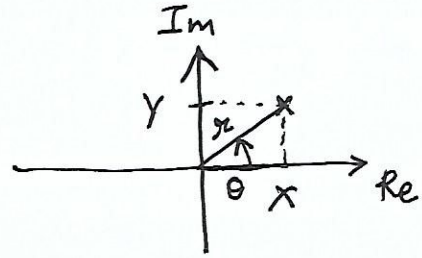
POLAR PLOTS

* The polar plot of a sinusoidal transfer function is a plot of the magnitude of $G(j\omega)$ versus the phase of $G(j\omega)$ in polar coordinates as ω is varied from 0 to ∞.

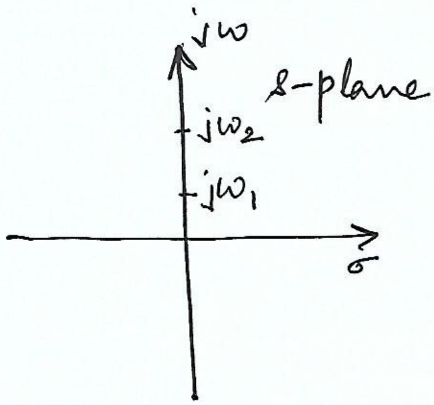
* A complex number may be expressed as $X+jY$, in Cartesian co-ordinates and as $re^{j\theta}$, in polar co-ordinates where $r=\sqrt{X^2+Y^2}$ and $\theta = \tan^{-1}\frac{Y}{X}$.

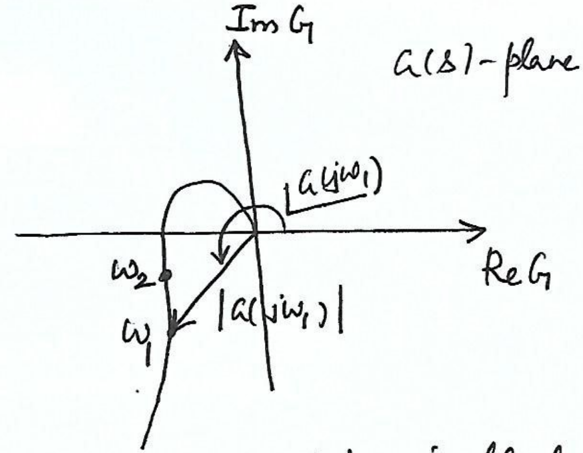


* In measuring the phase, counter clockwise is referred to as positive and clockwise as negative.

* From a mathematical point of view, polar plot may be regarded as mapping of the positive half of the imaginary axis of the s-plane onto the plane of the function $G(s)$.

* For any frequency $\omega=\omega_1$, the magnitude and phase angle of $G(j\omega_1)$ are represented by a phasor that has magnitude $|G(j\omega_1)|$ and phase angle $\angle G(j\omega_1)$, in the $G(s)$-plane



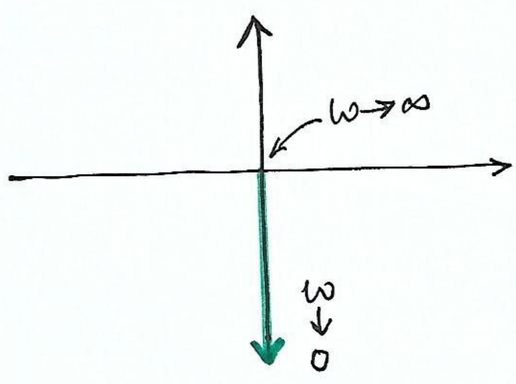


* An advantage in using a polar plot is that it depicts the frequency response characteristics of a system over the entire frequency range in a single plot.
* A disadvantage is that the plot does not clearly indicate the contributions of each of the individual factors of the OLTF.

① Polar plot of $\frac{1}{j\omega}$

→ When $\omega = 0$, mag $= \infty$ $\phi = -90^\circ$, $\infty \angle -90$

→ When $\omega \to \infty$, mag $= 0$ $\phi = -90$, $0 \angle -90$

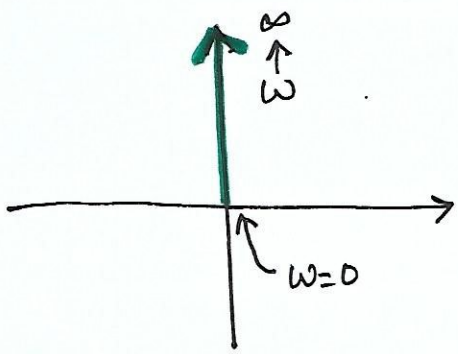


→ The polar plot of $G(j\omega) = \frac{1}{j\omega}$ is the negative imaginary axis. ~~since~~

② Polar plot of jw

→ When $w = 0$, mag $= 0$, $\phi = 90^\circ$, $0\angle 90$

→ when $w \to \infty$, mag $= \infty$, $\phi = 90^\circ$, $\infty \angle 90$



→ The polar plot of $G(jw) = jw$, is the positive imaginary axis

③ Polar plot of $\frac{1}{1+j\omega T}$

→ For the sinusoidal transfer function

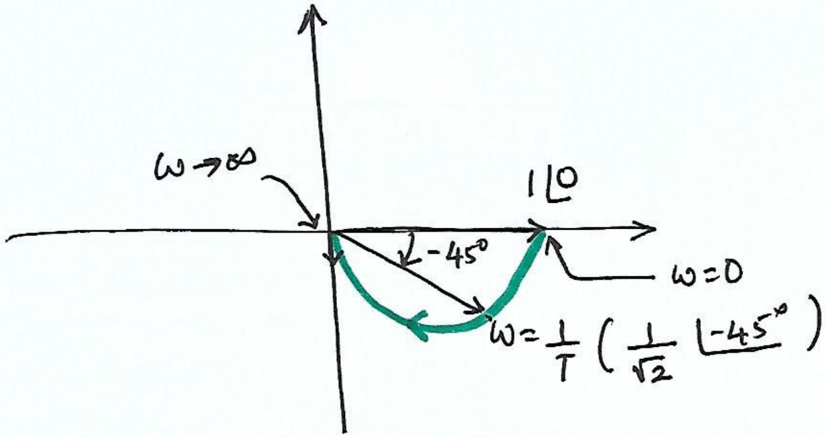
$$G(jw) = \frac{1}{1+j\omega T}, \quad Mag = \frac{1}{\sqrt{1+\omega^2 T^2}} \quad \phi = -\tan^{-1}\omega T$$

→ When $w = 0$, $1\angle 0$

→ When $w = \frac{1}{T}$, $\frac{1}{\sqrt{2}}\angle -45^\circ$

→ When $w \to \infty$, $0\angle -90^\circ$

→ The polar plot of $\frac{1}{1+j\omega T}$ is a semicircle as w varies from 0 to ∞



④ Polar plot of $1+j\omega T$

→ $G(j\omega) = 1+j\omega T$

→ When $\omega = 0$, $1\angle 0°$

→ When $\omega = \frac{1}{T}$, $\sqrt{2}\angle 45°$

→ When $\omega \to \infty$, $\infty \angle 90°$

→ The polar plot of the transfer function $1+j\omega T$ is simply the upper half of the straight line passing through $(1,0)$ in the complex plane and ~~of~~ parallel to the imaginary axis, as shown below