

# **Fuzzy Logic**

## **Human-like decision Making**

Dr. Emad Natsheh

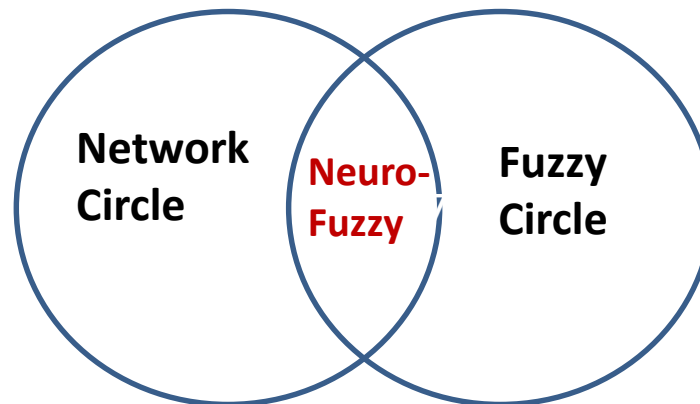
# Fuzzy Logic vs Neural Network

## ❖ Neural Network

- Is a powerful data modeling tool that is able to capture and represent complex input/output relationships
- It learns through examples
- The goal of this type of network is to create a model that correctly maps an input to the desired output using historical data

## ❖ Fuzzy Logic

- Is a mathematical method for answering questions with imprecise information
- It deals with reasoning that is approximate rather than fixed



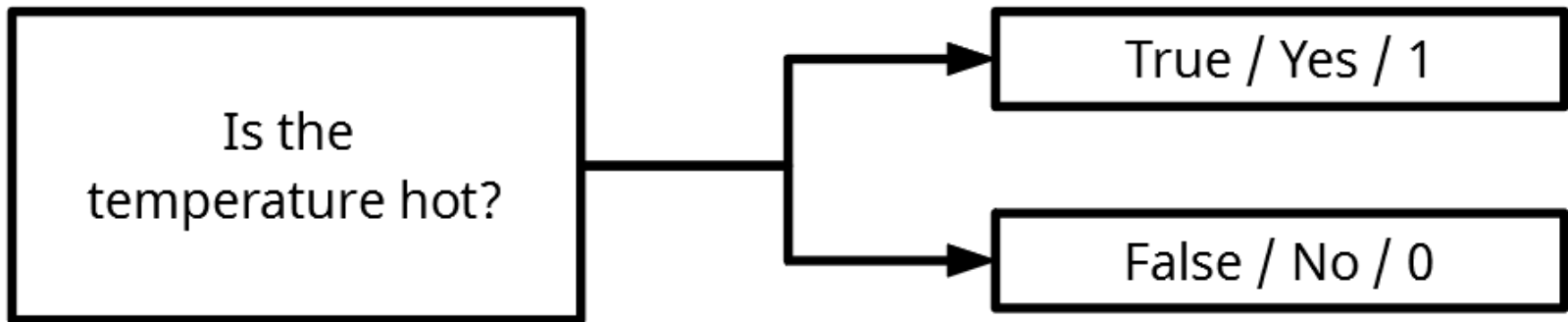
# Fuzzy Logic

- The opposite word (antonym) of fuzzy is crisp. **Fuzzy** means un-clear or ambiguous, and **crisp** means clear, clean, and sharp.
- Fuzzy logic was proposed by Zadeh in 1965, and was applied in steam engine control by Mamdani in 1974.
- Fuzzy logic was made practically useful in Japan in the 1990s.

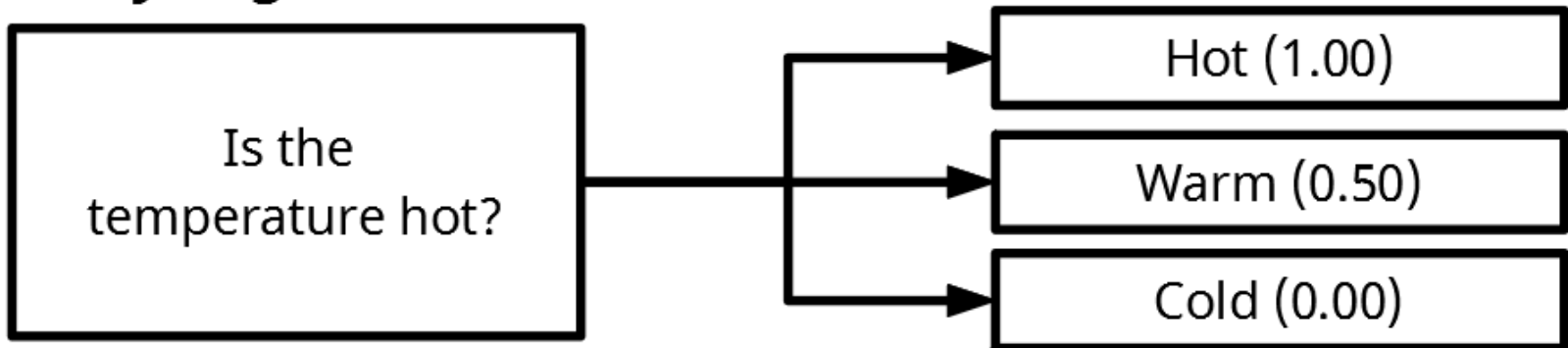


# Boolean (Crisp) Logic vs Fuzzy Logic

## Boolean Logic

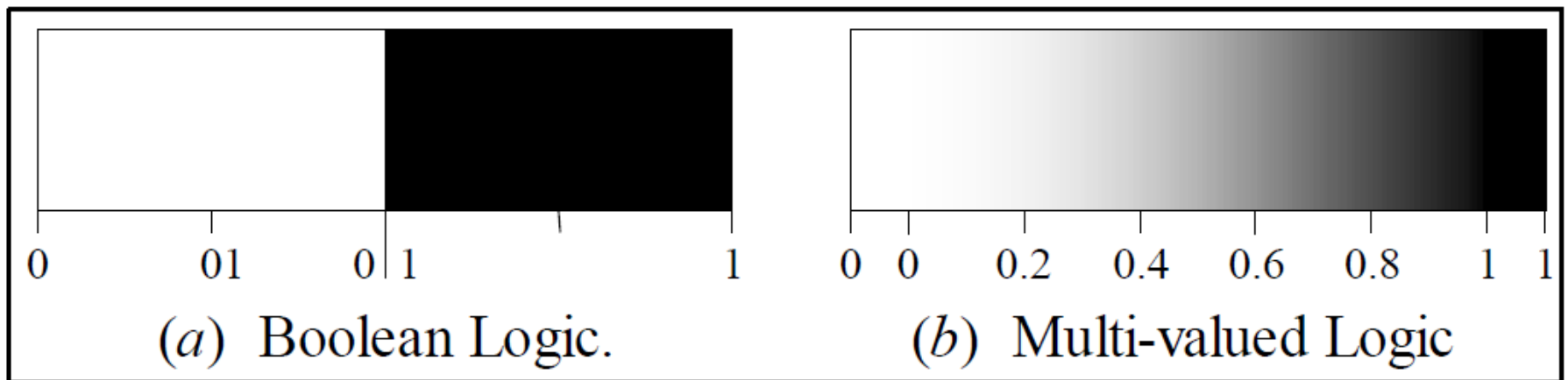


## Fuzzy Logic



# Boolean (Crisp) Logic vs Fuzzy Logic

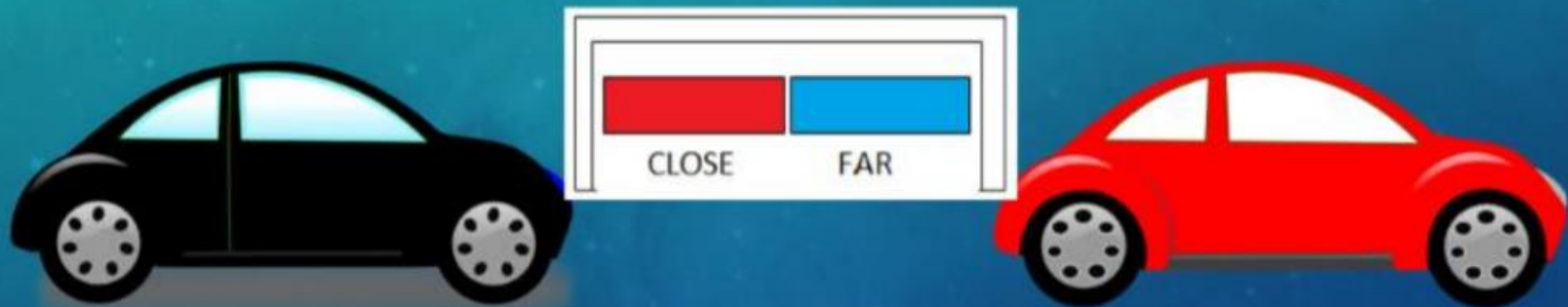
- Unlike Boolean logic, fuzzy logic is *multi-valued*
  - ▣ Fuzzy logic represents degrees of membership and degrees of truth
  - ▣ Things can be *part true* and *part false* at the same time



# Why Fuzzy Logic?

## Automatic Braking System

### Traditional Logic



Is car close? : 0 or 1 (NO or YES)

Brakes : 0 or 1 (OFF or ON)

# Why Fuzzy Logic?

Automatic Braking System

Fuzzy Logic



Is car close? : 0-1 (Range of No to Yes)

Brakes : 0-1 (Range of Off to On)

# Fuzzy Logic

- **Fuzzy Logic:** based on the idea that everything belong to a degree
- Fuzzy logic operations on fuzzy sets
  - Temp: {freezing, cool, warm, hot}
  - Cloud Cover: {overcast, partly cloudy, sunny}
  - Speed: {slow, fast}



# Example Fuzzy Set for Tall Men

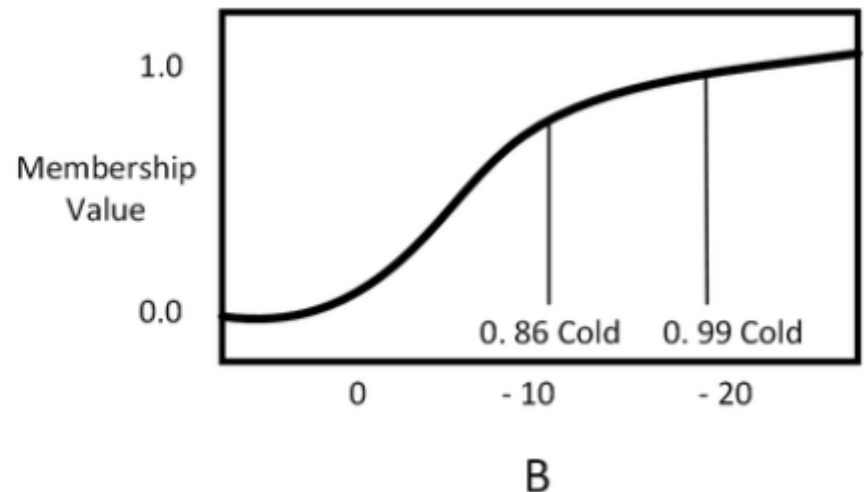
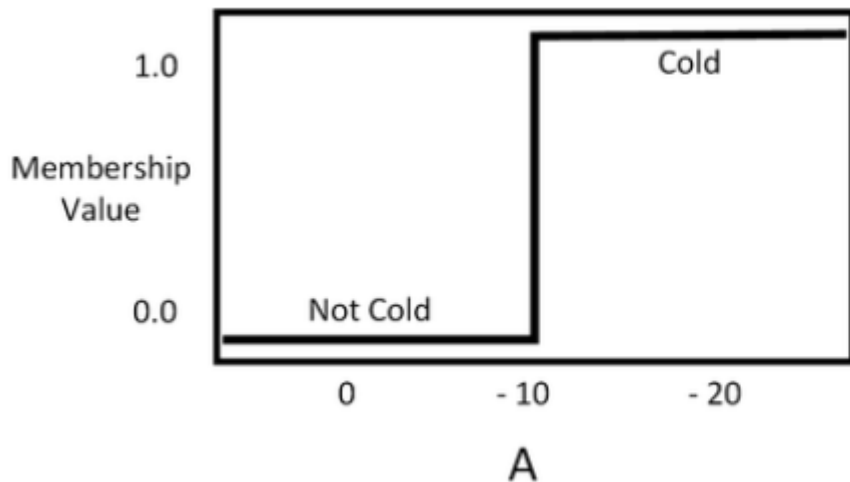
- A comparison of crisp and fuzzy sets depicting height

Crisp logic is the same as Boolean logic. Either a statement is true or it is not

Name	Height, cm	Degree of Membership	
		<i>Crisp</i>	<i>Fuzzy</i>
Chris	208	1	1.00
Mark	205	1	1.00
John	198	1	0.98
Tom	181	1	0.82
David	179	0	0.78
Mike	172	0	0.24
Bob	167	0	0.15
Steven	158	0	0.06
Bill	155	0	0.01
Peter	152	0	0.00

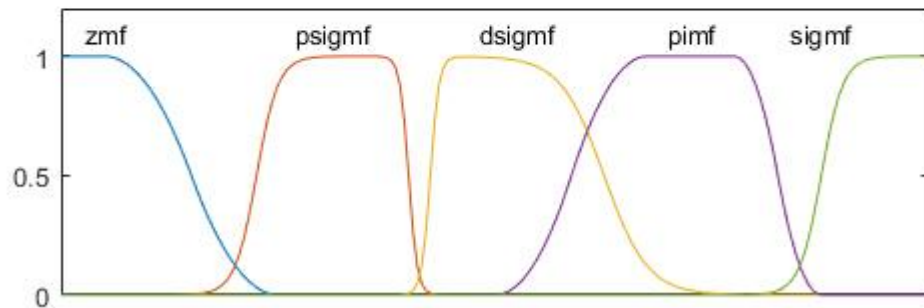
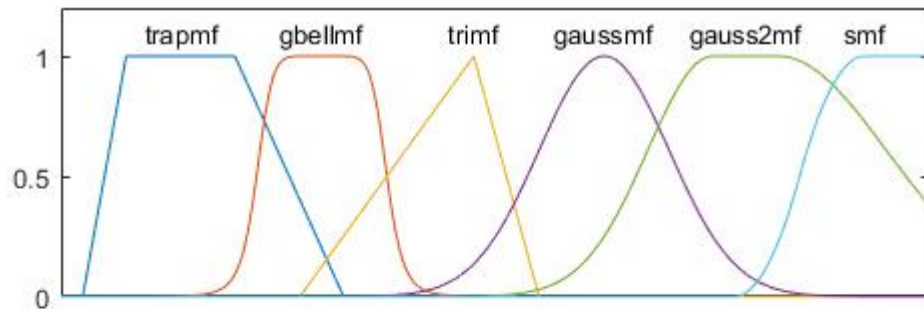
# Membership Function

- To represent a fuzzy set, we need to express it **as a function** and then to map the elements of the set to their degree of membership.



# Membership Function

- A ***membership function*** (MF) is a curve that defines how each point in the input space is mapped to a membership value (or degree of membership) between 0 and 1.

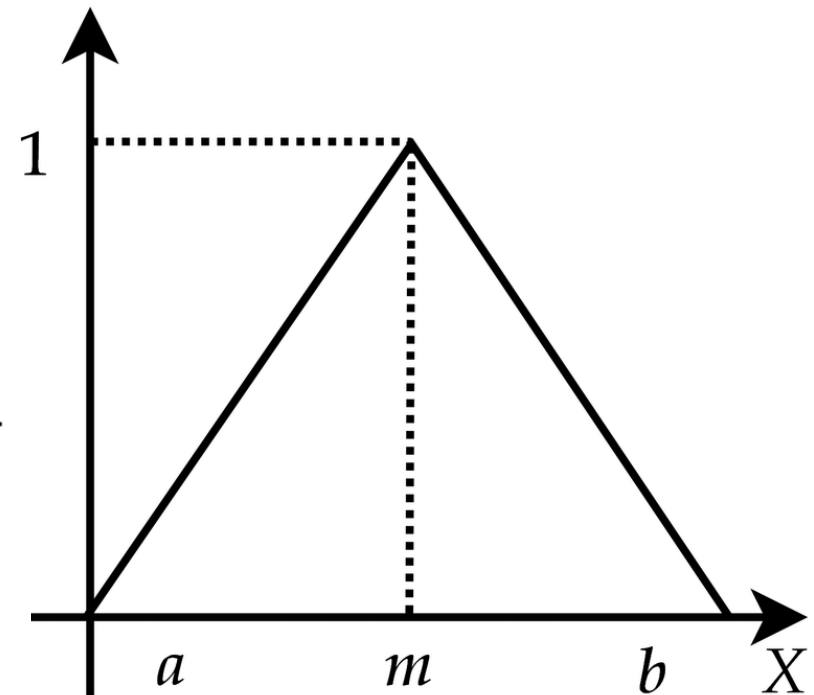


Membership Functions in Matlab

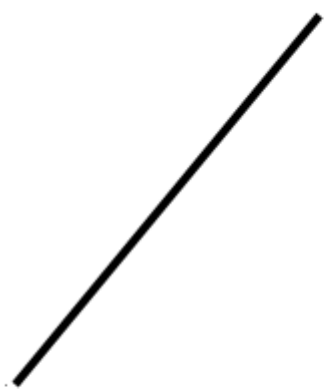
# Membership Function

## 1) Triangular MF

$$M(x) = \begin{cases} 0 & x \leq a \\ \frac{x - a}{m - a} & a < x \leq m \\ \frac{b - x}{b - m} & m < x < b \\ 0 & x \geq b \end{cases}$$



# Line Equation



(175,0)

(185,1)

$$Y = mX + b$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 0}{185 - 175} = \frac{1}{10}$$

$$Y = \frac{X}{10} + b$$

$$Y = \frac{X}{10} + b$$

$$1 = \frac{185}{10} + b$$

$$1 = 18.5 + b$$

$$b = -17.5$$

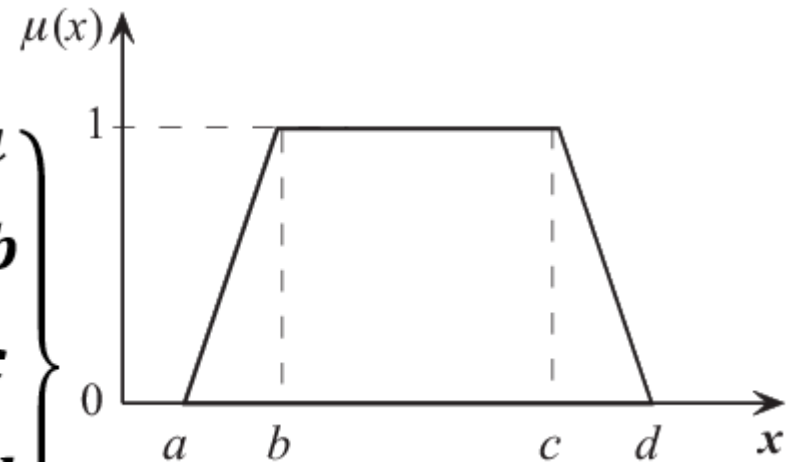
$$Y = \frac{X}{10} - 17.5$$

$$\frac{X - 175}{185 - 175} = \frac{X - 175}{10}$$

# Membership Function

## 2) Trapezoidal MF

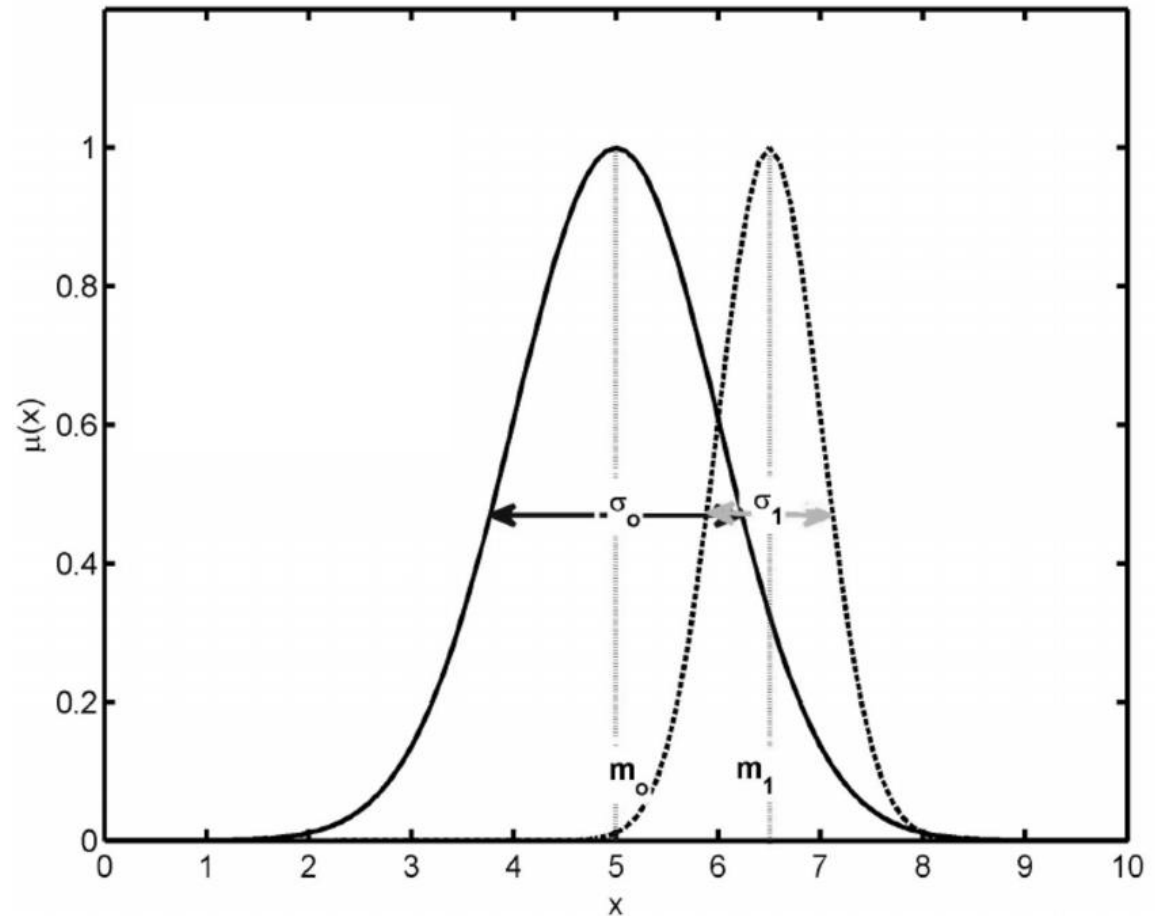
$$M(x) = \begin{cases} 0 & x \leq a \\ \frac{x-a}{b-a} & a < x < b \\ 1 & b \leq x \leq c \\ \frac{d-x}{d-c} & c < x < d \\ 0 & x \geq d \end{cases}$$



# Membership Function

## 3) Gaussian MFs

$$f(x; \sigma, c) = e^{\frac{-(x-c)^2}{2\sigma^2}}$$



# Membership Function

## 4) Sigmoid MF

$$f(x, a, c) = \frac{1}{1 + e^{-a(x-c)}}$$

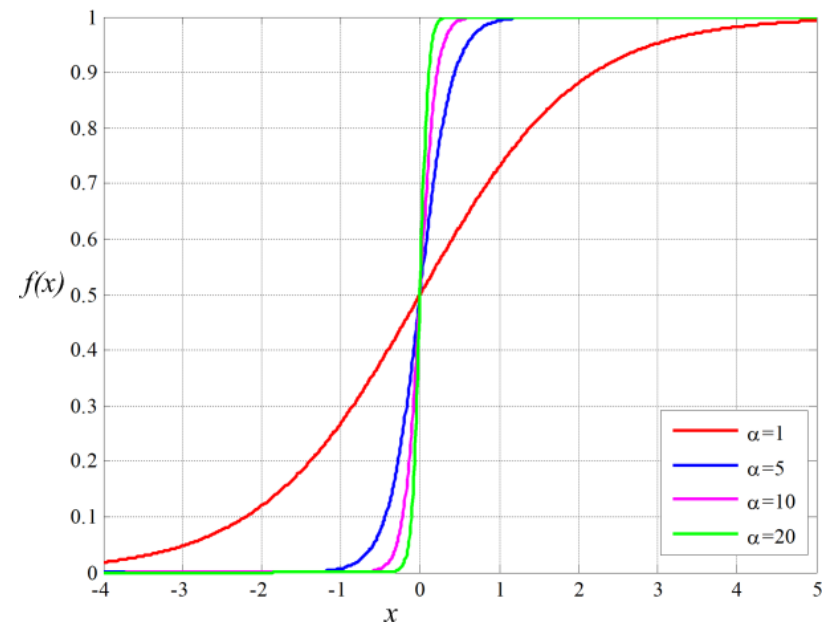
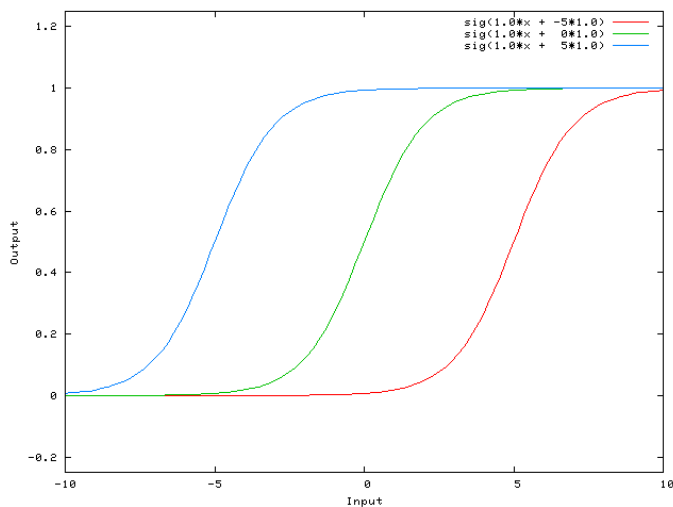


Figure 1. Sigmoid Functions for different  $\alpha$  values.



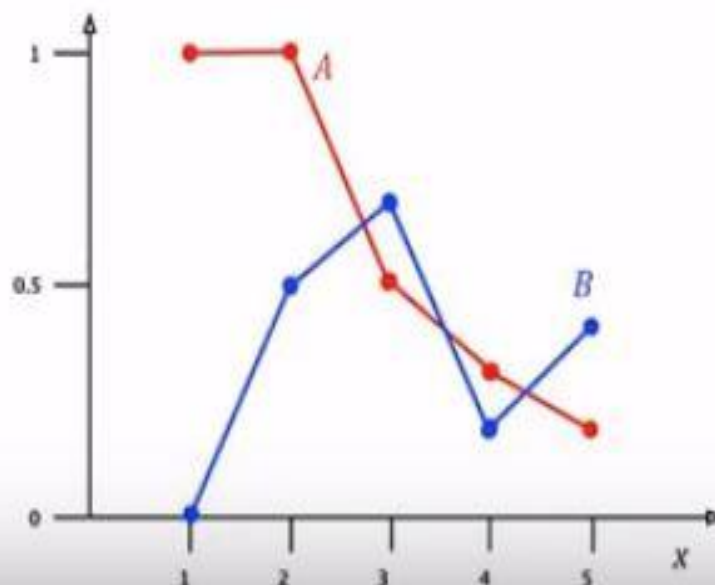
# Discrete Membership Function

$$A = \left( \frac{\mu(A_1)}{A_1}, \frac{\mu(A_2)}{A_2}, \dots, \frac{\mu(A_n)}{A_n} \right) \text{ or } A = \left( \frac{\mu(A_1)}{A_1} + \frac{\mu(A_2)}{A_2} + \dots + \frac{\mu(A_n)}{A_n} \right).$$

Example - here are two discrete membership functions in which the universe of discourse is defined as the integers from 1 to 5:

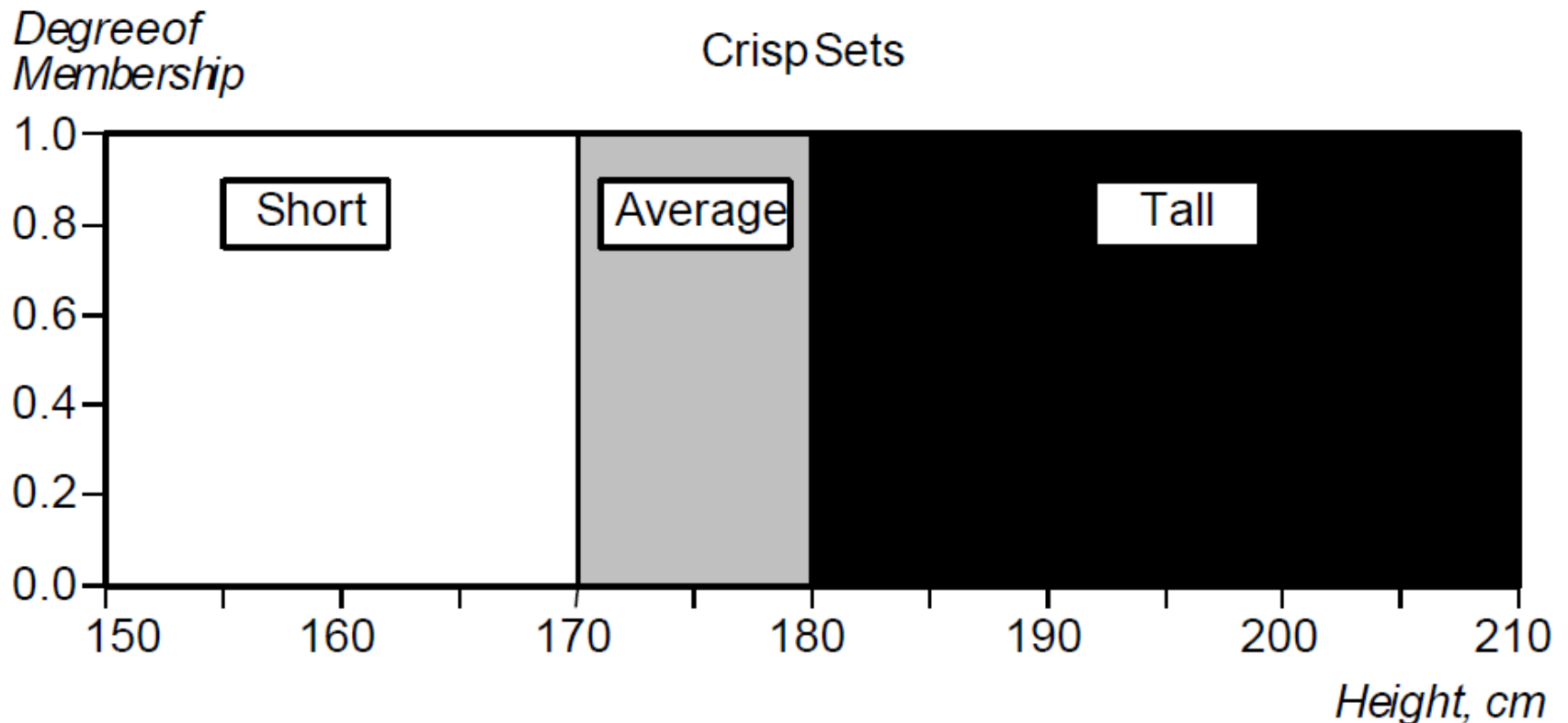
$$A = \left( \frac{1}{1}, \frac{1}{2}, \frac{0.5}{3}, \frac{0.3}{4}, \frac{0.2}{5} \right)$$

$$B = \left( \frac{0}{1}, \frac{0.5}{2}, \frac{0.7}{3}, \frac{0.2}{4}, \frac{0.4}{5} \right)$$



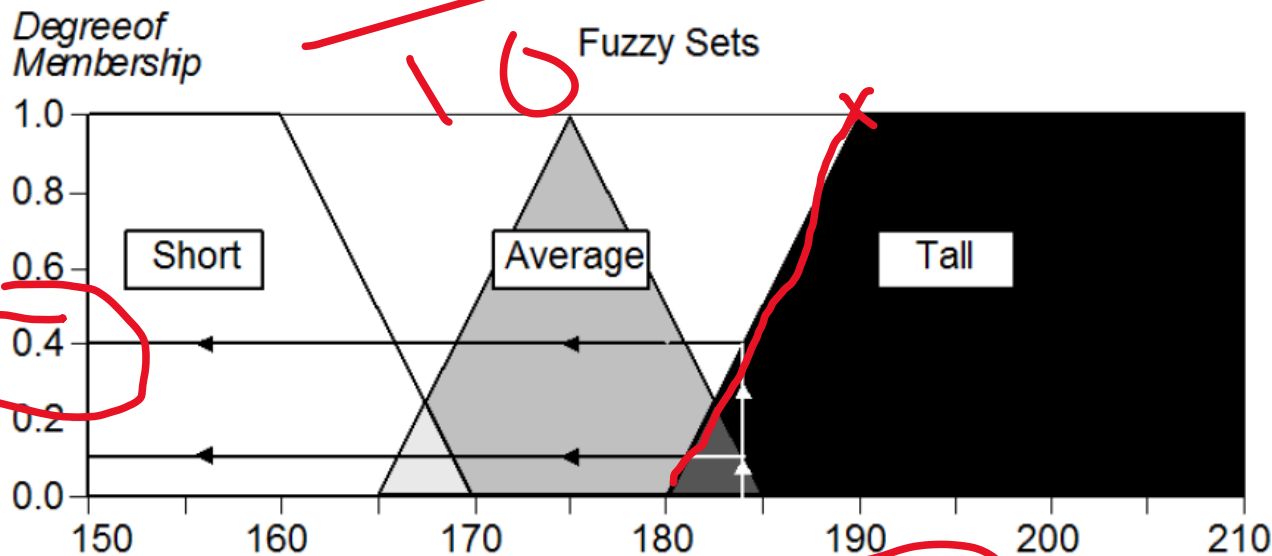
# Representing Crisp Sets

- Representing height using three crisp sets:



# Representing Fuzzy Sets

- Representing height using three fuzzy sets:



185 - X  
-----  
185 - 175

184

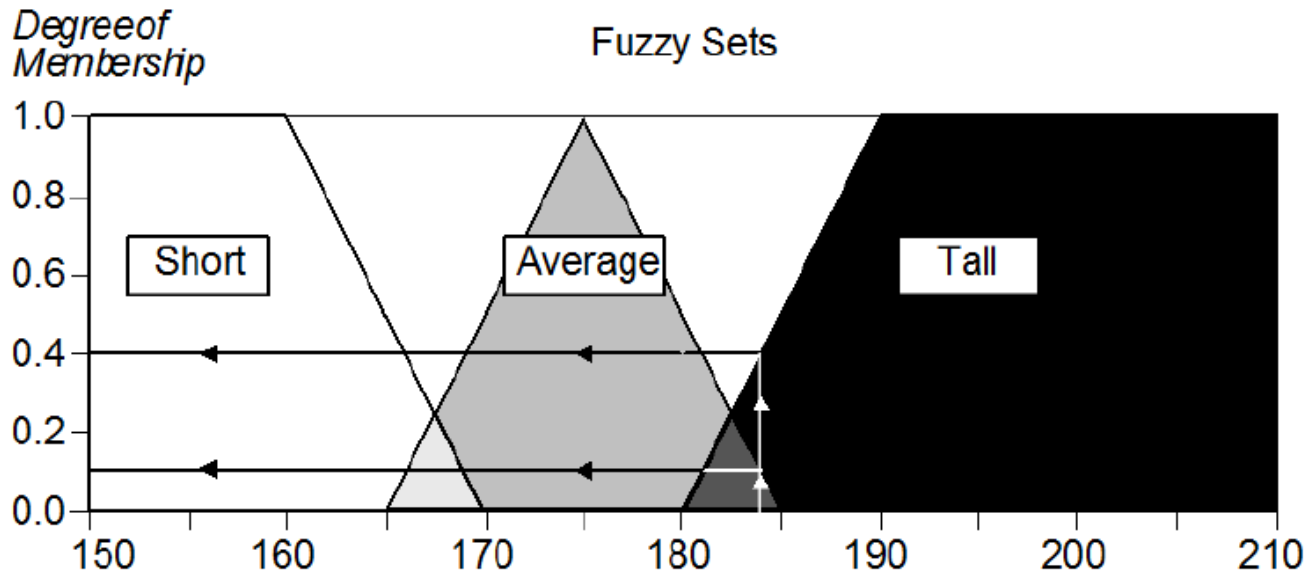
185 - 175  
-----  
10

184 - 175  
-----  
9

185 - 175  
-----  
10

# Exercise

- What's the degree of membership for Steven and Bob in *each* fuzzy set?
  - Write a function or method to calculate degree of membership (HINT: use analytic geometry)





Name	Height, cm
Chris	208
Mark	205
John	198
Tom	181
David	179
Mike	172
Bob	167
Steven	158
Bill	155
Peter	152

# Linguistic Variables



- linguistic variable is a fuzzy variable.
  - Wind is strong
  - Sailing is good
  - Speed is slow
- The range of possible values of a linguistic variable represents the universe of discourse of that variable.

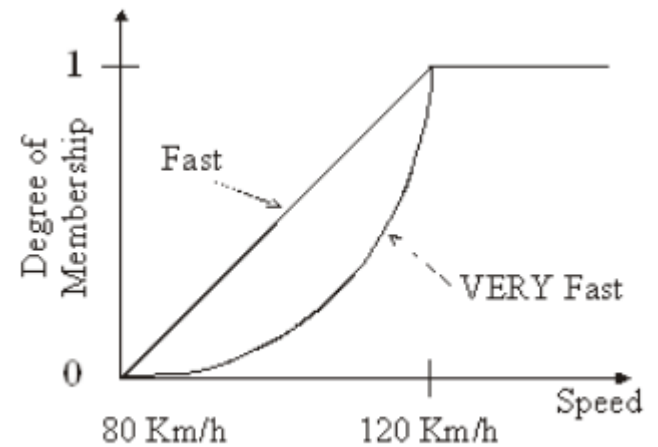
# Hedges

- Hedges are terms that modify the shape of fuzzy sets.
- They include adverbs such as **very, somewhat, quite, more or less and slightly.**

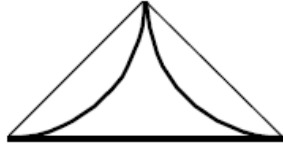
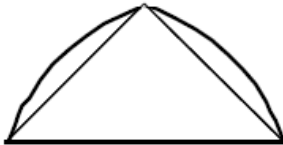
<i>Hedge</i>	<i>Mathematical Expression</i>	<i>Graphical Representation</i>
A little	$[\mu_A(x)]^{1.3}$	
Slightly	$[\mu_A(x)]^{1.7}$	

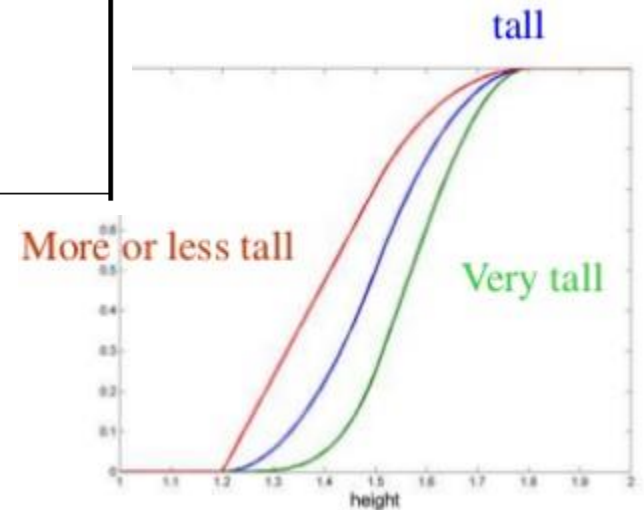
# Hedges

<i>Hedge</i>	<i>Mathematical Expression</i>	<i>Graphical Representation</i>
Very	$[\mu_A(x)]^2$	
Extremely	$[\mu_A(x)]^3$	



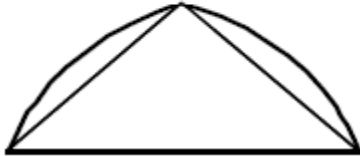

# Hedges

<i>Hedge</i>	<i>Mathematical Expression</i>	<i>Graphical Representation</i>
Very very	$[\mu_A(x)]^4$	
More or less	$\sqrt{\mu_A(x)}$	





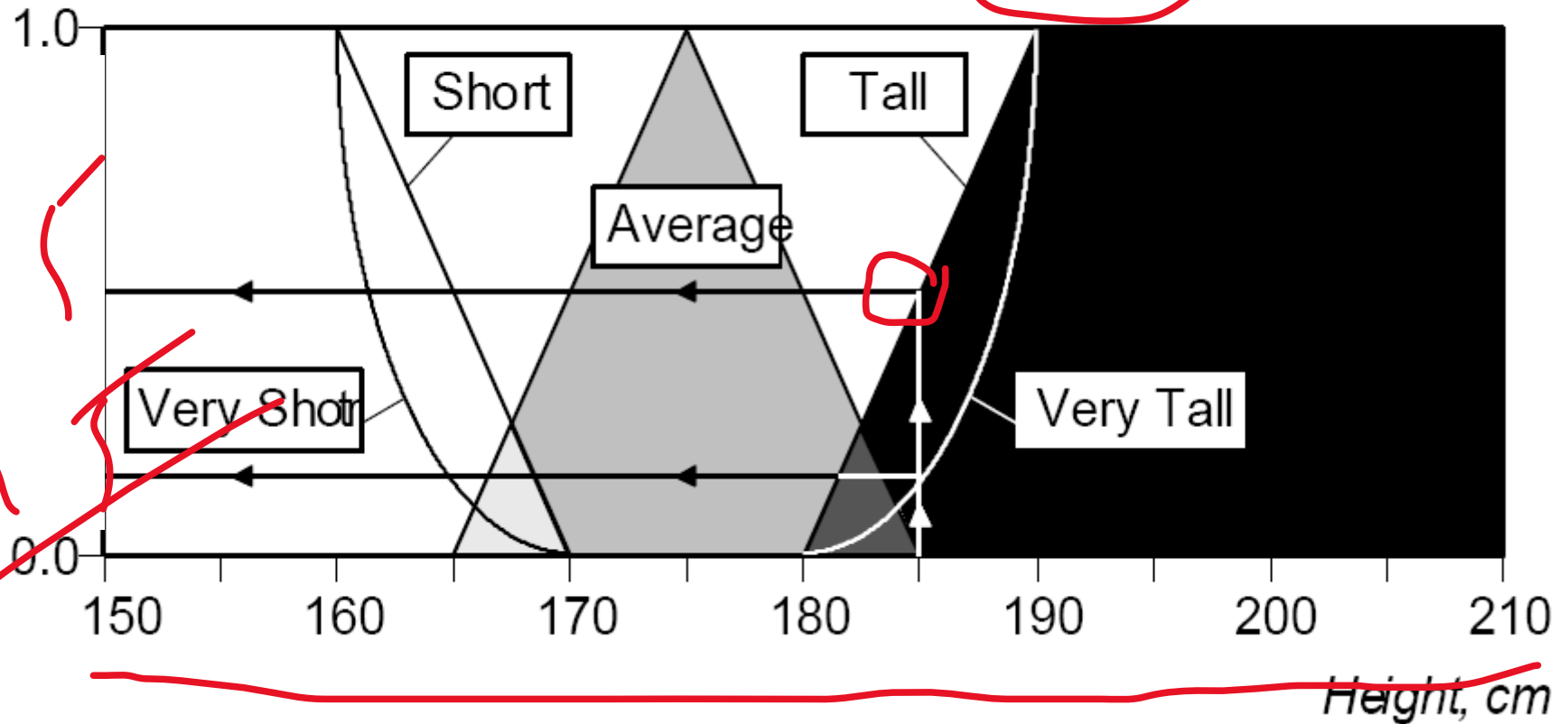
# Hedges

<i>Hedge</i>	<i>Mathematical Expression</i>	<i>Graphical Representation</i>
Somewhat	$\sqrt{\mu_A(x)}$	
Indeed	$2 [\mu_A(x)]^2$ if $0 \leq \mu_A \leq 0.5$ $1 - 2 [1 - \mu_A(x)]^2$ if $0.5 < \mu_A \leq 1$	

# Hedges

*Degree of Membership*

$$[\mu_A(x)]^2$$



# Fuzzy logical Operations

- AND, OR, NOT, etc.

- **NOT** A = A' =  $1 - \mu_A(x)$
- A **AND** B =  $A \cap B = \min(\mu_A(x), \mu_B(x))$
- A **OR** B =  $A \cup B = \max(\mu_A(x), \mu_B(x))$

From the following truth tables it is seen that fuzzy logic is a **superset** of Boolean logic.

min(A,B)

A	B	A and B
0	0	0
0	1	0
1	0	0
1	1	1

max(A,B)

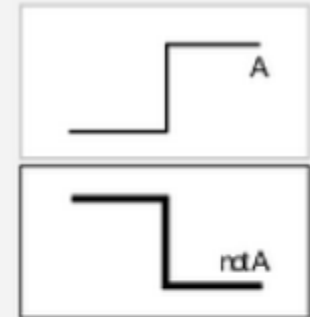
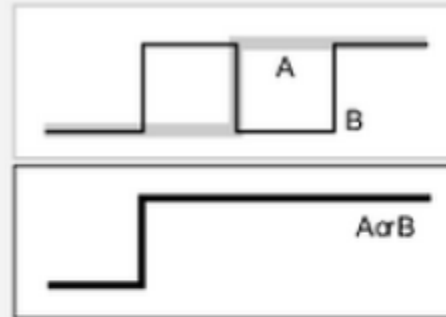
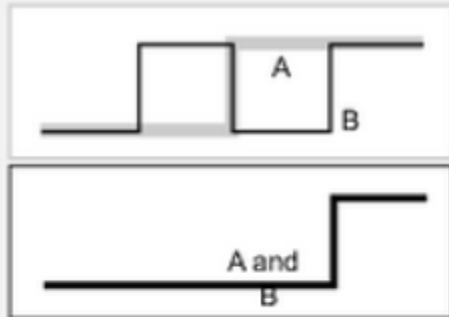
A	B	A or B
0	0	0
0	1	1
1	0	1
1	1	1

1-A

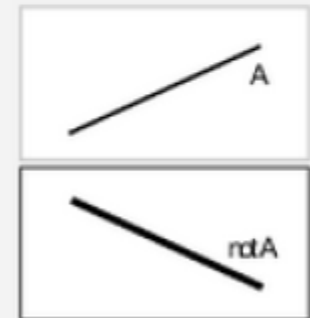
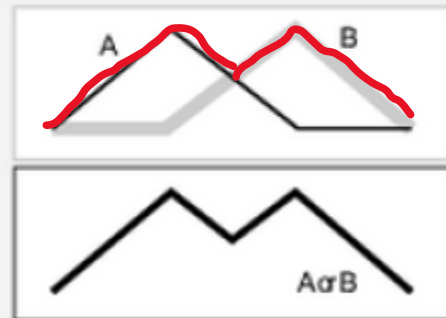
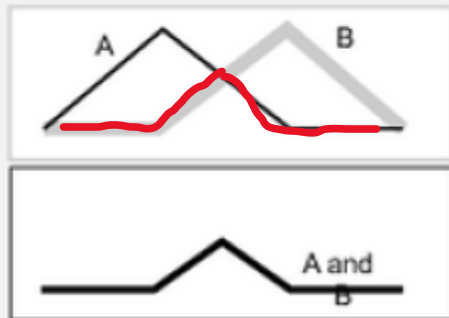
A	not A
0	1
1	0

# Fuzzy logical Operations

Two-valued  
logic



Multivalued  
logic



AND  
 $\min(A, B)$

OR  
 $\max(A, B)$

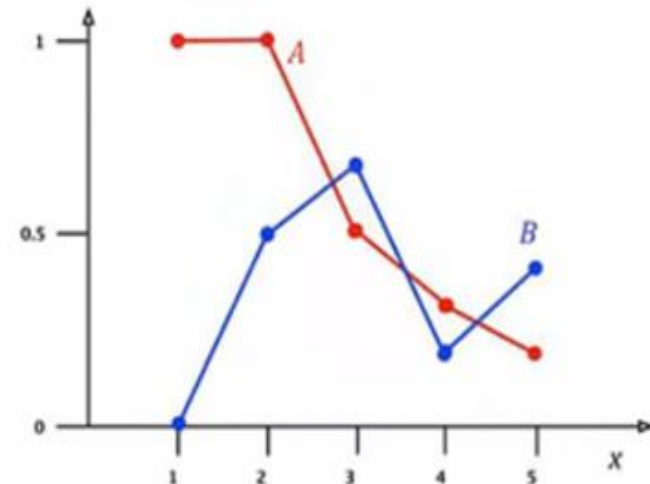
NOT  
 $(1 - A)$

# Fuzzy logical Operations

Example - here are two discrete membership functions in which the universe of discourse is defined as the integers from 1 to 5:

$$A = \left( \frac{1}{1}, \frac{1}{2}, \frac{0.5}{3}, \frac{0.3}{4}, \frac{0.2}{5} \right)$$

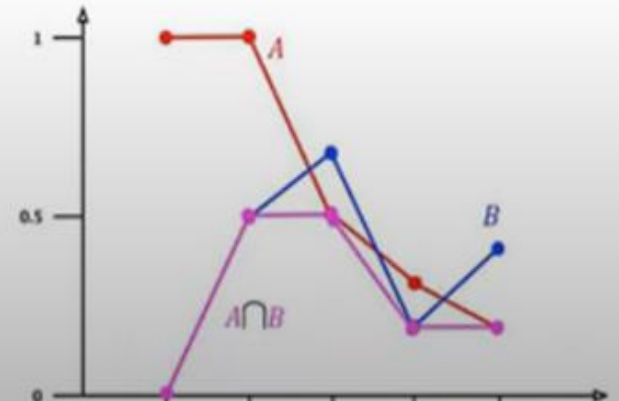
$$B = \left( \frac{0}{1}, \frac{0.5}{2}, \frac{0.7}{3}, \frac{0.2}{4}, \frac{0.4}{5} \right)$$



Then

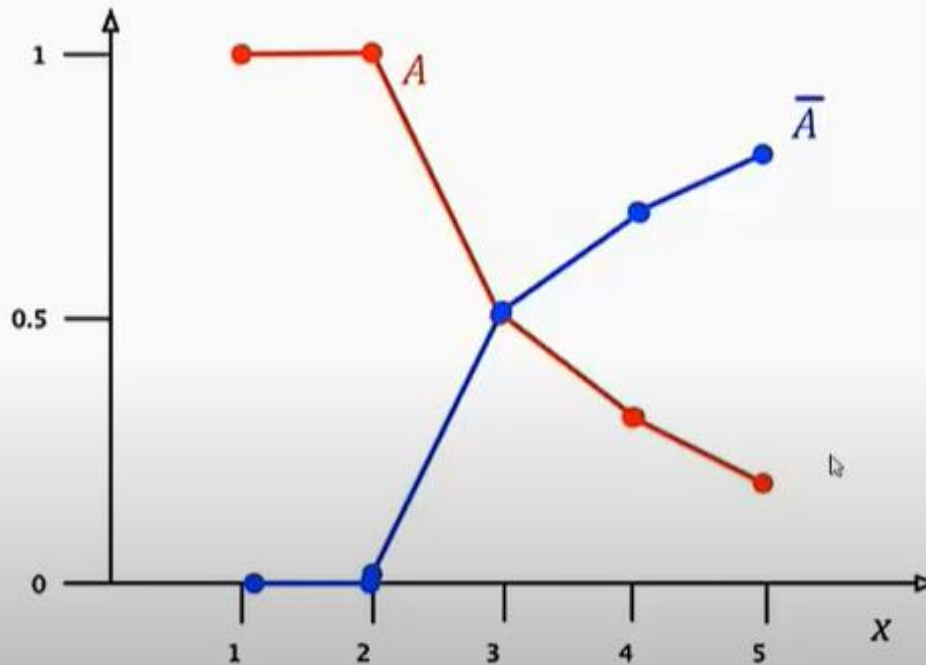
$$A \cap B = \left( \frac{\min(\mu(A_1), \mu(B_1))}{1}, \frac{\min(\mu(A_2), \mu(B_2))}{2}, \dots \right)$$

$$= \left( \frac{0}{1}, \frac{0.5}{2}, \frac{0.5}{3}, \frac{0.2}{4}, \frac{0.2}{5} \right)$$



# Fuzzy logical Operations

$$\triangleright \bar{A} = \left( \frac{1-\mu(A_1)}{1}, \frac{1-\mu(A_2)}{2}, \dots \right) = \left( \frac{0}{1}, \frac{0}{2}, \frac{0.5}{3}, \frac{0.7}{4}, \frac{0.8}{5} \right)$$



**Exercise Find  $A \cup B$**

# Fuzzy Rule

- A *fuzzy rule* is a conditional statement in the familiar form:

IF         $x$  is  $A$   
THEN     $y$  is  $B$

- ▣  $x$  and  $y$  are linguistic variables
- ▣  $A$  and  $B$  are linguistic values determined by fuzzy sets on the universe of discourses  $X$  and  $Y$ , respectively

# Fuzzy Rule

## Classical (binary logic)

- Rule: 1
  - ▣ IF speed is  $> 100$   
THEN stopping\_distance is long
- Rule: 2
  - ▣ IF speed is  $< 40$   
THEN stopping\_distance is short

## Fuzzy

- Rule: 1
  - ▣ IF speed is fast  
■ THEN stopping\_distance is long
- Rule: 2
  - ▣ IF speed is slow  
■ THEN stopping\_distance is short

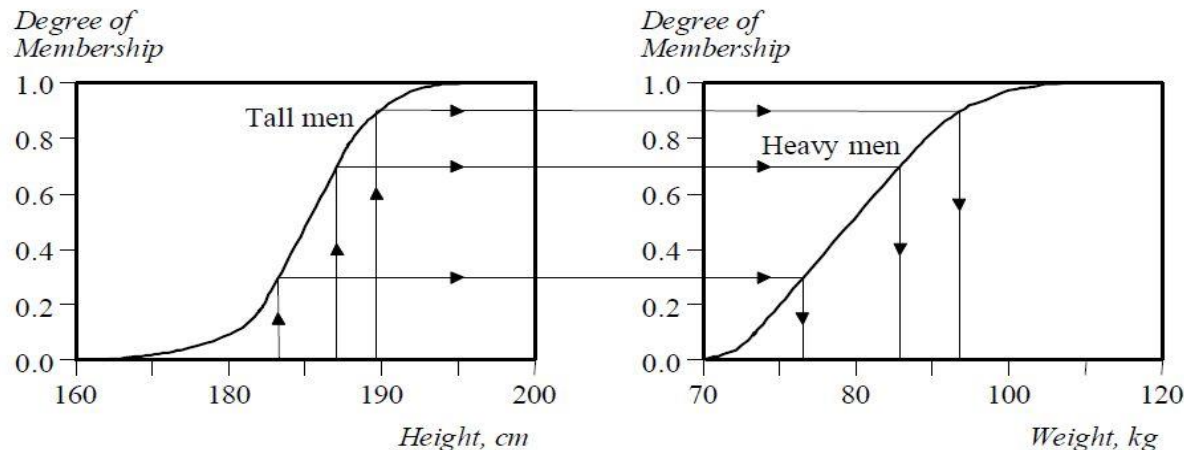


# Fuzzy Rule

- Reason with Fuzzy Rules: If the antecedent is true to some degree of membership, then the consequent is also true to that same degree.

IF height is tall

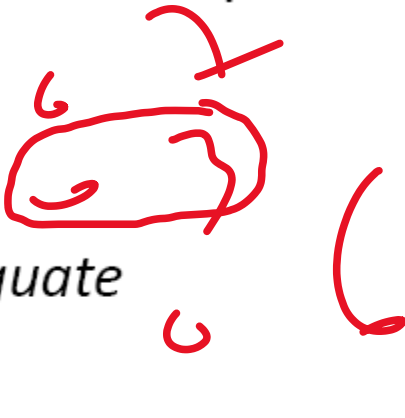
THEN *weight is heavy*



# Fuzzy Rule

- As a production rule, a fuzzy rule can have multiple antecedents, for example

□ e.g. **IF** *'project duration' is long*  
**AND** *'project staffing' is large*  
**AND** *'project funding' is inadequate*  
**THEN** *risk is high*



□ e.g. **IF** *service is excellent*  
**OR** *food is delicious*  
**THEN** *tip is generous*

- All parts of the antecedent are calculated simultaneously and resolved in a single number

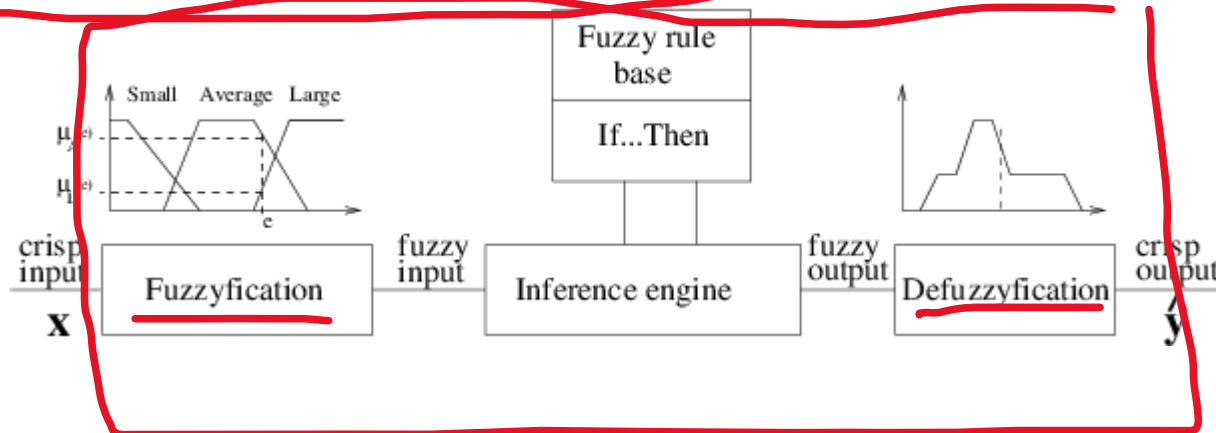
# Fuzzy Rule

- ▣ The consequent of a fuzzy rule can also include multiple parts, for instance
- ▣ e.g. **IF**            *temperature is hot*  
          **THEN**    *'hot water' is reduced;*  
                      *'cold water' is increased*
- ▣ In this case, all parts of the consequent are affected equally by the antecedent

# Fuzzy Inference

- Fuzzy inference can be defined as a process of mapping from a given input to an output, using the theory of fuzzy sets.

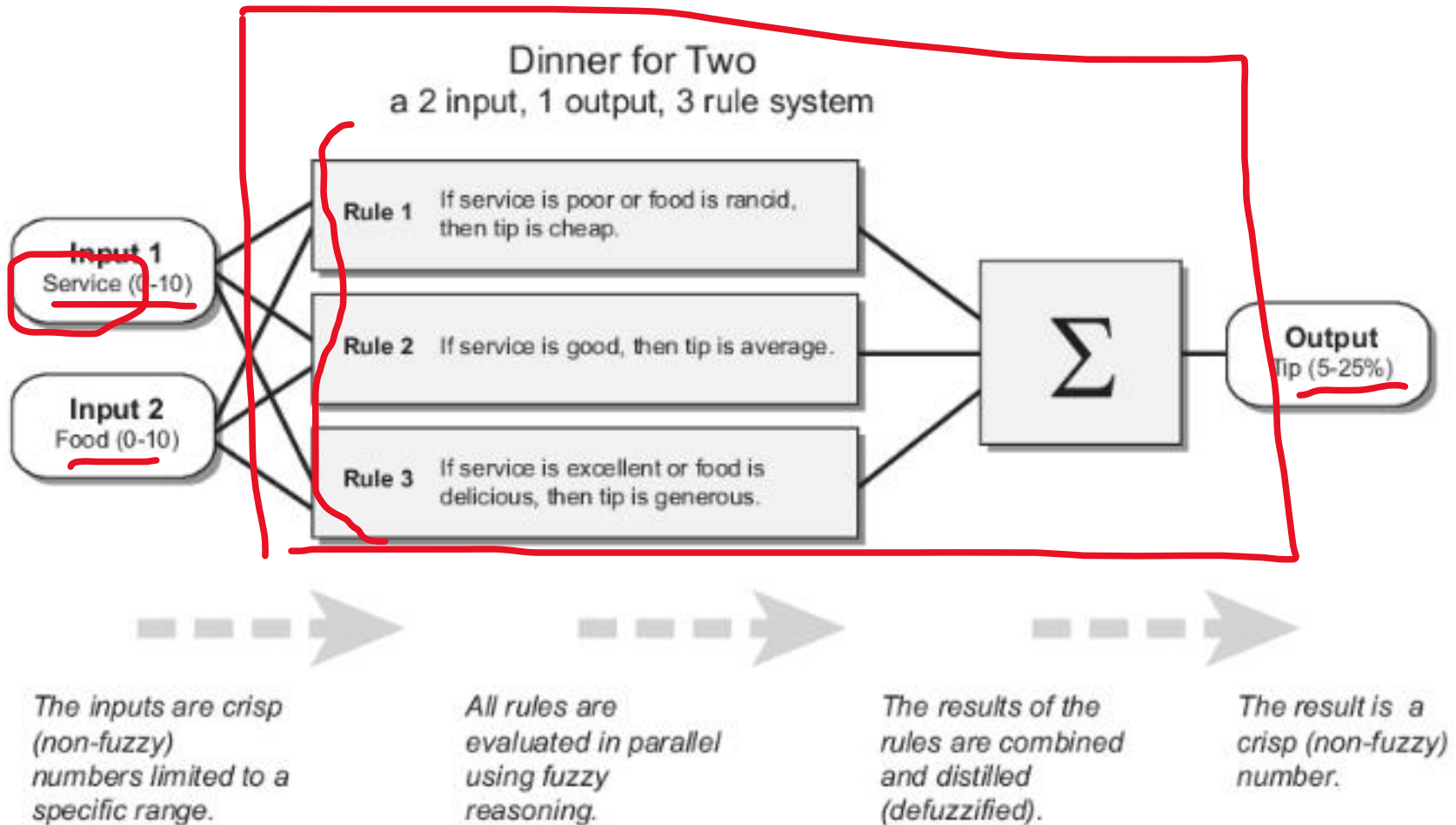
- Mamdani-style inference
- Sugeno-style inference



# Mamdani Method

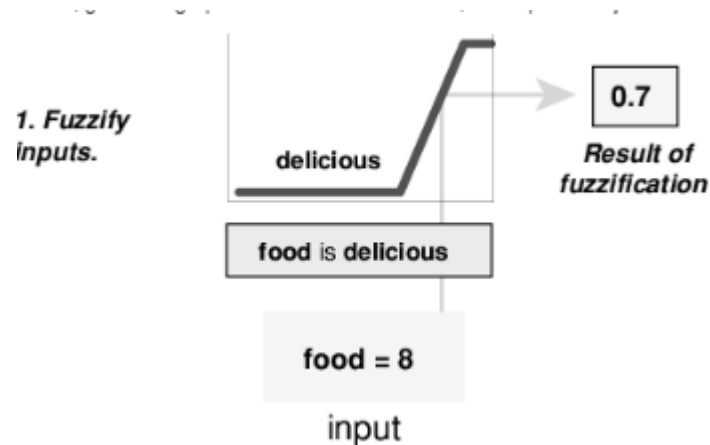
- The most commonly used fuzzy inference technique is the so-called **Mamdani method**. In 1975, Professor Ebrahim Mamdani of London University built one of the first fuzzy systems to control a steam engine and boiler combination.
- The Mamdani-style fuzzy inference process is performed in four steps:
  - 1) **Fuzzification** of the input variables : determine the degree to which these inputs belong.
  - 2) **Rule evaluation** : take the fuzzified input and apply them to the antecedents of the fuzzy rules. This number is then applied to the consequent membership function
  - 3) **Aggregation** of the rule outputs: is the process of unification of the outputs of all rules
  - 4) **Defuzzification**: The input for the defuzzification process is the aggregate output fuzzy set and the output is a single number

# Mamdani Fuzzy Inference Process (Tip Example)



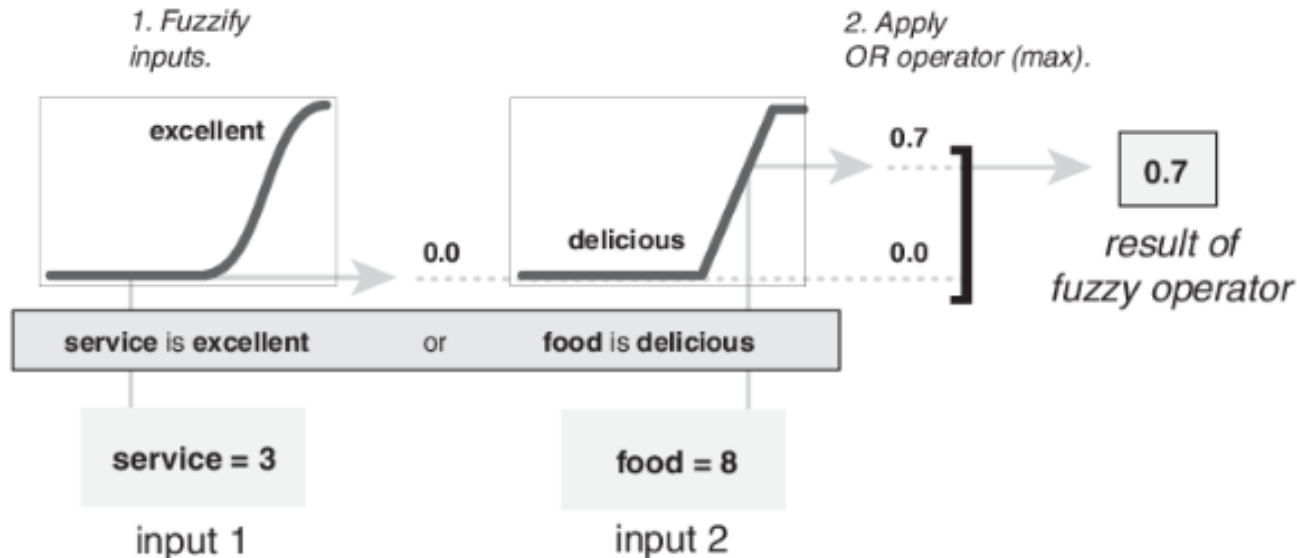
# Fuzzification of Inputs

- The first step is to take the inputs and determine the degree to which they belong to each of the appropriate fuzzy sets via membership functions (*fuzzification*).
  - service is poor, service is good, food is rancid, food is delicious, and so on.
- The output is a fuzzy degree of membership in the qualifying linguistic set (always the interval from 0 through 1).



# Apply Fuzzy Operator

- After the inputs are fuzzified, you know the degree to which each part of the antecedent is satisfied for each rule. If the antecedent of a rule has more than one part, the fuzzy operator is applied to obtain one number that represents the result of the rule antecedent. This number is then applied to the output function.





# Fuzzy Operator

- To evaluate the disjunction of the rule antecedents, we use the OR fuzzy operation. Typically, fuzzy expert systems make use of the classical fuzzy OR operation (**MAX**)

$$\mu_{A \cup B}(x) = \max [\mu_A(x), \mu_B(x)]$$

- However, there is another used OR (**probabilistic**)

$$\mu_{A \cup B}(x) = \text{prob} [\mu_A(x), \mu_B(x)] = \mu_A(x) + \mu_B(x) - \mu_A(x) \times \mu_B(x)$$

- Similarly, the **AND** fuzzy operator

- Classic (**min**)

$$\mu_{A \cap B}(x) = \min [\mu_A(x), \mu_B(x)]$$

- The **product**

$$\mu_{A \cap B}(x) = \text{prod} [\mu_A(x), \mu_B(x)] = \mu_A(x) \times \mu_B(x)$$

# Do different methods of the fuzzy operations produce different results?

Rule: 1

IF  $x$  is  $A3$  (0.0)

OR  $y$  is  $B1$  (0.1)

THEN  $z$  is  $C1$  (0.1)

$$\mu_{C1}(z) = \max [\mu_{A3}(x), \mu_{B1}(y)] = \max [0.0, 0.1] = 0.1$$

or

$$\mu_{C1}(z) = \text{probor} [\mu_{A3}(x), \mu_{B1}(y)] = 0.0 + 0.1 - 0.0 \times 0.1 = 0.1$$

Rule: 2

IF  $x$  is  $A2$  (0.2)

AND  $y$  is  $B2$  (0.7)

THEN  $z$  is  $C2$  (0.2)

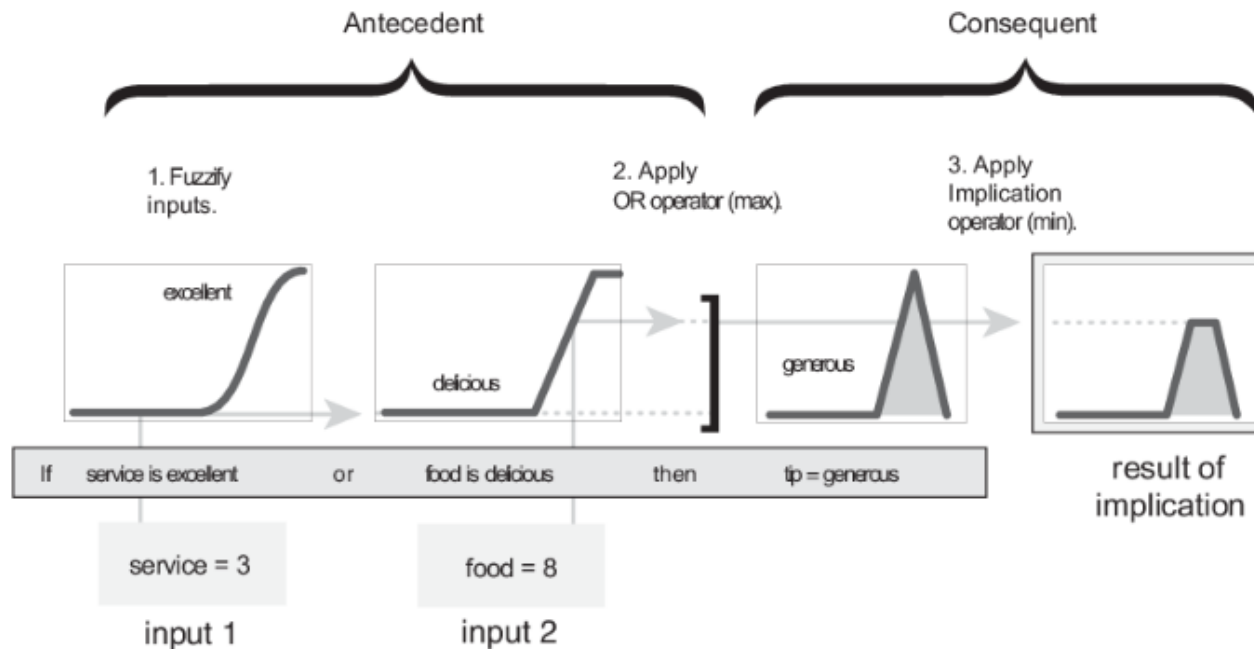
$$\mu_{C2}(z) = \min [\mu_{A2}(x), \mu_{B2}(y)] = \min [0.2, 0.7] = 0.2$$

or

$$\mu_{C2}(z) = \text{prod} [\mu_{A2}(x), \mu_{B2}(y)] = 0.2 \times 0.7 = 0.14$$

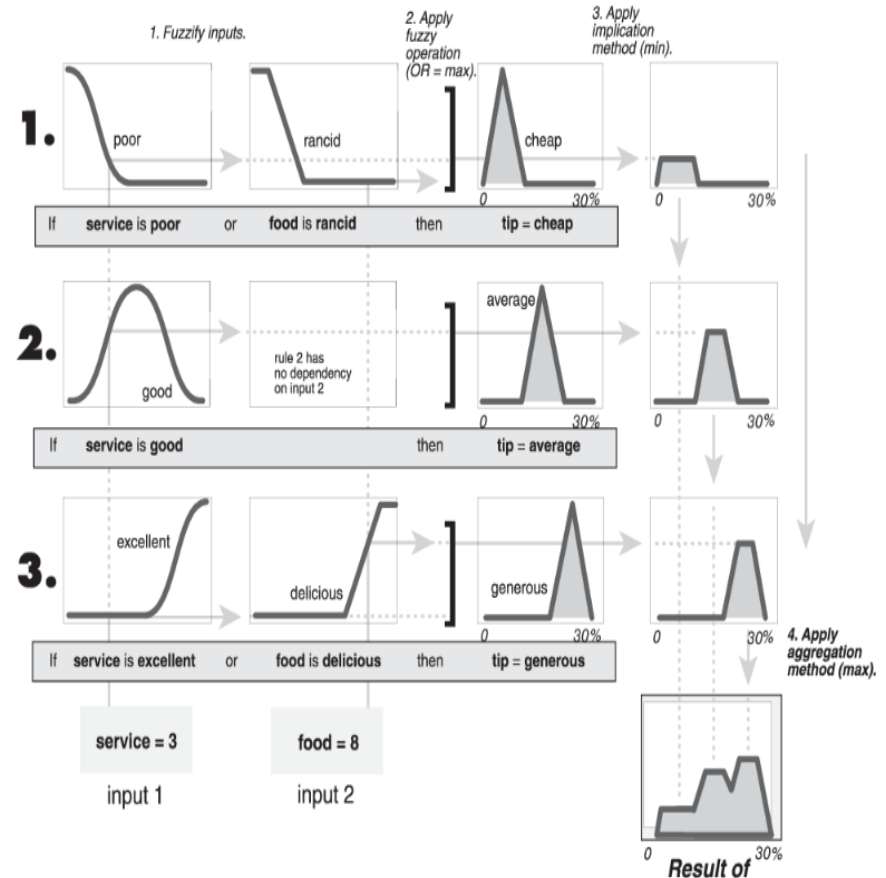
# Apply Implication Method

- The **input for the implication process is a single number** given by the antecedent, and the **output is a fuzzy set**. Implication is implemented for each rule.
- Two built-in methods are supported, and they are the same functions that are used by the AND method: ***min* (minimum)**, which truncates the output fuzzy set, and ***prod* (product)**, which scales the output fuzzy set.



# Aggregate All Outputs

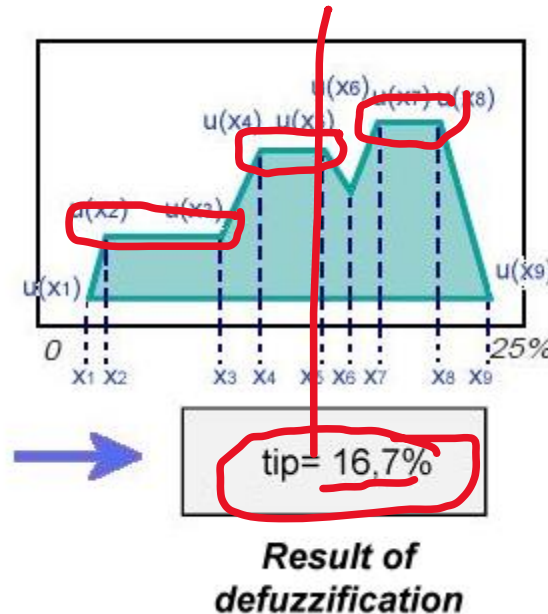
- Aggregation is the process by which the fuzzy sets that represent the outputs of each rule are combined into a single fuzzy set.
- The order in which the rules are executed is unimportant.
- Three built-in methods are supported:
  - max (maximum)
  - probor (probabilistic OR)
  - sum (sum of the rule output sets)

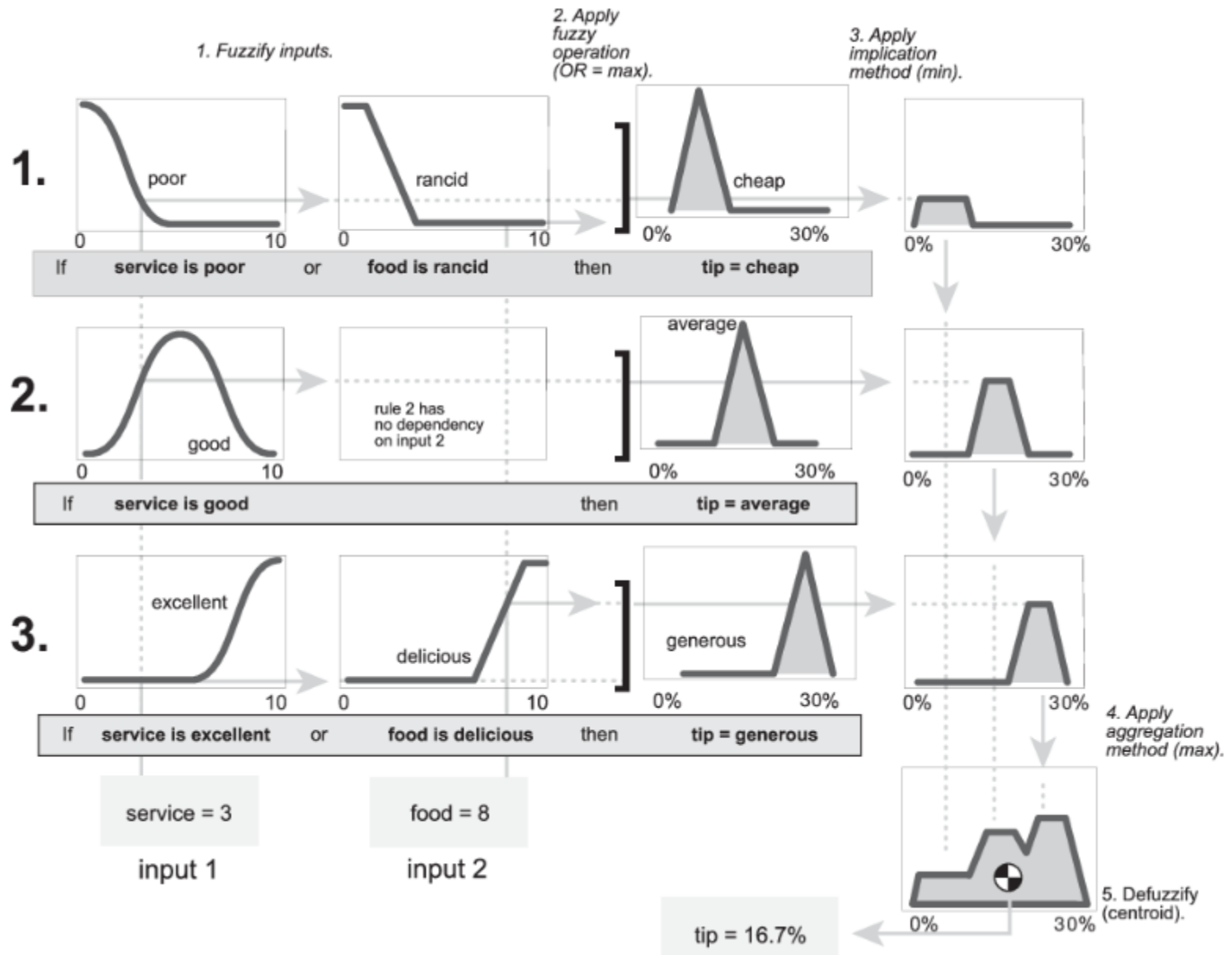


# Defuzzify

- The **input** for the defuzzification process is a **fuzzy set** (the aggregate output fuzzy set) and the **output is a single number**.
- There are five built-in defuzzification methods but the most popular defuzzification method is the **centroid calculation**, which returns the center of area under the curve, as shown in the following:

$$\text{COG} = \frac{\sum_{x=a}^b \mu_A(x) \cdot x}{\sum_{x=a}^b \mu_A(x)}$$





# Mamdani Fuzzy Inference Process (Project Risk Example)

- To see how everything fits together, we examine a simple two-input one output problem that includes three rules:

Rule: 1  
IF  $x$  is  $A3$   
OR  $y$  is  $B1$   
THEN  $z$  is  $C1$


Rule: 2  
IF  $x$  is  $A2$   
AND  $y$  is  $B2$   
THEN  $z$  is  $C2$

Rule: 3  
IF  $x$  is  $A1$   
THEN  $z$  is  $C3$

Rule: 1  
IF *project\_funding* is *adequate*  
OR *project\_staffing* is *small*  
THEN *risk* is *low*

Rule: 2  
IF *project\_funding* is *marginal*  
AND *project\_staffing* is *large*  
THEN *risk* is *normal*

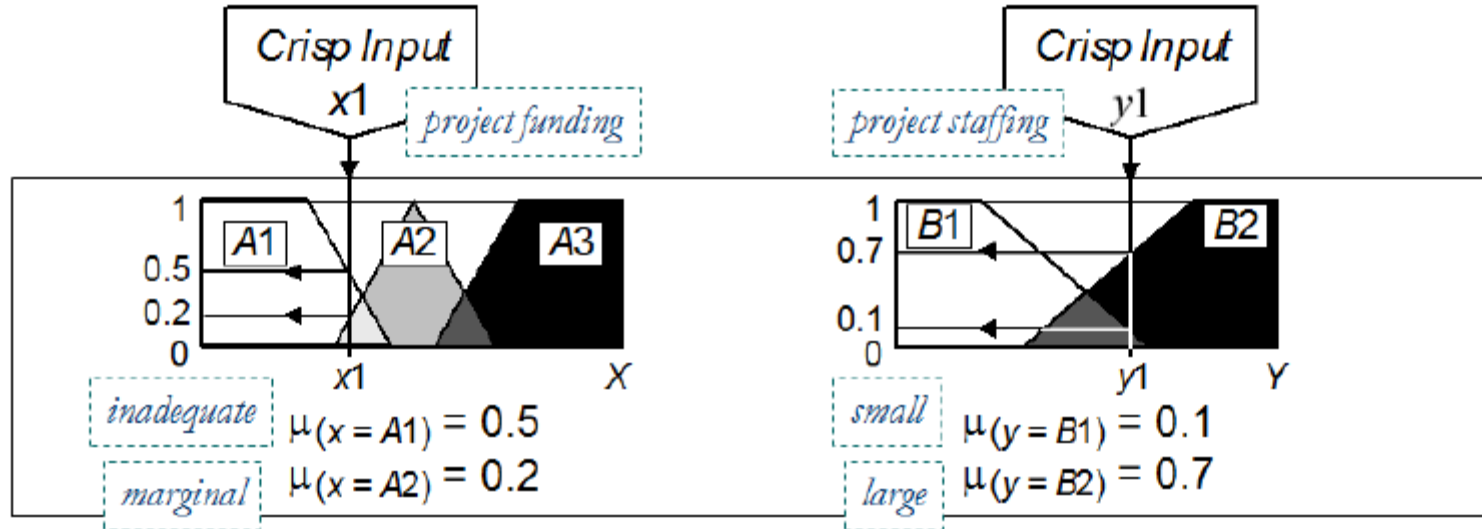
Rule: 3  
IF *project\_funding* is *inadequate*  
THEN *risk* is *high*



$x$ ,  $y$ , and  $z$  are linguistic variables;  $A1$ ,  $A2$ , and  $A3$  are linguistic values on  $X$ ;  $B1$  and  $B2$  are linguistic values on  $Y$   
 $C1$ ,  $C2$ , and  $C3$  are linguistic values on  $Z$

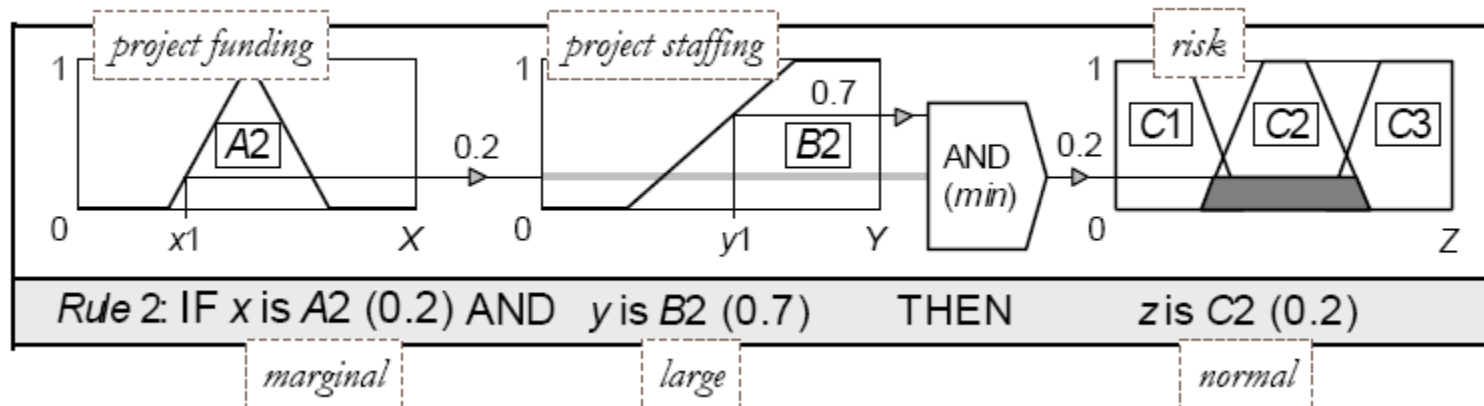
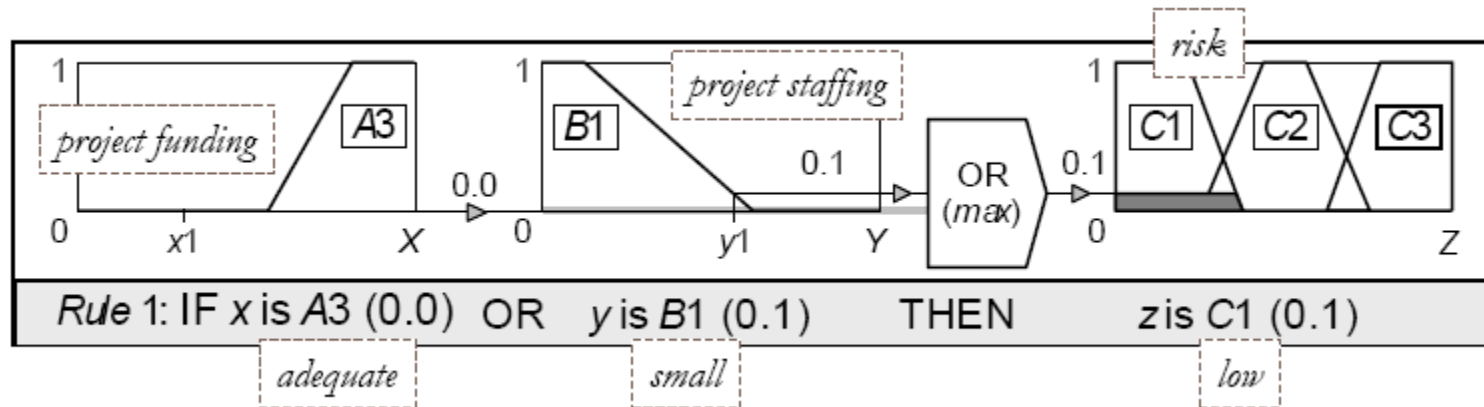
# Fuzzification of Inputs

- The first step is to take the crisp inputs,  $x_1$  and  $y_1$  (project funding and project staffing), and determine the degree to which these inputs belong to each of the appropriate fuzzy sets.

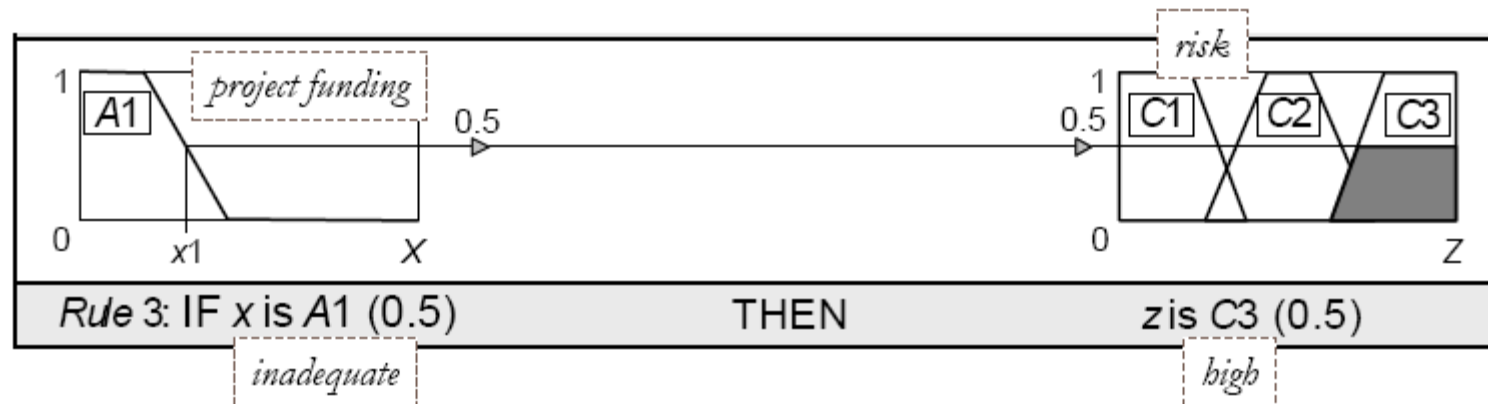




# Rule Evaluation and Implication

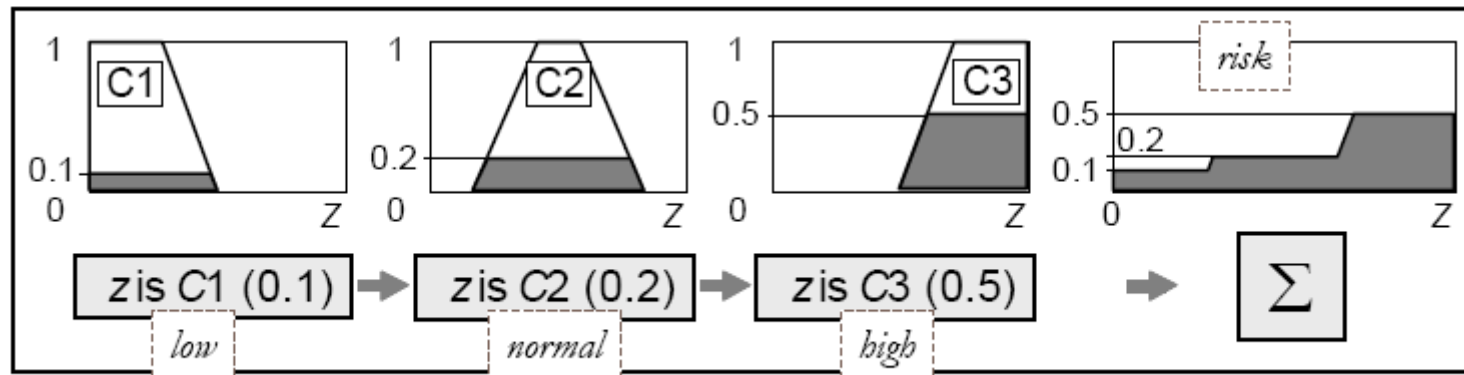


# Rule Evaluation and Implication



# Aggregation

- Aggregation is the process of unification of the outputs of all rules



# Defuzzification

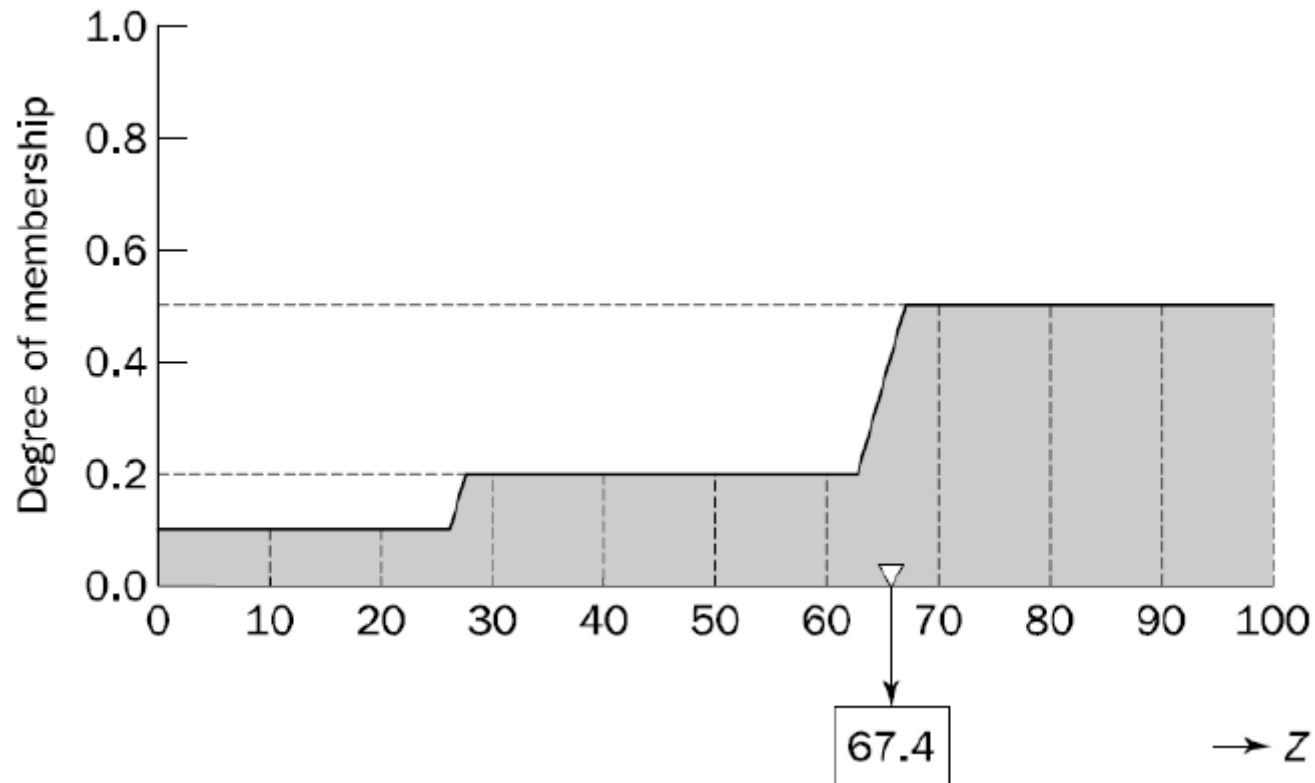
- Mathematically this centre of gravity (COG) can be expressed as

$$\text{COG} = \frac{\sum_{x=a}^b \mu_A(x)x}{\sum_{x=a}^b \mu_A(x)}$$

- Let us now calculate the centre of gravity for our problem

$$\begin{aligned}\text{COG} &= \frac{(0 + 10 + 20) \times 0.1 + (30 + 40 + 50 + 60) \times 0.2 + (70 + 80 + 90 + 100) \times 0.5}{0.1 + 0.1 + 0.1 + 0.2 + 0.2 + 0.2 + 0.2 + 0.5 + 0.5 + 0.5 + 0.5} \\ &= 67.4\end{aligned}$$

# Defuzzification

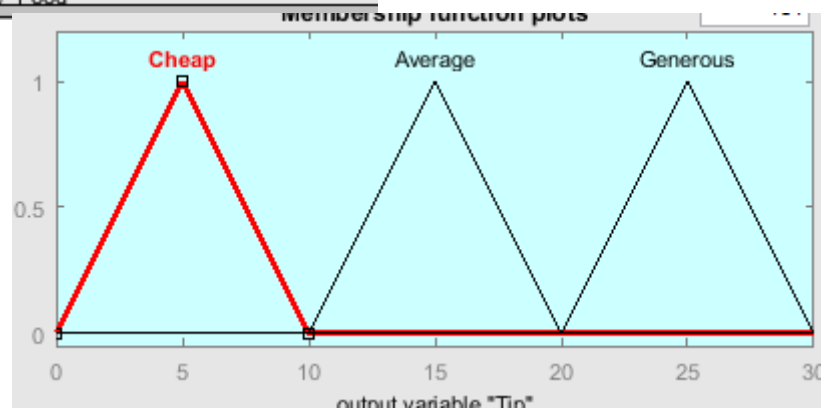
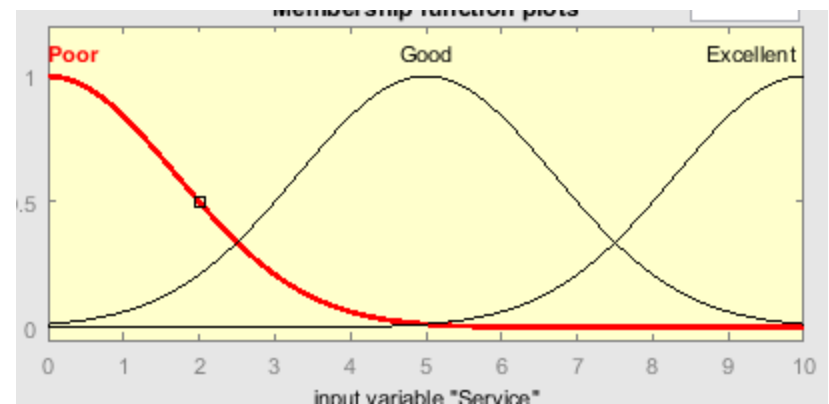
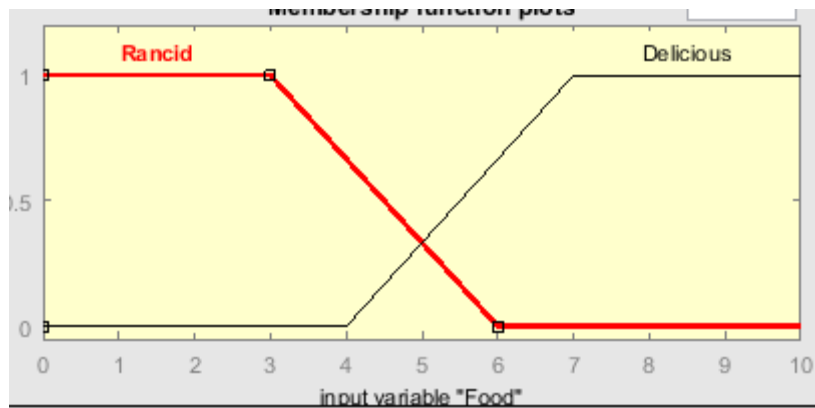


$$\text{COG} = \frac{(0 + 10 + 20) \times 0.1 + (30 + 40 + 50 + 60) \times 0.2 + (70 + 80 + 90 + 100) \times 0.5}{0.1 + 0.1 + 0.1 + 0.2 + 0.2 + 0.2 + 0.2 + 0.5 + 0.5 + 0.5 + 0.5}$$

$$= 67.4$$

# Tip Example (Calculation)

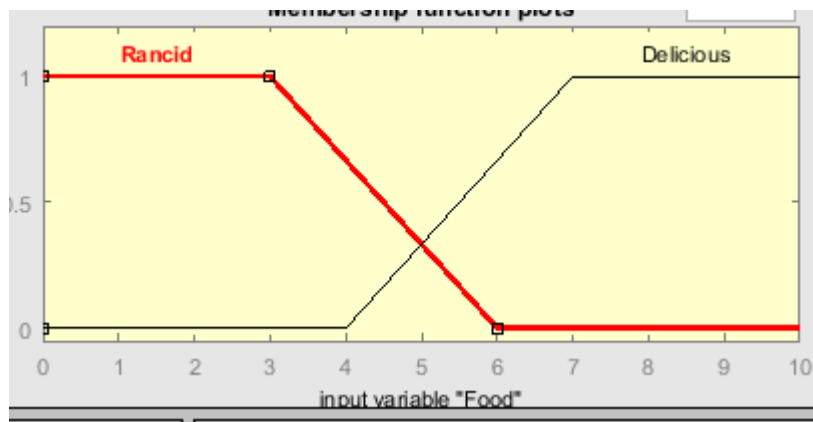
- Rules
  - If the service is poor or the food is rancid, then tip is cheap.
  - If the service is good, then tip is average.
  - If the service is excellent or the food is delicious, then tip is generous.



# Tip Example (Calculation)

- Using Mamdani method what the tip should be if Food= and service=8

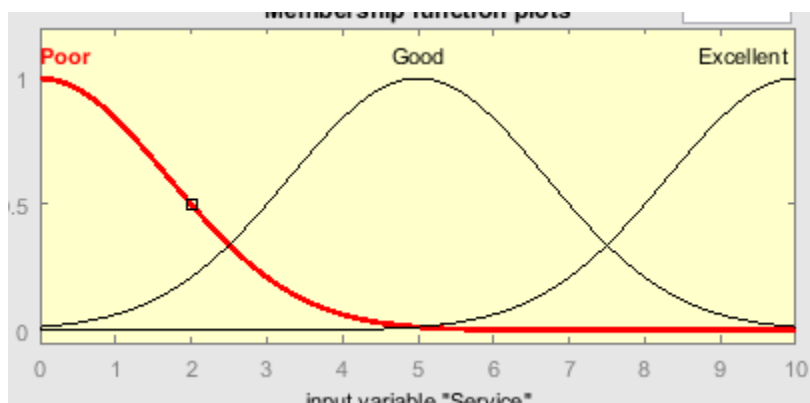
## Fuzzification



$$\text{Rancid}(6) = 0$$

$$\text{Delicious}(6) = (6-4)/(7-4) = 2/3 = 0.66$$

# Fuzzification



$$\text{Poor}(8) = 0$$

$$\text{Good}(8) = e^{-\frac{(8-5)^2}{2*1.7^2}} = 0.21$$

$$\text{Excellent}(8) = e^{-\frac{(8-10)^2}{2*1.7^2}} = 0.5$$

$$f(x; \sigma, c) = e^{\frac{-(x-c)^2}{2\sigma^2}}$$



# Rule Evaluation and Implication

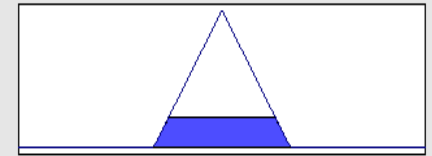
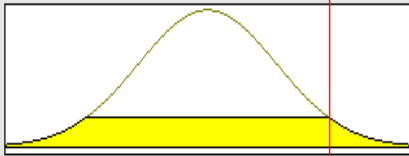
1) If the service is poor or the food is rancid, then tip is cheap.

$$(0) \max (0) \rightarrow 0$$

2) If the service is good, then tip is average.

$$(0.21) \rightarrow (0.21)$$

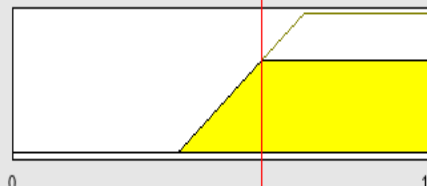
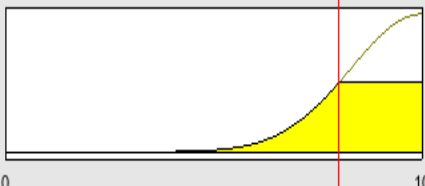
2



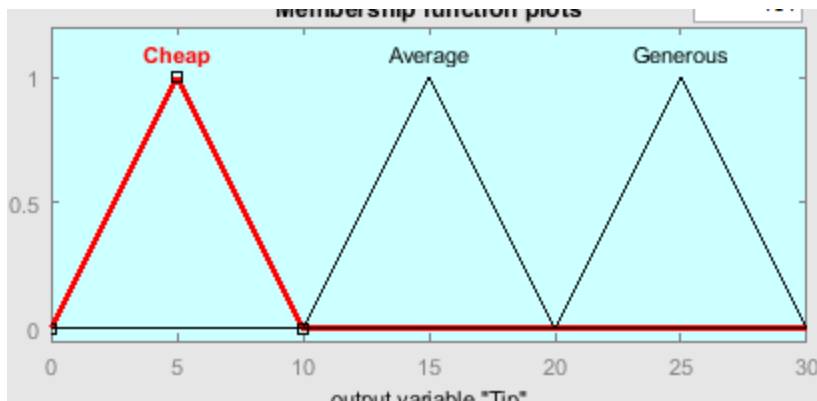
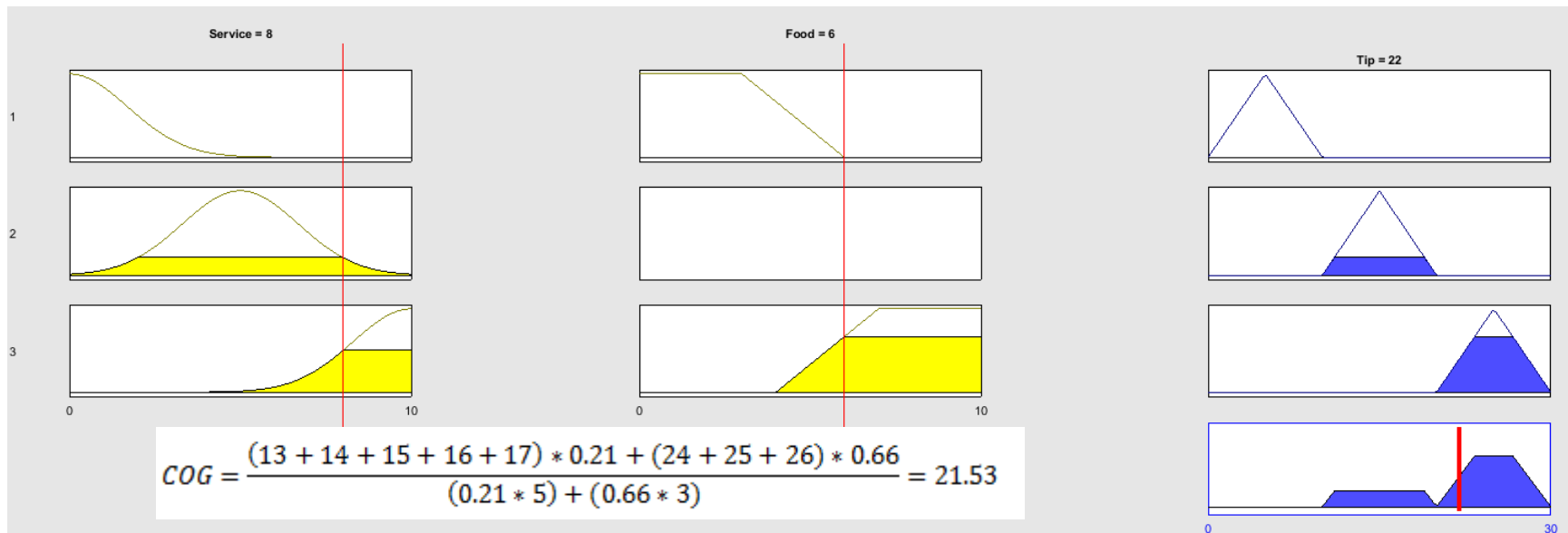
3) If the service is excellent or the food is delicious, then tip is generous.

$$(0.5) \text{ MAX } (0.66) \rightarrow (0.66)$$

3



# Aggregation and Defuzzification



# Sugeno-style Inference

- To shorten the time of fuzzy inference single spike, singleton, is used as the membership function of the rule consequent.
- A singleton, or more precisely a fuzzy singleton, is a fuzzy set with a membership function that is unity at a single particular point on the universe of discourse and zero everywhere else.
- Sugeno-style fuzzy inference is very similar to the Mamdani method. Sugeno changed only a rule consequent. Instead of a fuzzy set, he used a mathematical function of the input variable. The format of the Sugeno-style fuzzy rule is

IF  $x$  is  $A$

AND  $y$  is  $B$

THEN  $z$  is  $f(x,y)$

- where  $x$ ,  $y$  and  $z$  are linguistic variables;  $A$  and  $B$  are fuzzy sets on universe of discourses  $X$  and  $Y$ , respectively; and  $f(x,y)$  is a mathematical function

# Sugeno-style Inference

- The most commonly used **zero-order Sugeno** fuzzy model applies fuzzy rules in the following form:

IF  $x$  is  $A$   
AND  $y$  is  $B$   
THEN  $z$  is  $k$

where  **$k$  is a constant.**

- In this case, the output of each fuzzy rule is constant.
- In other words, all consequent membership functions are represented by singleton spikes

# Sugeno Fuzzy Inference Process (Project Risk Example)

Rule: 1

IF  $x$  is  $A3$   
OR  $y$  is  $B1$   
THEN  $z$  is  $K1$

Rule: 2

IF  $x$  is  $A2$   
AND  $y$  is  $B2$   
THEN  $z$  is  $K2$

Rule: 3

IF  $x$  is  $A1$   
THEN  $z$  is  $K3$

Rule: 1

IF *project\_funding* is *adequate*  
OR *project\_staffing* is *small*  
THEN *risk* is *low*

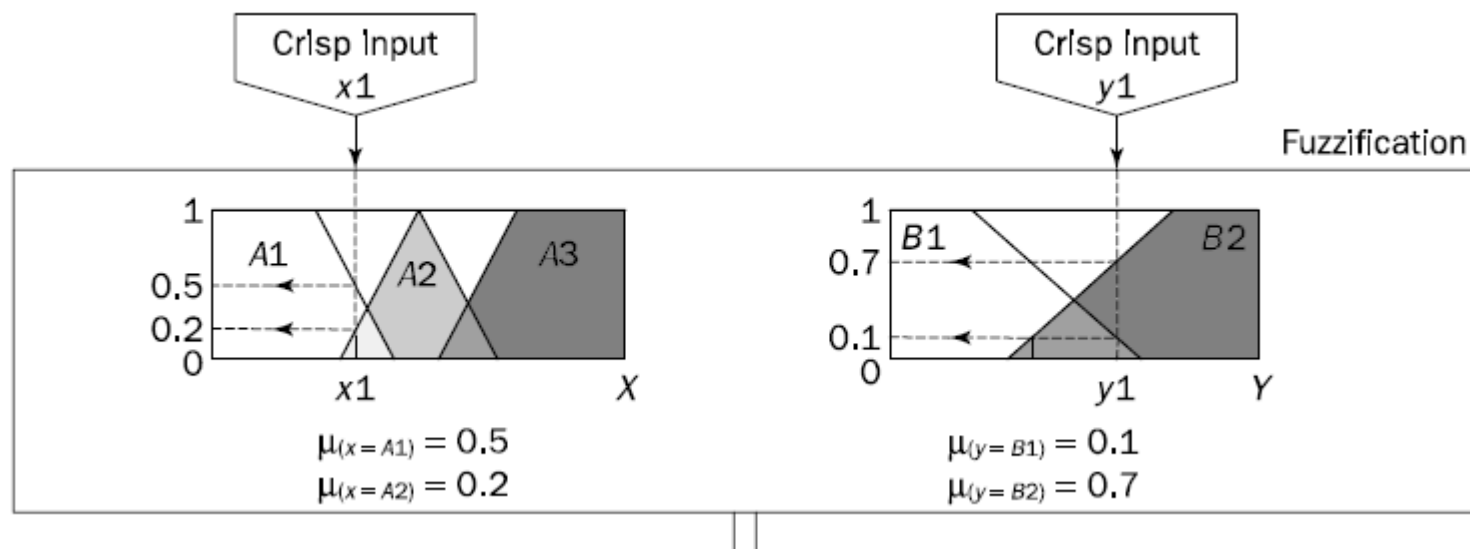
Rule: 2

IF *project\_funding* is *marginal*  
AND *project\_staffing* is *large*  
THEN *risk* is *normal*

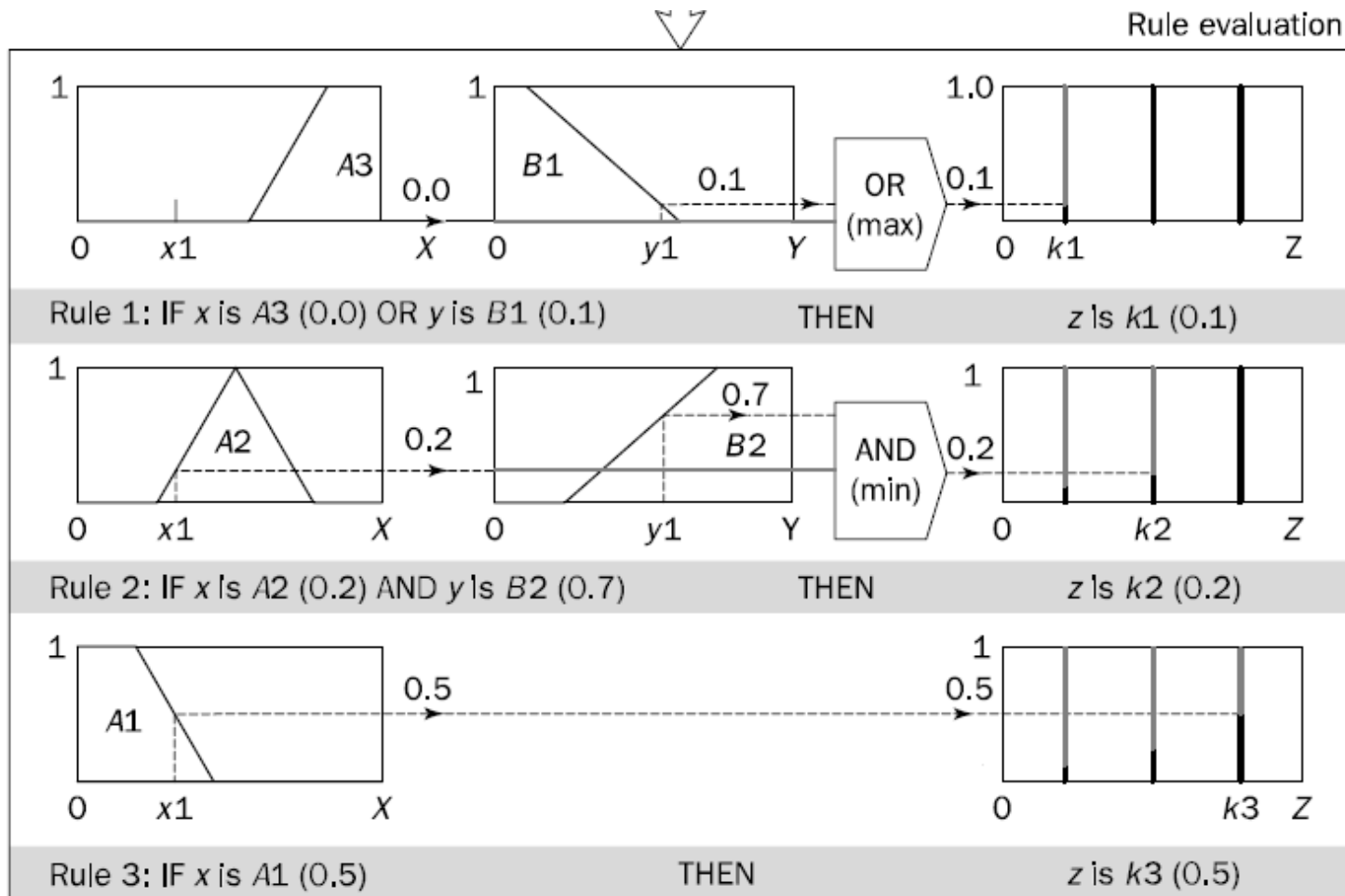
Rule: 3

IF *project\_funding* is *inadequate*  
THEN *risk* is *high*

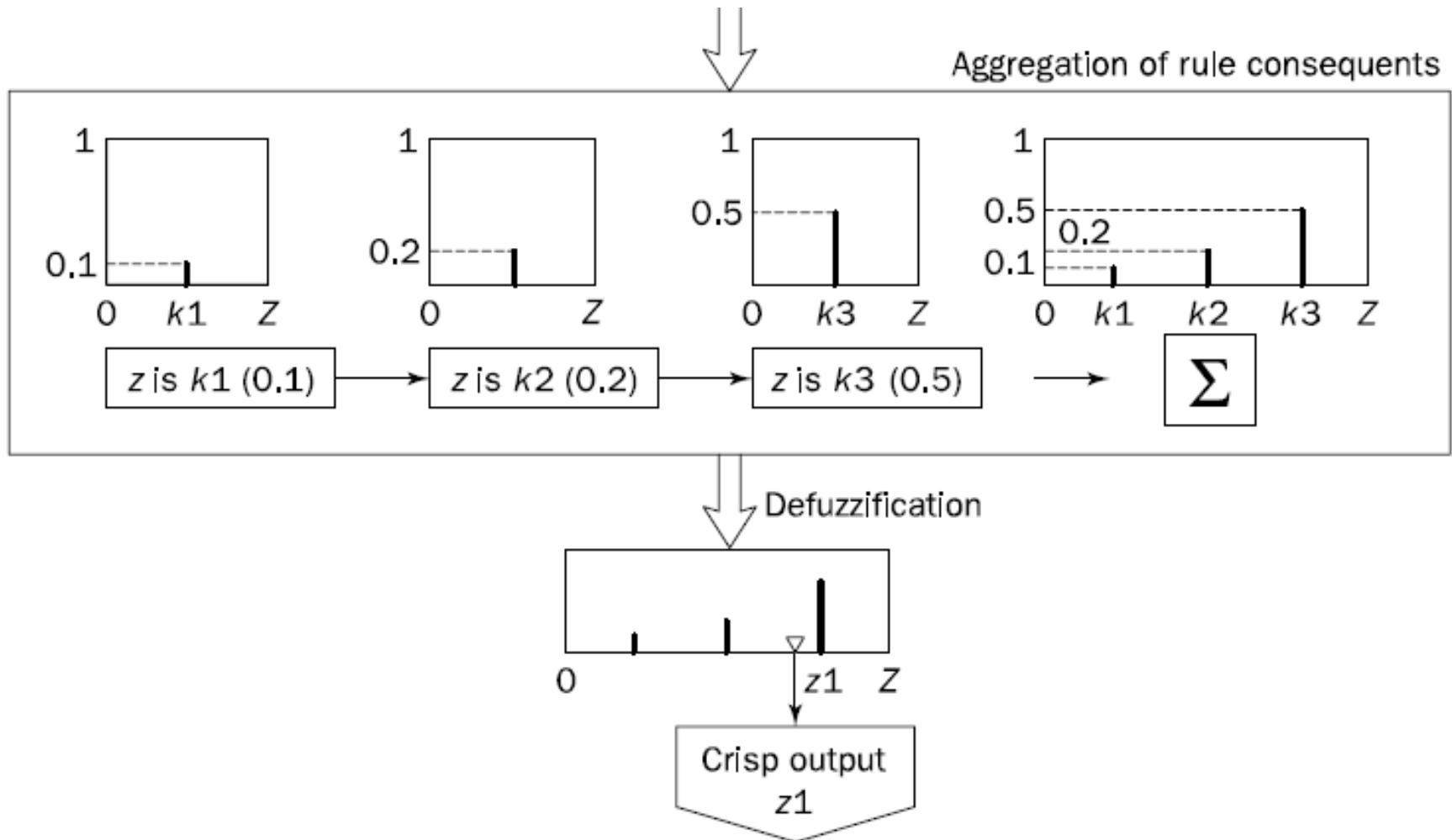
# Fuzzification



# Rule Evaluation and Implication



# Aggregation and Defuzzification





# Defuzzification

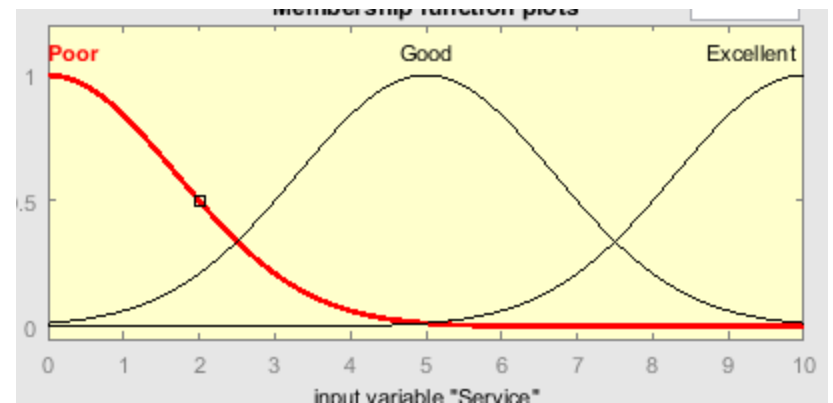
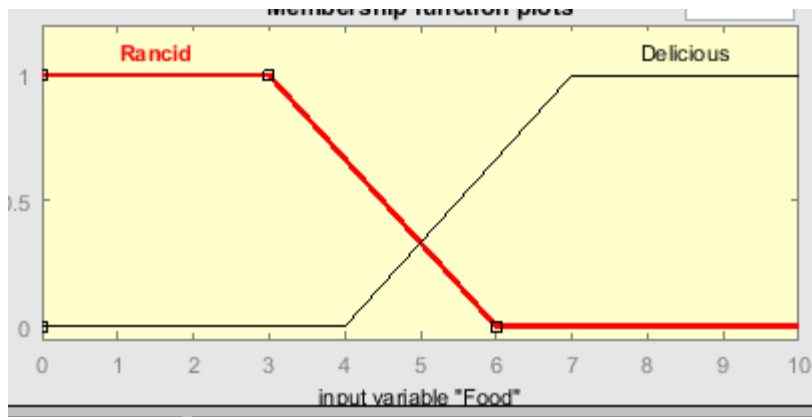
- the aggregation operation simply includes all the singletons. Now we can find the weighted average (WA) of these singletons

$$WA = \frac{\mu(k1) \times k1 + \mu(k2) \times k2 + \mu(k3) \times k3}{\mu(k1) + \mu(k2) + \mu(k3)} = \frac{0.1 \times 20 + 0.2 \times 50 + 0.5 \times 80}{0.1 + 0.2 + 0.5} = 65$$

- Thus, a zero-order Sugeno system might be sufficient for our problem's needs. Fortunately, singleton output functions satisfy the requirements of a given problem quite often

# Sugeno Fuzzy Inference Process (Tip Example)

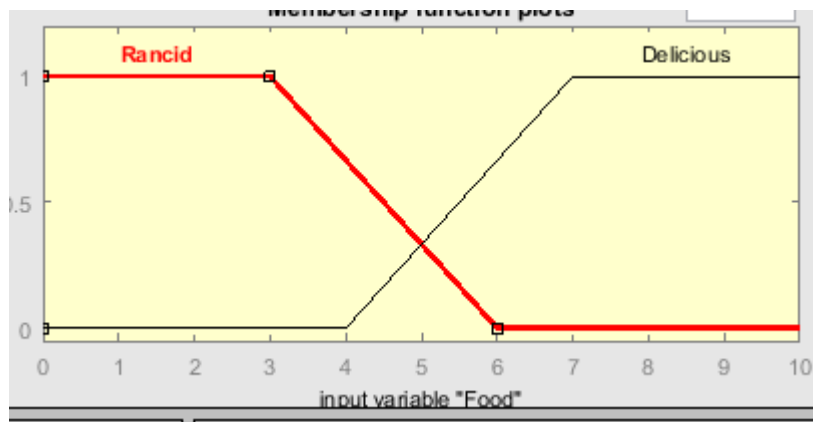
- Rules
  - If the service is poor or the food is rancid, then tip is cheap.
  - If the service is good, then tip is average.
  - If the service is excellent or the food is delicious, then tip is generous.



# Tip Example (Calculation)

- Using Sugeno method what the tip should be if Food= and service=8

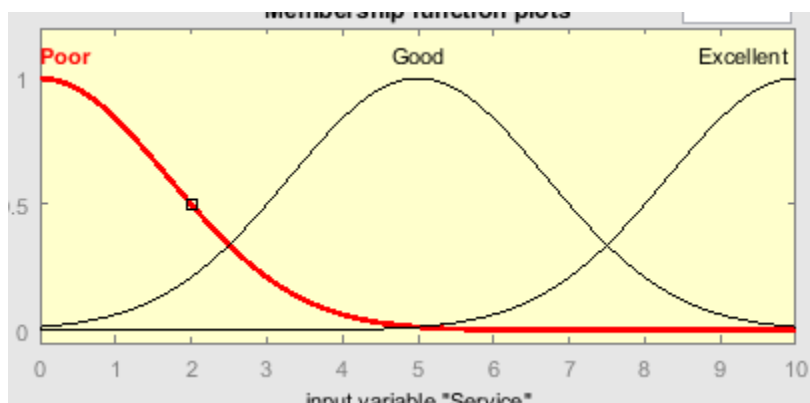
## Fuzzification



$$\text{Rancid}(6) = 0$$

$$\text{Delicious}(6) = (6-4)/(7-4) = 2/3 = 0.66$$

# Fuzzification



$$\text{Poor}(8) = 0$$

$$\text{Good}(8) = e^{-\frac{(8-5)^2}{2*1.7^2}} = 0.21$$

$$\text{Excellent}(8) = e^{-\frac{(8-10)^2}{2*1.7^2}} = 0.5$$

$$f(x; \sigma, c) = e^{\frac{-(x-c)^2}{2\sigma^2}}$$

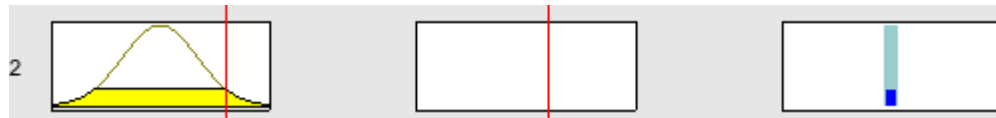
# Rule Evaluation and Implication

1) If the service is poor or the food is rancid, then tip is cheap.

$$(0) \max (0) \rightarrow 0$$

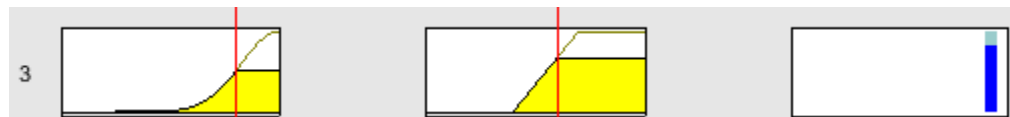
2) If the service is good, then tip is average.

$$(0.21) \rightarrow (0.21)$$

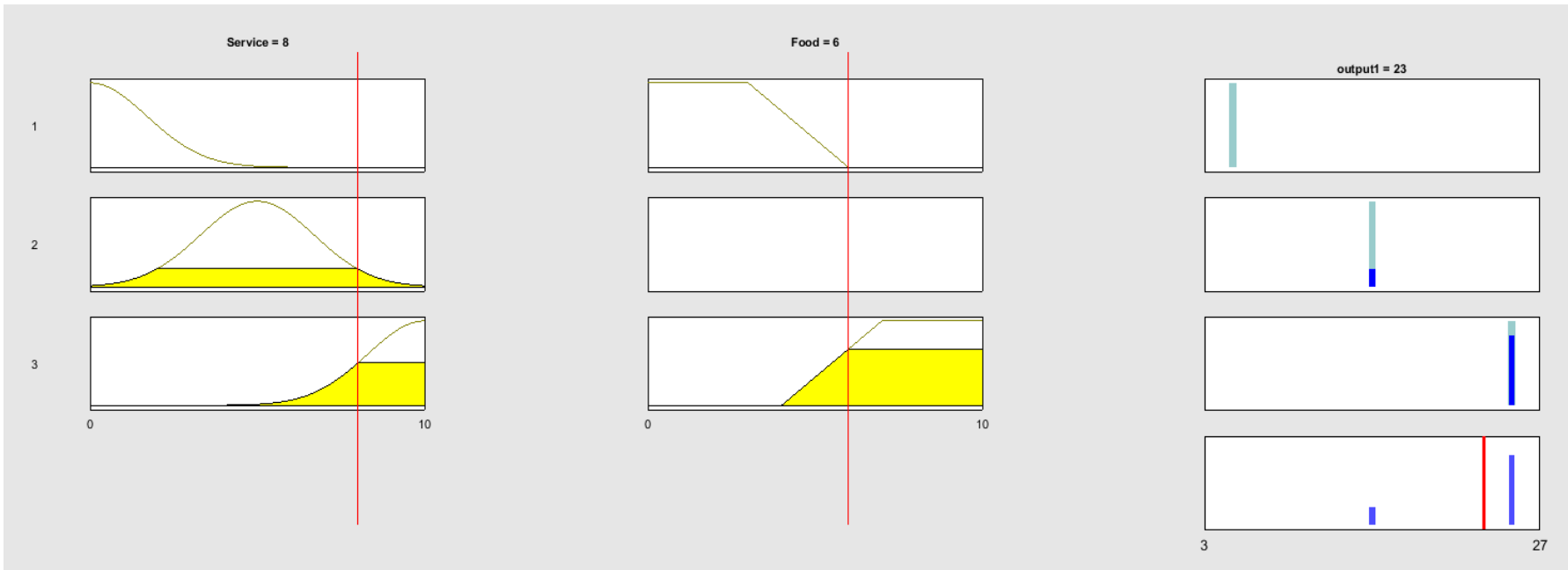


3) If the service is excellent or the food is delicious, then tip is generous.

$$(0.5) \text{ MAX } (0.66) \rightarrow (0.66)$$



# Aggregation and Defuzzification



$$WA = \frac{15 * 0.2107 + 25 * 0.666}{0.2107 + 0.666} = 22.59$$

# Mamdani or Sugeno?

- It was found that Mamdani method is widely accepted for capturing expert knowledge. It allows us to describe the expertise in more intuitive, more human-like manner. However, for systems with many parameters Mamdani-type fuzzy inference entails a substantial computational burden.
- On the other hand, the Sugeno method is computationally effective and works well with optimization and adaptive techniques, which makes it very attractive in control problems, particularly for dynamic nonlinear systems.

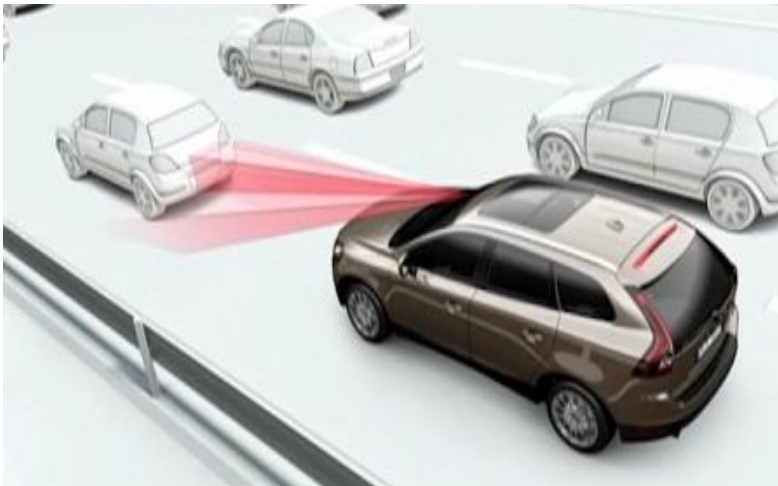
# Building a fuzzy expert system

- Specify the problem and define **linguistic variables**
- Determine fuzzy sets (membership function)
- Construct fuzzy rules
- Encode the fuzzy sets, and fuzzy rules to perform fuzzy inference into the expert system
- **Evaluate and tune the system**
  - Start with min number of FS, rules
  - adjust the FS range
  - add more rules
  - change the FS type
  - add more FS
  - change the method of implication and aggregations



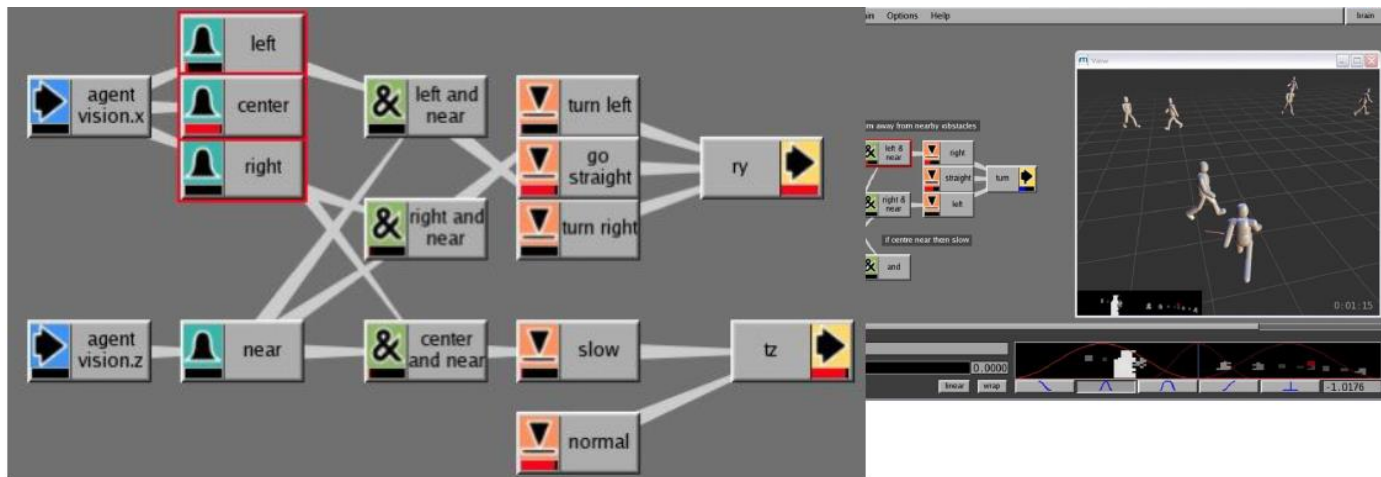
# Application of Fuzzy system

- Hardware: Control

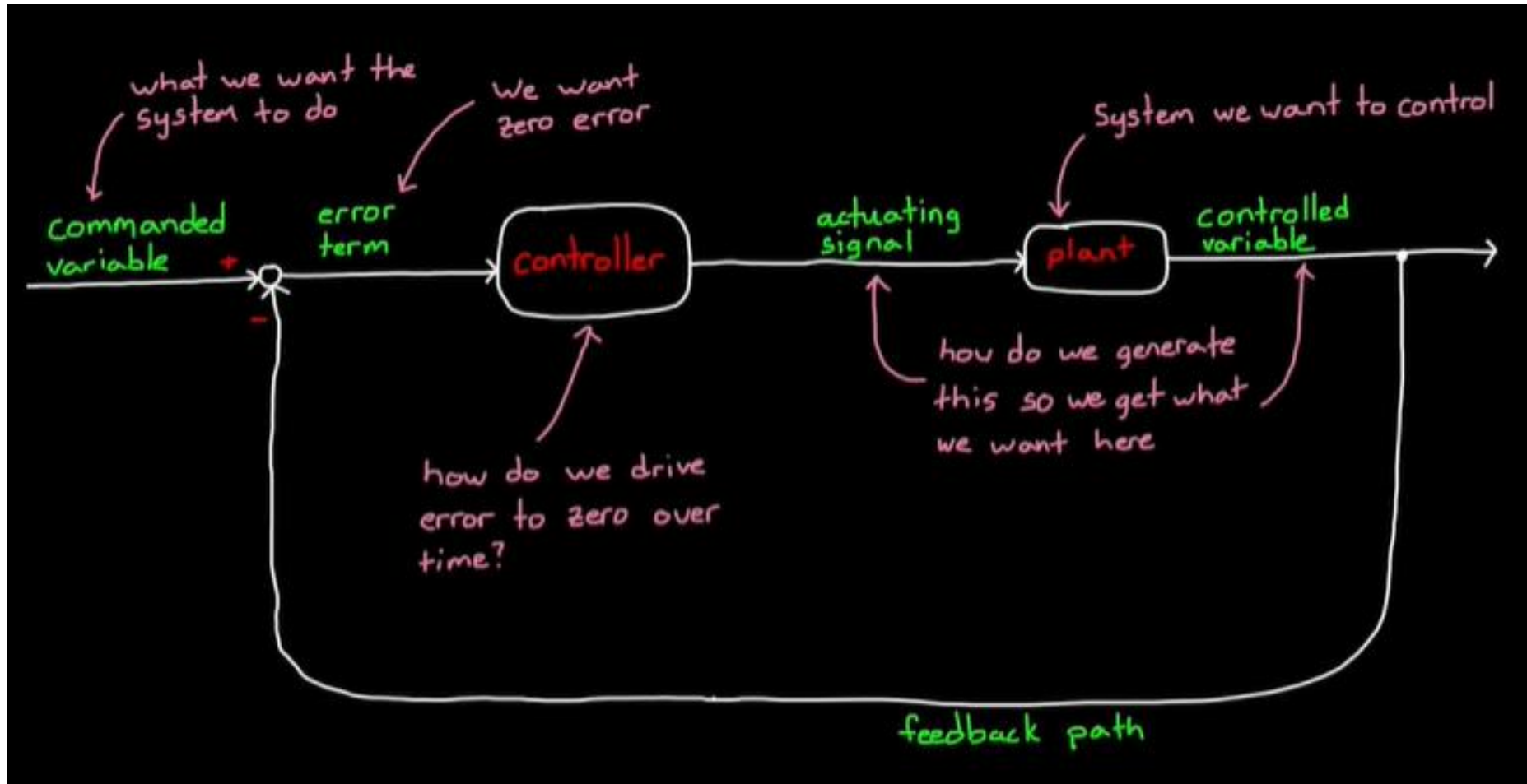


# Application of Fuzzy system

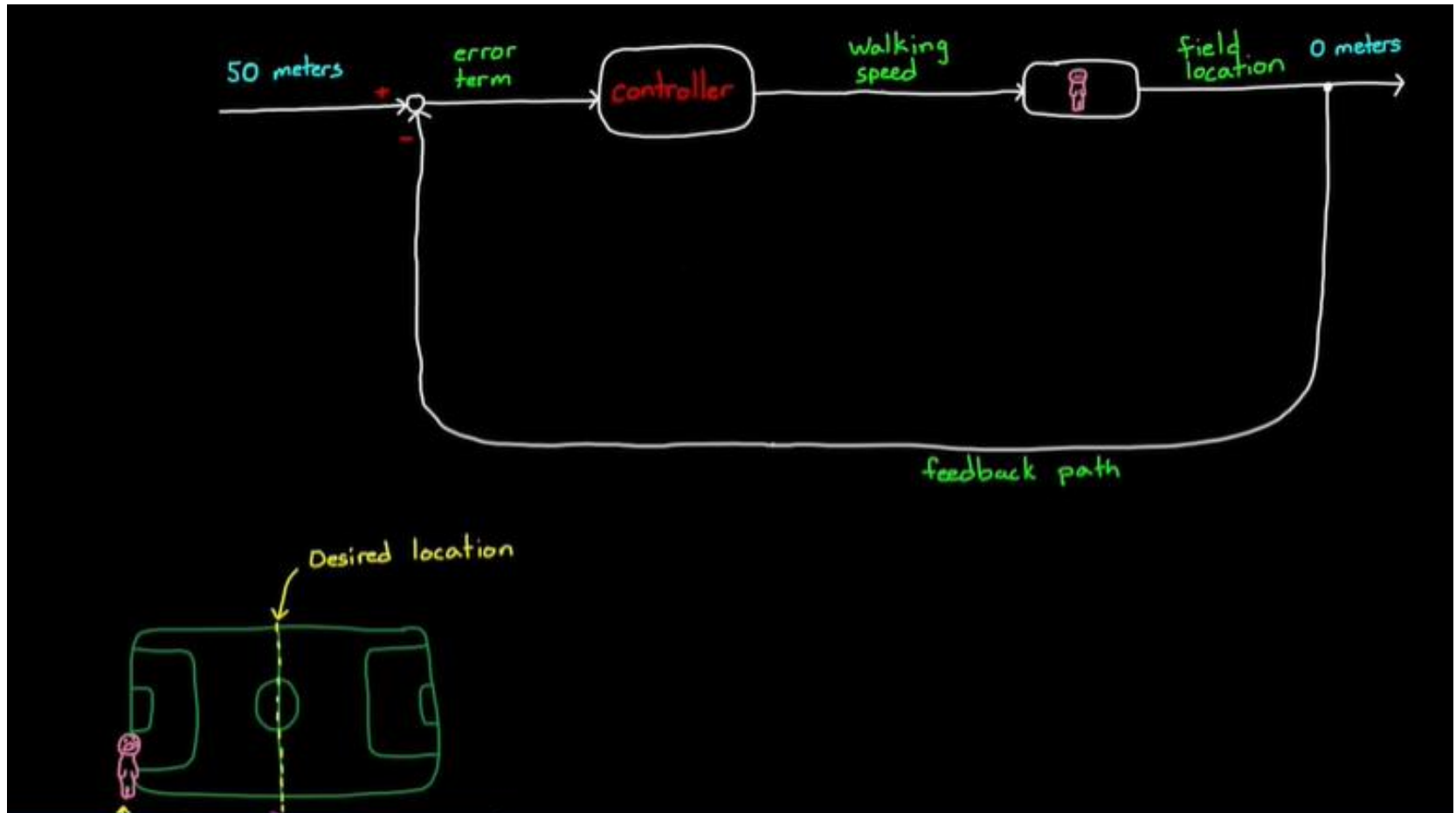
- Software: Decision support (E.g. MASSIVE)



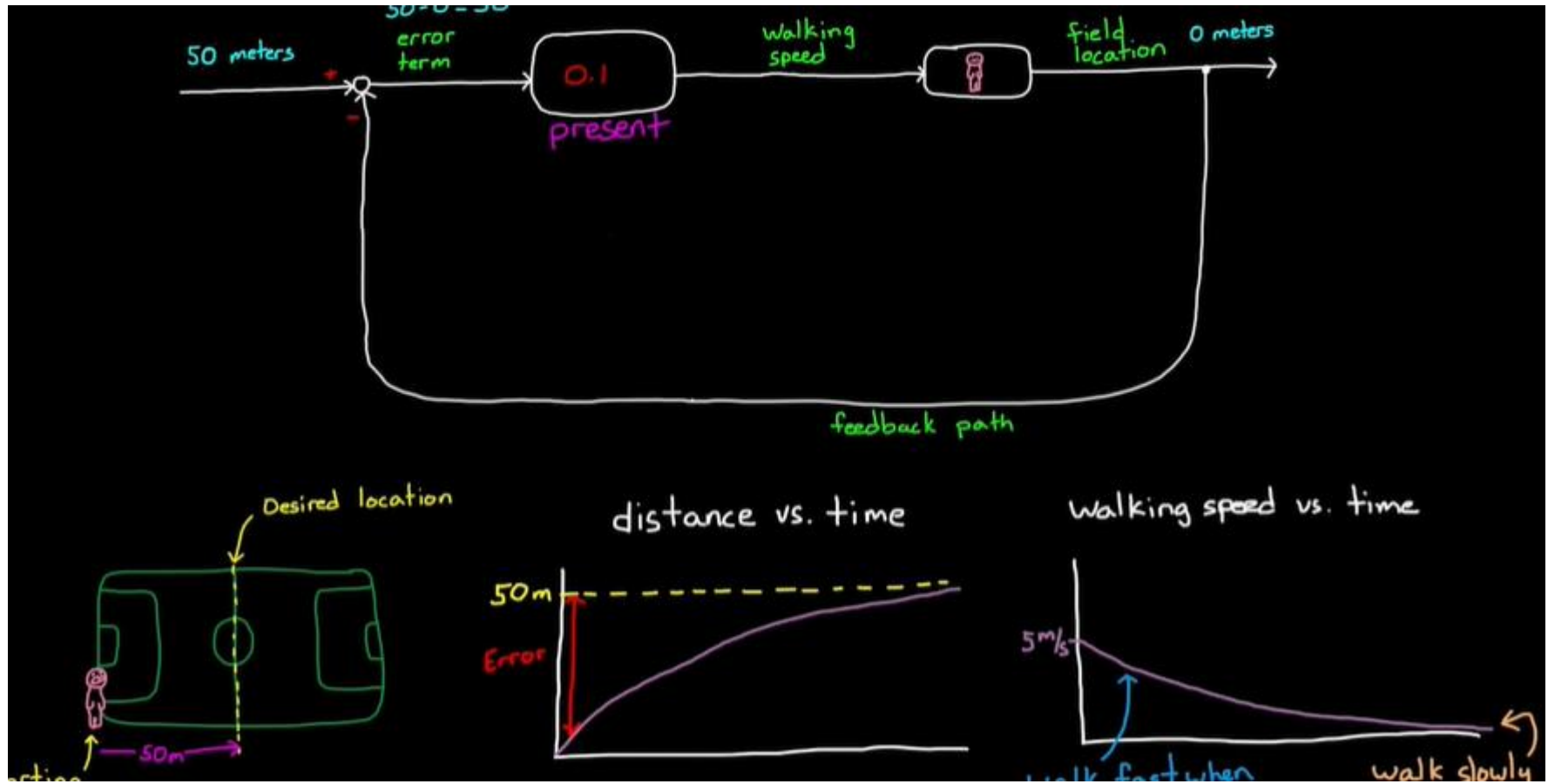
# Introduction to Control



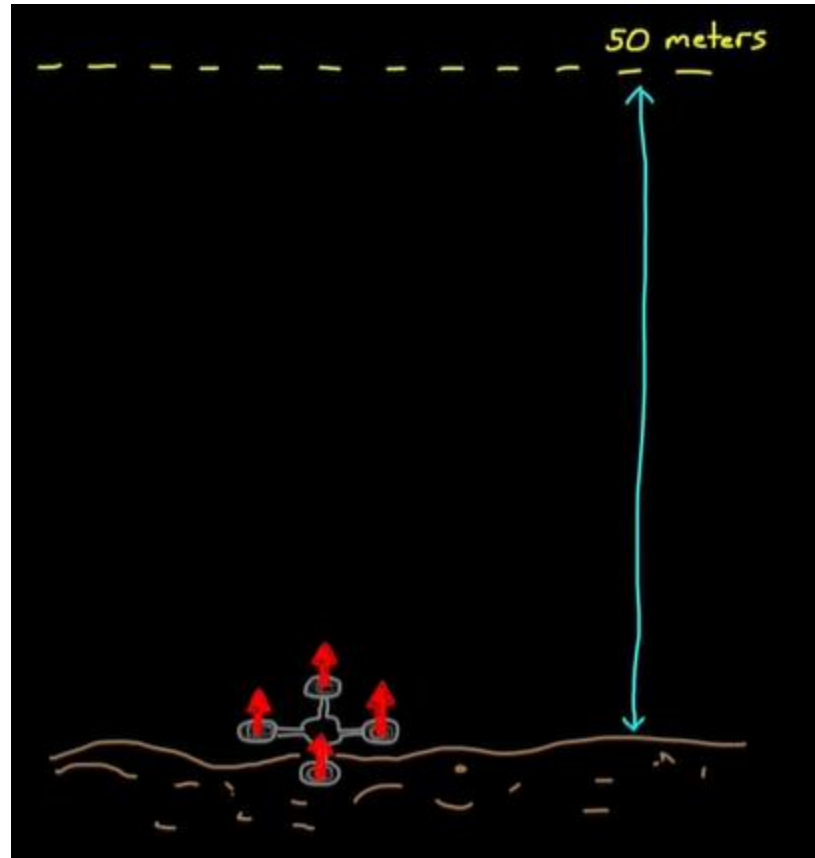
# Introduction to Control



# Introduction to Control

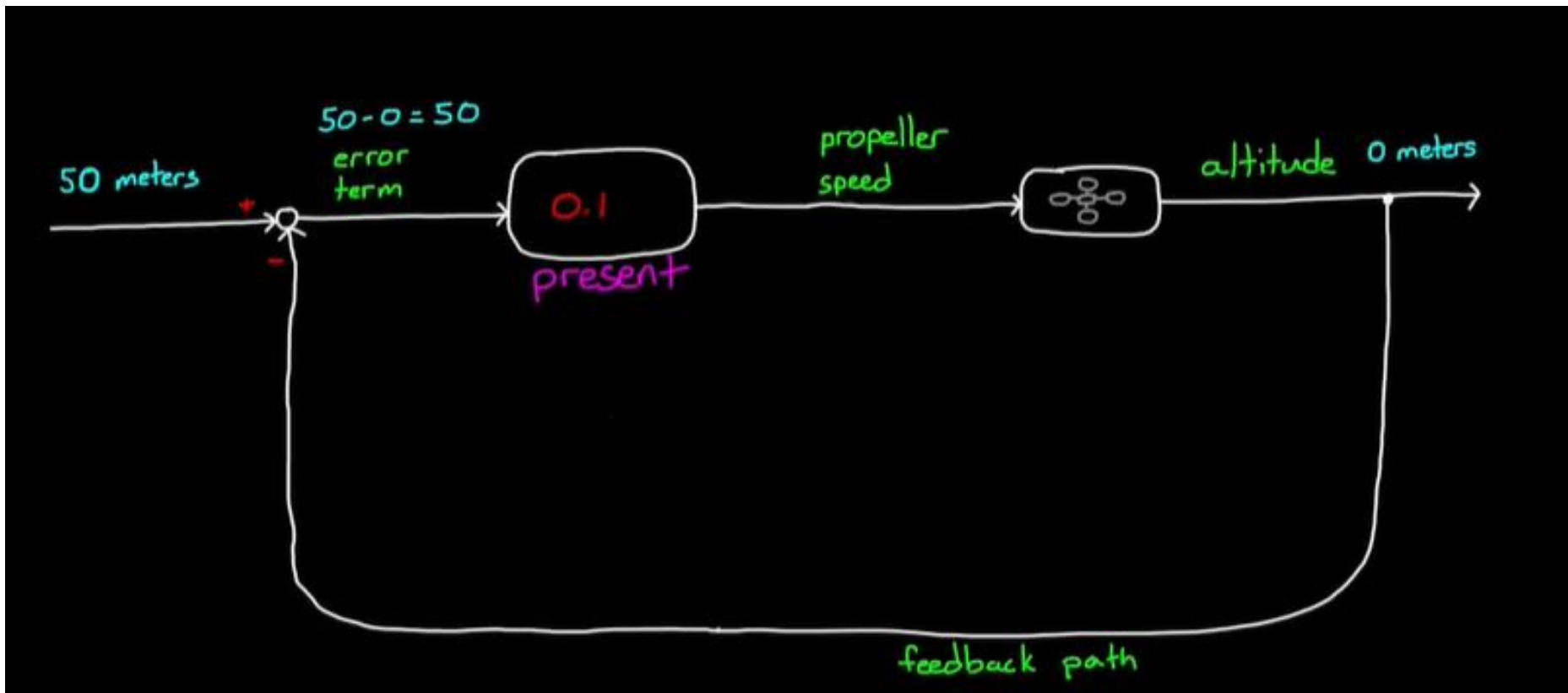


# Introduction to Control

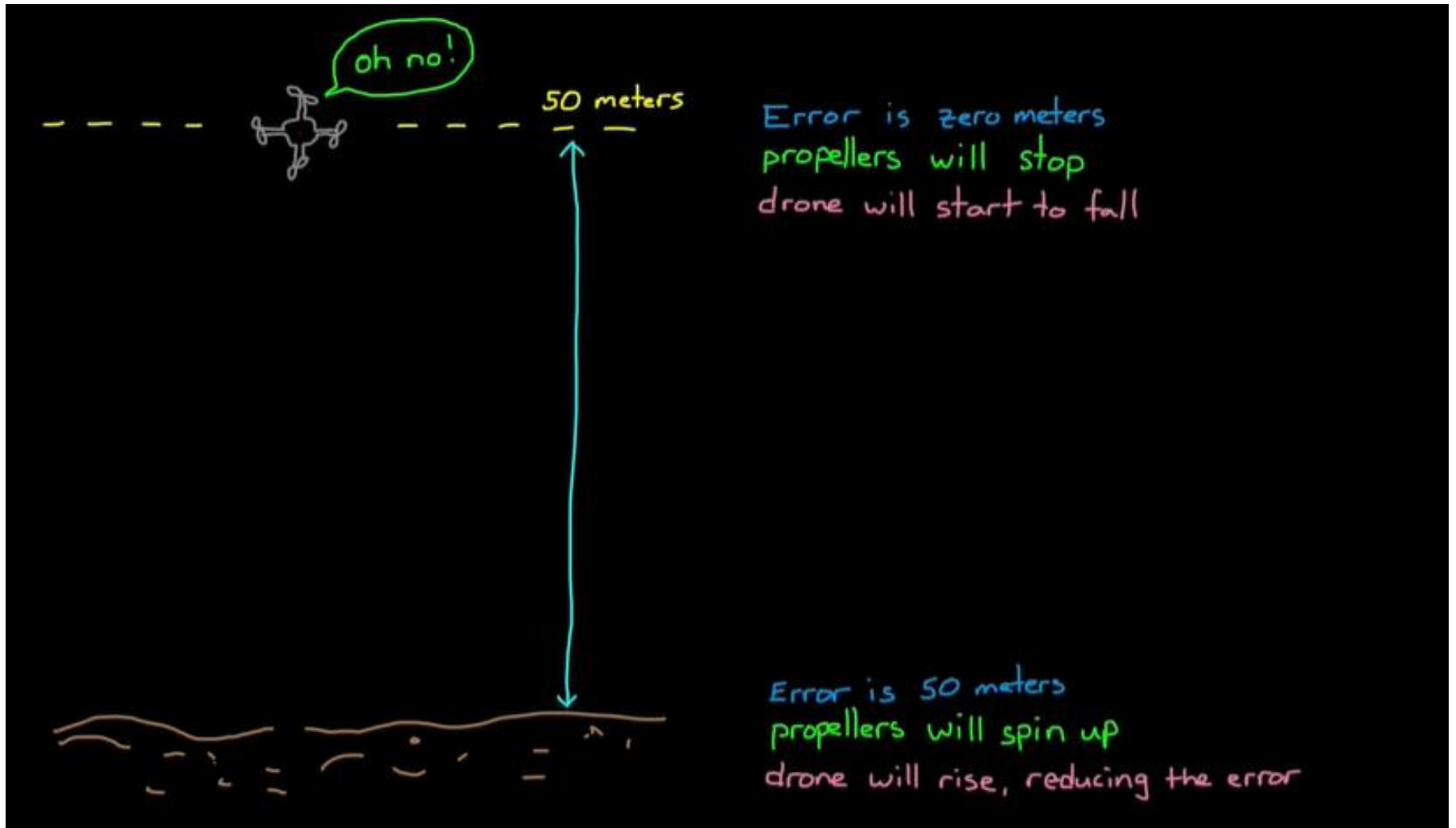




# Introduction to Control

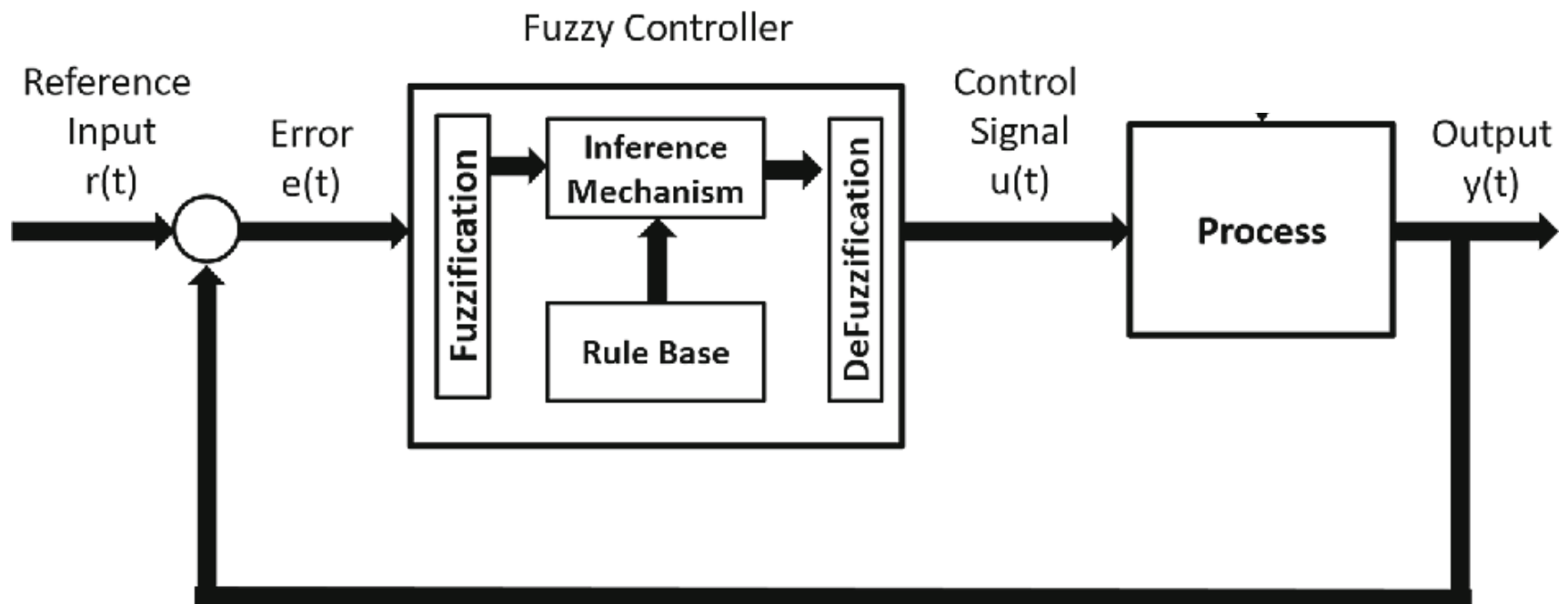


# Introduction to Control



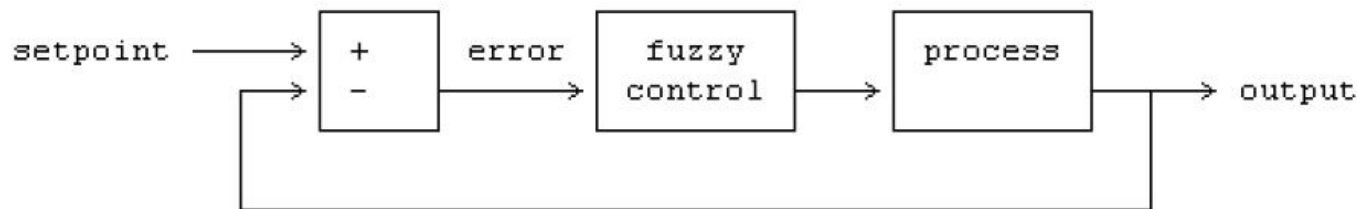


# Fuzzy Control

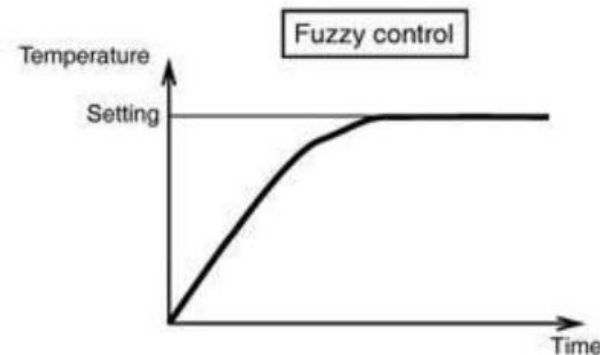


# Types of Fuzzy Controllers

- **Direct fuzzy Control**: the outputs of the fuzzy logic system are the command variables of the plant

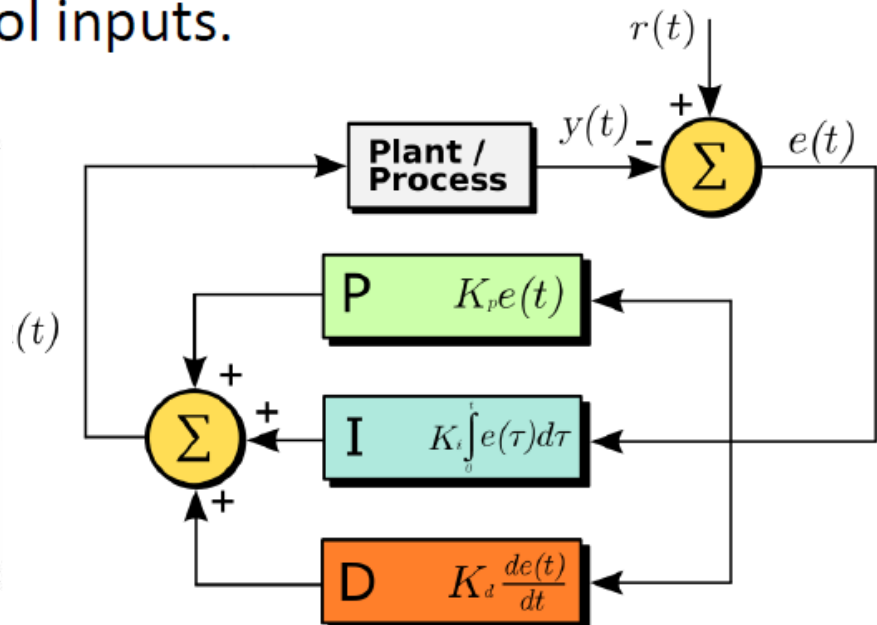
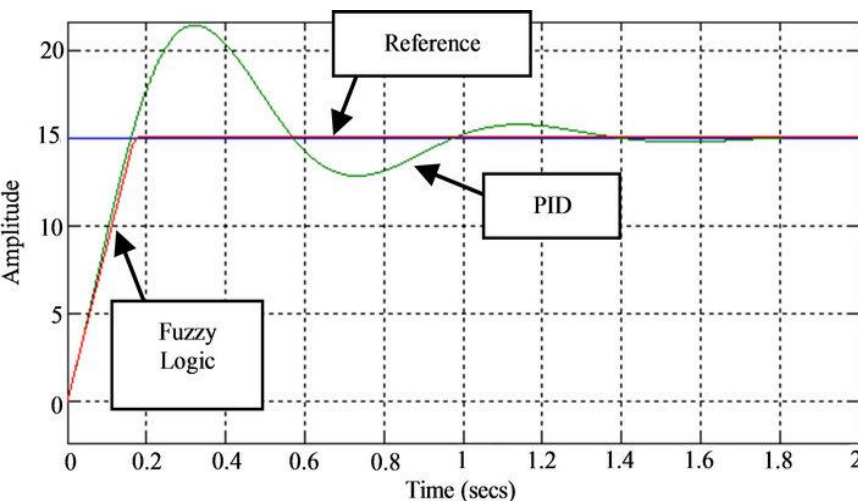


LP: large positive  
SP: small positive  
ZE: zero  
SN: small negative  
LN: large negative



# PID(proportional-integral-derivative) controller

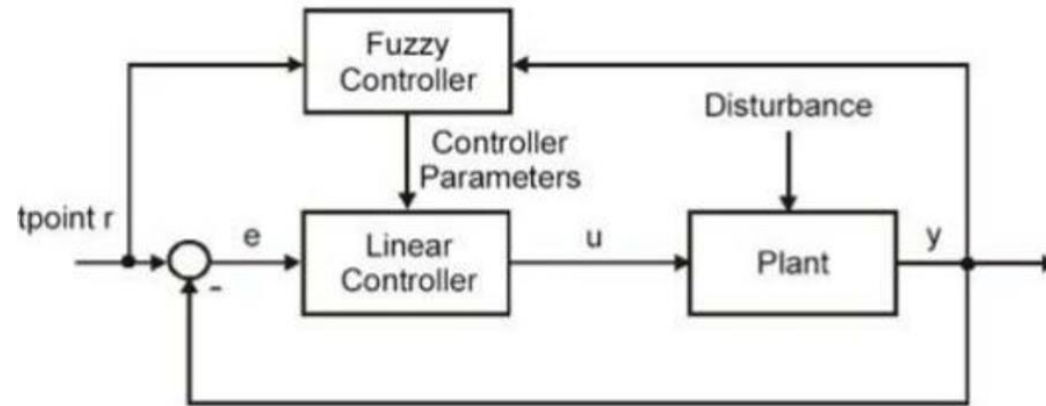
- PID controller is a control loop feedback mechanism (controller) widely used in industrial control systems
- A PID controller calculates an "error" value as the difference between a measured process variable and a desired set-point. The controller attempts to minimize the error in outputs by adjusting the process control inputs.



# Types of Fuzzy Controllers

- **PID Adaptation**

The fuzzy logic system analyzes the performance of the PID controller and optimizes it



- **Fuzzy Feedforward**

Fuzzy Logic Controller and PID Controller in Parallel

