FILTER DESIGN

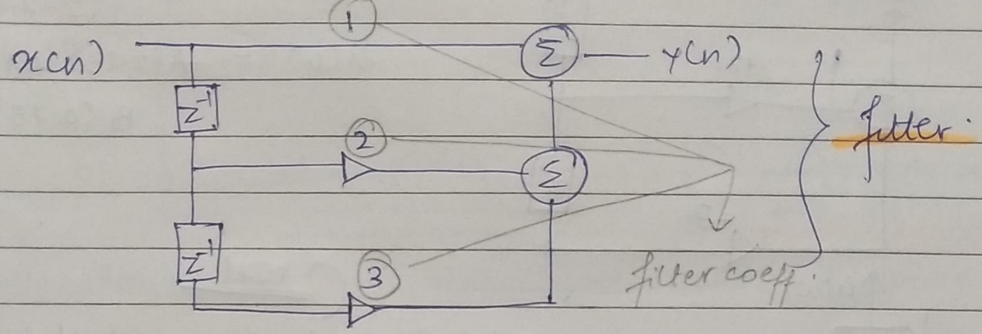
FIR

(i) $h[n] = [1\ 2\ 3]$ ↑ — filter coeff

order of system = 2

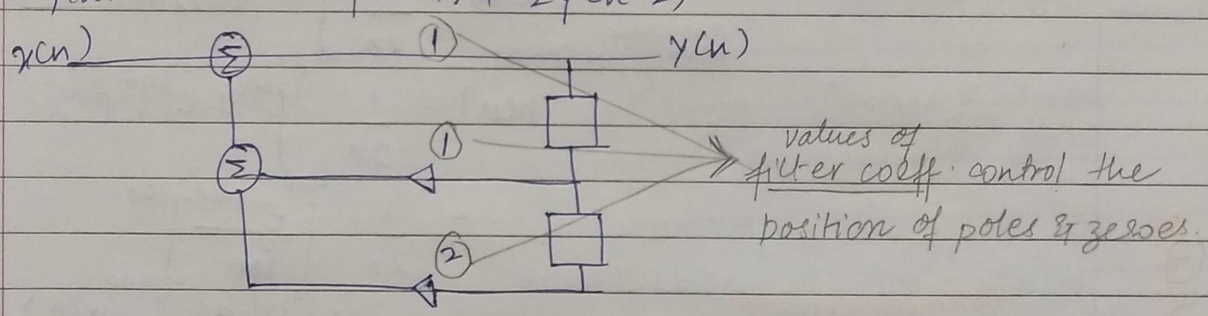
$$H(z) = 1 + 2z^{-1} + 3z^{-2} = \frac{z^{2} + 2z + 3}{z^{2}}$$

$$y(n) = x(n) + 2x(n-1) + 3x(n-2)$$



(ii)

$$y(n) = x(n) + y(n-1) + 2y(n-2)$$



$$H(z) = \frac{1}{1 - z^{-1} - 2z^{-2}}$$

$$= \frac{z^2}{z^2 - z - 2} \rightarrow h(n) = a^n u(n) + b^n u(n)$$

$$= \frac{z^2}{(z+1)(z-2)}$$

$$\{\ h(n) = A(-1)^n u(n) + B(2)^n u(n)\ \}$$

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IIR

$y(n) = x(n) + 2y(n-1) + 2y(n-2)$

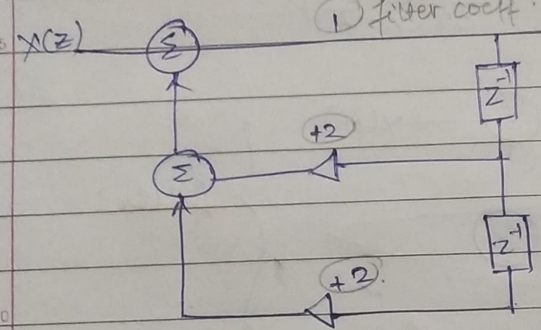
length $= \infty$
order $= 2$.

$Y(z) - 2z^{-1}Y(z) - 2z^{-2}Y(z) = X(z)$

$H(z) = \frac{1}{1-2z^{-1}-2z^{-2}} = \frac{z^2}{z^2-2z-2}$

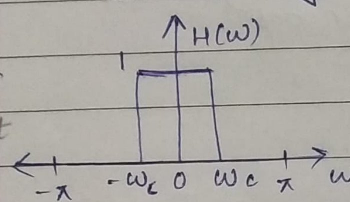
$= \frac{z^2}{(z-2.73)(z+0.73)}$

① filter coeff.



$h(n) = A(2.73)^n u(n) + B(0.73)^n u(n)$

Q.



will repeat after π → Periodic

real

$H(\omega) = 1 \; ; \; 0 \le |\omega| \le \omega_c$

real, Periodic, Continuous, even $\quad 0 \; ; \;$ elsewhere.

$h(n) = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} H(\omega)e^{j\omega n}\,d\omega$

Aperiodic $-\infty < n < \infty$

$h(n) = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} 1 \cdot e^{j\omega n}\,d\omega$

$= \frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn}\right]_{-\omega_c}^{\omega_c}$

$= \frac{1}{\pi n \cdot 2j}\left(e^{j\omega_c n} - e^{-j\omega_c n}\right)$

$h(n) = \frac{\sin \omega_c n}{\pi n}, \quad -\infty < n < \infty$

$H(z) = \sum_{n=\infty}^{\infty} h(n)z^{-n}$

$= \sum_{n=-\infty}^{\infty} \frac{\sin \omega_c n}{\pi n} z^{-n}$

not summable

no series exists & thus, we cannot do its IIR implementation.

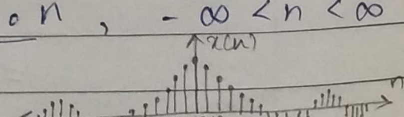
constraint ①.

Can we do FIR implementation?

→ length = no. of coeff] finite

→ order = length − 1

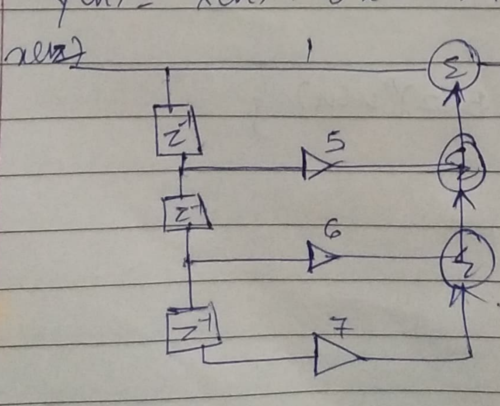
→ we'll have infinite filter which isn't possible. (length)

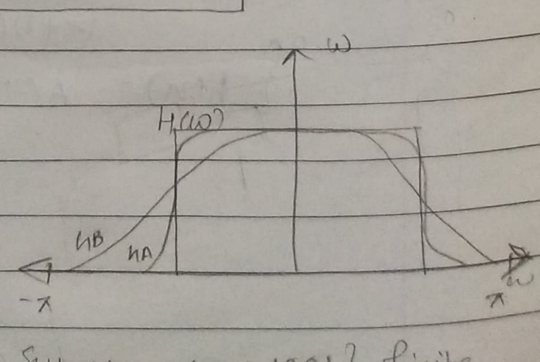


as $n \to \infty$, $h(n) \to 0$

Q. $h(n) = [\underset{\uparrow}{1}, 5, 6, 7]$

$y(n) = x(n) + 5x(n-1) + 6x(n-2) + 7x(n-3)$





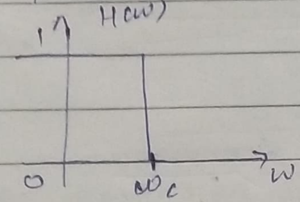
Suppose, $h_A \to 1001$, $h_B \to 101$ } finite

We can implement h_A, h_B but not $H(\omega)$.

Constraints: → length becomes ∞.
① Ideal low pass filter cannot be realized.
② Causality condition → provide delay → negative phase → Spectrum still be complex.

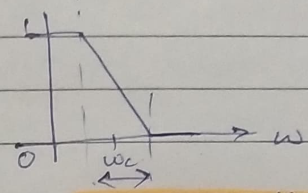
* H(ω) isn't practical. It is an ideal LPF. They cannot be realized practically.
* Practical filter - LPF doesn't remain ideal. ω > cutoff, around the practical filters allow for some extent.

ideal LPF



→ transition width = 0

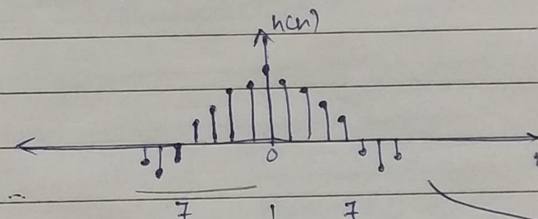
practical.



transition width.
→ should be as small as possible.

Q. If I cannot represent ideal filter, what is the effect on spectrum?

> Suppose → h(n) =



m = 15

* $h(n) = 0;\ n < 0$ → condition of causality

problem: not a causal system.

Soln: eg. $h(n) = [1, 2, 1]$ (↑ at 2)

$H(z) = z + 2 + z^{-1}$] × z^{-1} delay of 1

$H(\omega) = 2 + 2\cos\omega$

$h(n) = [1, 2, 1]$ (↑ at 1)

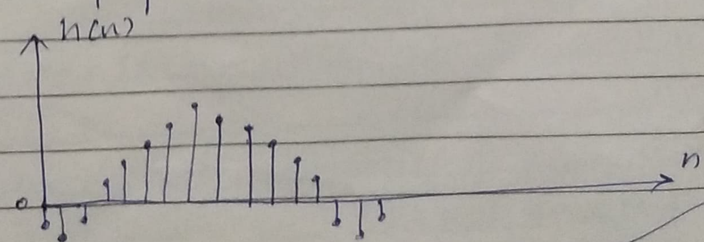
$H(\omega) = [2 + 2\cos\omega]e^{-j\omega}$ → complex

$\angle H(\omega) = -\omega$

↓

H(ω) changes.

→ provide delay of 7. → $n - 7$ = $n - \left(\frac{m-1}{2}\right)$



$H(\omega) = 1 \cdot e^{-j7\omega}$
$= 1 \cdot e^{-j\left(\frac{m-1}{2}\right)\omega}$

adds phase and thus, complex

To represent, they will always have negative phase (or) the slope should be negative.

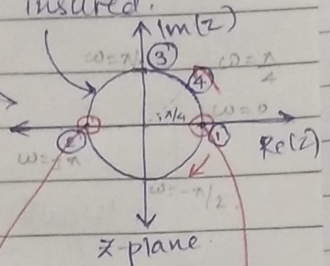
But, if h(n) changes, H(ω) will also change

problem

spectrum → real → complex
for non causal

[-7] -4 [-10]

$\left\{ \begin{array}{l} z = re^{j\omega} \\ z = e^{j\omega} \end{array} \right\}$ $r = 1 \longrightarrow$ considering unit circle
↳ stability is insured.



→ if zero is at $\omega = 0$.
$\omega \to -\pi$ to π

lowest $\omega = 0 \Rightarrow z = 1 = 1\angle 0°$ — ①
highest $\omega = \pm\pi \Rightarrow z = -1 = 1\angle 180°$ — ②

Cases: $\omega = \pi/2 \Rightarrow z = i = 1\angle 90°$ — ③
$\omega = \pi/4 \Rightarrow z = 1/\sqrt{2} + i\,1/\sqrt{2} = 1\angle 45°$ — ④

point of z-plane where the freq. is the lowest
Min Response

highest possible frequency in digital domain/ z-plane
Max response

When moving on the circle, moving away from ①, the frequency increases.
while, moving away from ②, the frequency decreases.

when $z = 2$, $2 + i0 \Rightarrow$ remains on $Re(z)$, the freq $= 0$.
$z = -2$, $-2 + i0 \Rightarrow$ " ——", freq $(\omega) = \pm\pi$

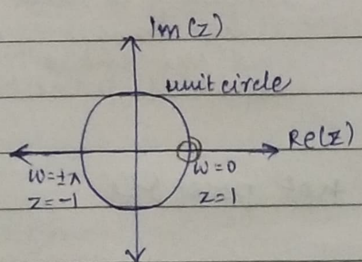
$H(z)$, $H(\omega)$
$H(z)$, $H(e^{j\omega})$

$\boxed{FIR}$

$H(z) = \dfrac{Num\cdot(z)}{Denom(z)}$ — zeroes / — poles.

Poles can't be used to design ∵ all are at origin

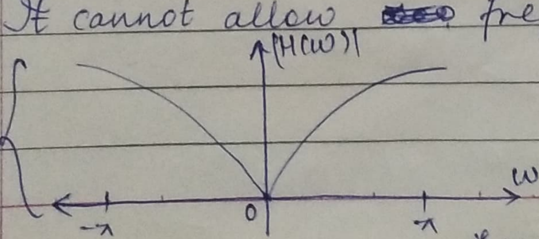
IIR is not possible.
Thus, we use FIR, all the poles are at origin.



if zero is at 1,
$H(z) = \dfrac{z-1}{z^{n}}$ → can be of any value
It'll keep adding delay.

Considering 1p, 1z $\Rightarrow$ $H(z) = \dfrac{z-1}{z} = 1 - z^{-1}$
$h(n) = (\underset{\uparrow}{1}, -1)$

It cannot allow frequencies to pass from $z = 0$.



at $\omega = 0$, the response is minimum.
as we move along the unit circle, response increases and the effect of zeroes diminishes.
Also, the graph is symmetrical since if we move clockwise/anticlockwise, it doesn't matter

worst high pass filter. It can never behave like LPF

$H(z) = 1 - z^{-1} \rightarrow h(n) = (1, -1)$

$H(\omega) = 1 - e^{-j\omega}$

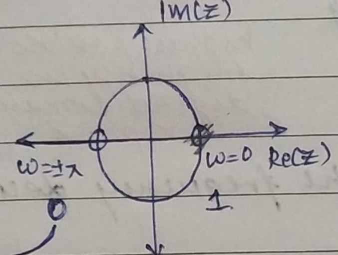
$= e^{-j\omega/2}[e^{j\omega/2} - e^{-j\omega/2}]$

$= 2j\sin\frac{\omega}{2} \cdot e^{-j\frac{\omega}{2}}$

$|H(\omega)| = |2\sin\frac{\omega}{2}|$

$\rightarrow$ if zero is at $\omega = \pm\pi$, $\underset{=}{\text{response}}$ $M = 2$

$H(z) = \frac{z+1}{z} = 1 + z^{-1} \Rightarrow h(n) = (1, 1)$ $\uparrow$



No freq can pass from $\omega = \pm\pi$. Meaning that it can behave like a HPF.

$\rightarrow$ Since we have zero at $\omega = \pm\pi$, the response increases when we move away from that point.

$\rightarrow$ The highest response is at $\omega = 0$.

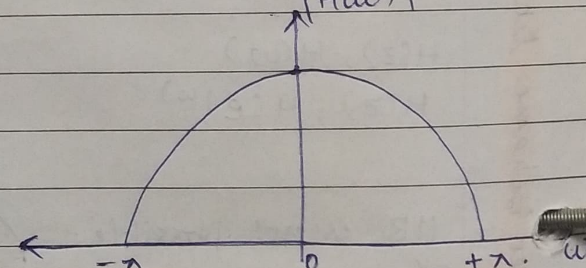
$\rightarrow$ Doesn't matter if we move clockwise or anti

$H(\omega) = 1 + e^{j\omega}$

$= e^{j\omega/2}[e^{j\omega/2} + e^{-j\omega/2}]$

$= 2\cos\frac{\omega}{2}\, e^{-j\omega/2}$

$|H(\omega)| = 2\cos\frac{\omega}{2}$.



comparing to ideal LPF, it has no cut off frequency and the entire band becomes transition width.

Q. Can $M < 2$?

$M = 0 \rightarrow$ not possible.

$M = 1 \rightarrow$ order $= M - 1 = 0 \rightarrow$ not possible.

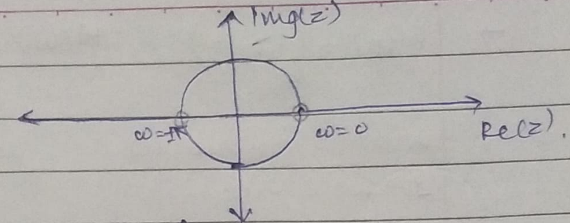
$M = \infty \rightarrow$ gives ideal filter.

$M = 2 \rightarrow$ order $= 2 - 1 = 1 \rightarrow$ 1st order system (lowest)

$\therefore$ lowest possible length $= 2$

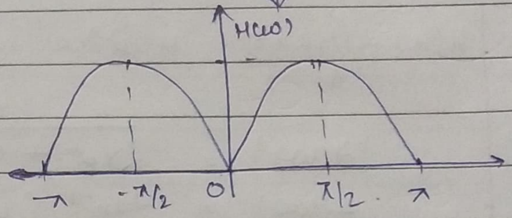
$\rightarrow$ worst filter (LPF)

$$H(z) = \frac{(z-1)(z+1)}{z^2}.$$



Cannot write 'z' in denominator as it will turn it non causal

$$= \frac{z^2-1}{z^2} = 1 - z^{-2}.$$



$$H(\omega) = 1 - e^{-2j\omega}$$
$$= e^{-j\omega}(e^{j\omega} - e^{-j\omega})$$
$$= 2\sin\omega \cdot e^{-j\omega}$$
$$|H(\omega)| = |2\sin\omega|$$

Worst ever bandpass filter.

① and ② technique can only be used when the system is either symmetric or antisymmetric

① $\rightarrow h(\omega) = (1, \underset{\uparrow}{2}, 1)$

$$H(\omega) = 1 + 2e^{-j\omega} + e^{-2j\omega}$$
$$= e^{-j\omega}[e^{j\omega} + 2 + e^{-j\omega}]$$
$$= e^{-j\omega}(2 + 2\cos\omega)$$

$\therefore |H(\omega)| = 2 + 2\cos\omega$

$\angle H(\omega) = -\omega$

③ $\rightarrow h(n) = (\underset{\uparrow}{1}, 2, 3)$

$$H(\omega) = 1 + 2e^{-j\omega} + 3e^{-2j\omega}$$
$$= (1 + 2\cos\omega + 3\cos 2\omega) + i(-2\sin\omega - 3\sin 2\omega)$$

$$|H(\omega)| = \sqrt{rp^2 + ip^2}$$
$$\angle H(\omega) = \tan^{-1}\left[\frac{ip}{rp}\right]$$

② $h(n) = [1, \underset{\uparrow}{0}, -1]$

$$H(\omega) = 1 + 0e^{-j\omega} - e^{-2j\omega}$$
$$= e^{-j\omega}[e^{j\omega} + 0 - e^{-j\omega}]$$
$$= e^{-j\omega}\, 2j\sin\omega$$
$$= e^{-j\omega} e^{j\pi/2}\, 2\sin\omega$$
$$= e^{-j(\omega + \pi/2)}\, 2\sin\omega$$

$|H(\omega)| = 2\sin\omega$

$\angle H(\omega) = \omega + \frac{\pi}{2}$

for ① and ②, the phase of $H(\omega)$ is linear functⁿ of 'ω' (frequency), it is called as linear phase system

③ $\rightarrow$ nonlinear phase system.

If $h(n)$ is symmetric or antisymmetric sequence, it will give linear phase.

Symmetric sequence $h(n)$ of length m (0 to $m-1$)

can be written as $\rightarrow h(n) = h(m-1-n)$

$h_0\ h_1\ h_2\ h_3\ h_4$ — to be symmetric, they need to be equal.

$$H(z) = \sum_{n=0}^{m-1} h(n) z^{-n} \quad \text{①}$$

antisymmetry $\rightarrow h(n) = -h(m-1-n)$

$$H(z) = \sum_{n=0}^{m-1} h(m-1-n) z^{-n}$$

if $m-1-n = p$

$-n = p - m + 1$

$n = 0 \rightarrow p = m-1$

$n = m-1 \rightarrow p = 0$

$$= \sum_{p=m-1}^{p=0} h(p)\, z^{p-(m-1)} = \sum_{p=0}^{m-1} [h(p) \cdot z^{p} z^{-(m-1)}]$$

$$H(z) = z^{-(m-1)} \sum_{p=0}^{m} h(p) z^{p} \cdot z^{-(m-1)} \cdot \sum_{p=0}^{m-1} h(p)(z^{-1})$$

But, z transform defⁿ ①,

$$H(z) = z^{-(m-1)} H(z^{-1})$$

- If antisymmetric, $H(z) = -z^{-(m-1)} H(z^{-1})$

$h(n) = h_0 \; h_1 \; h_2 \; h_1 \; h_0$

$z_0 \rightarrow$ zero then $1/z_0$ is also zero of $H(z)$

zero.

Case 1: $H(z) = \dfrac{(z - 1/2)}{z}$

$\therefore$ should also have zero at 2.

$H(z) = 1 - 1/2\, z^{-1}$

$\therefore \quad H(z) = \dfrac{(z - 1/2)(z-2)}{z^2}$

$h(n) = (1, -\frac{1}{2})$

$= \dfrac{z^2 - 2.5z + 1}{z^2}$

$H(\omega) = 1 - \frac{1}{2} e^{-j\omega}$

Case 2:

$z = 1$ (zero)

"1 is a reciprocal of itself and thus has linear phase"

$H(z) = \dfrac{z-1}{z}$

$H(z) = 1 - z^{-1}$

$h(n) = (1, -1)$

$\uparrow$

Case 3:

$z = -1$

$H(z) = \dfrac{z+1}{z}$

$(1, 1)$

$\uparrow$

Imp

Case 4: If I have a zero at $z = \frac{1}{2} e^{j\pi/4}$

$$\Rightarrow z = \frac{1}{2}\left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i\right]$$

$$= \frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} i$$

$$H(z) = \frac{\left(z - \frac{1}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} i\right)}{2}$$

$H(s) = \frac{(s-i)}{s}$

have you seen like this?

No, we have only seen s with real coeff.

So, $H(s) = \frac{(s-i)(s+i)}{s^2} = \frac{s^2+1}{s^2}$

We've never seen transfer functn having imaginary coefficients

Why?

multipliers in the system realization

Suppose $H(z) = 1 + 3z^{-1}$

To realize a system, we need the system to be real.

Thus, we cannot have imaginary multipliers.

$$H(z) = \frac{\left(z - \frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} i\right)\left(z - \frac{1}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} i\right)}{z^2}$$

always require conjugate if imaginary coefficients are required.

"Every root shall have its conjugate symmetry"

$$= \frac{z^2 - \frac{1}{2} e^{j\pi/4} z - \frac{1}{2} e^{-j\pi/4} z + \frac{1}{2} e^{j\pi/4} \cdot \frac{1}{2} e^{-j\pi/4}}{z^2}$$

$$= \frac{z^2 - 2 \times \frac{1}{2} \cos\frac{\pi}{4} \cdot z + \frac{1}{4}}{z^2}$$

$$= \frac{z^2 - \frac{1}{\sqrt{2}} z + \frac{1}{4}}{z^2}$$

~~Conjugate symmetry~~

$$H(z) = \frac{\left(z - \frac{1}{2}e^{j\pi/4}\right)\left(z - \frac{1}{2}e^{-j\pi/4}\right)}{z^2}$$

linear phase $\longrightarrow$ $\dfrac{\left(z-\frac{1}{2}e^{j\pi/4}\right)\left(z-2e^{-j\pi/4}\right)\left(z-\frac{1}{2}e^{-j\pi/4}\right)\left(z-2e^{j\pi/4}\right)}{z^4}$

$$= \frac{\left(z^2 - \frac{z}{\sqrt{2}} + \frac{1}{4}\right)\left(z^2 - 2\times 2\cos\frac{\pi}{4}\cdot z + 4\right)}{z^4}$$

$z = \frac{1}{2}e^{j\pi/4} \rightarrow$ 4 order

$z = i \rightarrow 2$

$\Rightarrow$ $\boxed{\text{Symmetric case}}$

$\boxed{m = \text{odd}}$

$H(z) = z^{-(m-1)} H\left(\frac{1}{z}\right)$

$m-1 \rightarrow$ even

I $H(z) = z^{-\text{even}} \cdot H\left(\frac{1}{z}\right)$

Suppose $z=1$ is zero.

$H(1) = 1^{-\text{even}} H(1/1)$

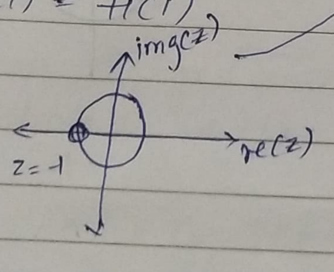
$H(1) = H(1)$

Let $z=-1$

$H(-1) = (-1)^{-\text{even}} H(-1/1)$

$H(-1) = 1\cdot H(-1)$

$H(-1) = H(-1)$



$\boxed{m \text{ is even}}$

II $m-1 \rightarrow$ odd.

$H(z) = z^{-\text{odd}} H\left(\frac{1}{z}\right)$

let $z=1$ be the zero

$H(1) = 1^{-\text{odd}} H(1) \Rightarrow H(1) = H(1)$

Let $z=-1$

$H(-1) = (-1)^{-\text{odd}} H(-1) \Rightarrow H(-1) = -H(-1)$.

$\therefore H(-1) = 0$

$c = -c$

$2c = 0 \Rightarrow \boxed{c=0}$

When m = even and we get $H(-1) = -H(-1)$, we get compulsory zero at $z = -1$.

Asymmetric

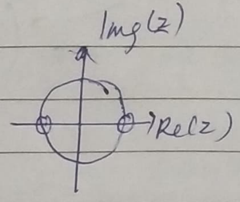
Antisymmetric with odd length, we compulsory get a zero at $H(1) \rightarrow z = \pm 1$.

$h(n) = -h(m-1-n)$

$H(z) = -z^{m-1} \cdot H\left(\frac{1}{z}\right)$

III $m = \text{odd}$, $m-1 \rightarrow \text{even}$

$H(z) = -z^{\text{even}}\, H\left(\frac{1}{z}\right)$



let $z = 1$

$H(1) = (1)^{\text{even}}\, H(1)$

$H(1) = -H(1)$

$\Rightarrow \boxed{H(1) = 0}$ compulsory zero.

let $z = -1$

$H(-1) = -(-1)^{\text{even}}\, H(-1)$

$H(-1) = -H(-1)$

$\Rightarrow \boxed{H(-1) = 0}$

IV $m = \text{even}$ $m-1 \rightarrow \text{odd}$

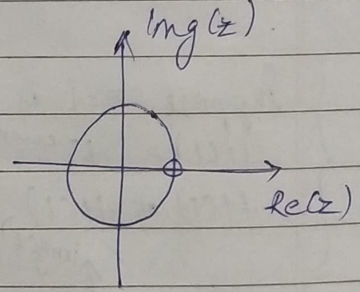
$H(z) = -z^{-\text{odd}}\, H(1/z)$

let $z = 1$

$H(1) = -(1)^{-\text{odd}} \cdot H(1)$

$H(1) = -H(1)$

$\Rightarrow \boxed{H(1) = 0}$



let $z = -1$

$H(-1) = -(-1)^{\text{odd}}\, H(-1) = H(-1)$

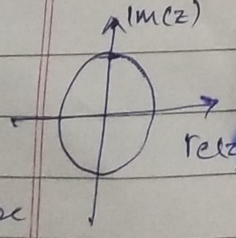
$H(-1) = H(-1)$

Summary:

* wherever zeroes are shown, there should be odd no. of zeroes and where the zeroes are not shown, ~~there~~ even no. of zeroes should be present.

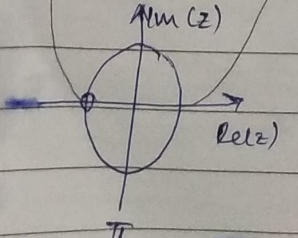
Symmetric

$m = \text{odd}$



Can be anything. I

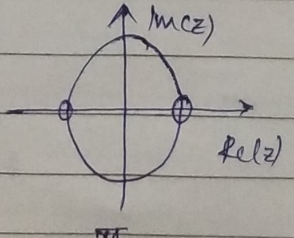
$m = \text{even}$



II Can't be HPF, BRF

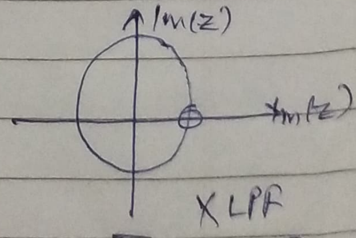
Antisymmetric

$m = \text{odd}$



III ✗HPF, ✗LPF, ✗BRF

$m = \text{even}$



IV ✗LPF ✗BRF

Q. $h(n) = [1, 2, 2, 1]$

$$H(z) = \frac{1 + 2z^{-1} + 2z^{-2} + z^{-3}}{z^3} = \frac{z^3 + 2z^2 + 2z + 1}{z^3}$$

$$= \frac{(z+1)(\_\_\_\_\_\_)}{z^3}$$

$z = -1$, compulsory zero.

Q. $h(n) = [1, 0, -1]$

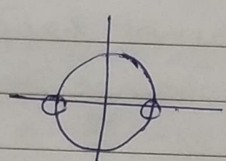
$$H(z) = 1 + 0z^{-1} - z^{-2}$$

$$= \frac{z^2 - 1}{z^2} = \frac{(z-1)(z+1)}{z^2}$$

Q. $h(n) = (1, 0, -1)$ Antisymmetric odd length.

$$H(z) = \frac{(z-1)(z+1)(z+1)}{z^2}$$

$$= \frac{(z^2-1)(z+1)}{z^2}$$



Antisymmetric even length.

$$= \frac{z^3 + z^2 - z - 1}{z^3}$$

$h(n) = (1, 1, -1, -1)$ → even length

The case changed. for it not to change, instead of adding 1 $(z+1)$, we'll add 2.

$$H(z) = \frac{(z-1)(z+1)(z+1)(z+1)}{z^4}$$

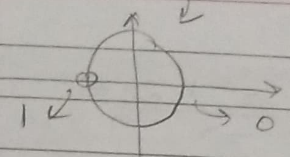
$$= \frac{(z^2-1)(z^2+2z+1)}{z^4}$$

$$= \frac{z^4 + 2z^3 + \cancel{z^2} - \cancel{z^2} - 2z - 1}{z^4}$$

$$= \frac{z^4 + 2z^3 - 2z - 1}{z^4}$$

$h(n) = (1, 2, 0, -2, -1)$

Case 2.

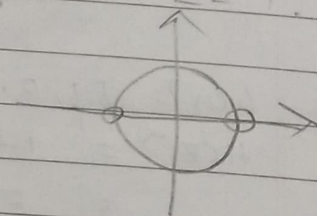


$(1,1) \quad \frac{z+1}{z} = (1+z^{-1})$

$H(z) = (1+z^{-1})$ Symmetric even length

Now, you've to <u>add</u> something

Suppose, we have to add 1 more zero at $z = 1$.



$$\frac{(z+1)(z-1)}{z^2}$$

$$= \frac{z^2-1}{z^2}$$

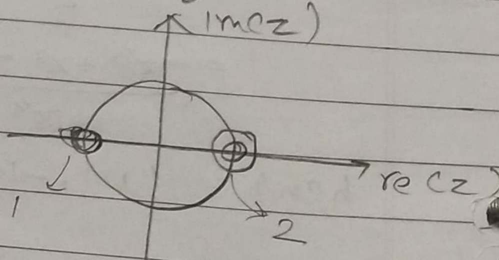
$h(n) = (1, 0, 1)$ $\qquad$ Antisymmetric odd length

$\uparrow$

Type 2 $\rightarrow$ Type 3

This cannot happen.

Not to change the type, we add 2 zeroes.



$$\frac{(z+1)(z-1)^2}{z^3}$$

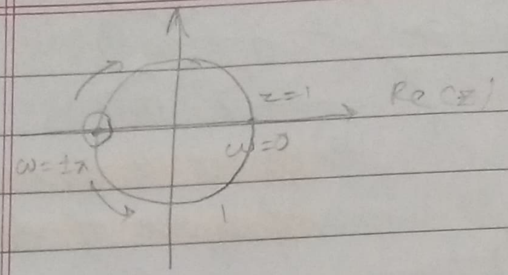
$$\frac{z^3 - z^2 - z + 1}{z^3}$$

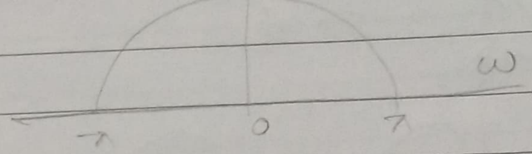
$h(n) = (1, -1, -1, 1)$ Symmetric even length.

$\uparrow$

* Type doesn't change.

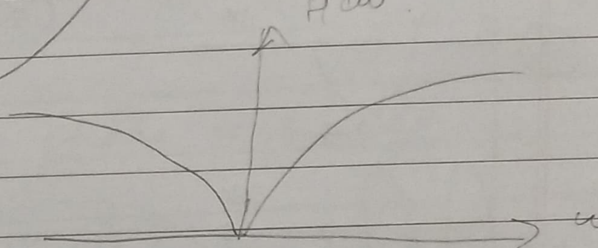
$\Rightarrow$

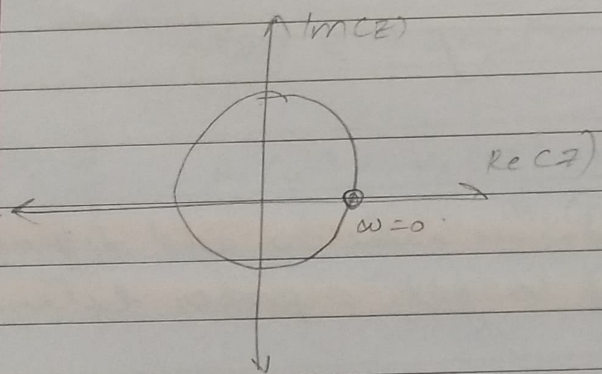




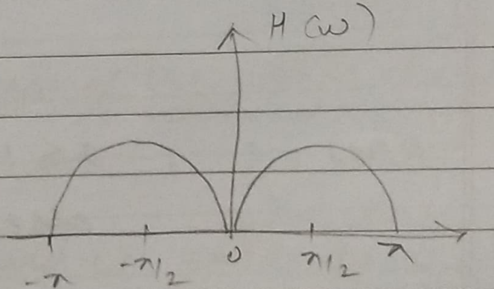
$$H(z) = \frac{z+1}{z} = 1+z^{-1}$$

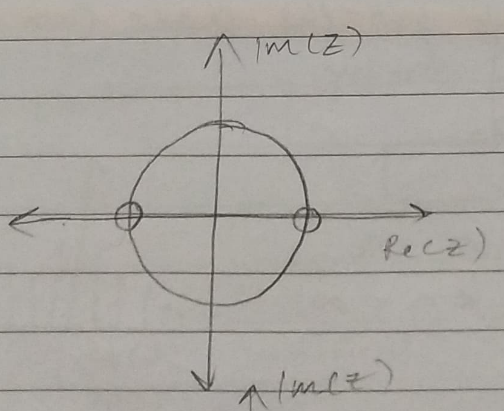
$$H(\omega) = 1+e^{-j\omega}$$
$$= e^{j\omega/2}\left[e^{j\omega/2}+e^{-j\omega/2}\right]$$
$$= e^{j\omega/2}\left(2\cos\omega/2\right)$$

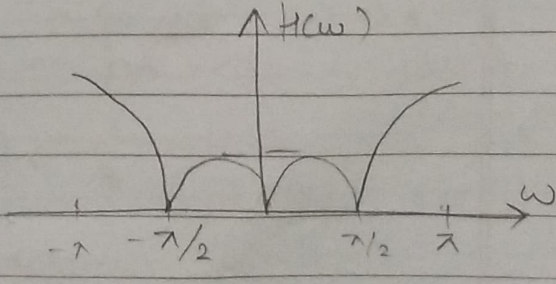


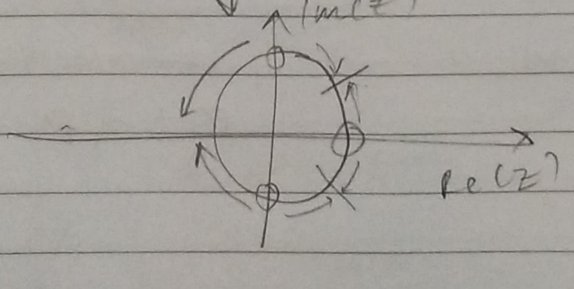


$$H(z) = \frac{z-1}{z}$$
$$= e^{-j\omega/2}\left(2\sin\omega/2\right)$$



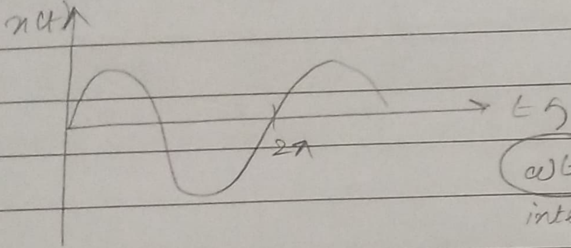






$\sin(\omega t + \phi)$

$x(t) = \sin \omega t$



ωt interchangeable

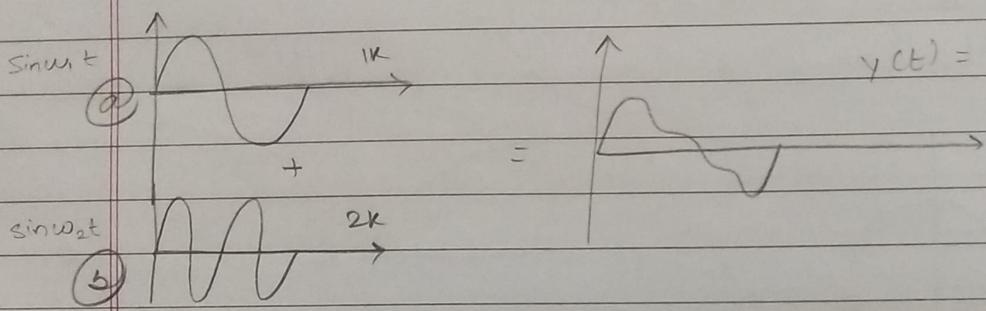
$\Rightarrow \phi = \omega t$ — instantaneous phase

$\rightarrow y = mx$

* Phase is linear function of frequency

ϕ = phase delay

$\hookrightarrow$ constant phase



$y(t) = \sin \omega_1 t + \sin \omega_2$

if delay is required, we have to add different phases in (a) and (b) to get a proper delayed resultant

$$\Rightarrow \left\{ \frac{d\phi}{d\omega} = \text{constant} \right\} \text{group delay}$$

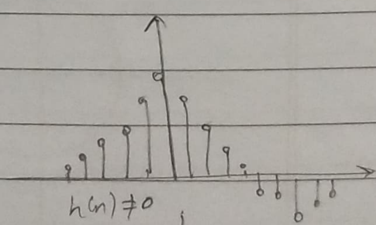
$\downarrow$

to get the shape same.

$H(\omega) = 1 \quad ; \quad 0 \le |\omega| \le \omega_c$.

$= 0$ elsewhere

$h(n) = \dfrac{\sin \omega_c n}{\pi n}, \quad -\infty < n < \infty$



$h(n) \neq 0$; $n < 0$

$\longleftrightarrow$ we shift

$\dfrac{M-1}{2} = P$

$H(\omega) = 1 \cdot e^{-j\left(\frac{M-1}{2}\right)\omega}$

$= 1 \cdot e^{-jP\omega}$

$x(t) \xrightarrow{L} X(s)$

$x(t-2) \xrightarrow{L} e^{-2s} X(s)$

$$\underbrace{[1\ 2\ \underset{\uparrow}{3}\ 2\ 1] \rightarrow e^{2j\omega} + 2e^{j\omega} + 3 + 2e^{j\omega} + e^{-2j\omega}}_{H_1(\omega)}$$

$$[\underset{\uparrow}{1}\ 2\ 3\ 2\ 1] \rightarrow 1 + 2e^{-j\omega} + 3e^{-2j\omega} + 2e^{-3j\omega} + e^{-4j\omega}$$

↳ $H_2(\omega)$

$\frac{M-1}{2}$

$$H_2(\omega) = e^{-j(2)\omega} \cdot H(\omega)$$

$$h_2(n) = h_1(n-2)$$

$\phi(\omega) = -2\omega$ → phase is linear

if $H_2(\omega) = H_1(\omega)\tan^{-1}(\omega)$ cannot be written

$h_2(n) = h_1(n-2)$ ↳ ϕ isn't linear with ω.

can't be written

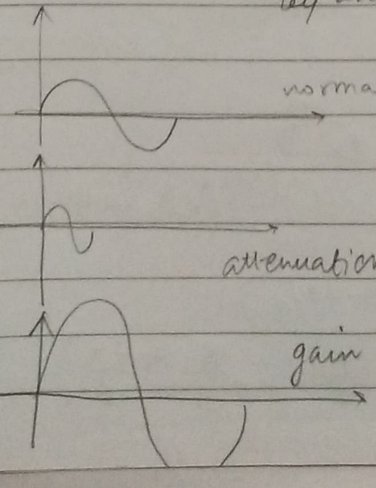
If the shape is being retained, it means the ϕ is linear funct^n of ω.

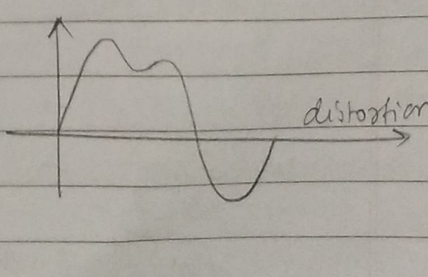
linear phase system

any signal
Composite → System → Shape of the signal should be retained

different freq.

phase introduced by the system should be linear funct^n of freq





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$\sin\omega_0 n$ $x(n) = [1, 2, 3, 4]$ $\longrightarrow X(z)$

$h(n) = [5, 6, 7, 8]$ $H(z)$

$y(n) = x(n) * h(n)$

$Y(z) = X(z) \times H(z)$

Inv. $\hookrightarrow y(n)$

if

$x(n) = \sin\omega_0 n$

$h(n) = [5, 6, 7, 8]$

$h(n) \to H(\omega) = \sum h(n) e^{-j\omega n}$ —— (a)

$x(n) = Ae^{j\omega_0 n} = A[\cos\omega_0 n + i\sin\omega_0 n]$ —— (b)

$y(n) = \sum_k x(n-k) \cdot h(k)$

$= \sum_k A e^{j\omega_0 (n-k)} \cdot h(k)$

$y(n) = A e^{j\omega_0 n} \left(\sum h(k) \cdot e^{-j\omega_0 k} \right)$ from (a) (b),

$y(n) = x(n) H(\omega)\Big|_{\omega=0}$ $H(\omega)\Big|_{\omega=0}$

response of the input at ω_0.

Eg

$H(\omega) = 1 \cdot e^{-j7\omega}$; $0 \le |\omega| \le \pi/2$

$= 0$; elsewhere

$x(n) = 7\sin\left(\frac{\pi}{3}n + \frac{\pi}{4}\right) + 4\cos\left(\frac{\pi}{6}n - \frac{\pi}{3}\right)$

$y(n) = ?$

$\rightarrow$

$\omega_1 = \pi/3 \rightarrow H(\omega)|_{\pi/3} = 1 e^{-j\frac{7\pi}{3}}$

$\omega_2 = \frac{\pi}{6} \rightarrow H(\omega)|_{\pi/6} = 1 \cdot e^{-j7\pi/6}$

$y(n) = 7 \cdot \sin\left(\frac{\pi}{3}n + \pi/4 - 7\pi/3\right)$

$+ 4\cos\left(\frac{\pi}{6}n - \frac{\pi}{3} - \frac{7\pi}{6}\right)$

$H(\omega) = 2e^{\ldots}$; $0 \le |\omega| < \frac{\pi}{4}$

$= 0$; elsewhere

$x(n) = 5\sin\left(\frac{\pi}{3}n + \frac{\pi}{4}\right) - 6\cos\left(\frac{\pi}{2}n + \frac{\pi}{18}\right) + 7\sin\left(\frac{\pi n}{6} + \frac{\pi}{18}\right)$

$y(n) = ?$

$\rightarrow \omega_1 = \pi/3 \rightarrow H(\omega)\big|_{\pi/3} = 2\cdot e^{-j\frac{3\pi}{3}}$

$\omega_2 = \pi/2 \rightarrow \times$ not in the range

$\omega_3 = \pi/6 \rightarrow H(\omega)\big|_{\pi/6} = 2e^{-5\pi/6}$

$y(n) = 10\sin\left(\frac{\pi}{3}n + \frac{\pi}{4} - \frac{5\pi}{3}\right) + 14\sin\left(\frac{\pi n}{6}\right.$

$x(t) = \sin(2\pi 50 t)$

Hint:

$y(t) = x^2(t)$ non linear	$y(t) = x(2t)$ time variant
$Y(t) = 0, 100$ f	$Y(t) = 100$ f

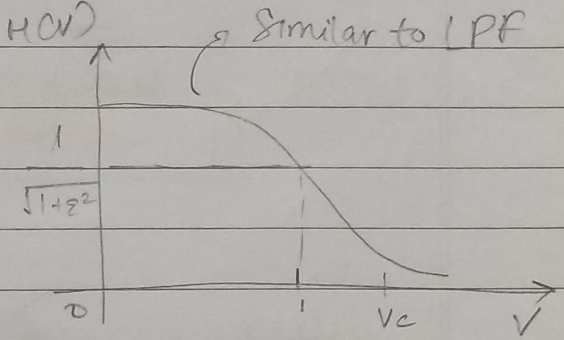
New frequency can not be generated in the o/p if the system is LTI system.

+Scmitt trigger. Oscillators generate new frequencies and thus, is not a non linear / Time variant system.

i/p (existing freq) → LTI system → o/p (existing freq from i/p (some may be cancelled))

i/p (existing freq) → non linear/ Time invariant → o/p (new frequencies can be added)

FIR → we plot zero.
If we consider poles,



$$|H(V)|^2 = H(V) \cdot H^*(V)$$

Analog LPF

$$|H(V)|^2 = \frac{1}{1+\varepsilon^2 V^{2n}}$$

$\frac{1}{s+1}$ is a specific ex. of this functⁿ only

$V = \frac{\Omega}{\Omega_c}$ → omega: analog freq
Ω_c → cut off.

should be very high for good filter
As → attenuation in stop band
Ap → attenuation in pass band.
should be very low for good filter.

$$A(V) = 10\log(1+\varepsilon^2 V^{2n})$$

$$A(1) = A_p = 10\log(1+\varepsilon^2)$$

$$A_s = A(V_s) = 10\log(1+\varepsilon^2 V_s^{2n})$$

$$\log(1+\varepsilon^2 V_s^{2n}) = 0.1A_s$$

$$1+\varepsilon^2 V_s^{2n} = 10^{0.1A_s}$$

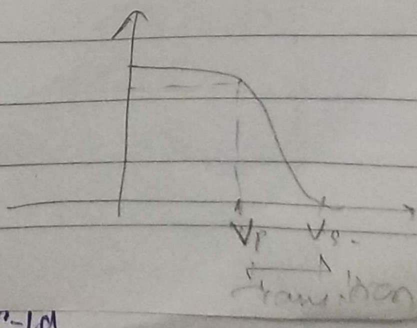
$$\varepsilon^2 V_s^{2n} = 10^{0.1A_s} - 1$$

$$V_s^{2n} = \frac{10^{0.1A_s} - 1}{\varepsilon^2}$$

$$\varepsilon^2 = 10^{0.1A_p} - 1$$

$$V_s^{2n} = \frac{10^{0.1A_s} - 1}{10^{0.1A_p} - 1}$$

$$\boxed{n = \frac{\log\left[\frac{10^{0.1A_s} - 1}{10^{0.1A_p} - 1}\right]}{2\log V_s}}$$



$\Omega \to$ omega

$$|H(V)|^2 = \frac{1}{1+\varepsilon^2 V^{2n}} = H(V) \cdot H^*(V)$$

suppose, $\varepsilon = 1$.

$$H(V) = \frac{1}{1+V^{2n}}$$

$$V = \frac{\Omega}{\Omega_c} \qquad s = j\Omega$$

$$\therefore V = \frac{s}{j\Omega_c}$$

$$H(V) = \frac{1}{1+\left(\frac{s}{j\Omega_c}\right)^{2n}} = H(s)$$

we'll get 'n' roots

poles?

$$\left(\frac{-s^2}{\Omega_c^2}\right)^{n} = -1 = e^{\frac{j\pi(2k+1)}{2\pi}} \qquad k = 0, 1, \ldots (n-1)$$

$$\Rightarrow \left(\frac{-s^2}{\Omega_c^2}\right)^n = e^{\frac{j2\pi(k+1)}{2n}}$$

$$\frac{-s_k^2}{\Omega_c^2} = e^{\frac{j2\pi(2k+1)}{2n}}$$

$$-s_k^2 = \Omega_c^2 e^{\frac{j2\pi(2k+1)}{2n}}$$

$$s_k^2 = \Omega_c^2 \cdot (-1) \cdot e^{\frac{j2\pi(k+1)}{2n}}$$

$$s_k^2 = \Omega_c^2 e^{j\pi} e^{\frac{j2\pi(k+1)}{2n}}$$

$$s_k = \Omega_c e^{j\pi/2} e^{\frac{j2\pi(k+1)}{2n}}$$

location of poles $\left\{ s_k \to \Omega_c \angle \frac{\pi}{2} + \frac{2\pi(k+1)}{2n}. \right.$

let $\Omega_c = 1$, $n = 1$

how many poles? $\to 1$

$$\frac{-S_k^2}{\Omega_c^2} = e^{j(2k+1)\pi}$$

$$\frac{-S_k^2}{\Omega_c^2} = e^{\frac{j(2k+1)\pi}{n}}$$

$$\Rightarrow S_k^2 = -\Omega_c^2 e^{\frac{j(2k+1)\pi}{n}}$$

$$S_k^2 = \Omega_c^2 e^{j\pi} e^{\frac{j(2k+1)\pi}{n}}$$

$$S_k = \Omega_c e^{\frac{j\pi}{2}} e^{\frac{j(2k+1)\pi}{2n}}$$

Pole location $\left\{ \boxed{S_k \rightarrow \Omega_c \angle \frac{\pi}{2} + \frac{(2k+1)\pi}{2n}} \right.$ $k = 0, 1, \ldots (n-1)$

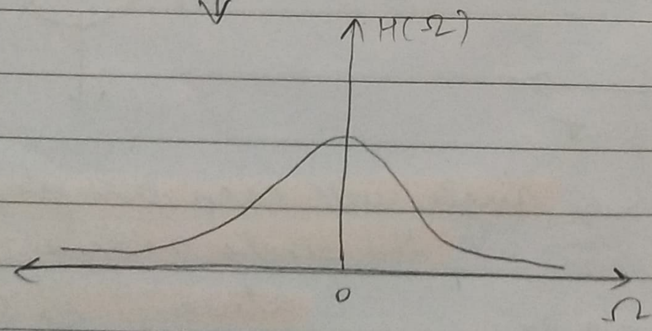
let $\Omega_c = 1$, $n = 1$

$$S \rightarrow 1 \angle \pi = -1$$

$$\therefore H(s) = \frac{1}{s+1} \Big\} \text{ LPF}$$

$$H(j\Omega) = \frac{1}{j\Omega + 1}$$

$$|H(j\Omega)| = \frac{1}{\sqrt{1+\Omega^2}}$$



Q.

$A_p = 1\text{dB}$ $A_s = 20$ dB. $V = 2$. $n = ?$

$$n = \frac{\log\left[\frac{10^{0.1A_s}-1}{10^{0.1A_p}-1}\right]}{2\log V_s}$$ $n = 4.28$ ↓ 5

for eg. 4.01 → 5 ... next possible integer

Q. $n = 2$;

when ε is considered, $s_k \to \Omega_c \left(\frac{1}{\varepsilon}\right)^{1/n}$

when $s_k = 1$, $H(s) = \frac{1}{s+1}$

$s_k \neq 1$ $s \to \frac{s}{|s_k|}$ to make highest value of LPF = 1

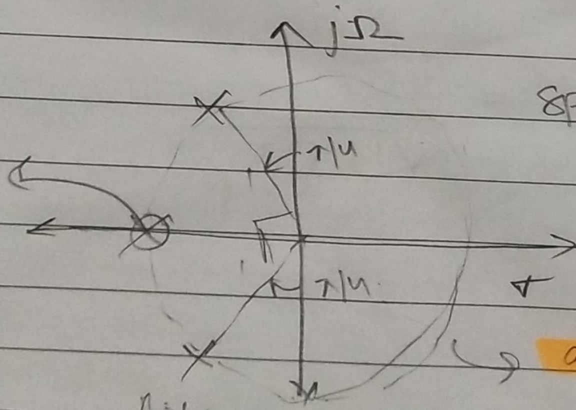
$$S_k = \Omega_c \angle \frac{\pi}{2} + \frac{(2k+1)\pi}{2n}$$

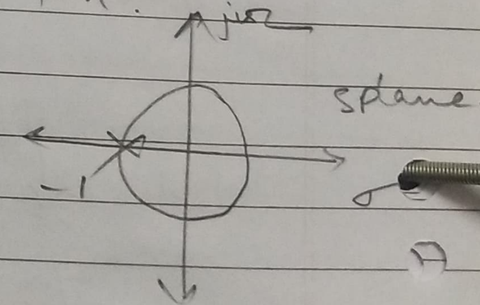
$n=2$ $\quad S_2 = \Omega_c \angle \frac{\pi}{2} + \frac{3\pi}{4}, \frac{\pi}{2} + \frac{\pi}{4}$

$= \Omega_c \angle \frac{5\pi}{4}, \frac{3\pi}{4} = 1 \angle \frac{5\pi}{4}, \frac{3\pi}{4}.$

$n = 1$ $\quad S_1 = \Omega_c \angle -\pi. = 1 \angle \pi.$

diff b/w angle $= \frac{\pi}{n}$





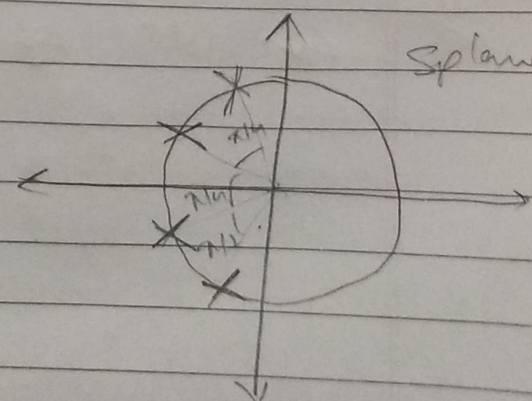
angle will keep changing but the circle radius will always be Ω_c.

$$\frac{1}{\left(s + e^{j\frac{\pi}{2}+\frac{\pi}{4}}\right)\cdot\left(s + e^{j\frac{\pi}{2}+\frac{3\pi}{4}}\right)}$$

$n = 4$ $\quad \Omega_c \angle \frac{\pi}{2} + \frac{(2k+1)\pi}{8}$

$S_1 = \angle \frac{\pi}{8}$ $\quad S_3 = \angle \frac{5\pi}{8}$

$S_2 = \angle \frac{3\pi}{8}$ $\quad S_4 = \angle \frac{7\pi}{8}$



$N = 5$. $\frac{\pi}{2} + \frac{(2k+1)\pi}{10}$

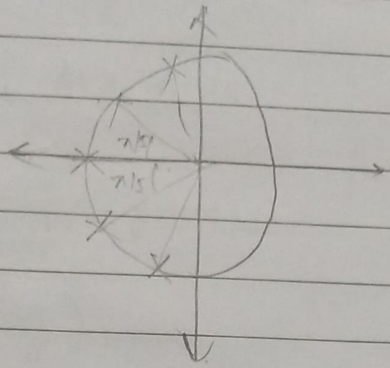
$S_1 \rightarrow \pi/10$

$S_2 \rightarrow 3\pi/10$

$S_3 \rightarrow \pi/2$

$S_4 \rightarrow 7\pi/10$

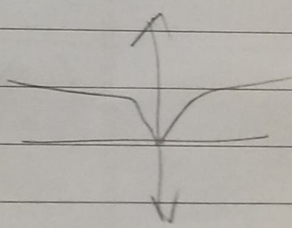
$S_5 \rightarrow 9\pi/10$



→ poles always are on the left side thus, stable.

→ angle b/w the poles $\frac{\pi}{n}$.

for HPF, use $S \rightarrow \frac{1}{S}$ $H(S) = \frac{S}{S+1}$



* Read the question carefully. Understand which kind of specification is given and compute accordingly.

$z = e^{sT}$

$\Omega = \frac{\omega}{T}$

Q2: impulse invariance method

Q1 Design a digital LP Butterworth filter by conformal mapping for following specifications.

$H(s) \rightarrow H(z)$

$\left\{ s = \frac{2}{T}\left[\frac{z-1}{z+1}\right] \right\}$

$A_p \leq 1 dB$, $A_s \geq 40 dB$

analog specification: $f_p = 4 KHz$, $f_{stop} = 6 KHz$ $f_{samp} = 24 KHz$

$\Omega = \frac{2}{T}\tan\left(\frac{\omega}{2}\right)$ — analog freq, digital freq, freq relation

Step 1: Convert given data in proper form.

$$\Omega = \frac{2}{T}\tan\left(\frac{\omega}{2}\right)$$

$$\Omega_p = \frac{2}{T}\tan\left(\frac{2\pi f_p}{2 f_{samp}}\right) = \frac{2}{T}\tan\left(\frac{2\pi}{2}\cdot\frac{4}{24}\right)$$

$$\Omega_p = \frac{2}{T}\tan\left(\frac{\pi}{6}\right) = \frac{2}{T\sqrt{3}} = \frac{1.15}{T}$$

$$\Omega_s = \frac{2}{T}\tan\left(\frac{2\pi f_s}{2 f_{samp}}\right) = \frac{2}{T}\tan\left(\frac{2\pi}{2}\,\frac{6}{24}\right)$$

$$= \frac{2}{T}\tan\left(\frac{\pi}{4}\right) = \frac{2}{T}$$

Step 2: find V_s

$$V_s = \frac{\Omega_s}{\Omega_p} = \frac{2}{T} \times \frac{T\sqrt{3}}{2} = \sqrt{3}$$

Step 3: find order

$$n = \frac{\log\left[\frac{10^{0.1A_s} - 1}{10^{0.1A_p} - 1}\right]}{2\log V_s}$$

$$= \frac{\log\left[\frac{10^{0.1\cdot 40} - 1}{10^{0.1\cdot 1} - 1}\right]}{2\log(\sqrt{3})} = 9.613 \simeq \underline{\underline{10}}$$

Step 4: find $H(s)$

$$S_k \rightarrow \frac{1.15}{T}\left(\frac{1}{\varepsilon}\right)^{1/n} \angle \frac{\pi}{2} + \frac{(2k+1)\pi}{2n}$$

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Q.4 $A_p \leq 1dB$, $A_s \geq 40dB$

$\omega_p = 0.2\pi$, $\omega_s = 0.5\pi$ $\longrightarrow$ digital specifications

Step1: $\Omega_p = \frac{2}{T} \tan\left(\frac{\omega_p}{2}\right)$

$\Omega_s = \frac{2}{T} \tan\frac{(\omega_s)}{2}$.

Step 2: $V_s = \frac{\Omega_s}{\Omega_p}$

Step 3: n

Step 4: $H(s)$

Q5: $A_p \leq 1dB$

$A_s \geq 60dB$

analog specificatn $\begin{cases} f_p = 1KHz \\ f_{stop} = 5KHz \\ f_{samp} = 20KHz. \end{cases}$

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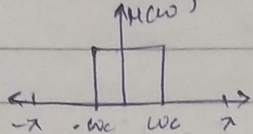
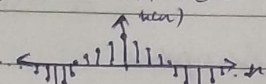
Lab 4:

script given is just to understand what happens when we use windows.

> ideal LPF

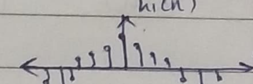


$\longrightarrow$ $h(n) = \dfrac{\sin 2\pi f_c n}{\pi n}$, $-\infty < n < \infty$



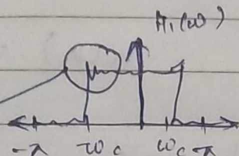
length ∞

truncation.



finite 'm'

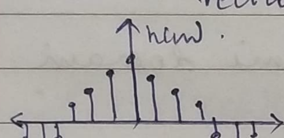
$\longleftarrow$



ripples

transition width

$h(n)$ * rectangular window :



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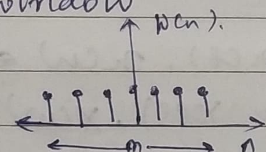


$H_1(\omega) = H(\omega) * W(\omega)$

$$H(z) = \frac{z-a}{z-1/a}$$

Condition for any filter to be all pass ?

$\rightarrow$ poles and zeroes should be reciprocal of each other.

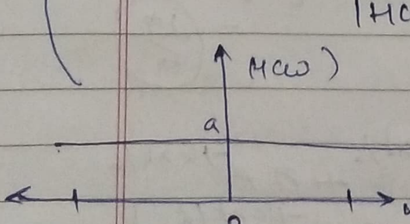
$$H(e^{-j\omega}) = H(z) = \frac{z(1-az^{-1})}{z(1-1/a\, z^{-1})}$$

allpass filter

$$H(\omega) = \frac{1-ae^{-j\omega}}{1-1/a\, e^{-j\omega}}$$

$$= \frac{(1-a\cos\omega) + i a \sin\omega}{(1-\frac{\cos\omega}{a}) + i \frac{1}{a}\sin\omega}$$

$$|H(\omega)| = \sqrt{\frac{(1-a\cos\omega)^2 + a^2 \sin^2\omega}{(1-\frac{1}{a}\cos\omega)^2 + \frac{1}{a^2}\sin^2\omega}}$$



$$= \left[\frac{(1-2a\cos\omega+a^2)}{(1-\frac{2}{a}\cos\omega+\frac{1}{a^2})}\right]^{1/2}$$

$$= \left[\frac{\cancel{(1-2a\cos\omega+a^2)}\,a^2}{\cancel{(1-2a\cos\omega+a^2)}}\right]^{1/2}$$

$$|H(\omega)| = a$$

$\downarrow$ 20 $\quad$ $\downarrow$ e^{-fi} $\quad$ $\downarrow$ -4 $\quad$ $\to$ c-id $\quad$ $\to$ a-ib

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Case 1:

$H(z) = \frac{1}{z} = z^{-1}$

$h(n) = [0, 1]$ delay by 1 unit

$H(\omega) = e^{-j\omega}$

$|H(\omega)| = 1$

$\angle H(\omega) = -\omega$

Case 2:

$H(z) = z$

$H(\omega) = e^{j\omega}$

$|H(\omega)| = 1$

$\angle H(\omega) = \omega$.

Suppose

$x(n) = [3, 2, 4]$ $h(n) = [0, 1]$.

$\Rightarrow X(z) = 3 + 2z^{-1} + 4z^{-2}$ $H(z) = z^{-1}$

$X(z) \times H(z) = 3z^{-1} + 2z^{-2} + 4z^{-3}$

$= [0, 3, 2, 4]$ • we created a d

Q: We can use $x(t)$, $x(n)$ ~~and~~ time domain.
Why to use $X(s)$, $X(\omega)$, $X(z)$?

→ Spectrum: to know what all is present and how much?
→ to convert differential eqn to linear

Q: Is there any loss while converting time to frequency domain?

$x(t) = t \xrightarrow{L} X(s) = \frac{1}{s^2} \xrightarrow{Inv.} x(t) = t$

No loss. Nothing is happening to the signal.

$x_p(n) = [3, 2, 1, 0, 1, 2]$

$\frac{1}{N} \sum |x(n)|^2 = \frac{1}{6}[9+4+1+1+4] = \frac{19}{6}$

$X(k) = \frac{1}{6} \sum_{n=0}^{6} x(n) e^{-j2\pi nk/N}$

$= \frac{1}{6}\left[3 + 2^{-j2\pi k/6} + 1e^{-j4\pi k/6} + 0e^{-j6\pi k/6} + 1e^{-j8\pi k/6} + 2e^{-j10\pi k/6}\right]$

$= \frac{1}{6}\left[3 + 4\cos\left(\frac{\pi k}{3}\right) + 2\cos\left(\frac{2\pi k}{3}\right)\right]$

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$X_0 = \frac{1}{6}(3+4+2) = \frac{9}{6}$

$X_1 = \frac{1}{6}(3+\cdot \quad = \frac{2}{3}$

$X_2 = \frac{1}{6}(3 \quad = 0.$

$X_3 \quad = \frac{1}{6}.$

~~X_4~~

$X_4 \quad = 0$

$X_5 \quad = 2/3.$

$P = \Sigma |X(k)|^2 = \left[\left(\frac{9}{6}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{1}{6}\right)^2 + \left(\frac{2}{3}\right)^2 \right]$

$= \frac{19}{6}$

Conclusion: You be in any domain, the main signal will remain the same even when converted.

Energy $E = \Sigma |x(n)|^2$

Power $P = \frac{1}{N} \Sigma |x(n)|^2.$

Energy or power &
in time and frequency domain, they remain the same.

★ But, it may happen that if used with a system, it may change.

If not, there is no loss.

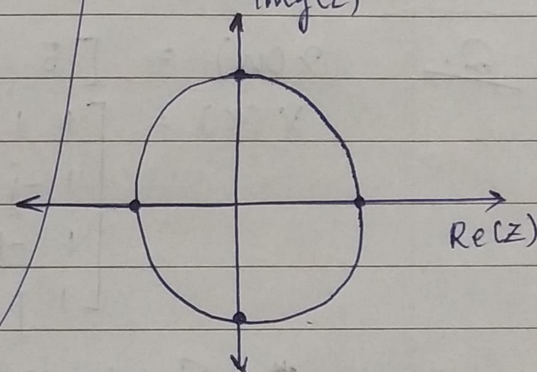
20 a-ib

$x_p(n) = [1\ 2\ 3\ 4]$

$$X_k = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \qquad X_k = \begin{bmatrix} 1+2+3+4 \\ 1-2j-3+4j \\ 1-2+3-4 \\ 1+2j-3-4j \end{bmatrix} = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

$\omega = \frac{2\pi k}{N}$. $\omega = \frac{2\pi k}{4} = \frac{\pi}{2} k$.

$$\begin{array}{cccc} [\,10 & -2+2j & -2 & -2-2j\,] \\ \uparrow & \uparrow & \uparrow & \uparrow \\ X_0 & X_1 & X_2 & X_3 \\ X(0) & X(\pi/2) & X(\pi) & X(-\pi/2) \end{array}$$



$$\begin{array}{c} n = \quad 0 \quad 1 \quad 2 \quad 3 \end{array}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{array}{l} \to k=0 \\ \to k=1 \\ \to k=2 \\ \to k=3 \end{array}$$

(spectrum formula) $X(k) = \sum_k x(n) e^{-j2\pi k}$ $\qquad k = 0$ to 3

complex and conjugate

Q. what if value of N is large? $\quad X_0\ X_1\ X_2\ X_3\ X_4\ X_5\ X_6\ X_7$

Real $\qquad \frac{N}{2}$ Real

→ Suppose $N = 8$.

Method 1: Use Matrix → $[\quad]_{8\times 8}$

$[1\ 2\ 3\ 4\ 1\ 2\ 3\ 4]$

$X(k) = \sum_{n=0}^{7} x(n) e^{-j\frac{2\pi nk}{N}}$

$X_0 = \sum x(n)$

$X_1 = \sum x(n) e^{-j\frac{2\pi n}{N}}$

$= x_0 + x_1 e^{-j\frac{2\pi}{8}} + x_2 e^{-j\frac{2\pi 2}{8}} + \ldots$

Method 2: formula

Method 3: Zplane circle.

$X_4 = \sum (-1)^n x(n) = -4$

$$X(k) = [\underset{\substack{\downarrow \\ 20}}{?},\ a+ib,\ \underset{\substack{\downarrow \\ e-fj}}{?},\ c+id,\ \underset{\substack{\downarrow \\ -4}}{?},\ \underset{\substack{\downarrow \\ c-id}}{?},\ e+fj,\ \underset{\substack{\downarrow \\ a-ib}}{?}]$$

Q. $x(n) = [5 \;\; 8]$

$\rightarrow$ $X(K) = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 5 \\ 8 \end{bmatrix} = \begin{bmatrix} 13 \\ -3 \end{bmatrix}$

Q. $x(n) = [5 \;\; 8 \;\; 5 \;\; 8]$

$\rightarrow$ $X(K) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 5 \\ 8 \\ 5 \\ 8 \end{bmatrix} = \begin{bmatrix} 26 \\ 0 \\ -6 \\ 0 \end{bmatrix}$

* Upsampling (addition of zeros)
* twice the signal. $13 \rightarrow 26$, $-3 \rightarrow -6$

Q. $x(n) = [5 \;\; 8] \longrightarrow [5 \; 0 \; 8 \; 0]$

$X(K) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 5 \\ 0 \\ 8 \\ 0 \end{bmatrix} = \begin{bmatrix} 13 \\ -3 \\ 13 \\ -3 \end{bmatrix}$

* Spectrum repeats.

When the signal repeats itself, the spectrum is upsampled and is twice the original spectrum. When the signal is upsampled, the spectrum is repeated.