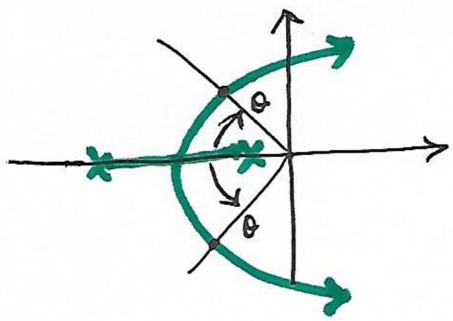
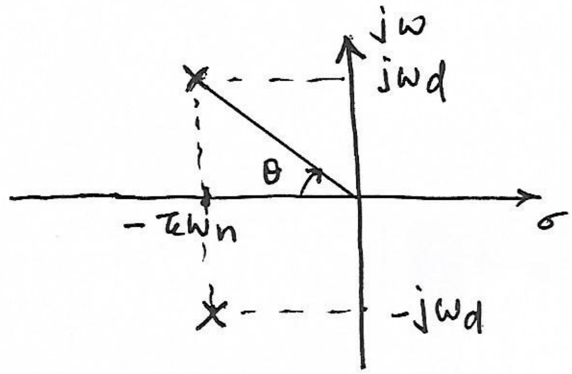
DAMPING RATIO FROM THE ROOT LOCUS

- The value of K for a given damping ratio ζ or vice versa may be found from the root locus.
- Draw a line at the origin with an angle $\pm\theta$ wrt to the negative real axis where $\theta = \cos^{-1}\zeta$.



- Find the value of K at the point where this line meets the root locus.
- This value of K will give the required damping ratio ζ.

$s^2+2\zeta\omega_n s+\omega_n^2=0$, $\left.\begin{array}{l} s_1=-\zeta\omega_n+j\omega_d \\ s_2=-\zeta\omega_n-j\omega_d \end{array}\right\}$ for $\zeta<1$



$$\cos\theta = \frac{\zeta\omega_n}{\sqrt{(\zeta\omega_n)^2+(\omega_d)^2}} = \frac{\zeta\omega_n}{\sqrt{(\zeta\omega_n)^2+(\omega_n\sqrt{1-\zeta^2})^2}}$$

$$\cos^2\theta = \frac{\zeta^2\cancel{\omega_n^2}}{\cancel{\zeta^2\omega_n^2}+\cancel{\omega_n^2}-\cancel{\omega_n^2\zeta^2}} \qquad \therefore \cos\theta=\zeta$$

$$\therefore \boxed{\theta=\cos^{-1}\zeta}$$

Pb (34) Draw the root locus for

$$G(s)H(s) = \frac{K}{(s+2)^3}$$

Find the value of K which will give a damping ratio of 0.5 in the system.

Solution

(1) $K=0$ points $\quad s=-2,\ s=-2,\ s=-2$

(2) $K=\infty$ points $\quad s=\infty,\ s=\infty,\ s=\infty$

(3) Number of branches $= 3$ $\quad (n=3,\ m=0)$

(4) Symmetry of the root locus

(5) Angles of asymptotes

$l = 0, 1, 2$

$\theta_0 = 60^\circ,\ \theta_1 = 180^\circ,\ \theta_2 = 300^\circ$

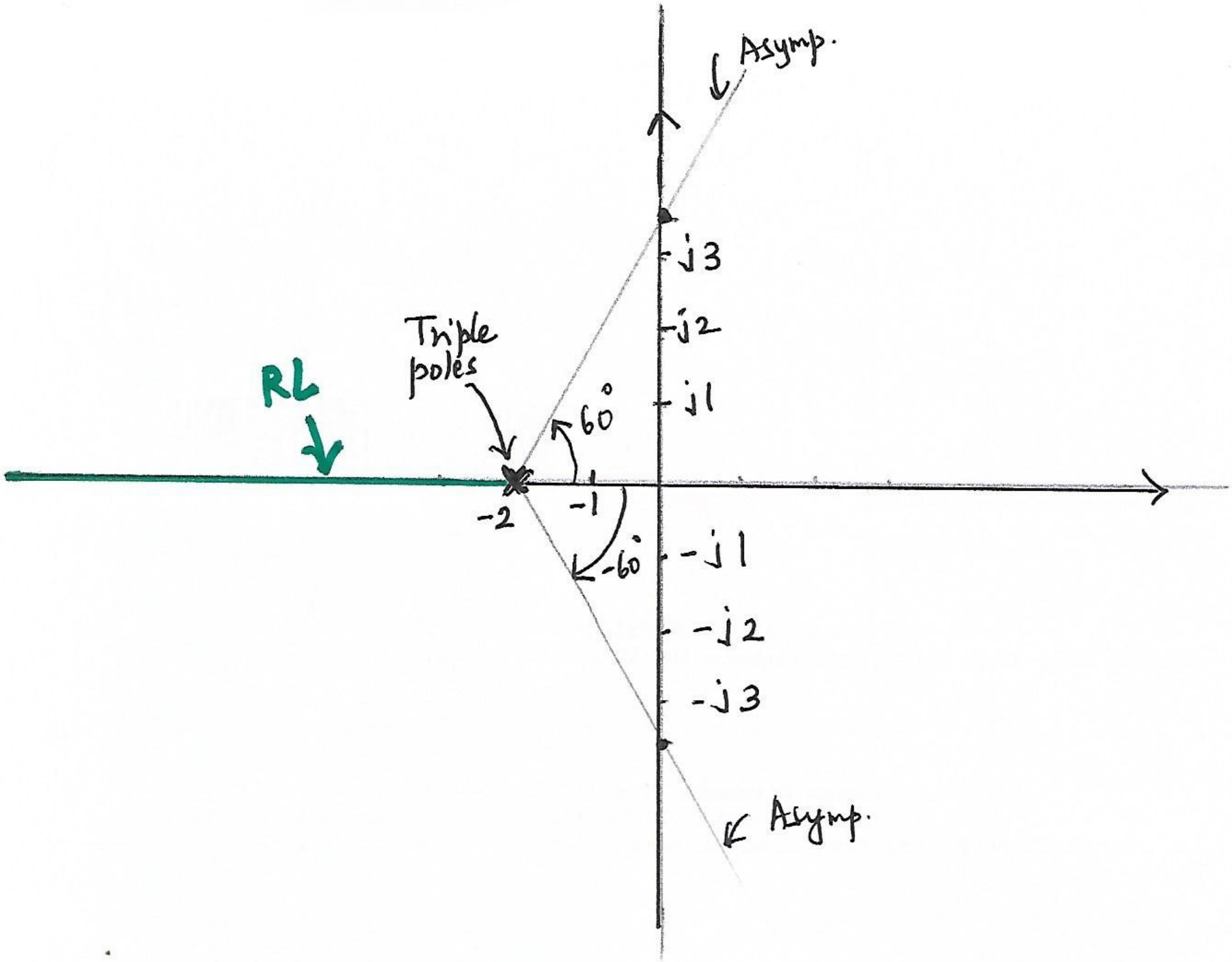
(6) Centroid

$$\frac{(-2-2-2)-(0)}{3-0} = -2$$

Since the centroid is at the pole, the asymptotes themselves are the root loci. This is an exact root locus plot. (No rough sketch)

(7) Root locus on the Real axis

Root locus may be found on the Real axis to the left of -2



⑧ Angles of departure and arrival need not be found as there are no complex conjugate poles or zeros.

⑨ Intersection of the root locus with the Imaginary axis:

$$1 + \frac{K}{(s+2)^3} = 0$$

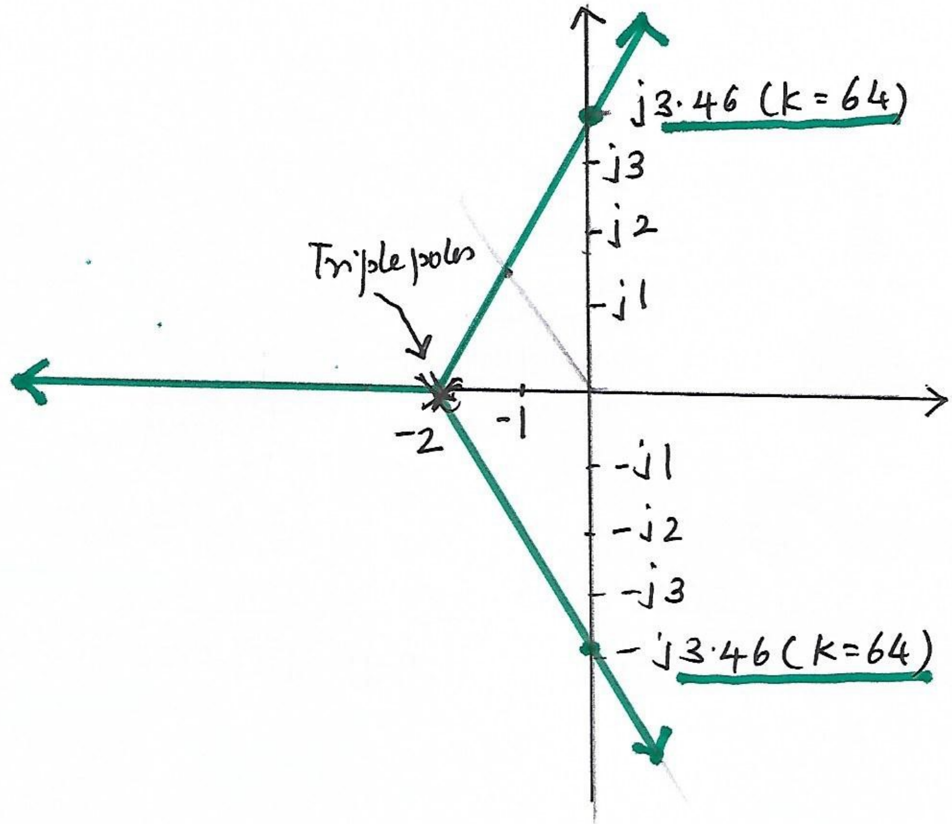
$$s^3 + 6s^2 + 12s + 8 + K = 0$$

s^3	1	12
s^2	6	$8+K$
s	$\frac{64-K}{6}$	
s^0	$8+K$	

$K = 64$

$s = \pm j3.46$

⑩ There are no breakaway points



To find the value of K which will give a damping ratio of 0.5

→ Draw a line with an angle $\theta = \cos^{-1}\zeta$

$= \cos^{-1} 0.5$

$= 60^\circ$

wrt to the negative real axis (from the origin)

→ This line meets the root locus at

$s = -1 + j1.75$

→ Check the angle condition at this point

$\angle G_1(s)H_1(s) = \left[0 - 3\angle s+2\right]_{s = -1+j1.75}$

$\rightarrow \quad \angle G_1(s)H_1(s) = [0 - 3\angle{-1+1.75}]$

$= [0 - 3(60.26)]$

$\simeq -180^{\circ}$

$\rightarrow$ To find the value of K at $s = -1+j1.75$ use magnitude condition

$$|G_1(s)H_1(s)| = \left|\frac{1}{K}\right|$$

$$\left|\frac{1}{(s+2)^3}\right|_{s=-1+j1.75} = \left|\frac{1}{K}\right|$$

$$\frac{1}{|(2)^3|} = \left|\frac{1}{K}\right| \quad \therefore \boxed{K = 8}$$