Pb 22

SOLUTION

$$s^4+8s^3+24s^2+32s+K=0$$

s^4	1	24	K
s^3	8	32	
s^2	20	K	
s	$\frac{640-8K}{20}$		
s^0	K		

- For the polynomial to have roots with zero real part $\frac{640-8K}{20}=0$ ie $8K=640$ ie $\boxed{K=80}$
- The sign of the coefficient above the zero and below the zero term is the same.
- Hence there are a pair of imaginary roots.
- The location of these roots can be found from the auxiliary equation

$$20s^2+K=0$$

$$20s^2+80=0$$

$$s^2=-4$$

$$\boxed{s=\pm j2}$$

Pb (23) The open loop transfer function of a unity feedback control system is $G(s) = \dfrac{K}{(s+2)(s+4)(s^2+6s+25)}$.

By applying Routh's criterion, discuss the stability of the closed loop system as a function of K. Determine the value of K which will cause sustained oscillations in the closed-loop system. What are the oscillation frequencies?

Ans Range of K for stability $-200 < K < 666.25$

$K = 666.25$ will cause sustained oscillations

$s = \pm j4.06$, oscillation frequency is 4.06 r/s

Pb (23) SOLUTION

- Characteristic eqn is $1 + G(s)H(s) = 0$
- Given system has unity feedback
- $1 + G(s) = 0$

$$1 + \frac{K}{(s+2)(s+4)(s^2+6s+25)} = 0$$

- characteristic Eqn is

$$s^4 + 12s^3 + 69s^2 + 198s + 200 + K = 0$$

s^4	1	69	$200+K$
s^3	12	198	
s^2	52.5	$200+K$	
s^1	$\frac{7995-12K}{52.5}$		
s^0	$200+K$		

- $\frac{7995-12K}{52.5} > 0$ ie $7995 > 12K$ ie $K < 666.25$

- $200+K > 0$ ie $K > -200$

- Range of K for stability is

$$\boxed{-200 < K < 666.25}$$

- The value of K which will cause sustained ~~osc~~ oscillations is obtained from

$$\frac{7995-12K}{52.5} = 0$$

$$7995 = 12K \quad \text{ie} \quad \boxed{K = 666.25}$$

- The location of imaginary roots can be found from

$$52.5s^2 + 200+K = 0$$

$$52.5s^2 + 200 + 666.25 = 0$$

$$52.5s^2 = -866.25$$

$$s^2 = -16.5$$

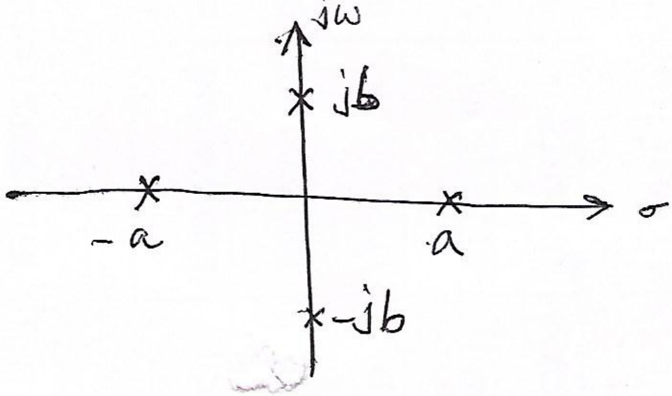
$$s = \pm j\,4.06$$

The oscillation frequency is 4.06 rad/sec

Special Case (2)

- If all the coefficients in any derived row are zero, it indicates that there are roots of equal magnitude lying radially opposite in the s-plane.

ie two real roots with equal magnitude and opposite signs and/or two conjugate imaginary roots.



- In such a case the evaluation of the rest of the array can be continued by forming an auxiliary polynomial with the coefficients of the previous row and by using the coefficients of the derivative of this polynomial in the next row

Eg $$s^5 + 2s^4 + 24s^3 + 48s^2 + 25s + 50 = 0$$

The array of coefficients is

s^5	1	24	25
$\longrightarrow s^4$	2	48	50
s^3	0	0	0

- The term in the s^3 row are all zero.

(*) An auxilliary polynomial is then formed from the coefficients of the s^4 row

$$A(s) = 2s^4 + 48s^2 + 50$$

$$\frac{dA(s)}{ds} = 8s^3 + 96s$$

(*) The term in the s^3 row are replaced by the coefficients of the previous equation.

	s^5	1	24	25
	s^4	2	48	$\cancel{\ }$50
Coeff. of $\frac{dA(s)}{ds}$ $\longrightarrow$	s^3	0 (8)	0 (96)	
	s^2	24	50	
	s^1	79.33		
	s^0	50		

(*) If there are no sign changes in the first column of the new array, the system is LIMITEDLY STABLE.

(*) If there are sign changes in the first column of the new array, the system is UNSTABLE

Pb 24

The characteristic equation of a system is $s^5 + 2s^4 + 4s^3 + 8s^2 + 5s + 10 = 0$

Check whether the system is stable, unstable or limitedly stable.

Pb 24

SOLUTION

$$s^5 + 2s^4 + 4s^3 + 8s^2 + 5s + 10 = 0$$

s^5	1	4	5
$\longrightarrow s^4$	2 (1)	8 (4)	10 (5)
s^3	0 (4)	0 (8)	
s^2	2	5	
s	-2		
s^0	5		

$$A(s) = s^4 + 4s^2 + 5$$

$$\frac{dA(s)}{ds} = \underline{4}s^3 + \underline{8}s$$

Two sign changes, UNSTABLE

Pb (25) Discuss the stability of the system whose Characteristic equation is given by

$$s^6 + 3s^5 + 5s^4 + 9s^3 + 8s^2 + 6s + 4 = 0$$

Pb (25)

SOLUTION

$$\begin{array}{l|llll}
s^6 & 1 & 5 & 8 & 4 \\
s^5 & 3 & 9 & 6 & \\
\rightarrow s^4 & 2 & 6 & 4 & \\
s^3 & 0\,(8) & 0\,(12) & & \\
s^2 & 3 & 4 & & \\
s^1 & 4/3 & & & \\
s^0 & 4 & & &
\end{array}$$

$$A(s) = 2s^4 + 6s^2 + 4$$

$$\frac{dA(s)}{ds} = 8s^3 + 12s$$

There are no sign changes in the first column of the array. Hence the system is LIMITEDLY STABLE