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Q1.] As we have seen in our lectures, where we took limejuice example, we saw that lime juice was represented as weighted sum of ingredients.
Similarly, a function can be represented as weighted sum of complex exponential.
The function taken by Keith is sinusoid function of 200 Hz.
Considering the signal to be continuous,

$$x(t) = A\sin(400\pi t) \qquad \left[\because x(t) = A\sin(2\pi f t) \text{ and } f = 200\text{Hz}\right]$$

It can be represented as

$$x(t) = A\left[\frac{e^{j400\pi t} - e^{-j400\pi t}}{2j}\right]$$

$$\left[\text{Since, } \sin x = \frac{e^{jx} - e^{-jx}}{2j}\right]$$

Now, $\because$ $x(t)$ is continuous & periodic; its spectrum will be discrete and aperiodic

$$X(K) = \frac{1}{T}\int_T x(t)\cdot e^{-j2\pi K f t}\,dt$$

$$= \frac{A}{T}\int_T \sin(400\pi t)\cdot e^{-j400\pi K t}\,dt$$

$$= \frac{A}{T}\int_T \left(\frac{e^{j400\pi t} - e^{-j400\pi t}}{2j}\right)\cdot e^{-j400\pi K t}\,dt$$

$$= \frac{A}{2jT}\int_T e^{j400\pi(1-K)t} - e^{-j400\pi(1+K)t}\,dt$$

$$= \frac{200A}{2j}\left[\frac{e^{j400\pi(1-K)t}}{j(400\pi)(1-K)} + \frac{e^{-j400\pi(1+K)t}}{-j(400\pi)(K+1)}\right]_0^{1/200}$$

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$$\Rightarrow X(K) = \frac{50A}{2j}\left[\frac{e^{2\pi j(1-K)}}{j(400\pi)(1-K)} + \frac{e^{-2\pi j(K+1)}}{j(400\pi)(K+1)} - \frac{2}{j(400\pi)(K^2-1)}\right]$$

Hence, we can see that the spectrum can be represented as sum of complex exponentials

i.e., $x(t) = \sum_{K} X(K) \cdot e^{j2\pi f K t}$; $K = 0, 1, 2, 3, \ldots$

If the same question had to be solved using DT,

$$X(K) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j2\pi nK/N}$$

Considering CT domain, the drawback is the infinite possible values of K.

Now,

$x(n) = 0.5 + 0.5\cos\left(\frac{2\pi n}{7}\right)$ [The signal is periodic & N=7]

$$X(K) = \sum_{n=0}^{6}\left(0.5 + 0.5\cos\left(\frac{2\pi n}{7}\right)\right) \cdot e^{-j2\pi nK/7}$$

$$= \sum_{n=0}^{6} 0.5 \cdot e^{-2\pi nK/7} + \sum_{n=0}^{6} 0.5\cos\left(\frac{2\pi n}{7}\right) e^{-j2\pi nK/7}$$

$$= 0.5\left(1 + e^{-2\pi K/7} + e^{-4\pi K/7} + e^{-6\pi K/7} - e^{\pi K/7} - e^{3\pi K/7} - e^{5K\pi/7}\right)$$
$$+ 0.5\left(1 + e^{-2\pi K/7}\cos\frac{2\pi}{7} + \ldots\right)$$

$$= 0.5\left[e^{-2\pi K/7}\left(1 - \cos\frac{2\pi}{7}\right) + e^{-4\pi K/7}\left(1 - \cos\frac{4\pi}{7}\right) + e^{-6\pi/7}\left(1 - \cos 6\pi/7\right) + e^{-\pi K/7}\left(1 + \cos\frac{\pi}{7}\right) - e^{3\pi K/7}\left(1 + \cos\frac{3\pi}{7}\right) - e^{5\pi/7}\left(1 + \cos\frac{5\pi}{7}\right)\right]$$

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Hence, the filter is allowing only low frequency components to pass. Hence, it is a low pass filter.

Q2-] Chintan claims that only real part of frequency response can be completely specified by only real part of frequency response.

Let $x(n)$ be complex.

$$x(n) = x_r(n) + i x_i(n)$$

$$X(\omega) = \sum_n [x_r(n) + i x_i(n)][\cos(\omega n) - i \sin(\omega n)]$$

$$= \sum_n [x_r(n) \cos\omega n - i x_r(n) \sin\omega n + i x_i(n) \cos\omega n - i^2 x_i(n) \sin\omega n]$$

$$[i^2 = -1]$$

$$= \sum_n [x_r(n) \cos\omega n + x_i(n) \sin\omega n]$$

$$+ i \sum_n [-x_r(n) \sin\omega n + x_i(n) \cos\omega n]$$

If $x(n)$ is real,

$$x(n) = x_r(n)$$

$$x_i(n) = 0$$

$$X(\omega) = X_R(\omega) + i X_i(\omega)$$

Hence, $X_R(\omega) = \sum_n x(n) \cos\omega n = X_R(-\omega)$

$$X_I(\omega) = -\sum_n x(n) \sin(\omega n) = -X_I(-\omega)$$

But if, $\omega = -\omega$, $X(-\omega) = X_R(-\omega) + i X_I(-\omega)$

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Hence, we can say that for a real signal, +ve & -ve side of frequency have same plots.
$\cos\theta$ is an even function & $\sin\theta$ is an odd function
i.e., $\cos(-\omega) = \cos\omega$
& $\sin(-\omega) = -\sin(\omega)$

~~Hence, we I agree with Chintan & oppose Mahesh's idea.~~

i) Further, if we consider $x(n)$ to be real & even;
we get $X(\omega)$ as real & even.

~~& if x(n)~~ i.e., $X_R(\omega) = \Sigma(\overset{\text{even}}{x(n)} \cdot \cos(\omega n))$
= something

$$X_I(\omega) = 0$$
Hence, $X(\omega) = X_R(\omega)$
and $X(\omega) = X(-\omega)$

ii) If $x(n)$ is real & odd

$$X_R(\omega) = \Sigma[x(n) \cdot \cos(\omega n)] = 0$$
$$X_i(\omega) = \Sigma[x(n) \cdot \sin(\omega n)] = \Sigma[\text{even}] = \text{something}$$

Hence, $X(\omega) = -X(-\omega)$
$X(\omega) = i\, X_i(\omega)$

Hence, $X(\omega)$ is imaginary & odd.

Hence, this is the reason for their argument. Both are correct at their respective places. Suppose if I pass $x(n) = n^3 + n^2$, then it is neither odd nor even. Hence, this $x(n)$ would be enough to pacify both of them.

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Q4] a.) i) $x(n) = e^{(jn\pi/2)}$

$y(n) = 0.25 e^{(jn\pi/2)}$

$\therefore y(n) = 0.25\, x(n)$

Let $x_1(n) = a\,x(n)$ & $x_2(n) = b\,x(n)$

$y_1(n) = 0.25\, a\, x(n)$
$y_2(n) = b\,(0.25)\, x(n)$

$\therefore y_1(n) + y_2(n) = (a+b) \cdot 0.25\, x(n)$
$= (a+b)\, y(n)$

$\therefore$ we can see that it is linear & time invariant.

$y(z) = 0.25\, x(z)$

$\therefore H(z) = \dfrac{Y(z)}{X(z)} = 0.25$

Let $z = e^{j\omega}$

$\therefore H(\omega) = 0.25$ (It is linear phase & there's only a scale in input)

ii) $y(n) = \dfrac{\sin(n\pi/3)}{n\pi}$, $x(n) = \dfrac{\sin(n\pi/6)}{n\pi}$

We know, $\cos\left(\dfrac{n\pi}{6}\right) = \sqrt{1-\sin^2\left(\dfrac{n\pi}{6}\right)}$

$\sin\left(\dfrac{n\pi}{3}\right) = 2\sin\left(\dfrac{n\pi}{6}\right) \cdot \cos\left(\dfrac{n\pi}{6}\right)$

$\therefore y(n) = \dfrac{2\, \cancel{\pi}/n\, x(n)\, \sqrt{1-(\pi n)^2 x^2(n)}}{\cancel{n\pi}}$

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Let $x_1(n) = ax(n)$ & $x_2(n) = b\,x(n)$

$y_1(n) = 2a\,x(n)\sqrt{1-(\pi n)^2 a^2 x^2(n)}$

$y_2(n) = 2b\,x(n)\sqrt{1-(n\pi)^2 b^2 x^2(n)}$

$y_1(n) + y_2(n) \neq (a+b)\,y(n)$

Hence, it is not linear.

If $n = n - n_0$

$y_1(n-n_0) = 2a\,x(n-n_0)\sqrt{1-\pi^2(n-n_0)^2 x^2(n-n_0)}$

If we delay the input, $x(n) = x(n-n_0)$

$y_1(n) = 2x(n-n_0)\sqrt{1-\pi^2(n^2)\cdot x^2(n-n_0)}$

$\therefore y_1(n-n_0) \neq y_1(n)$

Hence, it is time variant.

Also, transfer function exists only for LTI. Hence, no transfer function here.

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Q3.] a.) TMS320 is a DSP processor made by Texas Instruments as opposed to a general microprocessor.

Points of Contrast:-

i) TMS320 have address generators and are often used for real time computation as opposed to general microprocessor.

ii) TMS320 has an efficient external interface, powerful functional unit, used in image processing, audio & speech processing, etc. as opposed to low memory bandwidth, cheap cost, less robust hardware and limited uses in ~~ato~~ analog domain for general purpose microprocessors.