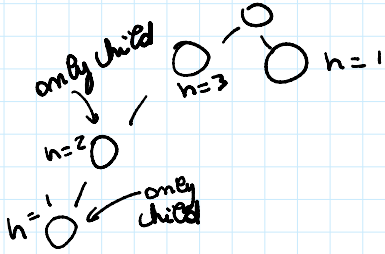# 1. AVL Tree:

a. The difference in heights of the right and left subtree can be at most 1.

If $n = 1$:
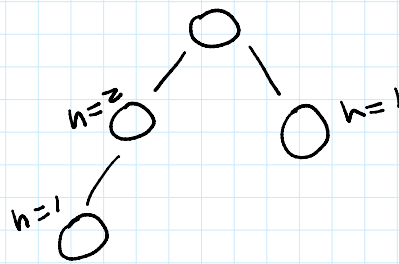
○

There are <u>no</u> only children.

An only child's parent CANNOT be an only child, because this would break the property of the AVL tree:
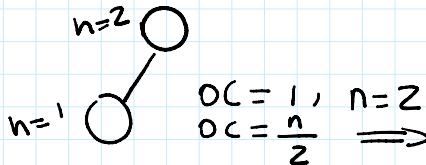
only child ○ ← ○ $n=3$ ○ $h=1$
$n=2$ ○
$h=1$ ○ ← only child

violates
AVL
properties

$n=2$ ○
$n=1$ ○   ○ $h=1$
$n=1$ ○

Valid
AVL   tree

If the parent is not the root, for every only child, the parent must have a sibling ⟹ there must be two nodes with a sibling for every only child, so the number of only children would be $< \frac{n}{2}$.

If the parent is the root :

$n=2$ ○
$h=1$ ○ $OC = 1, n = 2$
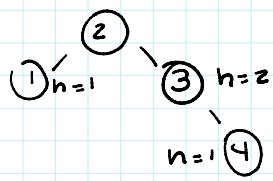$OC = \frac{n}{2}$ ⟹ The number of only children is at most $n/2$.

b. No the statement is false.

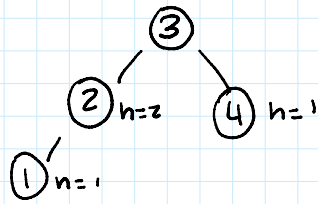Let's take the following as a counter example :

2, 1, 3, 4   3, 2, 1, 4

2, 1, 3, 4

3, 2, 1, 4

2 ① Root has to be black.

② A red node cannot have a red child.

③ Black height must remain unchanged.

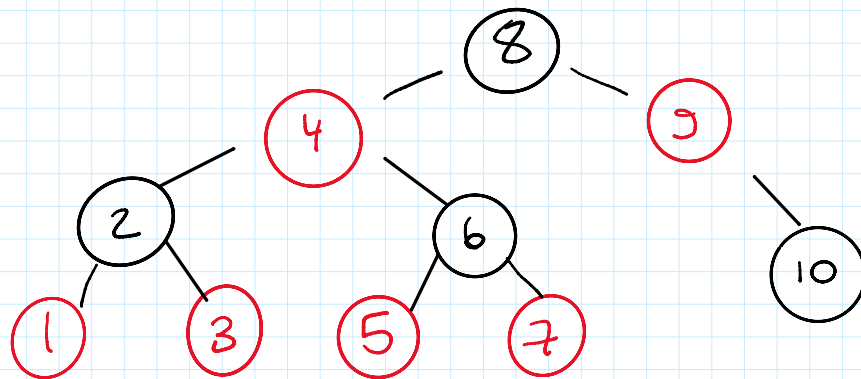A red black tree is a BST $\Longrightarrow$ In order traversal will sort the numbers.

$$R \ B \ R \ R \ R \ B \ R \ B \ R \ B$$
$$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10$$

cannot be the root (③ will be violated)

cannot be the root (③ will be violated)

In order traversal: 1 - 2 - 3 - 4 - 5 - 6 - 7 - 8 - 9 - 10