Pb (A1) Sketch the Nyquist ~~path~~ plot of

$$G(s)H(s) = \frac{1}{s+1}$$

and determine the stability of the closed loop transfer function.

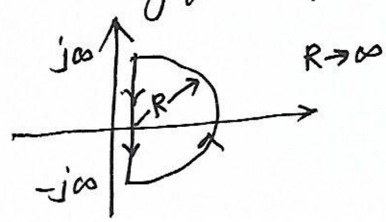
Solution Step (1)

Find the number of open loop poles in RHSP.

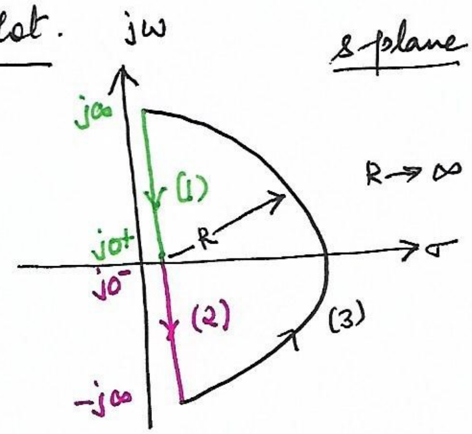
$P = 0$

Step (2)

Choose the Nyquist path



Step (3) Sketch the Nyquist plot.



(a) Section (1) $s = j\omega$, ω varies from ∞ to 0

$$G(j\omega)H(j\omega) = \frac{1}{j\omega+1}$$

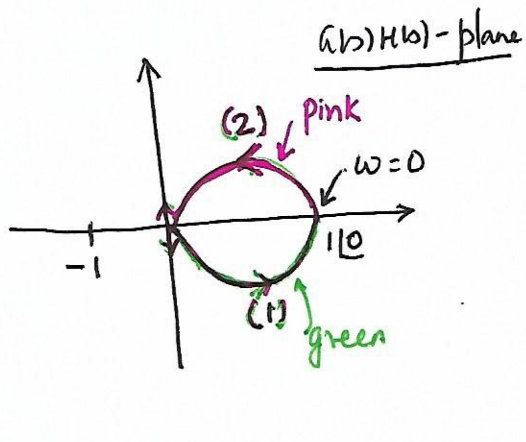
$\omega \to \infty$, $0\angle{-90}$

$\omega = 0$, $1\angle 0$ $\left(0\angle{-90} \text{ to } 1\angle 0\right)$

(b) Section (2) $s = -j\omega$

Mirror image of Section (1)

$1\angle 0$ to $0\angle 90$



(C) Section (3)

$s = Re^{j\theta}$, θ varies from -90 to 90

$R \to \infty$

$$G(s)H(s) = \frac{1}{Re^{j\theta}+1} \simeq \frac{1}{Re^{j\theta}}$$

$$= 0e^{-j\theta}$$

$$= 0\angle 90 \text{ to } 0\angle -90$$

$N = 0$

$N = Z - P$

$N = Z - 0$

$\therefore Z = 0$

The closed loop system will be stable

Pb 42 For the system given by OLTF

$$G(s)H(s) = \frac{52}{(s+2)(s^2+2s+5)},$$

determine closed loop stability.

Solution

$$G(s)H(s) = \frac{52}{(s+2)(s^2+2s+5)}$$

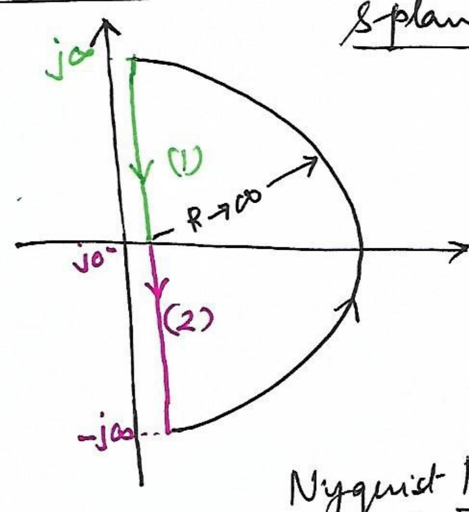
Poles are at $-2,\ -1+j2,\ -1-j2$

Step 1 $P=0$

Since there are no poles in RHSP

Step 2 Choose the Nyquist path.

Since there are no poles on the imaginary axis or at the origin, the Nyquist path is chosen as shown.



Nyquist Path

Step 3 Sketch the Nyquist Plot

Section (1) $s = j\omega$, ω varies from ∞ to 0

$$G(j\omega)H(j\omega) = \frac{52}{(j\omega+2)((j\omega)^2+2j\omega+5)}$$

$\omega \to \infty$, $0\angle{-270}$

$\omega = 0$, $5.2\angle{0}$ $\qquad 0\angle{-270}$ to $5.2\angle{0}$

Section (2) $s = -j\omega$ Mirror image of section (1)

$5.2\angle 0$ to $0\angle 270$

Section (3) $s = Re^{j\theta}$, $R \to \infty$, θ varies from -90 to 90

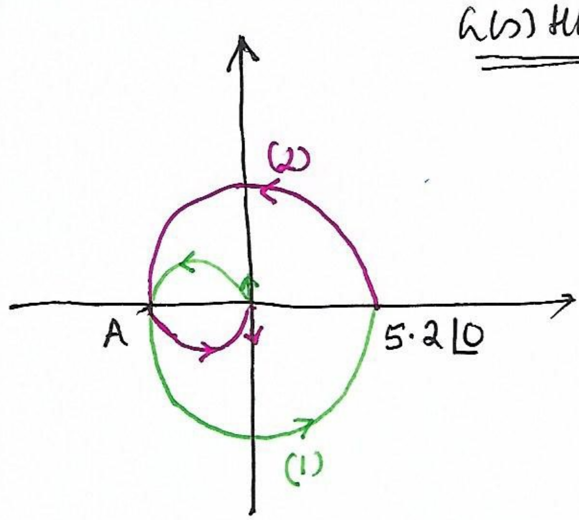
$$G(s)H(s) = \frac{52}{(Re^{j\theta} + \cancel{2})(R^2 e^{j2\theta} + \cancel{2Re^{j\theta}} + \cancel{5})}$$

$$= \frac{52}{R^3 e^{j3\theta}} \simeq 0e^{-j3\theta}$$

$0\angle 270$ to $0\angle -270$

Which implies that the Nyquist plot for section (3) is described by a phasor with infinitesimally small magnitude which rotates around the origin from $270°$ to $-270°$ in a clockwise direction.

$G(s)H(s)$ - plane



The co-ordinates of point A can be found as follows:

$$G(j\omega)H(j\omega) = \frac{52}{(j\omega+2)\left[(j\omega)^2 + 2(j\omega) + 5\right]}$$

$$G(j\omega)H(j\omega) = \frac{52}{(10-4\omega^2)+j(9\omega-\omega^3)}$$

Rationalize

$$G(j\omega)H(j\omega) = \frac{52\left[(10-4\omega^2)-j(9\omega-\omega^3)\right]}{(10-4\omega^2)^2+(9\omega-\omega^3)^2}$$

At point A, imaginary part of $G(j\omega)H(j\omega) = 0$

$$\frac{-(9\omega-\omega^3)}{(10-4\omega^2)^2+(9\omega-\omega^3)^2} = 0$$

$$-(9\omega-\omega^3) = 0$$

$$\omega = \pm 3 \text{ rad/sec}$$

$$G(j3)H(j\omega) = \frac{52[10-4\times 9]}{[10-4\times 9]^2}$$

$$= \frac{52[10-36]}{[10-36]^2}$$

$$= -2$$

The coordinates of point A are $(-2, 0)$.
Thus the $G(s)H(s)$ plot encircles $(-1, j0)$ point twice in the counterclockwise direction.

$$N = 2$$
$$P = 0$$
$$\therefore Z = 2$$

There are two zeros of $1+G(s)H(s)$ in the RHSP

Hence the closed loop system is unstable.