Number of Cases

I. Introduction

'Order' is the key to differentiate between permutation and combination. Permutation does care about the order of placing items while combination does not. For both,

$$n \geq r$$

is always satisfied where n is the total number of items and r is the particular number that we are interested in.

II. Permutation

$$_nP_r = n(n-1)(n-2)(n-3)\cdots(n-r+1) = \frac{n!}{(n-r)!} = (n)_r$$

a. Example

Let's say we have 10 different colored balls in a bag and we want to take 4 balls out of it and place them on a table. At the first place, 10 possible balls can be placed and 9 balls at the second and so forth. Hence, the final answer is $10 \times 9 \times 8 \times 7 = 5040$. There are total 5040 possible scenarios we can expect. By examining combination case, the difference between permutation and combination becomes clearer.

III. Combination

$$_nC_r = \frac{n!}{r!\,(n-r)!} = \binom{n}{r}$$

a. Example

I have a bag that contains 5 different balls labeled with number from 1 to 5. I want to extract two numbers out of the bag and what is the number of total possible combinations? Answer is 10.

$$(1,2), (1,3), (1,4), (1,5)$$
$$(2,3), (2,4), (2,5)$$
$$(3,4), (3,5)$$
$$(4,5)$$

We could apply the above formula to find out the answer.

$$\binom{5}{2} = \frac{5!}{2!\,(3!)} = 10$$

IV. Permutation with repetition

From the same example that we find in III, if we place the extracted ball back in the bag every time we take out a ball, we can find

$$_n\Pi_r = n^r$$

V. References

[1] https://en.wikipedia.org/wiki/Permutation
[2] https://en.wikipedia.org/wiki/Combination