* The roots of the equation (Characteristic)

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

are
$$s_1 = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$s_2 = \frac{-R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

* For the underdamped case,

$$s_1, s_2 = \frac{-R}{2L} \pm j\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

* Thus for the underdamped case the roots are complex conjugate in nature.

* For the critically damped case and the overdamped case, the roots are real.

* The characteristic equation of the standard second order system can be written as

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

* For the underdamped case, the roots are

$$-\zeta\omega_n \pm j\omega_d \quad , \quad \omega_d = \omega_n\sqrt{1-\zeta^2}$$

* For the critically damped case, the roots are

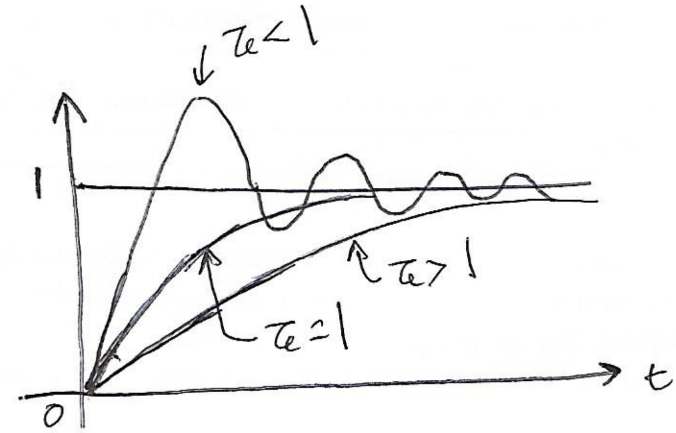
$$-\zeta\omega_n$$

* For the overdamped case, the roots are

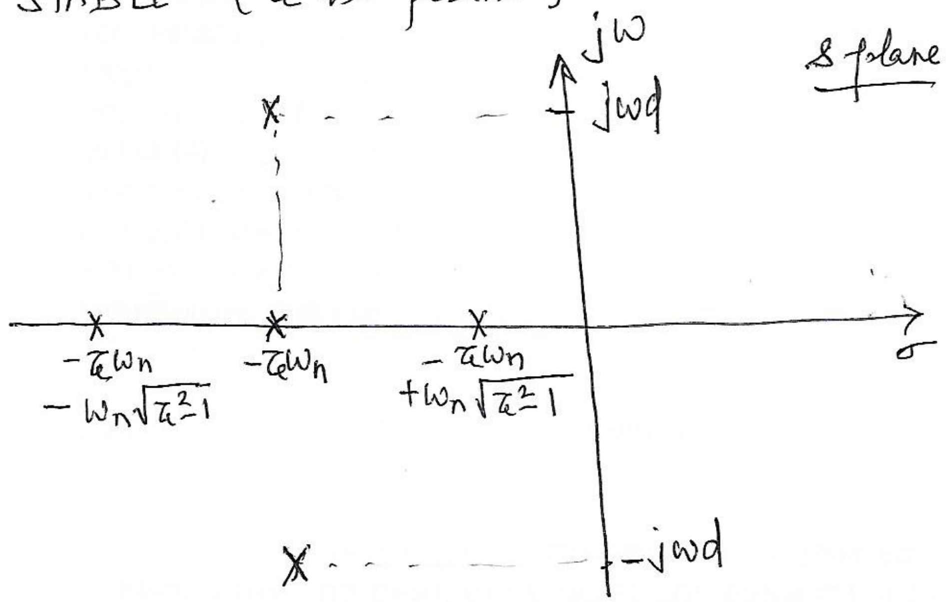
$$-\zeta\omega_n \pm \omega_n\sqrt{\zeta^2-1}$$

* For all the three cases of damping, the value of ζ is positive (positive damping) and the roots have negative real parts.

* It is also seen that the response for all the three cases of damping gives a STABLE output.



* Thus we can conclude that if the roots of the characteristic equation have negative real parts ie they lie in the left-half of the s-plane (LHSP), the system will be STABLE (ζ is positive)



* When the system is undamped the roots are

$\pm j\omega_n$

* That is the roots lie on the $j\omega$ axis.

* The system breaks into oscillations and the nature of the output is that of sustained oscillations at a frequency of ω_n.

* The system is said to be limitedly stable or marginally stable.

* Now imagine if a system has negative damping (opposite of what damping would do)

* The roots lie in the right half of s-plane (RHSP)

$$\zeta\omega_n \pm j\omega_d \quad , \quad \zeta\omega_n, \quad \zeta\omega_n \pm \sqrt{\zeta^2 - 1}$$

* Since the damping is negative, the oscillations will increase with time

* The system will be ~~unstable~~ unstable

THE ROOTS OF THE CHARACTERISTIC EQUATION ARE ALSO THE POLES OF THE SYSTEM

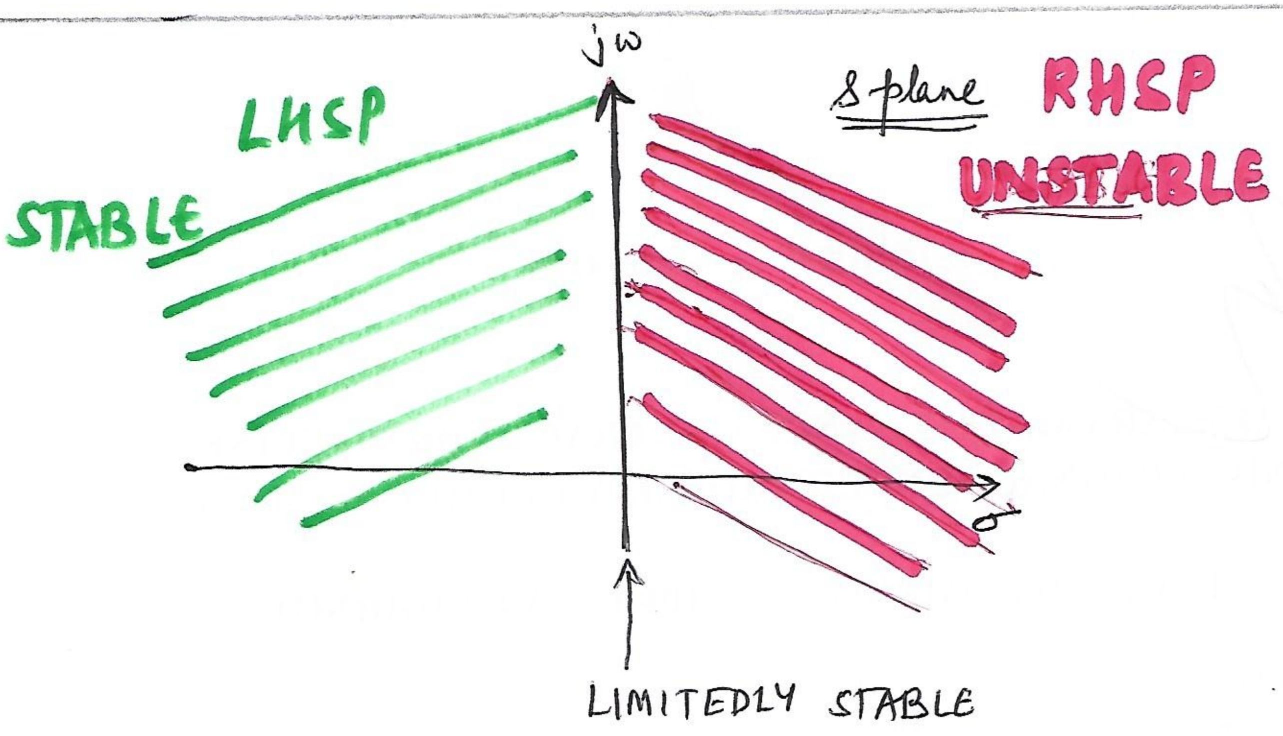
~~(*) Thus in conclusion we may say~~

(*) Thus it can be concluded that,

→ if the characteristic equation roots lie in the LHSP, the system is STABLE

→ if the characteristic equation roots (poles) lie on the jw axis, the system is LIMITEDLY STABLE

→ if the characteristic equation roots (poles) lie in the RHSP, the system is UNSTABLE



ROUTH'S STABILITY CRITERION

- A control system is stable if and only if all closed-loop poles lie in the LHSP.
- Since most linear control systems have closed-loop transfer function of the form

$$\frac{C(s)}{R(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_{m-1} s + b_m}{a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n} = \frac{B(s)}{A(s)}$$

where a's and b's are constants, it is necessary to factorize the polynomial $A(s)$ in order to find the closed-loop poles.

- This process may be very time consuming for a polynomial of degree greater than two.
- A simple criterion known as <u>Routh's Stability Criterion</u> enables us to determine the number of closed loop poles which lie in the RHSP without having to factorize the polynomial.
- The stability criterion applies to polynomials with only a finite number of terms.
- When the criterion is applied to a control system, information about stability can be obtained directly from the coefficients of the characteristic Equation.

PROCEDURE

(1) Write the polynomial in s in the following form:

$$a_0 s^n + a_1 s^{n-1} + \cdots\cdots + a_{n-1} s + a_n = 0$$

Where the coefficients are real quantities.

It is assumed that $a_n \neq 0$.

(2) For the roots to be in the left half of s-plane all the coefficients must be positive.

(3) If all the coefficients are positive, arrange the coefficients of the polynomial in rows and columns according to the following pattern.

s^n	a_0	a_2	a_4	a_6
s^{n-1}	a_1	a_3	a_5	a_7
s^{n-2}	b_1	b_2	b_3	b_4
s^{n-3}	c_1	c_2	c_3	c_4
$\vdots$	$\vdots$			
s^2	$\vdots$			
s^1	d_1			
s^0	e_1			

The coefficients b_1, b_2, b_3 etc are evaluated as follows:

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1}$$

$$b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1}$$

$$b_3 = \frac{a_1 a_6 - a_0 a_7}{a_1}$$

The evaluation of b's is continued until the remaining ones are all zero.

- The same pattern of cross multiplying the coefficients of the two previous rows is followed in evaluating c's, d's, e's etc.

$$C_1 = \frac{b_1 a_3 - a_1 b_2}{b_1}$$

$$C_2 = \frac{b_1 a_5 - a_1 b_3}{b_1}$$

$$d_1 = \frac{c_1 b_2 - b_1 c_2}{c_1}$$

- The process is continued until the nth row has been completed.
- The complete array of coefficients is triangular
- The array thus formed is called the

<u>ROUTH'S ARRAY</u>