Ring

I. Introduction

Performing addition and multiplication among integers seems trivial. I mean if I wanted to add 1 to 2, I would end up with a number, 3. Note that the two numbers, 1 and 2, were integers and the result, which is 3, was also an integer. Applying multiplication case would also result in the same manner.

The most familiar example of a ring is the set of all integers, $\mathbb{Z}$:

$$\mathbb{Z} = \{\cdots, -2, -1, 0, 1, 2, \cdots\}$$

Integers are closed under addition and multiplication.

II. Definition

A Ring is a set $\mathcal{R}$ equipped with two binary operations, addition and multiplication, satisfying the three ring axioms:

(1) The set is an abelian group under addition.
    a. There's the additive identity, zero.
(2) The set is a monoid under multiplication.
    a. There's the multiplicative identity in the set. (such that for every a in $\mathcal{R}$, $a \cdot 1 = 1 \cdot a = a$)
(3) Multiplication is *distributive* with respect to addition.

A rng (or, pseudo-ring) is a structure that satisfies all the axioms of ring except that there is no multiplicative identity.

III. Properties

Ring addition is commutative.

Ring multiplication doesn't have to be commutative; however, if it is commutative, then the set is called "commutative ring". (The set of all integers is commutative ring.)

IV. Homomorphism

A homomorphism from a ring $(R, +, \cdot)$ to another ring $(S, \ddagger, *)$ is a function f from R to S that preserves the ring operations. The following should hold for all $a \; and \; b$:

(1) $f(a + b) = f(a) \ddagger f(b)$
(2) $f(a \cdot b) = f(a) * f(b)$
(3) $f(1_R) = 1_S$

a. Isomorphism

A homomorphism is said to be an isomorphism if there exists inverse homomorphism for f. Any ***bijective*** ring homomorphism is a ring isomorphism.

The term "bijective" already ensures that there is an inverse homomorphism because bijection guarantees ***one-to-one correspondence***.

V. References

[1] https://en.wikipedia.org/wiki/Ring_(mathematics)

[2] https://en.wikipedia.org/wiki/Bijection