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TY BTech EXTC
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MST - DCS - Sem V

Q1.] a.) Here, we are given

$V_1 = (1000)$ V_2 (1010)

Their linear combinations are $a_1, a_2 = \{0, 1\}$

Now, by the linear combinations, we'll find out the value of $a_1 V_1 + a_2 V_2$

$\therefore$ $0.(1000) + 0.(1010)$ $[a_1 = 0, a_2 = 0]$
$= (0000) + (0000)$
$= 0000$

$0.(1000) + 1.(1010)$ $[a_1 = 0, a_2 = 1]$
$= (0000) + (1010)$
$= 1010$

$1 (1000) + 0.(1010)$ $[a_1 = 1, a_2 = 0]$
$= 1000 + 0000$
$= 1000$

$1(1000) + 1(1010)$ $[a_1 = a_2 = 1]$
$= (1000) + (1010)$
$= (0010)$

Here, we can see that $a_1 V_1 + a_2 V_2 \neq 0$
Only when $a_1 = a_2 = 0$, $a_1 V_1 + a_2 V_2 = 0$

Hence, they are linearly independent.

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Q1.] b.) $P(x) = x^3 + x^2 + 1$

$\because m = 3$

$\therefore n = 2^m - 1 = 7$

$\therefore$ It should divide a polynomial ~~given~~ with degree 7.

$$
\begin{array}{r l}
 & x^4 + x^3 + x^2 + 1 \\
\hline
x^3 + x^2 + 1 \;) & x^7 \qquad\qquad\qquad + 1 \\
 & x^7 + x^6 + x^4 \\
 & (-) \\
\hline
 & x^6 + x^4 \qquad + 1 \\
 & x^6 + x^5 + x^3 \\
 & (-) \\
\hline
 & x^4 + x^5 + x^3 + 1 \\
 & x^4 + x^5 \qquad + x^2 \\
 & (-) \quad (-) \\
\hline
 & x^3 + x^2 + 1 \\
 & x^3 + x^2 + 1 \\
 & (-) \; (-) \; (-) \\
\hline
 & 0
\end{array}
$$

$\because$ The remainder $= 0$,

$\therefore P(x) = x^3 + x^2 + 1$ is primitive.

Also, for GF(2) let us consider $\{0, 1\}$

$\therefore P(0) = 0^3 + 0^2 + 1 \neq 0$

$P(1) = 1 + 1 + 1 = 3 \neq 0$

Hence, the polynomial is irreducible as well.

Q1.] c.) $GF(8) = GF(2^3) = GF(2^m)$

$\therefore m = 3$

$\therefore GF(8) = \{0, 1, \alpha, \alpha^2, \alpha^3, \alpha^4 \ldots\ldots\ldots \alpha^{2^m-2}\}$

$\therefore GF(8) = \{0, 1, \alpha, \alpha^2, \alpha^3, \alpha^4, \alpha^5, \alpha^6\}$

Given primitive polynomial is $P(X) = X^3 + X + 1$

Putting $x = \alpha$ ($\because \alpha$ is a primitive element)

$\therefore \alpha^3 + \alpha + 1 = 0$

$\therefore \alpha^3 = \alpha + 1$

$\therefore \alpha^4 = \alpha^3 \cdot \alpha = \alpha(\alpha+1) = \alpha^2 + \alpha$

$\therefore \alpha^5 = \alpha^4 \cdot \alpha = \alpha(\alpha^2+\alpha) = \alpha^3 + \alpha^2 = 1 + \alpha + \alpha^2$

$\therefore \alpha^6 = \alpha^5 \cdot \alpha = \alpha(1+\alpha+\alpha^2) = \alpha + \alpha^2 + \alpha^3$

$= \alpha + \alpha^2 + \alpha + 1$

$= 1 + \alpha^2$

$\therefore GF(8) = \{0, 1, \alpha, \alpha^2, 1+\alpha, \alpha^2+\alpha, 1+\alpha+\alpha^2, 1+\alpha^2\}$

$\therefore$ Multiplicative inverse of α will be α^6.

$\alpha^6 = 1 + \alpha^2$

$\therefore$ In binary form, $\alpha^6 = [1\ 0\ 1]$

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Q2.]

a.) The conjugacy class for α^3 can be found by the following method:-

Let $\beta = \alpha^3$

$\therefore (\beta)^{2^i}$ for $i = 1, 2, 3, \ldots$ is to be found out.

$\therefore (\alpha^3)^{2^1} = \alpha^6$

$(\alpha^3)^{2^2} = \alpha^{12}$

$(\alpha^3)^{2^3} = \alpha^{24}$

$(\alpha^3)^{2^4} = \alpha^{48} = \alpha^{31} \cdot \alpha^{17} = \alpha^{17}$

$(\alpha^3)^{2^5} = \alpha^{96} = (\alpha^{31})^3 \cdot \alpha^3 = \alpha^3$ $\Rightarrow$ Repetition starts

$\therefore$ Conjugacy class for

$\alpha^3 = \{\alpha^3, \alpha^6, \alpha^{12}, \alpha^{17}, \alpha^{24}\}$

Q2.] b.). $F = \{0, 1, 2, 3, 4\}$

Additive Table :-

+	0	1	2	3	4
0	0	1	2	3	4
1	0	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

$\Rightarrow$ Commutative

i.e, $a+b = b+a$

$1+2 = 2+1 = 3$

$1+4 = 4+1 = 0$

$\therefore$ Commutative

Multiplicative Table :-

$\times$	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

$\Rightarrow$ Commutative

i.e, $a * b = b * a$

i.e, $2 * 3 = 3 * 2 = 1$

$4 * 3 = 3 * 4 = 2$

A) Here, we can see that closure property is followed, i.e, the result of addition or multiplication of two elements in set lies in the set.

B) Additive idenity $\rightarrow$ Let it be e.

$\therefore a+e = a$ Here, $e = 0$

Multiplicative identity $\rightarrow$ Let it be e

$\therefore a \cdot e = a$ where, $e = 1$

Q2.] (i) Order m for an extension field GF(n) is given by,

$$m = \log_2 n$$

Here, $n = 32$

$\therefore m = \log_2 32 = 5$

$\therefore$ order of this extension field GF(32) $= 5$

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Q3.] a.) We know,

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \rightarrow \text{given}$$

$m = [1\ 0\ 1] \longrightarrow$ given

Also, $c = m \cdot G$

$$= [1\ 0\ 1] \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= [1\cdot1 + 0\cdot0 + 1\cdot0 \quad 1\cdot0 + 0\cdot1 + 1\cdot0 \quad 1\cdot0 + 0\cdot0 + 1\cdot1 \quad 1\cdot1 + 0\cdot0 + 1\cdot1 \quad 1\cdot1 + 0\cdot1 + 1\cdot1 \quad 1\cdot0 + 0\cdot1 + 1\cdot1]$$

$$= [1\ 0\ 1\ 0\ 0\ 1]$$

Q3.] b.) Now, we know, $G = [I_k\ P_{n-k}]$
From given Generator matrix,

$$P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Also, $H = [P^T\ I_{n-k}]$

$$\therefore H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

To check if the codeword is valid, $S = r \cdot H^T = 0$

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$$\therefore S = [0\ 1\ 1\ 1\ 1\ 1\ 1]\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore S = [\ 0\cdot 1 + 1\cdot 0 + 1\cdot 1 + 1\cdot 1 + 1\cdot 0 + 1\cdot 0 \quad 0\cdot 1 + 1\cdot 1 + 1\cdot 1 + 1\cdot 0 + 1\cdot 1 + 1\cdot 0 \quad 0\cdot 0 + 1\cdot 1 + 1\cdot 1 + 1\cdot 0 + 1\cdot 0 + 1\cdot 1\ ]$$

$$\therefore S = [0\ 1\ 1]$$

$\because$ The syndrome is not $[0\ 0\ 0]$,
$\therefore$ The received codeword $[1\ 0\ 1\ 1\ 1\ 1\ 1]$ is not a valid codeword of the code set and has an error at one bit. ~~(first bit)~~.

Q3-] c.) Now, we have been given

$$X^{15} + 1 = (X+1)(X^4 + X^3 + X^2 + X + 1)(X^4 + X^3 + 1)\cdot(X^4 + X + 1)(X^2 + X + 1)$$

For a (n,k) systematic cyclic code, degree of generator polynomial $G(X)$ is $n-k$.

$\therefore$ Degree of $G(X) = 15 - 9 = 6$

$$\therefore G(X) = (X^4 + X + 1)(X^2 + X + 1)$$
$$= X^6 + X^5 + X^4 + X^3 + \cancel{X^2} + \cancel{X} + \cancel{X^2} + \cancel{X} + 1$$
$$= X^6 + X^5 + X^4 + X^3 + 1$$

$$\therefore G(X) = X^6 + X^5 + X^4 + X^3 + 1$$