

Alphabet: $\Sigma = \{0, 1, 2, 3\} \leftrightarrow \{A, T, G, C\}$
 Code length: ν symbol (even)
 Block length: $N = \nu n$ symbol (block = n inner codewords)
 Constraints (binary):
 run length: $\text{RL-}\rho$
 local balance: $\text{LB-}(\ell, \delta) \quad |w(\mathbf{b}_i^{i+\ell-1}) - \frac{\ell}{2}| \leq \delta$
 indicator func: $\mathbb{1}_{\rho, \ell, \delta}(\mathbf{b})$
 IDS channel:
 error prob: p_i, p_d, p_s
 drift vector:
 $\mathbf{d}^j = (d_0^j, \dots, d_N^j)$ (symbol level)
 $\tilde{\mathbf{d}}^j = (\tilde{d}_0^j, \dots, \tilde{d}_n^j)$ (word level) $\tilde{d}_i^j = d_{i\nu}^j$

Binary constraint vector set [SITA2024]

$\mathcal{B} \subset \mathbb{B}^\nu \quad (|\mathcal{B}| = 2^{k_a})$
 $\forall \mathbf{b} \in \mathcal{B}, w(\mathbf{b}) = \nu/2$ (LB: inside CW)
 $\forall \mathbf{b} \in \mathcal{B}, \forall i \in [\nu]_{\text{even}}, |\tilde{w}(\mathbf{b}_i^{\nu-1})| \leq 2\delta$ (LB: CW boundary)
 $\forall \mathbf{b} \in \mathcal{B}, \forall i \in [\nu]_{\text{odd}}, |\tilde{w}(\mathbf{b}_i^{\nu-1})| \leq 2\delta - 1$ (LB: CW boundary)
 $\forall \mathbf{b} \in \mathcal{B}, \lambda(\mathbf{b}) \leq \rho$ (RL: inside CW)
 $\forall \mathbf{b} \in \mathcal{B}, \lambda(\mathbf{b}, \nu-1) \leq \rho - 1$ (RL: CW boundary)
 $\mathbf{b} \in \mathcal{B} \rightarrow \bar{\mathbf{b}} \in \mathcal{B}$ (re-balance)
 $(01)^{\frac{\nu}{2}}, (10)^{\frac{\nu}{2}} \in \mathcal{B}$ (erasure symbol)

Composite symbol sets

$\Phi_0 \subseteq \{(\sigma_0, \sigma_1, \sigma_2, \sigma_3) \in \mathbb{Z}_{k+1}^4 \mid \sigma_0 + \sigma_1 = k, \sigma_2 = \sigma_3 = 0\}$ AT
 $\Phi_1 \subseteq \{(\sigma_0, \sigma_1, \sigma_2, \sigma_3) \in \mathbb{Z}_{k+1}^4 \mid \sigma_0 = \sigma_1 = 0, \sigma_2 + \sigma_3 = k\}$ GC
 $|\Phi_0| = |\Phi_1| = 2^{k_b} \leq k+1 \quad \Phi = \Phi_0 \cup \Phi_1$

Inner codebook

$\mathcal{C} = \{(c_0, \dots, c_{\nu-1}) \mid c_j \in \Phi_{b_j}, (b_0, \dots, b_{\nu-1}) \in \mathcal{B}\}$

Mapping

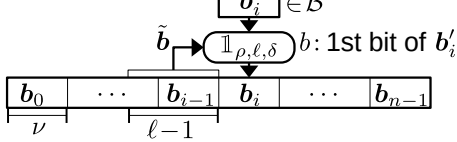
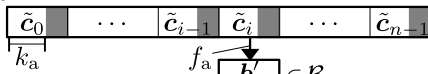
$f_a : \mathbb{B}^{k_a} \rightarrow \mathcal{B}$ (bijection)
 $f_i : \mathbb{B}^{k_b} \rightarrow \Phi_i \quad (i \in \mathbb{B})$ (bijection)
 $f_d : \Sigma \rightarrow \mathbb{B} \quad f_d(x) = \mathbb{1}[x \in \{2, 3\}]$

Encoding

input: $\tilde{\mathbf{c}} = (\tilde{c}_0, \dots, \tilde{c}_{n-1}) \quad \tilde{c}_i \in \mathbb{B}^{k_a}$
 $\hat{\mathbf{c}} = (\hat{c}_0, \dots, \hat{c}_{n-1}) \quad \hat{c}_i \in \mathbb{B}^{k_b \times \nu}$

output: $\mathbf{s} = (s_0, \dots, s_{n-1}) \quad s_i \in \Phi^\nu$

(1) encode $\tilde{\mathbf{c}}$ to $\mathbf{b} = (b_0, \dots, b_{n-1}) \in \mathcal{B}^n$



$b_i = \begin{cases} b'_i & (\mathbb{1}_{\rho, \ell, \delta}(\tilde{\mathbf{b}}, \mathbf{b}'_i) = 1) & \text{(unchanged)} \\ \bar{b}'_i & (\mathbb{1}_{\rho, \ell, \delta}(\tilde{\mathbf{b}}, \bar{\mathbf{b}}'_i) = 1) & \text{(invert)} \\ (\bar{b}b)^{\frac{\nu}{2}} & (\mathbb{1}_{\rho, \ell, \delta}(\tilde{\mathbf{b}}, (\bar{b}b)^{\frac{\nu}{2}}) = 1) & \text{(rebalance)} \\ (bb)^{\frac{\nu}{2}} & \text{(otherwise)} & \text{(rebalance)} \end{cases}$

(2) encode (\hat{c}_i, b_i) to $s_i \quad (i \in [n])$

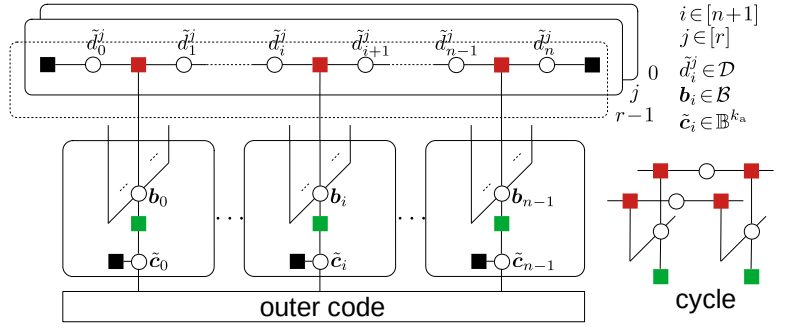
$s_{i,t} = f_{b_i,t}(\hat{c}_{i,t}^\top) \quad s_i = (s_{i,0}, \dots, s_{i,\nu-1}) \in \Phi^\nu$
 $(t \in [\nu]) \quad b_i = (b_{i,0}, \dots, b_{i,\nu-1}) \in \tilde{\mathcal{B}}$
 $\hat{c}_{i,t} : t \text{ th column of } \hat{c}_i$

(ex) $\nu = 6, |\mathcal{B}| = 16, k = 7, k_a = 4, k_b = 3$

\mathcal{B} : 0:001011 8:110100 Φ_0 : 0:0700 Φ_1 : 0:0007
 1:001101 9:110010 1:1600 1:0016
 2:001110 A:110001 2:2500 2:0025
 3:010011 B:101100 3:3400 3:0034
 -:010101 -:101010 4:4300 4:0043
 4:010110 C:101001 5:5200 5:0052
 5:011001 D:100110 6:6100 6:0061
 6:011010 E:100101 7:7000 7:0070
 7:011100 F:100011 (Lee distance?)

\tilde{c}_i : 0110(6)=011010 s_i : (0,3)=3400
 \hat{c}_i^\top : 011(3) (1,4)=0043
 100(4) (1,2)=0025
 010(2) (0,3)=3400
 011(3) (1,6)=0061
 110(6) (0,4)=4300
 100(4)

Decoding-1

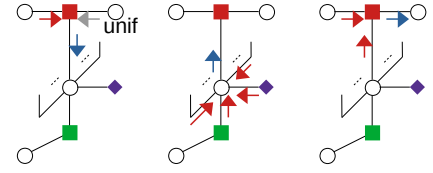


$$\begin{aligned}
 & p(\tilde{\mathbf{y}}^0, \dots, \tilde{\mathbf{y}}^{r-1}, \tilde{\mathbf{d}}^0, \dots, \tilde{\mathbf{d}}^{r-1}, \mathbf{b}, \tilde{\mathbf{c}}) \quad (\tilde{\mathbf{y}}^j = f_d(\mathbf{y}^j) \in \mathbb{B}^{N_j}) \\
 &= p(\tilde{\mathbf{y}}^0, \dots, \tilde{\mathbf{y}}^{r-1} | \tilde{\mathbf{d}}^0, \dots, \tilde{\mathbf{d}}^{r-1}, \mathbf{b}, \tilde{\mathbf{c}}) p(\tilde{\mathbf{d}}^0, \dots, \tilde{\mathbf{d}}^{r-1}) p(\mathbf{b} | \tilde{\mathbf{c}}) p(\tilde{\mathbf{c}}) \\
 &= p(\mathbf{b} | \tilde{\mathbf{c}}) p(\tilde{\mathbf{c}}) \prod_{j=0}^{r-1} p(\tilde{\mathbf{y}}^j | \mathbf{b}, \tilde{\mathbf{d}}^j) p(\tilde{\mathbf{d}}^j) \\
 &\simeq \left(\prod_{i=0}^{n-1} p(\mathbf{b}_i | \tilde{\mathbf{c}}_i) p(\tilde{\mathbf{c}}_i) \right) \prod_{j=0}^{r-1} \left(p(\tilde{\mathbf{d}}_0^j) \prod_{i=0}^{n-1} p(\tilde{\mathbf{y}}_{i\nu+\tilde{d}_i^j}^j | \mathbf{b}_i, \tilde{\mathbf{d}}_i^j, \tilde{\mathbf{d}}_{i+1}^j) p(\tilde{\mathbf{d}}_{i+1}^j | \tilde{\mathbf{d}}_i^j) \right)
 \end{aligned}$$

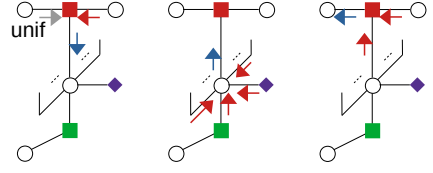
PF[Nseq][Ns+1][D]
 PB[Nseq][Ns+1][D]
 PD[Nseq][Ns][Q]
 PU[Nseq][Ns][Q]
 PI[Ns][Q]
 PO[Ns][Q]
 PM[Ns][Q]
 PUO[Ns][Q]

Scheduling

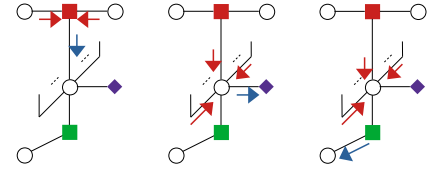
1. forward



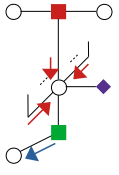
2. backward



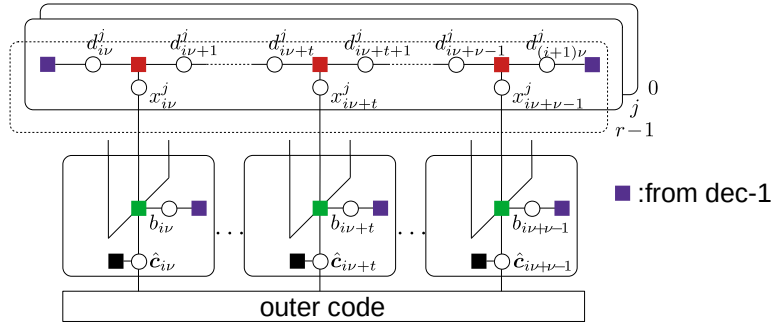
3. store



4. output

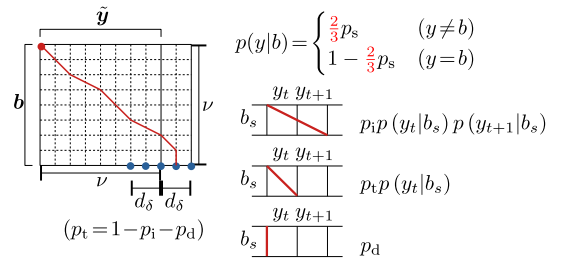


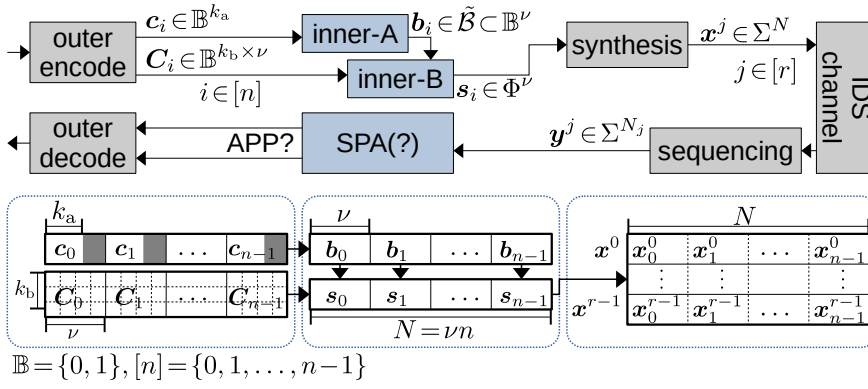
Decoding-2



$$\begin{aligned}
 & p(\mathbf{y}^0, \dots, \mathbf{y}^{r-1}, \mathbf{x}^0, \dots, \mathbf{x}^{r-1}, \mathbf{d}^0, \dots, \mathbf{d}^{r-1}, \mathbf{b}, \tilde{\mathbf{c}}) \\
 &= p(\mathbf{b}) p(\tilde{\mathbf{c}}) \prod_{j=0}^{r-1} p(\mathbf{y}^j | \mathbf{x}^j, \mathbf{d}^j) p(\mathbf{x}^j | \mathbf{b}, \tilde{\mathbf{c}}) p(\mathbf{d}^j) \\
 &\simeq \left(\prod_{t=0}^{\nu-1} p(\mathbf{b}_t) p(\tilde{\mathbf{c}}_t) \right) \prod_{j=0}^{r-1} \left(p(\tilde{d}_0^j) \prod_{t=0}^{\nu-1} p([\mathbf{y}^j]_{t+\tilde{d}_t^j}^{t+\tilde{d}_{t+1}^j} | \mathbf{x}_t, \mathbf{d}_t^j, \mathbf{d}_{t+1}^j) p(\mathbf{d}_{t+1}^j | \mathbf{d}_t^j) p(\mathbf{x}_t^j | \mathbf{b}_t, \tilde{\mathbf{c}}_t) \right)
 \end{aligned}$$

LUT





Alphabet: $\Sigma = \{0, 1, 2, 3\} \leftrightarrow \{A, T, G, C\}$
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 Block length: $N = \nu n$ symbol (block = n inner codewords)
 Constraints (binary):
 run length: RL- ρ
 local balance: LB- (ℓ, δ) $|w(b_i^{i+\ell-1}) - \frac{\ell}{2}| \leq \delta$
 indicator func: $\mathbb{I}_{\rho, \ell, \delta}(\mathbf{b})$
 IDS channel:
 error prob: p_i, p_d, p_s
 drift vector:
 $\mathbf{d}^j = (d_0^j, \dots, d_N^j)$ (symbol level)
 $\tilde{\mathbf{d}}^j = (\tilde{d}_0^j, \dots, \tilde{d}_n^j)$ (word level) $\tilde{d}_i^j = d_{i\nu}^j$

Binary constraint vector set [SITA2024]

$\mathcal{B} \subset \mathbb{B}^\nu$ ($|\mathcal{B}| = 2^{k_a}$)
 $\forall \mathbf{b} \in \mathcal{B}, w(\mathbf{b}) = \nu/2$ (LB: inside CW)
 $\forall \mathbf{b} \in \mathcal{B}, \forall i \in [\nu]_{\text{even}}, |\tilde{w}(\mathbf{b}_i^{\nu-1})| \leq 2\delta$ (LB: CW boundary)
 $\forall \mathbf{b} \in \mathcal{B}, \forall i \in [\nu]_{\text{odd}}, |\tilde{w}(\mathbf{b}_i^{\nu-1})| \leq 2\delta - 1$ (LB: CW boundary)
 $\forall \mathbf{b} \in \mathcal{B}, \lambda(\mathbf{b}) \leq \rho$ (RL: inside CW)
 $\forall \mathbf{b} \in \mathcal{B}, \lambda(\mathbf{b}, \nu-1) \leq \rho - 1$ (RL: CW boundary)
 $\mathbf{b} \in \mathcal{B} \rightarrow \bar{\mathbf{b}} \in \mathcal{B}$ (re-balance)
 $(01)^{\frac{\nu}{2}}, (10)^{\frac{\nu}{2}} \notin \mathcal{B}$ (erasure symbol)
 $\tilde{\mathcal{B}} = \mathcal{B} \cup \{(01)^{\frac{\nu}{2}}, (10)^{\frac{\nu}{2}}\}$ (re-balance)

Composite symbol sets

$\Phi_0 \subseteq \{(\sigma_0, \sigma_1, \sigma_2, \sigma_3) \in \mathbb{Z}_{k+1}^4 \mid \sigma_0 + \sigma_1 = k, \sigma_2 = \sigma_3 = 0\}$ AT
 $\Phi_1 \subseteq \{(\sigma_0, \sigma_1, \sigma_2, \sigma_3) \in \mathbb{Z}_{k+1}^4 \mid \sigma_0 = \sigma_1 = 0, \sigma_2 + \sigma_3 = k\}$ GC
 $|\Phi_0| = |\Phi_1| = 2^{k_b} \leq k+1$ $\Phi = \Phi_0 \cup \Phi_1$

Inner codebook

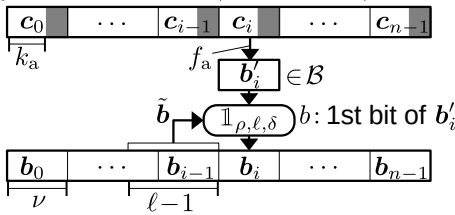
$\mathcal{C} = \{(c_0, \dots, c_{\nu-1}) \mid c_j \in \Phi_{b_j}, (b_0, \dots, b_{\nu-1}) \in \mathcal{B}\}$

Mapping $f_a: \mathbb{B}^{k_a} \rightarrow \mathcal{B}$ (bijection)
 $f_i: \mathbb{B}^{k_b} \rightarrow \Phi_i$ ($i \in \mathbb{B}$) (bijection)
 $f_d: \Sigma \rightarrow \mathbb{B}$ $f_d(x) = \mathbb{1}[x \in \{2, 3\}]$

Encoding

input: $\mathbf{c} = (c_0, \dots, c_{n-1})$ $c_i \in \mathbb{B}^{k_a}$
 $\mathbf{C} = (\mathbf{C}_0, \dots, \mathbf{C}_{n-1})$ $\mathbf{C}_i \in \mathbb{B}^{k_b \times \nu}$
 output: $\mathbf{s} = (s_0, \dots, s_{n-1})$ $s_i \in \Phi^\nu$

(1) encode \mathbf{c} to $\mathbf{b} = (b_0, \dots, b_{n-1}) \in \tilde{\mathcal{B}}^n$



$b_i = \begin{cases} b'_i & (\mathbb{I}_{\rho, \ell, \delta}(\tilde{\mathbf{b}}, \mathbf{b}'_i) = 1) & \text{(unchanged)} \\ \bar{b}'_i & (\mathbb{I}_{\rho, \ell, \delta}(\tilde{\mathbf{b}}, \bar{b}'_i) = 1) & \text{(invert)} \\ (\bar{b}b)^{\frac{\nu}{2}} & (\mathbb{I}_{\rho, \ell, \delta}(\tilde{\mathbf{b}}, (\bar{b}b)^{\frac{\nu}{2}}) = 1) & \text{(rebalance)} \\ (bb)^{\frac{\nu}{2}} & \text{(otherwise)} & \text{(rebalance)} \end{cases}$

(2) encode $(\mathbf{C}_i, \mathbf{b}_i)$ to \mathbf{s}_i ($i \in [n]$)

$s_{i,t} = f_{b_i,t}(\mathbf{C}_{i,t}^\top)$ $\mathbf{s}_i = (s_{i,0}, \dots, s_{i,\nu-1}) \in \Phi^\nu$
 $(t \in [\nu])$ $\mathbf{b}_i = (b_{i,0}, \dots, b_{i,\nu-1}) \in \tilde{\mathcal{B}}$
 $\mathbf{C}_{i,t}$: t th column of \mathbf{C}_i

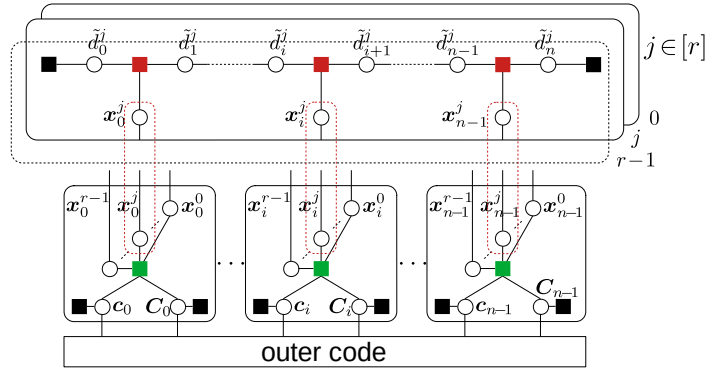
(ex) $\nu=6, |\mathcal{B}|=16, k=7, k_a=4, k_b=3$

B: 0:001011	8: 110100	Φ_0 : 0:0700	Φ_1 : 0:0007
1: 001101	9: 110010	1: 1600	1: 0016
2: 001110	A: 110001	2: 2500	2: 0025
3: 010011	B: 101100	3: 3400	3: 0034
-: 010101	-: 101010	4: 4300	4: 0043
4: 010110	C: 101001	5: 5200	5: 0052
5: 011001	D: 100110	6: 6100	6: 0061
6: 011010	E: 100101	7: 7000	7: 0070
7: 011100	F: 100011		

(Lee distance?)

c_i : 0110(6)=011010
 \mathbf{C}_i^\top : 011(3)
 \mathbf{s}_i : (0,3)=3400
 1,4)=0043
 1,2)=0025
 0,3)=3400
 1,6)=0061
 0,4)=4300

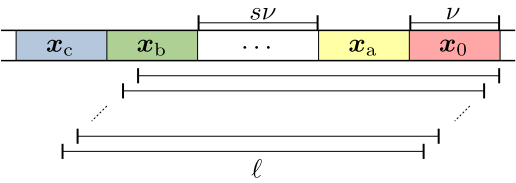
Decoding



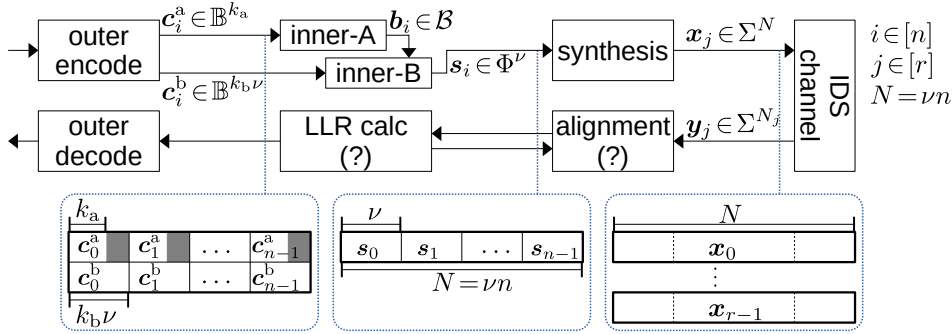
$p(\mathbf{y}^0, \dots, \mathbf{y}^{r-1}, \mathbf{x}^0, \dots, \mathbf{x}^{r-1}, \tilde{\mathbf{d}}^0, \dots, \tilde{\mathbf{d}}^{r-1}, \mathbf{c}, \mathbf{C})$
 $= p(\mathbf{y}^0, \dots, \mathbf{y}^{r-1} \mid \mathbf{x}^0, \dots, \mathbf{x}^{r-1}, \tilde{\mathbf{d}}^0, \dots, \tilde{\mathbf{d}}^{r-1}, \mathbf{c}, \mathbf{C}) \times$
 $p(\tilde{\mathbf{d}}^0, \dots, \tilde{\mathbf{d}}^{r-1}) p(\mathbf{x}^0, \dots, \mathbf{x}^{r-1} \mid \mathbf{c}, \mathbf{C}) p(\mathbf{c}, \mathbf{C})$
 $= p(\mathbf{c}) p(\mathbf{C}) \prod_{j=0}^{r-1} p(\mathbf{y}^j \mid \mathbf{x}^j, \tilde{\mathbf{d}}^j) p(\tilde{\mathbf{d}}^j) p(\mathbf{x}^j \mid \mathbf{c}, \mathbf{C})$
 $\simeq \left(\prod_{i=0}^{n-1} p(\mathbf{c}_i) p(\mathbf{C}_i) \right) \times$
 $\prod_{j=0}^{r-1} \left(p(\tilde{d}_0^j) \prod_{i=0}^{n-1} p([\mathbf{y}^j]_{i\nu+\tilde{d}_i^j}^{i\nu+\nu-1+\tilde{d}_{i+1}^j} \mid \mathbf{x}_i^j, \tilde{d}_i^j, \tilde{d}_{i+1}^j) p(\tilde{d}_{i+1}^j \mid \tilde{d}_i^j) p(\mathbf{x}_i^j \mid \mathbf{c}_i, \mathbf{C}_i) \right)$
 $\prod_{j=0}^{r-1} p(\mathbf{x}_i^j \mid \mathbf{c}_i, \mathbf{C}_i) = \prod_{j=0}^{r-1} p(\mathbf{b}_i^j \mid \mathbf{c}_i) p(\mathbf{x}_i^j \mid \mathbf{b}_i^j, \mathbf{C}_i) = \prod_{j=0}^{r-1} p(\mathbf{b}_i^j \mid \mathbf{c}_i) \prod_{t=0}^{\nu-1} p(\mathbf{x}_{i,t}^j \mid \mathbf{b}_{i,t}^j, \mathbf{C}_{i,t})$
 $\mathbf{b}_i^j = (f_d(x_{i,0}^j), \dots, f_d(x_{i,\nu-1}^j))$
 $p(\mathbf{x}_{i,t}^j \mid \mathbf{b}_{i,t}^j, \mathbf{C}_{i,t}) = \sigma_{x_{i,t}^j} / k$ ($x_{i,t}^j \in \Sigma, \mathbf{b}_{i,t}^j \in \mathbb{B}, \mathbf{C}_{i,t}^\top \in \mathbb{B}^{k_b}$)
 $(\sigma_0, \sigma_1, \sigma_2, \sigma_3) = f_{b_{i,t}^j}(\mathbf{C}_{i,t}^\top)$

- PF[Nseq][Ns+1][D]
- PB[Nseq][Ns+1][D]
- PD[Nseq][Ns][Q]
- PU[Nseq][Ns][Q]
- PI[Ns][Q]
- PO[Ns][Q]
- PM[Ns][Q]
- PU0[Ns][Q]

Encoding channel matrix (binary)



(1) $\ell = \nu$



Code length: ν (even)

Binary constraint vector set: $\mathcal{B} \subset \mathbb{B}^\nu$ ($|\mathcal{B}| \geq 2^{k_a}$)

Composite symbol set:

$$\Phi_0 \subseteq \{(\sigma_0, \sigma_1, \sigma_2, \sigma_3) \in \mathbb{Z}_{k+1}^4 \mid \sigma_0 + \sigma_1 = k, \sigma_2 = \sigma_3 = 0\} \text{ AT}$$

$$\Phi_1 \subseteq \{(\sigma_0, \sigma_1, \sigma_2, \sigma_3) \in \mathbb{Z}_{k+1}^4 \mid \sigma_0 = \sigma_1 = 0, \sigma_2 + \sigma_3 = k\} \text{ GC}$$

$$|\Phi_0| = |\Phi_1| = 2^{k_b} \leq k+1 \quad \Phi = \Phi_0 \cup \Phi_1$$

Inner codebook:

$$\mathcal{C} \subseteq \{(c_0, \dots, c_{\nu-1}) \mid c_j \in \Phi_{b_j}, (b_0, \dots, b_{\nu-1}) \in \mathcal{B}\}$$

Encode:

$$f_e^a : \mathbb{B}^{k_a} \rightarrow \mathcal{B} \quad (\text{lossy})$$

$$f_e^i : \mathbb{B}^{k_b} \rightarrow \Phi_i \quad (i \in \mathbb{B}) \quad (\text{bijection})$$

$$f_e(c_i^a, c_i^b) = s_i = (s_{i,0}, \dots, s_{i,\nu-1}) : \mathbb{B}^{k_a} \times \mathbb{B}^{k_b \nu} \rightarrow \Phi^\nu$$

$$s_{i,j} = f_e^{b_i}(c_{i,j}^b) \quad (b_0, \dots, b_{\nu-1}) = f_e^a(c_i^a)$$

$$c_{i,j}^b = [c_i^b]_{j\nu}^{j\nu+\nu-1}$$

(example) $\nu=6, |\mathcal{B}|=16, k=7, k_a=4, k_b=3,$

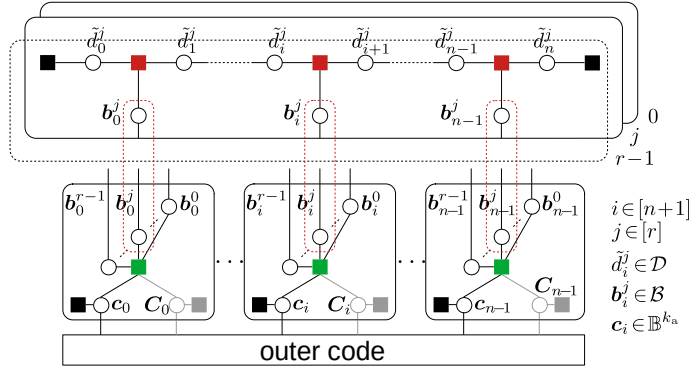
\mathcal{B} : 0:001011	8:110100	Φ_0 : 0:0700	Φ_1 : 0:0007
1:001101	9:110010	1:1600	1:0016
2:001110	A:110001	2:2500	2:0025
3:010011	B:101100	3:3400	3:0034
-:010101	-:101010	4:4300	4:0043
4:010110	C:101001	5:5200	5:0052
5:011001	D:100110	6:6100	6:0061
6:011010	E:100101	7:7000	7:0070
7:011100	F:100011		

Lee dist?

Decode:

Decoding (separate)

[approximation?]



$$\begin{aligned}
 & p(\mathbf{y}^0, \dots, \mathbf{y}^{r-1}, \mathbf{b}^0, \dots, \mathbf{b}^{r-1}, \tilde{\mathbf{d}}^0, \dots, \tilde{\mathbf{d}}^{r-1}, \mathbf{c}) \\
 &= p(\mathbf{y}^0, \dots, \mathbf{y}^{r-1} | \mathbf{b}^0, \dots, \mathbf{b}^{r-1}, \tilde{\mathbf{d}}^0, \dots, \tilde{\mathbf{d}}^{r-1}, \mathbf{c}) p(\tilde{\mathbf{d}}^0, \dots, \tilde{\mathbf{d}}^{r-1}) p(\mathbf{b}^0, \dots, \mathbf{b}^{r-1} | \mathbf{c}) p(\mathbf{c}) \\
 &= p(\mathbf{c}) \prod_{j=0}^{r-1} p(\mathbf{y}^j | \mathbf{b}^j, \tilde{\mathbf{d}}^j) p(\tilde{\mathbf{d}}^j) p(\mathbf{b}^j | \mathbf{c}) \\
 &\simeq \left(\prod_{i=0}^{n-1} p(c_i) \right) \prod_{j=0}^{r-1} \left(p(\tilde{d}_0^j) \prod_{i=0}^{n-1} p\left([\mathbf{y}^j]_{i\nu+\nu-1+\tilde{d}_{i+1}^j}^{\nu+\nu-1+\tilde{d}_{i+1}^j} | b_i^j, \tilde{d}_i^j, \tilde{d}_{i+1}^j \right) p(\tilde{d}_{i+1}^j | \tilde{d}_i^j) p(b_i^j | c_i) \right) \\
 &\propto \left(\prod_{i=0}^{n-1} p(c_i) \right) \prod_{j=0}^{r-1} \left(p(\tilde{d}_0^j) \prod_{i=0}^{n-1} p\left([\tilde{\mathbf{y}}^j]_{i\nu+\nu-1+\tilde{d}_{i+1}^j}^{\nu+\nu-1+\tilde{d}_{i+1}^j} | b_i^j, \tilde{d}_i^j, \tilde{d}_{i+1}^j \right) p(\tilde{d}_{i+1}^j | \tilde{d}_i^j) p(b_i^j | c_i) \right) \\
 &\quad \underbrace{\hspace{10em}}_{\text{sim}} \quad \tilde{\mathbf{y}}^j = f_d(\mathbf{y}^j) \in \mathbb{B}^{N_j} \\
 &\prod_{j=0}^{r-1} p(b_i^j | c_i)
 \end{aligned}$$