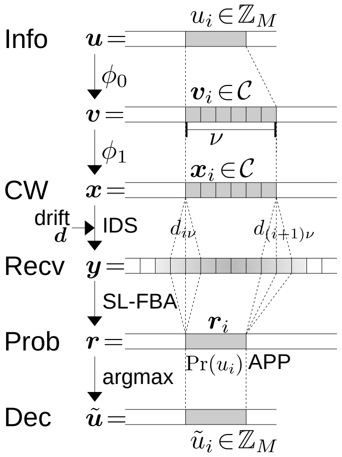
Constrained coding + Synchronization



block length: N (symbol)
$N\nu$ (bit)

[inner code] $\mathcal{C} \subset \mathbb{B}^{\nu}$ $(|\mathcal{C}| = M)$

code length: ν $(\leq \ell)$ (even)
balanced: $w(\boldsymbol{c}) = \nu/2$ $(\forall \boldsymbol{c} \in \mathcal{C})$
run-length: $\leq \rho$
(right-most) $\leq \rho-1$
invertible: $\boldsymbol{c} \in \mathcal{C} \to \overline{\boldsymbol{c}} \in \mathcal{C}$ $(\forall \boldsymbol{c} \in \mathcal{C})$
reset symbol: $(01)^{\frac{\nu}{2}}, (10)^{\frac{\nu}{2}} \in \mathcal{C}$

[encoding] $\phi = \phi_0 \circ \phi_1$ (lossy)

$\phi_0 : \mathbb{Z}_M \to \mathcal{C}$ (encoding)
$\boldsymbol{v}_i = \phi_0(\boldsymbol{u}_i) \in \mathcal{C}$ $(\boldsymbol{u}_i \in \mathbb{Z}_M)$

$\phi_1 : \mathcal{C}^{l_0} \times \mathcal{C} \to \mathcal{C}$ (constraint)

$\boldsymbol{x}_i = \phi_1(\boldsymbol{x}_{i-l_0}^{i-1}, \boldsymbol{v}_i)$ (priority) H

$$= \begin{cases} \boldsymbol{v}_i & (\mathbb{1}_{\rho,\ell,\delta}[\boldsymbol{x}_{i-l_0}^{i-1}, \boldsymbol{v}_i] = 1) \\ \overline{\boldsymbol{v}}_i & (\mathbb{1}_{\rho,\ell,\delta}[\boldsymbol{x}_{i-l_0}^{i-1}, \overline{\boldsymbol{v}}_i] = 1) \\ (\overline{v}v)^{\frac{\nu}{2}} & (\mathbb{1}_{\rho,\ell,\delta}[\boldsymbol{x}_{i-l_0}^{i-1}, (\overline{v}v)^{\frac{\nu}{2}}] = 1) \\ (v\overline{v})^{\frac{\nu}{2}} & (\mathbb{1}_{\rho,\ell,\delta}[\boldsymbol{x}_{i-l_0}^{i-1}, (v\overline{v})^{\frac{\nu}{2}}] = 1) \end{cases}$$ L

$l_0 = \left\lceil \frac{\ell-1}{\nu} \right\rceil$ $\quad v$: first bit of $\boldsymbol{v}_i$

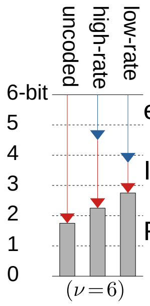
Constraint

run-length: ρ
local-balance: (ℓ, δ)
ℓ : even
$\left| w(\boldsymbol{x}_i^{i+\ell-1}) - \frac{\ell}{2} \right| \leq \delta$

IDS channel

$p_{\mathrm{i}}, p_{\mathrm{d}}, p_{\mathrm{s}}$: ins/del/sub probability
$d_{\min} < 0$: drift min
$d_{\max} > 0$: drift max
$\mathcal{D} = \{d \in \mathbb{Z} | d_{\min} \leq d \leq d_{\max}\}$

Performance measure



encode: $I(U;X)$

IDS channel

Rate: $R' = \frac{I(U;\tilde{U})}{\nu}$

SL-FBA

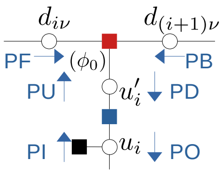
$p(\boldsymbol{y}, \boldsymbol{x}, \boldsymbol{u}, \boldsymbol{d})$ $\quad \left(\phi_0(u_i') = \boldsymbol{x}_{i\nu}^{(i+1)\nu-1}\right)$

$= p(\boldsymbol{y}|\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{d}) p(\boldsymbol{x}, \boldsymbol{u}) p(\boldsymbol{d}) = p(\boldsymbol{y}|\boldsymbol{x}, \boldsymbol{d}) p(\boldsymbol{x}|\boldsymbol{u}) p(\boldsymbol{u}) p(\boldsymbol{d}) = p(\boldsymbol{y}|\phi_0(\boldsymbol{u}'), \boldsymbol{d}) p(\boldsymbol{u}'|\boldsymbol{u}) p(\boldsymbol{u}) p(\boldsymbol{d})$

$= p(d_0) \prod_{i=0}^{N-1} p\left(\boldsymbol{y}_{i\nu+d_{i\nu}}^{(i+1)\nu+d_{(i+1)\nu}-1} \middle| \boldsymbol{x}_{i\nu}^{(i+1)\nu-1}, d_{i\nu}, d_{(i+1)\nu}\right) p\left(\boldsymbol{x}_{i\nu}^{(i+1)\nu-1} \middle| \boldsymbol{u}_0^i\right) p(u_i) p(d_{(i+1)\nu}|d_{i\nu})$

$\simeq p(d_0) \prod_{i=0}^{N-1} \underline{p\left(\boldsymbol{y}_{i\nu+d_{i\nu}}^{(i+1)\nu+d_{(i+1)\nu}-1} \middle| \phi_0(u_i'), d_{i\nu}, d_{(i+1)\nu}\right)} \underline{p(u_i'|u_i)} p(u_i) \underline{p(d_{(i+1)\nu}|d_{i\nu})}$

GX[Nu2][y][x] ECM[uin][uout] GD[d0][d1]



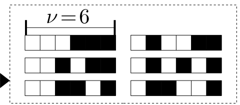
Nu2: 0...Nu*2
y: 0...2^Nu2-1
x: 0...Q-1
(approximation)
Nu2 range: Nu2min...max
d range: not bounded

uin: 0...M-1
uout: 0...M-1

d0: Dmin...Dmax
d1: Dmin...Dmax

Baseline

* marker
* watermark
* trivial



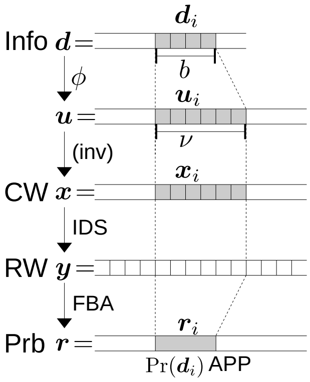
Example $(\nu=6, \rho=2, \binom{6}{3}=20)$

------	(1)	(2)	(3)
000111	-	-	-
001011	0	0	0
001101	1	1	1
001110	2	-	-
010011	3	2	2
010101	4	3	3
010110	5	4	4
011001	6	5	5
011010	7	6	6
011100	8	7	-
100011	9	8	-
100101	10	9	7
100110	11	10	8
101001	12	11	9
101010	13	12	10
101100	14	13	11
110001	15	-	-
110010	16	14	12
110100	17	15	13
111000	-	-	-

p_{i} :	insertion
p_{d} :	deletion
$p_{\mathrm{s}}(y\|x)$:	asymmetric error
$d_{\min} < 0$:	drift min
$d_{\max} > 0$:	drift max

$\mathcal{D} = \{d \in \mathbb{Z} | d_{\min} \leq d \leq d_{\max}\}$

Constrained coding + Synchronization



$\mathcal{C} \subset \mathbb{B}^\nu$ $(|\mathcal{C}| \le 2^b)$ ← Inner code (not bijective)

length: ν (even)

balanced: $w(\boldsymbol{u}) = \nu/2$

invertible: $\boldsymbol{u}\in\mathcal{C} \to \overline{\boldsymbol{u}}\in\mathcal{C}$ $(\forall \boldsymbol{u} \in \mathcal{C})$

$\phi : \mathbb{B}^b \to \mathcal{C}$

$\boldsymbol{d}_i \in \mathbb{B}^b$

$\boldsymbol{u}_i = \phi(\boldsymbol{d}_i) \in \mathcal{C}$

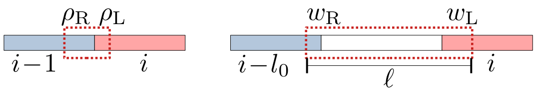
$l_0 = \lceil \frac{\ell-1}{\nu} \rceil$

$$f_i^0 = \begin{cases} 1 & ((\boldsymbol{x}_{i-l_0}^{i-1}, \boldsymbol{u}_i) \text{ satisfy the constraints}) \\ 0 & (\text{otherwise}) \end{cases}$$

$$f_i^1 = \begin{cases} 1 & ((\boldsymbol{x}_{i-l_0}^{i-1}, \overline{\boldsymbol{u}}_i) \text{ satisfy the constraints}) \\ 0 & (\text{otherwise}) \end{cases}$$

$$\boldsymbol{x}_i = \begin{cases} \boldsymbol{u}_i & (f_i^0 = 1) \\ \overline{\boldsymbol{u}}_i & (f_i^0 = 0 \land f_i^1 = 1) \\ \perp & (f_i^0 = f_i^1 = 0) \end{cases}$$ encoding failure

[run-length] [local balance]



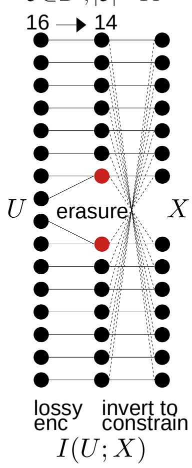
RunL[cw]
RunR[cw]
WtL[cw][idx]
WtR[cw][idx]

$$f^0(\boldsymbol{v}) = \begin{cases} 1 & ((\boldsymbol{x}_{i-l_0}^{i-1}, \boldsymbol{v}) \text{ satisfy the constraints}) \\ 0 & (\text{otherwise}) \end{cases}$$

	$\mathcal{C}_0 \subset \mathbb{B}^6$	$\rho_{\max}$	$\mathcal{C} \subseteq \mathcal{C}_0$
1	000111	3	------
2	001011	2	001011
3	001101	2	001101
4	001110	3	------
5	010011	2	010011
6	010101	1	010101
7	010110	2	010110
8	011001	2	011001
9	011010	2	011010
10	011100	3	------
11	100011	3	------
12	100101	2	100101
13	100110	2	100110
14	101001	2	101001
15	101010	1	101010
16	101100	2	101100
17	110001	3	------
18	110010	2	110010
19	110100	2	110100
20	111000	3	------

$\phi : \mathbb{B}^4 \to \mathcal{C}$

$\mathcal{C} \subset \mathbb{B}^6, |\mathcal{C}| = 14$



Constraint

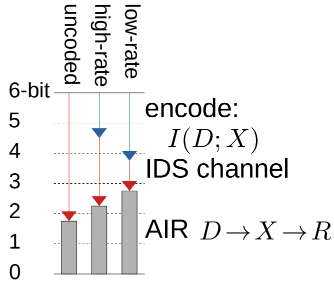
run-length: ρ

local-balance: (ℓ, δ)

ℓ : even

$\left| w(\boldsymbol{x}_i^{i+\ell-1}) - \frac{\ell}{2} \right| \le \delta$

Rate

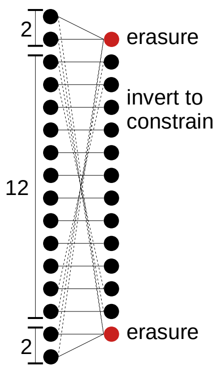


baseline: constraint only
IDS only

decoding: SL-FBA

outer code: NB-LDPC (?)

performance: code rate
AIR


2
erasure
invert to
constrain
12
erasure
2

Constrained non-binary IDS channel

* channel input/output alphabet:
$\Sigma = \{0,1,2,3\}$
* block length: n
* input: $\boldsymbol{x} = (x_0,\dots,x_{n-1}) \in \Sigma^n$,
* output: $\boldsymbol{y} = (y_0,\dots,y_{n'-1}) \in \Sigma^{n'}$
* input constraint:
  - run-length: $f_{\mathrm{R}}(\boldsymbol{x}) \le \rho$
  - local-balance: (ℓ,ϵ)
    $\max_i \left|\frac{1}{2} - f_{\mathrm{B}}(\phi_w(\boldsymbol{x}_i^{i+\ell-1}))\right| \le \varepsilon$
  - ...

* error model
  p_{i} : insertion
  p_{d} : deletion
  $p_{\mathrm{s}}(y|x)$: asymmetric error
  $d_{\min} < 0$: drift min
  $d_{\max} > 0$: drift max
  $\mathcal{D} = \{d \in \mathbb{Z} | d_{\min} \le d \le d_{\max}\}$
* performance measure:
  * code rate
  * mutual info (AIR)

* mappings
  $\phi_x : \mathbb{B} \times \mathbb{B} \to \Sigma$
  $\phi_w : \Sigma \to \mathbb{B}$
  $\phi_d : \Sigma \to \mathbb{B}$
  $\phi_x(\phi_w(x), \phi_d(x)) = x$

w	d	$\phi_x(w,d)$
0	0	0
0	1	1
1	0	2
1	1	3

* functions
  max run-length: $f_{\mathrm{R}}(\boldsymbol{v})$
  local-balance (binary):
  $f_{\mathrm{B}}(\boldsymbol{u}_i^{i+\ell-1}) = w(\boldsymbol{u}_i^{i+\ell-1})/\ell$:

Constrained non-binary WM

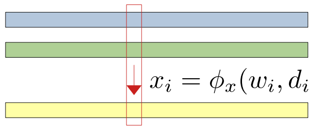
[baseline] rate=1/2 i

info: $\boldsymbol{d} =$ $\in \mathbb{B}^n$

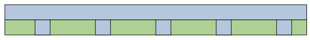
WM: $\boldsymbol{w} =$ $\in \mathbb{B}^n$

$x_i = \phi_x(w_i, d_i)$

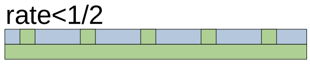
CW: $\boldsymbol{x} =$ $\in \Sigma^n$



rate>1/2



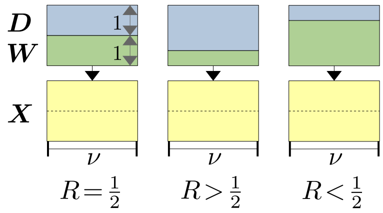
rate<1/2



WM: synchronization: ?
run-length: $f_{\mathrm{R}}(\boldsymbol{w}) \le \rho$
local-balance: $\max_i \left|\frac{1}{2} - f_{\mathrm{B}}(\boldsymbol{w}_i^{i+\ell-1})\right| \le \varepsilon$

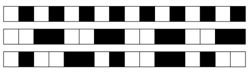
[decoding (detection)] SPA on factor graph

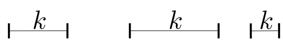
[generalize]
$\phi_x : \mathcal{M} \times \Sigma^\nu \to \Sigma^\nu \;\; (1 \le |\mathcal{M}| < 2^{2\nu})$
rate: $R = \frac{\log_2 |\mathcal{M}|}{2\nu}$
$I(\boldsymbol{X};\boldsymbol{W}) + I(\boldsymbol{X};\boldsymbol{D}) \le 2$



maximize $I(\boldsymbol{X};\boldsymbol{W})$?

WM design





index ↔ W: mutual info