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EWE - Tutorial II

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TY BTech EXTC

Q1.] The total electromagnetic energy given is

$$W = \frac{1}{2}\int (E \cdot D + H \cdot B)\, dv$$

From Maxwell's equations,

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{— (i)}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad \text{— (ii)}$$

By dotting (ii) on both sides with $\vec{E}$, we get,

$$\vec{E} \cdot (\nabla \times \vec{H}) = \vec{E} \cdot \vec{J} + \vec{E} \cdot \frac{\partial \vec{B}}{\partial t} \quad \text{— (iii)}$$

[Since, $\vec{A}$ and $\vec{B}$ are arbitrary vector,

$$\nabla \cdot (\vec{A} \times \vec{B}) = B \cdot (\nabla \times \vec{A}) - \vec{A}\,(\nabla \times \vec{B})$$]

Applying the above rule on LHS of (iii),

$$\vec{H} \cdot (\nabla \cdot \vec{E}) + \nabla \cdot (\vec{H} \times \vec{E}) = \vec{E} \cdot \vec{J} + \frac{1}{2}\frac{\partial}{\partial t}(\vec{B} \cdot \vec{E}) \quad \text{— (iv)}$$

From (i),

$$\vec{H} \cdot (\nabla \times \vec{E}) = \vec{H} \cdot \left(-\frac{\partial \vec{B}}{\partial t}\right) = \frac{1}{2}\frac{\partial}{\partial t}(\vec{B} \cdot \vec{H}) \quad \text{— (v)}$$

Substituting (v) in (iv), we get,

$$-\frac{1}{2}\frac{\partial}{\partial t}(\vec{B} \cdot \vec{H}) - \nabla \cdot (\vec{E} \times \vec{H}) = \vec{J} \cdot \vec{E} + \left(\frac{1}{2}\right)\frac{\partial}{\partial t}(\vec{D} \cdot \vec{E})$$

Hence, by rearranging the terms and taking volume integral on both sides, we get,

$$\int_V \nabla\cdot(\vec{E}\times\vec{H})\,dv = \frac{-\partial}{\partial t}\,\frac{1}{2}\int_V (\vec{E}\cdot\vec{D} + \vec{H}\cdot\vec{B})\,dv - \int_V \vec{J}\cdot\vec{E}\,dv$$

$$\Rightarrow \oint_S (\vec{E}\times\vec{H})\cdot ds = \frac{-\partial W}{\partial t} - \int_V \vec{J}\cdot\vec{E}\,dv$$

i.e., $$\boxed{\frac{\partial W}{\partial t} = -\oint_S (E\times H)\cdot ds - \int_V E\cdot J\,dv}$$

Hence, proved.

Q2.] For the antenna that radiates in free space, we are given that

$$H = \frac{12\sin\theta}{r}\cos(2\pi\times10^8 t - \beta r)\,a_\theta \ \ mA/m$$

By Maxwell's equations, $\nabla\times H = \sigma E + \varepsilon\frac{\partial E}{\partial t}$ $(\sigma = 0)$

$$\rightarrow \quad E = \frac{1}{\varepsilon}\int \nabla\times H\,dt$$

But,

$$\nabla\times H = \frac{1}{r\sin\theta}\frac{\partial H_\theta}{\partial\phi}a_r + \frac{1}{r}\frac{\partial}{\partial r}(rH_\theta)a_\phi$$

$$= \frac{12\sin\theta}{r}\beta\sin(2\pi\times10^8 t - \beta r)\,a_\phi$$

$$E = \frac{12\sin\theta}{\varepsilon_0}\beta\int\sin(2\pi\times10^8 t - \beta r)\,dt\,a_\phi$$

$$\Rightarrow \boxed{E = \frac{-12\sin\theta}{\omega\varepsilon_0 r}\beta\sin(\omega t - \beta r)a_\phi, \quad \omega = 2\pi\times10^8}$$

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