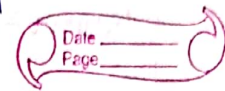


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Q1.] As we have seen in our lectures, where we took lime juice example, we saw that lime juice was represented as weighted sum of ingredients.

Similarly, a function can be represented as weighted sum of complex exponential.

The function taken by Keith is sinusoid function of 200 Hz.

Considering the signal to be continuous,

$$x(t) = A \sin(400\pi t) \quad [\because x(t) = A \sin(2\pi f t) \text{ and } f = 200 \text{ Hz}]$$

It can be represented as

$$x(t) = A \left[\frac{e^{j400\pi t} - e^{-j400\pi t}}{2j} \right]$$

$$\left[\text{Since, } \sin x = \frac{e^{jx} - e^{-jx}}{2j} \right]$$

Now, $\because x(t)$ is continuous & periodic; its spectrum will be discrete and aperiodic

$$\begin{aligned} X(K) &= \frac{1}{T} \int_T x(t) \cdot e^{-j2\pi K t} dt \\ &= \frac{A}{T} \int_T \sin(400\pi t) \cdot e^{-j400\pi K t} dt \\ &= \frac{A}{T} \int_T \left(\frac{e^{j400\pi t} - e^{-j400\pi t}}{2j} \right) \cdot e^{-j400\pi K t} dt \\ &= \frac{A}{2jT} \int_T e^{j400\pi(1-K)t} - e^{-j400\pi(1+K)t} dt \\ &= \frac{200A}{2j} \left[\frac{e^{j400\pi(1-K)t}}{j(400\pi)(1-K)} + \frac{e^{-j400\pi(1+K)t}}{-j(400\pi)(K+1)} \right]_0^{1/200} \end{aligned}$$

$$\Rightarrow X(K) = \frac{50A}{2j} \left[\frac{e^{2\pi j(1-K)}}{j(400\pi)(1-K)} + \frac{e^{-2\pi j(K+1)}}{j(400\pi)(K+1)} - \frac{2}{j(400\pi)(K^2-1)} \right]$$

Hence, we can see that the spectrum can be represented as sum of complex exponentials

$$\text{i.e., } x(t) = \sum_K X(K) \cdot e^{j2\pi f_K t} \quad ; \quad K = 0, 1, 2, 3, \dots$$

If the same question had to be solved using DT,

$$X(K) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j2\pi nK/N}$$

Considering CT domain, the drawback is the infinite possible values of K.

Now,

$$x(n) = 0.5 + 0.5 \cos\left(\frac{2\pi n}{7}\right) \quad \left[\begin{array}{l} \text{The signal is periodic} \\ \text{w/ } N=7 \end{array} \right]$$

$$\begin{aligned} X(K) &= \sum_{n=0}^6 \left(0.5 + 0.5 \cos\left(\frac{2\pi n}{7}\right) \right) \cdot e^{-j2\pi nK/7} \\ &= \sum_{n=0}^6 0.5 \cdot e^{-j2\pi nK/7} + \sum_{n=0}^6 0.5 \cos\left(\frac{2\pi n}{7}\right) e^{-j2\pi nK/7} \end{aligned}$$

$$\begin{aligned} &= 0.5 \left(1 + e^{-2\pi jK/7} + e^{-4\pi jK/7} + e^{-6\pi jK/7} + e^{-8\pi jK/7} + e^{-10\pi jK/7} + e^{-12\pi jK/7} \right) \\ &\quad + 0.5 \left(1 + e^{-2\pi jK/7} \cos\frac{2\pi}{7} + \dots \right) \end{aligned}$$

$$\begin{aligned} &= 0.5 \left[e^{-2\pi jK/7} \left(1 - \cos\frac{2\pi}{7} \right) + e^{-4\pi jK/7} \left(1 - \cos\frac{4\pi}{7} \right) + \right. \\ &\quad e^{-6\pi jK/7} \left(1 - \cos\frac{6\pi}{7} \right) + e^{-8\pi jK/7} \left(1 + \cos\frac{\pi}{7} \right) - \\ &\quad \left. e^{-10\pi jK/7} \left(1 + \cos\frac{3\pi}{7} \right) - e^{-12\pi jK/7} \left(1 + \cos\frac{5\pi}{7} \right) \right] \end{aligned}$$

Hence, the filter is allowing only low frequency components to pass. Hence, it is a low pass filter.

Q2-] Chintan claims that only real part of frequency response can be completely specified by only real part of frequency response.

Let $x(n)$ be complex.

$$x(n) = x_r(n) + i x_i(n)$$

$$X(\omega) = \sum_n [x_r(n) + i x_i(n)] [\cos(\omega n) - i \sin(\omega n)]$$

$$= \sum_n [x_r(n) \cos \omega n - i x_r(n) \sin \omega n + i x_i(n) \cos \omega n - i^2 x_i(n) \sin \omega n]$$

$$[i^2 = -1]$$

$$= \sum_n [x_r(n) \cos \omega n + x_i(n) \sin \omega n] + i \sum_n [-x_r(n) \sin \omega n + x_i(n) \cos \omega n]$$

If $x(n)$ is real,

$$x(n) = x_r(n)$$

$$x_i(n) = 0$$

$$X(\omega) = X_R(\omega) + i X_I(\omega)$$

$$\text{Hence, } X_R(\omega) = \sum_n x(n) \cos \omega n = X_R(-\omega)$$

$$X_I(\omega) = - \sum_n x(n) \sin(\omega n) = -X_I(-\omega)$$

$$\text{But if, } \omega = -\omega, \quad X(-\omega) = X_R(-\omega) + i X_I(-\omega)$$

Hence, we can say that for a real signal, +ve & -ve side of frequency have same plot.

$\cos \theta$ is an even function & $\sin \theta$ is an odd function
i.e., $\cos(-\omega) = \cos \omega$

$$\& \sin(-\omega) = -\sin(\omega)$$

Hence, we I agree with Chintan & oppose Maher's idea.

Further, if we consider $x(n)$ to be real & even;
we get $X(\omega)$ as real & even.

~~if~~ $x(n)$ i.e., $X_R(\omega) = \sum (x(n) \cdot \cos(\omega n))$ ^{even}
= something

$$X_I(\omega) = 0$$

$$\text{Hence, } X(\omega) = X_R(\omega)$$

$$\text{and } X(\omega) = X(-\omega)$$

ii) If $x(n)$ is real & odd

$$X_R(\omega) = \sum [x(n) \cdot \cos(\omega n)] = 0$$

$$X_I(\omega) = \sum [x(n) \cdot \sin(\omega n)] = \sum [\text{even}] = \text{something}$$

$$\text{Hence, } X(\omega) = -X(-\omega)$$

$$X(\omega) = j X_I(\omega)$$

Hence, $X(\omega)$ is imaginary & odd.

Hence, this is the reason for their argument. Both are correct at their respective places. Suppose if I pass

$$x(n) = n^3 + n^2, \text{ then it is neither odd nor even.}$$

Hence, this $x(n)$ would be enough to pacify both of them.

Q4] a) i) $x(n) = e^{jn\pi/2}$
 $y(n) = 0.25 e^{jn\pi/2}$

$\therefore y(n) = 0.25 x(n)$

Let $x_1(n) = a x(n)$ & $x_2(n) = b x(n)$

$y_1(n) = 0.25 a x(n)$

$y_2(n) = 0.25 b x(n)$

$\therefore y_1(n) + y_2(n) = (a+b) \cdot 0.25 x(n)$
 $= (a+b) y(n)$

\therefore We can see that it is linear & time invariant.

$y(z) = 0.25 x(z)$

$\therefore H(z) = \frac{y(z)}{x(z)} = 0.25$

Let $z = e^{j\omega}$

$\therefore H(\omega) = 0.25$ (It is linear phase & there's only a scale in input)

ii) $y(n) = \sin\left(\frac{n\pi}{3}\right)$, $x(n) = \sin\left(\frac{n\pi}{6}\right)$

We know, $\cos\left(\frac{n\pi}{6}\right) = \sqrt{1 - \sin^2\left(\frac{n\pi}{6}\right)}$

$\sin\left(\frac{n\pi}{3}\right) = 2 \sin\left(\frac{n\pi}{6}\right) \cdot \cos\left(\frac{n\pi}{6}\right)$

$\therefore y(n) = \frac{2 \sin\left(\frac{n\pi}{6}\right) \cos\left(\frac{n\pi}{6}\right)}{\sin\left(\frac{n\pi}{6}\right)} x(n)$

Let $x_1(n) = ax(n)$ & $x_2(n) = bx(n)$

$$y_1(n) = 2ax(n) \sqrt{1 - (\pi a)^2 x^2(n)}$$

$$y_2(n) = 2bx(n) \sqrt{1 - (\pi b)^2 x^2(n)}$$

$$y_1(n) + y_2(n) \neq (a+b)y(n)$$

Hence, it is not linear.

gt $n = n - n_0$

$$y_1(n - n_0) = 2ax(n - n_0) \sqrt{1 - \pi^2 (n - n_0)^2 x^2(n - n_0)}$$

gt we delay the input, $x(n) = x(n - n_0)$

$$y_1(n) = 2x(n - n_0) \sqrt{1 - \pi^2 (n)^2 \cdot x^2(n - n_0)}$$

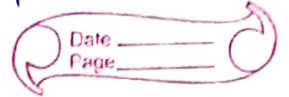
$$\therefore y_1(n - n_0) \neq y_1(n)$$

Hence, it is time variant.

Also, transfer function exists only for LTI. Hence, no transfer function here.

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Q3.] a.) TMS320 is a DSP processor made by Texas Instruments; as opposed to a general microprocessor.

Points of Contrast:-

- i) TMS320 have address generators and are often used for real time computation as opposed to general microprocessors.
- ii) TMS320 has an efficient external interface, powerful functional unit, used in image processing, audio & speech processing, etc. as opposed to low memory bandwidth, cheap cost, less robust hardware and limited use in ~~analog~~ analog domain for general purpose microprocessors.