Pb 32 Sketch the root locus for the system whose open loop transfer function is

$$G(s)H(s) = \frac{K}{(s+1)(s^2+4s+5)}$$

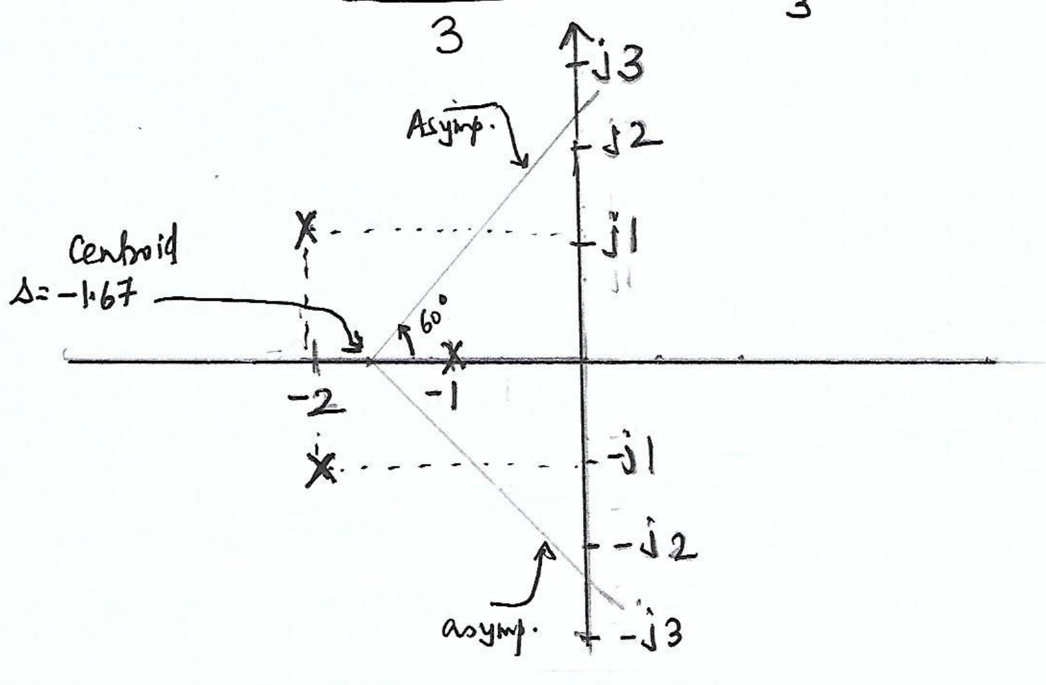
Solution

$$G(s)H(s) = \frac{K}{(s+1)(s+2-j)(s+2+j)}$$

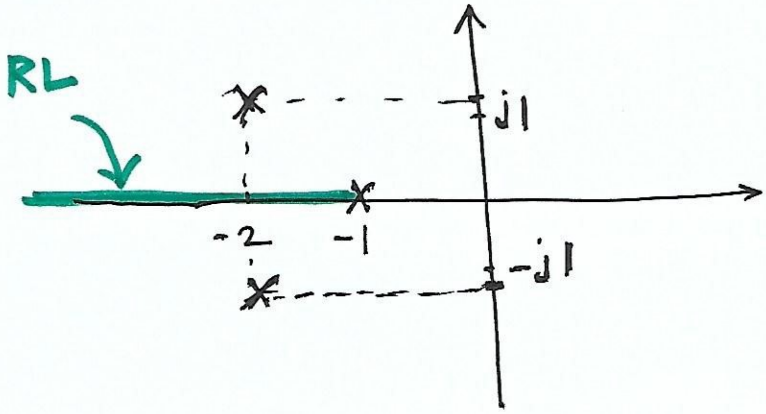
1. $K=0$ points are at $s=-1$, $s=-2+j$, $s=-2-j$
2. $K=\infty$ points are at $s=\infty$, $s=\infty$, $s=\infty$
3. Number of branches $= 3$ $(n=3,\ m=0)$
4. The root locus is symmetrical about the real axis.
5. Asymptotes $\quad \theta_l = \dfrac{(2l+1)\pi}{n-m} \quad l = 0,1,2$

$$\theta_0 = \frac{\pi}{3} = 60^\circ,\quad \theta_1 = \pi = 180^\circ,\quad \theta_2 = \frac{5\pi}{3} = 300^\circ$$

6. Centroid $\quad \dfrac{(-1-2-2)-0}{3} = -\dfrac{5}{3} = -1.67$



(7) Root locus on the real axis



(8) Angles of departure

Let the angle of departure at $s = -2+j$ be θ_{D1}

$$\left[0 - \left(\angle s+1 + \theta_{D1} + \angle s+2+j\right)\right]_{s=-2+j} = 180^\circ$$

$$\left[0 - \left(\angle -2+j+1 + \theta_{D1} + \angle -2+j+2+j\right)\right] = 180^\circ$$

$$\left[0 - \left(\angle -1+j + \theta_{D1} + \angle 0+2j\right)\right] = 180^\circ$$

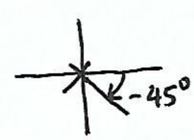
$$\left[0 - \left(135^\circ + \theta_{D1} + 90^\circ\right)\right] = 180^\circ$$

$$\left[0 - \left(225 + \theta_{D1}\right)\right] = 180$$

$$225 + \theta_{D1} = -180$$

$$\theta_{D1} = -180 - 225 = -405^\circ$$

$$\boxed{\theta_{D1} = -45^\circ}$$



Let the angle of departure at $-2-j$ be θ_{D2}

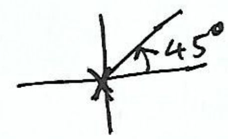
$$\left[0-\left(\angle s+1+\angle s+2-j+\theta_{D2}\right)\right]_{s=-2-j}=180$$

$$\left[0-\left(\angle -2-j+1+\angle -2-j+2-j+\theta_{D2}\right)\right]=180$$

$$\left[0-\left(\angle -1-j+\angle -2j+\theta_{D2}\right)\right]=180$$

$$0-(45+90+\theta_{D2})=180$$

$$0-(135+\theta_{D2})=180$$



$$\theta_{D2}=-315^\circ$$

$$\boxed{\theta_{D2}=45^\circ}$$

(9) Intersection of the root loci with the imaginary axis.

$$G(s)H(s)=\frac{K}{(s+1)(s^2+4s+5)}$$

$$1+G(s)H(s)=0$$

$$(s+1)(s^2+4s+5)+K=0$$

$$s^3+5s^2+9s+5+K=0$$

s^3	1	9
s^2	5	$5+K$
s	$\frac{45-(5+K)}{5}$	
s^0	$5+K$	

$$\frac{45-(5+K)}{5}=0$$

$$K=40$$

$$s=\pm j3$$

(10) There are no breakaway points.