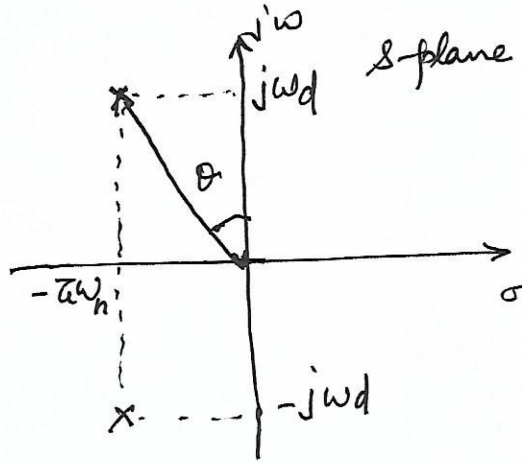
RELATIVE STABILITY

- For the following second order characteristic equation

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0 \qquad 0<\zeta<1$$

the roots are $\quad -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$

- $$\sin\theta = \frac{\zeta\omega_n}{\sqrt{(\zeta\omega_n)^2 + \omega_d^2}}$$



$$\sin^2\theta = \frac{\zeta^2\omega_n}{\zeta^2\omega_n^2 + \omega_n^2 - \zeta^2\omega_n^2}$$

$$\sin^2\theta = \zeta^2$$

$$\sin\theta = \zeta$$

- The angle θ is indicative of the damping ratio ζ.
- As θ becomes smaller so does the value of ζ
- Thus if the value of ζ reduces the system tends to become oscillatory and unstable.
- In other words if the closed loop poles are located near the $j\omega$ axis the system becomes less stable.
- The Nyquist plot can be used as a measure of relative stability of closed loop systems which are open loop stable.

- The stability information for such systems become obvious by inspection of the polar plot of the open loop function $G(s)H(s)$, since this stability criterion is merely the non-encirclement of $(-1, j0)$ point.

- Consider two different systems whose closed-loop poles are shown in the s-plane (Fig 1)

- Obviously system A is "more stable" than system B since its dominant closed-loop poles are located comparitively away to the left from the $j\omega$ axis.

- The open loop frequency response plots for the systems A and B are shown in Fig 2

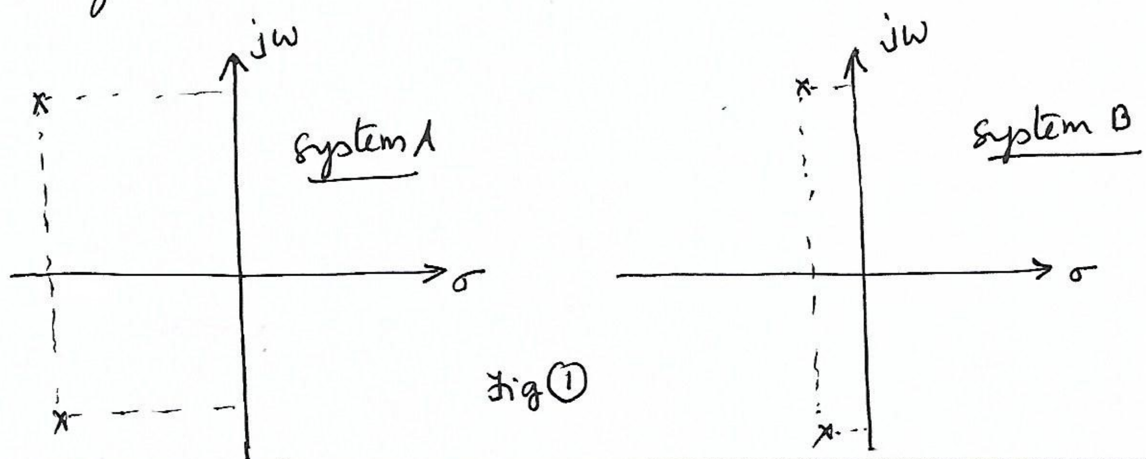


Fig 1

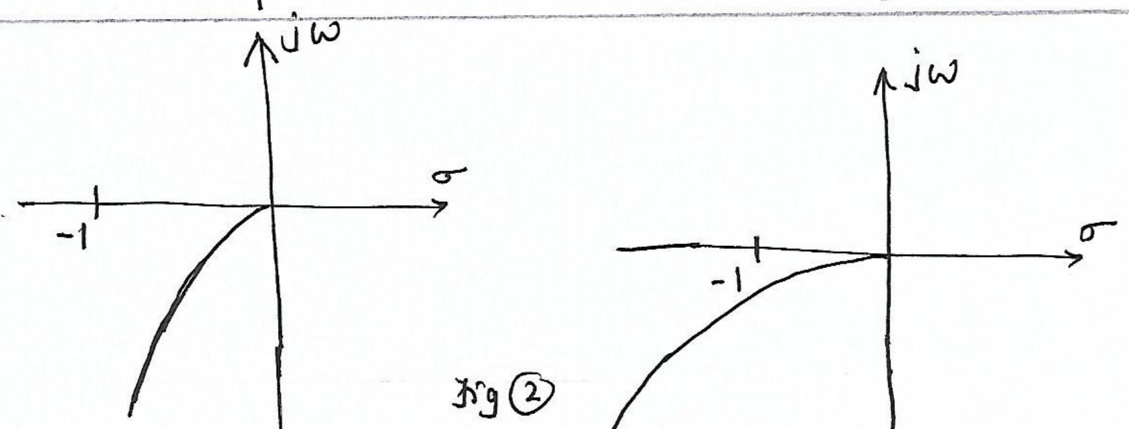


Fig 2

(iv) The comparison of the closed loop pole locations of these two systems with their corresponding polar plot reveals that a polar plot moves closer to $(-1, j0)$ point the system behaves relatively less stable and ~~so~~ vice versa.

MEASUREMENT OF RELATIVE STABILITY

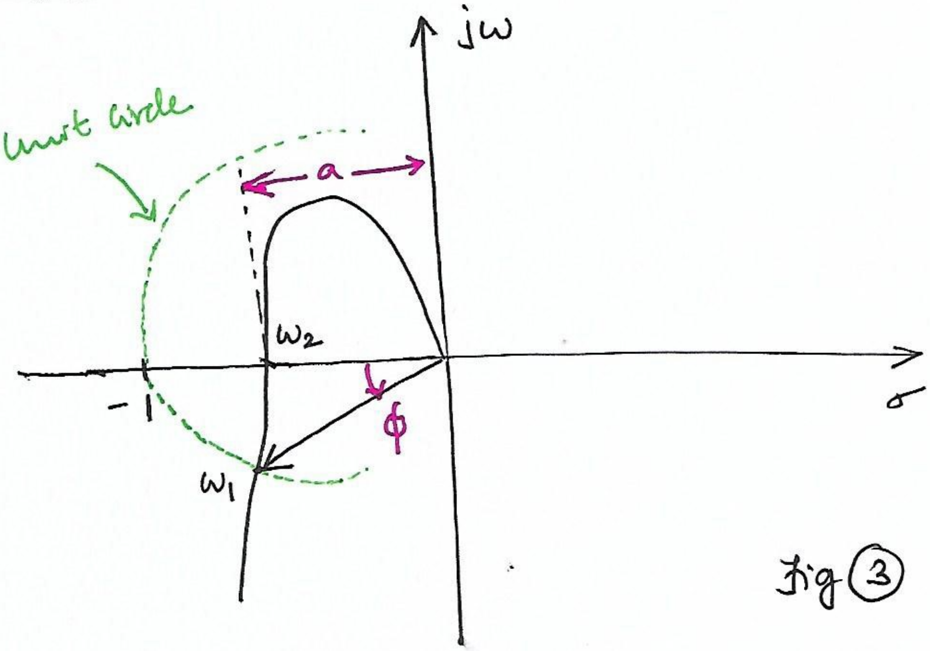


Fig (3)

- Above Fig shows a typical $G(j\omega)H(j\omega)$ plot which crosses the negative real axis at a frequency $\omega = \omega_2$ with an intercept of 'a'
- Let a unit circle centred at the origin intersect the $G(j\omega)H(j\omega)$ plot at a frequency $\omega = \omega_1$ and let the phasor $G(j\omega_1)H(j\omega_1)$ make an angle 'ϕ' with the negative real axis, measured positively in counterclockwise direction.
- As $G(j\omega)H(j\omega)$ locus approaches $(-1, j0)$ point, the relative stability reduces. Simultaneously the value of 'a' approaches unity and that of ϕ tends to zero.
- The relative stability could thus be measured in terms of intercept a and/or the angle ϕ

GAIN MARGIN

- It is the factor by which the system gain can be increased to drive it to the verge of instability.

- In Fig 3 it is seen that at $\omega = \omega_2$ the phase angle $\angle G(j\omega)H(j\omega)$ is 180° and $|G(j\omega)H(j\omega)| = a$.

- If the gain of the system is increased by a factor $\frac{1}{a}$ then $|G(j\omega)H(j\omega)|_{\omega=\omega_2}$ becomes $a \times \frac{1}{a} = 1$ and hence the $G(j\omega)H(j\omega)$ plot will pass through $(-1, j0)$ point, driving the system to the verge of instability.

- Thus GAIN MARGIN (GM) may be defined as the reciprocal of the gain at the frequency at which the phase angle becomes 180°

- The frequency at which the phase angle is 180° is called the phase cross-over frequency.

- $GM = \frac{1}{'a'}$. In decibels the $GM = 20 \log \frac{1}{a}$

$$GM = -20 \log a \text{ dB}$$

- Since 'a' is less than 1 for a stable system, [illegible] GM is positive