Mixer

I. Introduction

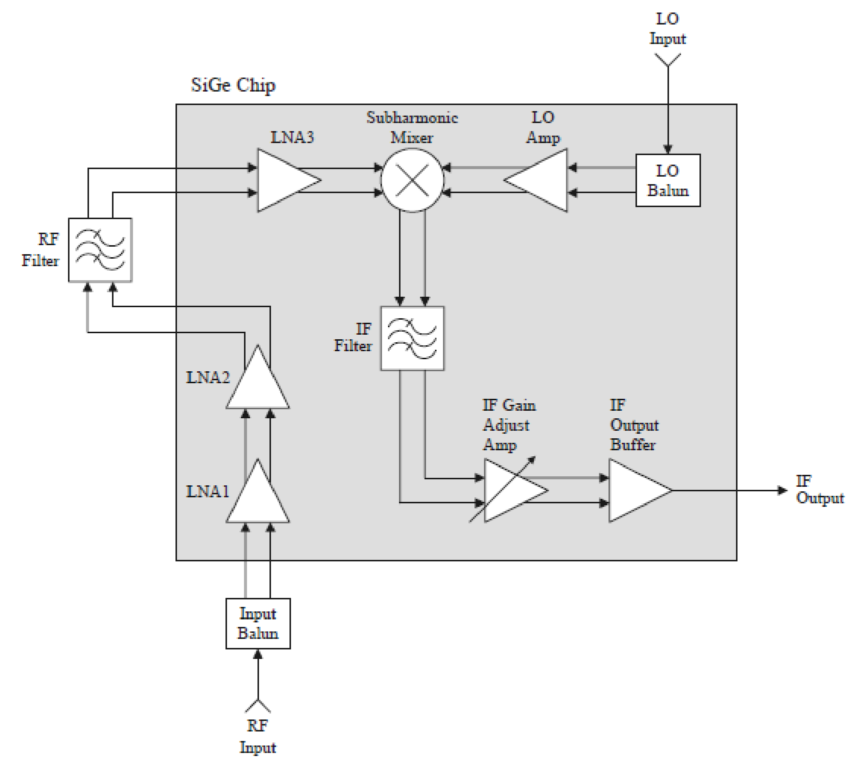
The idea of a mixer is similar to that of a frequency multiplier except that mixers utilize the fundamental frequency whereas frequency multipliers don't/can't. Candidates are:

Single-Ended Diode Mixer
Single-Ended FET Mixer
Image Reject Mixer
Differential FET Mixer
Gilbert Cell Mixer

TABLE 13.1 Mixer Characteristics

Mixer Type	Number of Diodes	RF Input Match	RF-LO Isolation	Conversion Loss	Third-Order Intercept
Single ended	1	Poor	Fair	Good	Fair
Balanced (90°)	2	Good	Poor	Good	Fair
Balanced (180°)	2	Fair	Excellent	Good	Fair
Double balanced	4	Poor	Excellent	Excellent	Excellent
Image reject	2 or 4	Good	Good	Good	Good

"Image reject" mixer shows overall good performances. "Gilbert Cell" mixers are used frequently as well (CMOS RFIC application). Now, let's look at real circuit diagram for down converter.



(b)

Figure. Millimeter wave Down Converter

Note that, in practice, there are many amplifiers. It might have been good if the design itself contains a good conversion gain but it is not in reality. In future example, IF signals become weak, which suggests that the advent of amplifier is necessary.

II. Characteristics

a. Transmitter side (Up Converter)

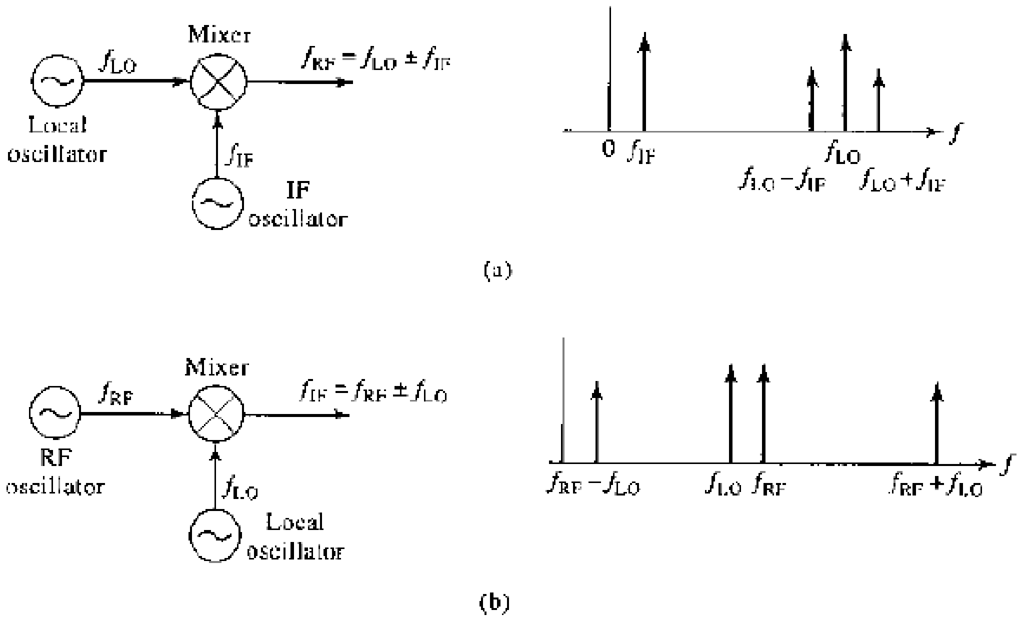


FIGURE 13.24 Frequency conversion using a mixer. (a) Up-conversion. (b) Down-conversion.

Note that in the up-conversion figure, the IF and LO are far from each other. In this case, the two RF frequencies would occur near the LO.

The Local Oscillator can be represented as:

$$v_{LO}(t) = \cos(2\pi f_{LO} t)$$

Usually the up-converter circuit is for transmitter system. Your signal may be lower than the operating frequency of the antenna (transmitter); therefore, you consider a mixer circuit to generate the operating frequency (RF frequency) to send out signal to the receiver.

Sidebands?

In the up-conversion figure,

$$f_{RF} = f_{LO} \pm f_{IF}$$

These are called "Sidebands of the carrier frequency f_{LO} signal". In this case, $f_{LO} + f_{IF}$ is called the upper sideband whereas $f_{LO} - f_{IF}$ is called the lower sideband. If a mixer contains both sidebands, then it is considered as Double Sideband (DSB). Single Sideband (SSB) can be produced by either (1) filtering or (2) using a single sideband mixer.

b.Receiver Side (Down Converter)

The RF signal from the transmitter (v_{RF}) will be collected at the receiver. Since the operating frequencies of many processor are lower than that of RF signal, we need to down-convert the RF signal to IF signal so that they can be read at the processor.

$$f_{IF} = f_{RF} \pm f_{LO}$$

However, since the LO and RF frequencies are close together in down converter, we are particularly interested in the difference between RF and LO, which is much smaller than RF. The LPF circuit will filter out the $f_{RF} + f_{LO}$ signal.

What is going on with $f_{RF} - f_{LO}$?

$$f_{RF} = f_{LO} + f_{IF}$$

Let's insert this to the equation:

$$f_{IF} = f_{RF} - f_{LO}$$
$$= f_{LO} + f_{IF} - f_{LO}$$

III. Design

a. Single Ended Diode Mixer

b. Single Ended FET Mixer

Before jumping on to the main topic, single ended FET mixer requires external matching network. Since there are total 3 ports (RF, LO, and IF) in it, requiring 3 different matching networks to match to, usually, 50-ohm impedance (in practice).

The basic idea here is that we are going to use nonlinearity properties of FET, which in turn leads to mixing characteristic. The transconductance of FET is the main key for mixing. We are considering Common-Source stage but it's not for amplifier circuit. Then, what makes it behave like mixer? Let's look the change in transconductance of FET:

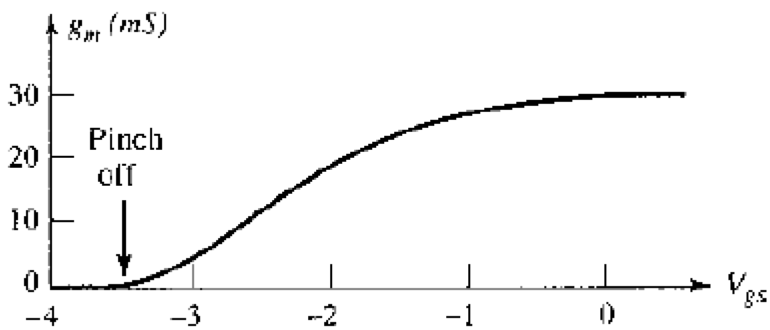


FIGURE 13.26 Variation of FET transconductance versus gate-to-source voltage.

What features are different from FET ***Amplifier***?

Recall that the Gate voltage is chosen close to near zero (slightly positive). This is because we want maximum transconductance, shown in the Figure 13.26. In addition, non-zero transconductance will ensure "linearity", meaning if input is zero, then the output is also zero.

Now, let's consider the zero transconductance (***pinch-off***) FET circuit. This circuit can be realized by considering negative Gate voltage. Zero transconductance means the circuit will have non-linear characteristics, meaning zero input may provide non-zero output. This sounds little bizarre but it is what it is.

The book describes it as "When the gate bias is near the *pinch-off* region, where the transconductance approaches zero, a small positive variation of gate voltage can cause a large change in transconductance, leading to a nonlinear response." [1]

This fact leads us to have LO circuit connected to the Gate of FET.

Let's look at single ended FET mixer circuit diagram.

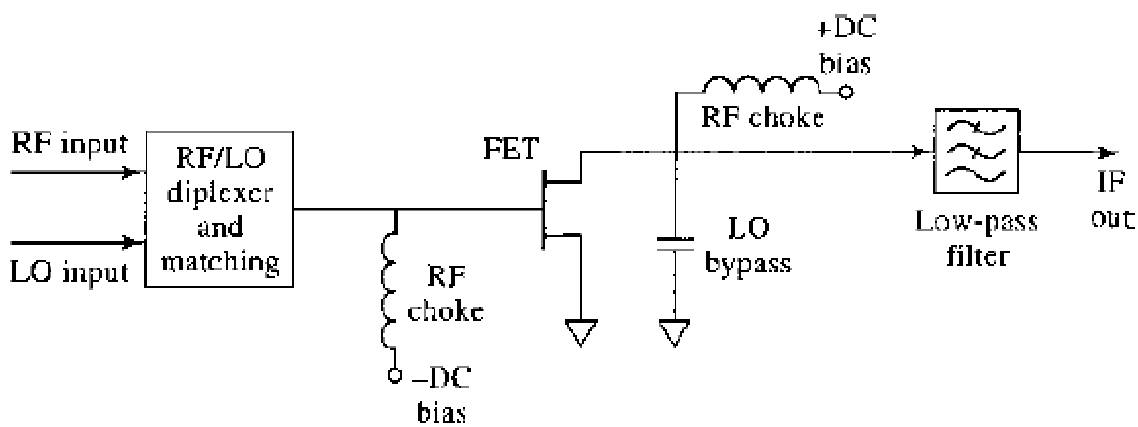


FIGURE 13.27 Circuit for a single-ended FET mixer.

Duplexing Coupler?
This unit will combine the RF and LO components.

Impedance Matching?
It's usually low input impedance of FET. Impedance matching between the inputs (RF and LO inputs) and the Gate of FET is required.

Biasing?
This can be done by considering "***RF choke***". RF choke is for AC block. You may use an analogy of DC blocking capacitor. It allows ***positive bias at Drain and negative bias at Gate***.

LO bypass capacitor?
We need a return path for LO signal, which can be done by using a capacitor. In addition, this will act like a low pass filter for the final IF signal.

Maximum Power Transfer?

Since a mixer consists of other important sub-circuits, we need to concern about the maximum power transfer between ports of each device. The maximum power transfer occurs from the conjugate matching. Note that RF and IF signals need the maximum power transfer whereas LO does not because LO is there for providing "frequency".

$$Z_{RF} = R_{RF} + jX_{RF} = Z_g$$
$$Z_{IF} = R_{IF} + jX_{IF} = Z_L$$
$$Z_{LO} = Z_0$$

The textbook argues that the transconductance can be expressed in terms of harmonics of LO signal since FET is driven by LO: (we can use Fourier series then!)

$$g(t) = g_0 + 2\sum_{n=1}^{\infty} g_n \cos(2\pi n f_{LO} t)$$

Now, we face a problem: we ***can't find the coefficients*** because there's no explicit formula for the transconductance. So, ***we must measure them***. (Typically, 10 mS) In addition, the down-conversion is solely due to the $n = 1$ term of harmonics. <<Why?>>

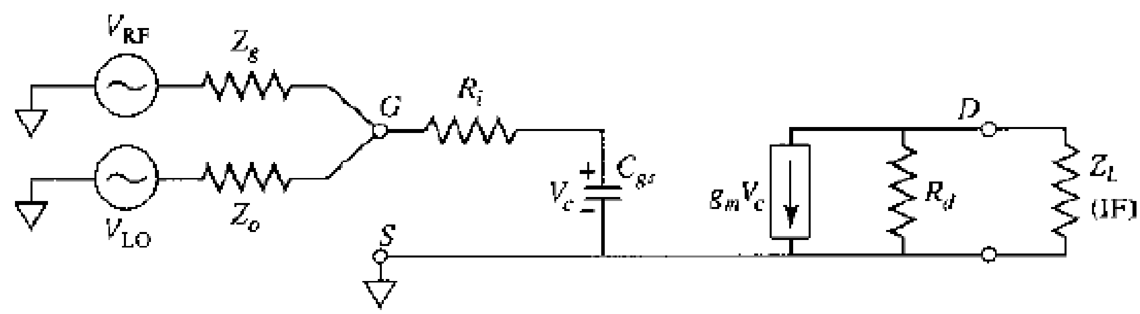


FIGURE 13.28 Equivalent circuit for the FET mixer of Figure 13.27.

The gate source voltage (phasor) results from the voltage division between $Z_g + R_i$ and C_{gs}:

$$V_c^{RF} = \frac{Z_{gs}}{(Z_g + R_i) + Z_{gs}} V_{RF}$$

$$= \frac{V_{RF}}{1 + j\omega_{RF} C_{gs}(R_i + Z_g)}$$

Which can be expressed in time-domain:

$$v_c^{RF}(t) = V_c^{RF} \cos(\omega_{RF} t)$$

Drain current is the product of the transconductance and the gate source voltage:

$$i_D(t) = g(t)v_c^{RF}(t) = \left[g_0 + 2\sum_{n=1}^{\infty} g_n \cos(n\omega_{LO}t)\right] V_c^{RF}\cos(\omega_{RF}t)$$

$$= g_0 V_c^{RF}\cos(\omega_{RF}t) + 2V_c^{RF} g_1 \cos(\omega_{LO}t)\cos(\omega_{RF}t) + 2V_c^{RF} g_2 cos(2\omega_{LO}t)cos(\omega_{RF}t) + \cdots$$

<<I think this is the point where RF and LO combined and split out IF frequency.>>

$$g(t)v_c^{RF}(t)|_{\omega_{IF}} = g_1 V_c^{RF}\cos(\omega_{RF}t - \omega_{LO}t)$$

$$= g_1 V_c^{RF}\cos(\omega_{IF}t)$$

c. Balanced Mixer

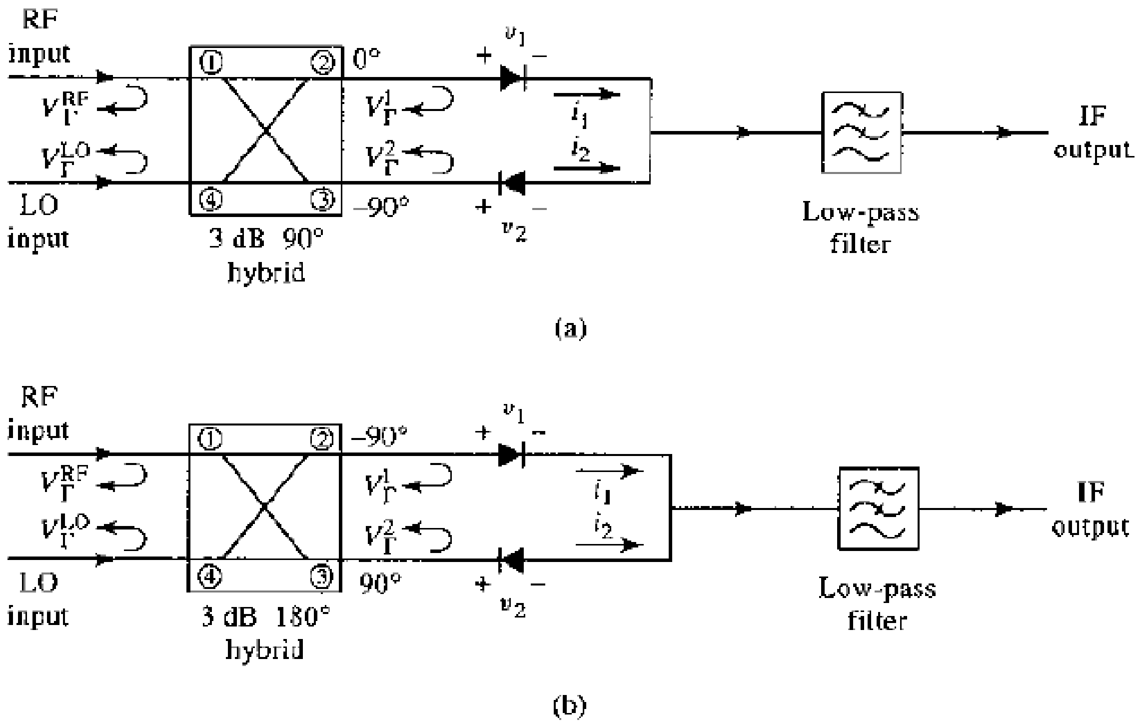


FIGURE 13.29 Balanced mixer circuits. (a) Using a 90° hybrid. (b) Using a 180° hybrid.

These Hybrid couplers are covered in another document. Just look for them. To design hybrid couplers, you might want to know about parameters of microstrip/strip line because they are usually designed on those types. <<I don't know how to design Coupler with transistors, are they any out there? >>

Why Balanced Mixer?
It provides ***better input matching*** and ***better isolation between RF and LO*** signals.

Basic structure of Balanced Mixer?
It consists of two single ended mixers with a hybrid junction.

When we want 90 Hybrid in it?
Ideally, it will provide perfect input match at RF port for broadband.
Like 180 Hybrid, all even-order intermodulation will be rejected.

When we want 180 Hybrid in it?
Ideally, it will provide perfect isolation between RF and LO for broadband.
Like 90 Hybrid, all even-order intermodulation will be rejected.

d.Examples

i. Single Balanced Mixer

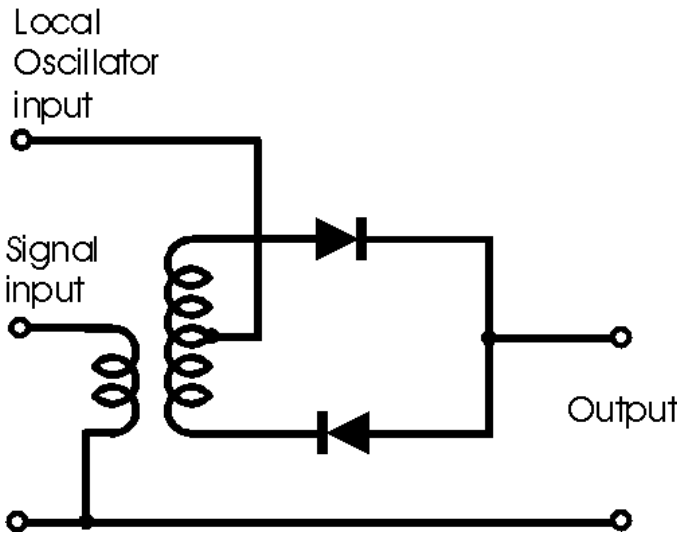


Figure. Single Balanced Mixer [2]

Instead of hybrid coupler, transformers are used to replace it.

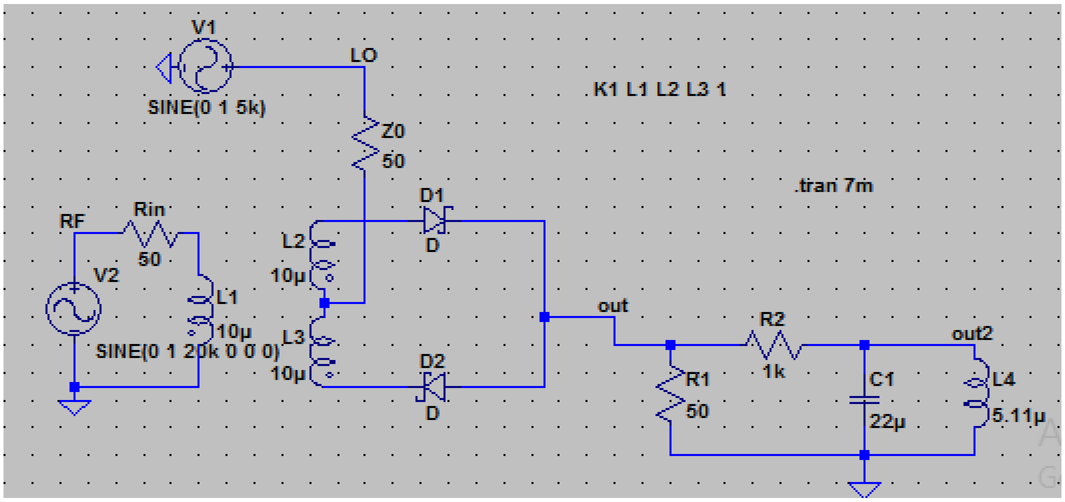


Figure. Single Balanced Diode Mixer

Schottky Didoes were used for the design. Transformer ratio also can be shown in the diagram.

On the right, a resonant circuit that oscillates at $15kHz$. Let's compare the signals at each node.

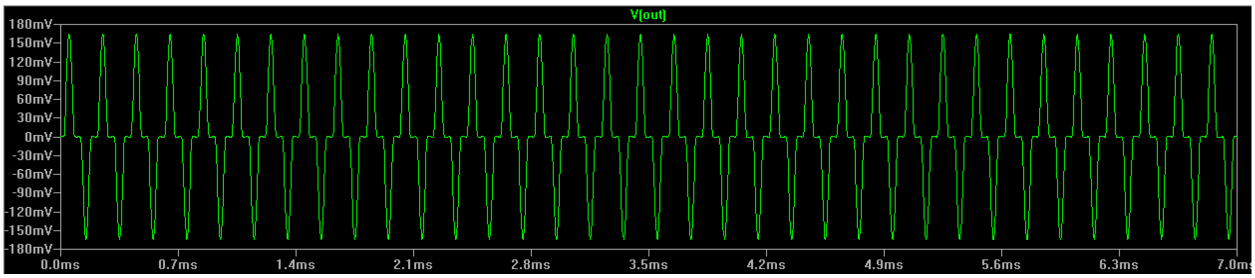


Figure. Transient Response at "out" node

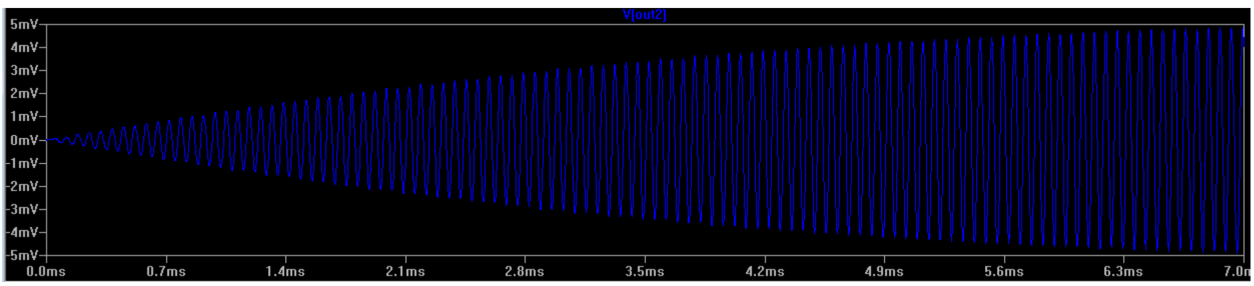


Figure. Transient Response at "out2" node

However, the results would not come out as expected when LO frequency $7kHz$ instead of $5kHz$..I mean the frequency response shows the maximum gain occurs around $13kHz$ but the "out2" signal doesn't look clear as it was in the case $LO = 5kHz\ and\ RF = 20kHz$.

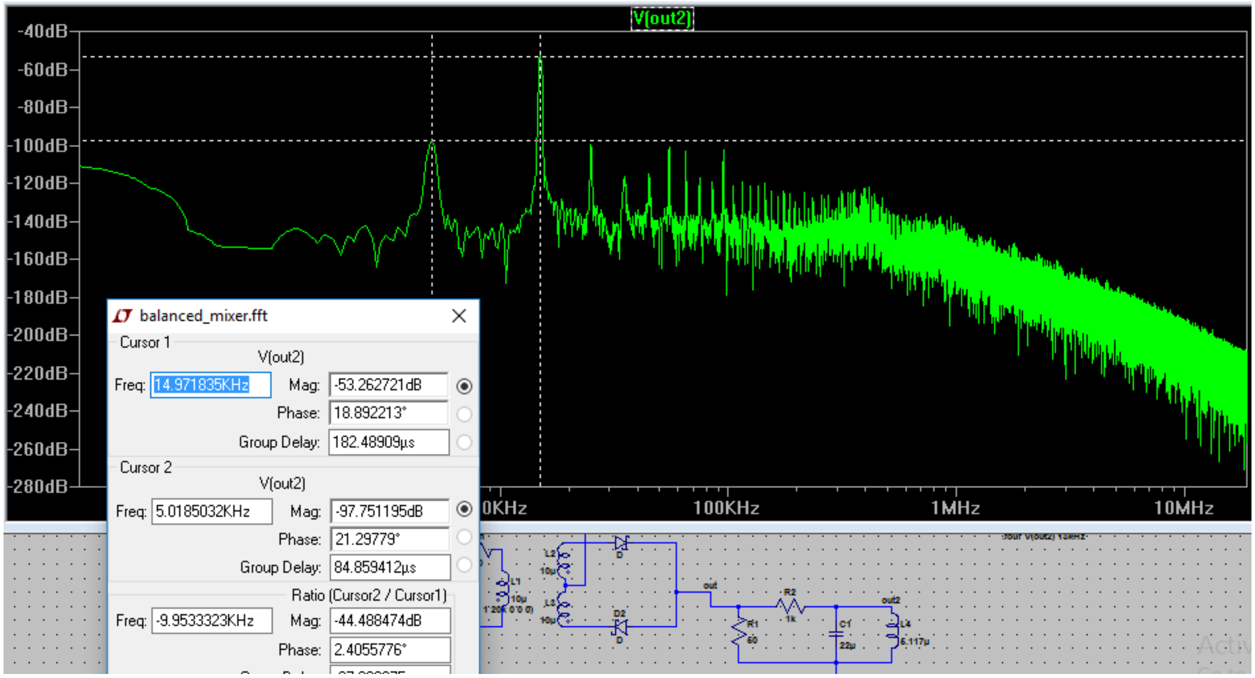


Figure. FFT at node "out2" when $LO = 5kHz\ and\ RF = 20kHz$

The level of signals from highest to lowest:

	Frequency	Amplitude
Strongest	$14.97kHz$	$-53.26dB$
Second strong	$5.02kHz$	$-97.75dB$
Third	$24.98kHz$	$-99.66dB$

Noise figure?

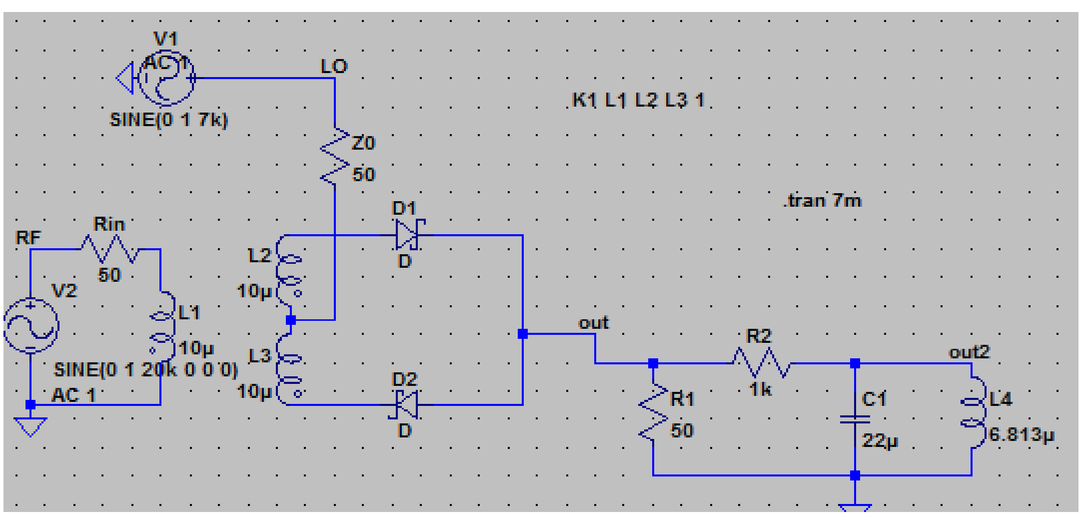


Figure. Single Balanced Diode Mixer when $LO = 7kHz$

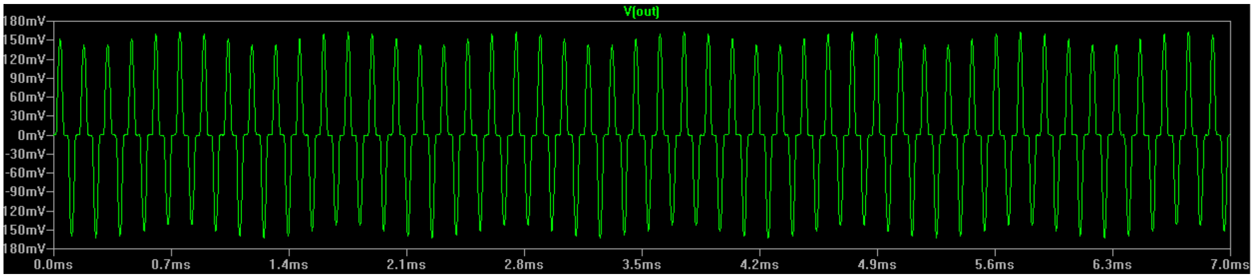


Figure. Transient Response at node “out”

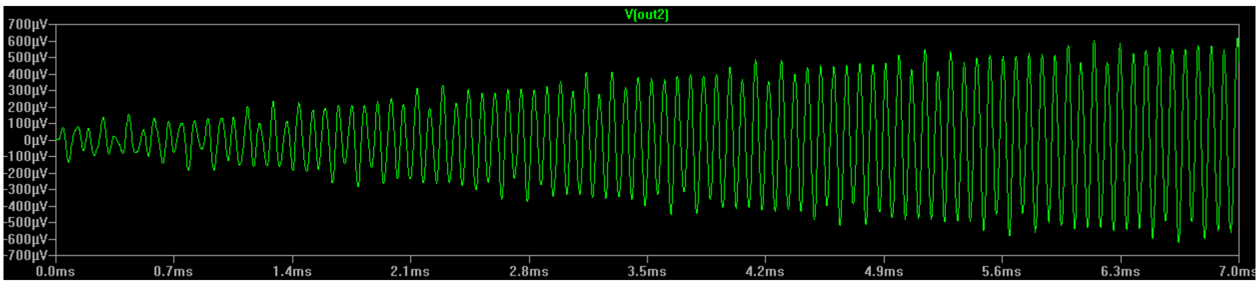


Figure. Transient Response at node “out2”

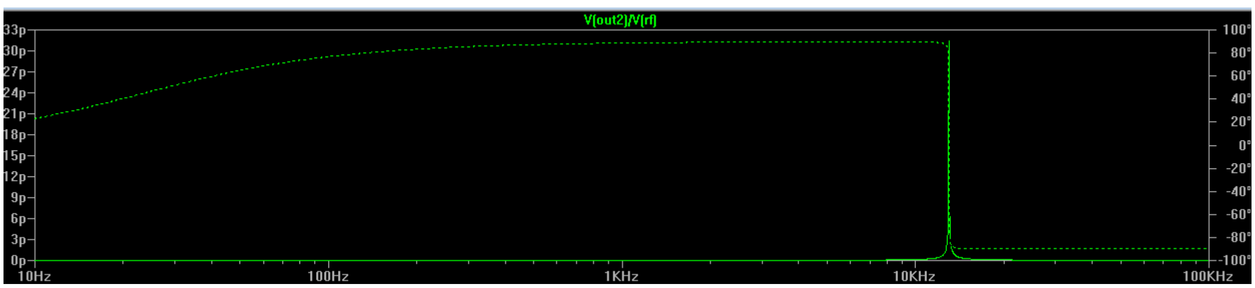


Figure. Frequency Response of $\frac{V_{out2}}{V_{RF}}$

If I’m reading correctly, the gain between RF signal and the final IF signal is 40pico, which is just zero. Fortunately (?), the maximum occurs at $13kHz$.

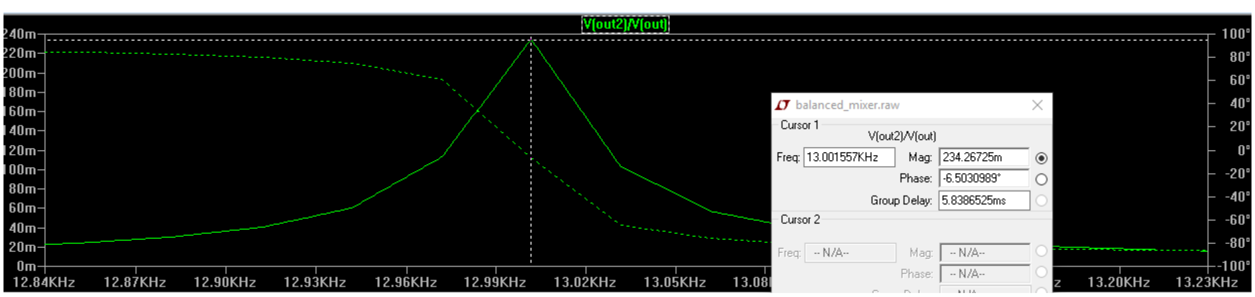


Figure. Frequency Response of $\frac{V_{out2}}{V_{out}}$

This time the gain is around 0.2, resonant circuit also contains huge loss. I’m not sure if this single ended diode mixer would be a good candidate for overall mixer design.

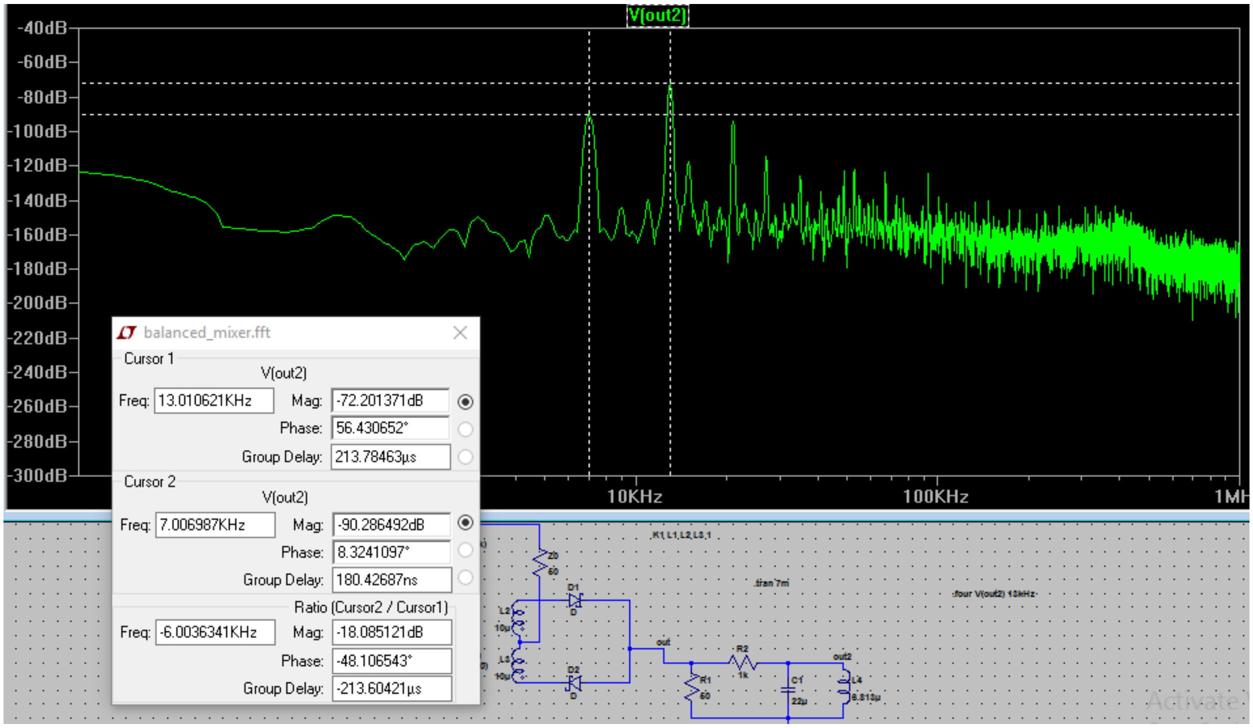


Figure. FFT of at node "out2" when $LO = 7kHz\ and\ RF = 20kHz$

The level of signals from highest to lowest:

	Frequency	Amplitude
Strongest	$12.093kHz$	$-72.86dB$
Second strong	$7.097kHz$	$-92.28dB$
Third	$20.99kHz$	$-96.64dB$
Fourth	$26.91kHz$	$-116.68dB$

To be honest, I have no clue where the $21kHz$ component came from.

In addition, explain about Noise Figure?

ii. Double Balanced Diode Mixer

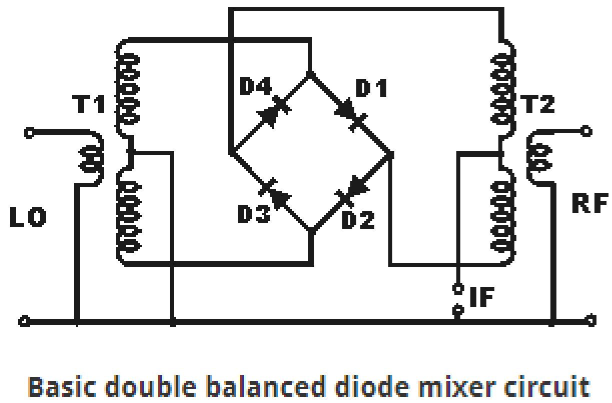


Figure. Transformer contained Mixer [3]

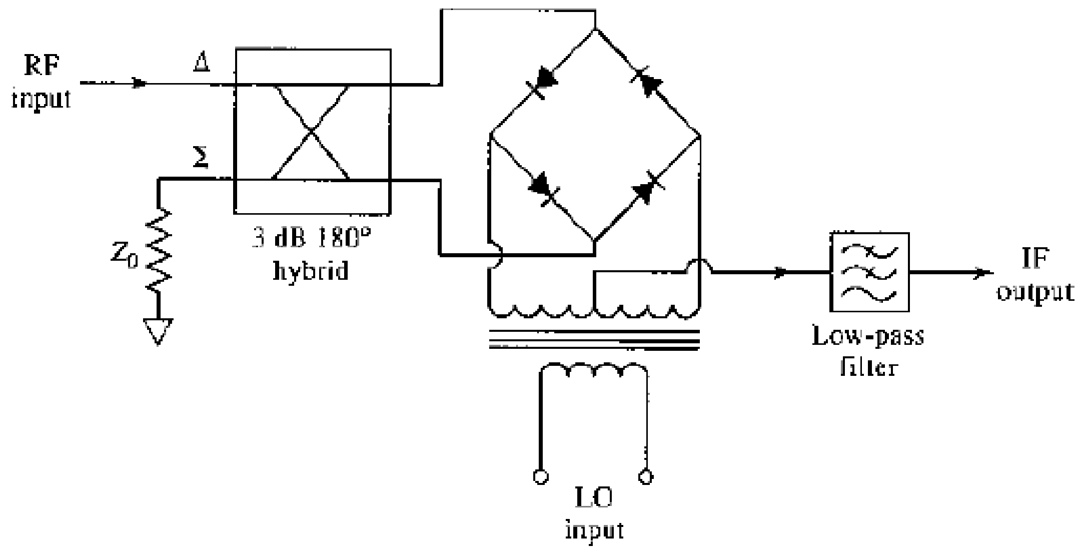


FIGURE 13.34 Double-balanced mixer circuit.

Figure. Hybrid Coupler contained Mixer

e. Image Reject Mixer

For down-converter case, both sideband might be considered as IF signal since they are the same. So, instead of sending both to a single output port, separate them into different output ports. For up-converter case, this technique will lead to ***single sideband modulator***. Designers may use the same circuit for both purposes: up-conversion and down-conversion.

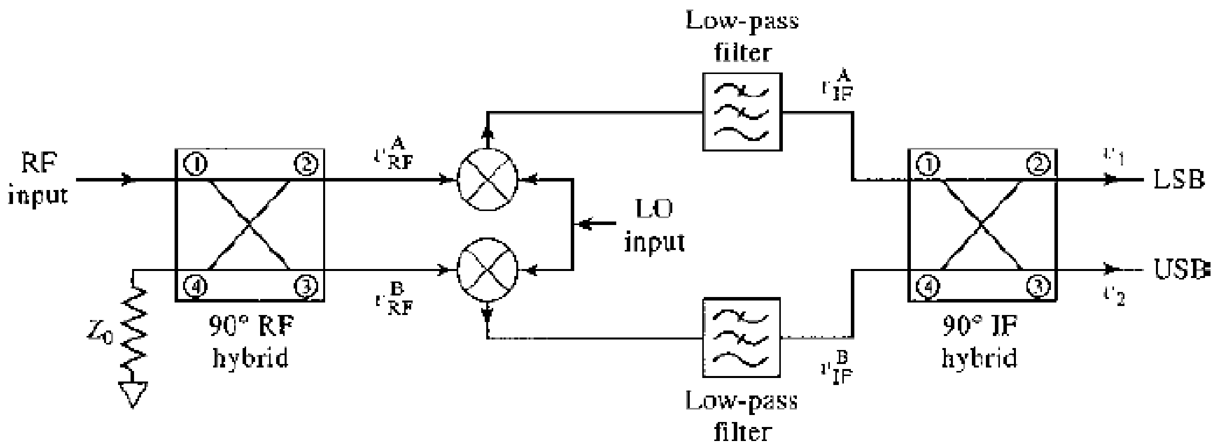


FIGURE 13.31 Circuit for an image reject mixer.

We already know that two RF signals that are

$$\omega_{RF} = \omega_{LO} \pm \omega_{IF}$$

Will down convert to ω_{IF} where the two distinct signals are expressed:

$$LSB = \omega_{LO} - \omega_{IF}$$

$$USB = \omega_{LO} + \omega_{IF}$$

In other mixers, both these two responses were detected at the output terminal. However, "Image Reject" Mixer can separate these two signals and detect each at each output port.

To analyze "Image Reject" mixer circuit,

"Small-Signal Approximation"

Will be considered.

Let's express the RF signal:

$$v_{RF}(t) = V_U \cos(\omega_{LO} + \omega_{IF})t + V_L \cos(\omega_{LO} - \omega_{IF})t$$

Where V_U and V_L represent the amplitudes of upper and lower sidebands, respectively.

This RF signals are going through the hybrid coupler, as shown in the above Figure.13.31. Using the S matrix properties of hybrid coupler (Quadrature Coupler in the figure), we find the voltage at node A (or port 2 of the first hybrid coupler):

$$v_{RF}^A(t) = \frac{V_U}{\sqrt{2}}\cos(\omega_{LO} + \omega_{IF} - 90°)t + \frac{V_L}{\sqrt{2}}\cos(\omega_{LO} - \omega_{IF} - 90°)\,t$$

$$= \frac{V_U}{\sqrt{2}}\sin(\omega_{LO} + \omega_{IF})\,t + \frac{V_L}{\sqrt{2}}\sin(\omega_{LO} - \omega_{IF})\,t$$

Likewise, the node B (or port 3 of the first hybrid coupler):

$$v_{RF}^{B}(t) = \frac{V_U}{\sqrt{2}}\cos(\omega_{LO} + \omega_{IF} - 180°)t + \frac{V_L}{\sqrt{2}}\cos(\omega_{LO} - \omega_{IF} - 180°)\,t$$

$$= -\frac{V_U}{\sqrt{2}}\cos(\omega_{LO} + \omega_{IF})\,t - \frac{V_L}{\sqrt{2}}\cos(\omega_{LO} - \omega_{IF})\,t$$

Recall the LO signal and diode response with the quadratic term only (for approximation):

$$v_{LO}(t) = V_{LO}\cos(\omega_{LO}t)$$

$$I(V) = \frac{v^2}{2}G_d' = \frac{Kv^2}{2}$$

Where

$$G_d' = \left.\frac{d^2 I}{dV^2}\right]_{V_0} = \left.\frac{dG_d}{dV}\right]_{V_0}$$

Its dimension would be

$$\left[\frac{1}{\Omega \mathrm{V}}\right]$$

(**??**)The product of $v_{RF}^{A}(t)$ and $v_{LO}(t)$: <<Considering the dimension of the product result, we shouldn't be doing this… unless the final result would be current like in the diode approximation >>

$$v_{RF}^{A}(t)v_{LO}(t) = \frac{V_{LO}}{\sqrt{2}}[V_U\sin(\omega_{LO} + \omega_{IF})t + V_L\sin(\omega_{LO} - \omega_{IF})t]\cos(\omega_{LO}t)$$

Alternate form of $\sin(A + B)\cos(A)$?

$$\sin(A + B)\cos(A) =$$

$$-\frac{1}{2}\sin^2(A)\sin(B) + \frac{1}{2}\cos^2(A)\sin(B) + \sin(A)\cos(A)\cos(B) + \frac{\sin(B)}{2}$$

<<Wolfram Alpha Result>>

Now, after Low-pass filter unit, the survived term will be:

$$\frac{\sin(B)}{2}$$

Likewise, for

$$\sin(A - B)\cos(A) =$$

$$\frac{1}{2}\sin^2(A)\sin(B) - \frac{1}{2}\cos^2(A)\sin(B) + \sin(A)\cos(A)\cos(B) - \frac{\sin(B)}{2}$$

After the LPF, the survived term:

$$-\frac{\sin(B)}{2}$$

(???)

The textbook solution at IF inputs:

$$v_{IF}^{A}(t) = \frac{KV_{LO}}{2\sqrt{2}}(V_U - V_L)\sin(\omega_{IF}t)$$

$$v_{IF}^{B}(t) = -\frac{KV_{LO}}{2\sqrt{2}}(V_U + V_L)\cos(\omega_{IF}t)$$

To predict the responses after the second hybrid coupler, one might want to work in the phasor domain to avoid heavy calculations. In the phasor domain, these voltages are:

$$V_{IF}^{A} = \frac{-jKV_{LO}(V_U - V_L)}{2\sqrt{2}}$$

$$V_{IF}^{B} = \frac{-KV_{LO}(V_U + V_L)}{2\sqrt{2}}$$

After the second hybrid coupler,

$$V_1 = V_{IF}^{A}\left(-\frac{j}{\sqrt{2}}\right) - \frac{V_{IF}^{B}}{\sqrt{2}}$$

$$V_2 = -\frac{V_{IF}^{A}}{\sqrt{2}} - V_{IF}^{B}\left(-\frac{j}{\sqrt{2}}\right)$$

<<to be fair, I was not able to follow the textbook solution. The only way I could understand was using the S matrix for the Quadrature coupler.>>

From the S matrix of Quadrature hybrid coupler, it is given as:

$$[S] = -\frac{1}{\sqrt{2}}\begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}$$

We could re-write the voltage responses as the follow:

$$\begin{bmatrix} V_{1A} \\ V_{2A} \\ V_{3A} \\ V_{4A} \end{bmatrix} = -\frac{1}{\sqrt{2}}\begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}\begin{bmatrix} V_{1B} \\ V_{2B} \\ V_{3B} \\ V_{4B} \end{bmatrix}$$

$$= -\frac{1}{\sqrt{2}}\begin{bmatrix} jV_{2B} + V_{3B} \\ jV_{1B} + V_{4B} \\ V_{1B} + jV_{4B} \\ V_{2B} + jV_{3B} \end{bmatrix}$$

Where 'A' refers to "After" and 'B' refers to "Before". For example, in this case, look at Figure13.31,

$$V_{1B} = V_{IF}^{A}$$
$$V_{4B} = V_{IF}^{B}$$
$$V_{2A} = V_1$$
$$V_{3A} = V_2$$

Hence, we ended up with the following relations:

$$V_{2A} = -\frac{1}{\sqrt{2}}(jV_{1B} + V_{4B})$$
$$V_{3A} == -\frac{1}{\sqrt{2}}(V_{1B} + jV_{4B})$$

Or equivalently,

$$V_1 = -\frac{1}{\sqrt{2}}(jV_{IF}^{A} + V_{IF}^{B})$$
$$V_2 = -\frac{1}{\sqrt{2}}(V_{IF}^{A} + jV_{IF}^{B})$$

Therefore,

$$V_1 = \frac{KV_{LO}V_L}{2}$$

$$V_2 = -\frac{jKV_{LO}V_U}{2}$$

Taking the inverse phasor form:

$$v_1(t) = \frac{KV_{LO}V_L}{2}\cos(\omega_{IF}t)$$

$$v_2(t) = \frac{KV_{LO}V_U}{2}\sin(\omega_{IF}t)$$

Important to note!

The two outputs $v_1(t)$ and $v_2(t)$ are having exactly **90°** ***phase shift between the two sidebands***.

Practical difficulties?

(1) Fabrication of hybrid coupler at relatively low IF frequency
(2) Losses are greater than simpler mixers. These losses cause greater noise figure. Hence, harder to detect signal.

IV. References

[1] Microwave Engineering, David Pozar, 4th Edition

[2] http://www.radio-electronics.com/info/circuits/diode_single_balanced_mixer/diode_single_balanced_mixer.php

[3] http://www.radio-electronics.com/info/rf-technology-design/mixers/double-balanced-mixer-tutorial.php