7.5 Trig Functions

0. Find the domain and range of each:

a. $f(\theta) = \cos\theta$

b. $g(\theta) = \sin\theta$

c. $h(\theta) = \tan\theta$

1. Beginning in standard position, run your finger around the unit circle in a positive direction multiple times. Concentrate only on the *height above* or *below* the x axis. Describe the behavior of the sine function with regard to intervals of increase and decrease.

2. a) Where in the rotation will the maximum of the sine function be achieved?

b) On a sheet of graph paper, label the x axis in increments of $\frac{\pi}{6}$. Label the vertical axis in increments of $\frac{1}{2}$.

c) For each Quadrantal angle, x, where $0 \le x \le 2\pi$, determine the corresponding sine value, y. Plot these points.

d) Plot points corresponding to $x = \frac{\pi}{6}$ and $x = \frac{\pi}{3}$.
How will sine values in Quadrant II compare? Plot ordered pairs for x values corresponding to Quadrant II angles.

e) How would values corresponding to Quadrant III and IV angles differ from Quadrants I and II? Complete the graph of one cycle.

f) Continue the graph over the interval $(-2\pi, 0)$. How are $\sin x$ and $\sin(-x)$ related? What kind of function does this fact tell us about the function $y = \sin x$?

3. Will $t(x)=\dfrac{\sin x}{x}$ be even, odd, or neither? What about $m(x)=(x^3-2x)sinx$?

4. Half the vertical distance between the maximum value and the minimum value in a sine graph is called its **amplitude**. A graph's **midline** is a line halfway between the maximum and minimum y-values on this graph.

For $y=\sin x$, the amplitude is 1 and the midline is $y=0$.

5. State the midline and amplitude of each, then sketch. Complete at least one curve in its entirety.

a) $y=1-\sin x$

b) $y=\dfrac{1}{2}\sin x-2$

c) $y=3\sin x$

6. Any function that has the property $f(x+p)=f(x)$ for a fixed p is called a periodic function. The length of the period here is represented by p.

What is the period of $y=\sin x$?

7. Graph $h(x)=\sin 2x$ graph $0\le\theta\le 2\pi$. What is its period?

How many cycles exist on $[0,2\pi]$? The number of cycles which occur in an interval of 2π radians is called the function's **frequency**. What is the relationship between the period and the frequency?

8. For $y = a\sin bx + d$, possibly in terms of a, b and/or d, and using absolute value where necessary, find an expression for the

a) period

b) equation of the midline

c) amplitude

d) range

9. Graph $f(\theta) = \cos(\theta)$ on the interval $[-2\pi, 2\pi]$

10. Graph $f(\theta) = 2\cos(\theta) - 1$ on the interval $[-2\pi, 2\pi]$

11. Graph $y = 2\sin\frac{x}{2} + 3$

12. Find all zeros of $y = 3\sin 4x - 3$

13. Graph $y = 3\cos\left(\frac{x}{2}\right)$

14. Write the equation of a sine function with period 3π and amplitude 3.

15. Write the equation of a sine function with a range of $[-1,9]$ and a period of $\pi/4$.

16. Exploring the graph of $h(\theta) = \tan\theta$

a. Evaluate $\lim_{\theta \to \frac{\pi}{2}^+} \tan\theta$ and. $\lim_{\theta \to \frac{\pi}{2}^-} \tan\theta$

b. on $[0,2\pi]$, for what values of θ does $\tan\theta = 1$? $\tan\theta = -1$?

c. What is the period of $h(\theta)$?

d. Sketch $h(\theta)$ on $[0,2\pi]$

17. Graph $y = \tan(3x)$

18. a. Solve $2\sin x - 1 = 0$ on $[-2\pi, 2\pi]$

b. Find the general solution (all solutions) to $2\sin x - 1 = 0$.
Consider $y = \sin x$ is a periodic function

19. Find the general solution to: $2\sin(2x) = -1$

20. Find all solutions on $[0, \pi]$ of $\cos^2(3x) - \cos(3x) = 0$

21. Find the general solution to $2\tan^2 x + \frac{3}{\cos x} = 0$