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TY-BTech EXTC

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DCS Tutorial -II

Q1:] Find the minimal polynomial of each elements from GF(16)

→ Conjugacy classes of GF(16) are

$\{\alpha, \alpha^2, \alpha^4, \alpha^8\}$ $\quad 0 \Rightarrow X$

$\{\alpha^3, \alpha^6, \alpha^9, \alpha^{12}\}$ $\quad 1 \Rightarrow X+1$

$\{\alpha^5, \alpha^{10}\}$

$\{\alpha^7, \alpha^{11}, \alpha^{13}, \alpha^{14}\}$

∴ Minimal polynomial for;

$$\alpha \Rightarrow P(X) = (X-\alpha)(X-\alpha^2)(X-\alpha^4)(X-\alpha^8)$$
$$= (X^2 - \alpha^2 X - \alpha X + \alpha^3)(X^2 - \alpha^8 X - \alpha^4 X + \alpha^{12})$$
$$= X^4 + \alpha^3 X^2 - \alpha^3 X^3 + \alpha^3 X^2 + \alpha^6 - \alpha^6 X$$
$$- \alpha^4 X^3 - \alpha^7 X + \alpha^7 X^2$$
$$= X^4 + X + 1$$

$$\alpha^3 \Rightarrow P(X) = (X-\alpha^3)(X-\alpha^6)(X-\alpha^9)(X-\alpha^{12})$$
$$= X^4 + \alpha^{21} X^2 - (\alpha^9 + \alpha^{12}) X^3 + \alpha^9 X^2 + \alpha^{30}$$
$$- (\alpha^{18} + \alpha^{21}) X - (\alpha^3 + \alpha^6) X^3$$
$$- (\alpha^{24} + \alpha^{27}) X + (\alpha^3 + \alpha^6)(\alpha^9 + \alpha^{12}) X^2$$
$$= X^4 + X^3 + X^2 + X + 1$$

$$\alpha^5 \Rightarrow P(X) = (X-\alpha^5)(X-\alpha^{10})$$
$$= (X^2 + \alpha^{15} - (\alpha^5 + \alpha^{10}) X)$$
$$= (X^2 + \alpha^3 + \alpha + \alpha^2 - (1 + \alpha + \alpha^2 + \alpha) X)$$
$$= (X^2 + 1 + \alpha^2 - (1 + \alpha^2) X)$$
$$= (X^2 + (1 + \alpha^2) - (1 + \alpha^2) X]$$
$$= X^2 + X + 1$$

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$\alpha^7 \Rightarrow P(X) = (X-\alpha^7)(X-\alpha^{11})(X-\alpha^{13})(X-\alpha^{14})$

$= (X^2 - (\alpha^7+\alpha^{11})X + \alpha^{18})$

$(X^2 - (\alpha^{13}+\alpha^{14})X + \alpha^{27})$

$= X^4 - (\alpha^{13}+\alpha^{14})X^3 + \alpha^{27}X^2 - (\alpha^7+\alpha^{11})X^3$

$+ (\alpha^{13}+\alpha^{14})(\alpha^7+\alpha^{11})X$

$- (\alpha^7+\alpha^{11})\alpha^{27} + \alpha^{18}X^2$

$- \alpha^{18}(\alpha^{13}+\alpha^{14})X + \alpha^{45}$

$= X^4 + X^3 + 1$

Q2:] Factorize the polynomial $X^{15}+1$ over $GF(2)$ using Q.1

→ Using the previous question, we factorize

$$X^{15}+1 \text{ as } \frac{X^{16}+X}{X}$$

as we found out the minimal polynomial of $GF(16)$ in the previous question we multiply them to get

$$(X^{15}+1) = \frac{(X^{16}+X)}{X}$$

$$= \frac{\cancel{X}(X+1)(X^2+X+1)(X^4+X+1)(X^4+X^3+1)(X^4+X^3+X^2+X+1)}{\cancel{X}}$$

$\therefore$ $(X^{15}+1) = (X+1)(X^2+X+1)(X^4+X+1)(X^4+X^3+1)(X^4+X^3+X^2+X+1)$

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Q3.] Find the generator polynomial for (15, 8) systematic cyclic code use Q 2

→ $X^{15}+1 = (X+1)(X^2+X+1)(X^4+X+1)(X^4+X^3+1)$
$(X^4+X^3+X^2+1)$

∴ $g(X)$ is of degree $n-k = 15-8 = 7$

∴ $g(X) = (X+1)(X^2+X+1)(X^4+X+1)$
∴ $g(X) = (X^3+X^2+X+X^2+X+1)(X^4+X+1)$
∴ $g(X) = (X^3+1)(X^4+X+1)$
$= X^7+X^4+X^3+X^4+X+1$
$= X^7+X^3+X+1$

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Q4] Find parity check polynomial using Q2.

$\rightarrow$ $h(x) = (x^4 + x^3 + 1)(x^4 + x^3 + x^2 + x + 1)$

$= x^8 + \cancel{x^7} + \cancel{x^6} + \cancel{x^5} + \cancel{x^4} + \cancel{x^7} + \cancel{x^6} + \cancel{x^5} + \cancel{x^4}$
$+ \cancel{x^3} + x^4 + \cancel{x^3} + x^2 + x + 1$

$= x^8 + x^4 + x^2 + x + 1$