Constraint

max run-length: ρ
local-balance: (ℓ, δ) ℓ : even
$$\left| w(\boldsymbol{x}_i^{i+\ell-1}) - \tfrac{\ell}{2} \right| \leq \delta$$

IDS channel

$p_{\mathrm{i}}, p_{\mathrm{d}}, p_{\mathrm{s}}$: ins/del/sub probability
$d_{\min} < 0$: drift min
$d_{\max} > 0$: drift max
$\mathcal{D} = \{d \in \mathbb{Z} | d_{\min} \leq d \leq d_{\max}\}$

Constrained coding + Synchronization

Info $\boldsymbol{u} =$ $u_i \in \mathbb{Z}_M$
  $\downarrow \phi_0$
  $\boldsymbol{v} =$ $v_i \in \mathcal{C}$ ν
  $\downarrow \phi_1$
CW $\boldsymbol{x} =$ $\boldsymbol{x}_i \in \mathcal{C}$
drift $\boldsymbol{d} \rightarrow$ IDS $d_{i\nu}$ $d_{(i+1)\nu}$
Recv $\boldsymbol{y} =$
  $\downarrow$ SL-FBA
Prob $\boldsymbol{r} =$ r_i
  $\downarrow$ maxlist $\Pr(u_i)$ APP
Dec $\tilde{\boldsymbol{u}} =$
  $\tilde{u}_i \in \mathbb{Z}_M^L$
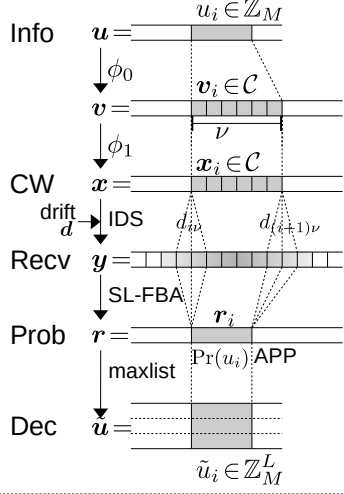
block length: N (symbol)
  $N\nu$ (bit)

[inner code] $\mathcal{C} \subset \mathbb{B}^\nu \ (|\mathcal{C}| = M)$
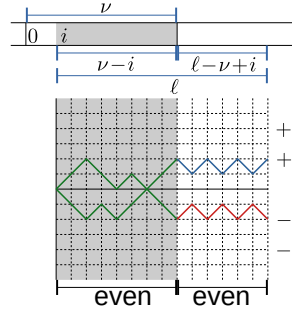 code length: $\nu \ (\leq \ell)$ (even)
  balanced: $w(\boldsymbol{x}) = \nu/2 \ (\forall \boldsymbol{x} \in \mathcal{C})$
    $\forall i \in [\nu]_{\mathrm{e}}, -2\delta \leq \tilde{w}(\boldsymbol{x}_i^{\nu-1}) \leq 2\delta$
    $\left[\forall i \in [\nu]_{\mathrm{o}}, -2\delta+1 \leq \tilde{w}(\boldsymbol{x}_i^{\nu-1}) \leq 2\delta+1\right] \vee \left[\forall i \in [\nu]_{\mathrm{o}}, -2\delta-1 \leq \tilde{w}(\boldsymbol{x}_i^{\nu-1}) \leq 2\delta-1\right]$

  run-length: $\leq \rho$ (right-most RL) $\leq \rho - 1$
  invertible: $\boldsymbol{x} \in \mathcal{C} \rightarrow \overline{\boldsymbol{x}} \in \mathcal{C} \ (\forall \boldsymbol{x} \in \mathcal{C})$
  reset symbol: $(01)^{\frac{\nu}{2}}, (10)^{\frac{\nu}{2}} \in \mathcal{C}$

[example] even $i \in [\nu]$ odd $i \in [\nu]$
$(2\delta = 2)$
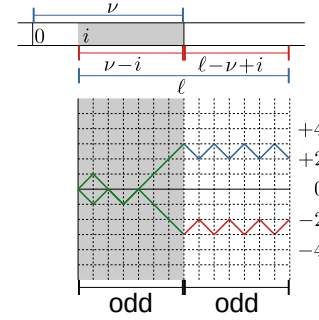
[encoding] $\phi = \phi_0 \circ \phi_1$ (lossy)

$\phi_0 : \mathbb{Z}_M \rightarrow \mathcal{C}$ (encoding)
  $\boldsymbol{v}_i = \phi_0(\boldsymbol{u}_i) \in \mathcal{C}$ $(\boldsymbol{u}_i \in \mathbb{Z}_M)$

$\phi_1 : \mathcal{C}^{l_0} \times \mathcal{C} \rightarrow \mathcal{C}$ (constraint)
  $\boldsymbol{x}_i = \phi_1(\boldsymbol{x}_{i-l_0}^{i-1}, \boldsymbol{v}_i)$ (priority) H

$$= \begin{cases} \boldsymbol{v}_i & (\mathbb{1}_{\rho,\ell,\delta}[\boldsymbol{x}_{i-l_0}^{i-1}, \boldsymbol{v}_i] = 1) \\ \overline{\boldsymbol{v}}_i & (\mathbb{1}_{\rho,\ell,\delta}[\boldsymbol{x}_{i-l_0}^{i-1}, \overline{\boldsymbol{v}}_i] = 1) \\ (\overline{v}v)^{\frac{\nu}{2}} & (\mathbb{1}_{\rho,\ell,\delta}[\boldsymbol{x}_{i-l_0}^{i-1}, (\overline{v}v)^{\frac{\nu}{2}}] = 1) \\ (v\overline{v})^{\frac{\nu}{2}} & (\mathbb{1}_{\rho,\ell,\delta}[\boldsymbol{x}_{i-l_0}^{i-1}, (v\overline{v})^{\frac{\nu}{2}}] = 1) \end{cases}$$

L

$l_0 = \left\lceil \frac{\ell-1}{\nu} \right\rceil$ v : first bit of $\boldsymbol{v}_i$

SL-FBA

$p(\boldsymbol{y}, \boldsymbol{x}, \boldsymbol{u}, \boldsymbol{d})$ $\left(\phi_0(u_i') = \boldsymbol{x}_{i\nu}^{(i+1)\nu-1} \right)$

$= p(\boldsymbol{y}|\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{d})p(\boldsymbol{x}, \boldsymbol{u})p(\boldsymbol{d}) = p(\boldsymbol{y}|\boldsymbol{x}, \boldsymbol{d})p(\boldsymbol{x}|\boldsymbol{u})p(\boldsymbol{u})p(\boldsymbol{d}) = p(\boldsymbol{y}|\phi_0(\boldsymbol{u}'), \boldsymbol{d})p(\boldsymbol{u}'|\boldsymbol{u})p(\boldsymbol{u})p(\boldsymbol{d})$

$= p(d_0) \prod_{i=0}^{N-1} p\left(\boldsymbol{y}_{i\nu+d_{i\nu}}^{(i+1)\nu+d_{(i+1)\nu}-1} \middle| \boldsymbol{x}_{i\nu}^{(i+1)\nu-1}, d_{i\nu}, d_{(i+1)\nu} \right) p\left(\boldsymbol{x}_{i\nu}^{(i+1)\nu-1} \middle| \boldsymbol{u}_0^i \right) p(u_i)p(d_{(i+1)\nu}|d_{i\nu})$

$\simeq p(d_0) \prod_{i=0}^{N-1} p\left(\boldsymbol{y}_{i\nu+d_{i\nu}}^{(i+1)\nu+d_{(i+1)\nu}-1} \middle| \phi_0(u_i'), d_{i\nu}, d_{(i+1)\nu} \right) p(u_i'|u_i) p(u_i)p(d_{(i+1)\nu}|d_{i\nu})$
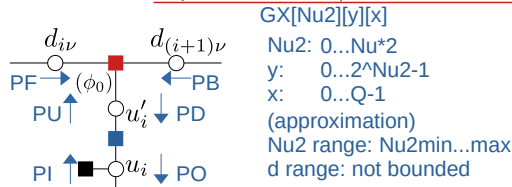
GX[Nu2][y][x]
Nu2: 0...Nu*2
y: 0...2^Nu2-1
x: 0...Q-1
(approximation)
Nu2 range: Nu2min...max
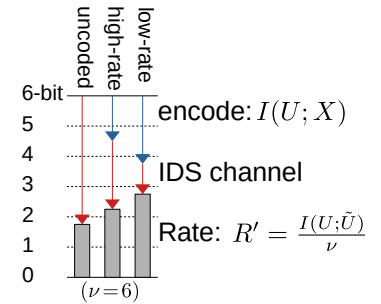d range: not bounded

ECM[uin][uout] GD[d0][d1]
uin: 0...M-1 d0: Dmin...Dmax
uout: 0...M-1 d1: Dmin...Dmax

$d_{i\nu}$ $d_{(i+1)\nu}$
PF $\rightarrow$ (ϕ_0) $\leftarrow$ PB
PU $\uparrow$ u_i' $\downarrow$ PD
PI $\uparrow$ u_i $\downarrow$ PO

Example $(\nu = 6, \rho = 3, \binom{6}{3} = 20)$

------	(1)	(2)	(3)	
000111	–	–	–	0+1+2+3+2+1
001011	0	0	0	0+1+2+1+2+1
001101	1	1	1	0+1+2+1 0+1
001110	2	–	–	0+1+2+1 0-1
010011	3	2	2	0+1 0+1+2+1
010101	4	3	3	0+1 0+1 0+1
010110	5	4	4	0+1 0+1 0-1
011001	6	5	5	0+1 0-1 0+1
011010	7	6	6	0+1 0-1 0-1
011100	8	7	–	0+1 0-1-2-1
100011	9	8	–	0-1 0+1+2+1
100101	10	9	7	0-1 0+1 0+1
100110	11	10	8	0-1 0+1 0-1
101001	12	11	9	0-1 0-1 0+1
101010	13	12	10	0-1 0-1 0-1
101100	14	13	11	0-1 0-1-2-1
110001	15	–	–	0-1-2-1 0+1
110010	16	14	12	0-1-2-1 0-1
110100	17	15	13	0-1-2-1-2-1
111000	–	–	–	0-1-2-3-2-1

Performance measure

6-bit
5
4
3
2
1
0
$(\nu = 6)$

uncoded · high-rate · low-rate

encode: $I(U; X)$
IDS channel
Rate: $R' = \frac{I(U; \tilde{U})}{\nu}$

Baseline

* marker
* watermark
* trivial $\rightarrow$

$\nu = 6$

Constrained coding + Synchronization

Info $\boldsymbol{d} =$
$\downarrow \phi$
$\boldsymbol{u} =$
$\downarrow$ (inv)
CW $\boldsymbol{x} =$
$\downarrow$ IDS
RW $\boldsymbol{y} =$
$\downarrow$ FBA
Prb $\boldsymbol{r} =$
$\mathrm{Pr}(\boldsymbol{d}_i)$ APP
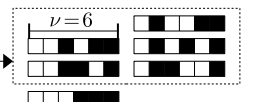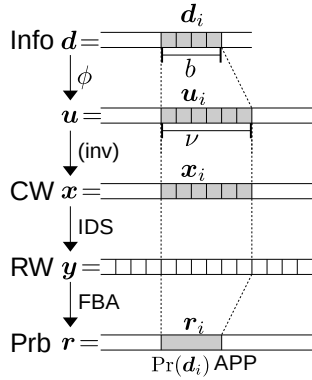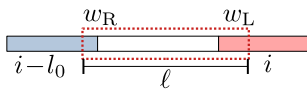
$\mathcal{C} \subset \mathbb{B}^\nu \ (|\mathcal{C}| \le 2^b) \leftarrow$ Inner code (not bijective)

$\phi : \mathbb{B}^b \to \mathcal{C}$

length: ν (even)

$\boldsymbol{d}_i \in \mathbb{B}^b$

balanced: $w(\boldsymbol{u}) = \nu/2$

$\boldsymbol{u}_i = \phi(\boldsymbol{d}_i) \in \mathcal{C}$

invertible: $\boldsymbol{u} \in \mathcal{C} \to \overline{\boldsymbol{u}} \in \mathcal{C}$

$l_0 = \left\lceil \frac{\ell - 1}{\nu} \right\rceil$

$(\forall \boldsymbol{u} \in \mathcal{C})$

$$f_i^0 = \begin{cases} 1 & ((\boldsymbol{x}_{i-l_0}^{i-1}, \boldsymbol{u}_i) \text{ satisfy the constraints}) \\ 0 & (\text{otherwise}) \end{cases}$$

$$f_i^1 = \begin{cases} 1 & ((\boldsymbol{x}_{i-l_0}^{i-1}, \overline{\boldsymbol{u}}_i) \text{ satisfy the constraints}) \\ 0 & (\text{otherwise}) \end{cases}$$

$$\boldsymbol{x}_i = \begin{cases} \boldsymbol{u}_i & (f_i^0 = 1) \\ \overline{\boldsymbol{u}}_i & (f_i^0 = 0 \wedge f_i^1 = 1) \\ \bot & (f_i^0 = f_i^1 = 0) \text{ encoding failure} \end{cases}$$
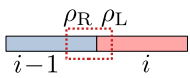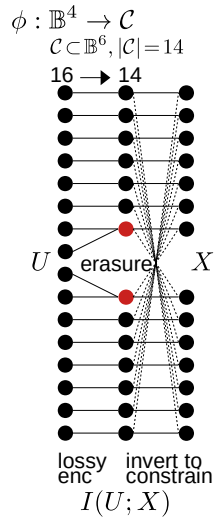
[run-length]   [local balance]

RunL[cw]
RunR[cw]
WtL[cw][idx]
WtR[cw][idx]

$\rho_\mathrm{R}, \rho_\mathrm{L}$

$i-1$ i

w_R w_L

$i-l_0$ i
ℓ

$$f^0(\boldsymbol{v}) = \begin{cases} 1 & ((\boldsymbol{x}_{i-l_0}^{i-1}, \boldsymbol{v}) \text{ satisfy the constraints}) \\ 0 & (\text{otherwise}) \end{cases}$$

	$\mathcal{C}_0 \subset \mathbb{B}^6$	$\rho_{\max}$	$\mathcal{C} \subseteq \mathcal{C}_0$
1	000111	3	------
2	001011	2	001011
3	001101	2	001101
4	001110	3	------
5	010011	2	010011
6	010101	1	010101
7	010110	2	010110
8	011001	2	011001
9	011010	2	011010
10	011100	3	------
11	100011	3	------
12	100101	2	100101
13	100110	2	100110
14	101001	2	101001
15	101010	1	101010
16	101100	2	101100
17	110001	3	------
18	110010	2	110010
19	110100	2	110100
20	111000	3	------

$\phi : \mathbb{B}^4 \to \mathcal{C}$
$\mathcal{C} \subset \mathbb{B}^6, |\mathcal{C}| = 14$
$16 \to 14$

U erasure X

lossy enc   invert to constrain

$I(U; X)$

Constraint

run-length: ρ
local-balance: (ℓ, δ)
   ℓ : even
   $\left| w(\boldsymbol{x}_i^{i+\ell-1}) - \frac{\ell}{2} \right| \le \delta$

Rate

uncoded
high-rate
low-rate

6-bit
5
4
3
2
1
0

encode:
   $I(D; X)$
IDS channel

AIR $D \to X \to R$

baseline: constraint only
               IDS only
decoding: SL-FBA
outer code: NB-LDPC (?)
performance: code rate
               AIR