FREQUENCY RESPONSE PLOTS

* The sinusoidal transfer function, a complex function of the frequency ω, is characterized by its magnitude and phase angle, with frequency as the variable parameter.

* There are three commonly used representations of the sinusoidal transfer function.

* They are
  1. Logarithmic plot or Bode plots
  2. Polar plot
  3. Log-magnitude versus phase plot

1. LOGARITHMIC PLOT OR BODE PLOT

* A logarithmic plot or Bode diagram consists of two graphs, one is a plot of the logarithm of the magnitude of a sinusoidal transfer function, the other is a plot of the phase angle, both are plotted against frequency in logarithmic scale.

* The standard representation of the logarithmic magnitude of $G(j\omega)$ is $20 \log |G(j\omega)|$

(*) Bode plot has the following unique characteristics:

→ Since the magnitude of $G(j\omega)$ in the Bode plot is expressed in decibels the product and division factors in $G(j\omega)$ become additions and subtractions respectively.
The phase relations are also added and subtracted from each other in a natural way.

→ The magnitude plot of the Bode plots of most functions encountered in control systems may be approximated by straight line segments (Asymptotic approximations)

→ This makes the construction of the Bode plot very simple.

(*) Consider the following transfer function to illustrate the construction of the Bode plot:

$$G(s)H(s) = \frac{K(1+T_1 s)(1+T_2 s)\,\omega_n^2}{s(1+T_a s)(s^2+2\zeta\omega_n s+\omega_n^2)}$$

(*) The magnitude of $G(j\omega)H(j\omega)$ in decibels is obtained by multiplying the logarithm to the base 10 of $|G(j\omega)H(j\omega)|$ by 20.

(*) $$G(j\omega)H(j\omega) = \frac{K(1+j\omega T_1)(1+j\omega T_2)\,\omega_n^2}{j\omega(1+j\omega T_a)\left[(j\omega)^2+2\zeta\omega_n j\omega+\omega_n^2\right]}$$

$$= \frac{K(1+j\omega T_1)(1+j\omega T_2)}{j\omega(1+j\omega T_a)\left[1+j2\zeta\frac{\omega}{\omega_n}+\left(\frac{j\omega}{\omega_n}\right)^2\right]}$$

* $|G(j\omega)H(j\omega)|\,dB = 20\log|G(j\omega)H(j\omega)|$

$$= 20\log K + 20\log|1+j\omega T_1| + 20\log|1+j\omega T_2|$$
$$- 20\log|j\omega| - 20\log|1+j\omega T_a|$$
$$- 20\log|1-u^2+j2\zeta u|$$

* The phase of $G(j\omega)H(j\omega)$ is written as

$$\angle G(j\omega)H(j\omega) = \angle K + \angle 1+j\omega T_1 + \angle 1+j\omega T_2 - \angle j\omega$$
$$- \angle 1+j\omega T_a - \angle 1+j2\zeta u-u^2$$

* Thus in general ~~f~~ $G(j\omega)H(j\omega)$ may contain just four types of factors:

1. Constant factor K
2. Poles or zeros at origin $(j\omega)^{\pm 1}$
3. Simple poles or zeros $(1+j\omega T)^{\pm 1}$
4. Complex poles or zeros $(1+j2\zeta u-u^2)^{\pm 1}$

* The Bode plot of each of the four types of factors listed may be considered as a separate plot, the individual plots are then added or subtracted accordingly to yield the total magnitude in decibels and the phase plot of $|G(j\omega)H(j\omega)|$ may be obtained by adding phase angle of individual factors.

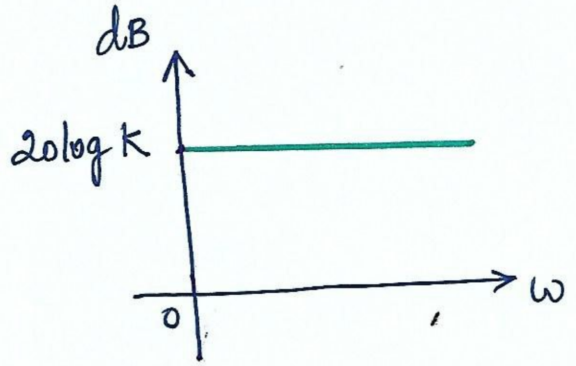
① Constant term K

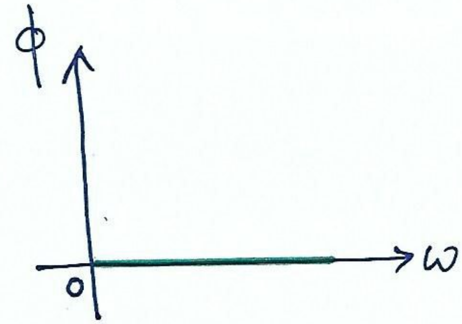
- A number greater than unity has a positive value in decibels while a number smaller than unity has a negative value.
- The log magnitude curve for a constant gain K is a horizontal straight line at the magnitude of $20\log K$ dB.
- The phase angle of the gain K is zero.
- The effect of varying K in the transfer function is that it raises or lowers the logmagnitude curve of the transfer function by the corresponding constant amount.

  But it has no effect on the phase angle.
- When expressed in dB, the reciprocal of a number differs from its value only in sign.

  ie $\quad 20\log K = -20\log \frac{1}{K}$

- The Bode plot of K is shown below:





② Pole at the origin $\left(\frac{1}{j\omega}\right)$

→ The log magnitude of $\frac{1}{j\omega}$ in dB is

$$20\log\left|\frac{1}{j\omega}\right| = -20\log\omega \text{ dB}$$

→ The phase angle of $\frac{1}{j\omega}$ is a constant and equal to -90°

→ In logarithmic plots, frequency ratios are expressed ~~as~~ in terms of octaves or decades.

→ An octave is a frequency band from ω_1 to $2\omega_1$, where ω_1 is any frequency value.

→ A decade is a frequency band from ω_1 to $10\omega_1$

→ If the log magnitude of $-20\log\omega$ dB is plotted against ω on a logarithmic scale it is a straight line.

→ At ω, $LM = -20\log\omega$ dB

At 10ω,
$$\begin{aligned} LM &= -20\log 10\omega \\ &= -20[\log 10 + \log\omega] \\ &= -20\log 10 - 20\log\omega \\ &= -20 - 20\log\omega \text{ dB} \end{aligned}$$

→ The dB magnitude has dropped by 20dB

→ At 100ω, $LM = -20 \log 100\omega$ dB

$$= -20[\log 100 + \log \omega]$$

$$= -20[2 + \log \omega]$$

$$= -40 - 20 \log \omega \text{ dB}$$

→ The dB magnitude has dropped by 40dB

→ In other words, the slope of the line is -20dB per decade, denoted as -20 dB/dec

→ If we consider an octave of frequency, ie a frequency band from ω to 2ω,

At ω, $LM = -20\log \omega$ dB

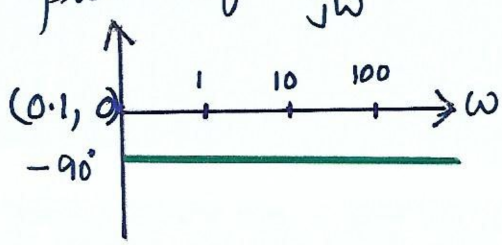
At 2ω, $LM = -20 \log 2\omega$

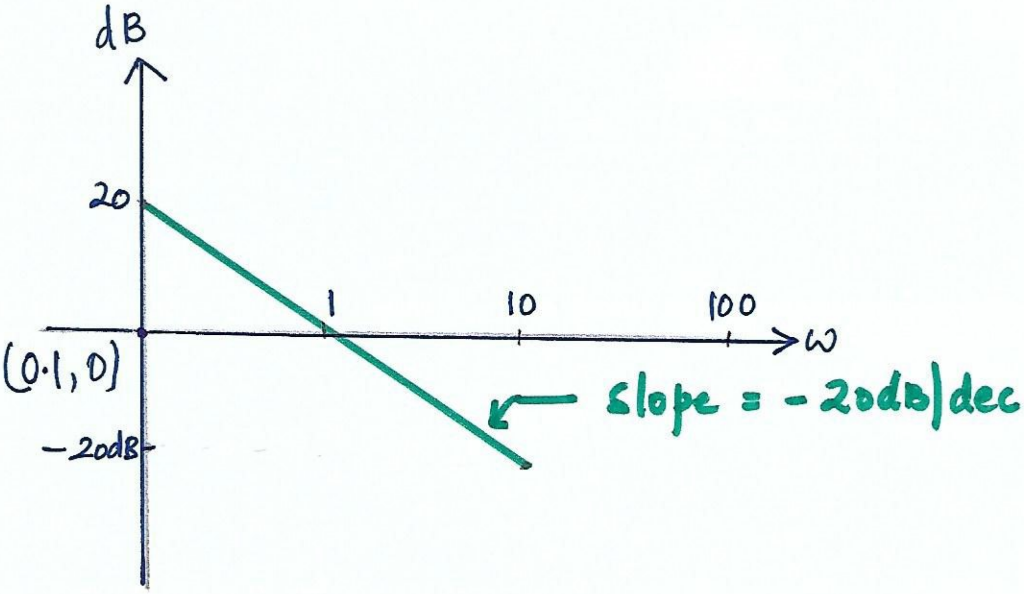
$$= -20 \log 2 - 20 \log \omega$$

$$= -6 - 20 \log \omega$$

→ The slope of the line is -6dB/octave

→ The phase plot of $\frac{1}{j\omega}$ is shown below




dB
20
1
10
100
ω
(0.1, 0)
-20dB
slope = -20dB/dec