⑤ Polar plot of $\left[1+2\zeta\left(\frac{j\omega}{\omega_n}\right)+\left(\frac{j\omega}{\omega_n}\right)^2\right]^{-1}$

→ $G(j\omega) = \dfrac{1}{1+2\zeta\left(\frac{j\omega}{\omega_n}\right)+\left(\frac{j\omega}{\omega_n}\right)^2}$

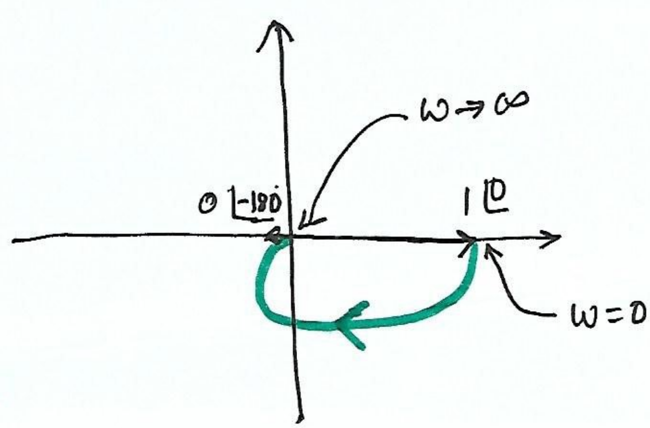
→ When $\omega = 0$, $1\angle 0$

→ When $\omega \to \infty$, $0\angle -180^\circ$

→ Thus the polar plot starts at $1\angle 0$ and ends at $0\angle -180^\circ$ as ω increases from 0 to ∞.

→ The negative real axis is tangent to the high frequency portion of $G(j\omega)$.

→ The values of $G(j\omega)$ in the frequency range of interest can be calculated directly at different values of ω.

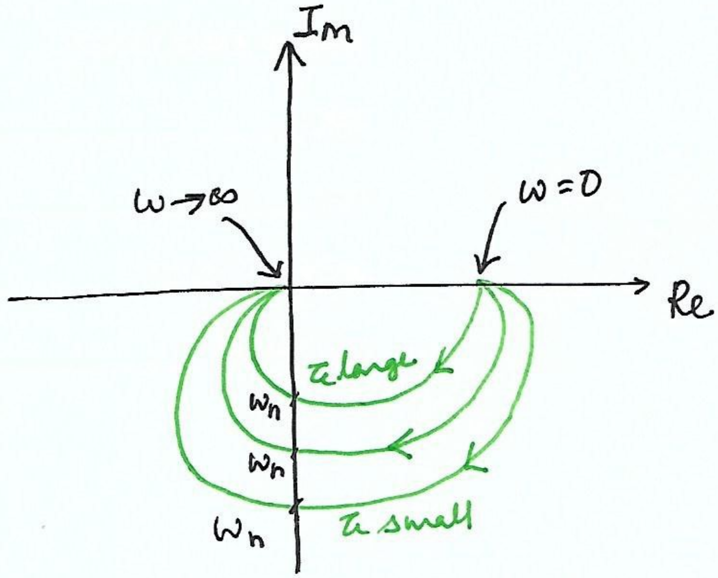


→ The exact shape of the polar plot depends on the value of ζ.

→ For the underdamped case at $\omega = \omega_n$,

$$G(j\omega_n) = \frac{1}{j2\zeta} \quad \text{ie} \quad \frac{1}{2\zeta} \angle -90^\circ$$

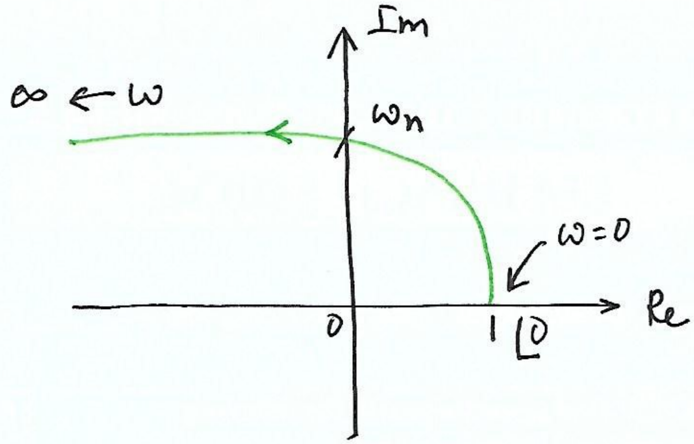
→ Thus the frequency at which the plot intersects the imaginary axis is the undamped natural frequency, ω_n.



⑥ POLAR PLOT OF $\left[1 + 2\zeta\left(\frac{j\omega}{\omega_n}\right) + \left(\frac{j\omega}{\omega_n}\right)^2\right]^{+1}$

→ When $\omega = 0$, $1 \angle 0^\circ$

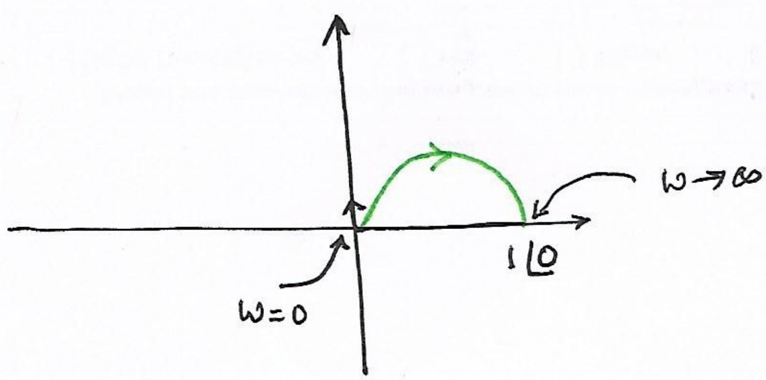
→ when $\omega \to \infty$, $\infty \angle 180^\circ$


Im
∞ ← ω
ωn
ω=0
0
1 ∠0
Re

Pb (37) Sketch the polar plot of $\frac{j\omega T}{1+j\omega T}$

Solution
(i) when $\omega = 0$, $0 \angle 90$

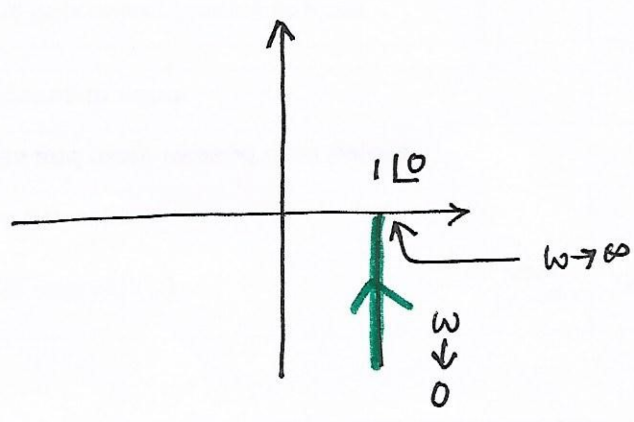
(ii) when $\omega \to \infty$, $1 \angle 0$



Pb (38) Sketch the polar plot of $\frac{1+j\omega T}{j\omega T}$

Solution
(i) when $\omega = 0$, $\infty \angle -90$

(ii) when $\omega \to \infty$, $1 \angle 0$



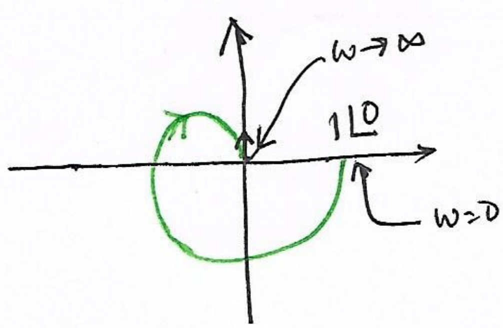
Pb ③⑨ Sketch the polar plot of

$$\frac{1}{(1+j\omega T_1)(1+j\omega T_2)(1+j\omega T_3)}$$

Solution

⊙ When $\omega = 0$, $1\angle 0$

⊙ When $\omega \to \infty$, $0\angle -270$



Pb ④⓪ Given the transfer function

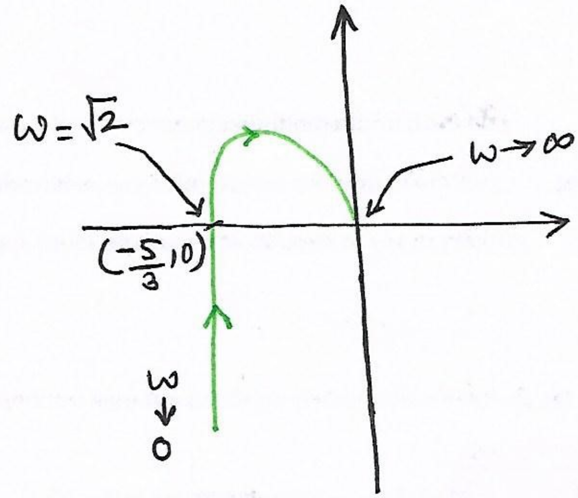
$$G(s)H(s) = \frac{10}{s(s+1)(s+2)},$$

make a rough sketch of the polar plot of $G(j\omega)H(j\omega)$. Does the plot intersect any of the axes? If so, what are the coordinates of the point of intersection?

Solution ⊙ $G(j\omega)H(j\omega) = \dfrac{10}{j\omega(1+j\omega)(2+j\omega)}$

⊙ When $\omega = 0$, $\infty\angle -90$

⊙ when $\omega \to \infty$, $0\angle -270$



- To find the point of intersection of the $G(j\omega)H(j\omega)$ curve on the real and imaginary axis of the $G(j\omega)H(j\omega)$ plane, it has to be rationalized.

- $$G(j\omega)H(j\omega) = \frac{10(-j\omega)(1-j\omega)(2-j\omega)}{j\omega(1+j\omega)(2+j\omega)(-j\omega)(1-j\omega)(2-j\omega)}$$

- $$G(j\omega)H(j\omega) = \frac{10(-j\omega-\omega^2)(2-j\omega)}{\omega^2(1+\omega^2)(4+\omega^2)}$$

$$= \frac{10(-j2\omega-\omega^2-2\omega^2+j\omega^3)}{\omega^2(1+\omega^2)(4+\omega^2)}$$

$$= \frac{-30\omega^2 - j\,10\omega(2-\omega^2)}{\omega^2(1+\omega^2)(4+\omega^2)}$$

- Put Imaginary part $= 0$

$$\frac{-10\omega(2-\omega^2)}{\omega^2(1+\omega^2)(4+\omega^2)} = 0$$

- $$-10w(2-w^2) = 0$$

$$2-w^2 = 0$$

$$w = \pm\sqrt{2}$$

- The point of intersection with the real axis is found by substituting $w=\sqrt{2}$ in the real part.

- $$\frac{-30w^2}{w^2(1+w^2)(4+w^2)} = 0$$

$$\frac{-30(2)}{2(3)(6)} = \frac{-5}{3}$$

- The plot intersects the negative real axis at $\left(-\frac{5}{3}, 0\right)$

NYQUIST STABILITY CRITERION

- Nyquist analysis is a graphical procedure for determining absolute and relative stability of closed loop control systems.

- For a system with CLTF $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$ to be stable, all the roots of the characteristic equation $1+G(s)H(s) = 0$ must lie in the LHSP.

- The Nyquist stability criterion is one which relates the open loop frequency response $G(j\omega)H(j\omega)$ to the number of zeros and poles of $1+G(s)H(s)$ that lie in the RHSP.

- Nyquist stability criterion is based on a theorem from the theory of complex variables.

- Consider the following:

→ ① Given the OLTF, $G(s)H(s)$, Nyquist stability criterion helps to predict about closed loop stability.

→ ② ⊙ For a closed loop system to be stable, the poles of $\frac{G(s)}{1+G(s)H(s)}$ must lie in the LHSP.

⊙ In other words the zeros of $1+G(s)H(s)$ must lie in the LHSP.

- If $G(s)$ and $H(s)$ are known individually, we can find CLTF as $\dfrac{G(s)}{1+G(s)H(s)}$.

- But if $G(s)H(s)$ is given as a single transfer function, we cannot find out CLTF.

- But we can find $1+G(s)H(s)$ and get information about zeros of $1+G(s)H(s)$.

- The zeros of $1+G(s)H(s)$ are nothing but the closed loop poles.

→ (3) The poles of $G(s)H(s)$ and the poles of $1+G(s)H(s)$ are the same.

eg $$G(s)H(s) = \frac{K(s+1)}{(s+2)(s+3)(s+4)}$$

$$1+G(s)H(s) = \frac{(s+2)(s+3)(s+4)+K(s+1)}{(s+2)(s+3)(s+4)}$$