Homework 4

Wednesday, March 16, 2022 8:53 AM

1. Heap

Step 1: we create any 4 pairs and compare the two numbers of each pair $\Longrightarrow$ 4 comparisons

n_1 n_2 → $n_2 > n_1$

n_3 n_4 → $n_3 > n_4$

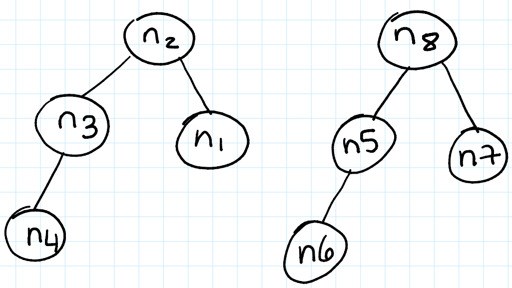
n_5 n_6 → $n_5 > n_6$

n_7 n_8 → $n_8 > n_7$

Step 2: we compare the biggest numbers of every two pairs $\Longrightarrow$ 2 comparisons

n_2 n_3 → $n_2 > n_3$

n_5 n_8 → $n_8 > n_5$



Step 3: we compare n_2 and n_8 to determine the root of the max heap $\Longrightarrow$ 1 comparison

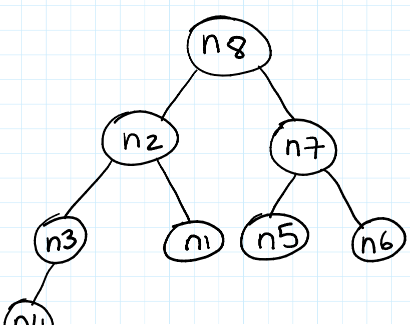
n_2 n_8 → $n_8 > n_2$

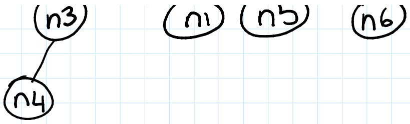
$\Longrightarrow$ n_8 is going to be the root, and n_2 is going to be one of its children.

Step 3: to determine the second child of the root, we compare n_5 and n_7 $\Longrightarrow$ 1 comparison

n_5 n_7 → $n_7 > n_5$ $\Longrightarrow$ n_7 is the second child of the root, and $n_7 > n_5$ implies $n_7 >$ children of n_5.

Now we can build the max heap:





This method works for any 8 numbers $n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8$.

$\Longrightarrow$ We can create a max heap of 8 numbers using:

$4+2+1+1 = \boxed{8 \text{ pairwise comparisons}}$

2. Hashing

631, 23, 33, 19, 44, 195, 64

a)
$h(631) = 1$
$h(23) = 3$
$h(33) = 3$
$h(19) = 9$
$h(44) = 4$
$h(195) = 5$
$h(64) = 4$

0		
1	631	
2		
3	23	→ 33 → ∅
4	44	→ 64 → ∅
5	195	
6		
7		
8		
9	19	

b)
$h_0(631) = h(631)\,\%\,10 = 1$
$h_0(23) = h(23)\,\%\,10 = 3$
$h_0(33) = h(33)\,\%\,10 = 3$
$\Rightarrow h_1(33) = [h(33)+1]\,\%\,10 = 4$
$h_0(19) = h(19)\,\%\,10 = 9$
$h_0(44) = h(44)\,\%\,10 = 4$
$\Rightarrow h_1(44) = [h(44)+1]\,\%\,10 = 5$
$h_0(195) = h(195)\,\%\,10 = 5$
$\Rightarrow h_1(195) = [h(195)+1]\,\%\,10 = 6$
$h_0(64) = h(64)\,\%\,10 = 4$
$\Rightarrow h_1(64) = [h(64)+1]\,\%\,10 = 5$
$\Rightarrow h_2(64) = [h(64)+2]\,\%\,10 = 6$
$\Rightarrow h_3(64) = [h(64)+3]\,\%\,10 = 7$

0	
1	631
2	
3	23
4	33
5	44
6	195
7	64
8	
9	19

c)
$h_0(631) = h(631)\,\%\,10 = 1$
$h_0(23) = h(23)\,\%\,10 = 3$
$h_0(33) = h(33)\,\%\,10 = 3$
$\Rightarrow h_1(33) = [h(33)+1^2+1]\,\%\,10 = 5$
$h_0(19) = h(19)\,\%\,10 = 9$

0	
1	631
2	
3	23

$\Rightarrow h_1(33) = [h(33)+1^2+1] \% 10 = 5$

$h_0(19) = h(19) \% 10 = 9$

$h_0(44) = h(44) \% 10 = 4$

$h_0(195) = h(195) \% 10 = 5$

$h_1(195) = [h(195)+1^2+1] = 7$

$h_0(64) = h(64) \% 10 = 4$

$\Rightarrow h_1(64) = [h(64)+1^2+1] \% 10 = 6$

2	
3	23
4	44
5	33
6	64
7	195
8	
9	19

d) $H_0(631) = h(631) \% 10 = 1$

$H_0(23) = h(23) \% 10 = 3$

$H_0(33) = h(33) \% 10 = 3$

$\Rightarrow H_1(33) = [h(33) + h'(33)] \% 10$

$= [3 + (7 - 33 \% 7)] \% 10$

$= [3 + (7 - 5)] \% 10 = 5$

$H_0(19) = h(19) = 9$

$H_0(44) = h(44) = 4$

$H_0(195) = h(195) = 5$

$\Rightarrow H_1(195) = [h(195) + h'(195)] \% 10$

$= [5 + (7 - 195 \% 7)] \% 10$

$= [5 + (7 - 6)] \% 10 = 6$

$H_0(64) = h(64) = 4$

$H_1(64) = [h(64) + h'(64)] \% 10$

$= [4 + (7 - 64 \% 7)] \% 10$

$= [4 + (7 - 1)] \% 10 = 0$

0	64
1	631
2	
3	23
4	44
5	33
6	195
7	
8	
9	19