1.
a)

Our strategy is to roll until we hit a number greater than 2.

Reasoning:
We work by cases:

Case Rolling 1 at n=1:

We have a 5/6 probability of being better off by rolling any number greater than 1.

```
0.6
>>> payoff([1,1])
0.36
>>> payoff([1,2])
0.72
>>> payoff([1,3])
1.08
>>> payoff([1,4])
1.44
>>> payoff([1,5])
1.7999999999999998
>>> payoff([1,6])
2.16
```

Chance of profit = ~83.34%

Probability 5/6

Case Rolling 2 at n=1;

In this case we'll only be better off if we roll a 4,5 or 6 after making our max profit percentage chance (1/2) or 50%
However, getting a 4 increases our profit by 10%.
Getting a 5 increases our profit by 50%, and a 6 increases it by 80%. At the same time getting a 1or a 2 leads to 40% loss and a 3 leads to 10% loss. So our incentive for profit is more. Therefore we must roll the dice again.

Case Roll a 3 at n=1:

1/6 chance of being better off only if we get a six after this roll.
Mathematically speaking, every other number we get leads to a loss. So 1/6 profit probability is too less to move forward.

Case 4/5/6 at n=1:

No way of being better off so it is smarter to stop at this turn. Any other number we get after this would lead to a loss.

b)

Expected Value of our stopping time = summation(n*(2/6)^(n-1)*(2/3)) = **1.5**
Which means in discrete values T = 1.

c)

Although our strategy (after 10^7 simulations) reduces tau to **~1.3496** because we're assuming a rational gamblers technique.

Giving us a payoff of **2.29**

sum(k=3,4,5,6) (3/5)^n * (1/3)^(n-1) * (1/6) = **2.25**

~close to our simulated payoff of **2.29**

2.

Marginal benefit >= Marginal cost

(3/5)*(New X) >= (Old X)

Only then do we have an incentive to play again.

Consider number of possible beneficial outcomes X_2 to be y. Then, y=N-X_2+1

Chances of profit = y/N

We will only take the next step if y/N >= 1/2

Now, considering the probabilities, a successive iteration is beneficial only if:
y/N*(3/5)*(new outcome) is >= ((N-y)/N)*(old outcome)

Hence, our strategy is to first decide X_2 with the formula (3/5)*X2 is >= X1 then find y using: N-X2+1 then calculating y/N if y/N is >= 1/2 we play again else we stop playing.

This formula: y/N*(3/5)*(new outcome) is >= ((N-y)/N)*(old outcome) calculates the probability of that profit percentage with new outcome.