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TY BTech EXTC

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DCS - Class Test 2

Q1.] Write the basis vector for the vector space $V6$ over $GF(2)$

→ Basis vector for $V6$ over $GF(2)$ is

$$V6 = \{100000, 010000, 001000, 000100, 000010, 000001\}$$

Q2.] Write the linear combination of vectors for given set of vectors 10111 and 00110 over $GF(2)$.

→ $a_1 V_a + a_2 V_b = V$; $V_a = 10111$

$V_b = 00110$

where,

$a_1 = \{0, 1\}$

$a_2 = \{0, 1\}$

$V_1 = 0 \cdot V_a + 0 \cdot V_b = 00000$

$V_2 = 0 \cdot V_a + 1 \cdot V_b = 00110$

$V_3 = 1 \cdot V_a + 0 \cdot V_b = 10111$

$V_4 = 1 \cdot V_a + 1 \cdot V_b$

$= 10111 + 00110$

$= (1+0)\ (0+0)\ (1+1)\ (1+1)\ (1+0)$

$V_4 = 10001$

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Q3] $m = [1\ 0\ 1\ 1]$

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$ $\rightarrow$ systematic form

$c = m \cdot G$

$$= [1\ 0\ 1\ 1] \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$$= [1\ 0\ 1\ 1\ 1\ 0\ 0]$$

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Q4.] $(n,k) = (7,4)$

$r = [0\ 0\ 1\ 0\ 1\ 1\ 1]$

$$P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$H = [P^T \ \ I_{n-k}]$$

$$= \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ Parity check matrix, $S = r \cdot H^T$

$$\Rightarrow S = [0\ 0\ 1\ 0\ 1\ 1\ 1] \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow S = [0 \quad 0 \quad 0]$$

Since, $S = [0\ 0\ 0]$, the received codeword is valid.

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Q5] $X^{15}+1 = (X+1).(X^4+X^3+X^2+X+1).(X^4+X^3+1).$
$(X^4+X+1).(X^2+X+1)$

$(n, k) = (15, 4)$

We know that,

$$X^n+1 = g(X)\,h(X)$$

$g(X) \Rightarrow$ Generator polynomial
$h(X) \Rightarrow$ Parity check polynomial.

(i) Degree of generator polynomial is

$$n-K = 15-4 = 11$$

$$\therefore g(X) = (X+1)(X^4+X^3+1)(X^4+X+1)(X^2+X+1)$$

$$\therefore g(X) = (X^8+X^5+\cancel{X^4}+X^7+\cancel{X^4}+X^3+X^4+X+1).$$
$$(X^3+X^2+\cancel{X^2}+\cancel{X}+\cancel{X}+1)$$

$$= X^{11}+X^3+X^{10}+X^7+X^8+X^5+X^7+$$
$$X^4+X^6+X^3+X^4+X^3+1$$

$$\therefore g(X) = X^{11}+X^{10}+X^6+X^5+X+1$$

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(ii) Now, $h(x) = \dfrac{x^{15}+1}{g(x)}$

$$\therefore h(x) = \frac{(x+1)(x^4+x^3+1)(x^4+x+1)(x^2+x+1)\cdot(x^4+x^3+x^2+x+1)}{x^{11}+x^{10}+x^6+x^5+x+1}$$

$$\therefore h(x) = x^4+x^3+x^2+x+1$$

(iii) Degree of $g(x)$ [Generator Polynomial] $\Rightarrow$ 11

& degree of $h(x)$ [parity check polynomial) $\Rightarrow$ 4