Pb (36)

Sketch the Bode plot for the system with

$$G(s)H(s) = \frac{64(s+2)}{s(s+0.5)(s^2+3.2s+64)}$$

Solution

(i) The rearrangement of the transfer function gives

$$G(s)H(s) = \frac{64\left(1+\frac{s}{2}\right)2}{s\left(1+\frac{s}{0.5}\right)\left(1+\frac{3.2s}{64}+\frac{s^2}{64}\right)64\times0.5}$$

$$= \frac{4\left(1+\frac{s}{2}\right)}{s\left(1+\frac{s}{0.5}\right)\left(1+0.05s+\frac{s^2}{64}\right)}$$

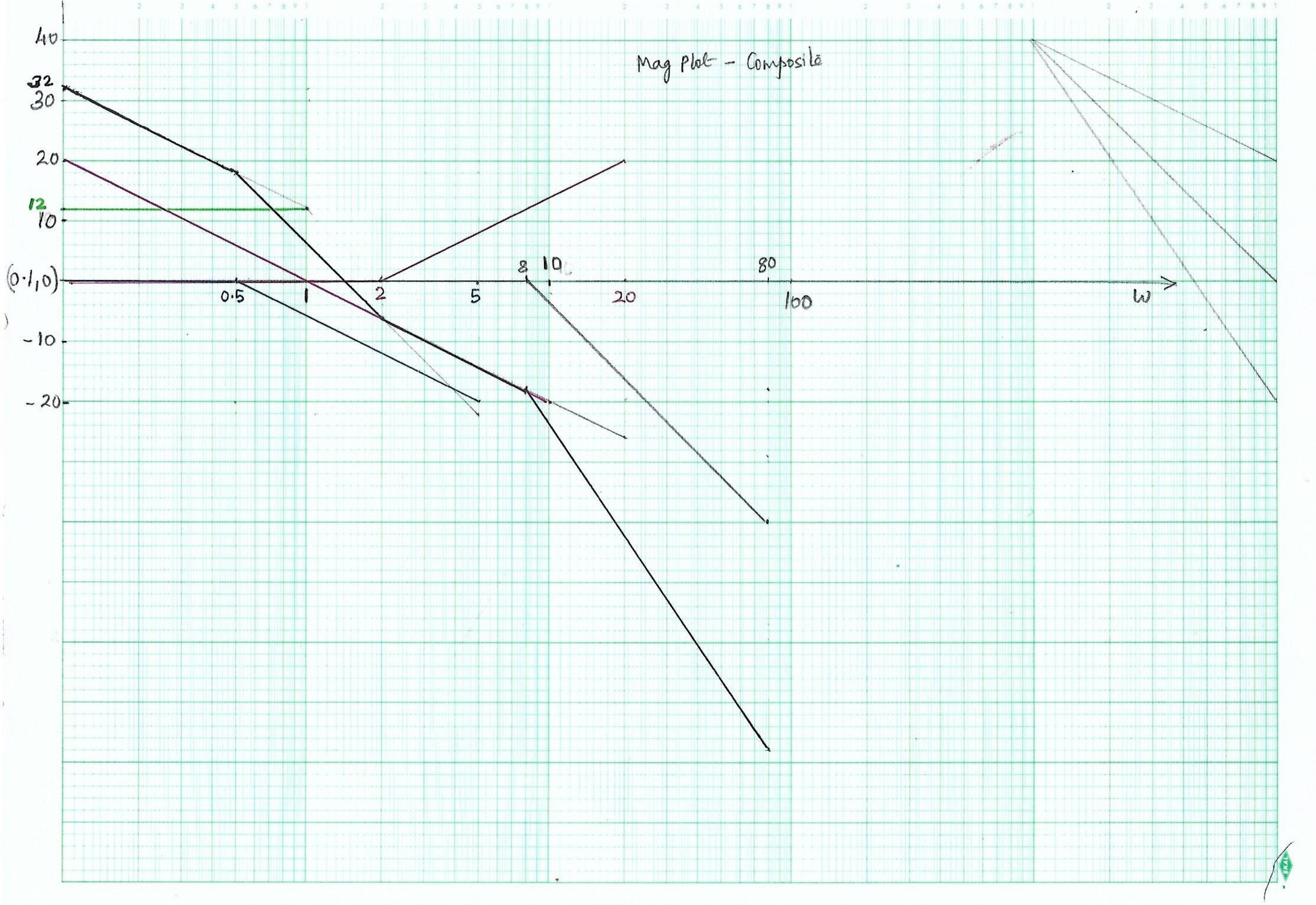
(ii) The sinusoidal transfer function is

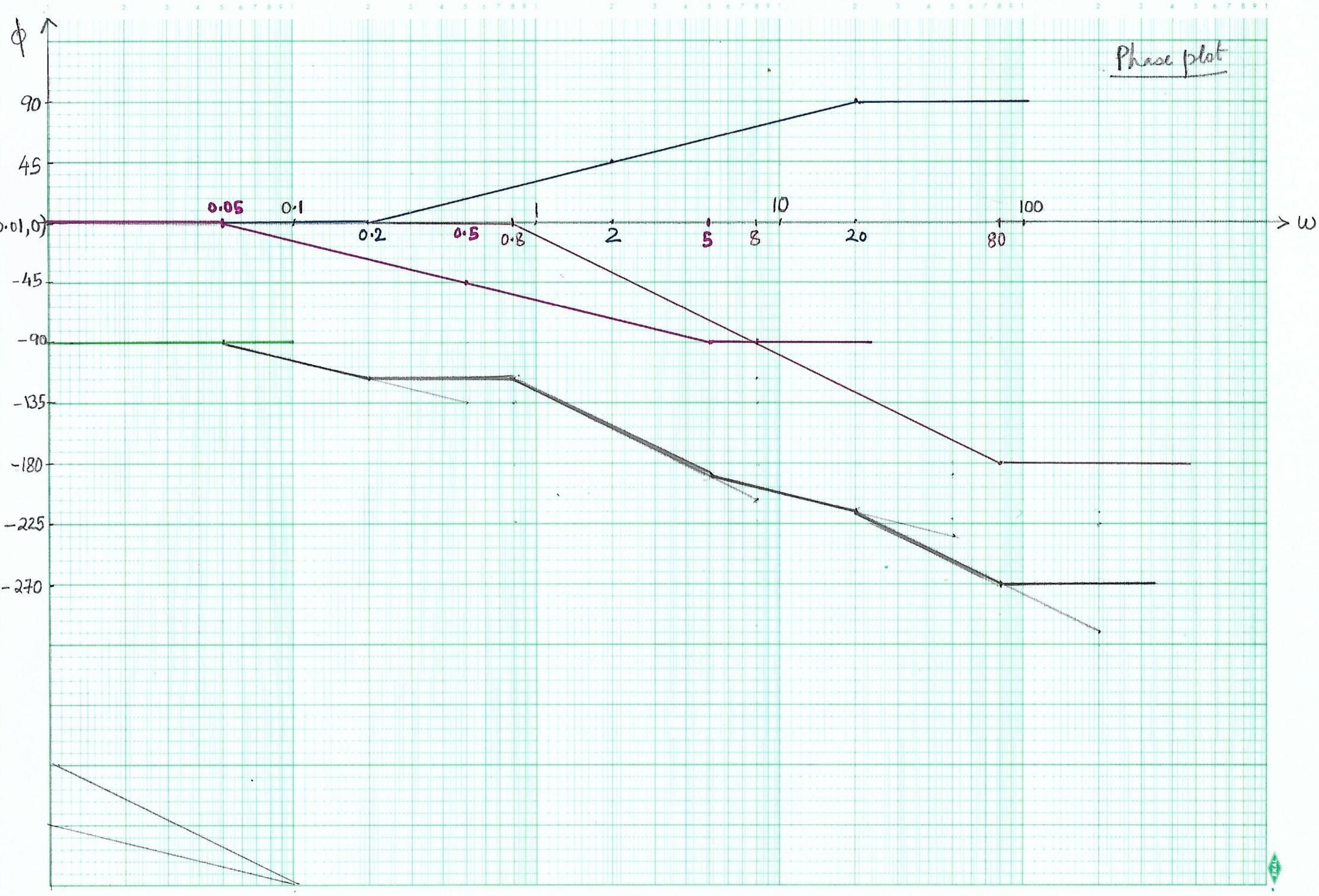
$$G(j\omega)H(j\omega) = \frac{4\left(1+\frac{j\omega}{2}\right)}{j\omega\left(1+\frac{j\omega}{0.5}\right)\left(1+0.05j\omega-\frac{\omega^2}{64}\right)}$$

(iii) Prepare a table as shown:

S.NO	Factor	Corner freq	LM	ϕ
1)	4	–	Constant 12dB	0°
2)	$(j\omega)^{-1}$	–	Constant slope −20dB/dec	-90°
3)	$\left(1+\frac{j\omega}{0.5}\right)^{-1}$	0.5	0dB upto cf −20dB/dec above cf	$\omega\le0.05, \phi=0^\circ$ $\omega=0.5, \phi=-45^\circ$ $\omega\ge5, \phi=-90^\circ$

S.NO	Factor	Corner freq	LM	ϕ
4	$\left(1+\frac{j\omega}{2}\right)^{+1}$	2	0dB upto cf +20 dB/dec > Cf	$\omega \leq 0.2, \phi=0$ $\omega=2, \phi=45$ $\omega \geq 20, \phi=90$
5	$\left[1-\frac{\omega^2}{64}+j0.05\omega\right]^{-1}$	8	0dB upto cf -40 dB/dec > Cf	$\omega \leq 0.8, \phi=0$ $\omega=8, \phi=-90^\circ$ $\omega \geq 80, \phi=-180^\circ$


Mag Plot - Composite
40
32
30
20
12
10
(0.1,0)
-10
-20
0.5
1
2
5
8
10
20
80
100
ω



Phase plot
ϕ
ω
90
45
-45
-90
-135
-180
-225
-270
(0.01,0)
0.05
0.1
0.2
0.5
0.8
1
2
5
8
10
20
80
100

RELATIONSHIP BETWEEN SYSTEM TYPE AND LOG-MAG CURVE

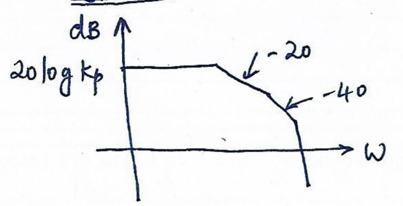
- The type of the system determines the slope of the log-magnitude curve at low frequencies.

- The static position, velocity and acceleration error coefficients describe the low frequency behaviour of type 0, type 1 and type 2 systems respectively.

- Thus information concerning the existence and magnitude of the steady state error of a control system to a given input can be determined from the observation of the low frequency region of the log-mag curve.

(1) POSITIONAL ERROR CONSTANT

- Fig below shows an example of the log-mag plot of a type 0 system.



- The positional error constant is defined as

$$K_p = \lim_{s \to 0} G(s) H(s)$$

i.e. $$K_p = \lim_{\omega \to 0} G(j\omega) H(j\omega)$$

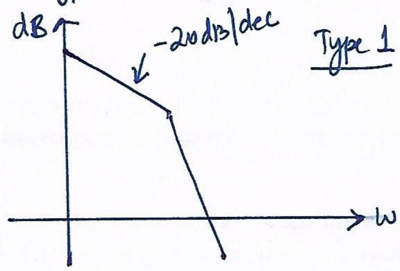
* At low frequencies,

$$20 \log K_p = 20 \log |G(j\omega) H(j\omega)|$$

* The LFA is a straight line at $20 \log K_p$.

② <u>VELOCITY ERROR CONSTANT</u>

* Fig below shows an example of the log-mag plot of a type 1 system.



* The velocity error constant is defined as

$$K_v = \lim_{s \to 0} s\, G(s) H(s)$$

* $$K_v = \lim_{\omega \to 0} (j\omega)\, G(j\omega) H(j\omega)$$

* At low frequencies ($\omega \to 0$)

$$K_v = (j\omega)\, G(j\omega) H(j\omega)$$

$$\therefore G(j\omega) H(j\omega) = \frac{K_v}{j\omega}$$

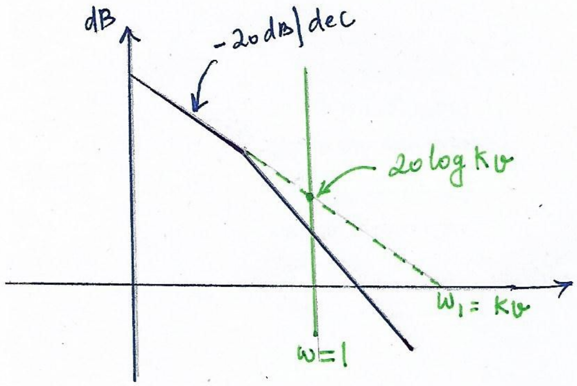
* Thus $20 \log |G(j\omega) H(j\omega)| = 20 \log \left| \frac{K_v}{j\omega} \right|$

* At $\omega = 1$, $20 \log |G(j\omega) H(j\omega)| = 20 \log \left| \frac{K_v}{1} \right| = 20 \log K_v$

* This means the intersection of the initial -20dB/dec segment (or its extension) with the line $\omega = 1$ has the magnitude $20 \log K_v$.

* The intersection of the initial -20dB/dec segment or its extension with the 0dB line has a frequency numerically equal to K_v.

* This can be proved as follows:
  - Let the line intersect 0dB at ω_1
  - $20 \log \left| \frac{K_v}{j\omega_1} \right| = 0\,dB = 20 \log 1$
  - $\left| \frac{K_v}{j\omega_1} \right| = 1$ or $\boxed{K_v = \omega_1}$



③ ACCELERATION ERROR CONSTANT

- The acceleration error constant is defined as

$$K_a = \lim_{s \to 0} s^2 G(s) H(s)$$

ie $$K_a = \lim_{\omega \to 0} (j\omega)^2 G(j\omega) H(j\omega)$$

- At low frequencies $(\omega \to 0)$

$$K_a = (j\omega)^2 G(j\omega) H(j\omega)$$

$$G(j\omega) H(j\omega) = \frac{K_a}{(j\omega)^2}$$

- $$20 \log |G(j\omega) H(j\omega)| = 20 \log \left| \frac{K_a}{(j\omega)^2} \right|$$

- At $\omega = 1$,

$$20 \log |G(j\omega) H(j\omega)| = 20 \log \left| \frac{K_a}{1} \right| = 20 \log K_a$$

- Thus the intersection of the initial -40 dB/dec segment or its extension with the $\omega = 1$ line has the magnitude $20 \log K_a$.

- The frequency ω_2 at the intersection of the initial -40 dB/dec segment or its extension with the 0 dB line gives the square root of K_a numerically.

(8) This is because,

$$20 \log \left| \frac{K_a}{(j\omega_2)^2} \right| = 0\,dB = 20 \log 1$$

$$\left| \frac{K_a}{(j\omega_2)^2} \right| = 1$$

$$K_a = \omega_2^2$$

$$\boxed{\omega_2 = \sqrt{K_a}}$$