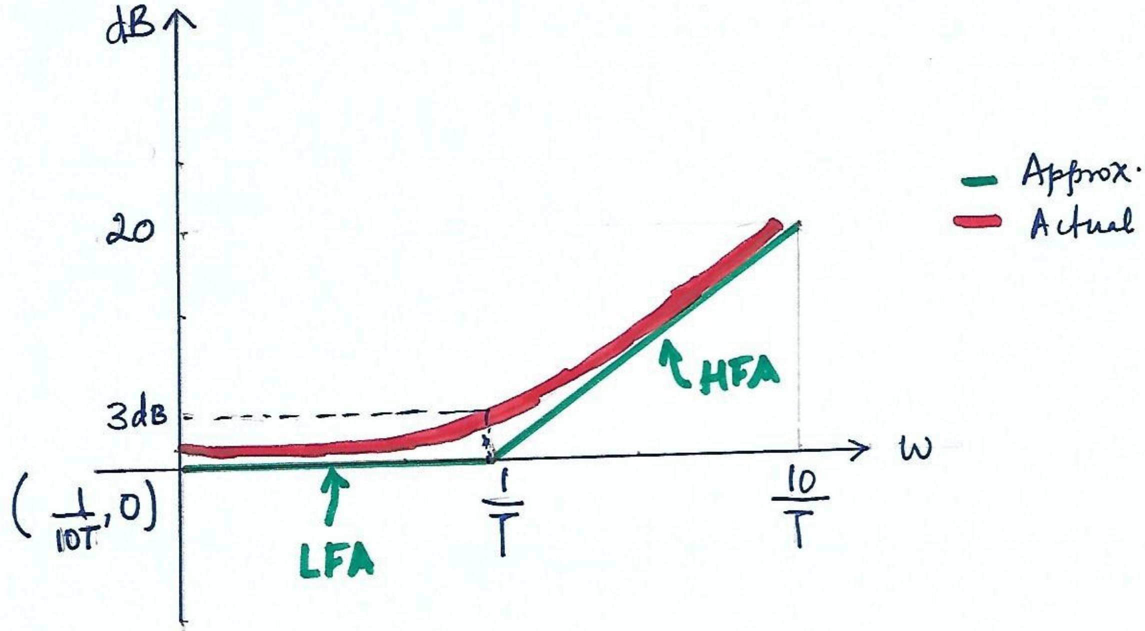
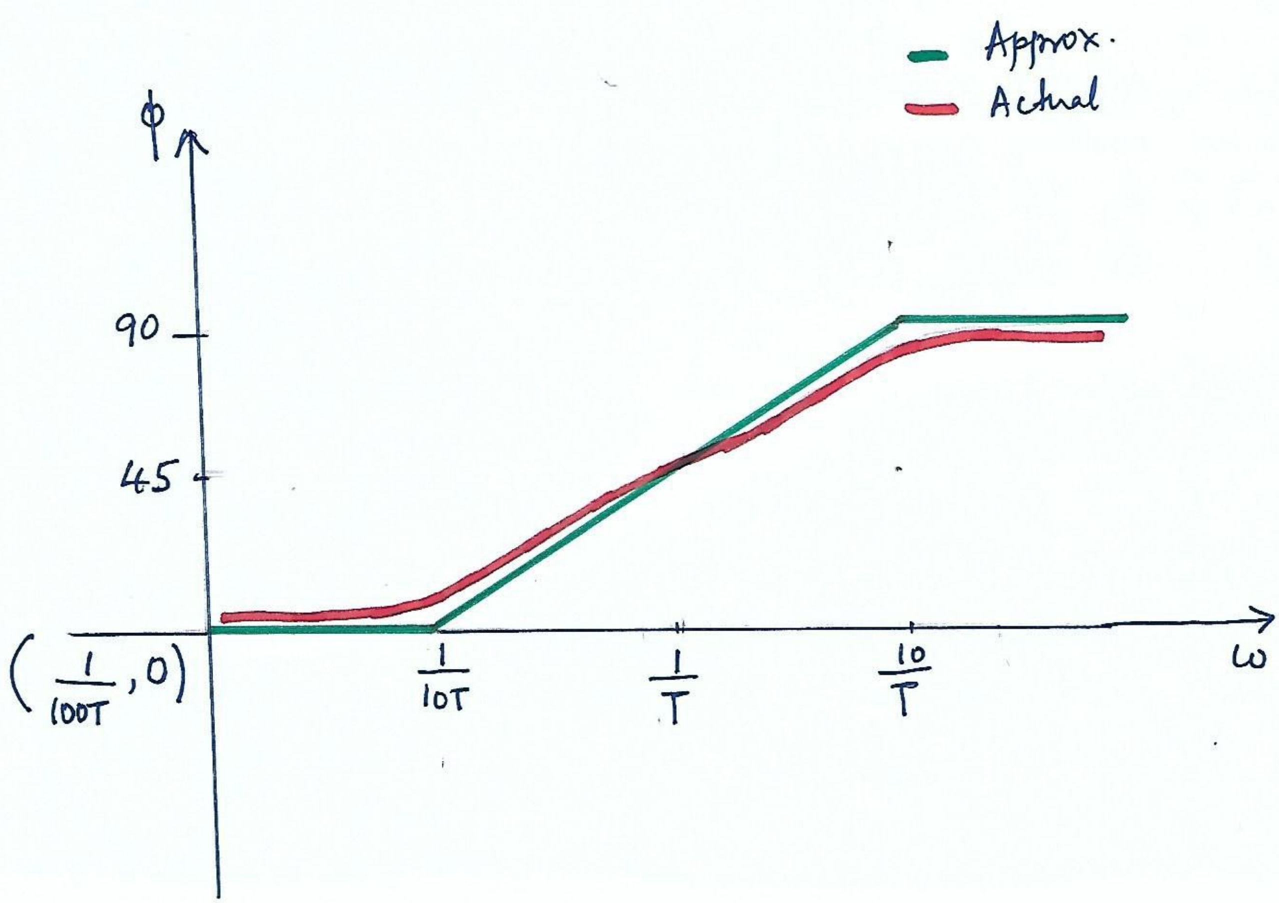
- The slope of the HFA of $1+j\omega T$ is 20dB/dec
- The phase angle varies from 0 to 90° as the frequency varies from zero to infinity.





COMPLEX POLES AND ZEROS

$$\left[1 + 2\zeta\left(\frac{j\omega}{\omega_n}\right) + \left(\frac{j\omega}{\omega_n}\right)^2\right]^{\pm 1}$$

* Control systems often possess quadratic factors of the form

$$\frac{1}{\left[1 + 2\zeta\left(\frac{j\omega}{\omega_n}\right) + \left(\frac{j\omega}{\omega_n}\right)^2\right]}$$

* If $\zeta > 1$ this quadratic factor can be expressed as a product of two first order ones with real poles.

* If $0 < \zeta < 1$, this quadratic factor is the product of two complex conjugate factors. Asymptotic approximations to the frequency response curves are not accurate for a factor with low values of ζ.

* The asymptotic frequency response curve may be obtained as follows:

$$20 \log \left| \frac{1}{1 + 2\zeta\left(\frac{j\omega}{\omega_n}\right) + \left(\frac{j\omega}{\omega_n}\right)^2} \right|$$

$$= -20 \log \sqrt{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}$$

- For low frequencies such that $\omega \ll \omega_n$,

$$LM = -20\log 1 = 0\,dB$$

- For high frequencies such that, $\omega \gg \omega_n$, the LM $= -20\log \frac{\omega^2}{\omega_n^2}$

$$= -40\log \frac{\omega}{\omega_n}\ dB$$

- The equation for the HFA is a straight-line having a slope of -40dB/dec since

$$-40\log \frac{10\omega}{\omega_n} = -40 - 40\log\frac{\omega}{\omega_n}$$

- The high frequency asymptote intersects with the low frequency one at $\omega = \omega_n$, since at this frequency $-40\log\frac{\omega}{\omega_n} = -40\log 1 = 0\,dB$
- This frequency (ω_n) is the corner frequency for the quadratic factor.
- A resonant peak occurs near the corner frequency.
- The damping ratio ζ determines the magnitude of this resonant peak.
- Therefore, error exists in the approximation by straight line asymptotes.

To find error due to approximation

* Actual $LM = -20\log\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}$

At $\omega = \omega_n$,

$$LM = -20\log\sqrt{(2\zeta)^2}$$

$$= -20\log 2\zeta$$

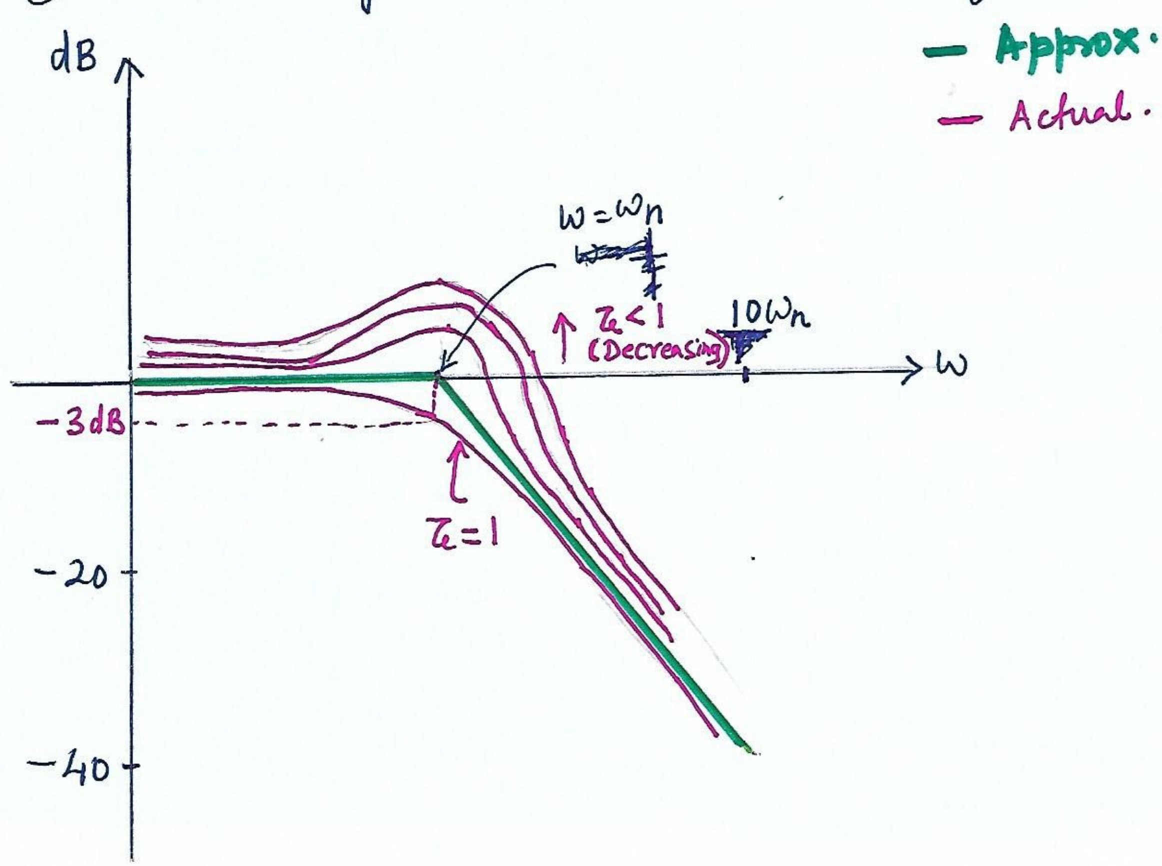
* For $\zeta = 1$, $LM = -20\log 2$

$$= -3\,dB$$

* For $\zeta < 1$, LM is a positive value

* The magnitude of error depends on the value of ζ.

* It is large for small values of ζ.



PHASE ANGLE OF THE QUADRATIC FACTOR

- The phase angle of the quadratic factor

$$\left[1 + 2\zeta\left(\frac{j\omega}{\omega_n}\right) + \left(\frac{j\omega}{\omega_n}\right)^2\right]^{-1} \quad \text{is}$$

$$\phi = -\tan^{-1}\left[\frac{2\zeta\frac{\omega}{\omega_n}}{1-\frac{\omega^2}{\omega_n^2}}\right]$$

- The phase angle is a function of both ω_n and ζ
- At $\omega = 0$, $\phi = 0^\circ$; $\omega = \omega_n$, $\phi = -90^\circ$; $\omega = \infty$, $\phi = -180^\circ$

- $$\lim_{\omega\to\infty} -\tan^{-1}\left[\frac{2\zeta\frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right]$$

- Let $\omega = \omega_n \tan\theta$, $\therefore$ as $\omega \to \infty$, $\theta \to \frac{\pi}{2}$

- $$\lim_{\omega\to\infty} -\tan^{-1}\left[\frac{2\zeta\frac{\cancel{\omega_n}\tan\theta}{\cancel{\omega_n}}}{1-\frac{\cancel{\omega_n^2}\tan^2\theta}{\cancel{\omega_n^2}}}\right]$$

Assuming $\zeta \cong 1$

- $$\lim_{\omega\to\infty} -\tan^{-1}\left[\frac{2\tan\theta}{1-\tan^2\theta}\right]$$

(x) $\lim_{\omega \to \infty} -\tan^{-1}[\tan 2\theta]$

(x) $\lim_{\omega \to \infty} -\tan^{-1}[\tan 2\theta] = \lim_{\theta \to \frac{\pi}{2}} -(2\theta)$

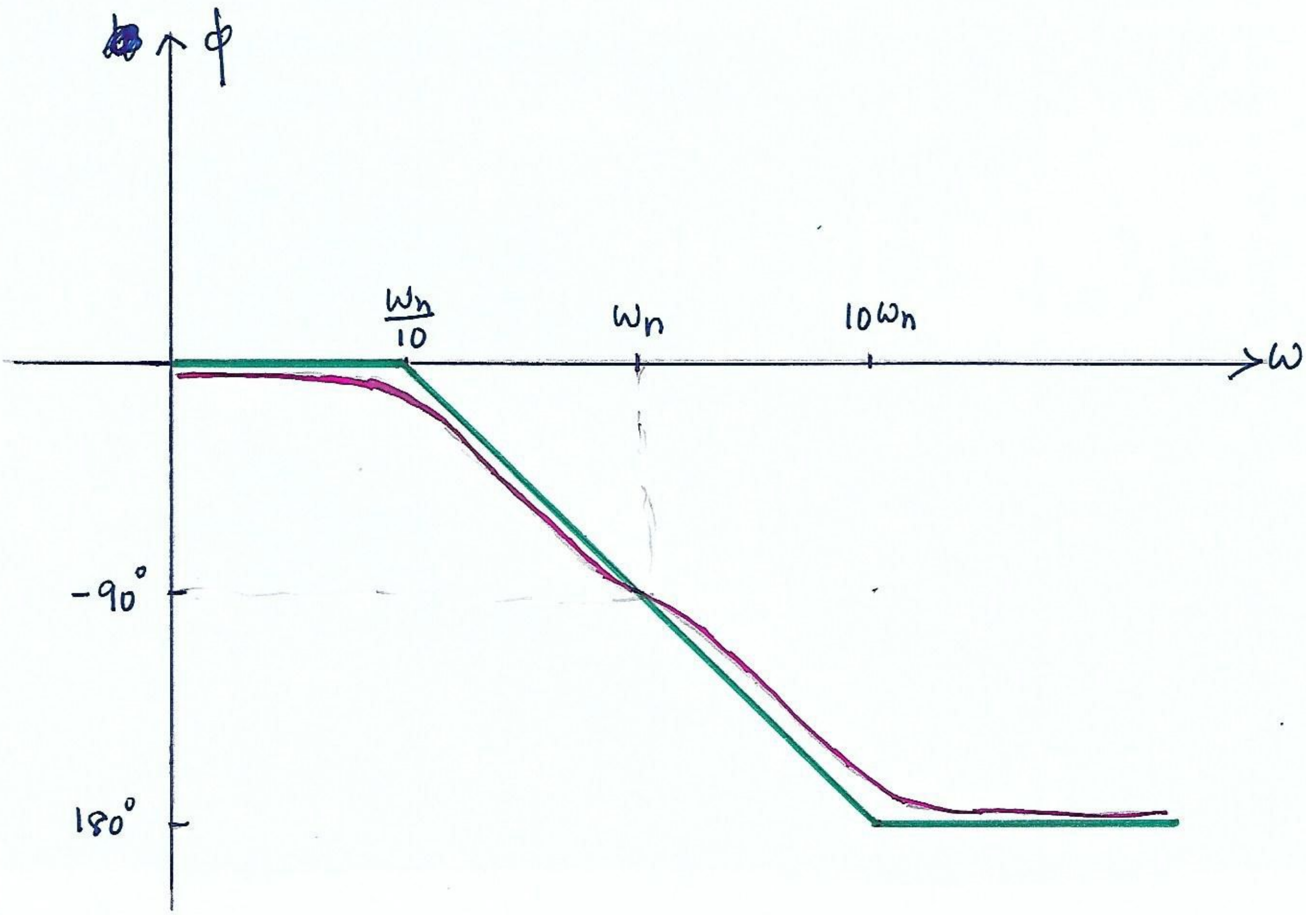
$= -2\left(\frac{\pi}{2}\right) = -\pi = -180^\circ$

Approximation of the phase plot

$\omega \leq \frac{\omega_n}{10}, \quad \phi = 0^\circ$

$\omega = \omega_n, \quad \phi = -90^\circ$

$\omega \geq 10\omega_n, \quad \phi = -180^\circ$



(x) The frequency response curves for the factor

$$1 + 2\zeta \frac{j\omega}{\omega_n} + \left(\frac{j\omega}{\omega_n}\right)^2$$

Can be obtained by merely reversing the sign of the log-mag and that of the phase angle of the factor.