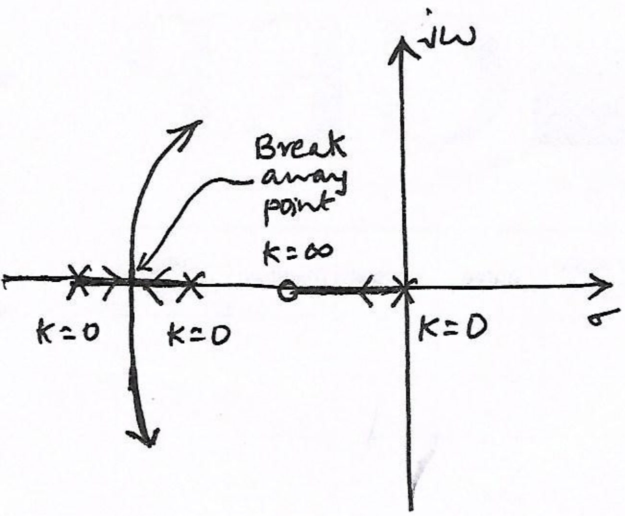
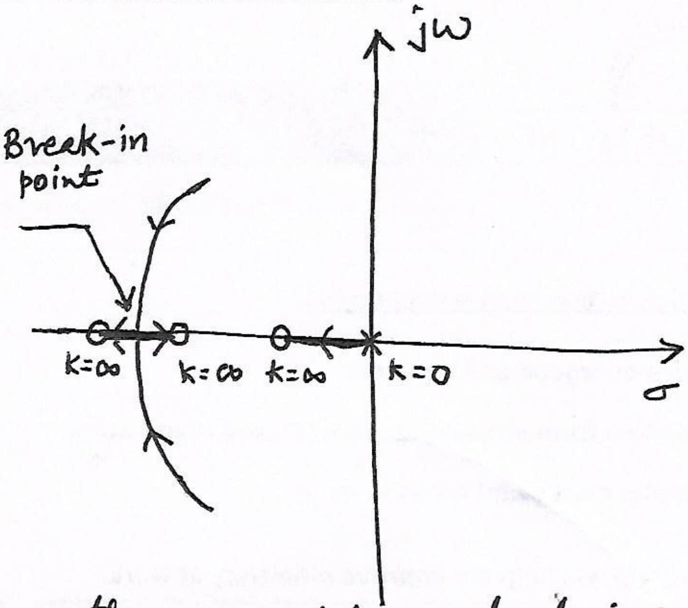
(10) Breakaway points of the root loci





The breakaway points on the complete root loci may be found from

$$\frac{d}{ds}\left[G_1(s)H_1(s)\right] = 0$$

Eg Rule (9) Intersection with imaginary axis.

Ch. eqn $s(s+2)(s+3) + k(s+1) = 0$

$$s^3 + 5s^2 + (6+k)s + k = 0$$

s^3	1	$6+k$
s^2	5	k
s	$\frac{5(6+k)-k}{5}$	
s^0	k	

$$\frac{5(6+k)-k}{5} = 0$$

$$30 + 5k - k = 0$$

$$k = -\frac{30}{4}$$

The root locus does not intersect the ~~s~~ imaginary axis

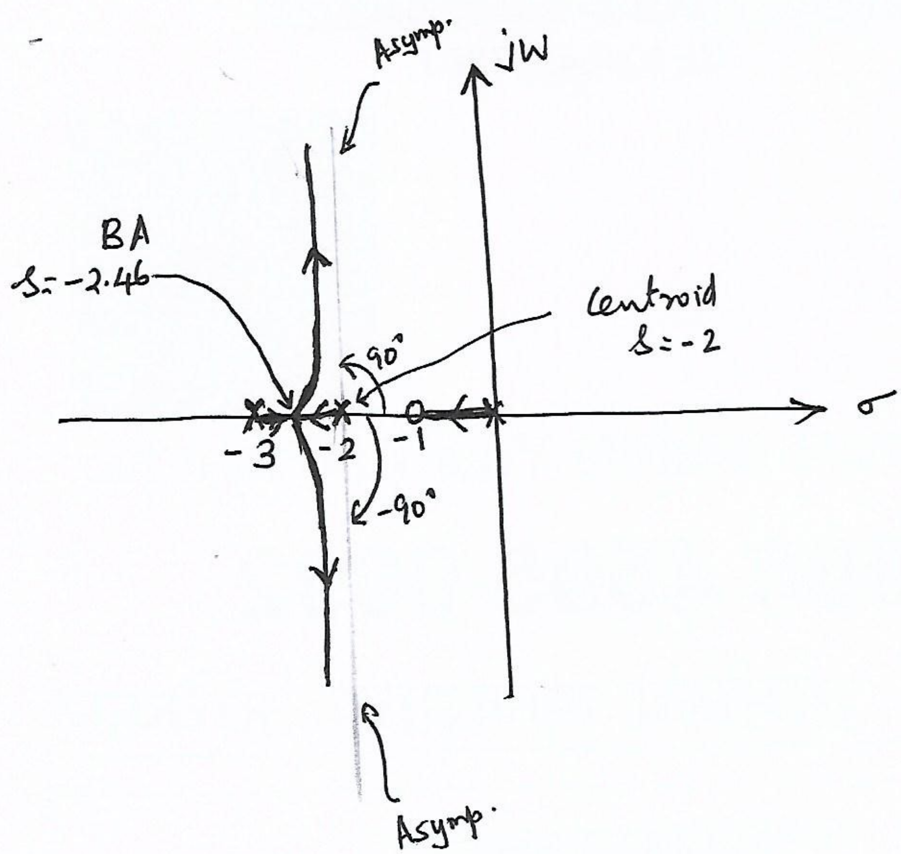
Eg Rule (10) Breakaway points.

$$\frac{d}{ds}\left[G(s)H(s)\right] = 0$$

$$\frac{d}{ds}\left[\frac{s+1}{s(s+2)(s+3)}\right] = 0$$

$s = -2.46$, $s = -0.76$

$s = -2.46$ is the breakaway point.



Pb 30 Sketch the root locus for the closed loop system whose open loop transfer function is

$$G(s)H(s) = \frac{K}{s(s+2)(s+4)} \quad (K \geq 0)$$

Solution

$$G(s)H(s) = K\,G_1(s)H_1(s)$$

$$\therefore G_1(s)H_1(s) = \frac{1}{s(s+2)(s+4)}$$

① The $K=0$ points are at $s=0$, $s=-2$, $s=-4$

② The $K=\infty$ points are at $s=\infty$, $s=\infty$, $s=\infty$

③ Number of branches $= 3$

④ The root locus is symmetrical about the real axis.

⑤ Angles of asymptotes

$$\theta_l = \frac{(2l+1)\pi}{n-m} \qquad l = 0, 1, 2 \cdots\cdots |n-m|-1$$

$$|3-0|-1$$

$$l = 0, 1, 2$$

$$\theta_0 = \frac{\pi}{3} = 60^\circ$$

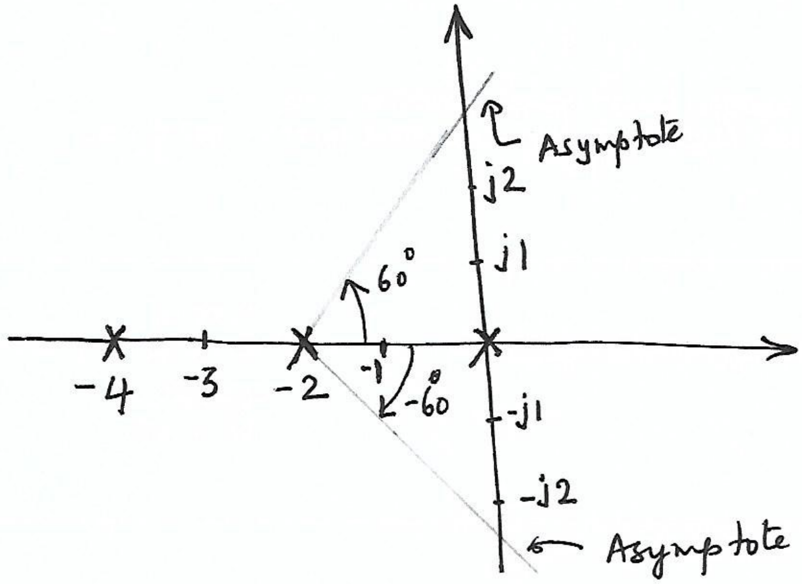
$$\theta_1 = \pi = 180^\circ$$

$$\theta_2 = \frac{5\pi}{3} = 300^\circ$$

⑥ Centroid (Point of intersection of asymptotes)

$$\frac{\sum \text{real part of poles of } G(s)H(s) - \sum \text{real part of zeros of } G(s)H(s)}{n-m}$$

$$\frac{(0-2-4)-0}{3-0} = \frac{-6}{3} = -2$$

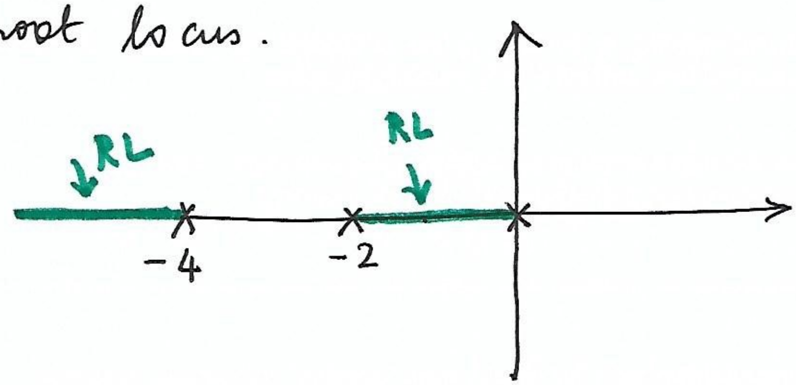


(7) Root loci on the Real axis

→ Between sections 0 to -2, there is only one pole to the right of the section ie odd, so the section includes the roots locus.

→ Between -2 and -4 there are two poles to the right of the section ie even, so the section does not form part of the root locus.

→ Beyond -4 the poles and zeros to the right of the section is 3 ie odd. Hence the section forms part of the root locus.



8. Angles of departure/arrival are found only for complex conjugate pole/zero pair.

9. Intersection of the root locus with the imaginary axis

$$s(s+2)(s+4)+K=0$$

$$s^3+6s^2+8s+K=0$$

s^3	1	8
s^2	6	K
s	$\frac{48-K}{6}$	
s^0	K	

$$\frac{48-K}{6}=0$$

$$K=48$$

$$6s^2+48=0$$

$$s=\pm j2.828$$

10. Breakaway points.

$$\frac{d}{ds}\left[G_1(s)H_1(s)\right]=0$$

$$\frac{d}{ds}\left[\frac{1}{s(s+2)(s+4)}\right]=0$$

$$3s^2+12s+8=0$$

$$s=-3.15 \quad \text{or} \quad s=-0.846$$

Since the breakaway point lies between 0 and −2, $s=-0.846$ is the breakaway point.

⑧ Angles of departure/arrival are found only for complex conjugate pole/zero pair.

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⑩ Breakaway points.

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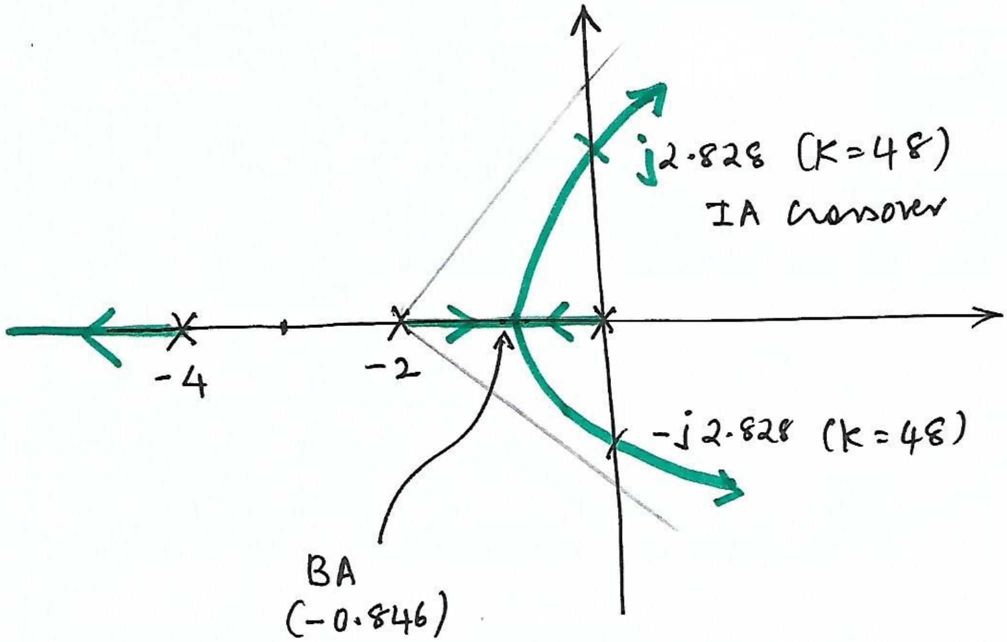

j2.828 (K=48)
IA crossover
-4
-2
-j2.828 (k=48)
BA
(-0.846)