Multi-Robot Formation Control using Graph Theory

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***Abstract*— The paper is an attempt to achieve the decentralized multi-robot formation control with the help of graph theory. In particular, we have studied different characteristics of multi-robot systems and formation control strategies associated with them. The paper consists of of three experiments involving the square formation control of four unicycle robots using decentralized coordination. While, first experiment involves a simple leader follower approach, the second experiment involves the graph theory for consensus building. The third experiment, is an extension of the second experiment to study the case of interference in prior interaction.**

***Index Terms*— Multi-Robot System (MRS), Graph Theory, Decentralized System, Consensus, Bias.**

I. INTRODUCTION

In the recent emerging trends in the field of robotics, Multi-Robot Systems (MRS) are finding their application in wide variety of tasks ranging from civilian to military applications. In addition, MRSs play a greater role in applications, as complicated as exploration, surveillance or rescue robotics, where a single robot is less effective. These systems rely on the abundant computational power in the embedded systems of autonomous vehicles enabling efficient information exchange and improved operational effectiveness [1]. A substantial amount of investment and research have been put on the operation of multi-agent systems for their practical characteristics of being flexible, redundant, fault tolerant, and being able to work with distributed sensing and actuation in various applications. A large amount of literature exists on the aspects of localization, task allocation, formation control, multi-sensor fusion, communication, coordination and cooperation of MRS which can help us understand and develop advanced multi-agent systems [12].

This paper is an attempt to present a solution to the problem of bringing a multi-agent system to a predefined formation using consensus algorithm. Consensus problems mainly arises due to limited interaction within the system between individual robots [14]. This may lead to deviation from the desired final state of the formation. A previous research from [14][15] provides information of implementing Graph theory for our work. In order to solve the consensus problem, we have combined kinematic model of a holonomic robot with the local position information of the robots.

The paper first introduces the important characteristics of the MRS in Section II. These characteristics gives us an idea about the pre-requisites that are to be considered when designing the control algorithm for the MRS. The background research for the paper has been explained in Section III where the literature review of previous researches in multi-robot formation control has been outlined. While Section IV establishes the group formation control of unicycle robot; Section V illustrates the problem statement and the experiments and simulations during the coarse of our project and the results associated with them.All the experiments conducted are analyzed in the section VI. Finally, Section VII and VIII concludes the paper with conclusion and future work respectively.

II. CHARACTERISTICS OF MULTI-ROBOT SYSTEMS

In order to realize a Multi-Robot System with advanced computational power, various factors have to be considered and incorporated in the development phase.

A. *Localization*

Localization defines the exact position and orientation of the robot in an obscure environment [8]. Individual robots need information on changes in position and distances of associate robots in order to coordinate in a task. Localization is interdependent to Mapping. Mapping generates consistent map of the surrounding for robots to draw its path of action. This helps to obtain a broader picture for the MRS to operate in the known surrounding [7][9].

B. *Task Allocation*

Task allocation can be defined as the process of assigning tasks to an individual robot in the MRS such that the overall system can accomplish the result with better efficiency, cost reduction, robust operation and better resource management. Initially, robots communicate within the group to distribute the tasks based on the surrounding, position of robots and energy factor to decide which robot is efficient to carry out the tasks based on the task requirement [10].

C. *Formation Control*

Formation control aims at driving and organizing the group of robots in realizing said states of individual robots with respect to its associate. Based on the architectures of sensor systems and the interaction topology defined in the robots, a wide range of formation control strategy can be designed [2][4].

D. *Multi-Sensor Fusion*

Multi-sensor fusion technology (MSFT) is the brain of robot intelligence. It gathers data for the MRS to detect elements in the environment. Sensors with various functionality are integrated into the robots for its internal assessment and collecting the surrounding characteristics. MSFT helps the robots develop skills, such as data collection, signal process, visual perception and image manipulation for its individual valuation and also coordinate the information as a group.

It helps to phase out redundancy and uncertainty in the information [10].

E. Communication

In order to work in a group to achieve a specific task, it is necessary to have an organized communication medium between the members and defined control structures. As the robots are simple, they have limited communication resources which can be seen in the lower bandwidth at which it operates. The global and local communication strategies have the major impact on the information exchange among the robots [3][9].

The domain of control in multi-robot systems can be explored in different ways: Centralized Systems and Decentralized Systems. While Centralized systems (Fig. 1.B) depend on a single central controller to exercise the control over the lower-level components of a systems through direct or hierarchical structures. On the other hand, Decentralized systems (Fig. 1.A) depend on the abstraction of lower level components to achieve a combined objective without any commanding influence from any other controller. Decentralized Systems can be very well depicted by the robots having perception sensors but not the communication devices. Nonetheless, it is also important to discuss the concept of Distributed Systems (Fig. 1.C) which involves multiple decentralized systems, however still consisting a central controller to achieve a global objective.

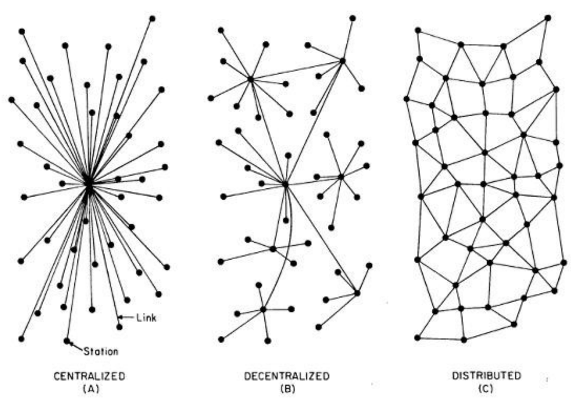

Fig. 1. Types of Systems

F. Co-operation and Coordination

Coordination and cooperation in MRS is the association of the individual robots in joint operation. Cooperation among the robots is the understanding between them to assist co-members during need which can help improve the overall efficiency of the task. Coordination helps in maintaining the formation among the robots which in turn is the result of the cooperative system of the MRS [3][10].

III. Background

A. Graph Theory

Graph theory is a tool to imitate the interaction topology among a set of connected robots in any formation control. The interaction between two systems can be defined using an un-directed graph and directed graph. While un-directed graph represents mutual interaction among two robots, directed graph illustrates the one way perception. The paper [11] discusses the decentralized approach of coordination among the multiple robots assuming the limited perception and limited range potentials of the individual robots. Based on these assumptions, the paper [11] shows the formulation of configuration space along with graph-based models. Further, the concept of connectivity graph has been discussed in order to bridge the gap between the graph theory and the configuration space for MRS. It has also shown the convergence of formation topology from connectivity graphs. Finally, the local rules and communication strategies are converged to get desired global formation.

The robots are considered as the nodes and their interaction with one another are edges [5][12]. In an un-directed graph with N number of robots is defined as a pair $G = (V, E)$, where

- V defines the set of Vertices or Nodes,
- E defines the set of Edges.

While designing an un-directed graph, it is considered that elements of E are un-ordered pairs of elements, i.e. $(v_i, v_j) \in E \Leftrightarrow (v_j, v_i) \in E$. A graph is said to be strongly connected if every node is connected to every other node representing the strength of the interaction among the nodes.

B. Formation Control Strategies

The formation control strategies implemented in multi-robot systems are basically based on the sensing and interaction properties of the robots [4]. They are further distinguishes as:

- Position-based: In this system, individual robots consider global coordinates using GPS to determine their absolute position with minimal mutual interaction. This system requires advanced sensors to interact with the surrounding. The global co-ordinate system provides feedback which is beneficial during disturbances [2].
- Displacement-based: The local co-ordinates of the robot are considered for its orientation. The system operates through the interactions among the individual robots to determine or sense relative displacement and attain constant speed. It is a decentralized formation control. Most systems follow leader-follower approach so that the speed of every robot is well coordinated and remains constant [2].
- Distance-based: In this strategy, distance between individual robots are always controlled and kept constant. The relative positions are set based on the local coordinate system. There has to be interaction among the robots as there is no common sense of orientation. For this reason, the interaction graph from graph theory must be rigid. This is the base strategy of our experiment [2].

C. Consensus Problem

The task of operating individual robots in a multi-robot system to attain a common final state forming a predefined formation is known an Consensus Problem [14]. Moreover, in order to understand the consensus behaviour in the MRS, it is first important to define the individual entity(robots) of the control system using a mathematical model. In order to define the problem, a holonomic robot with single degree of motion is considered

$$\dot{x}_i = u_i \tag{1}$$

where $\dot{x}_i$ is the state of the i-th robot[1][15]. Similarly, the kinetic model for N-number of robots can be represented using Laplacian feedback model in a consensus problem as

$$\dot{x}_i = u_i = -\mathcal{L}x_i \tag{2}$$

where $\mathcal{L}$ is the Laplacian matrix representing the connectivity among the robots in a graphical representation. This is further explained in Section IV-B where the Laplacian matrix for our model is drawn [13][14][15].

IV. Group Formation Control of Unicycle Robots

A. Unicycle Robot

Since, a robotic system can be as complex it can be, therefore for the sake of simplicity most of the previous research have used unicycle robots 2 to validate their consensus algorithm. This increases the computational efficiency of the system. Unicycle robots consist of differential wheels which facilitates in forward motion as well as rotation [13][14][15].

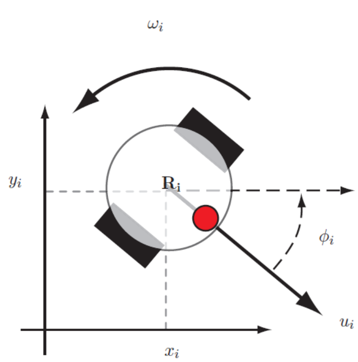

Fig. 2. Unicycle Robot

The kinematic equation (3) representing the real unicycle robot is as below

$$\begin{cases} \dot{x}_i = u_i \cos\phi \\ \dot{y}_i = u_i \sin\phi \\ \dot{\phi}_i = \omega_i \end{cases} \tag{3}$$

where u_i and ω_i are the linear and rotational motion of the robot and x_i, y_i and ϕ_i represent the degree of freedom in which the robots can move [5] (see Fig. 1).

Further, [5] evaluates the linear and rotational velocities of the unicycle robot while following a trajectory using:

$$u = \gamma\, \delta\, cos(\Delta\phi) \tag{4}$$

$$\omega = k\, \Delta\, \phi\, \dot{\phi}_d \tag{5}$$

where, k, $\gamma > 0$ and δ accounts for the change in the distance from the current position $P_i(x, y, \phi)$ of the robot to that of the desired location $P_d(x_d, y_d, \phi_d)$ i.e, if $\Delta x = x_d - x$, $\Delta y = y_d - y$, $\Delta\phi = \phi_d - \phi$ and $\phi_d = \arctan(\Delta y, \Delta x)$; then, $\delta = \sqrt{\Delta x^2 + \Delta y^2}$.

B. Consensus algorithm

As explained in Section III-C, we know the kinematic model of a non-holonomic robot. When the values of this model from all the individual robots is combined with the local coordinate system along with its orientation is used to generate an algorithm using Lyapunov function [6], we can converge and stabilize the system in a desired formation. It

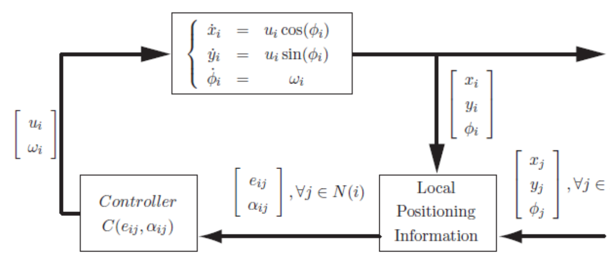

Fig. 3. Feedback diagram for consensus algorithm

can be mathematically defined as:

$$\bar{e}_{x,i} = \frac{1}{\Delta_i + 1} \sum_{j=1}^{\Delta_i} [-\mathcal{L}_{i,j} \cdot e_{i,j} \cdot \cos\alpha_{i,j}] \tag{6}$$

$$\bar{e}_{y,i} = \frac{1}{\Delta_i + 1} \sum_{j=1}^{\Delta_i} [-\mathcal{L}_{i,j} \cdot e_{i,j} \cdot \sin\alpha_{i,j}] \tag{7}$$

$$\bar{e}_i = \sqrt{\bar{e^2}_{x,i} + \bar{e^2}_{y,i}} \tag{8}$$

$$\bar{\alpha}_i = \arctan 2(\bar{e}_{y,i}, \bar{e}_{x,i}) \tag{9}$$

where Δ_i is the robot count connected to Robot R_i, $e_{i,j}$ is the distance between Robot R_i and R_j and $\alpha_{i,j}$ is the angle of R_j with respect to R_i; and the relative error in the position with respect to global positioning is defined by the Eq. (6) and Eq. (7) represented as c_i=$[e_i, \alpha_i]^T$ (see Fig. 3). [14], [15] has described "Bias" as the predefined distance which has to be maintained in all circumstances to constrain a formation. This maintains a steady formation leaving minimal room for deviation of robots from the path defined.The introduction of bias has been illustrated in Fig. 4. Further, it can be mathematically defined as:

$$\bar{e}_{x,i} = \frac{1}{\Delta_i + 1} \sum_{j=1}^{\Delta_i} [-\mathcal{L}_{i,j} \cdot (e_{i,j} - b_{i,j}) \cdot \cos\alpha_{i,j}] \tag{10}$$

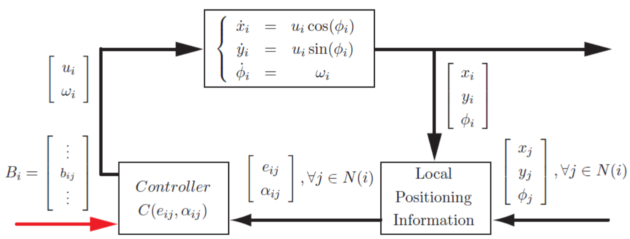

Fig. 4. Feedback diagram with bias input at the controller in consensus algorithm

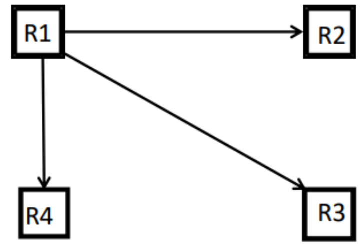

Fig. 6. Leader-Follower arrangement

$$\bar{e}_{y,i} = \frac{1}{\Delta_i + 1}\sum_{j=1}^{\Delta_i}[-\mathcal{L}_{i,j}\cdot(e_{i,j} - b_{i,j})\cdot\sin\alpha_{i,j}] \quad (11)$$

where $b_{i,j}$ is the desired set distance between the robots R_i and R_j which drives all the robots to the equilibrium state by matching the desired distances between the robots.

V. Simulation

A. Problem Statement

The problem statement consists of achieving a square formation control between 4 unicycle robots using decentralized coordination theory on pre-defined trajectory (see Fig. 5). It is further extended to test the controller for the lost interaction between two robots.

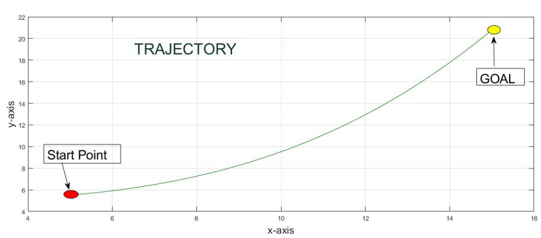

Fig. 5. Trajectory definition for the experiment

B. Simulation and Experiment

1) Decentralized Formation Control with Leader Follower Strategy: In our first experiment, we worked on coordinating the robot set through a single robot called the Leader robot. The path trajectory for our experiment was passed on to the Leader robot as the reference trajectory for the experiment. This path information was then broadcasted to the rest of the follower robots. From Fig. 6, we can observe information passing from Leader robot R_1 to Follower robots R_2, R_3, R_4 in a uni-direction [8].

The controller of each follower robot receives the reference trajectory and continuously generates feedback such that follower robots always maintain constant distance from the centroid of the square formation during the motion (see Fig. 7). The result of the experiment has been illustrated in Fig. 8.

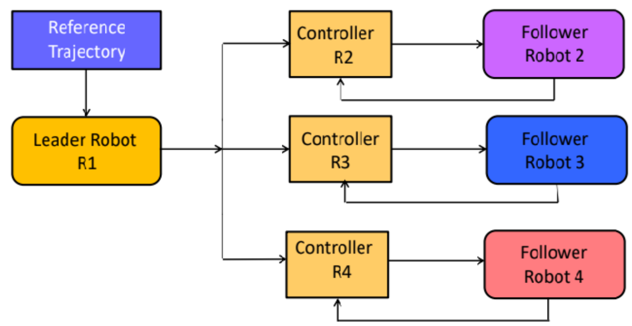

Fig. 7. Information Broadcasting and Feedback

2) Decentralized Formation Control using Graph Theory: The experiment is focused on achieving a decentralized control between the robots, where the global objective (trajectory) is achieved by local perception and computation on each robot. The perception sensors help a robot to indirectly interact with each other. This interaction or the coordination among the robots at the logical level has been defined using Graph Theory. Here, the graph between 4 robots has been represented by Fig. 9.

Mathematically, the interaction can be defined using Laplasian Matrix which states: "Given a graph $G = (V, E)$ with N nodes, the Laplacian matrix $L \in R^{N*N}$ of G is defined as $L = D - A$, where D is the degree of each node and A refers to adjacency" [17][18].

$$\mathcal{L}(i,j) = \begin{cases} deg(n_i), \text{if } i = j \\ -1, \text{if } i \neq j, and (n_i, n_j) \in E \\ 0, \text{otherwise} \end{cases}$$

where i,j are the robots and n_i,n_j are node representation of the robots.

The consensus control algorithm represented in Eq. (10) and Eq. (11) receives the perceived input (here, in the form of distance) from distance sensor and consists of predefined bias inside the robot controller. The controller of each robot tries to converge to the centroid (which is also control points on the trajectory) of the square formation considering the perceived position of other robots; simultaneously, trying to track the trajectory as well.

Considering the problem statement, the Laplacian and Bias

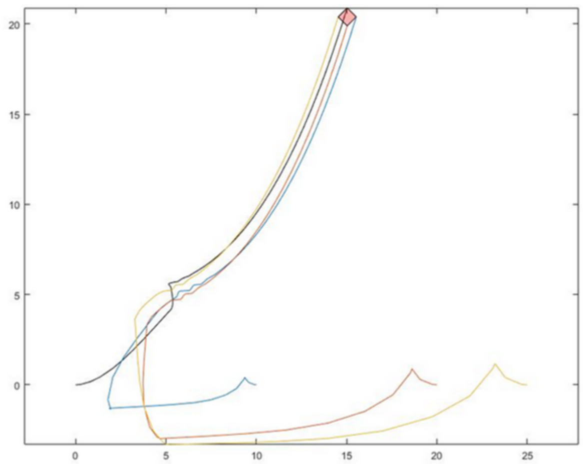

Fig. 8. Path Trajectory in Experiment 1

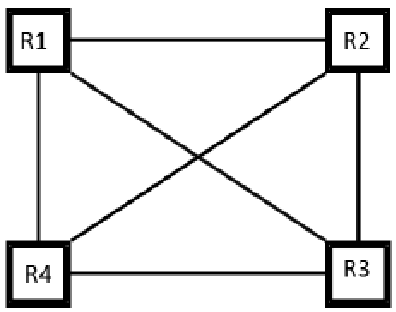

Fig. 9. Graph based connectivity

in a matrix formcan be represented as follows:

$$\mathcal{L} = \begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix} \tag{12}$$

$$B = \begin{bmatrix} 0 & 1/\sqrt{2} & 1 & 1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} & 1 \\ 1 & 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 1/\sqrt{2} & 1 & 1/\sqrt{2} & 0 \end{bmatrix} \tag{13}$$

The final output of the results of this experiment could be seen in the path diagram Fig. 11 obtained by simulating the model in Simulink.

3) Decentralized Formation Control with Measurement Error: The aim of the experiment is to enable to successfully maneuver the MRS in case of loss in interaction. This scenario could occur due to the presence of an obstacle between the robots or loss of sensor. Formation control with lack of interaction can be achieved through data transfer through wireless hops. The experiment extends the previous case eliminating the interaction between robots R_1 and R_2. Further, alternate connectivity was tested wherein the information from Leader robot R_1 was bypassed through follower robots R_3 or R_4 to be later accessed by R_2. In this experiment partial communication network among the robots was maintained with sensors working on the major

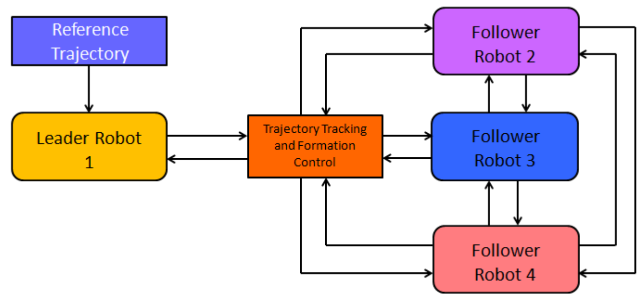

Fig. 10. Interaction between robots in Experiment 2

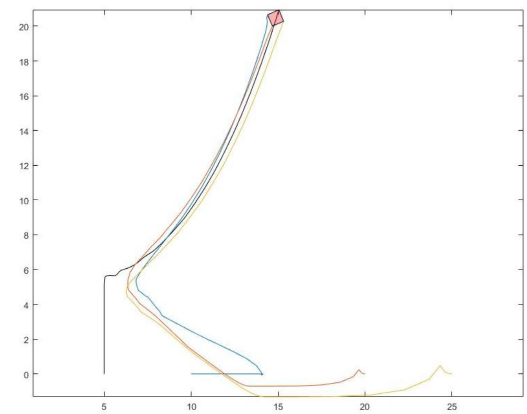

Fig. 11. Path trajectory in Exp.2

connectivity task. This is better represented in Fig. 12. Also the controller design is represented in the Fig. 13.

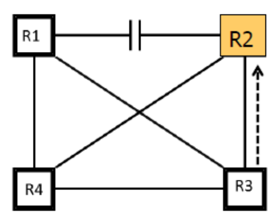

Fig. 12. Graph Based Connectivity with measurement error

During the execution of the simulation model in this experiment, initially robot R_2 deviated from its directed path assigned to it due to error in connectivity with Leader robot R_1. But with time and alternate path of information flow, robot R_2 could attain its desired position creating the exact square formation of the multi-robot system (Fig. 14).

VI. ANALYSIS

The experiments showcased in the previous section are aimed at presenting different methodologies and robot interaction in a decentralized scheme of control. The first experiment (Section V.B.1) only focuses on the formation control by initializing separate path to other three robots through a leader robot. The experiment does not explore the perception opportunities of the robots and hence due to the absence of interaction among the robots, the robots are

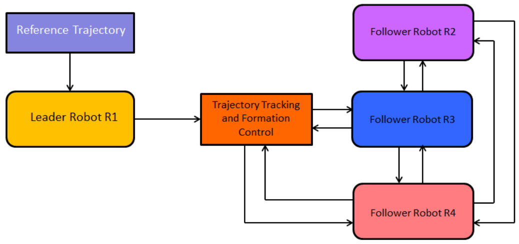

Fig. 13. Interaction between robots in Experiment3

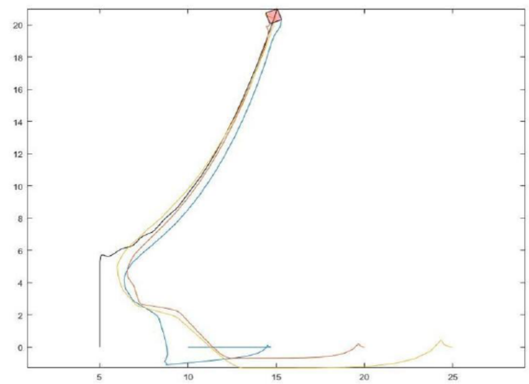

Fig. 14. Path Trajectory in Exp.3

unresponsive to the change in behaviour of the other robots in the formation.

The second experiment extends the leader-follower control strategy in the first experiment; however also enabling the robots to perceive distance (using distance sensors) of the obstacle or other robots. The perception capability equips the the robots to have interaction (not communication) with one another thus allowing us to introduce convergence algorithm to implement the formation control and centroid tracking. The control strategy in this experiment is found to be stable as well. However, the control strategy could have its drawback if the robots are not in the sensor range.

The lack of interaction (due to the sensor range) has been depicted in experiment 3. The experiment evolves the arrangement in the previous experiment by adding communication channels such as Bluetooth or Wi-Fi. It has been observed that the use of two hops to broadcast data is sufficient for a fast convergence and stability of the robotic system. "Broadcasting implicitly adds communication links and makes the communication network more stable; it proved to be particularly efficient when the connectivity was unstable or when obstacles could occlude lines-of-sight" [14].

VII. CONCLUSION

The paper has illustrated experiments to successfully demonstrated that non-holonomic unicycle robots could be driven in group creating a predefined formation. With the application of Graph theory, different behaviors can be achieved by changing the weights on the edges of the communication graph. Also the multi-robot model was operated under the test condition of breakdown in information exchange between two robots wherein an alternate path of information passage was established with the help of other members of the multi-robot system. We have not included the time, the robots take to achieve the formation.

VIII. FUTURE WORK

Though the bias factor allows us to keep a safe distance between the robots. It would be interesting to also notice the behaviour in case of an obstacle. We have mainly restricted ourselves to the development and simulation aspects on the MATLAB; however, a real development will help to understand the concept on a deeper level.

ACKNOWLEDGMENT

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