STATEMENT

Routh's stability criterion states that the number of roots with positive real part is equal to the number of changes in sign of the coefficients of the first column of the array.

- The exact values of the terms in the first column need not be known, instead only the signs are needed.
- Thus the necessary and sufficient condition that all roots lie in the LHSP is that all the coefficients of the equation be positive and all terms in the first column of the array have positive signs.

Pb (17) Find the number of positive real roots that the following polynomial has.

$$s^4+2s^3+3s^2+4s+5=0$$

Solution

s^4	1	3	5
s^3	2	4	0
s^2	1	5	
s^1	-6		
s^0	5		

The number of sign changes in the first column of the array is two. Hence there are two roots with positive real parts.

Pb (18) The characteristic equation for a closed loop system is given by

$$s^4+3s^3+3s^2+7s+6=0$$

Find the number of roots in the RHSP.

Solution

s^4	1	3	6
s^3	3	7	
s^2	$2/3$	6	
s	-20		
s^0	6		

There are two sign changes in the first column of the array.

Hence there are two roots in the RHSP.

Pb (19) Is the system with characteristic equation $s^4 + 8s^3 + 18s^2 + 16s + 15 = 0$ stable?

s^4	1	18	15
s^3	8	16	
s^2	16	15	
s	8.5		
s^0	15		

The elements of the first column of the array are all positive and hence the system is stable

Pb (20) The characteristics equation of a given system is $s^4 + 6s^3 + 11s^2 + 6s + K = 0$

What restrictions must be placed on the parameter K in order to ensure that the systems is stable?

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SOLUTION

s^4	1	11	K
s^3	6	6	
s^2	10	K	
s	$\frac{60-6K}{10}$		
s^0	K		

$\frac{60-6K}{10} > 0$, $K > 0$

$60 - 6K > 0$

$60 > 6K$

$10 > K$

$0 < K < 10$

SPECIAL CASE 1

- If a first column term in any row is zero but the remaining terms are not zero or there are no remaining terms, then the zero term is replaced by a very small positive number ε and the rest of the array is evaluated.

Eg:

$$s^3 + 2s^2 + s + 2 = 0$$

The array coefficients are

s^3	1	1
s^2	2	2
s	$0 \simeq \varepsilon$	
s^0	2	

- If the sign of the coefficient above the zero (ε) is the same as that below it, it indicates that a pair of roots lie on the imaginary axis. The system is LIMITEDLY STABLE

- If however the sign of the coefficient above the zero (ε) is opposite to that below it, it indicates that there is one sign change. The system is unstable.

Eg $s^3 + 3s + 2 = 0$

$+$	s^3	1	3
$+$	s^2	$0 \simeq \varepsilon$	2
$-$	s^1	$\frac{3\varepsilon - 2}{\varepsilon}$	
$+$	s^0	2	

There are two sign changes in the coefficients of the first column of the array.

Hence the system is UNSTABLE

Pb (21) Discuss the stability of the system whose characteristic equation is

$$s^5 + 2s^4 + 4s^3 + 8s^2 + 10s + 6 = 0$$

Ans There are two sign changes in the first column of the array. Hence there are two roots with positive real part. The system is unstable.

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SOLUTION

$$s^5 + 2s^4 + 4s^3 + 8s^2 + 10s + 6 = 0$$

+	s^5	1	4	10
+	s^4	2	8	6
+	s^3	$0 \simeq \varepsilon$	7	
−	s^2	$\frac{8\varepsilon - 14}{\varepsilon}$	6	
+	s^1	y		
+	s^0	6		

$$y = \frac{7\left(\frac{8\varepsilon - 14}{\varepsilon}\right) - 6\varepsilon}{\left(\frac{8\varepsilon - 14}{\varepsilon}\right)}$$

There are two sign changes in the first column of the array. Hence there are two roots with positive real part. The system is unstable.