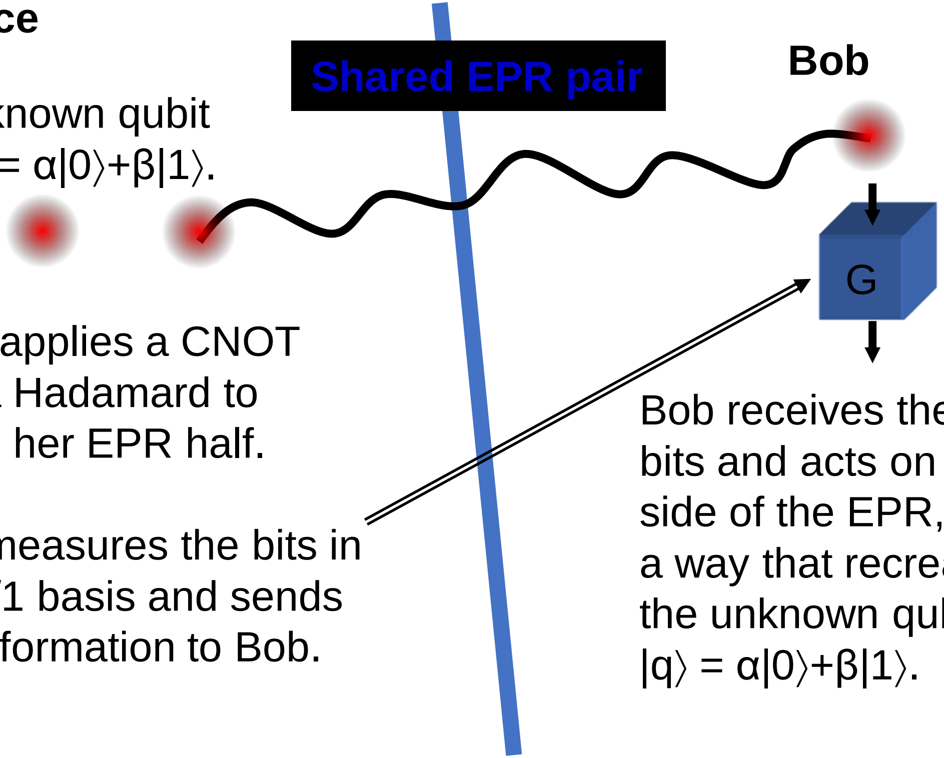
Teleportation and Superdense Coding

Dr. Moshiur Rahman

Teleportation

Alice

unknown qubit
$|q\rangle = \alpha|0\rangle+\beta|1\rangle$.



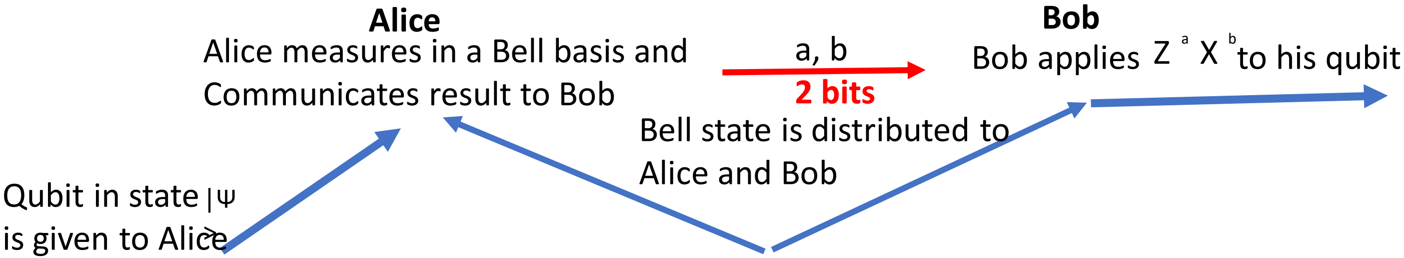
Alice applies a CNOT
and a Hadamard to
q and her EPR half.

She measures the bits in
the 0/1 basis and sends
the information to Bob.

Bob

Bob receives the two
bits and acts on his
side of the EPR, in
a way that recreates
the unknown qubit
$|q\rangle = \alpha|0\rangle+\beta|1\rangle$.

Entanglement and Teleportation



Entanglement and Teleportation

Alice

Alice measures in a Bell basis and
Communicates result to Bob

a, b

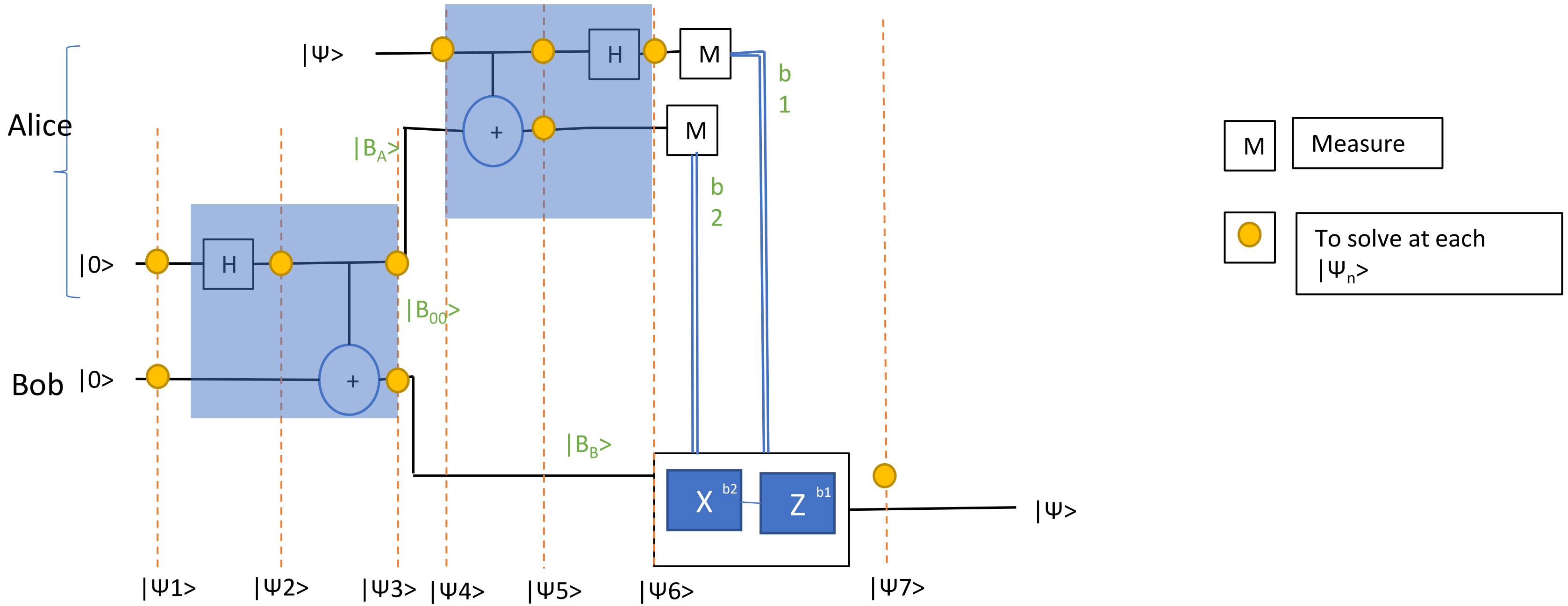
2 bits

Bob

Bob applies $Z^a X^b$ to his qubit

Qubit in state |Ψ> is given to Alice

Bell state is distributed to Alice and Bob



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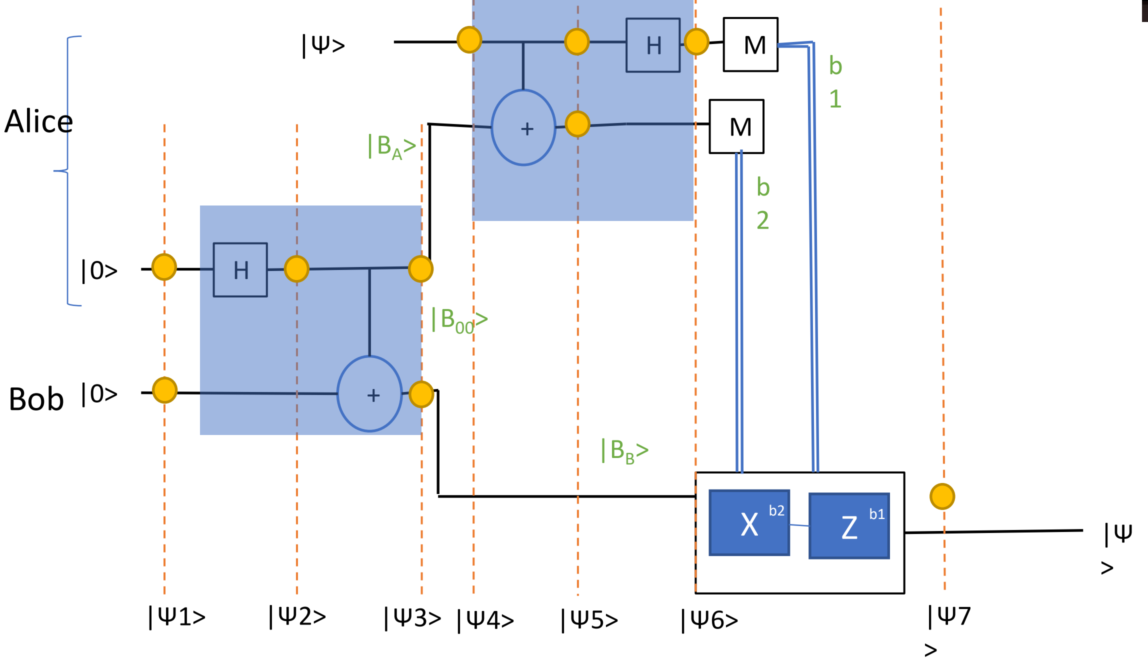
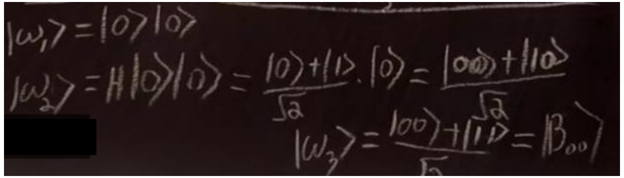
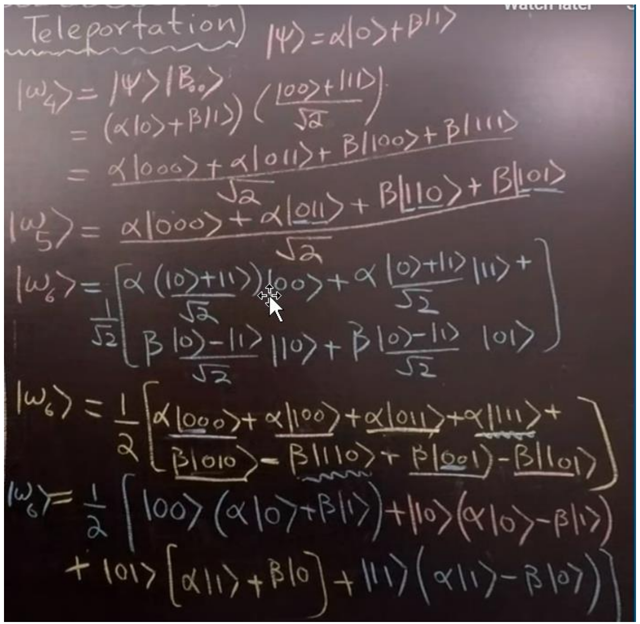
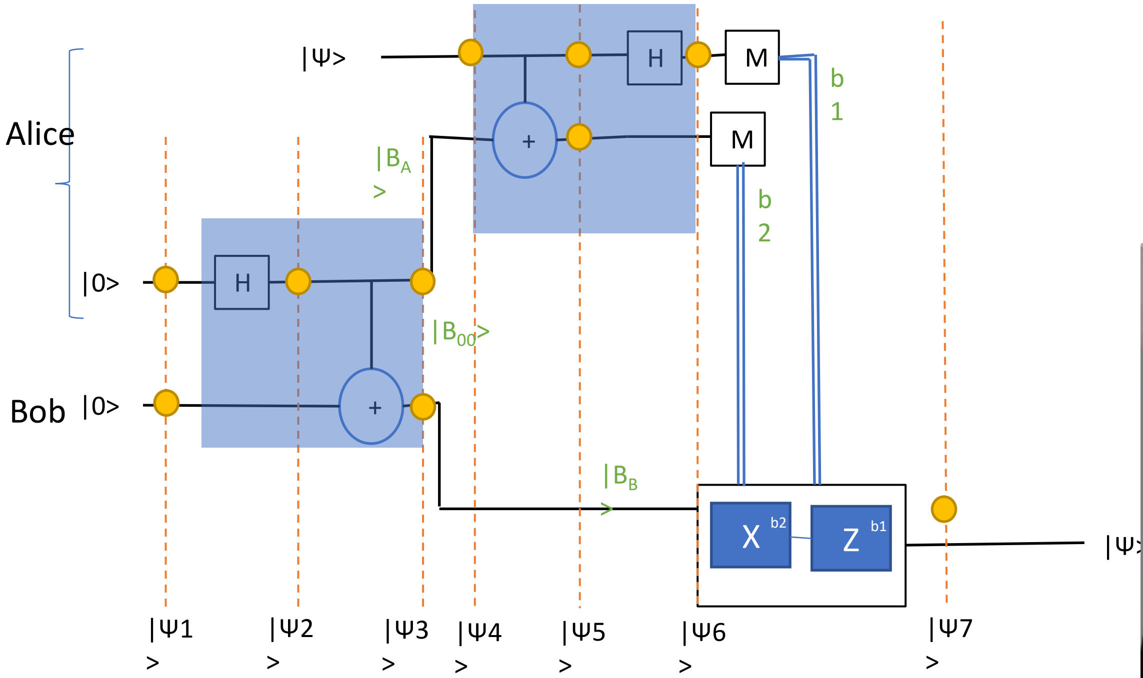
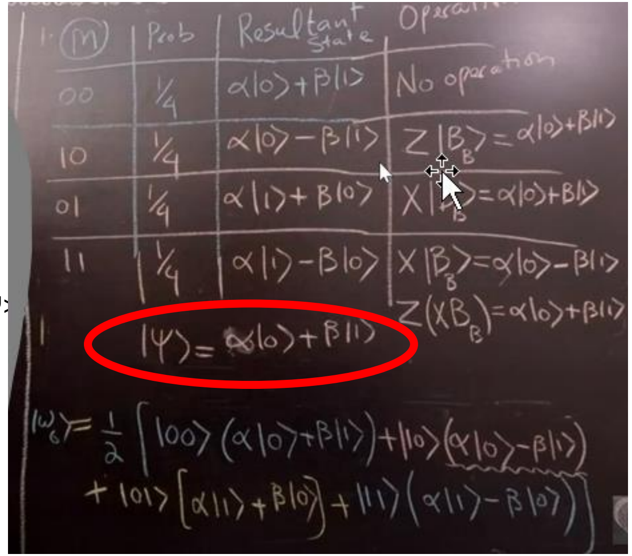






Entanglement and Teleportation Circuit





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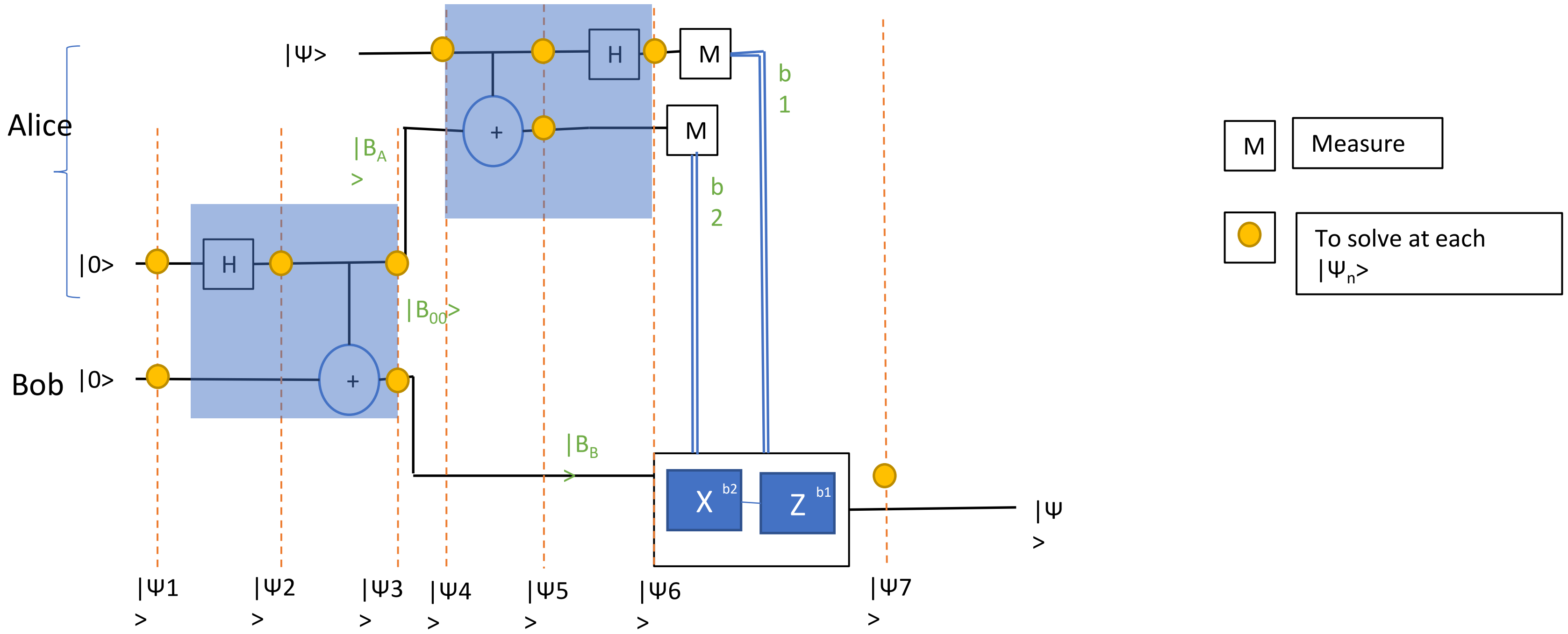

What is Teleportation?

It is not like how you see it in the movies! We can teleport *information*, but not matter. Specifically, we can transfer the state of one qubit to another over long distances.

Recently, a Chinese team used a satellite to teleport a qubit 1400 kilometers, setting the world record! Using fiber optic cables, the maximum range is around 100 kilometers because of *decoherence*.

Decoherence is when a qubit interacts too much with its environment and collapses prematurely. The superposition is destroyed and information is lost.

Why is Teleportation Needed?

In classical computers, there is no such thing as teleportation. Computers can make a copy of the data and send the copy. This is impossible in quantum computers because of the *no-cloning theorem*.

It is impossible to create a copy of an arbitrary quantum state because we would know the position and momentum of a particle which violates the Heisenberg uncertainty principle.

To move the state of a qubit over long distances without physically transporting it, we can teleport the original state to the target qubit.

Overview of the Teleportation Process

There are 3 qubits used in the process of teleportation:

The qubit whose state will be teleported, called the *input*

The qubit who will receive the state, called the *output*

An auxiliary qubit to help the process, called the *ancilla*

The ancilla and output qubits are entangled at **location A** and the output is sent to the destination, **location B**. The teleportation can now occur any time after this.

The teleportation process is started at **location A**, then information is sent via a classical channel to **location B** to finish the process.

Since we are sending information classically, teleportation is not faster than light.

The Teleportation Process

1) At **location A**, we have a single-qubit state: $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$
   where α and β are unknown. If they were known, we could just send them classically and re-create the quantum state at the destination.

1) Entangle the ancilla with the output qubit, forming a bell state:
   $|\vartheta\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle \right)$
   Then send the output qubit to **location B**.

1) Create a tensor product between the input and entangled state:
   $\omega_1 = \Psi \otimes \vartheta = \frac{1}{\sqrt{2}}\left(\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle \right)$

The Teleportation Process cont.

4) The two qubits in **location A** are sent through a CNOT gate.

$$\omega_1 = \frac{1}{\sqrt{2}} (\alpha|000\rangle + \alpha|011\rangle + \beta|110\rangle + \beta|101\rangle)$$

5) The input qubit is sent through a Hadamard gate.

$$\omega_2 = ½ [|00\rangle(\alpha|0\rangle + \beta|1\rangle) + |01\rangle(\alpha|1\rangle + \beta|0\rangle) + |10\rangle(\alpha|0\rangle - \beta|1\rangle) + |00\rangle(\alpha|1\rangle - \beta|0\rangle)]$$

Using the identity matrix, I and the three Pauli matrices, X, Y, and Z:

$$\omega_2 = ½ [|00\rangle\, I|\Psi\rangle + |01\rangle\, X|\Psi\rangle + |10\rangle\, Z|\Psi\rangle + |00\rangle\, XZ|\Psi\rangle]$$

The Teleportation Process cont.

6) Measure the input and ancilla qubits and send the result to **location B** via a classical channel. Check the value and apply gates as shown

Value	Gates
00	I
01	X
10	Z
11	ZX

The result of the measurement in **location A** returns 00, 01, 10, or 11.
This corresponds to the output qubit's state being:
$(\alpha|0\rangle + \beta|1\rangle)$, $(\alpha|1\rangle + \beta|0\rangle)$,
$(\alpha|0\rangle - \beta|1\rangle)$, or $(\alpha|1\rangle - \beta|0\rangle)$

Applying the gates shown in the table will transform the output qubit into $(\alpha|0\rangle + \beta|1\rangle)$.