Pb 33

Sketch the root locus for the system given by

$$G(s)H(s) = \frac{K(s^2-2s+5)}{(s^2+1.5s-1)}$$

Solution

$s^2-2s+5=0$, $\quad s = +1 \pm j2$

$s^2+1.5s-1=0$, $\quad s=0.5$, $s=-2$

$$G(s)H(s) = \frac{K(s+j2-1)(s-j2-1)}{(s-0.5)(s+2)}$$

(1) $K=0$ points are at $s=0.5$, $s=-2$

(2) $K=\infty$ points are at $s=1+j2$, $s=1-j2$

(3) Number of branches of the root loci = 2

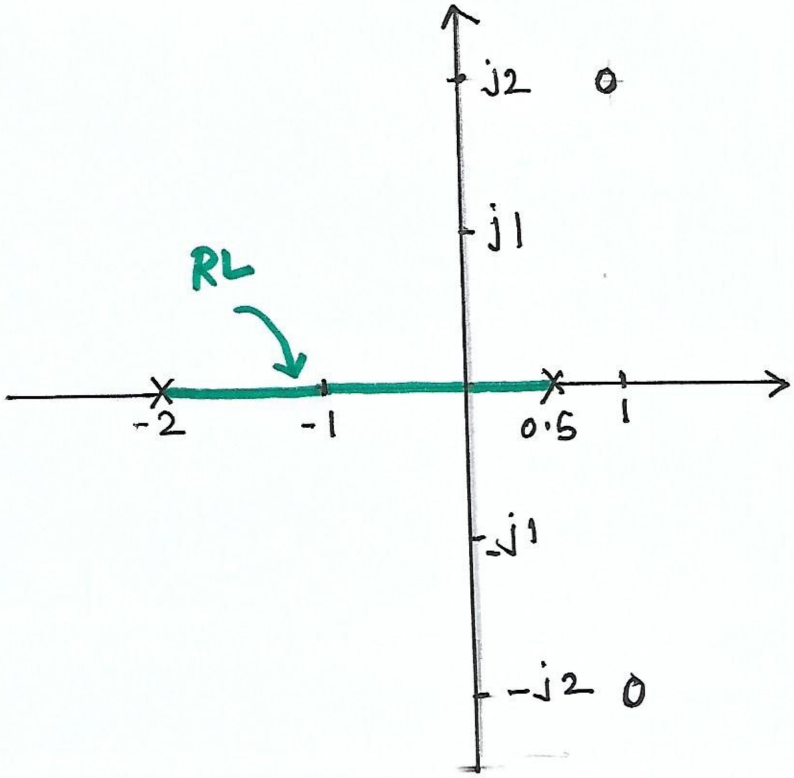
(4) The root locus is symmetrical about the real axis

(5) No asymptotes

(6) No centroid

(7) <u>Root locus on the real axis</u>

Between ~~sections~~ $s=0.5$ and $s=-2$ there is only one pole to the right of the section. Hence the root locus lies in this region.



⑧ Angles of arrival

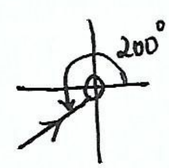
Let the angle of arrival at $s = 1+j2$ be θ_{A1}

$$\left[\left(\angle s-1+j2 + \theta_{A1}\right) - \left(\angle s-0.5 + \angle s+2\right)\right]_{s=1+j2} = 180$$

$$\angle 1+j2-1+j2 + \theta_{A1} - \angle 1+j2-0.5 - \angle 1+j2+2 = 180$$

$$90 + \theta_{A1} - 75.96 - 33.69 = 180$$

$$\boxed{\theta_{A1} = 199.6 \simeq 200^\circ}$$



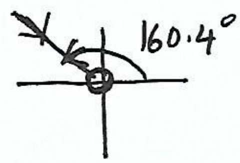
Let the angle of arrival at $s = 1-j2$ be θ_{A2}

$$\left[\left(\theta_{A2} + \angle s-1-j2\right) - \left(\angle s-0.5 + \angle s+2\right)\right]_{s=1-j2} = 180$$

$$\left[\left(\theta_{A2} + \angle 1-j2-1-j2\right) - \left(\angle 1-j2-0.5 + \angle 1-j2+2\right)\right] = 180$$

$$\left(\theta_{A2} - 90\right) - \left(-75.96 - 33.69\right) = 180$$

$$\left(\theta_{A2} - 90\right) - \left(-109.65\right) = 180$$



$$\boxed{\theta_{A2} = 160.4}$$

⑨ Intersection of the root locus with imaginary axis

$$1 + G(s)H(s) = 0, \quad 1 + \frac{K(s^2-2s+5)}{s^2+1.5s-1} = 0$$

$$s^2(1+K) + s(1.5-2K) + (5K-1) = 0$$

s^2	$1+K$	$5K-1$
s	$1.5-2K$	
s^0	$5K-1$	

$$1.5-2k=0$$
$$k=0.75$$

$$s^2(K+1) + 5K-1 = 0$$

$$s^2(0.75+1) + 5(0.75) - 1 = 0$$

$$s = \pm j\,1.25$$

⑩ Breakaway points

$$\frac{d}{ds}\left[\frac{s^2-2s+5}{s^2+1.5s-1}\right] = 0$$

$$3.5s^2 - 12s - 5.5 = 0$$

$s = 3.83 \quad , \quad s = -0.4$

Breakaway point is $s = -0.4$