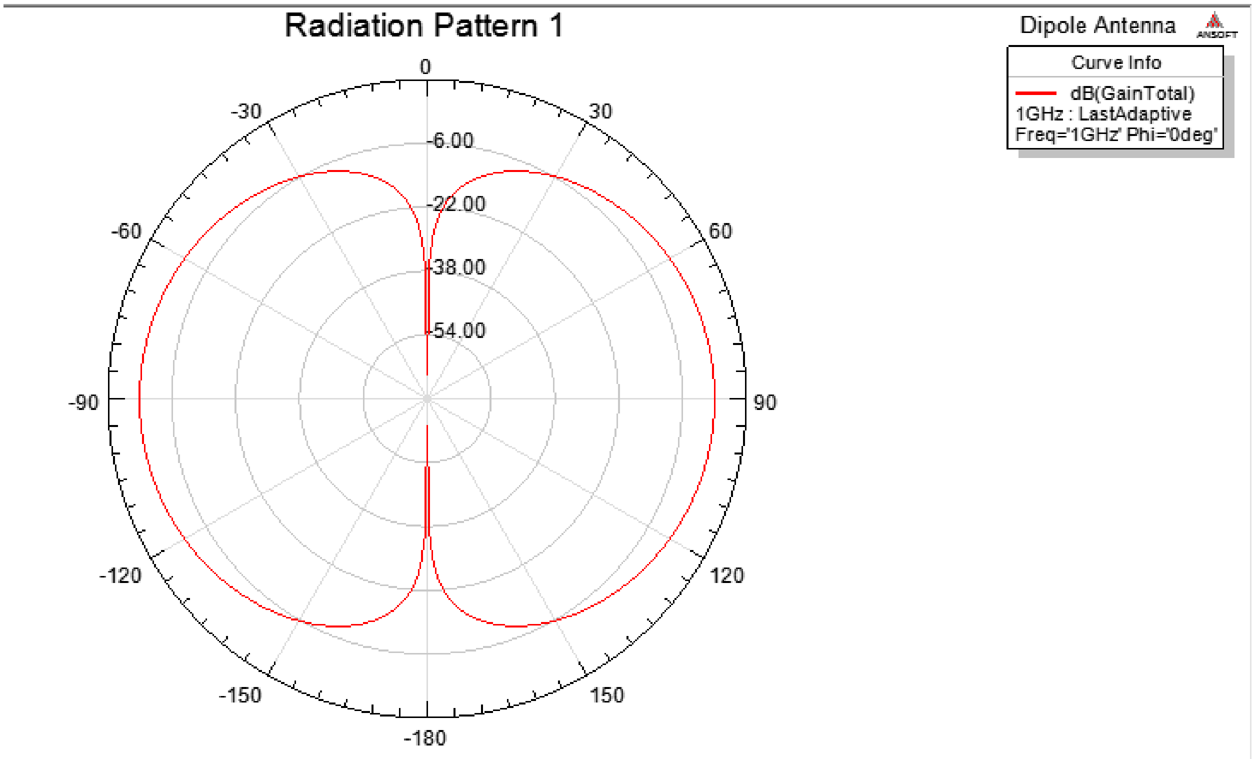


Figure. Radiation pattern of Dipole Antenna with P Plane

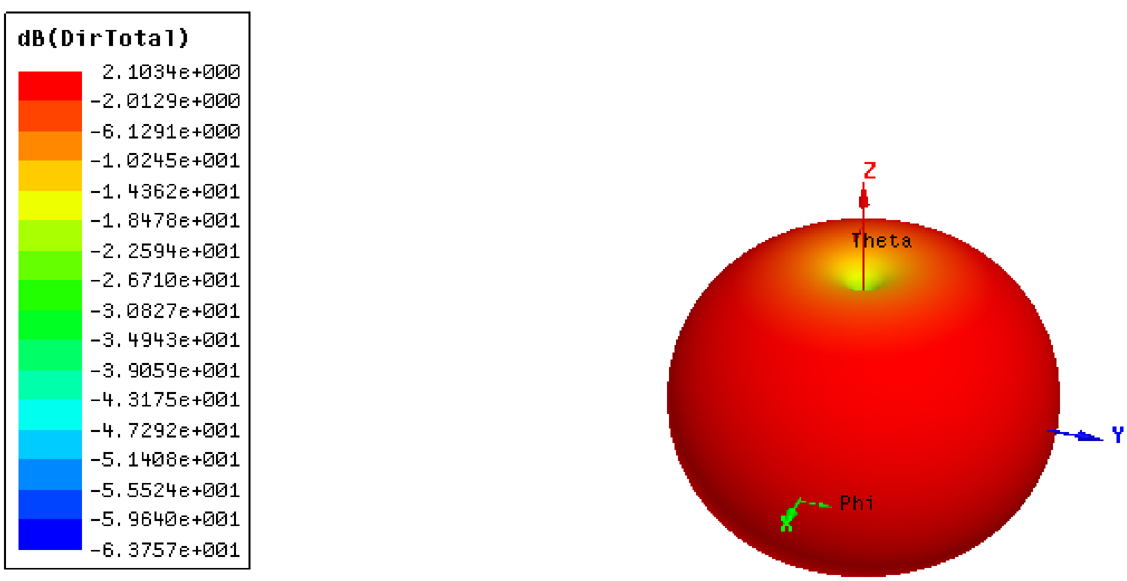


Figure. Directivity of Dipole Antenna

IV. References

[1] Antenna Theory and Analysis, Balanis

[2] “Ansys How to Videos” – Youtube Channel

[3] https://en.wikipedia.org/wiki/Del_in_cylindrical_and_spherical_coordinates

[4] https://en.wikipedia.org/wiki/Poynting%27s_theorem

[5] https://en.wikipedia.org/wiki/Wave_impedance