(4) Symmetry of the complete root loci

- The complete root loci are symmetrical wrt the real axis of the s-plane.
- In general the complete root loci are symmetrical wrt to the axes of symmetry of the poles and zeros of $G(s)H(s)$

(5) Asymptotes of the complete Root loci

(Behaviour of the Root Loci at $s=\infty$)

- For large values of s, the root loci (for $K \geq 0$) are asymptotic to straight lines or asymptotes with angles given by

$$\theta_l = \frac{(2l+1)\pi}{n-m}$$

Where $l = 0, 1, 2, \ldots\ldots |n-m|-1$

- For complementary root loci, $K \leq 0$ the angles of the asymptotes are

$$\theta_l = \frac{2l\pi}{n-m}$$

Where $l = 0, 1, 2, \ldots\ldots |n-m|-1$

(6) Intersection of asymptotes (Centroid)

- The intersection of the asymptotes of the complete root loci lies on the real axis of the s-plane.

$$\sigma_1 = \frac{\sum \text{real parts of poles of } G(s)H(s) - \sum \text{real parts of zeros of } G(s)H(s)}{n-m}$$

eg

$$s(s+2)(s+3) + K(s+1) = 0$$

$$1 + \frac{K(s+1)}{s(s+2)(s+3)} = 0$$

$$G(s)H(s) = \frac{K(s+1)}{s(s+2)(s+3)}$$

$$\sigma_1 = \frac{(0-2-3)-(-1)}{3-1} = \frac{-5+1}{2} = -\frac{4}{2}$$

$$\sigma_1 = -2$$

ie the asymptotes meet at -2

In other words the asymptotes can be drawn at -2

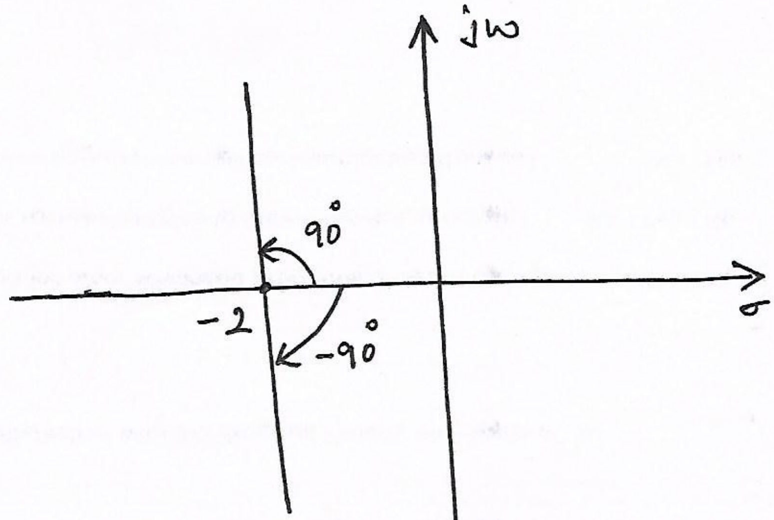
$$\theta_l = \frac{(2l+1)\pi}{n-m} \qquad l = 0, 1, \ldots \ldots |n-m|-1$$

$$|n-m|-1 = |3-1|-1 = 2-1 = 1$$

$$\therefore l = 0, 1$$

$$\theta_0 = \frac{\pi}{2} = 90^\circ$$

$$\theta_1 = \frac{3\pi}{2} = 270^\circ$$



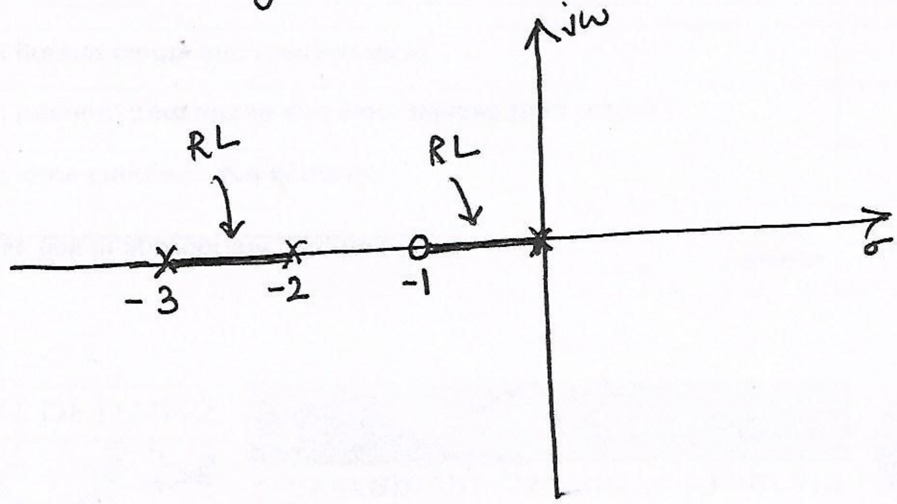
⑦ Root loci on the Real axis

(a) On a given section of the real axis, root loci ($K \geq 0$) may be found in the section only if the total number of real poles and zeros of $G(s)H(s)$ to the right of the section is ODD

(b) Complementary root loci

On a given section of the real axis, complementary root loci ($K \leq 0$) may be found in the section only if the total number of poles and zeros of $G(s)H(s)$ to the right of the section is EVEN

eg



⑧ **Angles of departure (from poles) and the angles of arrival (at zeros) of the complete root loci**

⊛ The angles of departure (arrival) of the complete root locus at a pole (zero) of $G(s)H(s)$ denotes the behaviour of the root loci near that pole (zero).

⊛ For the root loci ($K \geq 0$) these angles can be determined by the use of the equation

$$\sum_{i=1}^{m} \angle s+z_i - \sum_{j=1}^{n} \angle s+p_j = (2k+1)\pi$$

$$k = 0, 1, 2 \ldots\ldots$$

⊛ For complementary root loci ($K \leq 0$), these angles can be determined by the use of the equation

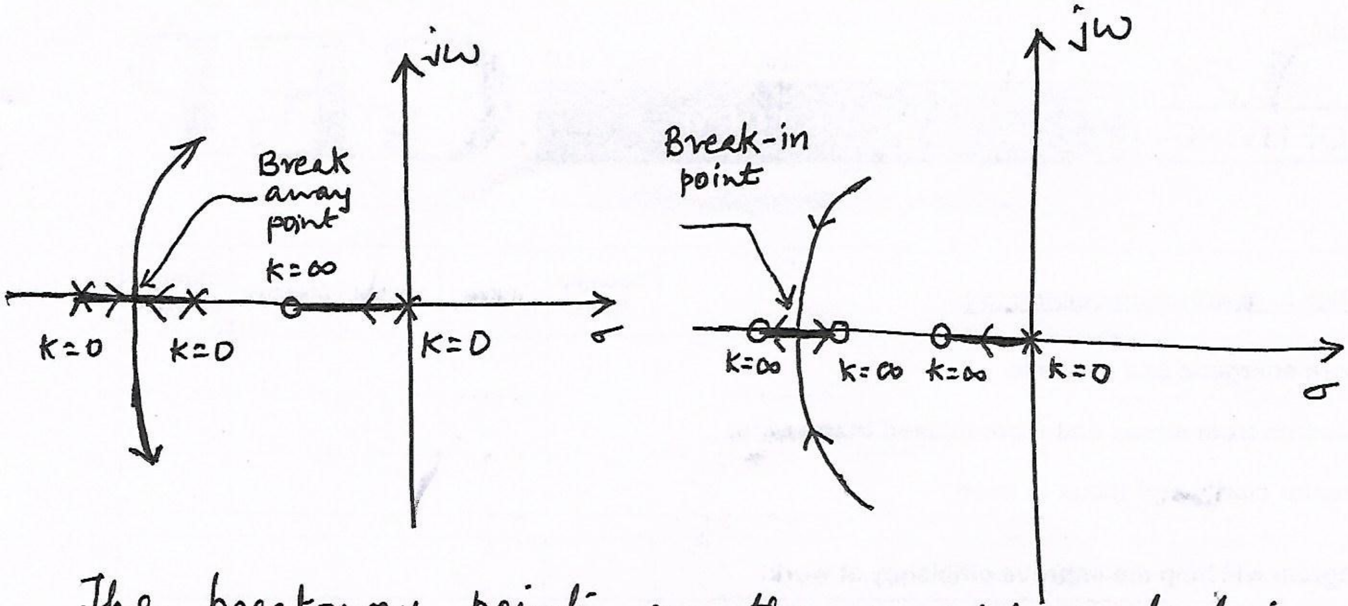
$$\sum_{i=1}^{m} \angle s+z_i - \sum_{j=1}^{n} \angle s+p_j = 2k\pi$$

$$k = 0, 1, 2 \ldots\ldots$$

⑨ **Intersection of the Root loci with the imaginary axis**

The points where the complete root loci intersect the imaginary axis of the s-plane and the corresponding values of K may be determined by means of the Routh's Criterion.

(10) Breakaway points of the root loci



The breakaway points on the complete root loci may be found from

$$\frac{d}{ds}\left[G_1(s)H_1(s)\right] = 0$$