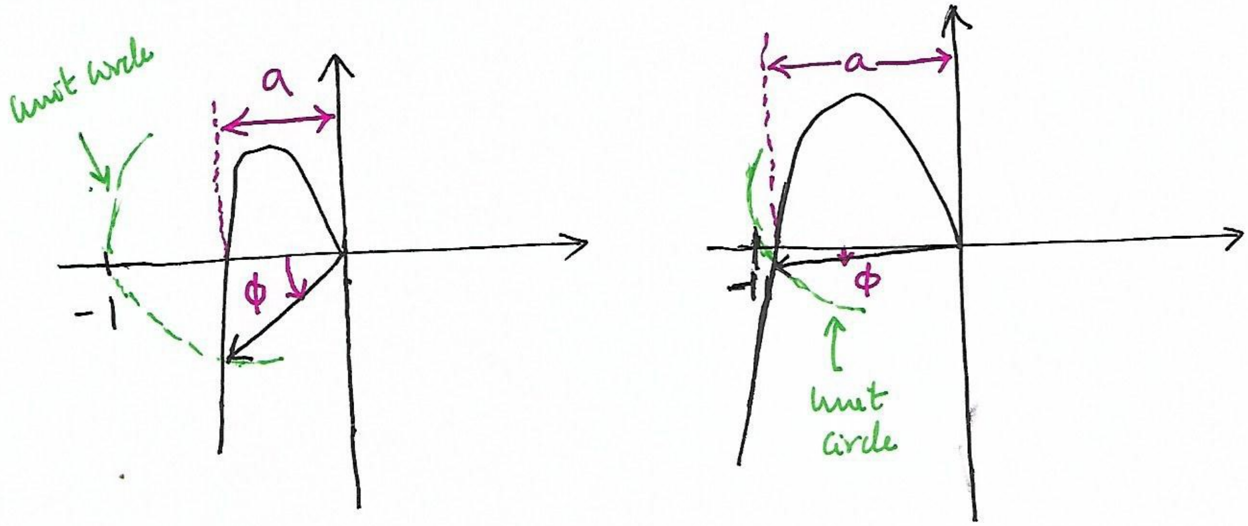
PHASE MARGIN

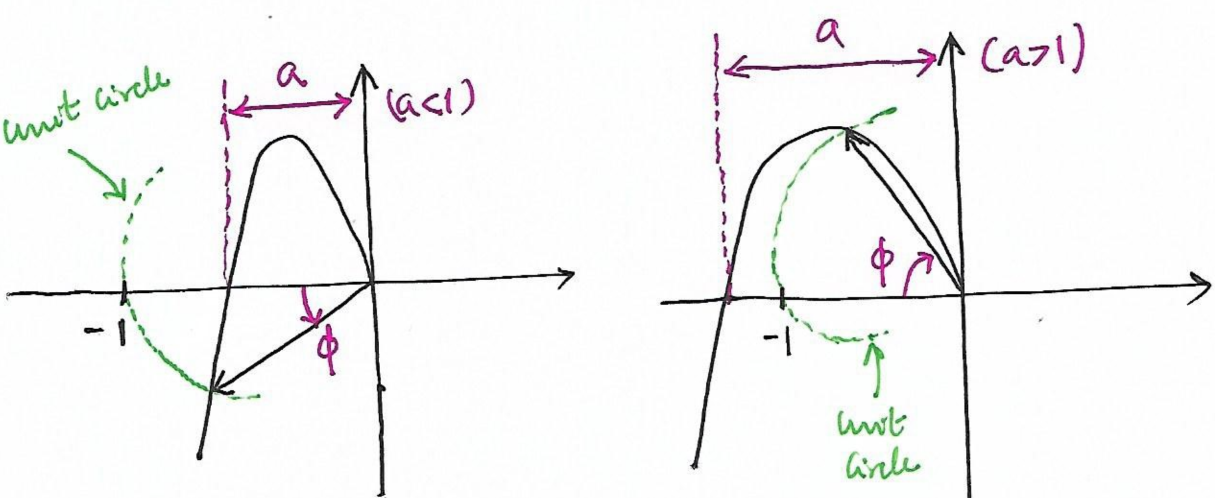
- The frequency at which $|G(j\omega)H(j\omega)| = 1$ is called the gain crossover frequency.
- It is given by the intersection of the $G(j\omega)H(j\omega)$ plot and a unit circle centered at the origin as shown in Fig 3
- At this frequency the phase angle is $\angle G(j\omega_1)H(j\omega_1)$ is equal to $(-180^\circ + \phi)$

- If an additional phase lag equal to ϕ is introduced at the gain crossover frequency, the phase angle $\angle G(j\omega_1)H(j\omega_1)$ will become -180° while the magnitude remains unity.
- The $G(j\omega)H(j\omega)$ plot will then pass through the $(-1, j0)$ point driving the system to the verge of instability.
- This additional phase lag ϕ is known as the PHASE MARGIN (PM)

(*) The phase margin is thus defined as the amount of additional phase lag at the gain crossover frequency, required to bring the system to the verge of instability.



(*) Fig below shows the GM and PM of both stable and an unstable system in polar (Nyquist) plots



$GM = 20\log\frac{1}{a}$ is +ve

PM is positive

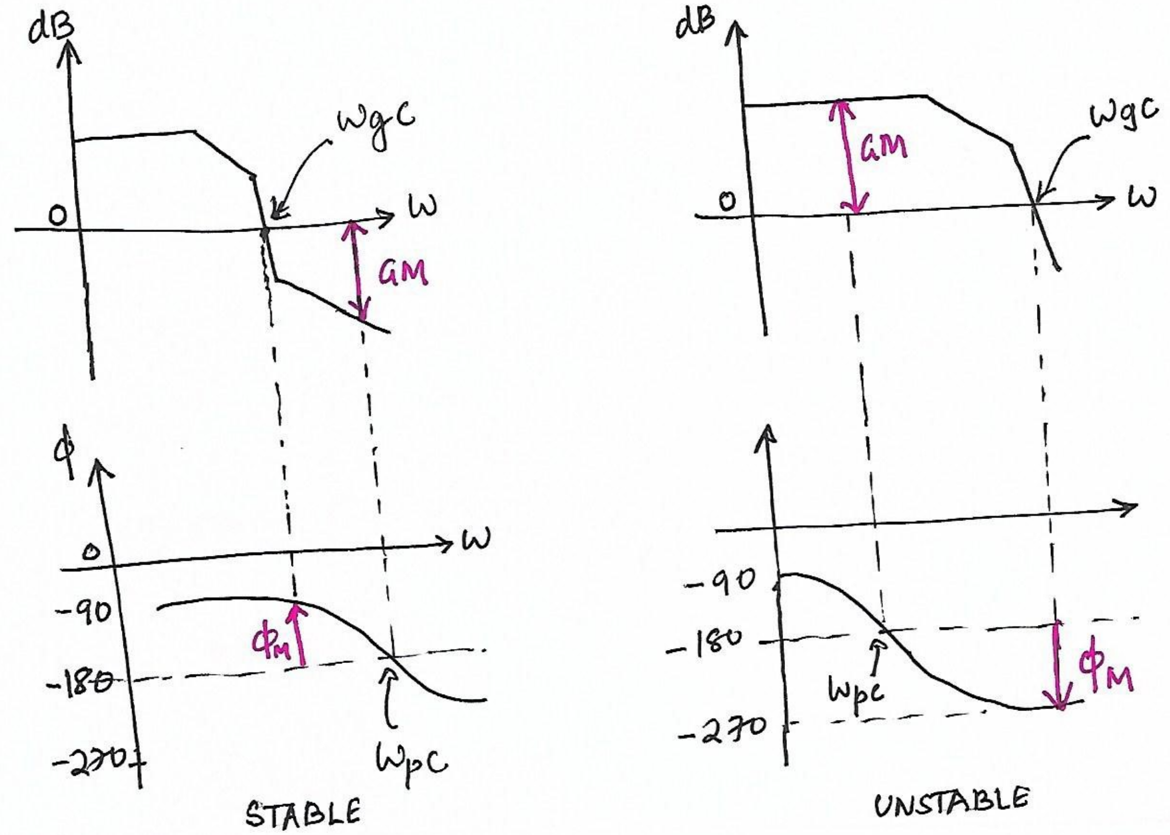
<u>STABLE</u>

$GM = 20\log\frac{1}{a}$ is −ve

PM is negative

<u>UNSTABLE</u>

(X) Fig below shows the GM and PM of both Stable and unstable system in Bode plots.



GAIN AND PHASE MARGINS FROM THE ROOT LOCUS

* The gain margin is the factor by which the design value of gain factor K can be multiplied before the closed loop system becomes unstable.

* It can be determined from the root locus using the following formula:

$$\text{Gain Margin} = \frac{\text{Value of K at imaginary axis crossover}}{\text{design value of K}}$$

* To find the phase margin it is necessary to find the point $j\omega_1$ on the $j\omega$ axis for which $|G(j\omega_1)H(j\omega_1)| = 1$.

* The phase margin is then computed as

$$PM = 180^\circ + \angle G(j\omega_1)H(j\omega_1)$$

Pb 34 Cont...

For $k=8$ find the gain margin and the phase margin of the system.

Solution

GM For $k=8$, $GM = \frac{64}{8} = 8$

$$\boxed{GM = 8}$$

PM Let ω_1 be the frequency at which

$|G(j\omega_1)H(j\omega_1)| = 1$

$$\left| \frac{8}{(j\omega_1+2)^2} \right| = 1$$

ie $\omega_1 = 0$

$\angle G(j\omega_1)H(j\omega_1) = 0^\circ$

$\therefore PM = 180 + 0^\circ$

$$\boxed{PM = 180^\circ}$$