Pb (43) Sketch the root locus for the system with open loop transfer function

$$G(s)H(s) = \frac{K}{s(s+4)^2}$$

For $K = 24$, find the Gain Margin and Phase Margin of the system.

Solution

① $K = 0$ points are at $s = 0, s = -4, s = -4$

② $K = \infty$ points are at $s = \infty, s = \infty, s = \infty$

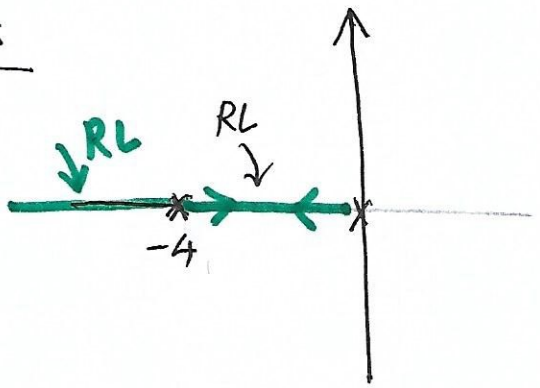
③ Number of branches $= 3$

④ The root locus is symmetrical about the real axis

⑤ Centroid ... $\frac{(0-4-4)-(0)}{3} = -\frac{8}{3} = -2.66$

⑥ Asymptotes ... $60^\circ, 180^\circ, 300^\circ$

⑦ Root locus on Real axis



⑧ Angles of departure / arrival — NA

As there are no complex conjugate pole or zero pairs

⑨ Intersection with Imaginary axis

$$1 + \frac{K}{s(s+4)^2} = 0 \quad , \quad s^3 + 8s^2 + 16s + K = 0$$

$s^3 + 8s^2 + 16s + k = 0$

s^3	1	16
s^2	8	K
s	$\frac{128-k}{8}$	
s^0	K	

$k_{max} = 128$

$8s^2 + 128 = 0$

$s = \pm j4$

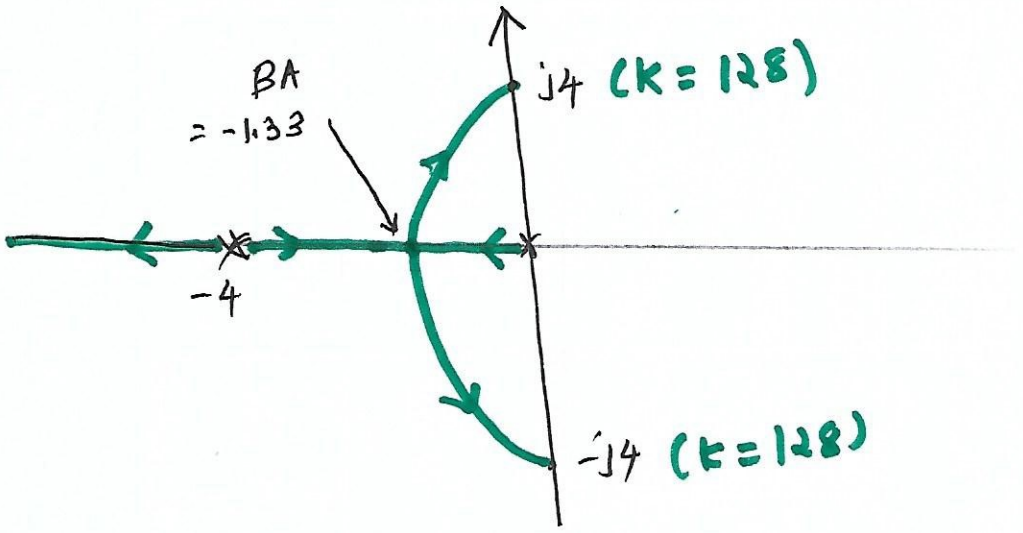
(10) Breakaway point

$$\frac{d}{ds}\left[\frac{1}{s(s+4)^2}\right] = 0$$

$3s^2 + 16s + 16 = 0$

$s = -4$ or -1.33

Breakaway point is -1.33



Gain Margin $GM = \frac{\text{Value of K at Im. axis crossover}}{\text{Design value of K}}$

$= \frac{128}{24} = 5.33$

$GM = 5.33$

Phase Margin

At $\omega = \omega_{gc}$, $|G(j\omega)H(j\omega)| = 1$

$$|G(j\omega)H(j\omega)| = \left|\frac{24}{j\omega(j\omega+4)^2}\right|$$

$$|G(j\omega_{gc})H(j\omega_{gc})| = \frac{24}{\omega_{gc}^2(\omega_{gc}^2+16)} = 1$$

$$\omega_{gc}^3 + 16\omega_{gc} - 24 = 0$$

$$\omega_{gc} = 1.347 \text{ rad/sec}$$

$$\underline{|G(j\omega_{gc})H(j\omega_{gc})} = -90 - 2\tan^{-1}\left(\frac{1.347}{4}\right)$$

$$= -90 - 2(18.61)$$

$$= -127.22$$

$$PM = 180 - 127.22$$

$$= 52.78$$

$$\boxed{PM = 52.78^\circ}$$