

# EC330 Applied Algorithms and Data Structures for Engineers Spring 2022

## Homework 1

**Out:** January 26, 2022

**Due:** February 7, 2022

*This homework has a written part and a programming part. Both are due at 11:59 am (by noon) on February 7. You should submit both parts on Gradescope.*

*This is an **individual** assignment. See course syllabus for policy on collaboration.*

### 1. Sums [15 pt]

Provide a closed-form solution to the following problems. Make sure you show the steps.

- a)  $\sum_{i=1}^{33} \left(\frac{1}{2}\right)^i$
- b)  $\sum_{i=0}^{\infty} \left(\frac{2}{5}\right)^i$
- c)  $\sum_{i=1}^N (i^3 + 3i^2 - 6i + 9)$

### 2. Exponents and Logs [15 pt]

Simplify the following expressions. Make sure you show the steps.

- a)  $x^1 \cdot x^2 \cdot x^3 \dots x^{330}$
- b)  $\log_x x^{330x}$
- c)  $\log_{330}(330^{330} \cdot 330)$

### 3. Combinatorics [5 pt]

How many integer solutions of  $x_1 + x_2 + x_3 = 10$  satisfy  $x_1 \geq 2$ ,  $x_2 \geq 1$  and  $x_3 \geq -2$ ?

### 4. Induction [15 pt]

Consider the function  $f$  defined as follows.

$$\begin{aligned} f(x) &= x & x &= 1, 2, 3 \\ f(x) &= f(x-1) + f(x-2) + f(x-3) & x &\in \mathbb{N} \text{ and } x > 3 \end{aligned}$$

Show that  $\forall x \in \mathbb{N}, f(x) < 2^x$ .

### 5. Program Understanding [10 pt]

- a) Describe the behavior of the following function foo (e.g. what it returns) for all possible values of x. Note that its behavior can be different depending on what x is.

```
int foo(int x) {
    if (x==1) return 1;
    else return 2*foo(int(x/2));
}
```

- b) What is the value of sum after the double-loop exits in the following program? Express your answer as a function of n. Show your steps.

```
int sum = 0;
for (int i = 0; i < n; i++) {
    sum = sum + (1 << i);
}
```

## 6. Programming [40 pt]

*Make sure to acknowledge any source you consult at the top of your program.*

*Do not include a main in your submitted files.*

- a) You are given an array of lower-case letters (e.g., {b, b, x}). Suppose every letter appears even number of times except for one. Write a C++ program that finds this odd-appearing letter. Your program should run in time  $O(n)$  where n is the size of the input array. **[20 pt]**

Example #1:

Input: {b, b, x}

Output: x

Example #2:

Input: {c, b, d, c, c, d, b, b, b}

Output: c

Your job is to implement the function *findOdd* in *findOdd.cpp*.

Submit your completed *findOdd.cpp* file on Gradescope.

- b) Consider the following sequence of numbers  $A_i$  such that  $A_0 = 1$  and  $A_{n+1} = \sum_{i=0}^n A_i A_{n-i}$  for  $n \geq 0$ . For example, the second number in the sequence  $A_1 = \sum_{i=0}^1 A_i A_{1-i} = A_0 A_0 = 1$ , and the third number  $A_2 = \sum_{i=0}^2 A_i A_{2-i} = A_0 A_2 + A_1 A_1 = 2$ , and the fourth number  $A_3 = 5$  (try to work this out yourself on paper). Write a C++ program that generates  $n^{\text{th}}$  number (i.e.  $A_{n-1}$ ) in this sequence. **[20 pt]**

Example:

Input: 3 (explanation: 3<sup>rd</sup> number in the sequence, i.e.  $A_2$ )

Output: 2

Your job is to implement the function *genA* in *genA.cpp*. Try to make your algorithm as efficient as you can. You can assume the input is a natural number. Submit your completed *genA.cpp* file on Gradescope.