4/11/2020

$$Z = \frac{1}{2} e^{j\pi/4} \quad \text{order 1}$$

$$Z = \frac{1}{2}\left\{\cos\frac{\pi}{4} + j\sin\pi/4\right\}$$

$$Z = \frac{1}{2}\left(\frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}}\right)$$

$$Z = \frac{1}{2\sqrt{2}} + j\frac{1}{2\sqrt{2}}$$

$$H(Z) = \frac{\left(Z - \frac{1}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}i\right)\left(z - \frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}i\right)}{2}$$

→ We have always seen s with real coefficients or conjugate pair

Q Why Real ?

$$H(Z) = 1 + 3z^{-1}$$

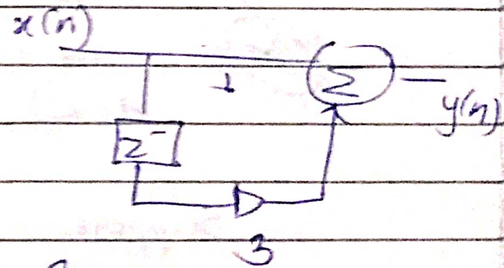
1, 3 are multipliers / real coefficients / h(n)

$$\frac{Y(z)}{X(z)} = 1 + 3z^{-1}$$

$$Y(z) = X(z)\left(1 + 3z^{-1}\right)$$

conjugate symmetry

$$y(n) = x(n) + 3x(n-1)$$



Thus coefficients must be real

$$\therefore \quad H(z) = \frac{\left(z - \frac{1}{2}e^{j\pi/4}\right)\left(z - \frac{1}{2}e^{-j\pi/4}\right)}{z^2}$$

$$-\frac{1}{2}e^{j\pi/4}\cdot z$$

$$-\frac{1}{2}e^{j\pi/4} z$$

$$+\frac{1}{2}e^{j\pi/4}\cdot\frac{1}{2}e^{-j\pi/4}$$

$$-z\frac{1}{2}\left(e^{j\pi/4} + e^{-j\pi/4}\right)$$

$$H(z) = \frac{z^2 - 2\times\frac{1}{2}\cos\frac{\pi}{4}z + \frac{1}{4}}{z^2}$$

$$H(z) = \frac{z^2 - 1/\sqrt{2}\,z + 1/4}{z^2}$$

$z^2 - 2e^{j\pi/4} \cdot 2e^{-j\pi/4}$

$\left(z - \frac{1}{2}e^{j\pi/4}\right) \longrightarrow$ reciprocal $\rightarrow \left(z - 2e^{-j\pi/4}\right)$

$\left(z - \frac{1}{2}e^{-j\pi/4}\right) \longrightarrow \left(z - 2e^{j\pi/4}\right)$

minimum

$$\frac{\left(z - \frac{1}{2}e^{j\pi/4}\right)\left(z - 2e^{-j\pi/4}\right)\left(z - \frac{1}{2}e^{-j\pi/4}\right)\left(z - \frac{1}{2}2e^{j\pi/4}\right)}{z^4}$$

$$\frac{\left(z^2 - \frac{1}{\sqrt{2}} + \frac{1}{4}\right)\left(z^2 - 2\times 2\cos\frac{\pi}{4} \cdot z + 4\right)}{z^4}$$

$i \rightarrow$ conjugate is i & reciprocal is also $-i$

$z = \frac{1}{\sqrt{2}} e^{j\pi/4} \longrightarrow 4$ order

$z = i \longrightarrow 2$

$H(z) = z^{-(m-1)} H\left(\frac{1}{z}\right)$ [For Symmetric case]

(I) S

$m \rightarrow$ odd

$m-1 \rightarrow$ even

$H(z) = z^{-\text{even}} H(1/z)$

Suppose $z = 1$ is the zero

$H(1) = 1^{-\text{even}} H(1/1)$

$H(1) = 1 \cdot H(1)$

$\rightarrow H(1) = H(1)$

Let $z = -1$

$H(-1) = (-1)^{-\text{even}} H(1/-1)$

$H(-1) = 1 \cdot H(-1)$

$\rightarrow H(-1) = H(-1)$

(II) If $m \rightarrow$ even

$m-1 \rightarrow$ odd

$H(z) = z^{\text{odd}} H(1/z)$

Let $z = 1$ be the zero

$H(1) = 1^{\text{odd}} H(1/1)$

$H(1) = 1 \, H(1)$

$H(1) = H(1)$

Let $z = -1$

$H(-1) = (-1)^{-\text{odd}} H(1/(-1))$

$= (-1) H(-1)$

$H(-1) = -H(-1)$

$\therefore 2H(-1) = 0$

as $2 \neq 0$ $\boxed{H(-1) = 0}$ must be

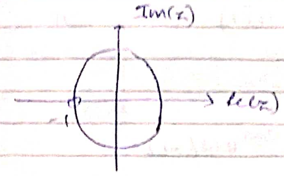
$c = -c$

$c + c = 0$

$2c = 0$

$c = 0$

$2 \neq 0$



If $H(z)$ is symmetric & when m is even then $H(-1) = 0$ (compulsory zero) when m is odd it is not compulsory case

For Antisymmetric case

(III)

$h(n) = -h(m-1-n)$

$H(z) = -z^{(m-1)} H(1/z)$

$m = $ odd

$m-1 = $ even

$H(z) = -z^{\text{even}} H(1/z)$

Let $z = 1$

$H(1) = -1^{\text{even}} H(1/1)$

$H(1) = -H(1)$

$H(1) = -H(1)$

$\therefore \boxed{H(1) = 0}$

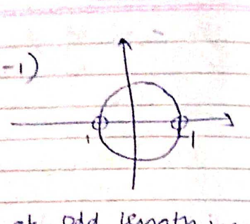
Let $z = -1$

$H(-1) = -(-1)^{even} H(1/-1)$

$= -1 \; H(-1)$

$H(-1) = -H(-1)$

$\boxed{H(-1) = 0}$



If $H(z)$ is antisymmetric at odd length i.e

$m = odd$

$H(1)$ & $H(-1) = 0$ (compulsory)

IV

$m = even \Rightarrow m-1 = odd$

$H(z) = -z^{-odd} H(1/z)$

Let $z = 1$

$H(1) = -(1)^{-odd} H(1)$

$H(1) = -H(1)$

$\therefore \boxed{H(1) = 0}$

Let $z = -1$

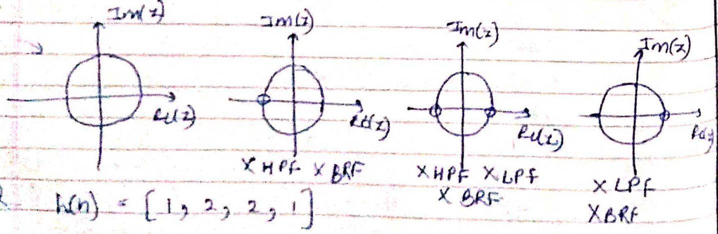
$H(-1) = -(-1)^{-odd} H(1/-1)$

$H(-1) = H(-1)$

No compulsory condition

I	II	III	IV
Symmetric	Symmetric	Antisymmetric	Antisymmetric
$m =$ odd	$m =$ even	$m =$ odd	$m =$ even

can be any filter



Q $h(n) = [1, 2, 2, 1]$

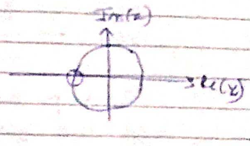
$H(z) = \frac{1 + 2z^{-1} + 2z^{-2} + z^{-3}}{z^3}$

$H(z) = \frac{z^3 + 2z^2 + 2z + 1}{z^3}$

$H(z) = \frac{(z+1) \dots}{z^3}$

Even symmetric $h(n) = [\underset{\uparrow}{1}, 1]$

$H(z) = 1 + z^{-1} = \frac{z+1}{z}$



III $h(n) = [1, 0, -1]$

$H(z) = 1 + 0z^{-1} - z^{-2}$

$H(z) = \frac{z^2 - 1}{z^2} = \frac{(z-1)(z+1)}{z^2} = \frac{z^2 - 2z - 1}{z^2}$

$H(z)$

III $h(n) = [\underset{\uparrow}{1}, 2, 0, -2, -1]$

$H(z) = 1 + 2z^{-1} - 2z^{-3} - z^{-4}$

$H(\omega) = 1 + 2e^{-j\omega} - 2e^{-3j\omega} - e^{-4j\omega}$

$= e^{-2j\omega}\left[e^{2j\omega} + 2e^{j\omega} - 2e^{-j\omega} - e^{-2j\omega}\right]$

$= e^{-2j\omega}\left[2j(\sin 2\omega + 2\sin\omega)\right]$

at $\omega = 0 \longrightarrow H(\omega) = 0$

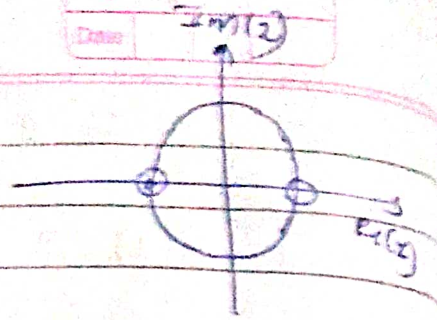
at $\omega = \pi \longrightarrow H(\omega) = 0$

Considering case III

$h(n) = [1, 0, -1]$ ← Added one zero

$H(z) = \frac{(z-1)(z+1)(z+1)}{z^2 \cdot z}$

$$= \frac{(z^2-1)(z+1)}{z^3}$$

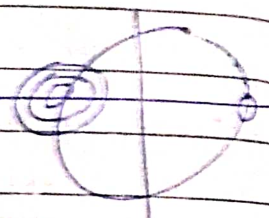


$$= \frac{z^3+z^2-z-1}{z^3}$$

$h(n) = [\underset{\uparrow}{1}, 1, -1, -1]$ Length = even

When one zero added then length becomes even ⟶ Antisymmetric ~~odd~~ even

To keep Antisymmetric odd then I must add two zero



$$= \frac{(z^3+z^2-z-1)(z+1)}{z^4}$$

$$= \frac{z^4+z^3-\cancel{z^2}-z+z^3+\cancel{z^2}-z-1}{z^4}$$

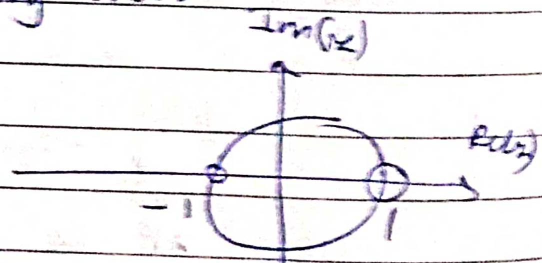
$$= \frac{z^4+2z^3-2z-1}{z^4}$$

$h(n) = [\underset{\uparrow}{1}, 2, 0, -2, -1]$

For case (III)   Symmetry even

$h(n) = (1, 1)$

$= \frac{z+1}{z}$



$H(z) = 1 + z^{-1}$

$= \frac{(z+1)(z-1)}{z^2}$

$= \frac{z^2-1}{z^2}$

$= (1, 0, -1)$ ⟶ Antisymmetric odd

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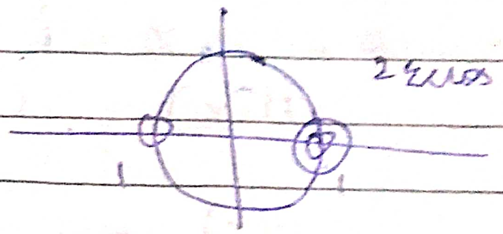
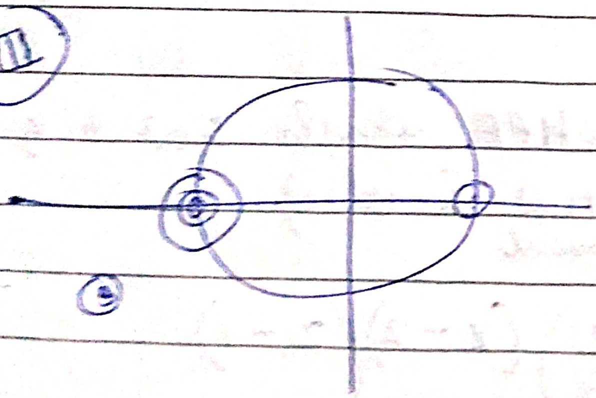
$$= \frac{(z^2-1)(z-1)}{z^3}$$

$$= \frac{z^3 - z^2 - z + 1}{z^3}$$

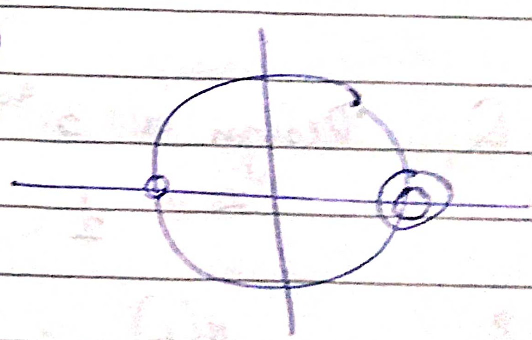


$h(n) = (\underset{\uparrow}{1}, -1, -1, 1) \longrightarrow$ Symmetry even

III



II



⊛ Type-1 Sequence

$z = \frac{1}{2} e^{j\pi/3}$ & zero at $z=1$

$\Downarrow$

$2 e^{-j\pi/3}$ 2 zero at 1

$\frac{1}{2} e^{-j2\pi/3}$ at type 1 sequence

$2 e^{j\pi/3}$

zero $\rightarrow$ 6

⊛ Type-3 sequence

$z = i$ & 2 zeros at $z = 1$

$z = i$ zeros $\to 6$

$$H(z) = \frac{(z-i)(z+i)}{z^2}$$

$$H(z) = \frac{z^2 + iz - iz - i^2}{z^2}$$

$$H(z) = \frac{(z^2+1)(z-1)(z-1)(z-1)(z+1)}{z^6}$$

Q. Design a 3rd order HPF which has a zero at $z = \frac{1}{2}$. Find $H(z)$ & $h(n)$
$\frac{1}{2} \to$ Reciprocal

$\to$ $$H(z) = \frac{\left(z-\frac{1}{2}\right)(z-2)(z-1)}{z^3}$$

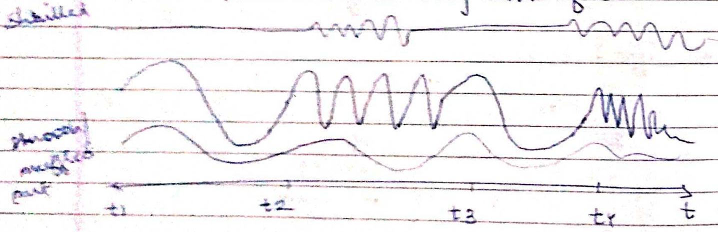
case IV

~~If~~ ~~as~~ 3rd order $\longrightarrow$ length is even

4/11/2020 Lab $\Longrightarrow$ Antisymmetric even

muffled / smooth $\longrightarrow$ Adjective of LPF

shrilled



0 to 4 kHz

0 to 22 kHz

Audio $\longrightarrow$ not by human being

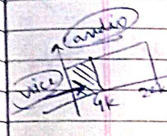
Ⓐ $x(t) = \sin(2\pi 200 t)$ } such 10 Harmonics
for synthetic
$x_2 = \cos(2\pi 5000 t)$ }
Add all of them then listen

VOICE 0 - 4K

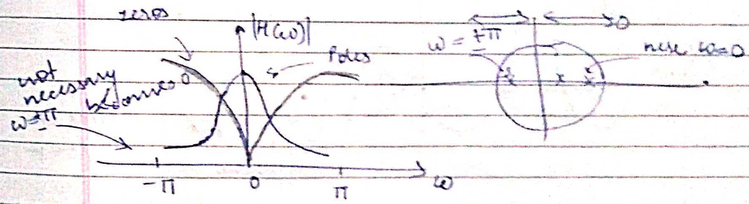
Non-voice (audio) 4k - 22k

Ⓑ Non synthetic
Agitating } voice
Singing

5/11/2020



FIR
① Zero placement (don't have control in poles)
② Symmetric / Antisymmetric $\Rightarrow$ Linear phase FIR
IF zero is taken at particular frequency it will block that frequency.
$\longrightarrow$ For causal & ~~BIBO~~ stable systems all poles must be inside unit circle.



Zeros can be anywhere
But poles must be inside the circle.

Why Linear Phase?

$$H_1(\omega) = e^{-j\left(\frac{m-1}{2}\right)\omega} \, |H(\omega)|$$

$$\frac{Y(\omega)}{X(\omega)} = C \cdot e^{-j\left(\frac{m-1}{2}\right)\omega}$$

$$= C\, e^{-jn_0\omega}$$

$$n_0 = \frac{m-1}{2}, \quad C = |H(\omega)|$$

Shape of $x(n)$ is retained → $Y(\omega) = C \cdot e^{-jn_0\omega} \cdot X(\omega)$

$$\boxed{y(n) = C \cdot x(n-n_0)}$$

$x(n)$

$x(n-n_0)$ will come unless and untill $e^{-jn_0\omega}$ will come & $e^{-jn_0\omega}$ will come unless and untill $\measuredangle H(\omega)$ - linear with freq^n

$x(t) \rightarrow X(s)$

$y(t) \rightarrow Y(s)$

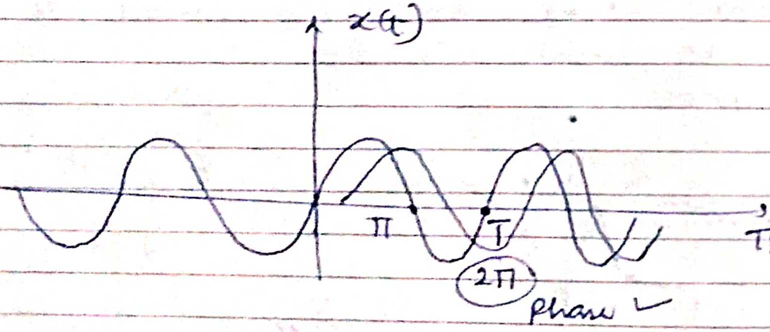
$y(t) = a \cdot x(t)$

$Y(s) = a\,X(s)$

$y(t) = a\,x(t-\tau)$

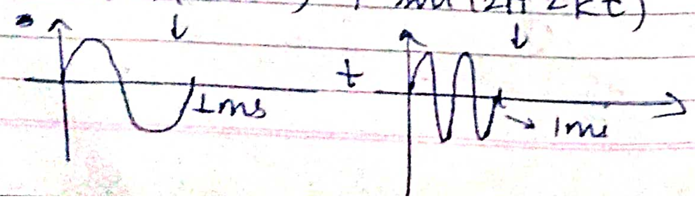
$Y(s) = a e^{-\tau s} X(s)$

$x(t) = \sin(100\pi t)$

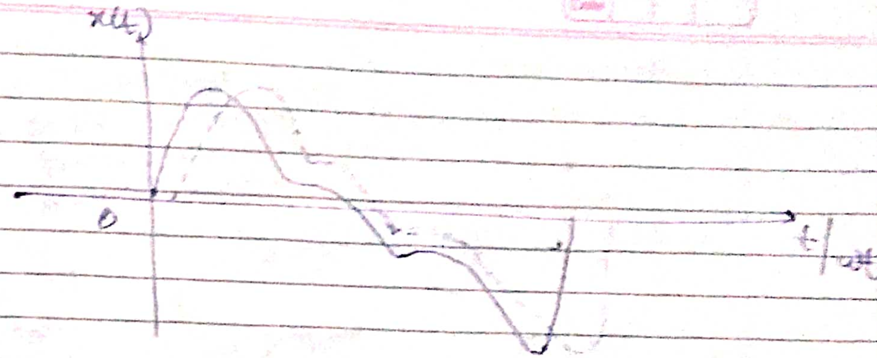


If sine is delayed by 30° then there will be change in phase

$x(t) = \sin(2\pi 1kt) + \sin(2\pi\, 2kt)$



Plot



IF $x(t)$ is delays then both component will get delay

IF delayed by same amount then will get same $x(t)$ but just delayed version without affecting shape change

$f_1 = 1000\,Hz$
$f_2 = 2000\,Hz$ } → must undergo same delay (constant)

In digital

$y(t) = \cancel{x(t)}\ x(t - \pi/6)$

$$\boxed{y(n) = x(n-n_0)}$$

$\theta = \omega t$ → time (θ: Phase, ω: Frequency)

$$\boxed{y = mx}$$

$\frac{d\theta(\omega)}{d\omega} = ?$ → constant t (Phase delay)

$2\pi F_1 t \longrightarrow \theta_1$
$2\pi F_2 t \longrightarrow \theta_2$ } group of phase

$\frac{d\theta(\omega)}{d\omega} \longrightarrow$ constant

For multiple it is called as Group delay

Q. O/P of system whose $h(n) = [\underset{\uparrow}{1}, 2, 3]$

$x(n) = [\underset{\uparrow}{1}, 5, 6, 7]$

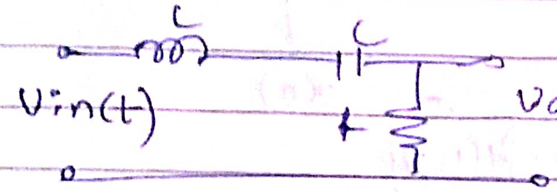
$H(z) = 1 + 2z^{-1} + 3z^{-2}$

$X(z) = 1 + 5z^{-1} + 6z^{-2} + 7z^{-3}$

$Y(z) = H(z) \cdot X(z)$

$= (1 + 2z^{-1} + 3z^{-2})(1 + 5z^{-1} + 6z^{-2} + 7z^{-3})$

Q



$v_o(t) = ?$ [using LT]

Q $h(n) = [\underset{\uparrow}{1}, 2, 3, 4]$

$-\infty < n < \infty$

$x(n) = \sin \omega_0 n$

$y(n) = x(n) * y(n)$

$Y(z) = X(z) \cdot Y(z)$

$h(n) \longrightarrow H(\omega)$

$x(n) = A \cdot e^{j\omega_0 n} = A[\cos \omega_0 n + j \sin \omega_0 n]$

$H(\omega)$ will have $= |H(\omega)| \angle H(\omega)$

$y(n) = h(n) * x(n)$

$= \sum_{k} h(k)\, x(n-k)$

$= \sum_{k} h(k) \cdot A e^{j\omega_0 (n-k)}$

$= A \sum_{k} h(k)\, e^{j\omega_0 n} \cdot e^{-j\omega_0 k}$

$$h(n) = \sum_{n} h(n)\, e^{-j\omega n} = H(\omega)$$

$$y(n) = A e^{j\omega_0 n} \left[\sum_{k} h(k) \cdot e^{-j\omega_0 k} \right]$$

$$y(n) = A e^{j\omega_0 n} \cdot H(\omega_0)$$

but $H(\omega_0) = |H(\omega_0)| \measuredangle H(\omega_0)$

$$= |H(\omega_0)| \cdot e^{j\measuredangle H(\omega_0)}$$

$$y(n) = \underline{A \cdot e^{j\omega_0 n}}\; H(\omega_0)$$

$$\boxed{y(n) = x(n) \cdot H(\omega_0)}$$

$\text{of } x(n)$ $\leftarrow h(n)$

mag $\rightarrow$ $A \cdot |H(\omega_0)|$

Phase $\rightarrow$ $e^{j\omega_0 n} \cdot e^{j\measuredangle H(\omega_0)}$

$$y(n) = A\, |H(\omega_0)| \cdot e^{j\omega_0 n}\, e^{j\measuredangle H(\omega_0)}$$

$$y(n) = A\, |H(\omega_0)|\, e^{j(\omega_0 n + \phi_0)}$$

where $\phi_0 = \measuredangle H(\omega_0)$

$\omega_0 \longrightarrow$ input freq^n

$x(n) = 2\cos(\pi/3\, n + \pi/4)$

$H(\omega) = 4e^{-3j\omega}$; $0 \le |\omega| \le \pi/2$

$= 0$; elsewhere

$y(n) = ?$

I/P Freq $\omega_0 = \pi/3$

$\therefore H(\omega_0) = H(\omega)\Big|_{\omega = \pi/3} = 4e^{-3j\pi/3} = 4e^{-j\pi}$

$|H(\omega)|_{\pi/3} = 4$, $\measuredangle H(\omega) = -\pi$

$\frac{\pi}{4} - \pi \qquad \frac{\pi - 4\pi}{4}$

$y(n) = 2\cos(\pi/3\, n + \pi/4) \cdot H(\omega_0)$

$y(n) = 2 \times 4 \cos(\pi/3\, n + \pi/4 - \pi)$

$y(n) = 8\cos\left(\frac{\pi}{3} n - \frac{3\pi}{4}\right)$

Q. $H(\omega) = 6e^{-j7\omega}$; $0 \le |\omega| \le \pi/4$

$= 0$; elsewhere

$x(n) = 3\sin\left(\frac{\pi}{8} n + \frac{\pi}{6}\right) - 5\cos\left(\frac{\pi}{2} n + \frac{\pi}{3}\right) + 6\cos\left(\frac{\pi}{6} n - \frac{\pi}{8}\right)$

$\omega_1 = \pi/8$, $\omega_2 = \pi/2$, $\omega_3 = \pi/6$

$H(\omega_1) = H(\omega_1)\Big|_{\omega_1 = \pi/8}$

$= 6e^{-j7\pi/8}$

$H(\omega_2) = 0$

$H(\omega_3) = 6e^{-j7\pi/6}$

$$y(n) = 18\sin\left(\frac{\pi}{8} n + \frac{\pi}{6} - \frac{7\pi}{8}\right) + 36\cos\left(\frac{\pi}{6} n - \frac{\pi}{8} - \frac{7\pi}{6}\right)$$

~~Sys~~system cannot generate any new frequency because it is a LTI system. Only non linear & Time variant system can generate new frequency

Ex:- $x(t) = \cos\omega t$

$y(t) = x^2(t)$

$y(t) = \cos^2\omega t$

$y(t) = \frac{1 + \cos 2\omega t}{2}$

6/11/2020

(*) condition of linear phase for $x(n)$

→ $x(n)$ must be Symmetric / antisymmetric

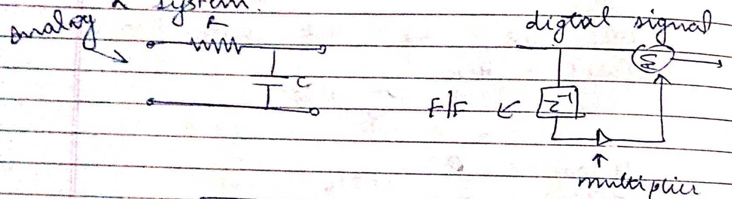
Analog system $H(s)$ ⟶ Digital system $H(z)$

$h(t) \longrightarrow h(n)$

Here $t = nT_s$ can't be used as it is

analog system

digital signal



$x(t) \longrightarrow \boxed{\frac{d}{dt}} \longrightarrow y(t)$

$$y(t) = \frac{d\,x(t)}{dt}$$

$$Y(s) = sX(s)$$

$$\frac{Y(s)}{X(s)} = s$$

$$H(s) = s \quad \text{——— (I)}$$

$$y(nT_s) = \frac{x(n) - x(n-1)}{T_s}$$ ⟸ distance between two samples

$$y(nT_s) = \frac{x(n) - x(n-1)}{T}$$

$$y(nT) = \frac{x(nT) - x(nT - T)}{T}$$

$$y(n) = \frac{x(n) - x(n-1)}{T}$$ (Backward differen[ce])

$$Y(z) = \frac{X(z) - z^{-1}X(z)}{T} \Rightarrow \frac{Y(z)}{X(z)} = \frac{1 - z^{-1}}{T}$$

$$\boxed{H(z) = \frac{1 - z^{-1}}{T}} \quad \text{——— (II)}$$

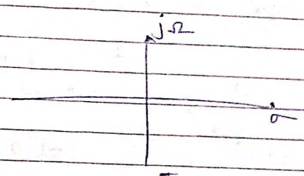
from (I) & (II)

$H(s)$ & $H(z)$ are equivalent to each other (i.e approximate)

$$s = \frac{1 - z^{-1}}{T}$$

Equivalent to every "s" but just derivation must be include

$$s = \frac{1 - z^{-1}}{T}$$

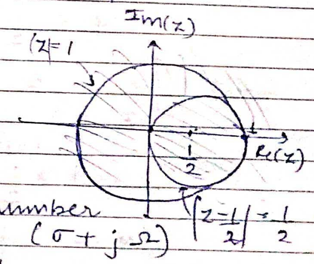


$$Ts = 1 - \frac{1}{z}$$

$$\frac{1}{z} = 1 - Ts$$

$$z = \frac{1}{(1 - Ts)}$$

$$\boxed{z = \frac{1}{1 - sT}}$$



$s \longrightarrow$ complex number $\sigma + j\omega$ $(\sigma + j\Omega)$

if $\sigma = 0 \Rightarrow z = \frac{1}{1 - j\Omega T}$

$$z - \frac{1}{2} = \frac{1}{1 - j\Omega T} - \frac{1}{2}$$

$$z - \frac{1}{2} = \frac{2 - 1 + j\Omega T}{2(1 - j\Omega T)}$$

$$z - \frac{1}{2} = \frac{1 + j\Omega T}{2(1 - j\Omega T)}$$ — conjugate pair

$$\left|z - \frac{1}{2}\right| = \frac{1}{2}$$ when $\sigma = 0$

$$\left|z - \frac{1}{2}\right| = \frac{1}{2}$$

$z = 1/2$ (radius of 1)

center $\rightarrow \frac{1}{2}$ $\quad \left(\frac{1}{2}, 0\right)$

Whatever the values of $j\Omega$ will come at a circle having radius $\frac{1}{2}$ & center $1/2$

for $\sigma = 0$

$\Rightarrow$ if $\sigma < 0$

$$z - \frac{1}{2} = \frac{1 - 0.8 + j\Omega}{1 + 0.8 - j\Omega}$$

$j\Omega$ will be inside the $\frac{1}{2}$ circle

as $z = \dfrac{1}{1-(-0.8 - j\Omega T)}$

$|z| < \frac{1}{2}$

$\Rightarrow$ if $\sigma > 0$

$$z = \frac{1}{1-(-1.2 - j\Omega T)}$$

$|z| > \frac{1}{2}$

$j\Omega$ will be outside the $\frac{1}{2}$ radius circle

using Forward Difference

$$y(n) = \frac{x(n+1) - x(n)}{T}$$

$$Y(z) = \frac{z\,X(z) - X(z)}{T}$$

$$Y(z) = \frac{X(z)\,(z-1)}{T}$$

$$H(z) = \frac{z-1}{T} \qquad \therefore \; s = \frac{z-1}{T}$$

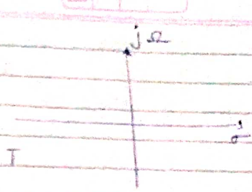
$z \rightarrow$ Complex number

$$s = \frac{z-1}{T}$$

$$sT = z - 1$$

$$z = 1 + sT$$

$$z = 1 + (\sigma + j\Omega)T$$



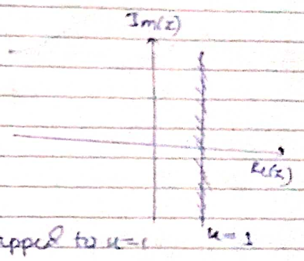
$\boxed{\sigma = 0}$ $\quad u + iv = 1 + (\sigma + j\Omega)T$

$z = \frac{1}{2}$ $\quad \rightarrow u = 1$

$u = \frac{1}{2}$ & $v = 0$

Doubt $\rightarrow$ $z = \frac{1}{2} \angle 45^\circ$

$\left(\frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}\right) + j$



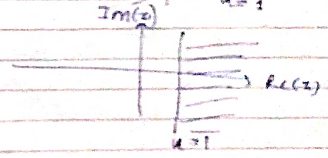
Here $j\Omega$ will get mapped to $u = 1$

$\boxed{\sigma < 0}$ $\quad u < 1$

$j\Omega$ will get mapped for $u < 1$

$\boxed{\sigma > 0}$ $\quad u > 1$

$j\Omega$ will mapped for $u > 1$



In Z domain

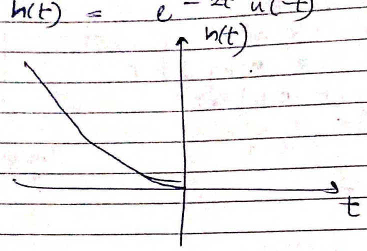
$\rightarrow$ System to be causal & stable poles must lies inside the unit circle

$\rightarrow$ System to be non causal & stable poles must lie outside the unit circle

Laplace revision

$H(s) = \frac{1}{s+2} = (s+2)^{-1}$ ROC $= ?$

$h(t) = e^{-2t}u(-t)$

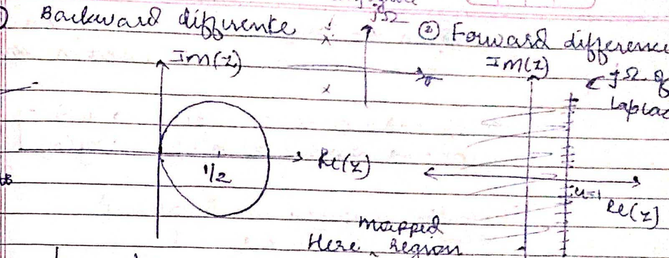


6/11/2020

LAB

① Backward difference ② Forward difference

Here digital to analog is difficult



$\left|z - \frac{1}{2}\right| = \frac{1}{2}$

Here mapped Region is not confined to $|z| \leq 1$

① Pole will be inside the circle
② Poles will be in the left side of u=1

Backward difference will always guarantee H(s) to be causal & stable H(z)

*** If H(s) is causal & stable then using $s = \frac{1-z^{-1}}{T}$ will give ~~stru~~ causal & stable H(z) always

If we are using derivative operator then can also use integration summation

x(t) → [H(s) / h(t)] → y(t)

x(t) ↓ t = nT sampling → x(n) →(z) X(z)

y(t) ↓ sampling t → nT → y(n) ↓ z → Y(z)

H(s) ↓ H(z)

$\frac{Y(z)}{X(z)} = H(z)$

Assumption → Response will not change
i.e it will remain same

$x(t) \longrightarrow$ provide to $h(t) \longrightarrow y(t)$] Approximately same
$x(n) \longrightarrow$ provide to $h(n) \longrightarrow y(n)$

Impulse response is invariant

$x(t) = \delta(t) \quad \therefore X(s) = 1$

$H(s) = \frac{1}{s+1} \quad , T = 1$

$Y(s) = \frac{1}{s+1} X(s)$

$Y(s) = \frac{1}{s+1} \cdot 1 = \frac{1}{s+1}$

$y(t) = e^{-t} u(t)$

$y(n) = e^{-nT} u(n)$

$y(n) = (e^{-T})^n u(n)$

$Y(z) = \frac{z}{z - e^{-T}}$

As $X(z) = 1$

$H(z) = \frac{z}{z - e^{-T}}$

This $H(z)$ is equivalent to

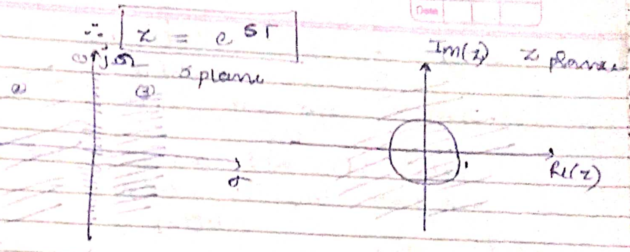
$H(s) = \frac{1}{s+1}$

IF $H(s) = \frac{1}{s+p}$ $\qquad s = -p$

$\therefore H(z) = \frac{z}{z - e^{-pT}}$ $\qquad z = e^{-pT}$

③ $\therefore \boxed{z = e^{sT}}$

For digital to Analog it is preferable



$z = re^{j\omega} = e^{(\sigma + j\Omega)T}$

$re^{j\omega} = e^{\sigma T} \cdot e^{j\Omega T}$

$\omega = \Omega T$

$r = e^{\sigma T}$

IF $\sigma = 0$ ——(I)

$\therefore r = 1$

IF $\sigma < 0$ ——(II)

$\therefore r < 1$

IF $\sigma > 0$ ——(III)

$\therefore r > 1$

IF

③ Third case

z - plane

Highest → $-\pi$ ~~to~~ & π

Lowest → 0

Range restricted

$\omega = 0$, $z = 1$ allowed

If I have to place pole/zero at $\omega = \pi$

③ Here entire region is available

$\frac{1}{2}x_3(n) + x_2(n)$

$[0, 1, \underset{\uparrow}{2}, -3] + [-1, \underset{\uparrow}{-1}, 3, 0]$

$= [0, 0, \underset{\uparrow}{1}, 0, 0] = \delta(n)$

$y_4(n) = \frac{1}{2}y_3(n) + y_2(n)$

$y_4(n) = [3, \underset{\uparrow}{2}, 0, 2, 2, 0] + [2, \underset{\uparrow}{3}, -1, 0, 1]$

$= [5, 5, -1, 2, 3, 0]$

When the system is LTI then

$x_1(n) * h(n) = y_1(n)$

8/10/2020

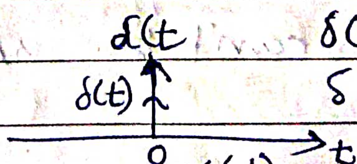
Plotting → Basic level
→ way of representation

Q Who does respresenter signals?
Q Signals are represented by whom?
Q How to search?

① Capable ④ Reach ⑦ Neutral
② Communicate ⑤ Regular
③ Handling ⑥ No ditch

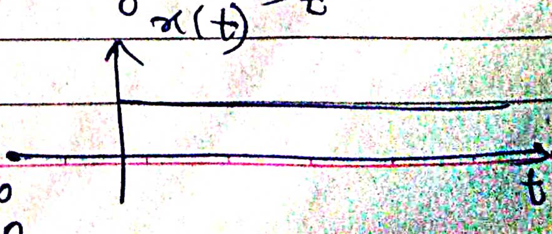
Signals

① Impulse →



$\delta(t) = \infty \; ; t = 0$
$\delta(t) = 0 \; ; t \neq 0$

② Unit step



$x(t) = u(t) = 1 \; ; t > 0$
$= 0 \; ; t < 0$

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$2u(t) - 3u(t-1)$

$x(t)-u(t) = 1 ; t>0$
$= 0 ; t<0$

$u(t) = ? \quad t=0$

Function is not define at $t=0$

Available nahi hai

Function is discontinuous at $t=0$

↓

not available

↓

ditch

90/10/2020

I don't want to ~~sepa~~ deal with signal separately

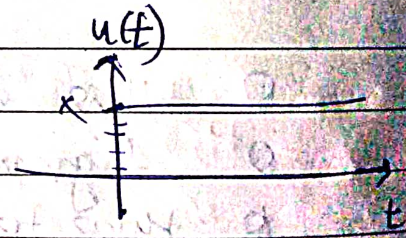
Impulse response → used for representing a system

Position of any signal can be determine using Impulse but cannot find it's value

At $t=0$ it can have any value

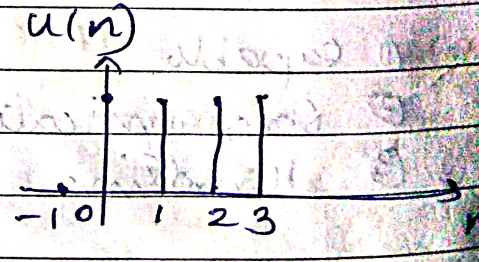
$u(t) = 1 ; t>0$
$0 ; t<0$

at $t=0$ $u(t)$ undefined



But

$u(n) = 1 ; n \geq 0$
$= 0 ; n<0$



*** $u(t)$ can't be used to represent signal second order derivative

Cubic signal

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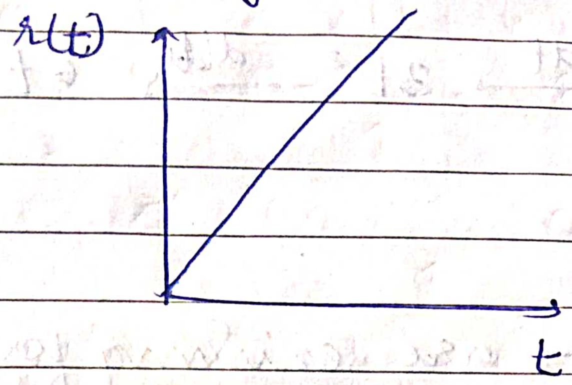
(8) Rampte signal



$$r(t) = t \;;\; t \geq 0$$
$$= 0 \;;\; t < 0$$

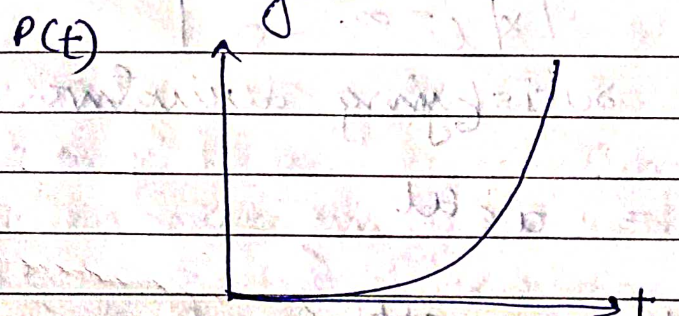
Constraints → must not go to infinity

$$x(t) = f(r(t)) \longrightarrow \boxed{\frac{d}{dt}[\;]} \longrightarrow y$$

h( ) H( )

$$y(t) = \frac{d}{dt} r(t) \Rightarrow u(t)$$

(9) Parabolic signal



$$P(t) = t^2 \;;\; t \geq 0$$
$$= 0 \;;\; t < 0$$

Const . . . t, t^2, t^3, t^4, . . . , t^n

t	t^2	t^3	t^4
	t	t^2	t^3
	const	t	t^2
		const	t
			const

$$\frac{d^n}{dt^n}$$

*** Polynomial can not be used in general to represent a signal.

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→ Problem of having discontinuity (Ditching)

$t^3 \xrightarrow{d/dt} 3t^2 \xrightarrow{d/dt} 6t \longrightarrow 6$

X ↑ Discontinuity

∴ two derivative are existing for t^3

$t^n \longrightarrow n-1$ derivative exist.

*** n^{th} order derivative must exist & $n \to \infty$

$e^t \to$ specific case

general case ← $e^{at} \longrightarrow$ Exponential Function $\infty > a \geq 0$

Limitation at $e^{\infty} = \infty$

$e^{-\infty} = 0$

$e^{\infty} = \infty$

$e^{-bt} \longrightarrow$ Limitation at $e^{-(-\infty)} = \infty$

$e^{-b|t|} \longrightarrow$ no Limitation but is discontinuous at $t = \infty$

$\boxed{\cancel{x}\ e^{-\infty} = 0}$

↑ Not satisfying derivative condition

$$\frac{d}{dt} e^{at} = a e^{at}$$

$$\frac{d}{dt} e^{-bt} = -b e^{-bt}$$

$$\int e^{-b|t|} \longrightarrow \begin{matrix} -b(-1)\, e^{at} \\ -b(1)\, e^{-at} \end{matrix}$$

$$e^{-a|t|} = e^{at} \cdot u(-t) + e^{at}\, u(t)$$

At $t = 0$ it is discontinuous

$e^{-\infty} = 0$

$e^{\infty} = 0$

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$x(t) = e^{at} \longrightarrow t$ +ve

$t \to \infty$, $x(t) \to \infty$

e^{-2t} $a < 0$

$t \to \infty$ $x(t) \longrightarrow 0$

for e^{at} | t +ve & $a > 0$ (1) |

13/10/2020

avg. htg = $\sum P(k)\, h(k)$

$h(0) \rightarrow$ height of student 1 $\quad P(k) \longrightarrow$ Probability [chance] weight

$h(1)$

$h(2)$

$\vdots$

$h(k)$

weight of ingredients

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$$L \cdot J = \sum_{k} \omega(k) \cdot I(k)$$

Ingredient

" L.J is represented as weighted sum of ingredient " ↓ equal to

$\boxed{\text{Another prism}}$

$$x(t) = \sum_{k} X(k) e^{j2\pi kt}$$

Signal can be represented as weighted sums of complex exponential

Argument ⟷ what is coming along with j i.e $2\pi kt$

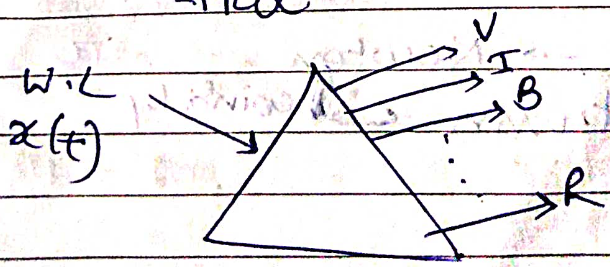
$$g(t) = \sum_{k} G(k) e^{j2\pi kt}$$

Q. How do I get the weights ?

$\boxed{\text{First prism}}$ $X(k) = \int x(t) e^{-j2\pi f kt} dt$

"weight can be represented as weighted sum of another exponential

What all are they? What are is their contributions?



$k=0 \rightarrow V$

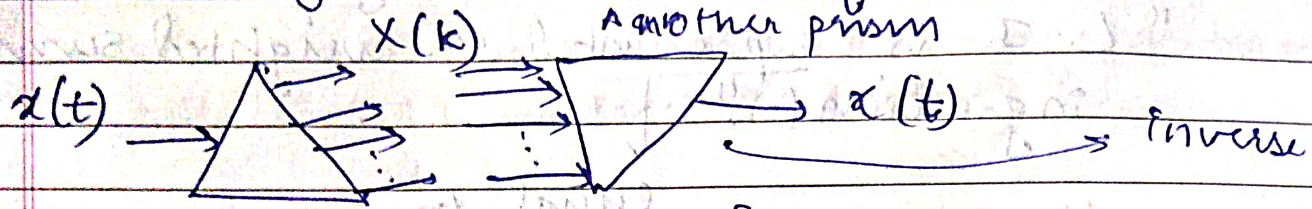
$k=1 \rightarrow I$

⋮

$k=c \rightarrow R$

$X(k) \longrightarrow$ contribution/weight of k^{th} component

$X(k)$ represents
→ k
→ weight / contribution of k



If we not allow Red to go then we will not get white light.

Make / Produce / Construct / generate / create
setup / form / compose / develop / Fabricate
synthesize

- In First prism I am trying to analyse
- In another prism I am synthesizing

$e^{-j2\pi}$ → complex exponential function is used to analyse every other function.

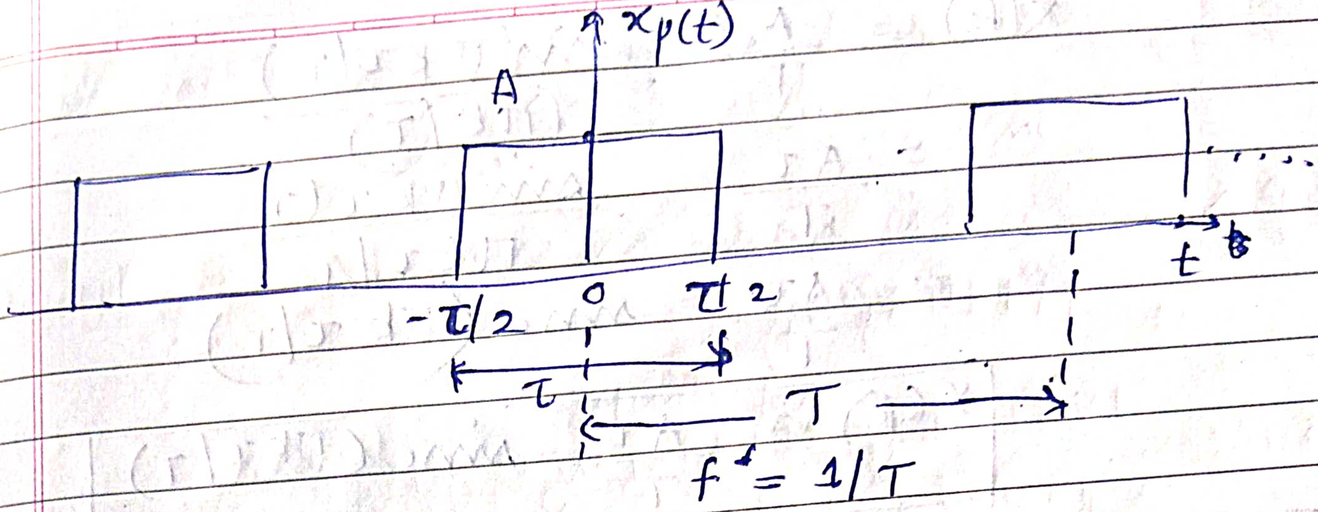
∴ Complex exponential function are ~~called as~~ "Analytic function" because they are used to analyse every other function

$$\left.\begin{aligned} \frac{du}{dx} &= \frac{dv}{dy} \\ \frac{du}{dy} &= -\frac{dv}{dx} \end{aligned}\right\} \text{CR Condition}$$

Cauchy Reimann condition

Prism → Newton
Spectrum ← coined by

Transfer function ka substitute



Q. who they are? how much they are?

$$\hookrightarrow X(k) = \frac{1}{T}\int x(t)\, e^{-j2\pi f k t}\, dt$$

To indicate that this integral is taken for one T

$$= \frac{1}{T}\int_{-\tau/2}^{\tau/2} A \cdot e^{-j2\pi F k t}\, dt$$

$$= \frac{A}{T}\int_{-\tau/2}^{\tau/2} e^{-j2\pi k t/T}\, dt$$

Here integration is used because signal is continuous

$$X(k) = \frac{A}{T}\left(\frac{e^{-j2\pi k t/T}}{\frac{-j2\pi k}{T}}\right)_{-\tau/2}^{\tau/2}$$

$$= \frac{A}{\cancel{T}} \times \frac{\cancel{T}}{-j2\pi k}\left(e^{-j2\pi k t/T}\right]_{-\tau/2}^{\tau/2}$$

$$= \frac{A}{-j2\pi k}\left[e^{-j2\pi k \tau/2T} - e^{j2\pi k \tau/2T}\right]$$

$$= \frac{A}{\pi k}\left[\frac{e^{j2\pi k \tau/2T} - e^{-j2\pi k \tau/2T}}{2j}\right]$$

$$X(k) = \frac{A}{T} \frac{\sin(\pi k \tau / T)}{(\pi k / T)}$$

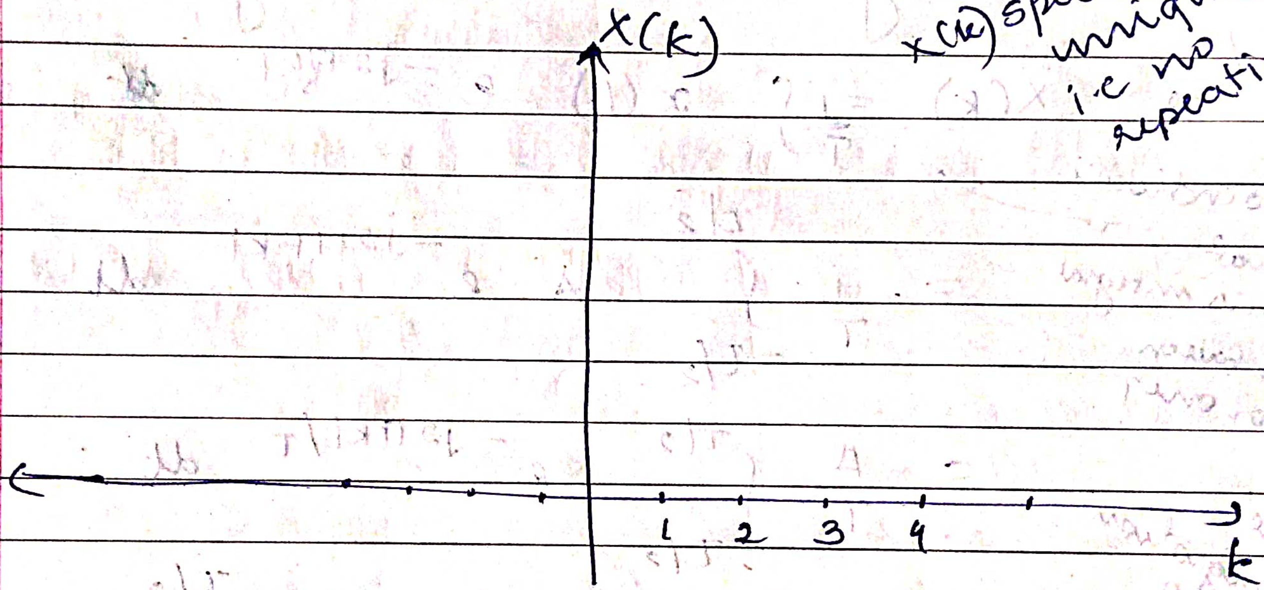
$$= \frac{A\tau}{T} \frac{\sin \pi k \tau / T}{\pi k \tau / T}$$

$$= \frac{A\tau}{T} \operatorname{sinc}(\pi k \tau / T)$$

$$\therefore \boxed{X(k) = \frac{A\tau}{T} \operatorname{sinc}(\pi k \tau / T)}$$

$\tau = 1$
$T = 10$

$-\infty < k < \infty$



here X(k) spectra is unique i.e no repeating

$$\frac{\sin(\pi \cdot k \times 0.1)}{0.1 \pi \times k}$$

LeHopital $\lim_{k \to 0} \frac{\sin(\pi k(0.1))}{\pi k(0.1)} = 1$

Case 1 $\rightarrow$ $\tau = 1$, $T = 10$

case 2 $\rightarrow$ $\tau = 1$, $T = 100$

Case 3 $\rightarrow$ $\tau = 1$, $T = 1000$

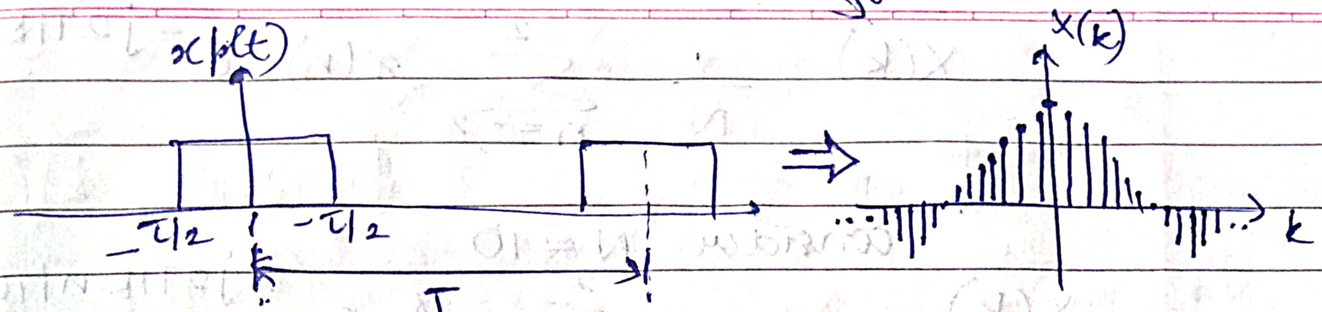
*** All periodic signal has <u>discrete</u> spectrum

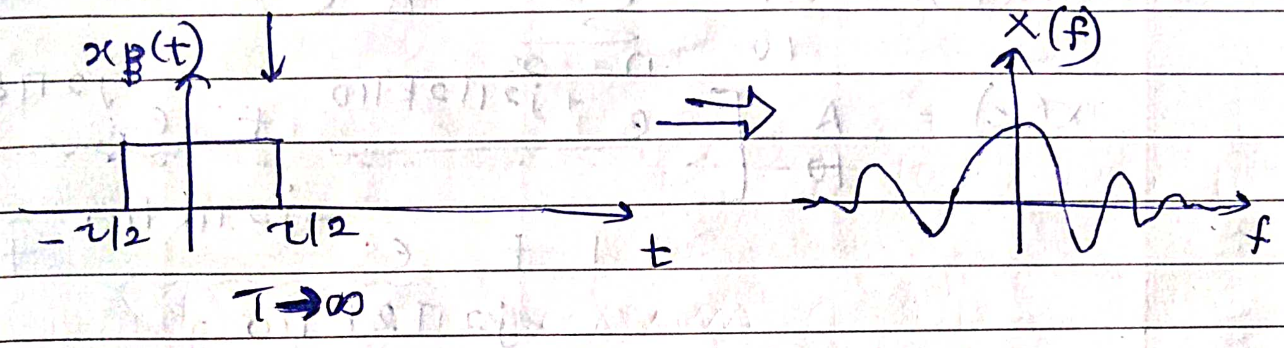
*** All aperiodic signal has <u>continuous</u> spectrum

	Signals	Spectrum
Continuous {	Periodic	Discrete
	Aperiodic	Continuous

Infinite no. of ingredients

5|10|2020



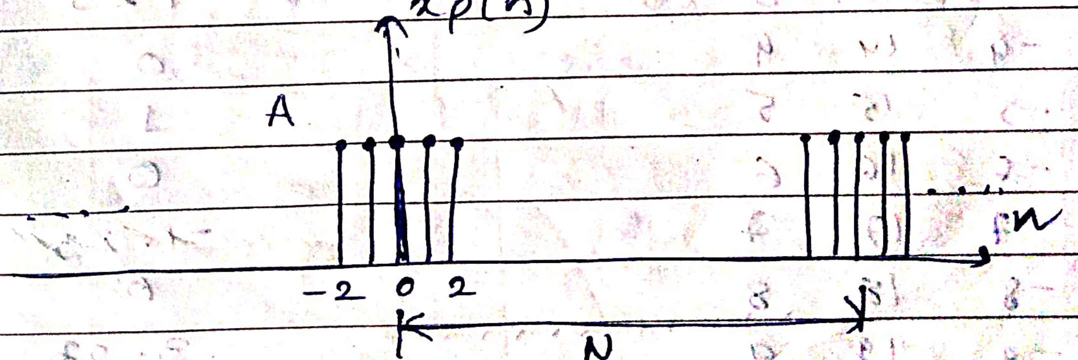


$x_p(t) \longrightarrow X(k)$

$x_B(t) \longrightarrow X(f)$

K ⟶ frequency (eg:- Spectrum)
graph will become continuous not signal

⟶ Spectrums tells about who all are present and about their contribution



width of the pulse ⟶ 5 (as 5 samples)

$$X(k) = \sum x(n)\, e^{-j2\pi kn/N}$$

$$f = \frac{1}{T} = \frac{1}{N} \qquad t \to n$$

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$$X(k) = \frac{1}{N} \sum_{n=-2}^{2} x(n)\, e^{-j2\pi kn/N}$$

consider $N = 10$

$$X(k) = \frac{1}{10} \sum_{n=-2}^{2} A\, e^{-j2\pi kn/10}$$

$$X(k) = \frac{A}{10}\left[e^{+j2\pi 2k/10} + e^{j2\pi k/10} + 1 + e^{-j2\pi k/10} + e^{-j2\pi 2k/10} \right]$$

$$= \frac{A}{10}\left[1 + 2\cos(2\pi k/10) + 2\cos(4\pi k/10) \right]$$

	$\pm k$		$X(k)$
	10	0	5
-1	11	1	3.23
-2	12	2	0
-3	13	3	-1.23
-4	14	4	0
-5	15	5	1
-6	16	6	0
-7	17	7	-1.23
-8	18	8	0
-9	19	9	3.23
-10	20	10	5
		11	3.23
		12	0
		13	-1.23

	SIGNAL		SPECTRUM
CT Periodic F.S	$x_p(t) = \sum_{k=-\infty}^{\infty} X(k) e^{j2\pi f kt}$ Inverse	Aperiodic Discrete	$X(k) = \frac{1}{T}\int_{T} x_p(t) e^{-j2\pi f kt} dt$ FS
CT Aperiodic F.T.	$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} df$ Inverse	Aperiodic Continuous	$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$ FT
DT Aperiodic D.T.F.T	$x(n) = \frac{1}{2\pi}\int_{2\pi} X(\omega) e^{j\omega n} d\omega$ Inverse	Periodic Continuous	$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$ DTFT
DT Periodic D.F.T	$x_p(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k) e^{j2\pi kn/N}$ Inverse	Periodic Discrete	$X(k) = \frac{1}{N}\sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N}$ DFT This is not written

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All are same

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$$X(\omega) = \sum_n x(n)\, e^{-j\omega n}$$

As here ω is used $\therefore$ continuous $\rightarrow$ Aperiodic

Limits of $n \longrightarrow -\infty$ to ∞

Let $x(n)$ be complex

$$x(n) = x_r(n) + i\, x_i(n)$$

$$X(\omega) = \sum_n [x_r(n) + i x_i(n)][\cos\omega n - i\sin\omega n]$$

$$= \sum_n [x_r(n)\cos\omega n - i\, x_r(n)\sin\omega n + i x_i(n)\cos\omega n - i^2 x_i(n)\sin\omega n]$$

$$= \sum_n [x_r(n)\cos\omega n - i\, x_r(n)\sin\omega n + i x_i(n)\cos\omega n + x_i(n)\sin\omega n]$$

$$= \sum_n [x_r(n)\cos\omega n + x_i(n)\sin\omega n] + i\sum_n [-x_r(n)\sin\omega n + x_i(n)\cos\omega n]$$

Let $x(n)$ be real

$$x(n) = x_r(n) \quad \text{--- (1)}$$

$$x_i(n) = 0$$

$$X(\omega) = X_R(\omega) + i X_I(\omega)$$

(2) $$\begin{cases} X_R(\omega) = \sum_n x(n)\cos\omega n = X_R(-\omega) \\ X_I(\omega) = -\sum_n x(n)\sin\omega n = -X_I(-\omega) \end{cases}$$

(Put $\omega = -\omega$) $$X(-\omega) = X_R(-\omega) + i X_I(-\omega)$$

③ & ④ are true when ① is true

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(*) for any real signal whatever happens at +ve frequency will same happen at -ve frequency

-ve freqⁿ is in mathematical domain ~~(not sure about word)~~

→ Relates reality of signal (i.e of real signal)

$$X(-\omega) = X_R(\omega) - i X_I(\omega)$$

$$\text{as } X_I(\omega) = -X_I(-\omega)$$

$$\therefore X(-\omega) = X^*(\omega) \quad —— ③$$

$$|X(\omega)| = \sqrt{X_R^2(\omega) + X_I^2(\omega)} = |X(-\omega)|$$

$$\angle X(\omega) = \tan^{-1}\left(\frac{X_I(\omega)}{X_R(\omega)}\right) = -\angle X(-\omega)$$

Arrow doesn't affect symmetry

(a) $(1, 2, 1)$ with the arrow under 2

Symmetric, Non causal, Finite

(b) $(1, 2, 1)$ with the arrow under the first 1

$x(n) = x(-n)$

symmetric [Even symmetric]

(c) $(1, 2, 1)$

where there is 2 it is symmetric point

(d) $(1\ 3\ 3\ 1)$

Point of symmetry is bet^n two 3's

(e) $(1\ 0\ -1)$

~~Asymmetric~~ Antisymmetric

(f) $(1\ 2\ 3)$

Asymmetric

$x(n) = x(-n)$

If point of symmetry is at origin

Antisymmetric	Symmetric
odd	Even symmetry
$x(n) = -x(-n)$	$x(n) = x(-n)$

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(1, 2, -1)
Neither symmetric ~~symmetric~~ nor anti symmetric.

1 2 2 1 Sum = 6
symmetric

-1 2 3 -2 1 Sum = 3
Neither symmetric nor anti symmetric

-1 2 0 -2 1 sum = 0
Anti symmetric

1 3 1 Sum = 5
symmetric

2 3 -3 -2 , Sum = 0
~~Anti~~ Anti symmetric

Integration / Sum of odd / Antisymmetric is going to be zero.

even/odd symmetric function

$\sin\theta = -\sin(-\theta)$ — odd symmetric

$\cos = \cos(-\theta)$ — Even symmetric

even x even = even
odd x odd = even
odd x even = odd
even + even =

Σ odd = 0
Σ even = something

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for 1 2 -1
$\uparrow$

$x(n) = -x(-n)$

Asymmetric $x(0) = -x(-0)$

$x(0) = -x(0)$

$x(0) + x(0) = 0$ as $2 \neq 0$

$2x(0) \neq 0$ $\therefore x(0) = 0 \rightarrow$ compulsory

$\therefore$ $x(0)$ must be 0 for x to be Antisymmetric -

for 1 0 -1

$x(n) = -x(-n)$

$x(1) = -x(-1)$

$\cancel{x(1)}\ -1 = -1$ ✓

$x(0) = -x(-0)$

$x(0) = -x(0)$

$x(0) + x(0) = 0$

$0 = 0$ ✓

Condition must satisfy for x to be antisymmetry

19/10/2020

1. Symmetric / Even symmetric
2. Antisymmetric / Odd symmetric
3. Neither symmetric nor Antisymmetric i.e Not symmetric

Spectrum

$$X(\omega) = \sum_n x(n)\, e^{-j\omega n}$$

$e^{-j\omega n} \rightarrow \cos\omega n - i\sin\omega n$

$x(n) \rightarrow$ signal

For real $x(n)$

$X(\omega) = X_R(\omega) + i X_I(\omega)$

$$X_R(\omega) = \sum_n x(n) \cos\omega n$$

$$X_I(\omega) = -\sum_n x(n) \sin\omega n$$

In general $X(\omega) \rightarrow$ complex quantity irrespective of $x(n)$

$X(-\omega) = X^*(\omega)$

$|X(\omega)| = |X(-\omega)| \rightarrow$ Magnitude spectrum

$\angle X(\omega) = -\angle X(-\omega) \rightarrow$ phase spectrum

Odd symmetric / anti symmetric

Even symmetric / symmetric

** For real signal my magnitude spectrum is even symmetric & phase spectrum / component are odd symmetric.

** $\therefore$ In general my spectrum is conjugate symmetry

① $x(n)$ is Real & Even ~~sy~~ symmetry

$X_R(\omega) = \sum \underbrace{x(n)}_{\text{even}} \underbrace{\cos\omega n}_{\text{even}} = \sum (\text{even}) = \text{something}$

$X_I(\omega) = -\sum x(n) \sin\omega n = \sum(\text{odd}) = 0$

$\therefore X(\omega) = X_R(\omega) + i \cdot 0$

$\therefore X(\omega)$ is real

$X(-\omega) = X_R(-\omega) + 0$

$= X_R(\omega)$

$\therefore X(-\omega) = X(\omega)$

Spectrum is real and even

② $x(n)$ is Real & odd

$X_R(\omega) = \sum x(n) \cos\omega n = \sum \text{odd even} = \sum \text{odd} = 0$

$X_I(\omega) = -\sum x(n) \sin\omega n = -\sum \text{odd odd} = \sum \text{even} = \text{something}$

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$$X(\omega) = X_R(\omega) + i\, X_I(\omega)$$
$$X(\omega) = 0 + i X_I(\omega) = i X_I(\omega)$$

$\Rightarrow$ Spectrum ~~is~~ $X(\omega)$ is Imaginary

$$|X(\omega)| = |X(-\omega)| \quad \& \quad \angle X(\omega) = -\angle X(-\omega)$$

$$X(\omega) = X_R(\omega) + i X_I(\omega)$$
$$X_R(-\omega) = X_R(\omega)$$
$$X_I(-\omega) = -X_I(\omega)$$

Q. $x(n) = [\underset{\uparrow}{1}, 2, 3, 4] \Rightarrow$ Discrete Aperiodic
$\Downarrow$ Periodic $\Downarrow$ continuous

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$
$$= \sum_{n=0}^{3} x(n)\, e^{-j\omega n}$$
$$= x(0)\, e^{0} + x(1)\, e^{-j\omega} + x(2)\, e^{-j\omega 2} + x(3)\, e^{-j\omega 3}$$
$$= 1 \cdot 1 + 2e^{-j\omega} + 3e^{-2j\omega} + 4e^{-3j\omega}$$
$$X(\omega) = 1 + 2e^{-j\omega} + 3e^{-2j\omega} + 4e^{-3j\omega}$$
$$= 1 + 2(\cos\omega - i\sin\omega) + 3(\cos 2\omega - i\sin 2\omega) + 4(\cos 3\omega - i\sin 3\omega)$$
$$= (1 + 2\cos\omega + 3\cos 2\omega + 4\cos 3\omega) - i(2\sin\omega + 3\sin 2\omega + 4\sin 3\omega)$$

$X_R(\omega)$ $\longrightarrow$ $(1 + 2\cos\omega + 3\cos 2\omega + 4\cos 3\omega)$

$X_I(\omega)$ $\longrightarrow$ $(2\sin\omega + 3\sin 2\omega + 4\sin 3\omega)$

$$|X(\omega)| = \sqrt{X_R^2(\omega) + X_I^2(\omega)}$$
$$\angle X(\omega) = \tan^{-1}\left(\frac{X_I(\omega)}{X_R(\omega)}\right)$$

$X(\omega) = 1 + 2e^{-j\omega} + 3e^{-2j\omega} + 4e^{-3j\omega}$ $a+ib$

complex exponential polar form, $re^{j\theta}$ → phase, r → magnitude

$x(n)$ is Purely imaginary

what all are frequency component, $[X(f)]$

20|10|2020

Spectrum → Who all are present ?
& how much ?
in the given signal

$x(t)$ CT Aperiodic
$x(n)$ DT Aperiodic
$x_p(t)$ CT Periodic
$x_p(n)$ DT Periodic

Q. $x(n) = [1, 2, 2, 1]$
Always prefer ω to be written in Polar form
$|X(\omega)|$
$\measuredangle X(\omega)$

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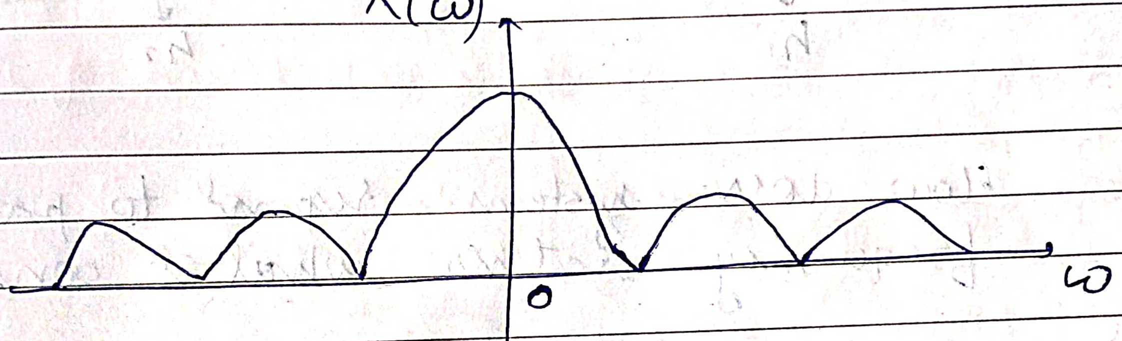
Extraction of information

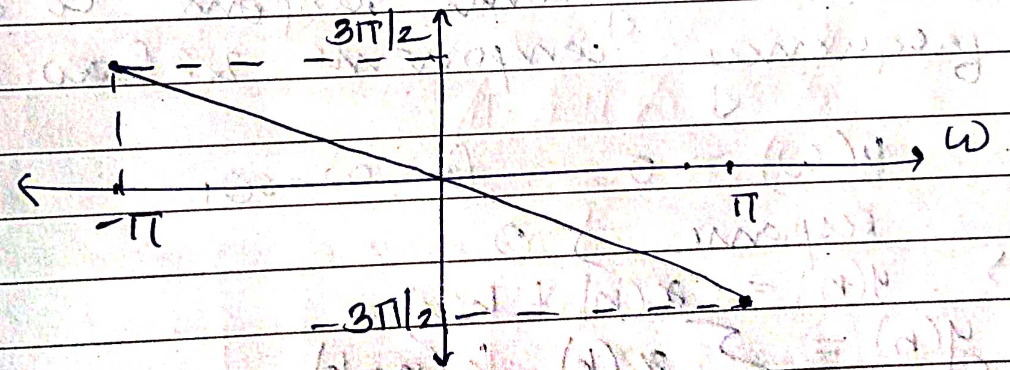
Here information is "Spectrum of signal"

$x(n) = (\underset{\uparrow}{1}, 2, 2, 1)$

$X(\omega) = 1 + 2e^{-j\omega} + 2e^{-2j\omega} + e^{-3j\omega}$

$X(\omega) = \left(4\cos \omega/2 + 2\cos 3\omega/2\right) e^{-j3\omega/2}$





System

$h(n) = (\underset{\uparrow}{1}, 2, 2, 1)$

$H(\omega) = ?$

Why did we require $h(n)$?

The form of the system was $H(\omega)$, $S(\omega)$ combination of or both

What is significance of $h(n)$?

We can compute $H(\omega)$

precipitate

$x(n) \rightarrow X(\omega)$ who all are present & how much

$h(n) \rightarrow H(\omega)$

$\omega_1\ \omega_2\ \omega_3\ \omega_4\ \omega_5$ ↓ h_1 $\omega_1\ \omega_2\ \omega_3\ \omega_4\ \omega_5$ ↓ h_2

How does system respond to particular frequency rather what it consists of.

→ How does system respond to different frequency component & how much

$H(\omega) = 0$ for $\omega = \omega_1$

Response $\Rightarrow 0$

$y(n) = x(n) * h(n)$

$y(n) = \sum x(k) \cdot h(n-k)$

$y(n) = \sum h(k)\, x(n-k)$

$x(n) \rightarrow$ signal

$X(\omega) \rightarrow$ Spectrum of $x(n)$

$h(n) \rightarrow$ system

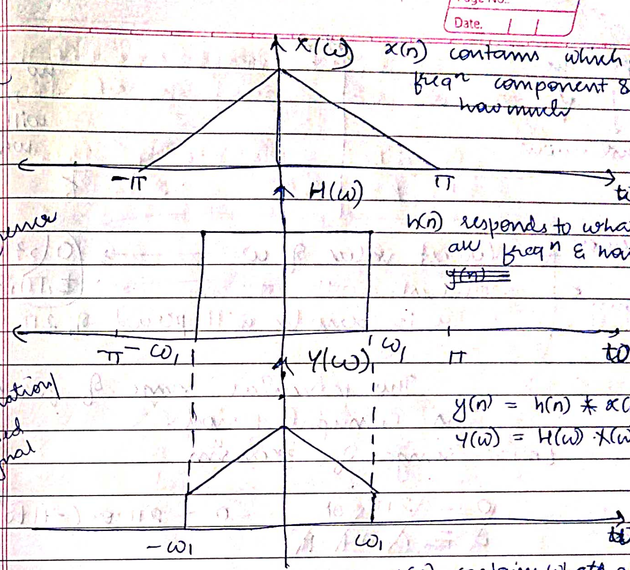
$H(\omega) \rightarrow$ Response of system

$y(n) \rightarrow$ o/p signal

$Y(\omega) \rightarrow$ Spectrum of O/P

$Y(\omega) = X(\omega) \cdot H(\omega)$

$X(\omega) \rightarrow$ undesired input



$x(n)$ contains which all freq^n component & how much

$h(n)$ responds to what all freq^n & how?

$y(n) = h(n) * x(n)$

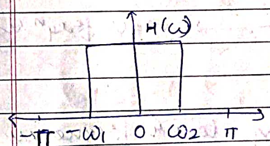
$Y(\omega) = H(\omega) \cdot X(\omega)$

$y(n)$ contains what all freq^n & how much

23/10/2020

① System can extract what info.

② signal contains what info.

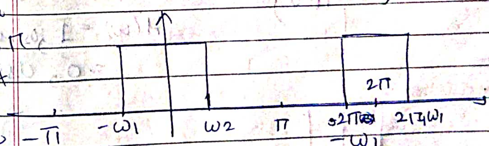


Impulse response $h(n) \rightarrow$ Aperiodic, DT

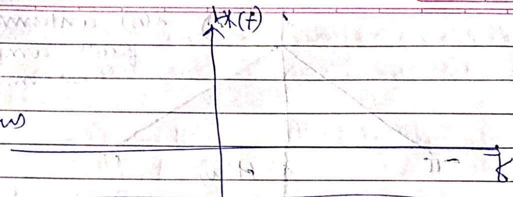
Aperiodic → Continuous in nature

$-\pi, \pi \rightarrow$ Periodic

If I write $-\pi$ to π thus it is periodic



Aperiodic ↓ continuous



ω : $-\pi$ to π will be written

0 region ← Lowest value of ω → 0, 2π, 4π ...

π region ← Highest —||— → $\pm\pi$, $\pm 3\pi$, $\pm 5\pi$

ω is periodic with period of 2π

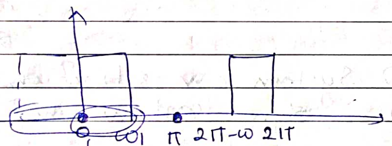
This particular range of freq is called as Principal Period.

Entire range of Spectrum

This is also a

0 – 2π OR ($-\pi$ to π)   0 – 2π or ($-\pi$ to π)

LPF



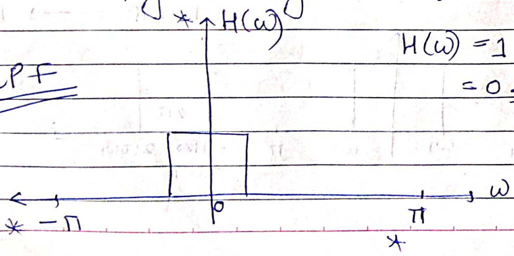
→ lower freqⁿ region

$\pm\pi \geq \omega > \omega_1$

$\omega > \omega_1$

It is simply allowing lower frequency

LPF

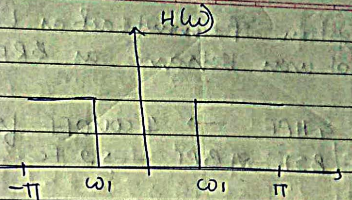


$H(\omega) = 1 ; 0 \leq |\omega| \leq \omega_1$

$= 0$; elsewhere

If I want precipitate

HPF

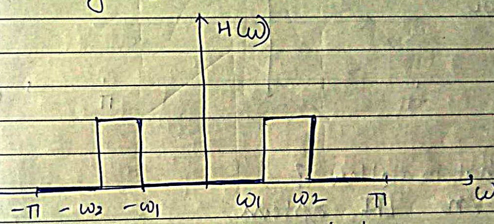


$H(\omega) = 1 ; \omega_1 \leq |\omega| \leq \pm\pi$

$= 0$; elsewhere

Range of spectrum → $-\pi$ to π

BPF



2 cutoff freqⁿ

$H(\omega) = 1 ; \omega_1 \leq |\omega| \leq \omega_2 ; \omega_2 > \omega_1$

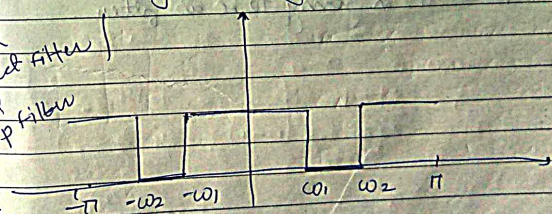
$= 0$; elsewhere

Basically allowing band of frequency

Band Reject Filter / Band stop filter

BSF

BRF



A selective reject

Role of ω_1 / ω_2 are important →

ω_1, ω_2 are cutoff frequencies

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Doubt

A system if cannot act as LPF therefore it can not even behave as BRF | BSF and BPF

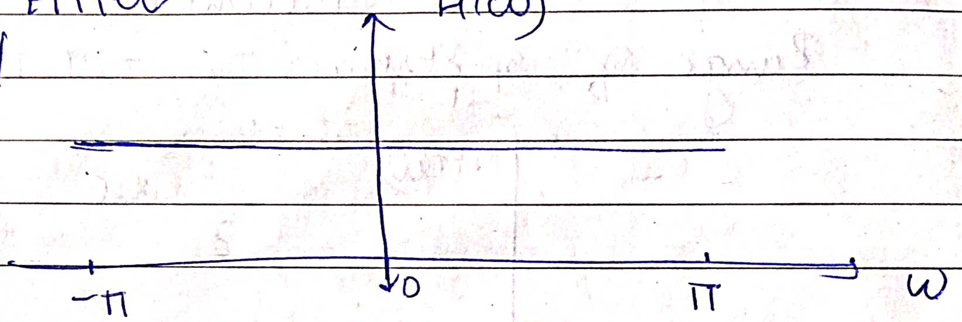
LPF & HPF ⟶ 1 cutoff freq^n

BRF | BSF or BPF ⟶ 2 fc

IF a system is not acting even a LPF nor as HPF therefore it cannot be BRF | BSF

All pass Filter

Flipflop / register



Q. What is a filter ?

⟶ An LTI system

$H(\omega)$ ⟶ from $h(n)$ ⟶ used for convolution ⟶ LT.I system.

Any LTI system whether you mean it or not it is going to be a filter.

$$= \frac{1}{4\pi^2 f^2}\left[e^{-j2\pi f} - 1 + e^{-j2\pi f}(j2\pi f)\right]$$

27/10/2020

FIR system

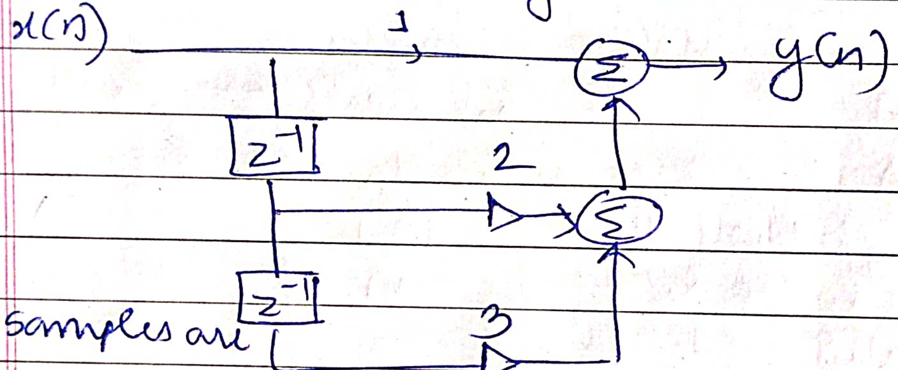
$h(n) = [\underset{\uparrow}{1}, 2, 3]$ — I.R

$H(z) = 1 + 2z^{-1} + 3z^{-2}$ — T.F

$\frac{Y(z)}{X(z)} = 1 + 2z^{-1} + 3z^{-2}$ — location of P. & Z

$y(n) = x(n) + 2x(n-1) + 3x(n-2)$ → D.E

Realization diagram



No. of samples are 3 therefore → Filter coefficient → 3 (1, 2, 3)

order of system → Second order

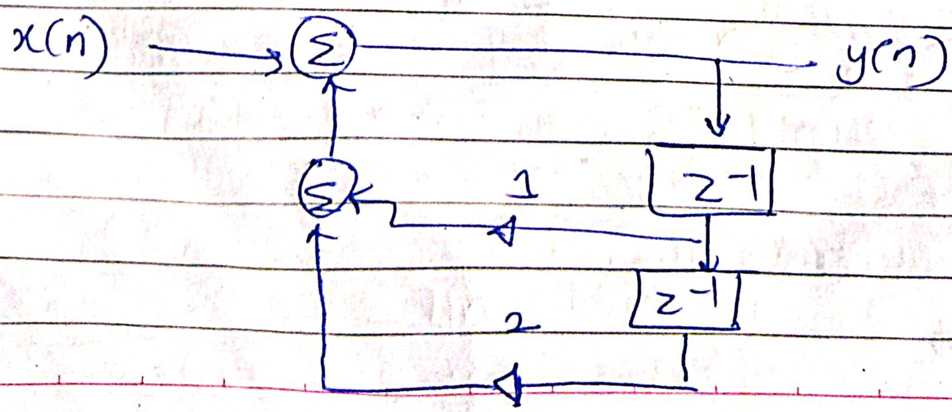
$$\frac{(z^2 + 2z + 3)}{z^2}$$

Length of Filter → 3

∴ Order is going to be → 2

if Length → 4 ∴ Order → 3

Q. $y(n) = x(n) + y(n-1) + 2y(n-2)$



$a^n u(n) + b^n u(n)$ — a & b all going to roots of this polynomial

$$Y(z) = X(z) + z^{-1}Y(z) + 2z^{-2}Y(z)$$

$$Y(z)\left[1 - z^{-1} - 2z^{-2}\right] = X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{1}{1 - z^{-1} - 2z^{-2}}$$

$$= \frac{z^2}{z^2 - z - 2} = \frac{1}{(z+1)(z-2)}$$

$$h(n) = A(-1)^n u(n) + B(2)^n u(n)$$

*** Filter coefficients they basically controls the location of poles & zeros of the system.

Q. $h(n) = \left(\frac{1}{2}\right)^n u(n)$

$$H(z) = \frac{z}{z - 1/2}$$

Order = 1   Length is ∞ (as IIR)

Q. $y(n) = x(n) + 2y(n-1) + 2y(n-2)$

Filter coefficients $\longrightarrow$ 1, +2, +2

$$Y(z) = X(z) + 2z^{-1}Y(z) + 2z^{-2}Y(z)$$

$$Y(z) - 2z^{-1}Y(z) - 2z^{-2}Y(z) = X(z)$$

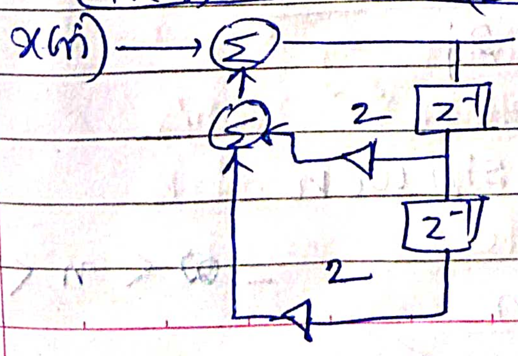
$$Y(z)\left(1 - 2z^{-1} - 2z^{-2}\right) = X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{1}{1 - 2z^{-1} - 2z^{-2}}$$

$$H(z) = \frac{z^2}{z^2 - 2z - 2} = \frac{z^2}{(z - 2.73)(z + 0.73)}$$

Length of IIR ∞

$$\boxed{h(n) = A(2.73)^n u(n) + B(-0.73)^n u(n)}$$



no. of filter coefficients $\rightarrow$ 3

Order $\longrightarrow$ 2

Samples $\longrightarrow \infty$

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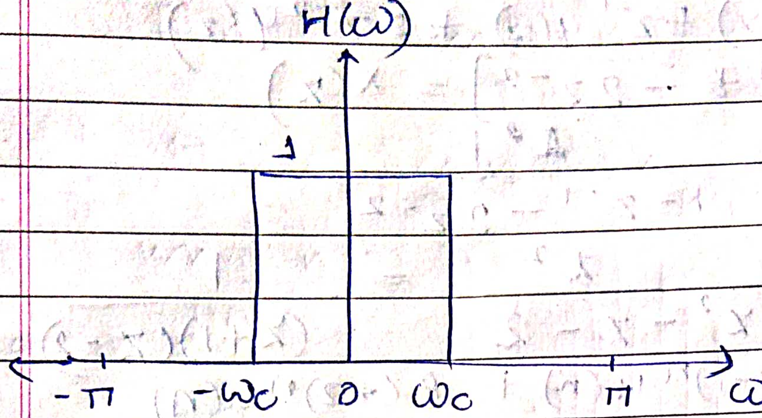
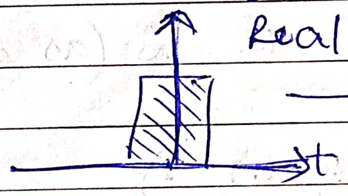


$H(\omega) = 1 \; ; \; 0 \le |\omega| < \omega_c$
$= 0 \; ; \;$ elsewhere

comment
→ Periodic ⟶ inverse discrete → h(n)
→ Even

LPF

H(ω) → Periodic ⟶ Discrete
Continuous ⟶ Aperiodic $(-\infty < n < \infty)$
LPF with cutoff ωc
Real ⟶ as H(ω) = 1



⟶ Pulse

$$h(n) = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} H(\omega)\, e^{j\omega n}\, d\omega$$

$$h(n) = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} 1 \cdot e^{j\omega n}\, d\omega$$

$$h(n) = \frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn}\right]_{-\omega_c}^{\omega_c}$$

$$h(n) = \frac{1}{2\pi jn}\cdot\left(e^{j\omega_c n} - e^{-j\omega_c n}\right)$$

$$h(n) = \frac{1}{\pi n}\,\frac{e^{j\omega_c n} - e^{-j\omega_c n}}{2j}$$

$$h(n) = \frac{1}{\pi n}\sin \omega_c n$$

$$h(n) = \frac{\sin \omega_c n}{\pi n} \qquad -\infty < n < \infty$$

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① $h(n) = [1, 5, 6, 7]$

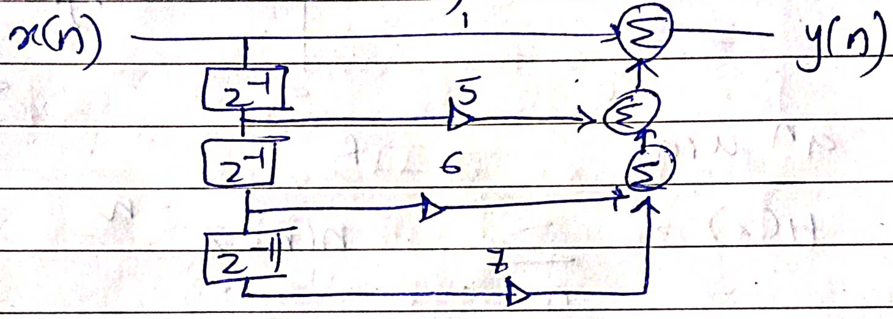
$$H(z) = 1 + 5z^{-1} + 6z^{-2} + 7z^{-3}$$

$$\frac{Y(z)}{X(z)} = 1 + 5z^{-1} + 6z^{-2} + 7z^{-3}$$

$$Y(z) = (1 + 5z^{-1} + 6z^{-2} + 7z^{-3})\,X(z)$$

$$Y(z) = X(z) + 5z^{-1}X(z) + 6z^{-2}X(z) + 7z^{-3}X(z)$$

$$y(n) = x(n) + 5x(n-1) + 6x(n-2) + 7x(n-3)$$



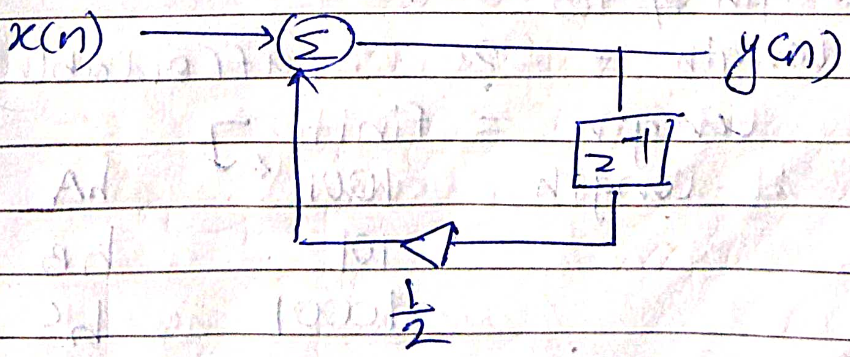
② $h(n) = \frac{1}{2}^{n} u(n)$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z}{z - \frac{1}{2}} = \frac{1}{1 - \frac{1}{2}z^{-1}}$$

$$Y(z)\left(z - \frac{1}{2}\right) = zX(z) \qquad \frac{Y(z)}{X(z)}\left(1 - \frac{1}{2}z^{-1}\right) = X(z)$$

$$Y(z)\,z - \frac{1}{2}Y(z) = zX(z) \qquad Y(z) - \frac{1}{2}z^{-1}Y(z) = X(z)$$

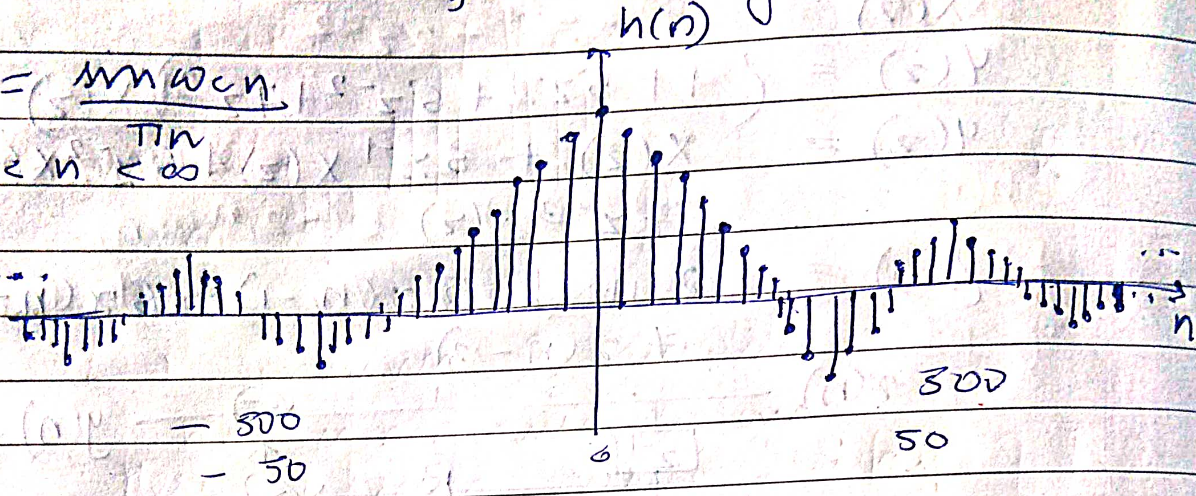
$$y(n+1) - \frac{1}{2}y(n) = x(n+1) \qquad y(n) = x(n) + \frac{1}{2}y(n-1)$$



(3) $h(n) = \dfrac{\sin \omega_c n}{\pi n}$, $-\infty < n < \infty$

Draw realization diagram

→ $h(n) = \dfrac{\sin \omega_c n}{\pi n}$
$-\infty < n < \infty$



LPF

$a^n u(n) \longrightarrow$ IIR

$$H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} \frac{\sin \omega_c n}{\pi n} z^{-n}$$

$$= \frac{1}{\pi} \sum_{n=-\infty}^{\infty} \frac{\sin \omega_c n}{n} z^{-n}$$

Not Summable

sinc do not have any series

→ If there is no series is existing
∴ we cannot implement IIR Implementation

→ Length is ∞
∴ We cannot realize using FIR Implementation.

Length $\neq \infty$

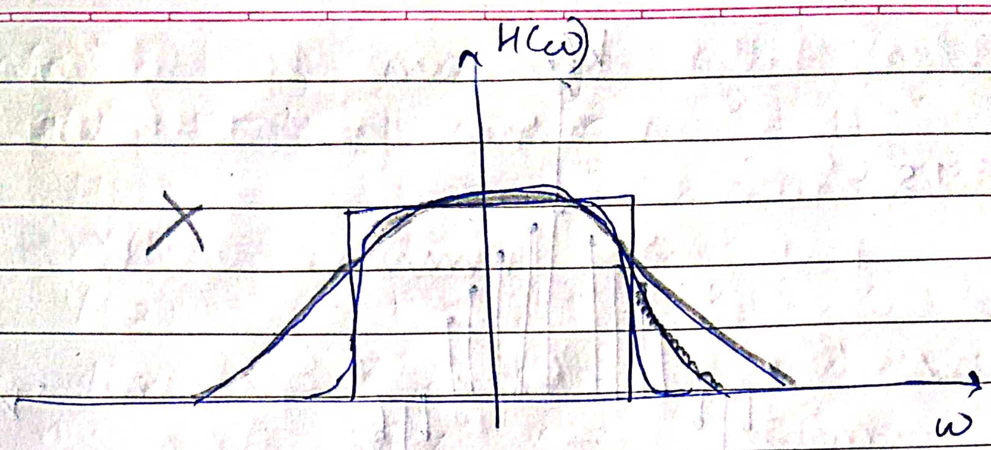
Length = ? (affordable)

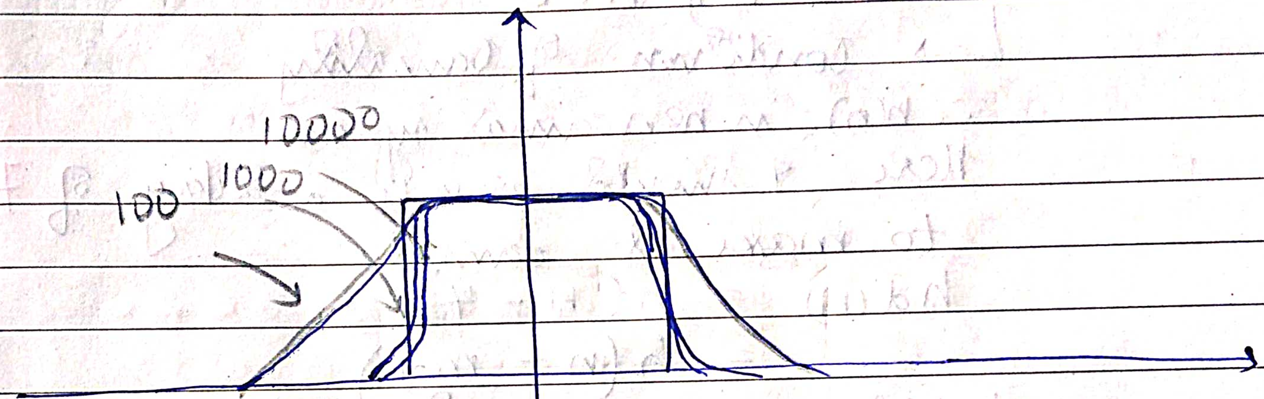
→ Length = Finite

Length = 1001   hA

101   hB

10001   hC





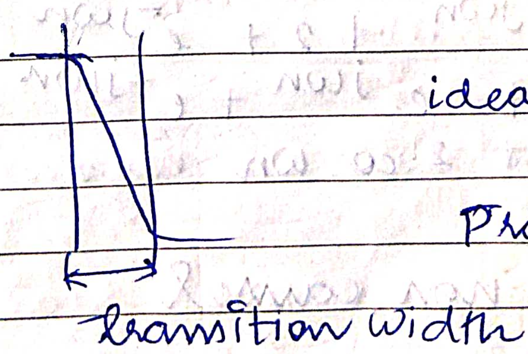
Ideal LPF → cannot be implemented

→ $H(\omega)$ → will not allowed outside ω_c

can be implemented
- hA → thoda ω_c ke bahar allow karegaa
- hB →
- hc →

hA, hB, hc → around cut off freqⁿ it will allow some

Practical



ideally → transition width = 0

Practically → Transition width must be less.

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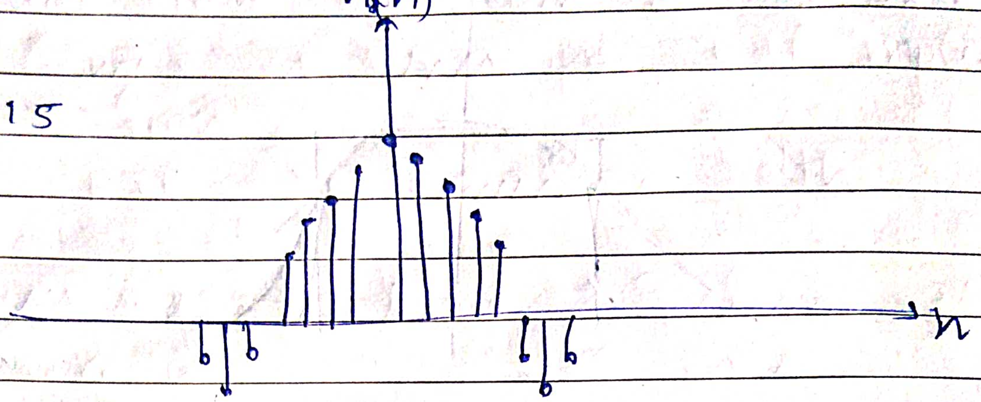
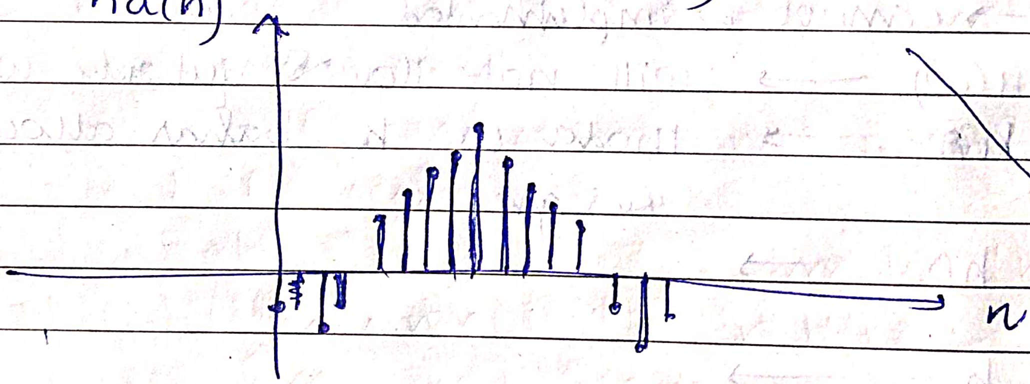
length = 15



$h(n) = 0 ; n < 0$
→ condition of causality
$\therefore h(n)$ is non causal systems
Here I should provide a delay of 7
to make it causal
$hd(n) = h(n-7)$
$= h\left(n - \frac{m-1}{2}\right)$



$h(n) = [1, \underset{\uparrow}{2}, 1]$

$H(\omega) = \sum_{n=-1}^{1} h(n) e^{-j\omega n}$

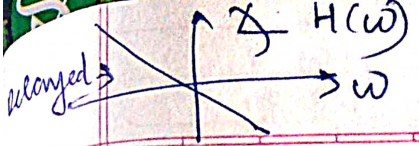
$H(\omega) = e^{j\omega n} + 2 + e^{-j\omega n}$

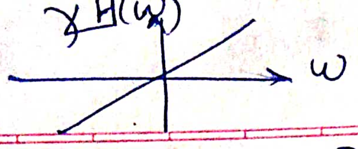
$H(\omega) = 2 + e^{j\omega n} + e^{-j\omega n}$

$H(\omega) = 2 + 2\cos\omega n$

$\therefore$ It is non causal

To make it causal we need to delay it by 1





IF spectrum is real then system h(n) is Non causal.

$$H_d(\omega) = [1, 2, 1]$$

$$H_d(\omega) = \sum_{n=0}^{2} h(n) \cdot e^{-j\omega n}$$

$$H_d(\omega) = h(0)e^{0} + h(1) \cdot e^{-j\omega} + h(2) \cdot e^{-2j\omega}$$

$$H_d(\omega) = 1 + 2e^{-j\omega} + e^{-2j\omega}$$

$$= 1 + 2e^{-j\omega} + e^{-j\omega} \cdot e^{-j\omega}$$

$$= e^{-j\omega}[e^{j\omega} + 2 + e^{-j\omega}]$$

$$= e^{-j\omega}[e^{j\omega} + e^{-j\omega} + 2]$$

$$= e^{-j\omega}\{2\cos\omega + 2\}$$

$$H_d(\omega) = (2 + 2\cos\omega)\, e^{-j\omega}$$

$$|H_d(\omega)| = |2 + 2\cos\omega|$$

$$\measuredangle H(\omega) = -\omega$$

$r e^{-j\theta}$: r → magnitude, θ → Phase

Sd → calibrated form of e^{jw}

$$H_d(\omega) = 1 \cdot e^{-j\left(\frac{M-1}{2}\right)\omega}$$

$$\measuredangle H(\omega) = -\left(\frac{M-1}{2}\right)\omega$$

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LP

① Ideal Filter ~~are~~ can be realized (why, no series for IIR implementa-tion)

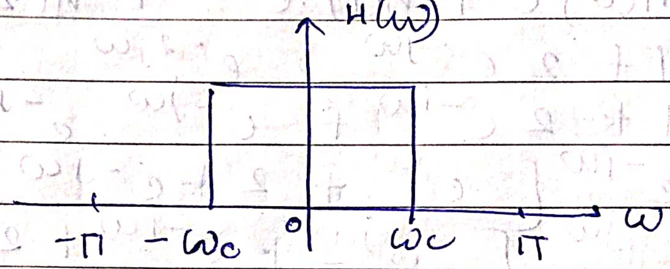
Length ↑ Transition width ↓

FIR → Transition width

② Causality → delay → Spectrum will be complex → -ve phase.

29/10/2020

ILPF



$$H(\omega) = 1 \; , \; 0 \le |\omega| \le \omega_c$$
$$= 0 \; , \; \text{elsewhere}$$

$$h(n) = \frac{\sin \omega_c n}{\pi n} \qquad -\infty < n < \infty$$

① h(n) has ∞ no. of complete samples but no series exist

∴ IIR realization not possible.

⇒ FIR realization

Take finite no. of samples 'm'

⇒ Transition width in H(ω)

↑m ⇒ ↓ Transition width

② $h(n) \neq 0 \; ; \; n < 0$

Non causal system

∴ make it causal by introducing a delay of $\left(\frac{m-1}{2}\right)$

⇒ it contributes a negative phase is H(ω)

H(ω) is no more real

$$h_{new}(n) = h\left(n - \frac{m-1}{2}\right)$$

$$H_{new}(\omega) = H(\omega)\, e^{-j\left(\frac{m-1}{2}\right)\omega}$$

$\angle H_{new}(\omega) = -\left(\frac{M-1}{2}\right)\omega$

Z is complex number

$Z = re^{j\omega}$

$z = 1 \cdot e^{j\omega}$ (if $r = 1$ i.e unit circle)

because when computing on unit circle it is consider that system is BIBO stable

$\omega \rightarrow -\pi$ to π $(1+j0)$

Lowest $\omega \Rightarrow 0$ $\therefore z = 1\angle 0°$

Highest $\omega \Rightarrow \pm\pi$ $(-1+j0)$ $1\angle 180°$

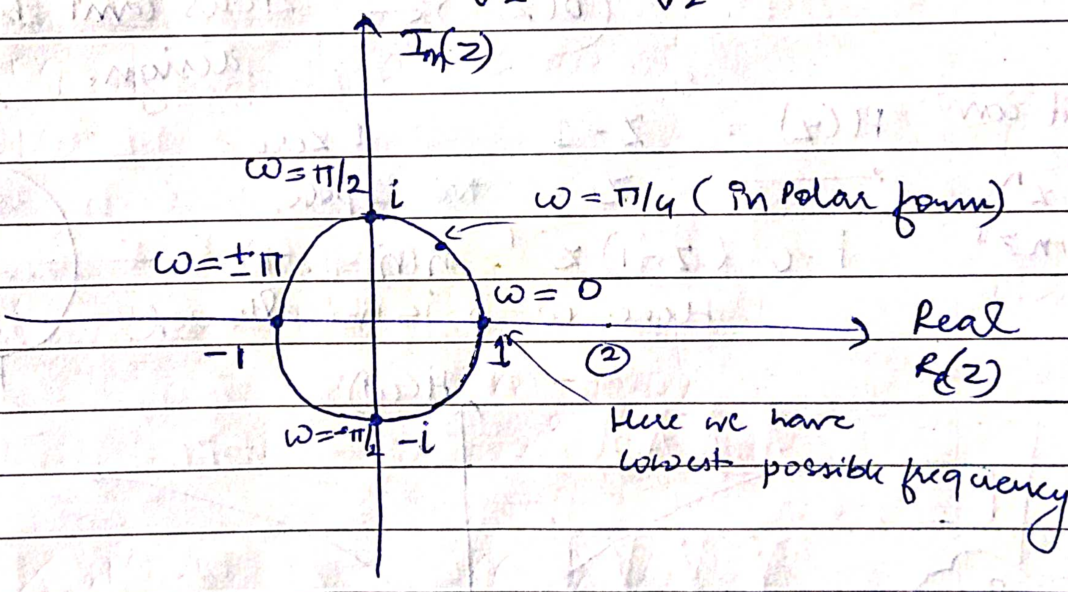
if $\omega=\pi \longrightarrow e^{j\pi} = -1$ $(\cos\pi + j\sin\pi)$

if $\omega = -\pi \longrightarrow e^{-j\pi} = -1$ $(\cos\pi - j\sin\pi)$

if $\omega = \pi/2 \Rightarrow z = i = 1\angle 90°$

$\omega = \pi/4 \Rightarrow \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}} = 1\angle 45°$

unit circle



If we move clockwise / anticlockwise from $\omega = 0$ I will reach to highest frequency.

if $z = 2$

$\therefore z = 2+j0 \longrightarrow \omega = 0$

if $z = -2$

$z = -2+j0 \longrightarrow \omega = \pm\pi$

$H(z)$ $H(\omega)$
$\downarrow$
$H(re^{j\omega})$

Designing $H(\omega)$ ⟶ find $h(n)$ → realization diagram

$H(z) = \frac{N(z)}{D(z)}$ → zeros, → Poles

$(\underset{\uparrow}{1}, 2, 1)$

3 Samples
$\downarrow$
FIR

$\frac{1}{2}^n u(n)$ ← coefficient is finite

∞ samples
$\downarrow$
Poles are not at origin

$H(z) = 1 + 2z^{-1} + z^{-2}$

$= \frac{z^2 + 2z + 1}{z^2}$

All poles are at origin → FIR

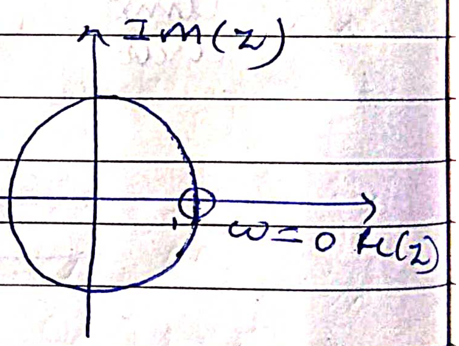
In FIR, $H(z) = \frac{N(z)}{D(z)}$ ← Poles can't be used in design.

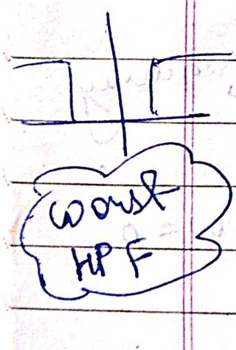
it can be z^n, $n \geq 1$

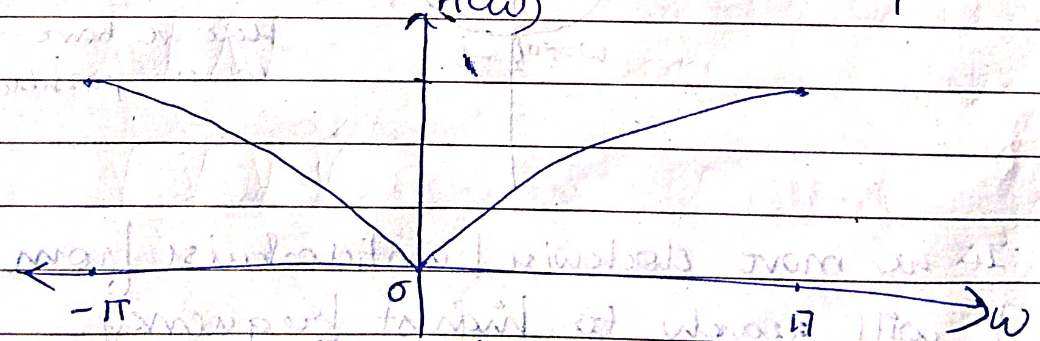
$H(z) = \frac{z-1}{z}$ → 1 zero, → 1 pole

i.e $(z-1)z^{-1}$ $h(n) = (1, -1)$

Here $\omega = 0$ is not allowed







$H(\omega) = 1 - e^{-j\omega}$

$= e^{-j\omega/2}\left(e^{j\omega/2} - e^{-j\omega/2}\right)$

$= 2j \sin\frac{\omega}{2} \cdot e^{-j\omega/2}$

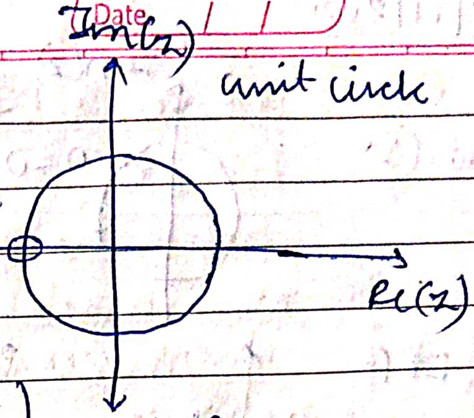
$|H(\omega)| = \left|2 \sin\frac{\omega}{2}\right|$

length is 1 → can u design 0 order system
order 0

order = 1
length = 2

$H(z) = \frac{z+1}{z}$

$H(z) = 1 + z^{-1}$ $h(n) = [1, 1]$



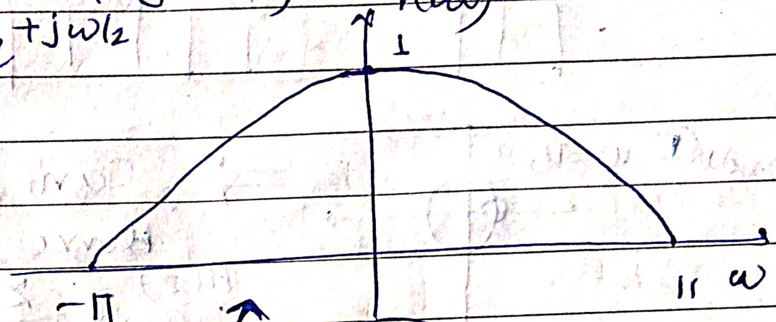
response → 0 at $\omega = \pm\pi$

$H(\omega) = 1 + e^{j\omega}$

$= e^{+j\omega/2}\left(e^{-j\omega/2} + e^{+j\omega/2}\right)$

$= 2\cos\frac{\omega}{2} \cdot e^{+j\omega/2}$

$H(\omega) = \left|2\cos\frac{\omega}{2}\right|$



$m \to \infty \to$ ILPF

$m \to 2 \to$

worst LPFilter

all pass filter

first order → Atleast 2 coefficients

10001 → 1001 → 101 → 2 (min)

Best

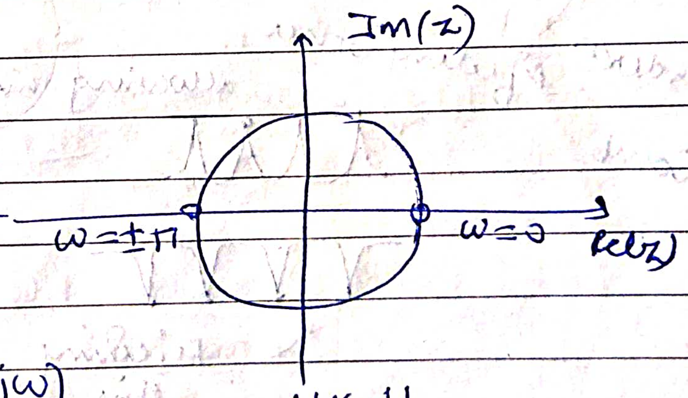
worst

→ If zero at $\omega = \pm\pi$ → It can never behave as HPF

→ If zero at $\omega = 0$ → It can never behave as LPF

$H(z) = \frac{(z-1)(z+1)}{z^2}$

$= \frac{z^2 - 1}{z^2} = 1 - z^{-2}$

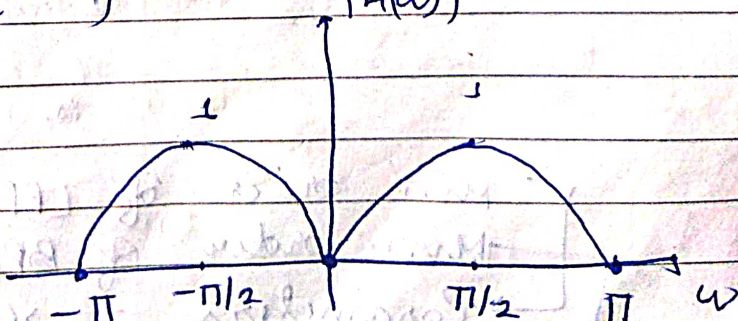


$H(\omega) = (1 - e^{-2j\omega})$

$= e^{-j\omega}\left(e^{j\omega} - e^{-j\omega}\right)$

$= e^{-j\omega}\, 2\sin\omega$

$|H(\omega)| = |2\sin\omega|$



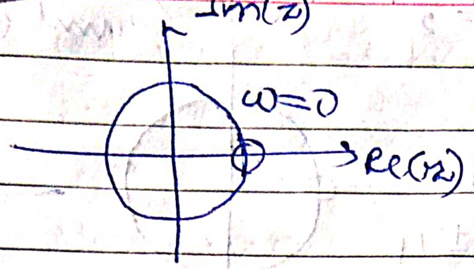
worst ever Band pass

worst BPF

$H(z) = \frac{z-1}{z} = 1 - z^{-1}, \; h(n) = (\underset{\uparrow}{1}, -1)$ Page No: Date.

3/11/2020

order (1)

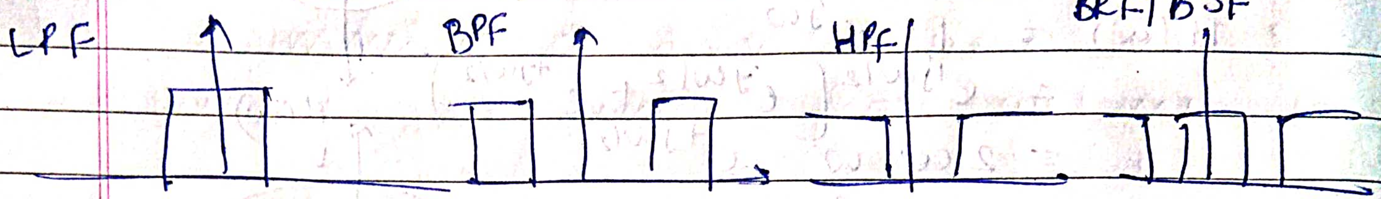


It will not allow $\omega = 0$

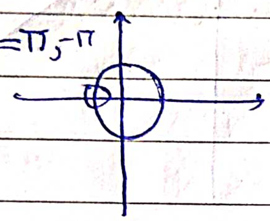
$\Rightarrow$ $\boxed{\text{HPF (worst HPF)}}$

It cannot be LPF

Hence It can't be BRF / BSF



Order (1) $\omega = \pi, -\pi$

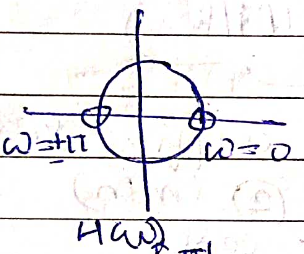


$\Rightarrow$ Can't be HPF

Hence can't be BRF / BSF

$H(z) = \frac{z+1}{z} = 1 + z^{-1}, \; h(n) = (\underset{\uparrow}{1}, 1)$

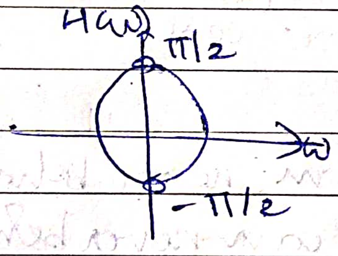
order (2)



$\Rightarrow$ Can't be LPF as well as HPF

Hence can't be BRF / BSF

$H(z) = \frac{(z-1)(z+1)}{z^2} = \frac{z^2-1}{z^2}, \; h(n) = (1, \underset{\uparrow}{0}, -1)$



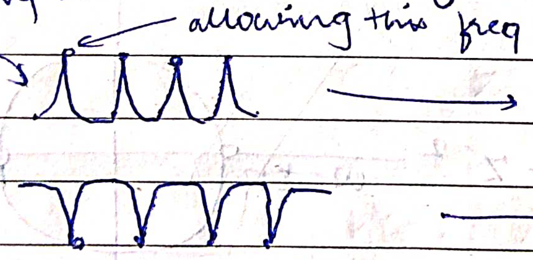
$\Rightarrow$ All frequency other than $\omega = \pm \pi/2$

Notch / BRF with $\omega = \pi/2$

It is best one freq^n reject filter but worst BRF

order (2) rejecting other

Comb



$\longrightarrow$ BRF

$\longrightarrow$ BRF

$\hookrightarrow$ Band

$\frac{(z-i)(z+i)}{z^2}$

$= \frac{z^2 - i^2}{z^2}$

$= \frac{z^2+1}{z^2}$

$= 1 + z^{-2}$

$h(n) = (1, \underset{\uparrow}{0}, 1)$

- Min. order of LPF / HPF $\longrightarrow$ 1
- Min. order of BPF / BRF $\longrightarrow$ 2
- Parameters $\longrightarrow$ ω_c
- Parameters $\longrightarrow$ ω_{c1}, ω_{c2}

$$H(z_1) = \sum_{p=0}^{m-1} h(p)\, z_1^{-p}$$

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*** Whenever I have a ~~linear~~ phase as a Linear function of frequency then it is called as Linear Phase otherwise it is Non linear phase

*** Symmetric or antisymmetric ($h(n)$) sequences will give linear phase systems

$m = 4$ $\quad (1\ 2\ 2\ 1)$ $\quad 1\ 2\ 3\ 4$

symmetric sequence $\quad n=1 \quad n=2 \quad h(1)$

Condition for symmetry $\quad h(n)$ of length m $(0 \text{ to } m-1)$ $\quad h(m-1+n)$

$$\rightarrow h(n) = h(m-1-n) \qquad h(4-1+0)$$

$$h(0) = h(5-1-0) = h(4) = h(3)$$

$$h(1) = h(5-1-1) = h(3)$$

Condition for antisymmetry $\rightarrow h(n) = -h(m-1-n) \qquad h(4-1+1)$

$$h(4)$$

$$H(z) = \sum_{n=0}^{m-1} h(n) \cdot z^{-n} \qquad h(4+1-1) \quad h(4-1+2)$$

$$h(4) \qquad h(4+1)$$

$$H(z) = \sum_{n=0}^{m-1} h(m-1-n)\, z^{-n} \quad \text{— for symmetric}$$

$$m-1-n = p \Rightarrow n=0 \quad p = m-1, \quad n = m-1 \quad p=0$$

$$-n = p - m + 1 = p-(m-1) \qquad 1\ 2\ 3\ 4\ 5$$

$$H(z) = \sum_{p=m-1}^{p=0} h(p)\, z^{p-(m-1)} \qquad 1\ 3\ 0\ -3\ -1$$

$m = 5$

1 to 5

$$= \sum_{p=0}^{p=m-1} h(p)\, z^{p} \cdot z^{-(m-1)} \qquad -h(5+1-n)$$

$$h(1) = -h(5+1-1)$$

$$= z^{-(m-1)} \sum_{p=0}^{m-1} h(p)\, z^{p} \qquad = -h(5)$$

$$h(2) = -h(5+1-2)$$

$$= z^{-(m-1)} \sum_{p=0}^{m-1} h(p)\, (z^{-1})^{-p} \qquad h(2) = -h(5-1)$$

$$= -h(4)$$

*** $$H(z) = z^{-(m-1)}\, H(z^{-1})$$

In general $\longrightarrow$ $H(z) = \pm z^{-(m-1)} H(z^{-1})$

$\longrightarrow$ For symmetric or antisymmetric sequences

$H(\omega) = z - 1 \Rightarrow z = 1$

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OR $H(1) = 0$

→ for Linear phase

$z = z_0$ is the zero of $H(z) \Rightarrow H(z_0) = 0$

This doesn't means z_0 is 0

$H(z) = \pm z^{-(m-1)} H\left(\frac{1}{z}\right)$

If $z = z_0$ be the zero of $H(z)$

$H(z_0) = \pm z_0^{-(m-1)} H\left(\frac{1}{z_0}\right) = 0$

$a \cdot b = 0$

either $a = 0$ or $b = 0$ or both $a = b = 0$

$z_0 \neq 0$

$H(1/z_0) = 0$ must always be zero

$H(z) = 0$

$H(z) = \frac{z-3}{z}$ ∴ $z = 3$ is zero of $H(z)$

∴ $1/z_0$ is zero of $H(z)$

If z_0 is zero then its reciprocal $1/z_0$ must also be zero (CS)

4/11/2020

$h(n) = h_0 h_1 h_2 h_1 h_0 \longrightarrow l = 5$

$H(z) = \frac{(z - 1/2)}{z}$

$H(z) = \frac{\left(z - \frac{1}{2}\right)(z-2)}{z^2}$

HPF

$H(z) = 1 - \frac{1}{2} z^{-1}$

$H(z) = \frac{z^2 - 2.5z + 1}{z^2}$

$h(n) = \left(\underset{\uparrow}{1}, \frac{-1}{2}\right) \longrightarrow$ Not symmetric

$H(\omega) = 1 - \frac{1}{2} e^{-j\omega}$

$H(z) = \frac{z-1}{z}$

$H(z) = 1 - z^{-1}$

$h(n) = (\underset{\uparrow}{1}, -1)$

This itself is Linear phase

∴ No need to write $(z-1)(z-1)$