Homework 6

Sunday, April 17, 2022 5:25 PM

1. Celebrity

The celebrity in the group does not have any edges leaving it. This means that as soon as the celebrity node is visited, no other edges or vertices can be visited. An eulerian cycle is a cycle that passes once through all edges of a graph. Since we cannot visited any edges after visiting the celebrity node, the graph G CANNOT HAVE AN EULERIAN CYCLE.

A Hamiltonian cycle visits every vertex once. Although visiting each vertex of the graph exactly once could be possible, a Hamiltonian cycle requires the start and end vertex to be the same, which isn't possible in this case since visiting the celebrity does not allow the visit of any edge or vertex afterwards. Therefore the graph G CANNOT HAVE A HAMILTONIAN CYCLE.

2. Bipartite Graph:

For a bipartite graph, ALL EDGES need to go from one set to another. By definition, a cycle starts and ends at the same vertex. Therefore, if we have two sets of vertices V_A and V_B, a cycle starting at V_1 would end at V_A.

To end at the same set that we started at, the bipartite graph would need a cycle with an EVEN number of edges ONLY, or else we would have edges within the same set of vertices, which violates the laws of bipartite graphs.

$\Rightarrow$ We need an even number of edges such that the edges are:

$$\{\underset{A}{v_1}, \underset{B}{v_2}\}, \{\underset{B}{v_2}, \underset{A}{v_3}\}, \{\underset{A}{v_3}, \underset{B}{v_4}\}, \ldots, \{\underset{B}{v_n}, \underset{A}{v_1}\}$$

One new distinct vertex for each edge $\Rightarrow$ even number of vertices.

$\Rightarrow$ A Bipartite graph can have an EVEN number of distinct vertices ONLY.

One new distinct vertex for each edge $\Rightarrow$ even number of vertices.

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