

$$i. \quad |\psi\rangle = \frac{1}{\sqrt{2}} |00\rangle + \frac{1}{2} |10\rangle - \frac{1}{2} |11\rangle$$

a) Probability of measuring both qubits as 0

$$= \left(\frac{1}{\sqrt{2}}\right)^2 \quad \text{since } |00\rangle \text{ is the only state with two 0 qubits.}$$

$$= \frac{1}{2}.$$

b) Probability of measuring first qubit as 1

$$= \left(\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2 \quad \text{since } |10\rangle \text{ and } |11\rangle \text{ are the only state with qubit starting with 1}$$

$$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

c) Probability of measuring second qubit as 0

$$= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 \quad \text{since } |00\rangle \text{ and } |10\rangle \text{ are the only states with 0 at second qubit}$$

$$= \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

d) After measuring the first qubit as 0, then we know that the only part of  $|\psi\rangle$  that has the first qubit as 0 is  $\frac{1}{\sqrt{2}} |00\rangle$ . However, we need to renormalise the state to make sure it has a probability of one. So the new state of the system after measuring the first qubit as 0 is  $|\psi'\rangle = |00\rangle$ .

e) After measuring the first qubit as 1, then we know that the only parts of  $|\psi\rangle$  that have the first qubit as 1 are  $\frac{1}{2} |10\rangle - \frac{1}{2} |11\rangle$ . However, we need to renormalize the state to make sure it has a probability of one. So, the new state of the system after the measurement is  $|\psi'\rangle = \frac{1}{\sqrt{2}} |10\rangle - \frac{1}{\sqrt{2}} |11\rangle$ .

2. For two fair coins: let us consider 0 as heads and 1 as tails. While they are in the air, the state of the two-coin system is:

$$|\psi\rangle = \frac{1}{2} |00\rangle + \frac{1}{2} |01\rangle + \frac{1}{2} |10\rangle + \frac{1}{2} |11\rangle.$$

where  $\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = 1.$

3. For two six-sided dice. let us consider 0 as the state for (2, 4, 6) and 1 for the odds (1, 3, 5) in the dice. There exist 36 combinations. But let us represent this in a two qubit system.

$$|\psi\rangle = \underbrace{\frac{1}{2} |00\rangle}_{\text{all even}} + \underbrace{\frac{1}{2} |01\rangle}_{\substack{\text{one even} \\ \text{one odd}}} + \underbrace{\frac{1}{2} |10\rangle}_{\substack{\text{one odd} \\ \text{one even}}} + \underbrace{\frac{1}{2} |11\rangle}_{\text{all odd}}.$$

Now, we only need probability for one even and other odd.

the probability is  $\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \boxed{\frac{1}{2}}.$