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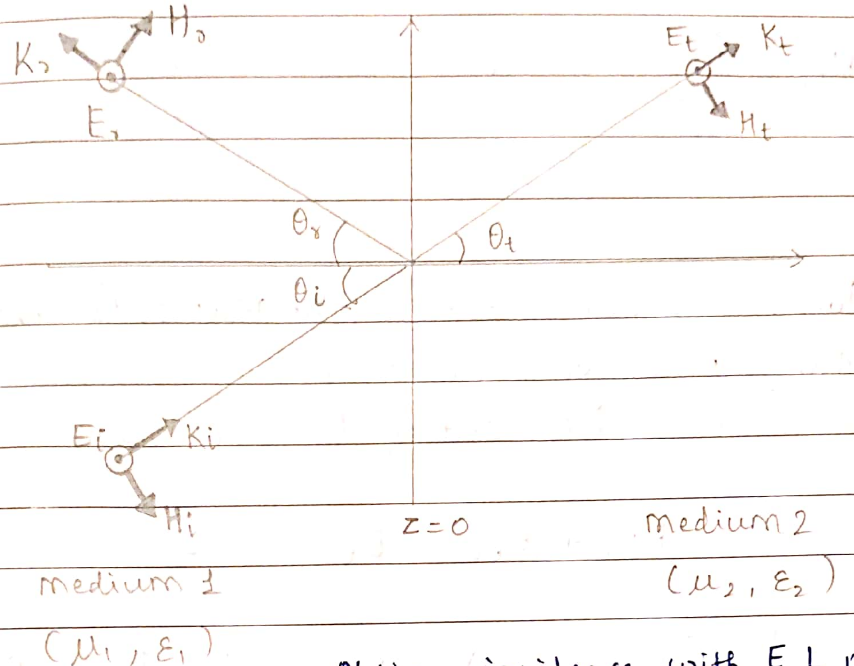
TY BTech EXTC

EWE Assignment :
Perpendicular Polarization

Q1] Derive the perpendicular polarization :



Oblique incidence with E ⊥ plane of incidence

Here, E ⊥ plane of incidence (xz-plane)
Also, H ∥ᵉˡ plane of incidence.

→ In medium 1, the incident & reflected fields are:-

$$E_{is} = E_{io}\, e^{-j\beta_1 (x\sin\theta_i + z\cos\theta_i)}\, a_y$$

$$H_{is} = \frac{E_{io}}{\eta_1}(-\cos\theta_i\, a_x + \sin\theta_i\, a_z)\, e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

$$E_{rs} = E_{ro}\, e^{-j\beta_1 (x\sin\theta_r - z\cos\theta_r)}\, a_y$$

$$H_{rs} = \frac{E_{ro}}{\eta_1}(\cos\theta_r\, a_x + \sin\theta_r\, a_z)\, e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}$$

→ In medium 2, the transmitted fields are :-

$$E_{ts} = E_{to}\, e^{-j\beta_2 (x\sin\theta_t + z\cos\theta_t)}\, a_y$$

$$H_{ts} = \frac{E_{to}}{\eta^2} (-\cos\theta_t\, a_x + \sin\theta_t\, a_z)\, e^{-j\beta_2 (x\sin\theta_t + z\cos\theta_t)}$$

→ Now, in order to satisfy the Maxwell's equation, the tangential components of E & H should be continuous at $z=0$ & i.e., setting $\theta_r = \theta_i$, we get,

$$E_{io} + E_{ro} = E_{to} \quad \text{— (i)}$$

$$\therefore \frac{1}{\eta_1} (E_{io} - E_{to}) \cos\theta_i = \frac{1}{\eta_2} E_{to} \cos\theta_t$$

→ Now, by expressing E_{ro} & E_{to} in terms of E_{io} fetches us

$$\Gamma_\perp = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} \quad \text{— (ii)}$$

∴ From (i) & (ii),

$$\tau_\perp = \frac{E_{to}}{E_{io}} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

i.e., $E_{to} = \tau_\perp E_{io}$

This is the Fresenel's equation for perpendicular polarization.

Hence, $E_{ro} = \Gamma_{\perp} E_{io}$

and, $E_{to} = \tau_{\perp} E_{io}$

$\therefore$ $\cancel{1+\Gamma_{\perp} = \tau_{\perp}}$

$\therefore 1 + \Gamma_{\perp} = \tau_{\perp}$ $\longrightarrow$ For normal incidence

For normal incidence, $\theta_i = \theta_t = 0$

→ For no reflection, $\Gamma_{\perp} = 0$ (i.e., $E_r = 0$)
Also, $\tau_{\perp} = 1$.

Lets replace θ_i with Brewster's angle $\theta_{B_\perp}$.

$$\therefore \eta_2 \cos\theta_{B_\perp} = \eta_1 \cos\theta_t$$

i.e., $$\boxed{\eta_2^{\,2}(1-\sin^2\theta_{B_\perp}) = \eta_1^{\,2}(1-\sin\theta_t^{\,2})}$$

→ For nonmagnetic media ($\mu_1 = \mu_2 = \mu_0$), $\sin^2\theta_B \rightarrow \infty$.
Also, if $\mu_1 \neq \mu_2$ & $\varepsilon_1 = \varepsilon_2$, then

$$\sin\theta_{B_\perp} = \sqrt{\frac{\mu_2}{\mu_1+\mu_2}}$$

Hence, proved.

Page No. ____
Date ____

Practice Problems – Sadiku

1.) Problem No. 10.10

If $E = (10a_y + 5a_z)\cos(\omega t + 2y - 4z)$ V/m in free space,

Find :-

a.) ω & λ :-

→ Now, $K = -2a_y + 4a_z$

$\therefore |K| = \sqrt{2^2+4^2} = \sqrt{20}$

$\omega = Kc = 3\times10^8\sqrt{20} = \underline{1.342\times10^9 \text{ rad/s}}$

$\therefore \lambda = 2\pi K = \underline{28.1 m}$

b.) Magnetic field component H :-

→ $H = \dfrac{a_K \times E}{\eta_0} = \dfrac{(-2a_y + 4a_z)}{\sqrt{20}\,(120\pi)} \times (10a_y + 5a_z)\cos(\omega t - K.r)$

$= \underline{-29.66\cos(1.342\times10^9 t + 2y - 4z)\,a_x \text{ mA/m}}$

c.) Time avg. power:-

→ $P_{avg} = \dfrac{|E_0|^2}{2\eta_0} a_K = \dfrac{125}{2(120\pi)} \dfrac{(-2a + 4a_z)}{\sqrt{20}}$

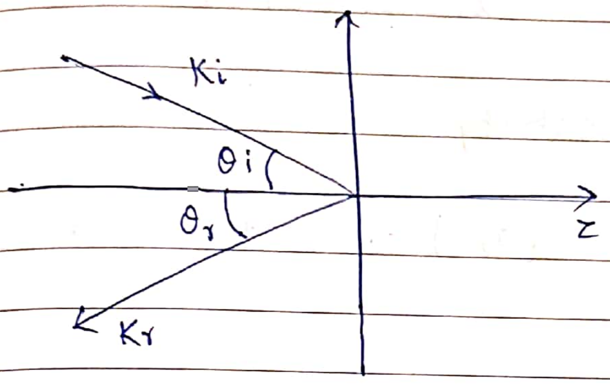
$= \underline{-74.15a_y + 148.9a_z \text{ W/m}^2}$

2.) **Problem No. 10.11 :-**

If the plane wave of practice problem 10.10 is incident on a dielectric medium having $\sigma = 0$, $\varepsilon = 4\varepsilon_0$, $\mu = \mu_0$ & occupying $z \geq 0$, calculate :-

(a.) **The angles of incidence, reflection, and transmission :-**

→



$$\tan\theta_1 = \frac{K_{iy}}{K_{iz}} = \frac{2}{4}$$

$$\therefore \theta_1 = 26.56^\circ = \theta_r$$

$$\therefore \sin\theta_t = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}} \Rightarrow \sin\theta_t = \frac{1}{2}\sin(26.56^\circ)$$

$$\therefore \boxed{\theta_t = 12.92^\circ}$$

(b.) **The reflection and transmission coefficients :-**

→ $\eta_{1} = \eta_0$, $\eta_2 = \eta_0/2$, E is parallel to the plane of incidence. Since $\mu_1 = \mu_2 = \mu_0$,

$$\Gamma_{11} = \frac{\tan(\theta_t - \theta_1)}{\tan(\theta_t + \theta_1)} = \frac{\tan(-13.64^\circ)}{\tan(39.48^\circ)} = -0.2946$$

$$\tau_{11} = \frac{2\cos(26.56^\circ)\sin(12.92^\circ)}{\sin(39.48^\circ)\cos(-13.64^\circ)} = 0.6474$$

Page No.
Date

(c.) The total E in the free space :-

$\rightarrow$ $K_r = -\beta_1 \sin\theta_r a_y - \beta_1 \cos\theta_r a_z$

Now, $K_r \cdot E_r = 0$ or $\nabla \cdot E_r = 0$.

Let $E_r = \pm E_{or}(-\cos\theta_r a_y + \sin\theta_r a_z)$
$\cos(\omega t + \beta_1 \sin\theta_r y + \beta_1 \cos\theta_r z)$

Only, (+) sign will satisfy

$E_t = E_{ot}(\cos\theta_t a_y + \sin\theta_t a_z) \cos(\omega t + 2y - 4z)$

$\because \theta_r = \theta_i$,

$E_{or} \cos\theta_r = \Gamma_{11} E_{ot} \cos\theta_t = 10\Gamma_{11} = -2.946$

$E_{or} \sin\theta_r = \Gamma_{11} E_{ot} \sin\theta_t = 5\Gamma_{11} = -1.473$

$\beta_1 \sin\theta_r = 2 \Rightarrow \beta_1 \cos\theta_r = 4$

$\therefore E_r = -(2.946 a_y - 1.473 a_z) \cos(\omega t + 2y + 4z)$

$E_1 = E_i + E_r = (10 a_y + 5 a_z) \cos(\omega t + 2y - 4z) +$
$(-2.946 a_1 + 1.473 a_z) \cos(\omega t + 2y + 4z)$

V/m

Page No. ______
Date ______

(d.) The total E field in the dielectric :-

$K_t = -\beta_2 \sin\theta_t a_y + \beta_2 \cos\theta_t a_z$

since, $K_r \cdot E_r = 0$, let,

$E_t = E_{ot} (\cos\theta_t a_y + \sin\theta_t a_z) \cos(\omega t + \beta_2 y \sin\theta_t - \beta_2 z \cos\theta_t)$

$\beta_2 = \omega\sqrt{\mu_2 \cdot \varepsilon_2} = \beta_1 \sqrt{\varepsilon_{r_2}} = 2\sqrt{20}$

$\sin\theta_t = \frac{1}{2} \sin\theta_i = \frac{1}{2\sqrt{5}}$, $\cos\theta_t = \frac{\sqrt{19}}{\sqrt{20}}$

$\beta_2 \cos\theta_t = 2\sqrt{20} \sqrt{\frac{19}{20}} = 8.718$

$E_{ot} \cos\theta_t = \tau_{11} E_{ot} \cos\theta_t = 0.6474 \sqrt{125} \sqrt{\frac{19}{20}}$

$= 7.055$

$E_{ot} \sin\theta_t = \tau_{11} E_{ot} \sin\theta_t = 0.6474 \sqrt{125} \sqrt{\frac{1}{20}}$

$= 1.6185$

Hence,

$$\boxed{E_2 = E_t = (7.055 a_y + 1.6185 a_z) \cos(\omega t + 2y - 8.718 z) \text{ V/m}}$$

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(e.) The Brewster Angle :-

$$\tan\theta_{B\parallel} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = 2$$

$$\Rightarrow \quad \underline{\underline{\theta_{B\parallel} = 63.43^\circ}}$$

-x-x-x-