

DCS Tutorial -II

01] Find the minimal polynomial of each elements from GF(16)

-> Conjugacy classes of GF(16) are

$$\begin{cases} \times, \times^{2}, \times^{4}, \times^{8} \end{cases} \qquad 0 \Rightarrow \times$$

$$\begin{cases} \times^{3}, \times^{6}, \times^{4}, \times^{12} \end{cases} \qquad 1 \Rightarrow \times + 1$$

$$\begin{cases} \times, \times^{3}, \times^{6}, \times^{13}, \times^{14} \end{cases}$$

$$\begin{cases} \times, \times^{2}, \times^{4}, \times^{8} \end{cases} \qquad 0 \Rightarrow \times$$

: Minimal polynomial for;

$$\lambda^{3} \Rightarrow P(x) = (X - \lambda^{3})(x - \lambda^{1})(x - \lambda^{9})(x - \lambda^{12})$$

$$= x^{4} + \lambda^{21} x^{2} - (\lambda^{9} + \lambda^{11}) x^{3} + \lambda^{9} x^{2} + \lambda^{30}$$

$$- (\lambda^{18} + \lambda^{21}) x - (\lambda^{3} + \lambda^{1}) x^{3}$$

$$- (\lambda^{24} + \lambda^{27}) x + (\lambda^{3} + \lambda^{6})(\lambda^{9} + \lambda^{12}) x^{2}$$

$$= x^{4} + x^{3} + x^{2} + x + 1$$

$x^{\frac{1}{2}} + f(x) = (x-a^{\frac{3}{2}})(x-a^{\frac{11}{2}})(x-a^{\frac{14}{2}})$ $= (x^{2} - (x^{3} + a^{\frac{11}{2}}) \times + a^{\frac{23}{2}})$ $= x^{4} - (x^{13} + a^{\frac{14}{2}}) \times \frac{18}{2} \times \frac{23}{2} \times 2 - (x^{\frac{3}{2}} + a^{\frac{11}{2}}) \times \frac{3}{2}$ $+ (x^{13} + a^{\frac{14}{2}})(x^{\frac{3}{2}} + a^{\frac{14}{2}}) \times \frac{3}{2} \times $		Page No.
$ (x^{2} - (x^{13} + x^{11}) \times + a^{23}) $ $ (x^{2} - (x^{13} + x^{11}) \times + a^{23}) $ $ = x^{4} - (x^{13} + x^{11}) (x^{3} + a^{23} + x^{2}) \times (x^{4} + x^{11}) \times x^{3} $ $ + (x^{15} + a^{11}) (x^{2} + a^{11}) \times x^{2} $ $ - (x^{13} + x^{11}) \times x^{2} + a^{14} \times x^{2} $ $ - (x^{15} + x^{11}) \times x^{4} \times x^{2} $ $ - x^{15} (x^{15} + a^{11}) \times x^{4} \times x^{4} $ $ = x^{4} + x^{3} + 1 $ $ 02. $ Factorize the polynomial $x^{15} + 1$ over $x^{15} + 1$		
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$= \frac{x^4 - (x^{13} + x^{14}) x^3 + x^{72} x^2 - (x^2 + x^{11}) x^2}{+ (x^{13} + x^{14}) (x^2 + x^{11}) x}$ $+ (x^{13} + x^{14}) (x^2 + x^{11}) x$ $- (x^3 + x^{11}) x^2 + x^{14} x^2$ $- x^{15} (x^{15} + x^{14}) x 4x^{45}$ $= x^4 + x^3 + 1$ Q2: Factorize the polynomial $x^{15} + 1$ over GF(2) using Q: $\Rightarrow \text{Using the previous question, use factorize}$ $x^{15} + 1 \text{ as } x^{16} + x$ x as we found out the minimal polynomial of GF(16) in the previous question we multiply them to get $(x^{15} + 1) = (x^{16} + x)$ x $- x(x + 1)(x^4 + x + 1)(x^4 + x + 1)(x^4 + x^3 + 1)$ $x(x^4 + x^3 + x^3 + x^4 + x^4 + 1)$ $(x^{15} + 1) = (x + 1)(x^2 + x + 1)(x^4 + x + 1)(x^4 + x^3 + 1)(x^4 + x^3 + 1)$ $(x^{15} + 1) = (x + 1)(x^2 + x + 1)(x^4 + x + 1)(x^4 + x^3 + 1)(x^4 + x^3 + 1)$		$= \left(\begin{array}{c} X^2 - \left(\frac{1}{2} + \frac{1}{2} \right) X + \frac{1}{2} \end{array} \right)$
$ + (x^{15} + x^{14})(x^{2} + x^{11})x^{2} + x^{16}x^{2} $ $ - (x^{3} + x^{11})x^{2} + x^{16}x^{2} $ $ - x^{16}(x^{13} + x^{14})x^{4}x^{5} $ $ = x^{4} + x^{3} + 1 $ $ = x^{16} + x $ $ = x^{15} + 1 $ $ = x^{16} + x $ $ = x^{16} + x^{16} + x $ $ = x^{16} + x^{16}$		$\frac{\left(\chi^{2}-\left(\chi^{1}+\chi^{11}\right)\chi+\alpha\right)}{22}$
$-(x^{3}+x^{3})^{2} + x^{16} \times 2$ $-x^{15}(x^{13}+x^{14}) \times 4x^{45}$ $= x^{4}+x^{3}+1$ 22] Factorize the polynomial $x^{15}+1$ over $GF(2)$ using Q_{1} $- y Using the pravious question, we factorize$ $- x^{15}+1 \text{ as } x^{16}+x$ $- x $ as we found out the minimal polynomial of $GF(16)$ in the previous question we multiply them to get $- (x^{15}+1) = (x^{16}+x)$ $- x^{16}+x$ $- x^{16}(x^{1}+x)$ $- x^{16}+x$ $- x$		$= x^4 - (x^{13} + x^{14}) x^3 + x^{27} x^2 - (x^2 + x^2) x$
$= \chi^{4} + \chi^{3} + 1$ $= \chi^{15} + 1 \text{ over } GF(1) \text{ using } Q1$ $\longrightarrow \text{ Using the previous question, we factorize}$ $\chi^{15} + 1 \text{ as } \chi^{16} + \chi$ χ $= \chi^{16} + \chi$ $= $		+ (x13+x14)(x7+x1)X
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O2:] Factorize the polynomial $X^{15}+1$ over GF(1) using Q3 -> Using the previous question, we factorize $X^{15}+1 \text{ as } X^{16}+X$ X as we found out the minimal polynomial of GF(16) in the previous question we multiply them to get $(X^{15}+1) = (X^{16}+X)$ X $= X(X+1)(X^{1}+X+1)(X^{4}+X+1)(X^{4}+X^{5}+1)$ $X(X^{15}+1) = (X+1)(X^{2}+X+1)(X^{4}+X+1)(X^{4}+X^{5}+1)(X^{4}+X^{5}+X^{4}+X^{4}+X^{4}+X^{5}+X^{4}+X^$		
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$\frac{\chi(\chi^{4} + \chi^{3} + \chi^{2} + \chi_{H})}{\chi}$ $= (\chi^{15} + 1) = (\chi^{+1})(\chi^{2} + \chi_{H})(\chi^{4} + \chi_{H})(\chi^{4} + \chi_{H}^{3} + \chi_{H}^{2} + \chi_{H}^{4})$		X
$\frac{\chi(\chi^{4} + \chi^{3} + \chi^{2} + \chi_{H})}{\chi}$ $= (\chi^{15} + 1) = (\chi^{+1})(\chi^{2} + \chi_{H})(\chi^{4} + \chi_{H})(\chi^{4} + \chi_{H}^{3} + \chi_{H}^{2} + \chi_{H}^{4})$		$- \times (x+1)(x^{2}+x+1)(x^{4}+x+1)(x^{4}+x^{3}+1)$
$(x^{15}+1) = (x+1)(x^{2}+x+1)(x^{4}+x+1)(x^{4}+x^{3}+1)(x^{4}+x^{3}+x^{2}+x+1)$		$\times (X^{4} + X^{3} + X^{2} + X + X)$
		(V1541) - GUIN (V2+V+1) (V4+ X+1) (X4+ X3+1) (X4+X3+X7X41)
		[2] 그렇게 되었다. 그런 그렇고 얼마나 하게 얼마가 되었다. 나는 나는 나는 나는 나는 나는 나는 나는 나는 그를 다 되었다. 그런 그를 다 나는 나는 그를 다 살았다.

	Page No
Q4]	Find parity check polynomial using Q2.
->	$b(x) = (x^4 + x^3 + 1)(x^4 + x^3 + x^2 + x + 1)$
	$= x^{8} + x^{4} + x^{6} + x^{5} + x^{4} + x^{5} + x^{6} + x^{5} = x^{4} + x^{5} + x^$
	$+ \chi^3 + \chi^4 + \chi^3 + \chi^2 + \chi + \chi$
	$= X^8 + X^4 + X^2 + X + 1$