RELATIVE STABILITY (Shifting of Origin)

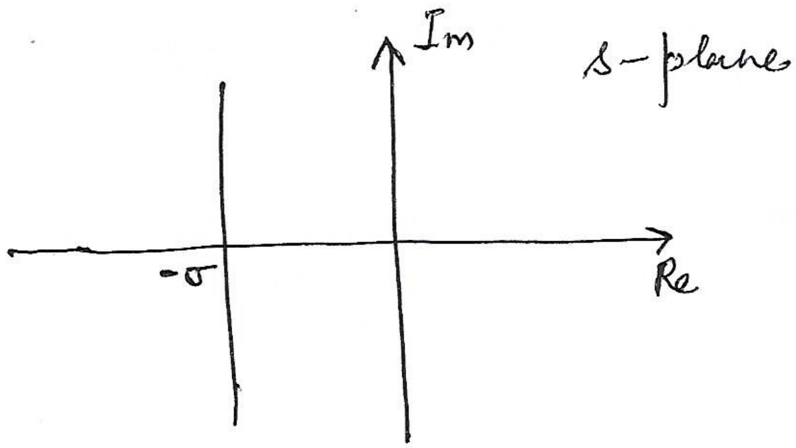
→ The Routh's Stability criterion ascertains absolute stability of a system by determining if all the roots of the characteristic equation lie in the LHSP.

→ Once a system is absolutely stable (all characteristic equation roots lie in the LHSP) it is desirable to determine its relative stability.

→ Routh's stability criterion can be extended for a preliminary relative-stability analysis.

→ To find out if all the roots of a given characteristic equation lie to the left of $s = -\sigma$ (as shown in Fig below), we substitute $s = \hat{s} - \sigma$ into the characteristic equation and write a polynomial in terms of $\hat{s}$.

→



→ Apply Routh's Stability Criterion to the new polynomial in $\hat{s}$.

→ If there are no changes in sign of the coefficients of the first column of the array developed for the polynomial in $\hat{s}$, it implies that all the roots of the original characteristic equation are more negative than $-\sigma$.

Pb 26 Consider a third order system with characteristic equation $s^3 + 7s^2 + 25s + 39 = 0$
Check if all the roots of this equation have real part more negative than -1

Solution Put $s = \hat{s} - 1$ in the characteristic eqn

$$(\hat{s}-1)^3 + 7(\hat{s}-1)^2 + 25(\hat{s}-1) + 39 = 0$$

$$\hat{s}^3 + 4\hat{s}^2 + 14\hat{s} + 20 = 0$$

Form an array with the coefficients of this equation.

$\hat{s}^3$	1	14
$\hat{s}^2$	4	20
$\hat{s}$	9	
$\hat{s}^0$	20	

Since there are no sign changes in the coefficients of the first column of the array, all the roots of the original characteristic equation are more negative than -1

b (27) The characteristic equation of a system is

$$s^2+2s+2+K_c\left(1+\frac{5}{s}\right)(s+3)=0$$

Determine the range of K_c for which the closed loop poles satisfy $Re(s) < -2$.

$(K_c > 0)$

Ans $K_c > 4.3892$

Pb (27)

SOLUTION

The characteristic equation is

$$s^2+2s+2+K_c\left(1+\frac{5}{s}\right)(s+3)=0$$

$$s^3+(2+K_c)s^2+(2+8K_c)s+15K_c=0$$

Let $s=\hat{s}-2$,

$$(\hat{s}-2)^3+(2+K_c)(\hat{s}-2)^2+(2+8K_c)(\hat{s}-2)+15K_c=0$$

$$\hat{s}^3+(K_c-4)\hat{s}^2+(4K_c+6)\hat{s}+(3K_c-4)=0$$

$\hat{s}^3$	1	$4K_c+6$
$\hat{s}^2$	K_c-4	$3K_c-4$
$\hat{s}^1$	$\dfrac{4K_c^2-13K_c-20}{K_c-4}$	
$\hat{s}^0$	$3K_c-4$	

(*) If there are no sign changes in the first column of the array, then all roots of the original characteristic equation satisfy $Re(s) < -2$

(*) Thus we require that K_c satisfies all the following conditions:

→ $K_c > 4$, $K_c > 4.3892$ or $K_c < -1.1392$,

$K_c > \frac{4}{3}$

→ The requirement $K_c < -1.1392$ is ~~disregarded~~ disregarded as K_c cannot be negative.

→ ∴ We have $Re(s) < -2$ for all closed loop poles provided $\boxed{K_c > 4.3892}$

HURWITZ STABILITY CRITERION

- The necessary and sufficient condition that all roots of the equation lie in the left half of the s-plane is that the polynomial's Hurwitz Determinants, D_K, $K = 1, 2, \ldots\ldots n$ must all be positive.

- Hurwitz determinants for

$$a_0 s^n + a_1 s^{n-1} + \ldots\ldots + a_{n-1} s + a_n = 0$$

are given by

$$D_1 = a_1, \quad D_2 = \begin{vmatrix} a_1 & a_3 \\ a_0 & a_2 \end{vmatrix}, \quad D_3 = \begin{vmatrix} a_1 & a_3 & a_5 \\ a_0 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{vmatrix}$$

$$D_n = \begin{vmatrix} a_1 & a_3 & a_5 & \cdots & a_{2n-1} \\ a_0 & a_2 & a_4 & \cdots & a_{2n-2} \\ 0 & a_1 & a_3 & \cdots & a_{2n-3} \\ 0 & a_0 & a_2 & \cdots & a_{2n-4} \\ 0 & 0 & a_1 & \cdots & a_{2n-5} \\ \vdots & \vdots & \vdots & & \\ 0 & 0 & 0 & \cdots & a_n \end{vmatrix}$$

- At a first glance the application of the Hurwitz determinants may seem to be formidable for higher order polynomials.

* Fortunately the rule was simplified by Routh into a tabulation, so one does not have to work with determinants.

* The relation between the elements in the first column of the Routh array and the Hurwitz determinants are

$$a_0 = a_0$$

$$a_1 = D_1$$

$$b_1 = \frac{D_2}{D_1}$$

$$c_1 = \frac{D_3}{D_2}$$

$$d_1 = \frac{D_4}{D_3}$$

* Therefore if all the Hurwitz determinants are positive, the elements in the first column would also be of the same sign.