Definition of inner code

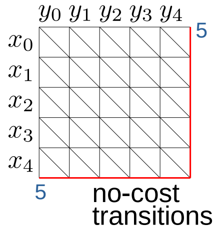
β : code length
ν : number of codes
$\mathcal{C}_i \subset \Sigma^\beta$: code $(i \in [\nu\rangle)$
$\boldsymbol{b} = (b_0, \ldots, b_{\nu-1})$: bit-width
$|\mathcal{C}_i| = 2^{b_i}$

$R_\mathrm{I} = \frac{1}{2n\beta} \sum_{i=0}^{\nu-1} b_i$: inner code rate

Metric

$\boldsymbol{x} = (x_0, \ldots, x_{\beta-1}) \in \mathbb{B}^\beta$
$\boldsymbol{y} = (y_0, \ldots, y_{\beta-1}) \in \mathbb{B}^\beta$
metric $d(\boldsymbol{x}, \boldsymbol{y})$:

insertion: (insert 0 or 1) + (delete the last bit)
deletion: (delete one bit) + (append 0 or 1)
substitution: invert one bit



Construction from binary code

$\boldsymbol{b}' = (b'_0, \ldots, b'_{\nu-1}) \in [\beta+1\rangle^\nu \quad (b_i = b'_i + \beta)$
even β : $2^{b'_i} \le \binom{\beta}{\beta/2}$
odd β : $2^{b'_i} \le \binom{\beta}{(\beta-1)/2} = \binom{\beta}{(\beta+1)/2}$

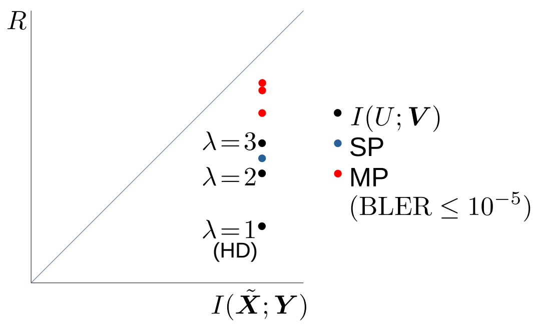
[computer search]

input: β, b', t
output: $\mathcal{B} \subset \mathbb{B}^\beta$
constraint: $|\mathcal{B}| = 2^{b'}, \forall \boldsymbol{u} \in \mathcal{B}, w(\boldsymbol{u}) = t$
objective:
(1) maximize: $\min_{\boldsymbol{x},\boldsymbol{y} \in \mathcal{B}} d(\boldsymbol{x}, \boldsymbol{y}) \quad (\boldsymbol{x} \neq \boldsymbol{y})$
(2) maximize:
(+ total sum?)

			b'					
β	t	$\binom{\beta}{t}$	1	2	3	4	5	6
2	$\{1\}$	2	$\frac{2}{2}$	–	–	–	–	–
3	$\{1,2\}$	3	$\frac{2}{3}$	–	–	–	–	–
4	$\{2\}$	6	$\frac{2}{6}$	$\frac{4}{6}$	–	–	–	–
5	$\{2,3\}$	10	$\frac{2}{10}$	$\frac{4}{10}$	$\frac{8}{10}$	–	–	–
6	$\{3\}$	20	$\frac{2}{20}$	$\frac{4}{20}$	$\frac{8}{20}$	$\frac{16}{20}$	–	–
7	$\{3,4\}$	35	$\frac{2}{35}$	$\frac{4}{35}$	$\frac{8}{35}$	$\frac{16}{35}$	$\frac{32}{35}$	–
8	$\{4\}$	70	$\frac{2}{70}$	$\frac{4}{70}$	$\frac{8}{70}$	$\frac{16}{70}$	$\frac{32}{70}$	$\frac{64}{32}$
9	$\{4,5\}$	126	$\frac{2}{126}$	$\frac{4}{126}$	$\frac{8}{126}$	$\frac{16}{126}$	$\frac{32}{126}$	$\frac{64}{126}$

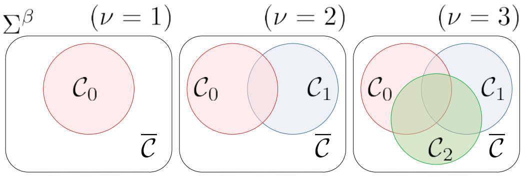
Evaluation of inner code

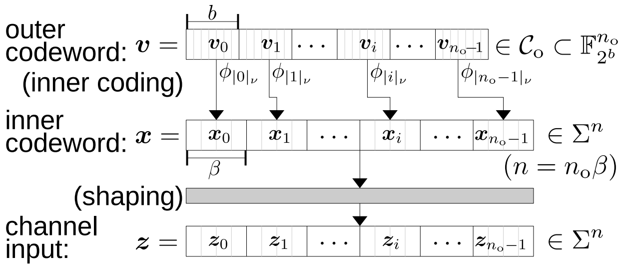
channel input: $u \in \mathbb{F}_{2^{b(i,\nu)}}$
channel output: $\boldsymbol{p} \in \mathbb{P}_{2^{b(i,\nu)}}$ (APP from inner code)
(a) HD+H(p): $\boldsymbol{v} = (v, Q_\lambda(H(\boldsymbol{p})))$
Q_λ : λ-level quantization
(b) list output: (list size λ) $\boldsymbol{v} = (v_0, \ldots, v_{\lambda-1})$
$= \arg\max(\boldsymbol{p}, \lambda) \in \mathbb{F}^{\lambda}_{2^{b(i,\nu)}}$
mutual info.: $I(U; \boldsymbol{V})$



Concatenated coding

channel alphabet: $\Sigma = \{\mathsf{A},\mathsf{T},\mathsf{G},\mathsf{C}\}$
outer code: $\mathcal{C}_\mathrm{o} \subset \mathbb{F}_{2^b}^{n_\mathrm{o}}$
  n_o : code length
  b : symbol size (bits)
inner code: $\mathcal{C}_i \subset \Sigma^\beta \quad (i \in [\nu\rangle, b < 2\beta)$
  β : code length (symbols)
  ν : number of code books
  $|\mathcal{C}_i| = 2^b$: number of codewords
  $R = b/2\beta$: inner code rate
  $\phi_i : \mathbb{F}_{2^b} \to \mathcal{C}_i$: encoding function





prior: $p(\boldsymbol{x}_i) = \begin{cases} 1/2^b & (\boldsymbol{x}_i \in \mathcal{C}_{|i|_\nu}) \\ 0 & (\boldsymbol{x}_i \notin \mathcal{C}_{|i|_\nu}) \end{cases}$

params:

β	b
3	4, 5
4	6, 7
5	8, 9
6	10, 11

$\nu = 2$

Constraint

RL-LB constrained coding $(\ell; w, \delta)$:
* run length coding: $f_\mathrm{R}(\boldsymbol{z}) \le \ell$
* local GC-balance: $\left|f_\mathrm{B}\left(\boldsymbol{z}_i^{i+w-1}\right)\right| \le \delta$ $\quad (\forall i \in [n-w\rangle)$
* synchronization: MIR?

vector over Σ : $\boldsymbol{u} = (u_0, \ldots, u_{n-1}) \in \Sigma^n$
weight: $w(\boldsymbol{u})_x = |\{i \in [n\rangle \mid u_i = x\}| \quad (x \in \Sigma)$
balance: $f_\mathrm{B}(\boldsymbol{u}) = w_\mathsf{G}(\boldsymbol{u}) + w_\mathsf{C}(\boldsymbol{u}) - w_\mathsf{A}(\boldsymbol{u}) - w_\mathsf{T}(\boldsymbol{u}) \in [-n, n]$
max run length: $f_\mathrm{R}(\boldsymbol{u}) \in [1, n]$

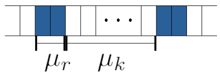
Inner codebook

$$\mathcal{C}_i = \left\{(\phi(x_0y_0), \ldots, \phi(x_{\beta-1}y_{\beta-1})) \middle| \boldsymbol{x} \in \mathcal{B}_i, \boldsymbol{y} \in \mathbb{B}^\beta\right\}$$

$\boldsymbol{x} = (x_0, \ldots, x_{\beta-1})$
$\boldsymbol{y} = (y_0, \ldots, y_{\beta-1}) \quad (i \in [\nu\rangle)$
$\mathcal{B}_i \subset \mathbb{B}^\beta$
$|\mathcal{C}_i| = 2^\beta |\mathcal{B}_i| = 2^b, \ |\mathcal{B}_i| = 2^{b-\beta}$

$\boldsymbol{v}$	$\phi(\boldsymbol{v})$
00	A
01	T
10	G
11	C

Marker



rate: $R_\mathrm{i} = \frac{\mu_k}{\mu_k + \mu_r}$

Rate

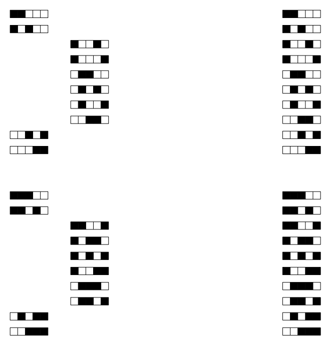
$R = R_\mathrm{o} R_\mathrm{i} R_\mathrm{c}$ [bits/symbol]
  $R_\mathrm{o} \le$ 1-cross entropy: inner APP
  $R_\mathrm{i} =$ inner code rate
  $R_\mathrm{c} \le$ bound of $(\ell; w, \delta)$ constraint
    bound for given $P(X)$

$R_\mathrm{o} R_\mathrm{i} \le I(X;Y)$:

(example)

□ : 0
■ : 1

		β	ν	R_I	ℓ	δ	b_i	$\mathcal{B}_i$	
1	3.4.2	3	2	4/6	2	2	4	$\mathcal{B}_0$	010 100
							4	$\mathcal{B}_1$	101 110
2	3.5.1	3	1	5/6	3	$*$	5	$\mathcal{B}_0$	010 100 101 110
3	4.5.2	4	2	5/8	2	2	5	$\mathcal{B}_0$	0101 1010
							5	$\mathcal{B}_1$	0110 1001
4	4.6.1a	4	1	6/8	2	2	6	$\mathcal{B}_0$	0101 0110 1001 1010
5	4.6.1b	4	1	6/8	4	$*$	6	$\mathcal{B}_0$	0011 0100 1011 1100
6	4.7.1	4	1	7/8	4	$*$	7	$\mathcal{B}_0$	0010 0011 0101 0110 1001 1010 1100 1101
7	5.7.2	5	2	7/10	5	5	7	$\mathcal{B}_0$	11000 10100 00101 00011
							7	$\mathcal{B}_1$	00111 01011 11010 11100
9	5.8.2	5	2	8/10	3	3	8	$\mathcal{B}_0$	00101 00110 01001 01010 01100 10001 10010 10100
							8	$\mathcal{B}_1$	11010 11001 10110 10101 10011 01110 01101 01011
10	5.9.1	5	1	9/10	4	$*$	9	$\mathcal{B}_0$	00101 00110 01001 01010 01011 01100 01101 01110 10001 10010 10011 10100 10101 10110 11001 11010
8	5.a.2	5	2	15/20	3	3	8	$\mathcal{B}_0$	00101 00110 01001 01010 01100 10001 10010 10100
							7	$\mathcal{B}_1$	11010 11001 01110 01011



(ex2b)

β	3		3	4		4	4	4	5		5		6	
b	4		5	5		6	6	7	8		9		10	
ν	2		1	2		1	1	1	2		1		1	
R_i	0.67		0.83	0.63		0.75	0.75	0.88	0.80		0.90		0.83	
	010 100	101 110	010 100 101 110	0101 1010	0110 1001	0101 0110 1001 1010 a	0011 0100 1011 1100 b	0010 0011 0101 0110 1001 1010 1100 1101	00101 00110 01001 01010 01100 10001 10010 10100	11010 11001 10110 10101 10011 01110 01101 01011	00101 00110 01001 01010 01011 01100 01101 01110	10001 10010 10011 10100 10101 10110 11001 11010	001101 001110 010011 010101 010110 011001 011010 011100	100011 100101 100110 101001 101010 101100 110001 110010
$(\ell; w, \delta)$	(2; 10, 2)		(3; $*$, $*$)	(2; 10, 2)		(2; 10, 2)	(4; $*$, $*$)	(4; $*$, $*$)	(3; 10, 2)		(4; $*$, $*$)			

* (multi-read)

NB-IDS channel

block length: n

alphabet: $\Sigma \quad (|\Sigma| = q)$

error prob.: p_{id} $(< \frac{1}{2})$ (ins/del)

p_{s} (sub)

input: $\boldsymbol{x} = (x_0, \ldots, x_{n-1}) = \Sigma^n$

output: $\boldsymbol{y} = (y_0, \ldots, y_{n'-1}) = \Sigma^{n'}$

$(n - D \leq n' \leq n + D)$

[transmission]

1) drift vector: $\boldsymbol{d} = (d_0, \ldots, d_{n-1}, d_n) \in \mathcal{D}^{n+1}$

2) intermediate vector: $\boldsymbol{z} = (z_0, \ldots, z_{n'-1}) \in \Sigma^{n'}$

$$z_j = x_i \;\; (j \in [i + d_i, i + d_{i+1}]) \quad i \in [n\rangle, \;\; n' = n + d_n$$

3) output vector:

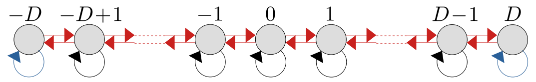
$$p(y_i|z_i) = \begin{cases} 1 - p_{\mathrm{s}} & (y_i = z_i) \\ \frac{p_{\mathrm{s}}}{q-1} & (y_i \neq z_i) \end{cases}$$

max drift: D

set of drift values: $\mathcal{D} = \{-D, \ldots, 0, \ldots, D\}$

drift vector: $\boldsymbol{d} = (d_0 = 0, \ldots, d_{n-1}, d_n) \in \mathcal{D}^{n+1}$

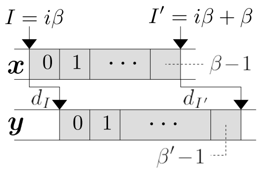
$$p(d_{i+1}|d_i) = \begin{cases} 1 - 2p_{\mathrm{id}} & (d_{i+1} = d_i, |d_i| < D) \\ 1 - p_{\mathrm{id}} & (d_{i+1} = d_i, |d_i| = D) \\ p_{\mathrm{id}} & (|d_{i+1} - d_i| = 1, |d_{i+1}| \leq D) \\ 0 & (\text{otherwise}) \end{cases}$$

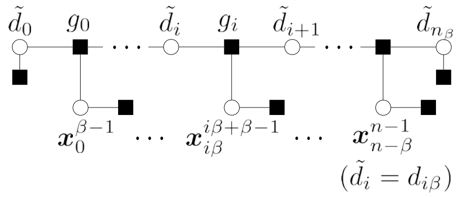


APP by FBA

$$p(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{d}) = p(\boldsymbol{y}|\boldsymbol{x}, \boldsymbol{d}) p(\boldsymbol{x}, \boldsymbol{d}) = p(\boldsymbol{y}|\boldsymbol{x}, \boldsymbol{d}) p(\boldsymbol{x}) p(\boldsymbol{d})$$

$$= p(d_0) \prod_{i=0}^{n-1} p\left(\boldsymbol{y}_{i+d_i}^{i+d_{i+1}} \middle| x_i, d_i, d_{i+1}\right) p(d_{i+1}|d_i) p(x_i)$$





segment level synchronization: $n = n_\beta \beta$

$$= p(d_0) \prod_{i=0}^{n_\beta - 1} p\left(\boldsymbol{y}_{I+d_I}^{I'-1+d_{I'}} \middle| \boldsymbol{x}_I^{I'-1}, d_I, d_{I'}\right) p(d_{I'}|d_I) \, p\left(\boldsymbol{x}_I^{I'-1}\right)$$

$$(I = i\beta, I' = i\beta + \beta)$$

[LUT] (for a large D)

$G\left(\beta', \boldsymbol{y}_0^{\beta'-1}\right)$ list of $\left[\boldsymbol{x}_0^{\beta-1}, \boldsymbol{p}_0\left(\boldsymbol{y}_0^{\beta'-1} \middle| \boldsymbol{x}_0^{\beta-1}, 0, d'\right) \boldsymbol{p}_0(d'|0) \; (> p_{\mathrm{th}})\right]$

$$\boldsymbol{p}_\alpha\left(\boldsymbol{y}_{\alpha'}^{\beta'-1} \middle| \boldsymbol{x}_\alpha^{\beta-1}, d, d'\right) \boldsymbol{p}_\alpha(d'|d) = \qquad \begin{matrix} (d' = \beta' - \beta) \\ (d = \alpha' - \alpha) \end{matrix}$$

$$\sum_{\delta \in \{-1,0,1\}} p\left(\boldsymbol{y}_{\alpha'}^{\alpha'+\delta} \middle| x_\alpha, d, d + \delta\right) p(d + \delta|d)$$

$$\times \, \boldsymbol{p}_{\alpha+1}\left(\boldsymbol{y}_{\alpha'+\delta+1}^{\beta'} \middle| \boldsymbol{x}_{\alpha+1}^{\beta-1}, d + \delta, d'\right) \boldsymbol{p}_{\alpha+1}(d'|d + \delta)$$

```
Gtable:
  int beta
  double Pid
  double Ps
  double Pth
  LIST[len][y]
```

```
LIST:
  int num
  XP[num]
```

```
XP:
  int x
  float p
```

[run-length of random sequence]

prob. of run-length ℓ: $p(\ell) = \left(\frac{1}{q}\right)^{\ell-1}\left(\frac{q-1}{q}\right) = \frac{q-1}{q^\ell}$

expectation: $E(\ell) = \sum_{\ell=1}^{n} \ell p(\ell) = \sum_{\ell=1}^{n} \frac{(q-1)\ell}{q^\ell}$

$= (q-1)\sum_{\ell=1}^{n} \frac{\ell}{q^\ell}$

$= \frac{q}{q-1}\left(1 - \frac{1}{q^n}\right) - \frac{n}{q^n}$

[local GC-balance of random sequence]

window size: w

prob. binary vector having weight t : $p_w(t) = \binom{w}{t}/2^w$

absolute GC-balance: $\sum_{t=0}^{w} p_w(t)\,|w-2t|$

[rate upper bound]

run length: constraint graph(?)

local GCB: $\frac{1}{w}\log_2\left(2^w \sum_{t\in\mathcal{T}}\binom{w}{t}\right) = 1 + \frac{1}{w}\log_2\left(\sum_{t\in\mathcal{T}}\binom{w}{t}\right)$
$(\mathcal{T} = \{t \mid |w-2t| \leq \delta\})$

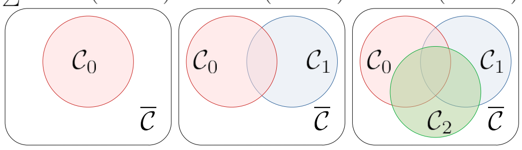
RL+LGCB: $(\ell; w, \delta)$ $\frac{1}{w}\log_2\left|\left\{\boldsymbol{u}\in\Sigma^w \,\middle|\, |f_{\mathrm{B}}(\boldsymbol{u})| \leq \delta, f_{\mathrm{R}}(\boldsymbol{u}) \leq \ell\right\}\right|$

Concatenated coding

channel alphabet: $\Sigma = \{\mathsf{A},\mathsf{T},\mathsf{G},\mathsf{C}\}$
outer code: $\mathcal{C}_{\mathrm{o}} \subset \mathbb{F}_{2^b}^{n_{\mathrm{o}}}$
- n_{o} : code length
- b : symbol size (bits)

inner code: $\mathcal{C}_i \subset \Sigma^\beta \quad (i \in [\nu\rangle, b < 2\beta)$
- β : code length (symbols)
- ν : number of code books
- $|\mathcal{C}_i| = 2^b$: number of codewords
- $R = b/2\beta$: inner code rate
- $\phi_i : \mathbb{F}_{2^b} \to \mathcal{C}_i$: encoding function

Σ^β $(\nu=1)$ $(\nu=2)$ $(\nu=3)$



outer codeword: $\boldsymbol{v} = \underbrace{\boldsymbol{v}_0}_{b}\ \boldsymbol{v}_1 \cdots \boldsymbol{v}_i \cdots \boldsymbol{v}_{n_{\mathrm{o}}-1} \in \mathcal{C}_{\mathrm{o}} \subset \mathbb{F}_{2^b}^{n_{\mathrm{o}}}$

(inner coding) $\phi_{|0|_\nu}$ $\phi_{|1|_\nu}$ $\phi_{|i|_\nu}$ $\phi_{|n_{\mathrm{o}}-1|_\nu}$

inner codeword: $\boldsymbol{x} = \underbrace{\boldsymbol{x}_0}_{\beta}\ \boldsymbol{x}_1 \cdots \boldsymbol{x}_i \cdots \boldsymbol{x}_{n_{\mathrm{o}}-1} \in \Sigma^n$ $(n = n_{\mathrm{o}}\beta)$

(shaping)

channel input: $\boldsymbol{z} = \boldsymbol{z}_0\ \boldsymbol{z}_1 \cdots \boldsymbol{z}_i \cdots \boldsymbol{z}_{n_{\mathrm{o}}-1} \in \Sigma^n$

prior: $p(\boldsymbol{x}_i) = \begin{cases} 1/2^b & (\boldsymbol{x}_i \in \mathcal{C}_{|i|_\nu}) \\ 0 & (\boldsymbol{x}_i \notin \mathcal{C}_{|i|_\nu}) \end{cases}$

params:

β	b
3	$4,5$
4	$6,7$
5	$8,9$
6	$10,11$

vector over Σ : $\boldsymbol{u} = (u_0,\ldots,u_{n-1}) \in \Sigma^n$
weight: $w(\boldsymbol{u})_x = |\{i \in [n\rangle \mid u_i = x\}| \quad (x \in \Sigma)$
balance: $f_{\mathrm{B}}(\boldsymbol{u}) = w_{\mathsf{G}}(\boldsymbol{u}) + w_{\mathsf{C}}(\boldsymbol{u}) - w_{\mathsf{A}}(\boldsymbol{u}) - w_{\mathsf{T}}(\boldsymbol{u}) \in [-n,n]$
max run length: $f_{\mathrm{R}}(\boldsymbol{u}) \in [1,n]$

Constraint

RL-LB constrained coding $(\ell; w, \delta)$:
* run length coding: $f_{\mathrm{R}}(\boldsymbol{z}) \le \ell$
* local GC-balance: $\left|f_{\mathrm{B}}\left(\boldsymbol{z}_i^{i+w-1}\right)\right| \le \delta \quad (\forall i \in [n-w\rangle)$
* synchronization: MIR?

[example 1]
forbidden set $\overline{\mathcal{C}} \subseteq \Sigma^\beta \setminus \cup_{i\in[\nu\rangle}\mathcal{C}_i \quad (|\overline{\mathcal{C}}| \le 2^{2\beta} - 2^b)$
- ϕ (empty)
- $\overline{\mathcal{C}}_{*,\lambda} = \left\{\boldsymbol{u} \in \Sigma^\beta \mid f_{\mathrm{R}}(\boldsymbol{u}) \ge \lambda\right\}$ (RL)
- $\overline{\mathcal{C}}_{\omega,*} = \left\{\boldsymbol{u} \in \Sigma^\beta \mid |f_{\mathrm{B}}(\boldsymbol{u})| \ge \omega\right\}$ (LB)
- $\overline{\mathcal{C}}_{\omega,\lambda} = \overline{\mathcal{C}}_{\omega,*} \cup \overline{\mathcal{C}}_{*,\lambda}$ (both)

[example 2a] $\beta = 4, b = 7$

$\mathcal{C}_i = \left\{(\phi(x_0y_0),\ldots,\phi(x_3y_3)) \middle| (x_0,\ldots,x_3) \in \mathcal{B}_i, (y_0,\ldots,y_3) \in \mathbb{B}^4\right\}$

$\mathcal{B}_0$: 0011,0101,0110,1001,1010,1100,0010,0100
$\mathcal{B}_1$: 0011,0101,0110,1001,1010,1100,1101,1011
$|\mathcal{C}_i| = 8 \times 16 = 128 = 2^b$

$\boldsymbol{v}$	$\phi(\boldsymbol{v})$
00	A
01	T
10	G
11	C

[example 2b] $\beta = 5, b = 8$

$\mathcal{C}_i = \left\{(\phi(x_0y_0),\ldots,\phi(x_4y_4)) \middle| (x_0,\ldots,x_4) \in \mathcal{B}_i, (y_0,\ldots,y_4) \in \mathbb{B}^5\right\}$

$\mathcal{B}_0$: 00011,00101,00110,01001,01010,01100,10001,10010,10100,11000
$\mathcal{B}_1$: 11100,11010,11001,10110,10101,10011,01110,01101,01011,00111
$|\mathcal{C}_i| = 8 \times 32 = 256 = 2^b$

$\boldsymbol{a} = (a_0,\ldots,a_4), \boldsymbol{c} = (c_0,\ldots,c_4) \in \mathcal{B}_0$
$\boldsymbol{b} = (b_0,\ldots,b_4), \boldsymbol{d} = (d_0,\ldots,d_4) \in \mathcal{B}_1$

RL: max RL in $\boldsymbol{a}$ and $\boldsymbol{b}$: 3 (by definition)
max RL in $(\boldsymbol{a},\boldsymbol{b})$: 3 (proof 1)
max RL in $(\boldsymbol{b},\boldsymbol{c})$: 3 (proof 2)

(proof 1) length of the last run of $\boldsymbol{a}$: 1 (run of 1)
2 (run of 0s)
length of the first run of $\boldsymbol{b}$: 2 (run of 1s)
1 (run of 0)

(proof 2) length of the last run of $\boldsymbol{b}$: 1 (run of 0)
2 (run of 1s)
length of the first run of $\boldsymbol{c}$: 2 (run of 0s)
1 (run of 1)

LB: even $w(\ge 10)$
$(\boldsymbol{a}_t^4, \boldsymbol{b}, \boldsymbol{c}_0^{t-1})$:

* (multi-read)

$\frac{1}{w}\log_2\left(\sum_{t\in\mathcal{T}}\left(2^w\binom{w}{t}-\rho_{\ell,w,t}\right)\right)$

$\rho_{\ell,w,t}=|\{\boldsymbol{u}\in\Sigma^w|w_{\mathsf{G}}(\boldsymbol{u})+w_{\mathsf{C}}(\boldsymbol{u})=t, f_{\mathrm{R}}(\boldsymbol{u})>\ell\}|$

$(\beta=3,b=4)$
$\mathcal{B}_0$: 010,100
$\mathcal{B}_1$: 101,110

$(\beta=3,b=5)$
$\mathcal{B}_0$: 001,010,100,101
$\mathcal{B}_1$: 110,101,011,010

$(\beta=4,b=6)$
$\mathcal{B}_0$: 0101,0110,1001,1010
$\mathcal{B}_1$: 0101,0110,1001,1010

$(\beta=4,b=7)$ [ex2a]
$\mathcal{B}_0$: 0011,0101,0110,1001,1010,1100,0010,0100
$\mathcal{B}_1$: 0011,0101,0110,1001,1010,1100,1101,1011

$(\beta=5,b=8)$ [ex2b]
$\mathcal{B}_0$: 00101,00110,01001,01010,01100,10001,10010,10100
$\mathcal{B}_1$: 11010,11001,10110,10101,10011,01110,01101,01011

$(\beta=5,b=9)$
$\mathcal{B}_0$: 00101,00110,01001,01010,01100,10001,10010,10100,...
$\mathcal{B}_1$: 11010,11001,10110,10101,10011,01110,01101,01011,...

position	max RL	pattern
(a_0,a_1,a_2,a_3,a_4)	3	10001
$(a_1,a_2,a_3,a_4;b_0)$	3	$*100;0$
$(a_2,a_3,a_4;b_0,b_1)$		
$(a_3,a_4;b_0,b_1,b_2)$		
$(a_4;b_0,b_1,b_2,b_3)$		
(b_0,b_1,b_2,b_3,b_4)	3	01110
$(b_1,b_2,b_3,b_4;c_0)$		
$(b_2,b_3,b_4;c_0,c_1)$		
$(b_3,b_4;c_0,c_1,c_2)$		
$(b_4;c_0,c_1,c_2,c_3)$		