Ronaldo Music
shakira

Page No:
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Discrete → sampled signal → discrete intervals → no. of samples are finite but values are from infinite set of numbers
i.e, 2 → 10 points

* Process of approximation / rounding off → Quantization
→ Values of finite no. of sample comes from finite set.
→ Naturally all signals around you are CTS
Temperature → continuous quantity
Fan → Runs continuously.

CT, & DT → cannot be processed & store in digital devices
& Digital → can be processed & store in digital devices
→ When we group CT & DT together as both cannot be process & store in digital devices then this both signals are collectively called as Analog signals.

Magnetic Tape, VCR, Gramophone

Analog → CT, DT } ASP (Analog) , CTSP, DTSP, Simple DP

→ * Analog signal can be processed in Analog devices. ↓ anything to extract

11/8/2020

→ some signals which are not useful for us but can be useful for others thus it is noise for us.
→ Noise is relative quantity i.e not absolute.

* We are not involve in one field. What we not want is noise

Page No: Opinion
$2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$

Footnote 1 :-

How would you decide no. of samples on x-axis in process of sampling ?
(it must be estimated) — Personal opinion / Fact (Might be)
Better Fellow → Michael Faraday

Footnote 2 :-

(process on no. of bit required)

How many intervals to be taken on the y-axis in the process of Quantization ?
1 to 10 → 5 (1, 3, 5, 7, 9) upto 8 levels
→ 20 (0.5 → 10) upto 32 levels
→ 2 (1 ... 10) Binary signal

→ What do you think of these three representation ?
- They all are different digital signals
- They all are digital signals

Footnote 3 :-

Every binary signal is digital signal but converse is not true.

13/8/2020.

Steps → Sampling & Quantization
n = bits .. upto 2^n representation

State of mind

Forms of ~~signals~~ systems :-
① Hardware
② Software
③ Combination of both
Education system → algorithm
To Extract information from DS

* System → may respond or may not
→ System will process ~~sin~~ signal
→ Place of processing is this system

digital device → system

The form of signal cannot be categorize as Hlw, Slw or both

Me (Hlw & Slw) & Soap (Hlw) & its fragrance (Signal)

Footnote 4 & 8 When the form of signal & systems are different how do you integrate to make them work together ?

→ Either by changing signal or systems

Signal is physical quantity

Q → signal ↓ From different system

A → signal cannot be calculated

Will categorizes on their response ↓

poke → for an instant or a moment

impulsive behaviour

poking of needle → impulse

Aah, Ouch → impulsive response

→ one of the ways It is that way to describe & categorize different system

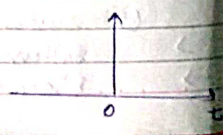
14/8/2020

Ouch → Pain is unbearable or unquantifiable
- unaware of situation
- (pain) Intensity tending to infinity

It lasted for t tending to zero

→ unbearable pain for an instant

⊕ Impulse signal &



Other examples → when powerful electric current

Intensity or Amplitude → $\delta(t) = \infty$ for $t = 0$

$\delta(t) \neq \infty$ for $t \neq 0$

$= 0$

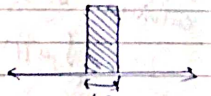
Lightening → Electric current / charge

When impulse is like a pulse (pain becomes bearable) thus intensity become finites

Integration / Area: if $t = 0$, $A = \infty$

if t = finite, A = finite

If h x width = Area = 1 (We are fixing)



$\delta(t) \leftarrow \nabla t \quad \tau = 1$

$\int_{-\infty}^{\infty} \delta(t)\,dt \leftarrow \int_{-\infty}^{\infty} \nabla t\,dt = 1$ — Dirac Delta Function

$\delta(t) \leftarrow \nabla t = \infty$ for $t = 0$

Original constraint: $A = 1$ (Area under the pulse)

$\int_{-\infty}^{\infty} \delta(t)\,dt \leftarrow \int_{-\infty}^{\infty} \nabla t\,dt = \int 1$ → unit Impulse signal

Perceive

⇓

Conceive → to deliver something physical form

→ Series of impulse ↑↑↑↑ (↔ min) ‖ (↔ 1 min) ‖ (↔ 1 sec)

→ constant signal → Step signal

(series of impulse with the distance bet^{n} two impulses = 0)

unit step signal → A = 1

⊕ ways for characterising systems

Impulse → Step → Ramp → Parabolic

→ is Integration

This idea is conceive → Dirac
The quality of being useful

Footnote 5 &

What is the utility of impulse response & physical significance of impulse?

18/8/2020

Evaluation parameter → whether you own it or not

3 day extended

$\int_{-\infty}^{\infty} \delta(t)\,dt = u(t) \Rightarrow$ step

$\int_{-\infty}^{\infty} u(t)\,dt = \int_{-\infty}^{\infty} dt = [t]_{-\infty}^{\infty} \Rightarrow$ Ramp

$\int_{-\infty}^{\infty} t\,dt = \left[\frac{t^2}{2}\right]_{-\infty}^{\infty} \Rightarrow$ Parabolic / Parabola

— wait for some speech to be deliver
Impulse, parabolic, step, ramp → like hello, hi, How r u?

↳ They are not actual signal which contains information.

footnote 8 &

How will you get the response?
What shall I do with the signals & system to get information?
Shubh, Diwana, Sayali.

It represents the system, characterize the systems, response of impulse

Juri members } $H(1), H(2), H(3) \ldots H(K)$
Juri

$x(1)\ H(1)$
$H(2)\ \ x(7\ -2) = y(n)$
$x(2)\ \ x(5) = y(n)$

$x \rightarrow$ participants (slg)
$n \rightarrow$ variable for the no. of people
$k \rightarrow$ Total no. of participants

$$\sum_k H(k)\ x(n-k) = y(n)$$

($H(k)$: Impulse Response; $x(n-k)$: Participants; $y(n)$: no. of samples we have)

① Queue / line up / Flip $(-k)$
② Shifting / Participants coming
③ Interaction / multiplication
④ Adding / Summing (Aggregate)

$$\sum_k x(k)\ H(n-k) = y(n)$$

(+) $(n+k)$ is also valid

21/08/2020

If $J_1, J_2, J_3 \longrightarrow$ L to R
then $P_1\ P_2\ P_3 \longrightarrow$ R to L
} either one must flip up

Synthesis → Same I/P to different systems
Analysis → Different I/P to same system

Judges ← $J = (J_1\ \ J_2\ \ J_3)$ → Here they are independent

Participants ← $P = (P_1\ P_2\ P_3\ P_4\ P_5 \ldots)$

$J_1 \quad J_2 \quad J_3$

$P_4\ P_3\ P_2\ P_1$
→

$P_4\ P_3\ P_2\ P_1$ $J_1 P_1$
→

$P_4\ P_3\ P_2\ P_1$ $J_1P_2 + J_2P_1$
→

$P_4\ P_3\ P_2\ P_1$ .....
→

Last one

↳ This process will continue unless and until P_4 goes out

$$y(n) = \sum_k h(k)\, x(n-k)$$

$$y(n) = \sum_k x(k)\, h(n-k)$$

Here participants are still & judges are moving (flip judges)

Answer is same (If you repeat)

Q. Do you love football

→ Shubh : No

System is unique

In synthesis if it is ask then we will get different responses. (sing)
o/p is unique → Analysis

Analysis → Fixed
Synthesis → different → o/p is different

System is not unique
soln is of synthesis is not unique
→ system which you have find out
I/P, o/P are decided we have to find H(s)
→ qualitative subjective one

& which one is easier Analysis or synthesis?
Analysis → less explorative
Synthesis → chance of exploration
In DC & AC Analysis → answer is unique
Design → synthesis

Q Charging & Discharging of capacitor
Analysis → Process is Fixed
→ whether you are convinence with equation
→ If one is trying to manipulate then what to do.

25/08/2020

Ananya in train → shabby & well uniformed

I am trying to investigate where to apply or where not (under some conditions)

doesn't make difference to sir

Sir deliver lecture at 11:30 am → understanding level is high almost 90%

11 → 3:00 am → lesser understanding

Prior understanding / dogma / pre concent notion / which have biased / cannot be applied to system where time is consider

Judgement is going to be different because event conducted at different point of time.

We cannot apply that equation to the system having prior misunderstanding / dogma / pre concive notion / which have biased / where time is consider (i.e o/p will depend on the time at which it is applied.)

bucks or rupees

$\sum H(k)\, x(n-k)$ → can be applied on unbiased system and o/p doesn't depend on varying time.

That is derived and formulated to get response from the actual I/P signal.

Flip → Shift → multiply → add (To convolve)
→ Process of convolution

Significance of convolution :-
To get response from the actual I/P.

If I am able to undo my things → unbiase

Sir giving 10 Rs to Ananya through Shubh
$x(n)$ $h(n)$

For sir 2 Rs is eaten by shubh

Ananya = $x(n) - (2)$ → bias

If sir ask 10 Rs back from Ananya he will get only 8 Rs i.e it will not get recovered.

$y = x^2$

$y = \pm x$ ← biase

- O/P depends on the time at which I/P is applied.
- O/P varies with time at which I/P is applied.

* Bias timing is non linear
* Unbias timing is linear / rational

Convolution is only applied to the system which is both linear / rational & time invariant i.e (LTI). It cannot be applied to non-linear time variant system / linear time variant / non linear time invariant system.

Convolution $\sum H(k)\, x(n-k)$ can only be applied to the LTI system (Linear Time invariant System)

27/08/2020

Scripture → religious books

We are culprits as we didn't consider some conditions / different aspects.

General Question

Q] Can I device or formulate equation that can be applied to my brain?

→ We must know constraints of the subjects

Linear system :- It does not have any bias

Time invariant :- where O/P is independent of time at which I/P is applied is Time invariant.

$ax + by = c$

$x^2 + 2xy + y^2 = 0$

- O/P can be function of time
- O/P can depends time at which I/P is applied

Q] How to find whether the system is Linear / Time invariant or not?

→ Process of linearity → Superposition

- When system is linear then superposition is used (Chaving resistor)
- no L & C are used in Linear circuit
- Multiple sources

$V = IR$

$y = mx$

Linearity → ① Superposition
② Scaling
(Principle of Homogeneity)

→ System which satisfies superposition and principle of homogeneity then that system is Linear system

① Differential equation (CTS)

② Difference equation : used in DTS and Digital signal (DS), which gives relation between Input and output, and that describe the behaviour of system.

① $y[n] = 2x[n]$

- Same time Doubles the input
- here no constant
- but has coefficients

coefficients → 1, -2 & 0

(Major - Constant)

$y_1[n] = 2a_1x_1[n]$

$y_2[n] = 2a_2x_2[n]$

$y_3[n] = 2a_1x_1[n] + 2a_2x_2[n]$ — ①

$y_4[n] = 2[a_1x_1[n] + a_2x_2[n]]$

$y_4[n] = 2a_1x_1[n] + 2a_2x_2[n]$ — ②

From ① & ② System is Linear

② $y[n] = x[n-1]$ $(1, -1, 0)$

↳ Badme ajana (kal aana)

Ex:- cable operator

same I/P Different time

After 1 unit of time 1 unit

③ $y[n] = x[n] - 4$ $(1, -1, 4)$

↳ bank is taking some charge

There is constant i.e 4

* Whenever any D.E contains any constant then that system is non-linear

(*) c without initial charge is linear component

② For superposition we always consider c without initial condition

(*) offset is basically → non linear term

Q] How you used "kal aana" in engineering?

→ "Buffer Amplier" → D F/F

↳ Delay

(Linear system)

④ $y[n] = \log x[n]$

log, sin, cos ——→ only non linear term

→ $y_1[n] = \log a_1x_1[n]$

$y_2[n] = \log a_2x_2[n]$

$y_3[n] = \log a_1x_1[n] + \log a_2x_2[n]$ — ①

$y_4[n] = \log[a_1x_1[n] + a_2x_2[n]]$ — ②

From ① & ② System is non linear

⑤ $y[n] = nx[n]$ $(n, 1, 0)$ ↳ here n is coefficient

$y_1[n] = a_1nx_1[n]$; $y_2[n] = a_2nx_2[n]$

$y_3[n] = a_1nx_1[n] + a_2nx_2[n]$ — ①

$y_4[n] = n(a_1x_1[n] + a_2x_2[n])$ — ②

From ① & ② ∴ System is Linear

⑥ $y[n] = x[n]$ $(1, 1, 0)$ (LS)

⑦ $y[n] = 2x[n]$ $(2, 1, 0)$ (LS)

⑧ $y[n] = x^2[n]$ (NLS) → $y[n] = \underbrace{x[n] \cdot x[n]}_{\text{coefficient}}$

⑨ $y[n] = x[n]$ (LS)

⑩ $y[n] = x^3[n] \longrightarrow \underbrace{x^2[n] \cdot x[n]}_{\text{coefficient}}$ (NLS)

$(x^2[n], -1, 0)$

⑪ $y[n] = nx[n]$ → LS

⑫ $y[n] = \sin x[n]$ → NLS

⑬ $y[n] = \sin[n] \cdot x[n]$ → LS

*** When coefficient is constant then that system is linear i.e. coefficients are either constant.

Basically may coefficients are functions of time but they there is distinct feature.

① Coefficients are explicit function of time

② Coefficients are implicit function of time

How do you identify openly non linear term ? tim

Tudos

① If you put the value of n & get numeric value then that coefficient is explicit Ex: (1) (2)

② whereas if you put the value of n & dosn't get numeric value then that coefficient is implicit Ex: ⑦ & ⑩

** Coefficients must be explicit function of time

— Openly Non linear term
— Transcendental functions
— log, sin, tan, cos, cot has got a series
— Even modulus has expansion

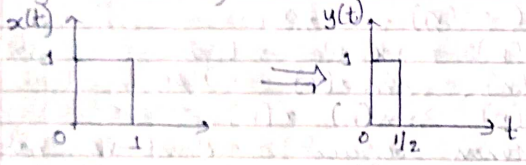
* $x(\ )$ → Index term

non linear system before → IF $y = x^2$ & $x^3 = z$ becomes
$\therefore y = z$ → Linear system after

28/8/2020

→ compression

Q. $y(t) = x(2t)$



To get $y(t) = x(2t)$

$2t = 0$ → $t = 0$ → Lower limit

$2t = 1$ → $t = 1/2$ → upper

* Time scaling

$y[n] = \sqrt{x[n]}$

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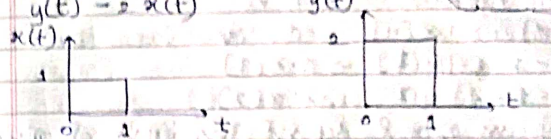
Date

(1 ka 2 kar raha hai?)

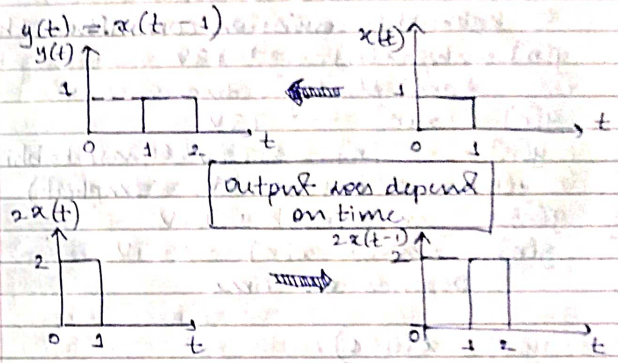
Q. $y(t) = 2x(t)$



Lab → kabhi bhi

* Amplitude scaling

Q. $y(t) = x(t-1)$



For linearity :-

① D.E. must not contain any constant
Ex:- $y[n] = x[n] - 4$

② Must not contain any openly non linear term Ex: log, sin, cosec etc

③ coefficient are either constant or explicit function of time.

④ Time invariant system
o/p does not depend on time at which I/p is applied.

① $y[n] = 2x[n] \longrightarrow$ TIV

② $y[n] = nx[n] \longrightarrow$ TV

$n=1$; $y[1] = 1 \cdot x[1]$

$n=2$; $y[2] = 2 \cdot x[2]$

System output depends on time at which IIP is applied (TV)

③ $y[n] = x[n-1] \longrightarrow$ TIV

④ $y[n] = x[n] - 4 \longrightarrow$ TIV

kabhi bhi ~~comission~~ commision khara

⑤ $y[n] = \log x[n] \longrightarrow$ TIV

kabhi bhi "log" dera

⑥ $y[n] = x[n] \longrightarrow$ TIV

⑦ $y[n] = x^2[n] \longrightarrow$ TIV (Square kabhi bhi)

⑧ $y[n] = x^3[n] \longrightarrow$ TIV (Implicit)

⑨ $y[n] = \sin(x[n]) \longrightarrow$ TIV

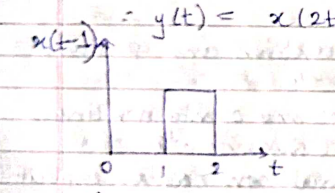
⑩ $y[n] = \sin(n)\, x[n] \longrightarrow$ TV

Depends on Time

$y(t) = x(t-1)$

Put $t = 2t$

$\therefore y(t) = x(2t-1)$

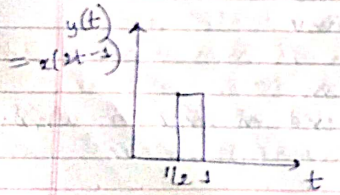


$2t = 1$

$t = \frac{1}{2}$

$2t = 2$

$t = 2/2 = 1$



For Time Invariant System :-

① If coefficient of eqn are explicit then system is Time variant therefore it must not be explicit function of time

② Whenever I do Time scaling that that makes the system Time variant i.e No time scaling.

For system to be LTI :-

coefficient must be constant

① $y[n] = 2x[n] \rightarrow$ LTIV

② $y[n] = x[n-1] \rightarrow$ LTIV

③ $y[n] = x[n] - 4 \rightarrow$ NLTIV

④ $y[n] = \log(x[n]) \rightarrow$ NLTIV

⑤ $y[n] = nx[n] \longrightarrow$ LTV

⑥ $y[n] = x[n] \longrightarrow$ LTIV

⑦ $y[n] = x^2[n] \rightarrow$ NLTIV

⑧ $y[n] = x^3[n] \rightarrow$ NLTIV

⑨ $y[n] = \sin x[n] \rightarrow$ NLTIV

⑩ $y[n] = \sin(n)\, x[n] \rightarrow$ LTV

3/9/2020

$\sin\theta = \frac{O \cdot S}{Hyp}$ $\quad \cos\theta = \frac{A \cdot S}{Hyp}$

Sine is not wave all the time it is a "signal".

Q] How to characterized sine signal ?

→ Sine :- $s(t) = A \sin \omega t$

There are three parameters

① Amplitude

② Frequency
③ Phase

$s(t) = A \sin \omega t$

$s(t) = A \sin(\omega t - \phi)$

↳ here ωt is itself a phase

$s(t)$ = instantaneous Amplitude
A = max / Peak Amplitude
ωt = instantaneous phase (IP)
ϕ = constant phase
Maximum amplitude is the specific value of an instantaneous amplitude

- instantaneous → occuring instantly
- instantaneous phase will take that value of phase constant.
- Phase constant is specific value of I.P.

4/9/2020

Q] What do you write on x axis?
→ time 0, π, 2π

Q] Why do we write 0, π, 2π ... on x axis?
→ It is time as well as phase as Independent variable

- Basically assume that ω will remain constant as long as ϕ plot.
  ω ⟶ constant
  On Horizontal axis ⟶ ω or t is there i.e time or phase?
  as ω = constant
  ∴ f = constant

In continuous time domain
Instantaneous amplitude

→ $x(t) = A \sin(2\pi f t + \theta)$ —①

$A \rightarrow$, $f \rightarrow$, $\theta \rightarrow$

sample it ⟹ $t = n T_s$
↳ values of sample

$x(nT_s) = A \sin(2\pi f n T_s + \theta)$

$x(n) = A \sin\left(2\pi f n \frac{1}{f_s} + \theta\right)$

Put $T_s = 1/f_s$

$x[n] = A \sin\left[2\pi \frac{f}{f_s} n + \theta\right]$

Put $F = f/f_s$

∴ $x[n] = A \sin(2\pi F n + \theta)$ —②

Instantaneous In discrete time domain
sample
Amplitude t & n = independent variable / related to each other

(Unit of Freq^n is Hertz) f = frequency of CT sinusoidal signal

(Unit is nothing) F = Freq^n of a sampled sinusoidal signal
i.e F is Unitless Quantity

*** There exist a concept of frequency where we don't have a unit

- $f = 50$ Hz, Range → 0 to ∞
  If we believe in idea of -ve frequency → Range: -∞ to ∞
  (same as +ve Freq^n)

$f_s \geq 2f_m$ or $2f$ ← $F = \frac{f}{f_s}$

$F_{min} = 0$ if $f_s = \infty$
$F_{max} = 0.5$ if $f_s = 100$ Hz, $f = 50$ Hz
$f/f_s = 1/2 = 0.5$ (by Nyquist)

Problem is with single freq ie nasty = unpleasant

$\therefore$ Fmin $\longrightarrow$ 0 (Lowest)
Fmax $\longrightarrow$ ± 0.5 (Highest)

capital "w" omega for capital F

we know that $\omega = 2\pi f$
$\therefore \Omega = 2\pi F$

Because of this
Fmax $\longrightarrow$ $\pm \pi$

$$-\frac{1}{2} \leq F \leq \frac{1}{2}$$

$\therefore$ For discrete as well as Digital signal

$$0 \leq |F| \leq \frac{1}{2}$$

As Quantization will not have effect on x axis.

4/9/2020 unit $\longrightarrow$ Radian / sec or Frequency

Harmonic signals $\longrightarrow$ when we deal with Freq^n

$x(t) = A \sin(2\pi F t)$
$x(t+T) = A \sin(2\pi F (t+T))$
$= x(t)$
signal repeats itself after T period!

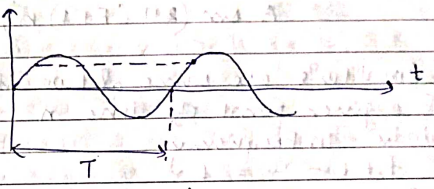
① Consider $T = 2$ sec $(f = 0.5$ Hz)
Nasty number $\rightarrow \sqrt{3}, e^5, \pi$
If I have a signal of $\sqrt{3}$ Hz $\longrightarrow$ still it is going to be periodic

F = any number of s/g is going to be periodic

Two instantaneous Amplitude $\rightarrow$ ① $x(t) = t_1$
② $x(t+T) = t_2$

Periodic $\longrightarrow$ At two different times $\rightarrow$ Amplitude are same
$t_2 - t_1 = T$

*** If Amplitudes are same at two different time at a given frequency then signal is Periodic in time.



$x(t) = x(t+T)$ any frequency

*** $\therefore$ For CTS to be periodic there is no requirement

Q] Does it remain same for DT sinusoidal signal

$x[n] = A \sin 2\pi F n$
if $x[n+N] = x[n]$?
where n, N = integer
$x[n+N] = A \sin(2\pi F(n+N))$ $\longrightarrow$ Independent equation
$\therefore x[n+N] = A \sin(2\pi Fn + 2\pi FN)$
$= A \sin 2\pi Fn$ iff $FN = k$ (integer)
$= x(n)$

if $x[n+N] \neq x[n]$ then signal will not be periodic

If I want DS to be periodic then it must be ratio of two integers.
i.e. $x[n] = x[n+N]$
iff $F = k/N$

10/9/2020 They discuss when there is assignment

Let
$x[n] = A\sin(2\pi Fn)$
Consider $(F+1) = F_2$
$x[n] = A\sin(2\pi F_2 n)$
$= A\sin(2\pi(F+1)n)$
$= A\sin(2\pi Fn + 2\pi n)$
$x[n] = A\sin 2\pi Fn$
$= A\sin(2\pi(F+1)n)$

*** If Amplitude are same at two different freq^n at a given point of time n then it is Periodic in Frequency.

$F+1 - F = 1$
$\Omega = 2\pi F$
$x[n] = A\sin(2\pi(F+m)n)$
$x[n] = A\sin(2\pi Fn + 2\pi mn)$

→ Discrete time Harmonic signal are going to be periodic in term of time if and only if F is the ratio of two integers
→ Discrete time Harmonic signal are always going to be periodic in terms of frequency with period of 2π.
→ Continuous time Harmonic signal are always periodic in time (as there no requirement)
→ Continuous time Harmonic signal are always of periodic in freq^n.

CT Harmonic signal	DT Harmonic signal
① Frequency has unit i.e Hertz.	① Frequency is unitless.
② $f_{min} \to 0$ $f_{max} \to -\infty$ to ∞	② $f_{min} \to 0$ $f_{max} \to -0.5$ to 0.5 $-\pi$ to π
③ Always periodic in time with periodic of $T = \frac{1}{F}$	③ If want to be periodic in time then Frequency must be ratio of two integers. $F = \frac{k}{n}$
④ In CT two frequency are always different and distinct	④ In DT two frequency can be different but can't be distinct
	→ DT harmonic signal are always going to periodic in terms of frequency with period 2π.

8/09/2020 Different & Distinct

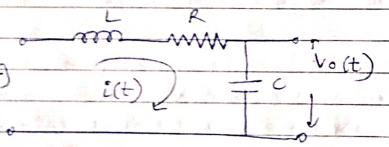
- Simultaneous eq^n
- Quadratic eq^n ⟶ specific example of polyno
- Transcendental eq^n (Function of that variable mi
- Differential eq^n

⟹ Trial & error method

⟶ 'n' order than it is describe with the coefficient of independent variable.

① $x^2 + 2x + 1 = 0$
Polynomial of degree 2
$(1, 2, 1)$

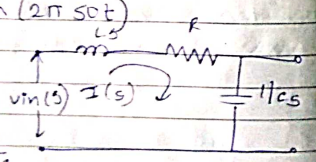
② $x + \log x = 0$
Transcendental equation
Instead of $\log x$ we can write series of $\log x$ ⟶ In Polynomial form but here number of coefficients are infinite
⟶ Transcendental eq^n consist of infinite coefficient.
⟶ All the things cannot be plot.



$V_{in}(t) = A \sin(2\pi 50t)$

$I(s) = \dfrac{V_{in}(s)}{Ls + R + \frac{1}{Cs}}$

$I(s) = \dfrac{V_{in}(s) \cdot Cs}{LCs^2 + RCs + 1}$



$c_1 e^{ax} + c_2 e^{bx}$
Integro - differential eq^n

$V_o(s) = \dfrac{1}{Cs} \times I(s)$

$V_o(s) = \dfrac{1}{Cs} \cdot \dfrac{V_{in}(s) \cdot Cs}{LCs^2 + RCs + 1}$

$V_o(s) = \dfrac{V_{in}(s)}{LCs^2 + RCs + 1}$

$V_i(t) = L\dfrac{di(t)}{dt} + Ri(t) + \dfrac{1}{C}\int i(t)\,dt$ —①

$V_o(t) = \dfrac{1}{C}\int i(t)\,dt$ —②

$\dfrac{dV_i(t)}{dt} = L\dfrac{d^2 i(t)}{dt^2} + R\dfrac{di(t)}{dt} + \dfrac{1}{C} i(t)$

④ Using formula of D.E. we will get Particular solution
⑤ Mathematics is having problem with D.E.
⑥ Difficult ⟶ D.E.

Convert D.E into a set of simultaneous equation

$V_i(s) = sL\,I(s) + R\,I(s) + \dfrac{1}{sC}\,I(s)$ —③

$V_o(s) = \dfrac{1}{sC}\,I(s)$ —④

③ & ④ is a set of simultaneous eq^n

Utility ⟶ To convert D.E into simulta - neous equation

① Ordinary D.E
② Partial D.E
③ Homogeneous D.E

Integrate → combine

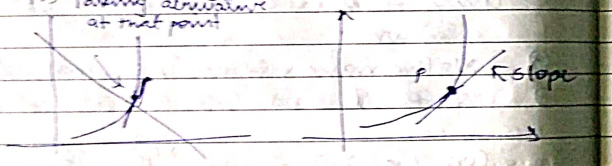
Widrow

→ CT system & DT systems
→ Most used mathematical tool → adder
DT → 2 & 3
CT → $2t$ & $2t^2$
we perform integration
Further summing / summation / adder
Integration is equivalent to adder

Q] How do you integrate? → capacitor
S] ——"—— differentiate? → Inductor
Resistor → scaling by can be done
as it can increase or decrease

rate of change =
++++ how much do the dependable variable change along with change in independent variable
(when)

→ Taking derivative at that point



At every point if we taking derivative we will get infinite slope

Ex :-
change in level of understanding with change in language

Now if independent variable is changing then how will the dependable variable change.

→ Dividing it into infinite intervals
Take a slope
uses of integration & derivative

Why it $\lim_{h \to 0}$? because we have minimum difference b/t two points i.e h tends to 0

What are the two successive numbers in continuous time domain

First no. h $\quad x+h \quad \lim_{h \to 0}$
First no. x $\quad x-h \quad \lim_{h \to 0}$

Two successive numbers in DT domain

$x+1$
$x \rightarrow x-1$ (Previous in digital domain)

$$f'(x) = \frac{df(x)}{dx}$$

$$= \lim_{h \to 0} \frac{f(x) - f(x-h)}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\begin{cases} f'(x) = f(x+1) - f(x) \\ = f(x) - f(x-1) \end{cases}$$

$$f''(x) = \underbrace{[f(x) - f(x-1)]}_{f'(x)} - [f(x-1) - f(x-2)]$$

$$= f(x) - f(x-1) - f(x-1) + f(x+1)$$

$$= f(x) - 2f(x-1) + f(x+2)$$

They are taking derivatives
In principle they are same but numerically it can be different
1) forward Difference
2) Back Difference
-ward

Find the Root of Polynomial & transcendental
↳ Regular falsi / False Position ; Newton Raphson

Q] where to use when ?

10/7/2020

→ Any eqⁿ which help give I/P & O/P relation of the system in DTS/DSP is Difference equation

→ Any eqⁿ which give I/P & O/P relation of the system in CTS is Differential eqⁿ

(*) We represent a system by impulse response, step / ramp / parabolic response (Difference eqⁿ)

One of the way

(*) Differential eqⁿ is another eqⁿ way to represent a system.

→ used to represent same system

Q] which natural question arises ?

Q If they are representing same system then what is relationship between them ?
↳ (ways)
(Differential / Difference / Impulse responses)

Q. $x^2 - 2 = 0$

$x^2 = 2$

$x = \pm\sqrt{2}$

Mathematical tool → Laplace
(DE to simultaneous eq)

Quantity → Vo(s), Vin(s)
We find Transfer function

$H(s) = \frac{Vo(s)}{Vin(s)}$

(*) ∴ Transfer function is the one of the way to represent system.

They discuss when there is assignment ?

→ Transfer function is in domain of simultaneous equation.

→ Transfer function is always value when we have set of simultaneous equation.

When I tried to come back into D.E by taking Laplace inverse of it we get h(t) impulse response.

Relationship between Differential eqⁿ and impulse response is Laplace Transform

Transfer function is impulse response of LT.

Q. What is transfer function ?

→ $\frac{y(t)}{x(t)}$ → X (wrong) $\frac{y(s)}{x(s)}$ → ✓ (Right)

*** Transfer function is the ratio of output to input in domain of simultaneously eqⁿ / trans-form
or
Transfer function gives the between output & input in domain of simultaneous eqⁿ / trans-form

As if signal is can be digital form

Q. can you convert any D.E into any set of simultaneous eqⁿ ?

→ Limitation of Laplace or Z transforms.
↳ Laplace does not exist
EXISTENCE OF LAPLACE TRANSFORM

Search about FT

Q. What is the validity of Transfer function?
→ It is only for LTI systems.

*** → Transfer function is transform of convolution.
→ Transfer function cannot be validated for non LTI systems

11/9/2020

Transfer function → Transform domain (LTI)

This is complex

Z.T. $X(z) = \sum_{n} x[n] \cdot z^{-n} < \infty$

$F(s) = \int_{-\infty}^{\infty} f(t) \cdot e^{-st}$ * (complex function)

** ∫ replaced by Σ ** e^{-s} is replaced by z.

Q $x[n] = \{4, 3, 1, 2\}$ (↑ position n = 0)

$X[z] = \sum_{n=0}^{3} x[n] \cdot z^{-n}$

one point $z \neq 0$

$X[z] = x[0] z^0 + x[1] z^{-1} + x[2] z^{-2} + x[3] z^{-3}$

$X[z] = 4 + \frac{3}{z} + \frac{1}{z^2} + \frac{2}{z^3} < \infty$ (if $z \neq 0$)

$X[z] = 4 + 3z^{-1} + z^{-2} + 2z^{-3}$ (OR) $(z \neq 0)$

Q $x[n] = \{4, 3, 1, 2\}$ (↑ n = 0)

$X[z] = \sum_{n=-3}^{0} x[n] \cdot z^{-n}$

one points $z \neq \infty$

$X[z] = x[-3] z^3 + x[-2] z^2 + x[-1] z^1 + x[0] \cdot z^0$

$X[z] = 4z^3 + 3z^2 + z + 2 \quad (< \infty)$ (if $z \neq \infty$)

Q. $x[n] = \{4, 3, 1, 2\}$ (↑ n = 0)

two points $z \neq 0$, $z = \infty$ valid for all points except

$X[z] = \sum_{n=-2}^{1} x[n] \cdot z^{-n}$

$X[z] = x[-2] z^2 + x[-1] z^1 + x[0] z^0 + x[1] z^{-1}$

$X[z] = 4z^2 + 3z + 1 + 2z^{-1}$ (if $z \neq 0$ & $z \neq \infty$)

Q. $x[n] = \left(\frac{1}{2}\right)^n u[n]$

$X[z] = \sum_{n=-\infty}^{\infty} (1/2)^n u[n] z^{-n}$ (first term; have amplitude 1 from 0 to ∞)

$\frac{a}{1-r}$ → common ratio; $|r| < 1$

$X[z] = \sum_{n=0}^{\infty} (1/2)^n z^{-n}$

$X[z] = \sum_{n=0}^{\infty} \left(\frac{1}{2} z^{-1}\right)^n$

Taylor $\sum \alpha^n = \frac{1}{1-\alpha}$

Taylor series

$$X[z] = \frac{1}{1-\frac{1}{2}z^{-1}} \qquad |x| < 1$$

$$X(z) = \frac{1}{1-\frac{1}{2}\frac{1}{z}}$$

$$X[z] = \frac{z}{z-\frac{1}{2}} \qquad |z| > 1/2 \quad \text{—(1)}$$

decrease

$|x| < 1$

valid ⟶ $|z| > \frac{1}{2}$ $\quad \frac{1}{2}\frac{1}{z} < 1$

Invalid ⟶ $|z| \leq \frac{1}{2}$ $\quad \frac{1}{2} < z$

Points invalid ⟶ infinite points

Q. $x(n) = \left(\frac{1}{2}\right)^n u(-n-1)$

$$-X(z) = \sum_{n=-1}^{-\infty} 1/2^n \cdot z^{-n}$$

increase

Put $n = -m$

$$-X(z) = \sum_{m=1}^{\infty} (1/2)^{-m} z^m$$

$$-X[z] = \sum_{m=1}^{\infty} \left(1/2^{-1} z\right)^m$$

$$-X[z] = \sum_{m=1}^{\infty} (2z)^m$$

$$-X[z] = \sum_{m=0}^{\infty} (2z)^m - 1 \qquad \text{for } m=0 \text{ ans is } 1$$

$$-X[z] = \frac{1}{1-2z} - 1 \qquad |2z| < 1$$

$u(-n)$ 0 to $-\infty$ $\quad u(-n+4) = 4$ to ∞

$u(-n-1)$ -1 to $-\infty$

$u(-n+1)$ 1 to $-\infty$

$$-X(z) = \frac{1-1+2z}{1-2z}$$

$$= \frac{2z}{1-2z} \qquad 2z < 1 \quad z < \frac{1}{2}$$

$$\therefore X(z) = \frac{+2z}{+(2z-1)}$$

$$X(z) = \frac{2z}{2z-1}$$

$|x| < 1$

$$= \frac{2z}{2\left(z-\frac{1}{2}\right)}$$

$\frac{1}{2}z^{-1} < 1$

$\frac{1}{2} < z$ $\qquad X(z) = \frac{z}{z-\frac{1}{2}}$; $|z| < \frac{1}{2}$ —(1)

From (1) & (1)

~~My condition is useless~~

My z transform is useless if ~~consider~~ condition is not consider

If condition is not consider recovery is of original input signal is difficult

Thus condition is to be written we if have to come back by taking z-inverse.

Continuous time convolution

$-2b$

$\frac{1}{s+2}$

$|s| > 2$

real $s > 2$

$\sigma > 2$

$\sigma + j\omega$

$\frac{2z}{1-2z}$

1×4

prodding ⟶ poke

provoke ⟶ irritating

$\left(\frac{1}{2}\right)^n = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$

Diff & ratio is same for all term

15/09/2020 → represent an Integration

$y(n) = x(n) + x(n-1)$ ← an amount of delayed version

summing → This is difference Equation as it give relation between input & output

Z transform of $y(n) \longrightarrow Y(z)$

$Y(z) = X(z) + z^{-1} X(z)$

→ $x(n+1) \longrightarrow z^{1} X(z)$

→ $x(n-8) \longrightarrow z^{-8} X(z)$

Transfer function → $Y(z) = X(z)(1 + z^{-1})$

$\frac{Y(z)}{X(z)} = 1 + z^{-1}$

$H(z) = 1 + z^{-1}$

we are going to take coefficient

~~Z transform~~ Z inverse is

$h(n) = (\underset{\uparrow}{1}, 1)$

$y(n) = a x(n) + b x(n-1)$

Impulse response → $h(n) = (\underset{\uparrow}{a}, b)$

$y(n) = a x(n) + b x(n-2)$

$h(n) = (\underset{\uparrow}{a}, 0, b)$

$h(n) \longrightarrow$ Impulse response

$H(z) \longrightarrow$ Transfer function

From difference eqⁿ we reached impulse response

Q $y(n) = x(n) - x(n-1)$ (Difference eqⁿ)

→ First order Differentiate at n

→ Subtract it

Q. $y(n) = x(n) + x(n-1) + x(n-2)$ (Difference eqⁿ)

→ Integration

→ limits of integration are different

Q. $y(n) = x(n) + y(n-1)$

$Y(z) = X(z) + z^{-1} Y(z)$

$(1 - z^{-1}) Y(z) = X(z)$

$\frac{Y(z)}{X(z)} = \frac{1}{1 - z^{-1}}$

$H(z) = \frac{1}{1 - \frac{1}{z}}$

$H(z) = \frac{z}{z - 1}$

$h(n)$ $\quad (1)^n u(n)$ $\quad |z| > 1$

$(1)^n u(-n-1)$ $\quad |z| < 1$

Q $y(n) = x(n) + x(n-1) + y(n-1)$

To the present input previous input & previous O/P are added

$Y(z) = X(z) + z^{-1} X(z) + z^{-1} Y(z)$

$Y(z) - z^{-1} Y(z) = (1 + z^{-1}) X(z)$

$(1 - z^{-1}) Y(z) = (1 + z^{-1}) X(z)$

$\frac{Y(z)}{X(z)} = \frac{1 + z^{-1}}{1 - z^{-1}}$

$H(z) = \frac{1}{1 - z^{-1}} + \frac{z^{-1}}{1 - z^{-1}}$

$\frac{1}{1 - \frac{1}{z}}$

$H(z) = \frac{z}{z - 1} + \frac{z}{z - 1} z^{-1}$

$\frac{1}{z - 1}$

z - Inverse

$h(n) = (1)^n u(n) + 1^{n-1} u(n-1)$

where ever you find it replace it by $n-1$

$x(n) \longleftrightarrow X(z)$

Q $y(n) = a x(n) - b x(n-1) + c x(n-2) + y(n-1)$

$Y(z) = a X(z) - b z^{-1} X(z) + c z^{-2} X(z) + z^{-1} Y(z)$

$Y(z) - z^{-1} Y(z) = a X(z) - b z^{-1} X(z) + c z^{-2} X(z)$

$Y(z)[1 - z^{-1}] = X(z)[a - b z^{-1} + c z^{-2}]$

$\frac{Y(z)}{X(z)} = \frac{a - b z^{-1} + c z^{-2}}{1 - z^{-1}}$

$= \frac{a}{1 - z^{-1}} - \frac{b z^{-1}}{1 - z^{-1}} + \frac{c z^{-2}}{1 - z^{-1}}$

$= \frac{a z}{z - 1} - b \frac{z}{z - 1} z^{-1} + c \frac{z}{z - 1} z^{-2}$

$h(n) = a(1)^n u(n) - b(1)^{n-1} u(n-1) + c(1)^{n-2} u(n-2)$

c times $\frac{z}{z - 1} z^{-2}$

$n - 1 = 0$

$n = 1$

Q $y(n) = x(n) + x(n-1)$

$y(n) = x(n) + x(n+1)$

It cannot be accomplished by changing any one of them

I can have access to change by changing $x(n)$

① $y(n) = x(n) + x(n \mp 1)$ — ①

$y(n) = x(n) + x(n+1)$ — ②

$y(n) = x(n) + x(n-1) + y(n+1)$ — ③

$y(n) = x(n) + y(n-1) + x(n-1)$ — ④

→ If we want to change O/P we can change $x(n)$ as we cannot change $x(n-1)$

Why ? As I already know $x(n-1)$

Because there is cause & effect → eq^n ① & ④

missing cause & effect relationship → ② & ③

1 & 4 ⟶ have cause & effect relationship
2 & 4 ⟶ dothave any cause & effect relationship

$x(n) = \{4, 3, 1, 2\}$

$y(n) = x(n) \; x(n-1)$

$n-1=0 \longrightarrow n=1 \quad x(n-1) = 4$

$n-1=1 \longrightarrow n=2 \quad x(n-1) = 3$

$n-1=2 \longrightarrow n=3 \quad = 1$

$n-1=3 \longrightarrow n=4 \quad = 2$

$\{0, 4, 3, 1, 2\}$

$x(n+1) \longrightarrow \{4, 3, 1, 2\}$

⟶ The system which have a cause & effect relationship is called as causal system

⟶ The system which don't have a cause & effect relationship is non causal system

① $x(t) = \cos(2\pi i t)$

② $x[n] = \cos(2 \cdot \pi i \cdot n)$

③ $y[n] = \cos(2 \cdot \pi \cdot 2\sqrt{3} \cdot n)$

$t = nTs$

$n = \frac{t}{1}$

$F = \frac{f}{f_s}$

$F = \frac{1}{1}$

$F = 1$

Eqn ① & ④

Cause → we already know $x(n-1)$ effect can be changed in $x(n)$

OR

As we know $x(n-1)$ then we can predict its cause and effect i.e it have cause effect relationship.

problems

① $y(n) = ax(n) + bx(n-1) + cx(n-3)$

$Y(z) = aX(z) + bz^{-1}X(z) + cz^{-3}X(z)$

$Y(z) = X(z)[a + bz^{-1} + cz^{-3}]$

$\frac{Y(z)}{X(z)} = a + bz^{-1} + cz^{-3}$

$H(z) = a + bz^{-1} + cz^{-3}$

$h(n) = [a, b, 0, 0, c]$ (Impulse response)

↑ we are going to normalise it by putting $n=0$

② $y(n) = ax(n) + bx(n+1) + cx(n-3)$

$Y(z) = aX(z) + bzX(z) + cz^{-3}X(z)$

$Y(z) = X(z)[a + bz + cz^{-3}]$

$H(z) = \frac{Y(z)}{X(z)} = a + bz + cz^{-3}$

$h(n) = [b, a, 0, 0, c]$ (Impulse response)

↑

③ $y(n) = x(n) + ay(n-1)$

$Y(z) = X(z) + az^{-1}Y(z)$

$Y(z)(1 - az^{-1}) = X(z)$

$\frac{Y(z)}{X(z)} = H(z) = \frac{1}{1 - az^{-1}}$

$= \frac{z}{z-a}$

$|z| > |a|$

$\frac{a}{z} < 1$

$a < z$

$\therefore h(n) = a^n u(n) \quad |z| > a$

$|a| < |z|$

$a < \frac{1}{2}$

④ $y(n) = x(n) + ay(n+1)$

$Y(z) = X(z) + azY(z)$

$Y(z)[1 - az] = X(z)$

$\frac{Y(z)}{X(z)} = H(z) = \frac{1}{1 - az}$

$= \frac{-zz^{-1}}{a(z - 1/a)} = \frac{-z}{a} \cdot \frac{z}{z - \frac{1}{a}}$

causality → signal & system relation

For impulse response of the system

$h(n) = 0$ for $n < 0$ → system is causal

If $h(n) = 0$ for all negative values then there is property of causality (presence)

ways of representation → Transfer function, Difference eqn

$h_1(n) = a^n u(n)$	$h_2(n) = a^n u(-n-1)$
$= [a^0, a^1, a^2, \ldots]$	$= [\ldots, a^{-3}, a^{-2}, a^{-1}, 0]$ ↑
$h_1(n)$ is causal because $h(n) = 0$ for $n < 0$	$h_2(n)$ is non causal because $h(n) \neq 0$ for $n < 0$
$H(z) = \frac{z}{z-a} \quad \lvert z\rvert > a$	$H(z) = \frac{z}{z-a} \quad \lvert z\rvert < a$

① $h_1(n) = [1, 2, 4, 3, 8]$ → causal

Point of reference is imp ↑

② $h_2(n) = [1, 3, 6, 7, 9]$ → Non causal

↑

here amplitude is infinite but samples are finite

Difference eqn ⇒ $y(n) = a(n+3) + 3x(n+2) + 6x(n+1) + 7x(n) + 9x(n-1)$

③ $h_3(n) = [2, 3, \infty, 4, 3]$ → causal

↑

Even its coefficient is ∞ value of $n > 0$

④ $h_4(n) = a^n u(n)$ → causal

⑤ $h_5(n) = b^n u(-n-1)$ ⟶ Non causal

① $h_8(n) = \frac{1}{2}^n u(n-3)$ ⟶ causal

⑦ $h_7(n) = -2^n u(-n)$ ⟶ Non causal

①②③④⑤⑥⑦ all are impulse response

①②③	④⑤⑥⑦
Finite number of samples are required to represent impulse response	Infinite numbers of samples are required to represent impulse response

↓ System representation

⟶ IF impulse response is represented in finite number of samples (irrespective of the values) than it is called as Finite impulse response (FIR)

⟶ IF impulse response is represented in infinite number of samples than it is called as Infinite impulse response (IIR)

① $h(n) = [3, 3, 3, 4, 5]$ ⟶ FIR, can't decide causality

⑧ $h(n) = \left(\frac{1}{2}\right)^n u(n-4)$ ⟶ IIR, Causal

4 to ∞

③ $h(n) = \frac{1}{2}^n u(n) - \frac{1}{2}^n u(n-5)$

start at n = 5

$\left[1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32} \ldots\right] - \left[\frac{1}{32}, \frac{1}{64}, \frac{1}{128}, \ldots\right]$

$= \left(1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}\right)$ 5 samples

∴ System is FIR

④ $h(n) = \frac{1}{2}^n u(n) - \frac{1}{8}^n u(n-3)$ ⟶ IIR, Causal

⑤ $h(n) = [2, 7, 13, 5, 6, 9]$ FIR, Causal

O/P depends on I/P but I/P does not depend on O/P

$y(n) = 2x(n) + 7x(n-1) + 13x(n-2) + 5x(n-3) + 6x(n-4) + 9x(n-5)$

⑥ $h(n) = 3^n u(n)$ ⟶ IIR, Causal

$H(z) = \frac{z}{z-3}$; $|z| > 3$

$H(z) = \frac{1}{1-3z^{-1}}$

$\frac{Y(z)}{X(z)} = \frac{1}{1-3z^{-1}}$

$Y(z) - 3z^{-1}Y(z) = X(z)$

$y(n) = x(n) + 3y(n-1)$

⟶ FIR system is a system where it depends on input and its versions

⟶ IIR system depends on any one output

$y(n) = x(n) + ax(n-1) + by(n-1)$

$Y(z) = X(z) + az^{-1}X(z) + bz^{-1}Y(z)$

$Y(z) - bz^{-1}Y(z) = X(z)(1 + az^{-1})$

$\frac{Y(z)}{X(z)} = \frac{1+az^{-1}}{1-bz^{-1}}$

output depends on input & input also depends on output

$H(z) = \frac{1}{1-bz^{-1}} + az^{-1}\frac{1}{1-bz^{-1}}$

$= \frac{z}{z-b} + az^{-1}\frac{z}{z-b}$

$h(n) = b^n u(n) + a.b^{n-1}u(n-1) \longrightarrow$ IIR

depends on I/P
- Fan → air (O/P) Manual speed control
- Ac → air (O/P) Automatic
  - Input does not depends on O/P
  - Input also depends on O/P

FIR → open loop systems (without F/B)

IIR → closed loop systems (with F/B)

18/9/2020

Q) $h(n) = [1, 2, 1, 3] \longrightarrow$ FIR, can't say about causality

Q) 1. $h_1(n) = [1, 2, 1]$ (↑) → FIR causal

$H_1(z) = 1 + 2z^{-1} + z^{-2}$

$= 1 + \frac{2}{z} + \frac{1}{z^2}$

$= \frac{z^2 + 2z + 1}{z^2}$ → zeros, → poles

pole at $z = 0$

① Impulse response
② Difference response
③ Transfer function

} How do we represent system?

Q.2] $h_2(n) = [1, \underset{\uparrow}{2}, 1]$ (Non causal, FIR)

$H_2(z) = z + 2 + z^{-1}$

$= z + 2 + \frac{1}{z}$

$H_2(z) = \frac{z^2 + 2z + 1}{z}$

Poles at $z = 0$

Q.3] $h_3(n) = [1, \underset{\uparrow}{2}, 1]$ (FIR, Non causal)

$H_3(z) = z^2 + 2z + z^{-1}$

$= \frac{z^2 + 2z + 1}{1}$

No poles →

①②③ → Poles are changing (Denominator is affected)

①② → $H_2(z) = z H_1(z)$

In Q.1] If we add pole at origin

$H_1(z) = \frac{z^2 + 2z + 1}{z^2 \cdot z} = \frac{z^2 + 2z + 1}{z^3}$

$= z^{-1} + 2z^{-2} + z^{-3}$

$= [0, 1, 2, 1]$ (↑)

Even after adding one pole we again get causal & FIR system.

→ When we remove a pole it becomes causal

$$\frac{z^2+2z+1}{z^2} = \frac{z^2+2z+1}{z^2}$$

$$= 1 + 2z^{-1} + z^{-2}$$

$$= (1, 2, 1)$$

↑

If no. of poles are greater than or equal to no. of zeros then system is causal

*** **no. of poles ≥ no. of zero's** gives me causality

① $h_1(n) = [1, 2, 1]$ ⟶ causal + FIR

↑

2 poles ⟶ $H_1(z) = \frac{z^2+2z+1}{z^2}$

If I remove a one pole

1 poles ⟶ $H_1(z) = \frac{z^2+2z+1}{z}$

$= z + 2 + z^{-1}$

$h_1(n) = (1, 2, 1)$ ⟶ Non causal + FIR

↑

If I add a pole

3 poles ⟶ $H_1(z) = \frac{z^2+2z+1}{z^3}$

$= z^{-1} + 2z^{-2} + z^{-3}$

$h_1(n) = (0, 1, 2, 1)$

↑

If I add a pole other than origin

$H(z) = \frac{1}{z-a} + \frac{2}{}$

$\frac{z}{z-a}$ ⟶ $a^n u(n)$ $\frac{1}{z-a}$ $a^{n-1} u(n-1)$ $a^{n+1} u(n+1)$

$H(z) = \frac{z^2+2z+1}{z^2(z-a)}$

$= \frac{A}{z} + \frac{B}{z^2} + \frac{C}{z-a}$

$h_1(n) = \delta(n-1) + \delta(n-2) + B\, a^n u(n)$

$n = n-1$

$n = n+1$

Here when we add a pole to them other then at origin system become IIR

*** If all poles are at origin then system is FIR

*** If any pole of the system is not at origin then system is IIR

*** **Poles must be at origin ⟹ FIR**

② I have a non causal system whose impulse response is given as

$h(n) = [1, 2, 3, 1]$

↑

Design a system which will make this h(n) causal.

⟶ $h(n) = [1, 2, 3, 1]$

↑

$H(z) = z + 2 + 3z^{-1} + z^{-2}$

$= z + 2 + \frac{3}{z} + \frac{1}{z^2}$

$= \frac{z^3 + 2z^2 + 3z + 1}{z^2}$

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Now causal → [?] → causal

$\frac{z^3+2z^2+3z+1}{z^2}$ → $\frac{1}{z}$ → $\frac{z^3+2z^2+3z+1}{z^3}$

$[1, \underset{\uparrow}{2}, 3, 1)$ ⇓ $[\underset{\uparrow}{1}, 2, 3, 1)$

System

$T(z) = \frac{1}{z} = z^{-1}$

$T(z) = z^{-1}$

$\frac{y(z)}{x(z)} = z^{-1}$

$t(n) = \underset{\uparrow}{(0,1)} = \delta(n-1)$ $y(n) = \underset{\text{kal aana}}{x(n-1)}$

Multiplication transform form is convolution in differential domain

Indus		1	2	3	1	$\underset{\uparrow}{(0,1)}$
	0	0	0	0	0	$(1,2,3,1)$
$0+(-1)$	1	1	2	3	1	$\underset{\uparrow}{(0,1,2,3,1)}$
$=-1$					-1	3
$=1+2$						
$=3$						

$H(z) = (z + 2 + 3z^{-1} + z^{-2}).z^{-1}$

$= z^0 + 2z^{-1} + 3z^{-2} + z^{-3}$

$= (\underset{\uparrow}{1}, 2, 3, 1)$

Q. Can you design it in other way use $\frac{1}{z^2}$ (Infinite solutions) (Many)

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$H(z) = (z + 2 + 3z^{-1} + z^{-2}).z^{-2}$

$= z^{-1} + 2z^{-2} + 3z^{-3} + z^{-4}$

$= [\underset{\uparrow}{0}, 1, 2, 3, 4)$

$T(z) = \frac{1}{z^2} = z^{-2}$

$t(n) = \delta(n-2)$

$T(z) = \frac{y(z)}{x(z)} = z^{-2}$

$y(z) = z^{-2} X(z)$

$y(n) = x(n-2)$

Q. $T(z) = z^{-1}$

T.F → $\frac{y(z)}{x(z)} = z^{-1}$ $h(n) = \delta(n-1)$ ← I.R

$y(z) = z^{-1} X(z)$

D.E → $y(n) = x(n-1)$ ⟶ Hardware

Ans → D Flip Flop (Hardware representation)

Q. Is F/F a system? yes

Transfer function of F/F →

Q. $h(n) = a^n u(n)$ ⟶ causal

$H(z) = \frac{z}{z-a}$; $|z| > a$

Q. $h(n) = -(b)^n \, a u(n-1)$ ⟶ Non causal

$H(z) = \frac{z}{z-b}$; $|z| < b$

Q. $H(z) = \frac{z}{z-2} + \frac{z}{z-3}$ ⟶ ?

Q. $H(z) = \frac{z}{z-a}$

$h(n) = a^n u(n)$ if $|z| > a$ → Causal

or $h(n) = -a^n u(-n-1)$ if $|z| < a$ → Non Causal

Q $H(z) = \frac{z}{z-2} + \frac{z}{z-3}$

① $|z| > 2$ & $|z| > 3 \Rightarrow |z| > 3$

$h(n) = 2^n u(n) + 3^n u(n)$ [Causal]

② $|z| < 2$ & $|z| < 3 \Rightarrow |z| < 2$

$h(n) = -2^n u(-n-1) - 3^n u(-n-1)$ [NC]

Q → analog
ω → digital

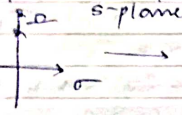
③ $|z| > 2$ & $|z| < 3 \Rightarrow 2 < |z| < 3$

$h(n) = 2^n u(n) - 3^n u(-n-1)$ [NC]

X ④ $|z| < 2$ & $|z| > 3$ → Invalid

$s = \sigma + j\Omega$

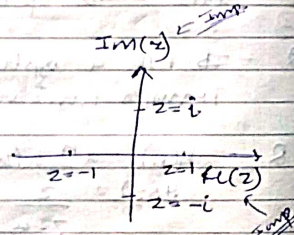
↓ Real part ↓ Imaginary part



s-plane → 2D plane

Laplace Transform

Z plane



Z Transform

$z = re^{j\omega}$

r → cannot be a real number

$j\omega$ → cannot be imaginary number

Eg: $z = 1 + j$ is in cartesian form & number is in polar form

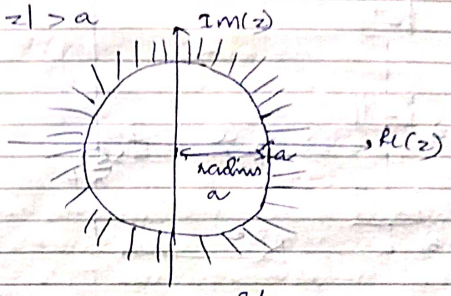
$z = Re(z) + j\,Im(z)$

$|z| = 1$ → (Infinite point)

↳ distance from origin

Difference between $|s| = 1$, v/s $|z| = 1$

Q. $|z| > a$



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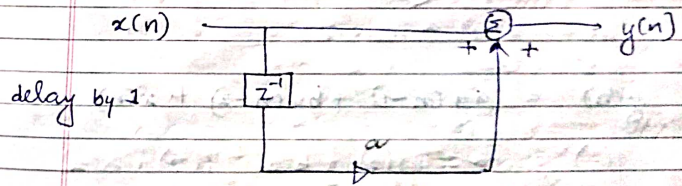
Q $y(n) = x(n) + a \cdot x(n-1)$ ← adder

$Y(z) = X(z) + a z^{-1} X(z)$

relationship between $x(n)$ & $x(n-1)$ ← delay

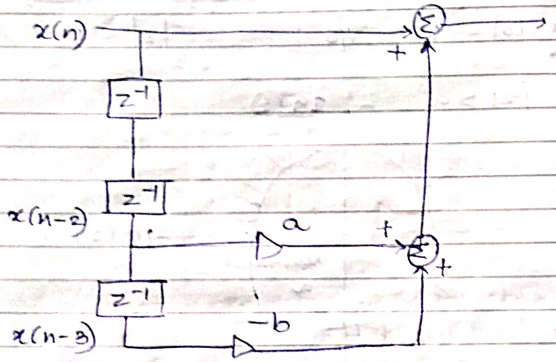
We can use a delay flip flop, to get $x(n-1)$ from $x(n)$



This is the hardware required to implement it.

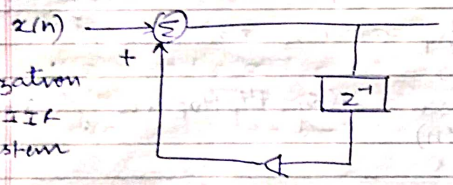
$a^n u(n)$ impulse response

Q $y(n) = x(n) + a x(n-2) - b x(n-3)$



Q $y(n) = x(n) + a y(n-1)$ $\quad Y(z)\left(1 - \frac{a}{z}\right) = X(z)$

$Y(z) = X(z) + a z^{-1} Y(z)$ $\quad \frac{Y(z)}{X(z)} = \frac{z}{z-a}$



Realization of IIR system

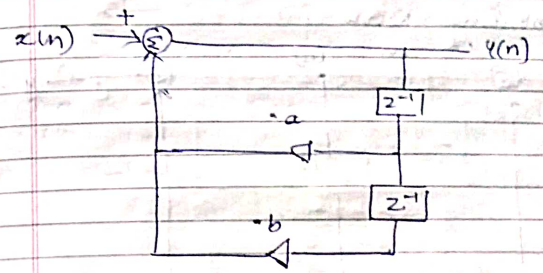
Q $y(n) = a y(n-1) + b y(n-2) + x(n)$

$Y(z) = a z^{-1} Y(z) + b z^{-2} Y(z) + X(z)$

$Y(z) - a z^{-1} Y(z) - b z^{-2} Y(z) = X(z)$

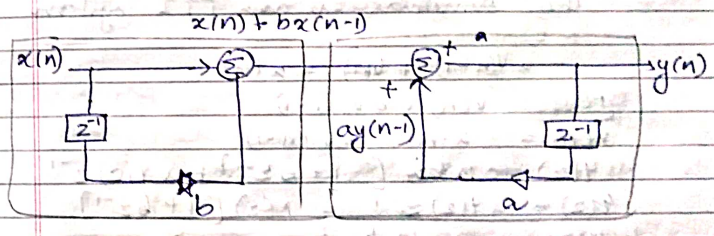
$Y(z)[1 - a z^{-1} - b z^{-2}] = X(z)$

$\frac{Y(z)}{X(z)} = \frac{1}{1 - a z^{-1} - b z^{-2}}$



Q $y(n) = x(n) + a y(n-1) + b x(n-1)$ causal system is IIR

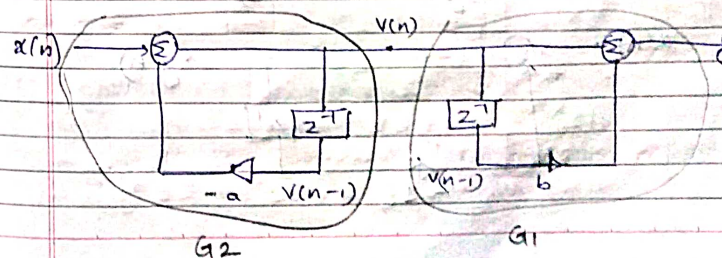
$= x(n) + b x(n-1) + a y(n-1)$



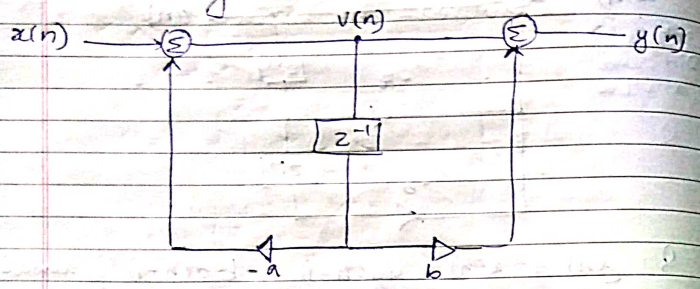
Can I swap G1 & G2 ?

$G = G_1 G_2$

$= G_2 G_1$



Combining them



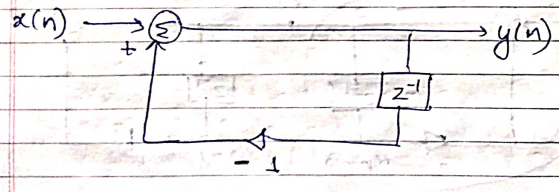
Achieving ⟶ reducing the delay component.

The structure is an IIR system

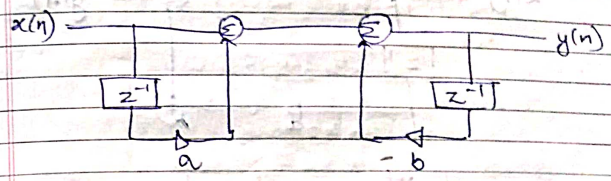
$$-a\,v(n-1) + v(n) = x(n)$$
$$y(n) = v(n) + b\,v(n-1)$$
$$v(n) = x(n) + a\,v(n-1)$$
$$Y(z) = X(z)\left[1 + bz^{-1}\right] + aY(z)z^{-1}$$
$$Y(z) - aY(z)z^{-1} = X(z)\left[1 + bz^{-1}\right]$$
$$\frac{Y(z)}{X(z)} = \frac{1 + bz^{-1}}{1 - az^{-1}}$$

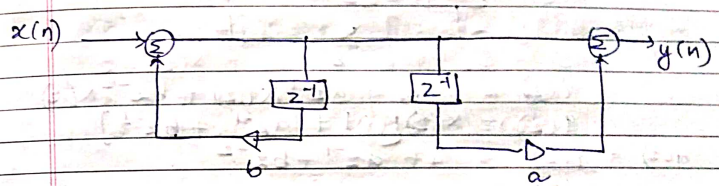
Q $y(n) = x(n) + y(n-1)$

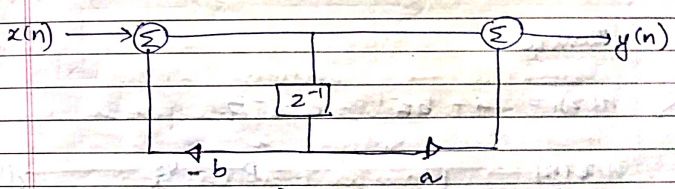
$$Y(z) = X(z) + z^{-1}Y(z)$$



Q. $y(n) = x(n) + ax(n-1) + by(n-1)$



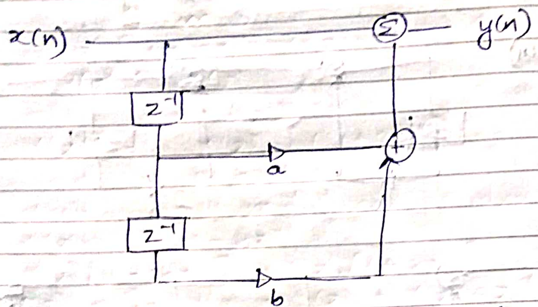




$$Y(z)\left[1 - bz^{-1}\right] = X(z)\left[1 + az^{-1}\right]$$
$$\frac{Y(z)}{X(z)} = H(z) = \frac{1 + az^{-1}}{1 - bz^{-1}}$$

Q $y(n) = x(n) + ax(n-1) + bx(n-1)$

There will be no F/B for FIR
There will be F/B for IIR



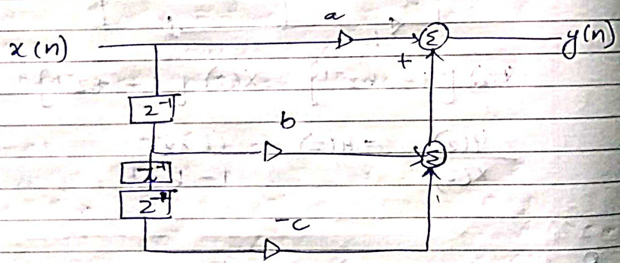
$$Y(z) = X(z) + az^{-1}X(z) + bz^{-2}X(z)$$

$$Y(z) = X(z)[1 + az^{-1} + bz^{-2}]$$

$$H(z) = \frac{Y(z)}{X(z)} = 1 + az^{-1} + bz^{-2}$$

Realize

Q $H(z) = a + bz^{-1} - cz^{-3}$



$$\frac{Y(z)}{X(z)} = a + bz^{-1} - cz^{-3}$$

$$Y(z) = X(z)[a + bz^{-1} - cz^{-3}]$$

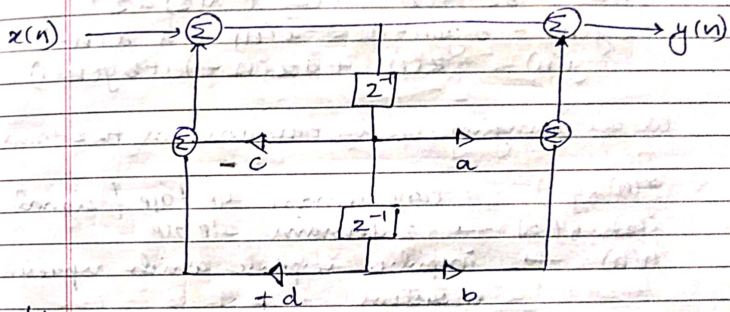
If the y(n) ... same sign ...

Q. $y(n) = x(n) + ax(n-1) + bx(n-2) + cy(n-1) - dy(n-2)$

$$\rightarrow Y(z) = X(z) + az^{-1}X(z) + bz^{-2}X(z) + cz^{-1}Y(z) - dz^{-2}Y(z)$$

$$Y(z) - cz^{-1}Y(z) + dz^{-2}Y(z) = X(z)(1 + az^{-1} + bz^{-2})$$

$$\frac{Y(z)}{X(z)} = \frac{1 + az^{-1} + bz^{-2}}{1 - cz^{-1} + dz^{-2}}$$



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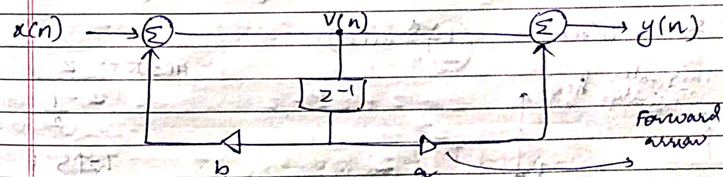
Q. $y(n) = x(n) + ax(n-1) + by(n-1)$



$V(n) = x(n) + bV(n-1)$ — ①
$y(n) = V(n) + aV(n-1)$ — ②
$V(z) = X(z) + bz^{-1}V(z)$ — ③
$Y(z) = V(z) + az^{-1}V(z)$ — ④

|z| is complex

$$Y(z) = V(z)[1 + az^{-1}] \quad -(5)$$

(3) in (5)

$$V(z)[1 - bz^{-1}] = X(z)$$

$$V(z) = \frac{X(z)}{1 - bz^{-1}}$$

$$Y(z) = \frac{X(z)}{1 - bz^{-1}}[1 + az^{-1}]$$

$$\frac{Y(z)}{X(z)} = \frac{1 + az^{-1}}{1 - bz^{-1}}$$

$$Y(z)[1 - bz^{-1}] = X(z)[1 + az^{-1}]$$

$$Y(z) - bz^{-1}Y(z) = X(z) + X(z)az^{-1}$$

$$y(n) - by(n-1) = x(n) + a\,x(n-1)$$

$$y(n) = x(n) + ax(n-1) + by(n-1)$$

We are bringing eqn in time domain to z domain

$x(n), y(n)$ ⟶ time domain I/P / O/P / signal

$X(z), Y(z)$ ⟶ z domain I/P O/P

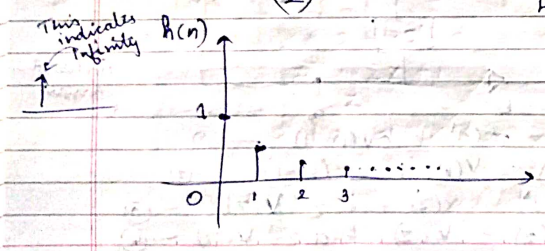
$h(n)$ ⟶ impulse response which represents a system

$H(z)$ ⟶ transfer function

Q. Diff btn axis & ~~syster~~ segments

Q. $h(n) = \left(\frac{1}{2}\right)^n u(n)$

This indicates Infinity



$$H(z) = \frac{z}{z - \frac{1}{2}}$$

$$|z| > \frac{1}{2}$$

Pole → 1 (1/2)

Zero → 1 (z=0)

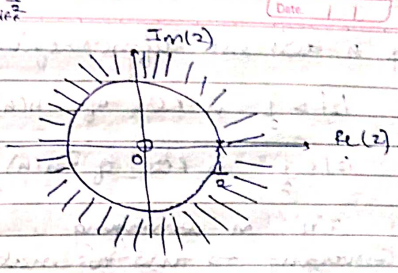
|z| = 1/2 is diff

|z| > 1/2 is diff

|z| ⟶ infinite set of numbers

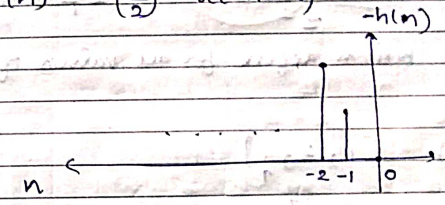
z - plane

Shaded region in the region where h(n) or H(z) is defined

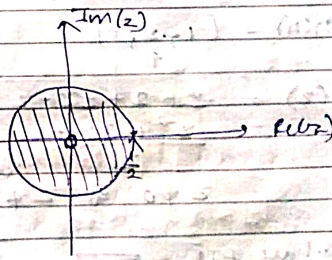


If my |z| take any value in shaded region then H(z) will be defined

Q. $h(n) = \left(\frac{1}{2}\right)^n u(-n-1)$



$$H(z) = \frac{z}{z - \frac{1}{2}} \qquad |z| < \frac{1}{2}$$



*** When region is asked I am asking values of z not of n

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*** n does not define a region

$|z| > \frac{1}{2}$ is ROC of $h(n) = \left(\frac{1}{2}\right)^n u(n)$

$|z| < \frac{1}{2}$ is ROC of $-h(n) = \left(\frac{1}{2}\right)^n u(-n-1)$

$h(n)$ is converging

Converging → tend to meet at point

Q. $h(n) = [1, 2, 1]$
↑

$H(z) = 1 + 2z^{-1} + z^{-2}$

$= \frac{z^2 + 2z + 1}{z^2}$ $z \neq 0$ is ROC

This $h(n)$ is define for all values of z except 0.

Q. $h(n) = [1, 2, 1]$
↑

$H(z) = \frac{z^2 + 2z + 1}{1}$; $z \neq \infty$ is ROC

$= z^2 + 2z$

Q. $h(n) = [1, 2, 1]$
↑

$H(z) = z + 2 + z^{-1}$

$= \frac{z^2 + 2z + 1}{z}$ — Two poles one is at origin & one at infinity

$z \neq 0$ & $z \neq \infty$ is ROC

All are FIR → nature (similarity)

$|z| > \frac{1}{2}$ → Im(z), Re(z)

ROC exist for both the system (IIR & FIR both)

Convergence → Time domain property
ROC → z domain.

*** Poles of the system is deciding the ROC
ROC is greater than the poles value & less than poles value except pole value

*** My region of convergence can never ever contain a pole.

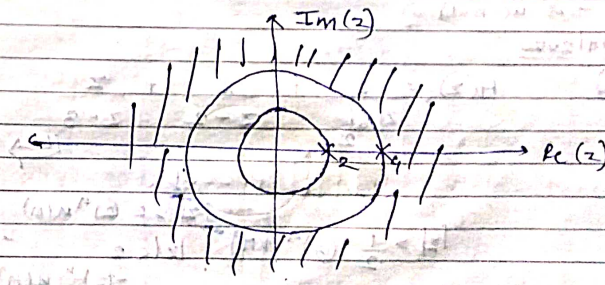
$H(z) = \frac{z}{z-2} + \frac{3z}{z-4}$ ← this condition will prevail

① $|z| > 2$ & $|z| > 4$ → $\boxed{|z| > 4}$ → $h(n) = 2^n u(n) + 3 \cdot 4^n u(n)$

② $|z| < 2$ & $|z| < 4$ → $\boxed{|z| < 2}$ → $h(n) = -2^n u(-n-1) - 3 \cdot 4^n u(-n-1)$

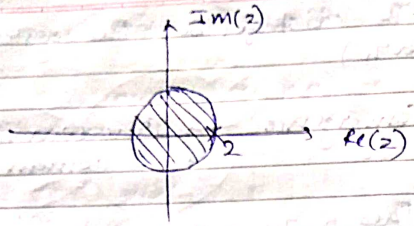
③ $|z| > 2$ & $|z| < 4$ → $\boxed{2 < |z| < 4}$ → $h(n) = 2^n u(n) - 3 \cdot 4^n u(-n-1)$

④

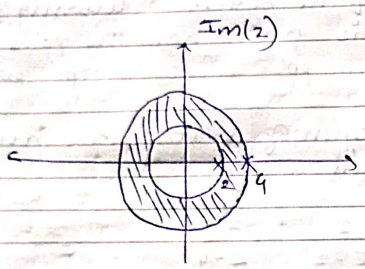


prevail → superior

(2)



(3)



$\frac{z}{z-a} + \frac{z}{z-b}$ — Both must satisfy

$\frac{z}{z-a}$ $|z| > a$ $a^n u(n)$ — $|z| > b$ $b^n u(n)$

OR

$\frac{z}{z-a}$ $|z| < a$ $-a^n u(-n-1)$ — $|z| < b$ $-b^n u(-n-1)$

15/07/2020

(1) $H(z) = \frac{z}{z-\frac{1}{2}} + \frac{z}{z-2} + \frac{z}{z-3}$

$|z| > \frac{1}{2}$ $(\frac{1}{2})^n u(n)$ — $|z| > 2$ $(2)^n u(n)$ — $|z| > 3$ $(3)^n u(n)$

$|z| < \frac{1}{2}$ $-(\frac{1}{2})^n u(-n-1)$ — $|z| < 2$ $-(2)^n u(-n-1)$ — $|z| < 3$ $-(3)^n u(-n-1)$

*** ROC can never contain a pole value

For zero's we need to solve the numerator then find out the poles & zero. as + sign is there in-between NM not multiplication

causal → $|z| > 3 \Rightarrow |z| > 2, |z| > \frac{1}{2}$

Non causal → $|z| < \frac{1}{2} \Rightarrow |z| < 2, |z| < 3$

For Finite Impulse response will have ROC of $z \neq 0$ & $z \neq \infty$ / $z \neq 0$ & $z \neq \infty$

ROC is entire z plane except $z = 0$ & $z = \infty$

*** There can other possibility other than ROC is in entire z plane except $z = 0$ / $z = \infty$ / $z = 0$ & $z = \infty$

→ If $|z| >$ max. pole value then that system is causal

→ & if $|z| <$ min. pole value then that system is Non causal

Q. $H(z) = \frac{z}{z-0.3} + \frac{z}{z-0.8}$

$|z| > 0.8$ → causal

$|z| < 0.3$ → Non causal

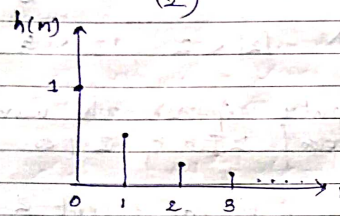
$0.3 < |z| < 0.8$ → Non causal

$|z|$ if lying between some pole then that system is still ✓ not causal

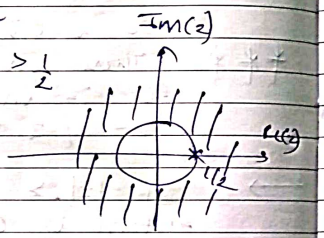
ROC gives the relation bt^n T & Z domain.

ROC is region where $h(n)$ is defined in z domain.

① $h(n) = \left(\frac{1}{2}\right)^n u(n)$

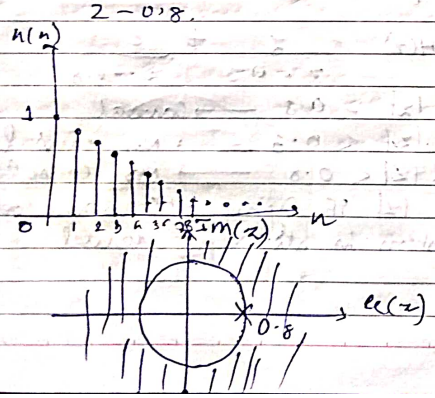


$H(z) = \frac{z}{z - \frac{1}{2}}$; $|z| > \frac{1}{2}$



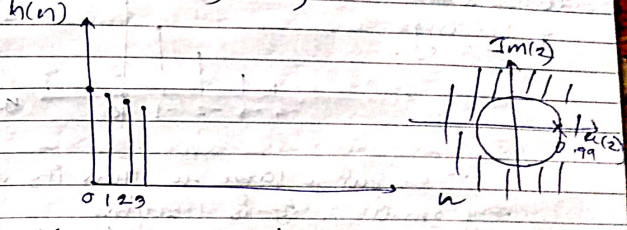
② $h(n) = 0.8^n u(n)$

$H(z) = \frac{z}{z - 0.8}$ $|z| > 0.8$.



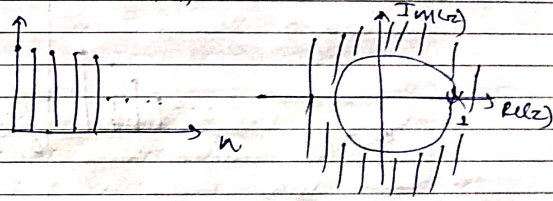
Decaying rate is more in ① than ②

③ $h(n) = (0.99)^n u(n)$

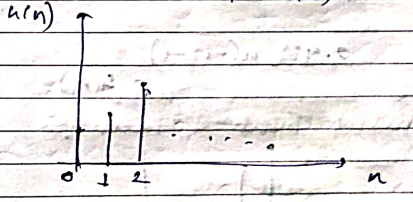


Very low decaying rate

④ $h(n) = 1^n u(n)$

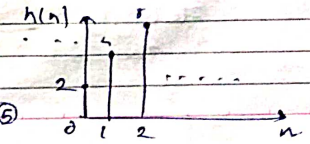


⑤ $h(n) = 1.01^n u(n)$



⑥ $h(n) = 2^n u(n)$

More decaying rate compare to ⑤



All have ROC

causal { When $h(n) = 1^n u(n)$ ⟶ Growing
When $h(n) < 1^n u(n)$ ⟶ Decaying

non causal { When $-h(n) > 1^n u(-n-1)$ ⟶ Decaying
When $-h(n) < 1^n u(-n-1)$ ⟶ Growing

$$y(n) = \sum_k h(k)\, x(n-k)$$

$$= \sum_k x(k)\, h(n-k)$$

represents a system

$h(n)$ ⟶ system / impulse response
$x(n), y(n)$ ⟶ signals

$y(n) = x(n) * h(n)$
↳ fairly controlled

∴ want to check $y(n)$ at different $h(n)$
∴ $y(n)$ depends on $h(n)$

Q. $y(n) = x(n) + a\,x(n-1) + b\,x(n-2) + c\,y(n-1) - d\,y(n-2)$

⟶ $Y(z) = X(z) + az^{-1}X(z) + bz^{-2}X(z) + cz^{-1}Y(z) - dz^{-2}Y(z)$

$$\frac{Y(z)}{X(z)} = H(z) = \frac{1 + az^{-1} + bz^{-2}}{1 - cz^{-1} + dz^{-2}}$$

Stability can be used
Stable cannot be used

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fairly controlled → Bounded → decaying

BIBO ~~sy~~ stable $\xrightarrow{for}$ stable
unstable $\xrightarrow{for}$ unstable itself

BIBO stability → ROC must include unit circle irrespective of system is causal or non causal.

(*) Condition for system being causal & BIBO stable

① Poles must be inside the unit circle
② It must include unit circle

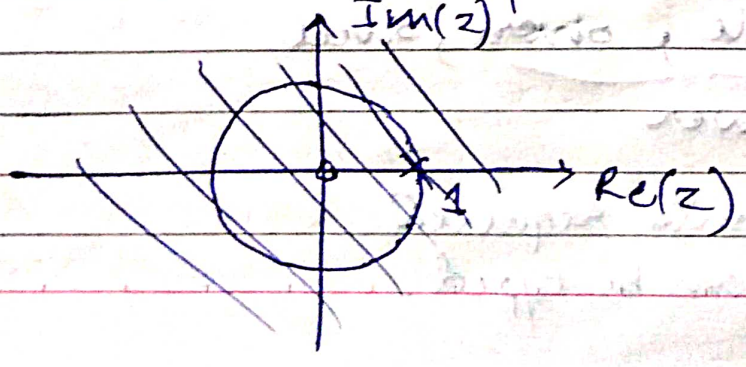
(*) Condition for system being non causal & BIBO stable

① Poles must be outside to the unit circle
② It must include unit circle

① $h(n) = [1, 2, 1]$
↑

- Causal
- FIR
- $H(z) = 1 + 2z^{-1} + z^{-2}$

ROC is entire z-plane except $z = 0$



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(2) $h(n) = (1, 2, 1)$
↑

$H(z) = \frac{z^2 + 2z + 1}{1}$

Pole is at ∞ $|z| \neq \infty$

For non causal & BIBO stable pole must be outside the unit circle

- Non causal
- FIR
- BIBO stable

*** All FIR are stable but converse is not true

*** IIR systems will be understudied for stability.

→ MST ⟶ 2 to 3 question
very similar to ~~that~~ assignment

→ Read the questions carefully

→ 90 min → Read

→ 1 & 1/2 page

→ If going beyond a page then there is something wrong then start thinking again.

→ Try to write what is ask not what is known

→ Simple, short, sweet answer

→ No formula required

→ Answer can be typed.