[Synchronization + Constrained coding (inner code)]

$\Sigma = \{0,1,2,3\}$ (A,T,G,C)

ℓ : constraint length

$\mathcal{Q} \subset \Sigma^\ell$: valid state set

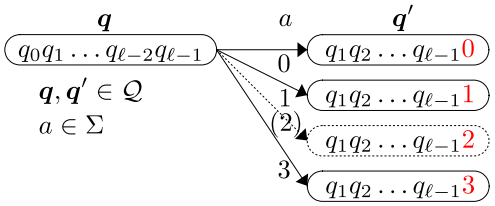
(ex) $\mathcal{Q}^{\mathrm{R}}_{\ell,k}$: maximum run-length k

$\mathcal{Q}^{\mathrm{B}}_{\ell,\varepsilon}$: local ε-balanced

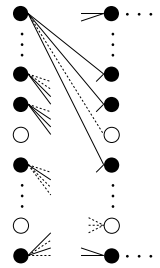
$\mathcal{Q}^{\mathrm{M}}_{\ell}$: motif (?)

$\mathcal{Q}^{\mathrm{S}}_{\ell}$: synchronization (?)

$$\mathcal{Q} = \mathcal{Q}^{\mathrm{R}}_{\ell,k} \cap \mathcal{Q}^{\mathrm{B}}_{\ell,\varepsilon} \cap \mathcal{Q}^{\mathrm{M}}_{\ell} \cap \mathcal{Q}^{\mathrm{S}}_{\ell} \cap \cdots$$



trellis:



state: $\boldsymbol{q} = q_0q_1\ldots q_{\ell-2}q_{\ell-1} \in \mathcal{Q}$

left-shift: $\overleftarrow{\boldsymbol{q}}\,a = q_1q_2\ldots q_{\ell-1}a \in \Sigma^\ell$

state transition: $t(\boldsymbol{q},a) = \{\boldsymbol{q}' = \overleftarrow{\boldsymbol{q}}\,a \,|\, \boldsymbol{q}' \in \mathcal{Q}\}$

$|t(\boldsymbol{q},a)| \in \{0,1\}$

edge label set: $\mathcal{L}(\boldsymbol{q}) = \{a | t(\boldsymbol{q},a) \neq \phi\}$

encoding: $f_{\mathrm{e}} : \mathcal{Q} \times \mathbb{B}^2 \to \Sigma$

$\mathcal{L}(\boldsymbol{q})$	00	01	10	11
$\{0,1,2,3\}$	0	1	2	3
$\{0,1,2\}$	0	1	2	2
$\{0,1,3\}$	0	1	3	3
$\{0,2,3\}$	0	0	2	3
$\{1,2,3\}$	1	1	2	3
$\{q_0,q_1\}$	q_0	q_0	q_1	q_1
$\{q\}$	q	q	q	q
ϕ	(invalidate $\boldsymbol{q}$)			

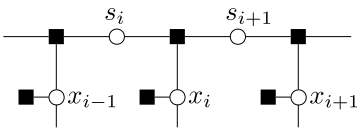
outer code: $\mathcal{C}_0$: left-bit

$\mathcal{C}_1$: right-bit

(rate) $R(\mathcal{C}_0) > R(\mathcal{C}_1)$

FBA: $s_i = (\boldsymbol{q}_i, d_i)$

$x_i = (x_i^0, x_i^1) \in \mathbb{B}^2$



[Synchronization + Constrained coding (inner code)]

$\Sigma = \{0,1,2,3\}$ (=A,T,G,C)

k : maximum run-length

δ : balance margin $\left|w_{01}(\boldsymbol{x}_0^{i-1}) - w_{23}(\boldsymbol{x}_0^{i-1})\right| \le \delta$ → local balance: $b = (2\delta + \ell')/\ell$

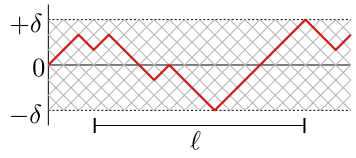
$\left|2w_{01}(\boldsymbol{x}_0^{i-1}) - i\right| \le \delta \quad (\forall i \in [n])$

$$\ell' = \begin{cases} (\ell - 2\delta)/2 & (\ell : \text{even}) \\ (\ell - 2\delta - 1)/2 & (\ell : \text{odd}) \end{cases}$$

$$b = \begin{cases} \frac{1}{2} + \frac{\delta}{\ell} & (\ell : \text{even}) \\ \frac{1}{2} + \frac{\delta}{\ell} - \frac{1}{2\ell} & (\ell : \text{odd}) \end{cases}$$

$$\varepsilon = \begin{cases} \frac{\delta}{\ell} & (\ell : \text{even}) \\ \frac{\delta}{\ell} - \frac{1}{2\ell} & (\ell : \text{odd}) \end{cases}$$

$w_{01}(\boldsymbol{x}_0^{i-1}) - w_{23}(\boldsymbol{x}_0^{i-1})$



[IDS channel]

p_{i} : insertion

p_{d} : deletion

$p_{\mathrm{s}}(y|x)$: asymmetric error

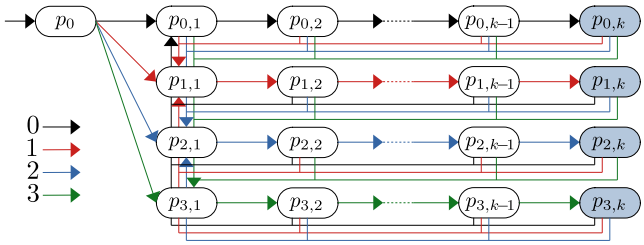
$d_{\min} < 0$: drift min

$d_{\max} > 0$: drift max

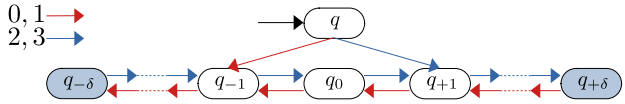
$\mathcal{D} = \{d \in \mathbb{Z} | d_{\min} \le d \le d_{\max}\}$

[state transition]

run-length: $t_{\mathrm{R}}(p,a) : \mathcal{P} \times \Sigma \to \mathcal{P} \cup \{\perp\}$

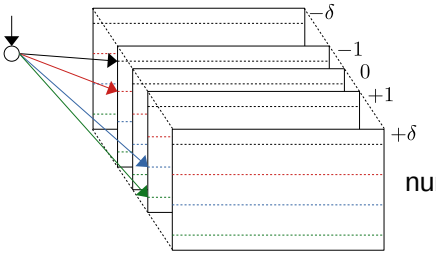


balance: $t_{\mathrm{B}}(q,a) : \mathcal{Q} \times \Sigma \to \mathcal{Q} \cup \{\perp\}$



run-length + balance:

$t(p,q,a) : \mathcal{P} \times \mathcal{Q} \times \Sigma \to (\mathcal{P} \times \mathcal{Q}) \cup \{\perp\}$



number of states: $4k(2\delta+1)+1$

$$t(p,q,a) = \begin{cases} (p',q') & (t_{\mathrm{R}}(p,a) = p', t_{\mathrm{B}}(q,a) = q') \\ \perp & (\text{otherwise}) \end{cases}$$

[information mapping]

$\mathcal{L}(p,q) = \{a \,|\, t(p,q,a) \neq \perp\}$

$f_{\mathrm{e}} : \mathcal{P} \times \mathcal{Q} \times \mathbb{B}^2 \to \Sigma$

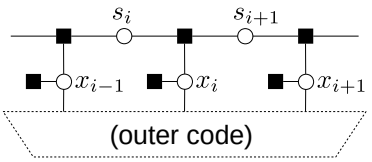
$\mathcal{L}(p,q)$	00	01	10	11
$\{0,1,2,3\}$	0	1	2	3
$\{0,1,2\}$	0	1	2	2
$\{0,1,3\}$	0	1	3	3
$\{0,2,3\}$	0	0	2	3
$\{1,2,3\}$	1	1	2	3
$\{q_0,q_1\}$	q_0	q_0	q_1	q_1
$\{q\}$	q	q	q	q
ϕ	(delete (p,q))			

erasures:
corrected by outer code

[FBA]

$s_i = (p_i, q_i, d_i) \in \mathcal{P} \times \mathcal{Q} \times \mathcal{D}$

$x_i = (x_i^0, x_i^1) \in \mathbb{B}^2$

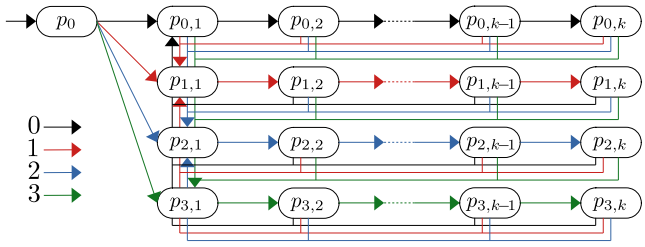


$$\begin{aligned} & p(\boldsymbol{y},\boldsymbol{x},\boldsymbol{p},\boldsymbol{q},\boldsymbol{d}) \\ &= p(\boldsymbol{y}|\boldsymbol{x},\boldsymbol{p},\boldsymbol{q},\boldsymbol{d}) p(\boldsymbol{x},\boldsymbol{p},\boldsymbol{q},\boldsymbol{d}) \\ &= p(\boldsymbol{x},\boldsymbol{p},\boldsymbol{q},\boldsymbol{d}) \textstyle\prod_{i=0}^{n-1} p(\boldsymbol{y}_{i+d_i}^{i+d_{i+1}} | x_i, p_i, p_{i+1}, q_i, q_{i+1}, d_i, d_{i+1}) \\ &= p(p_0)p(q_0)p(d_0) \textstyle\prod_{i=0}^{n-1} p(\boldsymbol{y}_{i+d_i}^{i+d_{i+1}} | x_i, p_i, p_{i+1}, q_i, q_{i+1}, d_i, d_{i+1}) p(x_i|p_i,q_i) p(p_{i+1}|p_i) p(q_{i+1}|q_i) p(d_{i+1}|d_i) \end{aligned}$$

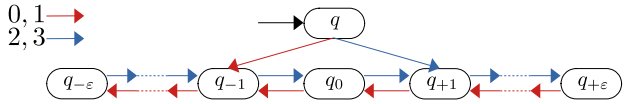
[evaluation]

* code rate
  - capacity of constrained channel
  - encoding rate (erasure prob.)
* mutual information: I(channel input, FBA output)
  - comparison to marker, watermark ?

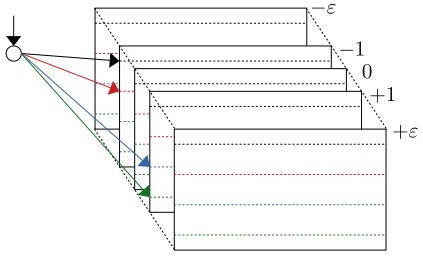
[run-length]



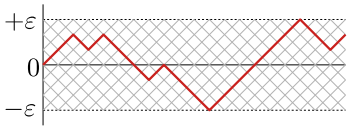
[balance]

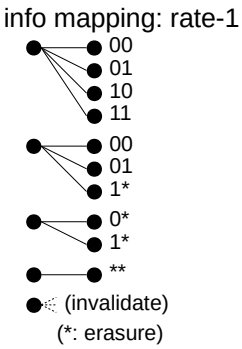


[run-length + balance]



number of states:
$4k(2\varepsilon + 1) + 1$




info mapping: rate-1
00
01
10
11
00
01
1*
0*
1*
**
(invalidate)
(*: erasure)