Homework 5.

PS1, $f(x) = \binom{3}{x}\left(\frac{1}{4}\right)^{x}\left(\frac{3}{4}\right)^{3-x}$ $\quad x = 0, 1, 2, 3$

The mean of X is.

$$E(X) = \sum_{\text{all } x} x\, f(x) = \sum_{x=0}^{3} x.\binom{3}{x}\left(\frac{1}{4}\right)^{x}\left(\frac{3}{4}\right)^{3-x}.$$

$$= 0 + 1.\binom{3}{1}\left(\frac{1}{4}\right)^{1}\left(\frac{3}{4}\right)^{2} + 2\binom{3}{2}\left(\frac{1}{4}\right)^{2}\left(\frac{3}{4}\right)^{1} + 3.\binom{3}{3}\left(\frac{1}{4}\right)^{3}\left(\frac{3}{4}\right)$$

$$= \frac{3}{4}$$

PS3, $f(x) = \begin{cases} 2(1-x) & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$

By definition, the average is.

$$E(X) = \int_{-\infty}^{+\infty} x\, f(x)\,dx = \int_{0}^{1} x.[2(1-x)]\,dx$$

$$= \int_0^1 x.(2-2x)\,dx = \int_0^1 (2x - 2x^2)\,dx = x^2 - \frac{2}{3}x^3 \Big|_0^1$$

$$= 1 - \frac{2}{3} = \frac{1}{3}.$$

Since the units are in \$5000, the the average profit is

Average profit $= \$\,\frac{5000}{3}$

PS2, $E(X) = \int_0^1 \left(x \frac{4}{\pi(1+x^2)}\right) dx$ $\quad u = x^2 \Rightarrow du = 2x\,dx$

$$= \int_0^1 \frac{2\,du}{\pi(1+u)} = \frac{2}{\pi} \ln(1+u) \Big|_0^1 = \frac{2\ln 2}{\pi}.$$

PS4, From PS3 we have

$$E(X) = 2\int_0^1 x(1-x)dx = 2\left(\frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_0^1 = \frac{1}{3}.$$

$$E(X^2) = 2.\int_0^1 x^2(1-x)dx = 2\left(\frac{x^3}{3} - \frac{x^4}{4}\right)\Big|_0^1 = \frac{1}{6}.$$

Hence.

$$Var(X) = E(X^2) - [E(X)]^2 = \frac{1}{6} - \left(\frac{1}{3}\right)^2$$

$$= \frac{1}{18}.$$

$$\delta = \sqrt{1/18} = 0,2357.$$

PS5, X, Y follow a joint distribution

$$f(x,y) = \begin{cases} 2 & , 0 < x < y < 1 \\ 0 & , \text{otherwise.} \end{cases}$$

Determine the correlation between X and Y

$$f(x) = \int_{-\infty}^{+\infty} f_{x,y}(x,y)\,dy = 2.$$

PS6,

x	-2	3	5
$f(x)$	0.3	0.2	0.5

X: discrete variables

$\rightarrow \mu(X) = \sum_{\text{all } x} x \cdot f(x) = (-2).(0,3) + 3.0.2 + 5.0,5 = 2,5 = E(X)$

$E(X^2) = (-2)^2 . 0,3 + 3^2 . 0.2 + 5^2 . 0,5 = 15,5$

So $\sigma^2 = E(X^2) - [E(X)]^2 = 15,5 - (2,5)^2 = \frac{37}{4}$

$\rightarrow$ The standard deviation of X $\sigma = \sqrt{\sigma^2} = \sqrt{\frac{37}{4}} = \frac{\sqrt{37}}{2}$

PS7,

x	0	1	2	3
$f(x)$	0.4	0.3	0.2	0.1

$\mu(X) = \sum_{\text{all } x} x f(x) = 0.0,4 + (1.0,3) + (2.0,2) + (3.0,1) = 1$

$E(X^2) = \sum_{\text{all } x} x^2 f(x) = (0.0,4) + (1.0,3) + (2^2 . 0,2) + (3^2 . 0,1) = 2$

$Var(X) = \sigma = E(X^2) - [\mu(X)]^2 = 2 - 1 = 1$

PS8,

Theorem 4.5. If a, b are constants, then

$$E(aX + b) = a E(X) + b.$$

Theorem 4.6. $E[g(X) \pm h(X)] = E[g(X)] \pm E[h(X)]$.

From PS7, $\mu(X) = E(X) = \sum_{\text{all } x} x f(x) = 1$

$\mu(Z) = E.(5X + 3) = 5.E(X) + 3 = 5.1 + 3 = 8.$

From PS7, $E(X^2) = 2.$

$E(Z^2) = E[(5X+3)^2] = 25 E(X^2) + 30 E(X) + 9 = 25.(2) + 30.1 + 9$

$= 89$

$\rightarrow \sigma^2(z) = Var(Z) = E(Z^2) - [E(Z)]^2$

$= 89 - 8^2 = 25.$

PS9) $\sigma^2 x = 5$ Find Var of $Z = -2X + 4Y - 3$ or $\sigma^2(z)$.

$\sigma^2 y = 3$

X, Y are independent random variables

$\sigma^2(Z) = \sigma^2(-2X + 4Y - 3) = (-2)^2 \sigma^2 x + 4^2 \sigma^2 y$

$= (-2)^2 (5) + 4^2 (3)$

$= 68$

PS11) X, Y: continuous joint distribution $f(x,y) = \begin{cases} 12y^2 & 0 < y < x < 1 \\ 0, & \text{otherwise} \end{cases}$

$E(XY) = \int_{x=0}^{1} \int_{y=0}^{x} xy\,12y^2\,dx\,dy = \int_{x=0}^{1} \int_{y=0}^{x} xy^3\,dx\,dy$

$= 12 \int_{x=0}^{1} \frac{xy^4}{4} \Big|_{y=0}^{x} = 3. \int_{x=0}^{1} x^5\,dx = \frac{x^6}{2} \Big|_0^1$

$= \frac{1}{2}.$

PS12) X: is the number of boys selected

Y = 8 − X ______ girls selected

$E(X - Y) = E(X - (8 - X))$

$= E(2X - 8)$

$= 2E(X) - 8$ → XS Chọn số bạn nam tr…

$E[X] = \sum_{i=1}^{8} \underset{\text{Boys}}{E(B)} = 8 . \underset{x \cdot f(x)}{\frac{10}{25}} = \frac{16}{5}$

$E(X - Y) = 2 . \frac{16}{5} - 8 = \frac{-8}{5}$

PS 10, If an integer between 1 and 100 is chosen at random
expected value?

- The probability for an integer chosen is always alw equal to $\frac{1}{100}$

$$E(X) = \sum_{x=1}^{100} x \times \frac{1}{100} = \frac{101}{2} = 505$$

PS13,

$$E(X(X-1)] = \mu(\mu-1) + \sigma^2$$

$$E(X^2 - X] = [E(X)]^2 - E(X) + \cancel{[V(X)]}\ \mathrm{Var}X$$

$$E(X^2) - E(X) = [E(X)]^2 - E(X) + \mathrm{Var}X$$

$$E(X^2) - [E(X)]^2 = \mathrm{Var}\ X \quad (\text{đpcm}).$$