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EWE - Tutorial III

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TY BTech EXTC

Q1:] We are given, for $\varepsilon = 9\varepsilon_0$ & $\mu = \mu_0$, there's a plane wave with,

$$H = 0.2\cos(10^9 t - Kx - K\sqrt{8}z)\, a_y \text{ A/m}$$

and is incident on an air boundary at $z=0$.

a) $\tan\theta_1 = \dfrac{K_{1x}}{K_{1z}} = \dfrac{1}{\sqrt{8}}$

$\therefore \boxed{\theta_1 = \theta_r = 19.47^\circ}$

$\sin\theta_t = \sin\theta_1 \sqrt{\dfrac{\varepsilon_{r_1}}{\varepsilon_{r_2}}} = \dfrac{1}{3}(3) = 1$

$\therefore \boxed{\theta_t = 90^\circ}$

b) $\beta_1 = \dfrac{\omega}{c}\sqrt{\varepsilon_{r_1}} = \dfrac{10^9}{3\times10^8} \times 3 = 10 = K\sqrt{1+8} = 3K$

$\therefore 3K = 10 \Rightarrow \boxed{K = 3.333}$

c) $\lambda = 2\pi/\beta$, $\lambda_1 = 2\pi/\beta_1 = 2\pi/10 = \underline{\underline{0.6283 \text{ m}}}$

$\beta_2 = \omega/c = 10/3$, $\lambda_2 = 2\pi/\beta_2 = 2\pi \times 3/10 = \underline{\underline{1.885 \text{ m}}}$

d) $E_1 = \eta_1 a_K \times H_1 = 40\pi \dfrac{(a_x + \sqrt{8}a_z)}{3} \times 0.2\cos(\omega t - Kr)\, a_y$

$= \underline{\underline{(-213.3 a_x + 75.4 a_z)\cos(10^9 t - Kx - K\sqrt{8}z) \text{ V/m}}}$

e) $\tau_{\parallel} = \dfrac{2\cos\theta_t \sin\theta_1}{\sin(\theta_1 + \theta_t)\cos(\theta_1 - \theta_t)} = \dfrac{2\cos 19.47^\circ \sin 90^\circ}{\sin 19.47^\circ \cos 19.47^\circ} = 6$

$\Gamma = \dfrac{-\cot 19.47^\circ}{\cot 19.47^\circ} = -1$

Let $E_t = -E_{to}\,(\cos\theta_t a_x - \sin\theta_t a_z)\cos(10^9 t - \beta_2 x\sin\theta_t$
$- \beta_2 z\cos\theta_t)$

where,

$E_t = -E_{to}\,(\cos\theta_t a_x - \sin\theta_t a_z)\cos(10^9 t - \beta_1 x\sin\theta_t - \beta_1 z\cos\theta_t)$

$\sin\theta_t = 1$, $\cos\theta_t = 0$, $\beta_2 \sin\theta_t = 10/3$

$E_{to}\sin\theta_t = \tau_{\parallel} E_{io} = 6(24\pi)(3)(1) = 1357.2$

Hence,

$$\boxed{E_t = 1357\cos(10^9 t - 3.333x)\,a_z \ \text{V/m}}$$

since, $\Gamma = -1$, $\theta_r = \theta_i$

$$\boxed{E_r = (213.3 a_x + 75.4 a_z)\cos(10^9 t - kx + k\sqrt{8}z)\ \text{V/m}}$$

f) $\tan\theta_{B\parallel} = \sqrt{\dfrac{\varepsilon_2}{\varepsilon_1}} = \sqrt{\dfrac{\varepsilon_0}{9\varepsilon_0}} = \dfrac{1}{3}$

$\therefore$ $\boxed{\text{Brewster's Angle, } \theta_{B\parallel} = 18.43^\circ}$

Q2:] We are given that, a uniform plane wave in a lossy nonmagnetic media has

$$E_s = (5a_x + 12a_y)e^{-\gamma z}, \quad \gamma = 0.2 + j3.4 \,/m$$

a:) $E = Re[E_s e^{j\omega t}] = (5a_x + 12a_y)e^{-0.2z} \cos(\omega t - 3.4z)$

At $z = 4m$, $t = T/8$, $\omega t = \frac{2\pi}{T} \cdot \frac{T}{8} = \frac{\pi}{4}$

$$E = (5a_x + 12a_y)e^{-0.8} \cos(\pi/4 - 13.6)$$

$$|E| = 13 \cdot e^{-0.8} |\cos(\pi/4 - 13.6)|$$

$\Rightarrow \boxed{|E| = 5.662}$

b.) $\text{Loss} = \alpha \Delta z = 0.2(3) = 0.6 \text{ Np}$ $\quad [1 \text{ Np} = 8.686 \text{ dB}]$

$\text{loss} = 0.6 \times 8.686 = \underline{5.212 \text{ dB}}$

c.) Let $x = \sqrt{1 + \left(\frac{\sigma}{\omega \varepsilon}\right)^2}$

$$\frac{\alpha}{\beta} = \left(\frac{x-1}{x+1}\right)^{1/2} = \frac{0.2}{3.4} = \frac{1}{17}$$

$\frac{x-1}{x+1} = \frac{1}{289} \Rightarrow \boxed{x = 1.00694}$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}}\sqrt{x-1} = \frac{\omega}{c}\sqrt{\varepsilon_r/2}\sqrt{x-1}$$

$$\sqrt{\frac{\varepsilon_r}{2}} = \frac{\alpha c}{\omega\sqrt{x-1}} = \frac{0.2 \times 3 \times 10^3}{10^8\sqrt{0.00694}} = 2.4$$

$$\Rightarrow \quad \varepsilon_r = (2.4)^2 \times 2$$

$$\Rightarrow \quad \boxed{\varepsilon_r = 11.52}$$

$$|\eta| = \frac{\sqrt{\frac{\mu_0}{\varepsilon_0} \cdot \frac{1}{\sqrt{\varepsilon_r}}}}{\sqrt{x}} = \frac{120\pi}{\sqrt{11.52 \times 1.00694}} = 32.5$$

$$\tan 2\theta_n = \frac{\sigma}{\omega\varepsilon} = \sqrt{x^2 - 1} = 0.118$$

$$\therefore \theta_n = 3.365^\circ$$

$$\eta = 32.5 \angle 3.365^\circ$$

$$H_s = \frac{a_k \times E_s}{\eta} = \frac{a_z \times (5a_x + 12a_y) \cdot e^{-\gamma z}}{\eta}$$

$$= \frac{(5a_x + 12a_y)}{|\eta|} \cdot e^{-j\,3.365^\circ} \cdot e^{-\gamma z}$$

$$H = (-369.2\,a_x + 153.8\,a_y)\, e^{-0.2z} \cos(\omega t - 3.4z - 3.365^\circ) \text{ mA}$$

$$P = E \times H = \begin{vmatrix} 5 & 12 & 0 \\ -369.2 & 153.8 & 0 \end{vmatrix} \times 10^{-3} e^{-0.4z} \cos(\omega t - 3.4z) \cdot \cos(\omega t - 3.4z - 3.365^\circ)$$

$$P = 5.2\, e^{-0.4z} \cos(\omega t - 3.4z) \cos(\omega t - 3.4z - 3.365^\circ)\, a_z$$

At $z = 4$, $t = T/4$

$$P = 5.2\, e^{-1.6} \cos(\pi/4 - 13.6) \cos(\pi/4 - 13.6 - 0.0587)\, a_z$$

$$\Rightarrow \quad \boxed{P = 0.9702\, a_z \ \text{W/m}^2}$$