BASIC CONDITIONS OF THE ROOT LOCI

* $$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

* The characteristic equation is obtained by setting $1+G(s)H(s)$ to zero

$$1+G(s)H(s)=0$$
$$G(s)H(s)=-1$$

* If both sides of $F(s)$ are divided by the terms that do not contain K then,

$$1+\frac{K(s^m+b_1s^{m-1}+\dots\dots+b_{m-1}s+b_m)}{(s^n+a_1s^{n-1}+\dots\dots+a_{n-1}s+a_n)}=0$$

* Therefore $$G(s)H(s) = \frac{K(s^m+b_1s^{m-1}+\dots\dots+b_{m-1}s+b_m)}{s^n+a_1s^{n-1}+\dots\dots+a_{n-1}s+a_n}$$

Where $G(s)H(s)$ is the open loop transfer function.

* The complete root loci is the loci of the points in the s-plane that satisfy $1+G(s)H(s)=0$ as K is varied from $-\infty$ to ∞.

* Let $G(s)H(s) = K\,G_1(s)H_1(s)$ where $G_1(s)H_1(s)$ does not contain the variable parameter K

Then $$G_1(s)H_1(s) = -\frac{1}{K}$$

* To satisfy this equation the following conditions must be met simultaneously.

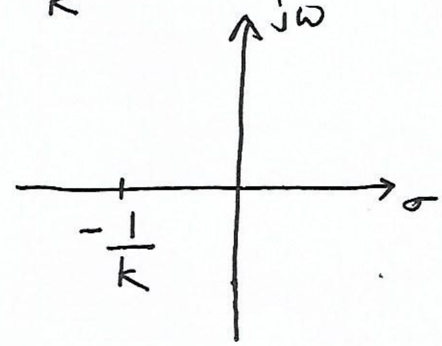
$$|G(s)H(s)| = \frac{1}{|K|} \qquad -\infty < K < \infty \rightarrow ①$$

$$\angle G(s)H(s) = (2k+1)\pi \qquad K \geq 0 \rightarrow ②$$

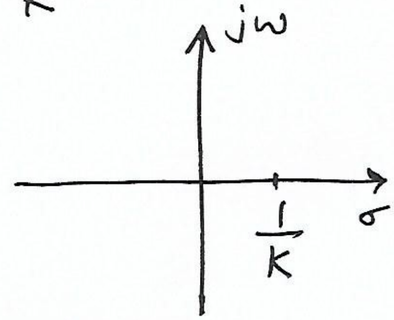
$$\angle G(s)H(s) = 2k\pi \qquad K \leq 0 \rightarrow ③$$

$$k = 0, \pm 1, \pm 2, \ldots\ldots$$

* When K is positive, $-\frac{1}{K}$ will be on the negative real axis. Angle on negative real axis is odd multiple of π



* When K is negative, $-\frac{1}{K}$ will be on the positive real axis. Angle on positive real axis is even multiple of π



Rules For Construction Of Root Loci

① K = 0 points

- The K=0 points on the complete root loci are at the poles of $G(s)H(s)$

$$|G_1(s)H_1(s)| = \frac{\prod_{i=1}^{m} |s+z_i|}{\prod_{j=1}^{n} |s+p_j|} = \frac{1}{|K|}$$

- As K approaches zero, $|G_1(s)H_1(s)|$ approaches ∞ and correspondingly s approaches the poles of $G_1(s)H_1(s)$, that is s approaches $-p_j$

- This property applies to both the root loci and the complementary root loci since the sign of K has no effect.

eg - Consider the following equation

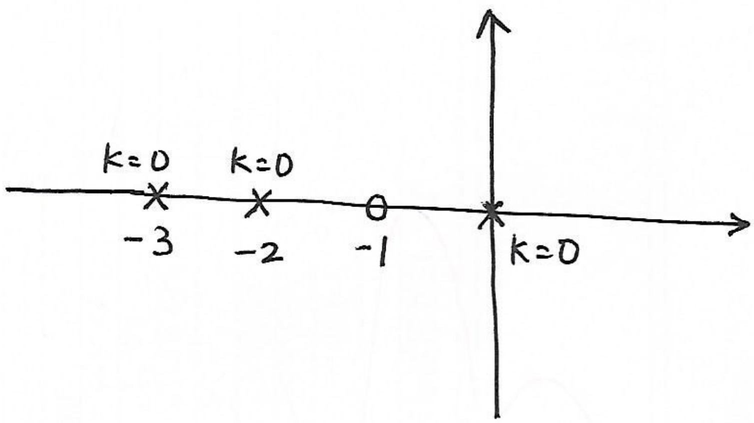
$$s(s+2)(s+3) + K(s+1) = 0$$

When K=0, the three roots of the equation are at $s=0$, $s=-2$ and $s=-3$

- $$1 + G(s)H(s) = 1 + \frac{K(s+1)}{s(s+2)(s+3)}$$

* These three points are the poles of the function $G(s)H(s)$

* The three $k=0$ points on the complete root loci are shown below



② $k = \pm\infty$ points

* The $k=\pm\infty$ points on the complete root loci are at the zeros of $G(s)H(s)$

* As k approaches $\pm\infty$ $|G_1(s)H_1(s)|$ approaches 0

* This corresponds to s approaching the zeros of $G(s)H(s)$ or s approaches $-Z_i$ $(i=1,2,\ldots\ldots m)$

Eg Consider the equation

$$s(s+2)(s+3)+k(s+1)=0$$

when k is very large, $k(s+1)=0$ which has a root at $s=1$

* $G(s)H(s)$ also has two other zeros located at infinity, because for a rational function the total number of poles and zeros must be equal if the poles and zeros at infinity are included.

* Therefore $K = \pm\infty$ points are at $s=-1$, ∞ and ∞

3) Number of branches of the complete root loci

* A branch of the complete root loci is the locus of one root when K takes on values between $-\infty$ and ∞.

* The number of branches of the root loci is equal to the greater of n and m.

eg $s(s+2)(s+3)+K(s+1)=0$ has three branches since $n=3$, $m=1$

* The equation is of third order in s, it must have three roots and therefore three branches of the root loci