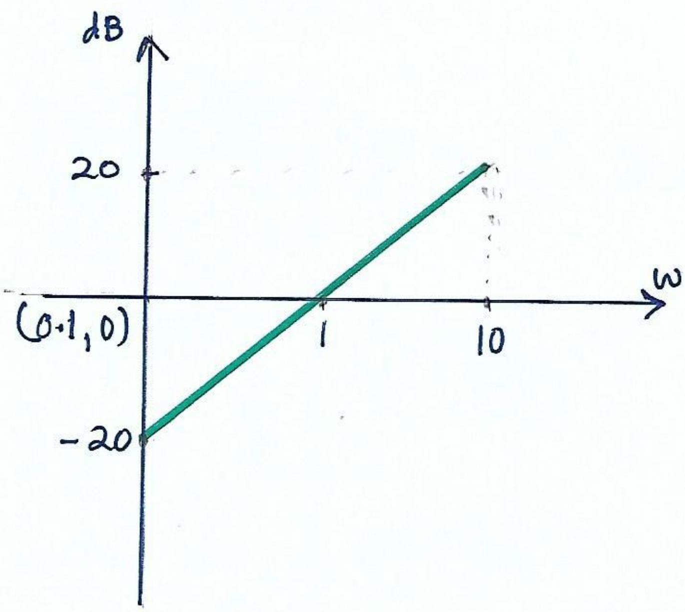
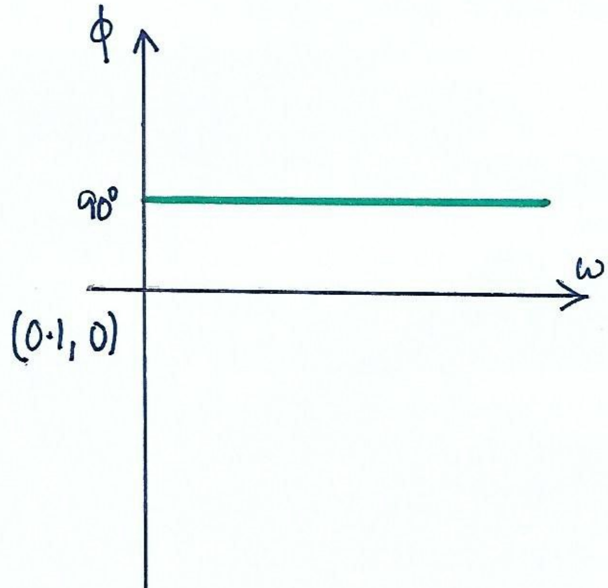
(3) Zero at the origin $(j\omega)^{+1}$

- The log magnitude of $j\omega$ in dB is
$$20\log|j\omega| = 20\log\omega \text{ dB}$$
- The phase angle of $j\omega$ is a constant and equal to 90°
- The log magnitude curve is a straight line with a slope of 20 dB/dec
- It is seen that the differences in the frequency responses of the factors $\frac{1}{j\omega}$ and $j\omega$ lie in the signs of the slopes of the log-mag curves and in the signs of the phase angles.
- Both log magnitudes become 0 dB at $\omega = 1$





(4) Simple Pole / Zero $\frac{1}{1+j\omega T}$, $1+j\omega T$

→ The log mag of $\frac{1}{1+j\omega T}$ is

$$20\log\left|\frac{1}{1+j\omega T}\right| = -20\log\sqrt{1+\omega^2T^2}\ \text{dB}$$

→ For low frequencies such that $\omega \ll \frac{1}{T}$, the log mag may be approximated as

$$-20\log\sqrt{1+\omega^2T^2} \simeq 20\log 1 = 0\text{dB}$$

→ Thus the log mag curve at low frequencies is the constant 0dB line, for $\omega \ll \frac{1}{T}$

→ It is called the Low frequency asymptote (LFA)

→ For higher frequencies such that $\omega \gg \frac{1}{T}$,

$$-20\log\sqrt{1+\omega^2T^2} \simeq -20\log\omega T\ \text{dB}$$

→ The value of $-20\log\omega T$ decreases by 20dB for every decade of frequency (ω).

→ The log magnitude curve is a straight line with a slope of -20dB/dec, for $\omega \gg \frac{1}{T}$.

→ It is called the high frequency asymptote (HFA)

→ This is known as Asymptotic approximation

→ The logarithmic representation of the frequency response curve of the factor $\frac{1}{1+j\omega T}$ can be approximated by two straight line asymptotes; one a straight line at 0dB for the frequency range $0<\omega<\frac{1}{T}$ and the other a straight line with a slope of -20dB/dec for the frequency range $\frac{1}{T}<\omega<\infty$

→ At $\omega=\frac{1}{T}$, the HFA has value $-20\log \omega T$

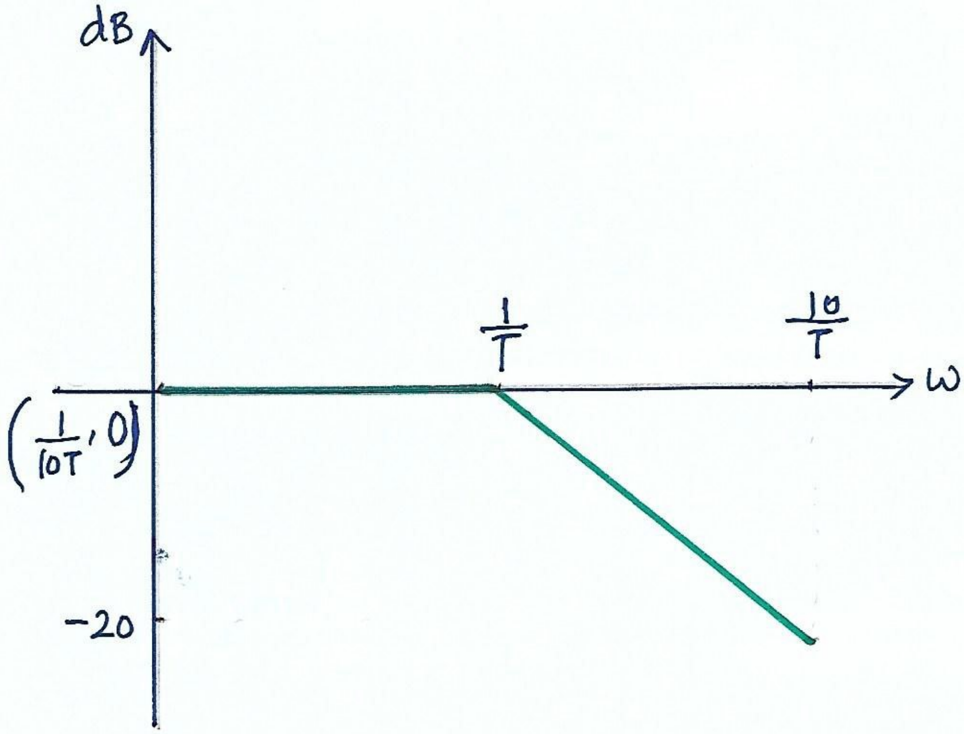
ie $-20\log \frac{1}{T}\cdot T = -20\log 1 = 0dB$

→ Thus the HFA meets the LFA at $\omega=\frac{1}{T}$

→ The frequency at which the two asymptotes meet is called the corner frequency or break frequency.

→ For the factor $\frac{1}{1+j\omega T}$, the frequency $\omega=\frac{1}{T}$ is called the corner frequency.

→ The corner frequency divides the frequency response curve into two regions, a curve for low frequency region and a curve for high frequency region.



→ <u>To find error due to approximation</u>

→ Actual log mag $-20 \log \sqrt{1+\omega^2 T^2}$ dB

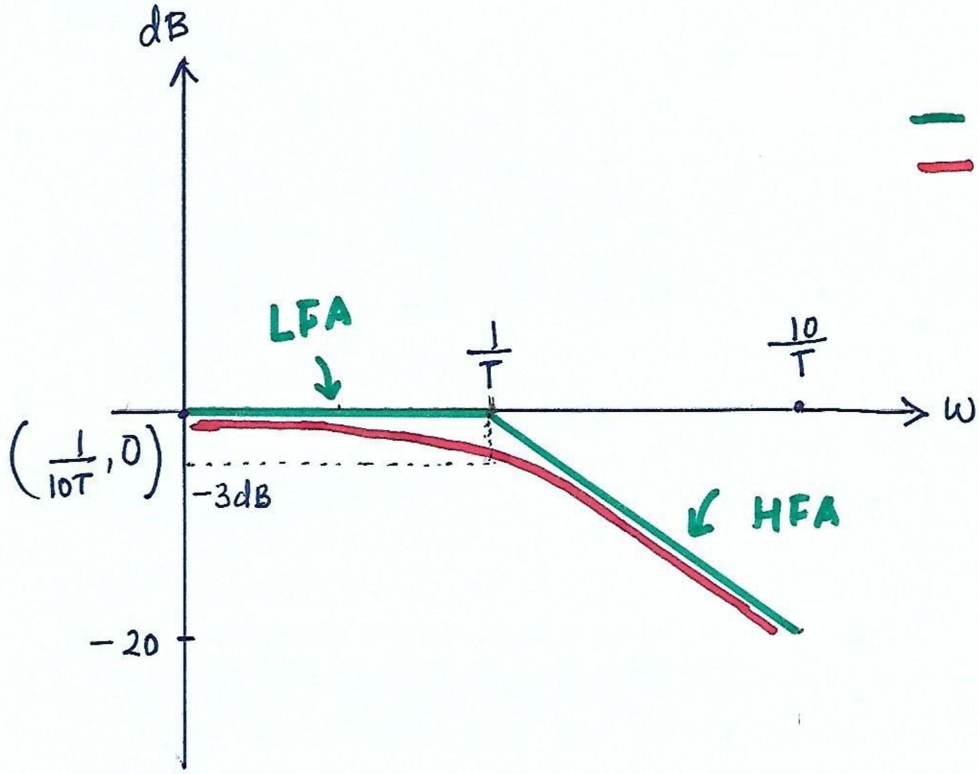
→ At $\omega = \frac{1}{T}$, $LM = -20 \log \sqrt{1+1^2}$

$$= -20 \log \sqrt{2}$$

$$= -3 dB.$$

→ But we have assumed LM at $\omega = \frac{1}{T}$ to be 0 dB

→ So there is an error of $-3 dB$ at the corner frequency.



PHASE PLOT

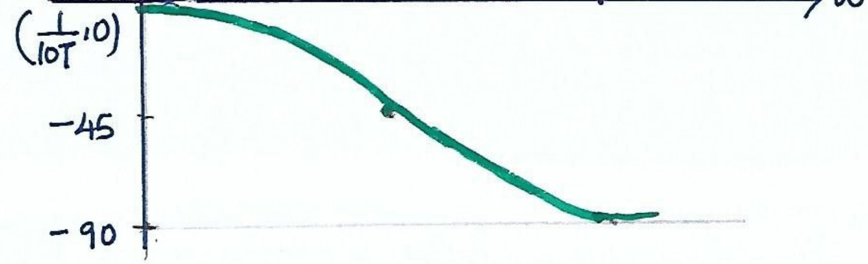
- The phase angle of the factor $\frac{1}{1+j\omega T}$ is

$$\phi = -\tan^{-1}\omega T$$

- At zero frequency the phase angle is zero
- At the corner frequency the phase angle is

$$\phi = -\tan^{-1}\frac{1}{T}\cdot T = -45^\circ$$

- At infinity the phase angle is -90°

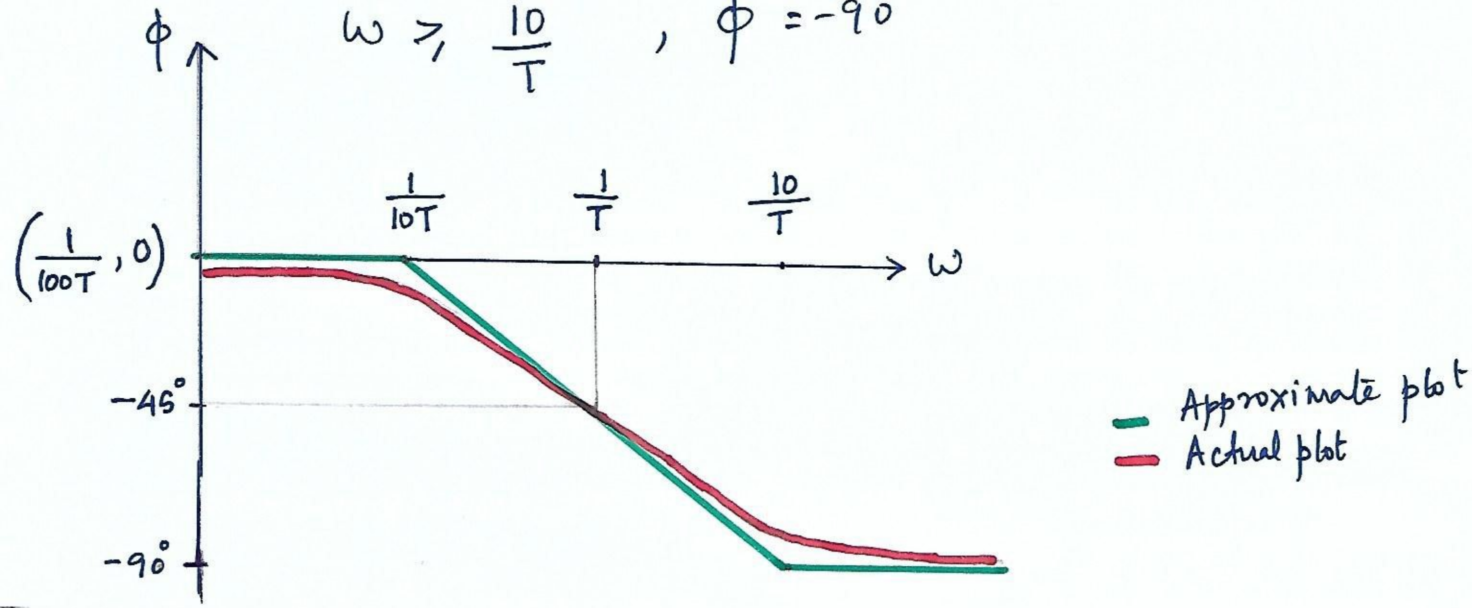


APPROXIMATION OF THE PHASE PLOT

For $\omega \leq \frac{1}{10T}$, $\phi = 0°$

$\omega = \frac{1}{T}$, $\phi = -45°$

$\omega \geq \frac{10}{T}$, $\phi = -90°$



Bode plot of $1+j\omega T$

* An advantage of the logarithmic representation is that, for the reciprocal factors the log mag and phase angle curves need only be changed in sign.

* LM for $1+j\omega T$ is $20 \log |1+j\omega T|$

  ie $20 \log \sqrt{1+\omega^2 T^2}$ dB

  ϕ for $1+j\omega T$ is $\tan^{-1} \omega T$

* The corner frequency is $\omega = \frac{1}{T}$