Definition of inner code

β : code length
ν : number of codes
$\mathcal{C}_i \subset \Sigma^\beta$: code $(i \in [\nu\rangle)$ $\big] |\mathcal{C}_i| = 2^{b_i}$
$\boldsymbol{b} = (b_0, \dots, b_{\nu-1})$: bit-width

$R_{\mathrm{I}} = \dfrac{1}{2n\beta} \sum_{i=0}^{\nu-1} b_i$: inner code rate

Metric

$\boldsymbol{x} = (x_0, \dots, x_{\beta-1}) \in \mathbb{B}^\beta$ $\rightarrow \downarrow : 1$
$\boldsymbol{y} = (y_0, \dots, y_{\beta-1}) \in \mathbb{B}^\beta$ $\searrow : \{0,1\}$
metric $d(\boldsymbol{x}, \boldsymbol{y})$:

  insertion: (insert 0 or 1) + (delete the last bit)
  deletion: (delete one bit) + (append 0 or 1)
  substitution: invert one bit
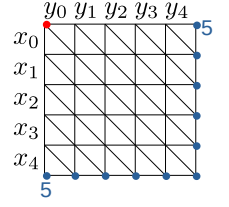
Construction from binary code

$\boldsymbol{b}' = (b'_0, \dots, b'_{\nu-1}) \in [\beta+1\rangle^\nu$
even β : $2^{b'_i} \leq \binom{\beta}{\beta/2}$ $(b_i = b'_i + \beta)$
odd β : $2^{b'_i} \leq \binom{\beta}{(\beta-1)/2} = \binom{\beta}{(\beta+1)/2}$

β	t	$\binom{\beta}{t}$	b' 0	1	2	3	4	5	6	7
2	1	2	T	T	$-$	$-$	$-$	$-$	$-$	$-$
3	1,2	3	T	T	F	$-$	$-$	$-$	$-$	$-$
4	2	6	T	T	T	F	$-$	$-$	$-$	$-$
5	2,3	10	T	T	T	T	F	$-$	$-$	$-$
6	3	20	T	T	T	T	T	F	$-$	$-$
7	3,4	35	T	T	T	T	T	T	F	$-$
8	4	70	T	T	T	T	T	T	T	F
9	4,5	126	T	T	T	T	T	T	T	F

[computer search: candidate codes]
input: β, b', t
output: $\mathcal{L} = \{\mathcal{B}^0, \dots, \mathcal{B}^{m-1}\}$ $(\mathcal{B}^j \subset \mathbb{B}^\beta)$
           (list of highest score codes)
constraint: $|\mathcal{B}^j| = 2^{b'}, \forall \boldsymbol{u} \in \mathcal{B}^j, w(\boldsymbol{u}) = t$
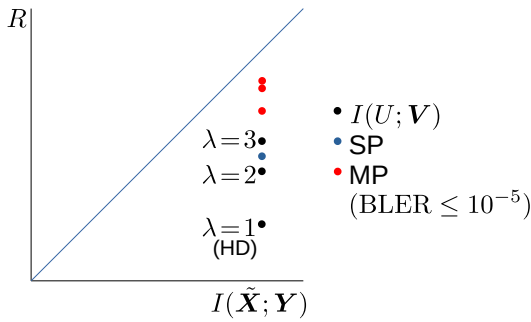objective: maximize
$$\min_{\boldsymbol{x}, \boldsymbol{y} \in \mathcal{B}^j} d(\boldsymbol{x}, \boldsymbol{y})   (\boldsymbol{x} \neq \boldsymbol{y})$$

[computer search: code sequence]
input: $(\mathcal{L}_0, \dots, \mathcal{L}_{\nu-1})$
output: $(\mathcal{B}_0, \dots, \mathcal{B}_{\nu-1})$
constraint: $\mathcal{B}_i \in \mathcal{L}_i$
objective: minimize $\max_{i \in [\nu\rangle} \left| \mathcal{B}_i \cap \mathcal{B}_{|i+1|_\nu} \right|$

Evaluation of inner code

channel input: $u \in \mathbb{F}_{2^{b(i,\nu)}}$
channel output: $\boldsymbol{p} \in \mathbb{P}_{2^{b(i,\nu)}}$ (APP from inner code)
(a) HD+H(p): $\boldsymbol{v} = (v, Q_\lambda(H(\boldsymbol{p})))$
                 Q_λ : λ-level quantization
(b) list output: $\boldsymbol{v} = (v_0, \dots, v_{\lambda-1})$
    (list size λ) $= \arg\max(\boldsymbol{p}, \lambda) \in \mathbb{F}_{2^{b(i,\nu)}}^\lambda$
mutual info.: $I(U; \boldsymbol{V})$

$\beta=4, \nu=1$		$\beta=4, \nu=3$		$\beta=5, \nu=1$		$\beta=5, \nu=2$	
$\boldsymbol{b}$	R_{I}	$\boldsymbol{b}$	R_{I}	$\boldsymbol{b}$	R_{I}	$\boldsymbol{b}$	R_{I}
(4)	$4/8$	$(4,4,4)$	$12/24$	(5)	$5/10$	$(5,5)$	$10/20$
(5)	$5/8$	$(5,4,4)$	$13/24$	(6)	$6/10$	$(6,5)$	$11/20$
(6)	$6/8$	$(5,5,4)$	$14/24$	(7)	$7/10$	$(6,6)$	$12/20$
$\beta=4, \nu=2$		$(6,4,4)$	$14/24$	(8)	$8/10$	$(7,5)$	$12/20$
$\boldsymbol{b}$	R_{I}	$(5,5,5)$	$15/24$			$(7,6)$	$13/20$
$(4,4)$	$8/16$	$(6,5,4)$	$15/24$			$(7,7)$	$14/20$
$(5,4)$	$9/16$	$(6,5,5)$	$16/24$			$(8,6)$	$14/20$
$(5,5)$	$10/16$	$(6,6,5)$	$17/24$			$(8,7)$	$15/20$
$(6,4)$	$10/16$	$(6,6,6)$	$18/24$			$(8,8)$	$16/20$
$(6,5)$	$11/16$						
$(6,6)$	$12/16$						

Concatenated coding

channel alphabet: $\Sigma = \{\mathsf{A}, \mathsf{T}, \mathsf{G}, \mathsf{C}\}$

outer code: $\mathcal{C}_\mathrm{o} \subset \mathbb{F}_{2^b}^{n_\mathrm{o}}$
  n_o : code length
  b : symbol size (bits)

inner code: $\mathcal{C}_i \subset \Sigma^\beta$ $(i \in [\nu\rangle, b < 2\beta)$
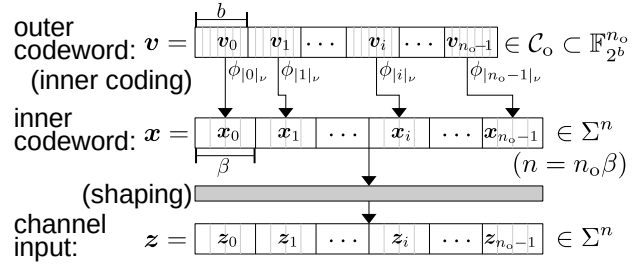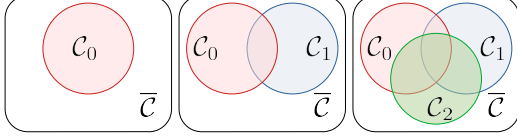
  β :   code length (symbols)
  ν :   number of code books
  $|\mathcal{C}_i| = 2^b$: number of codewords
  $R = b/2\beta$: inner code rate
  $\phi_i : \mathbb{F}_{2^b} \to \mathcal{C}_i$: encoding function

Σ^β $(\nu = 1)$ $(\nu = 2)$ $(\nu = 3)$

outer codeword: $\boldsymbol{v} = [\boldsymbol{v}_0\ \boldsymbol{v}_1\ \cdots\ \boldsymbol{v}_i\ \cdots\ \boldsymbol{v}_{n_\mathrm{o}-1}] \in \mathcal{C}_\mathrm{o} \subset \mathbb{F}_{2^b}^{n_\mathrm{o}}$
  (inner coding) $\phi_{|0|_\nu}\ \phi_{|1|_\nu}\ \phi_{|i|_\nu}\ \phi_{|n_\mathrm{o}-1|_\nu}$

inner codeword: $\boldsymbol{x} = [\boldsymbol{x}_0\ \boldsymbol{x}_1\ \cdots\ \boldsymbol{x}_i\ \cdots\ \boldsymbol{x}_{n_\mathrm{o}-1}] \in \Sigma^n$ $(n = n_\mathrm{o}\beta)$

(shaping)

channel input: $\boldsymbol{z} = [\boldsymbol{z}_0\ \boldsymbol{z}_1\ \cdots\ \boldsymbol{z}_i\ \cdots\ \boldsymbol{z}_{n_\mathrm{o}-1}] \in \Sigma^n$

prior: $p(\boldsymbol{x}_i) = \begin{cases} 1/2^b & (\boldsymbol{x}_i \in \mathcal{C}_{|i|_\nu}) \\ 0 & (\boldsymbol{x}_i \notin \mathcal{C}_{|i|_\nu}) \end{cases}$

params:

β	b
3	4, 5
4	6, 7
5	8, 9
6	10, 11

$\nu = 2$

Constraint

RL-LB constrained coding $(\ell; w, \delta)$:

 * run length coding: $f_\mathrm{R}(\boldsymbol{z}) \le \ell$
 * local GC-balance: $\left| f_\mathrm{B}\left(\boldsymbol{z}_i^{i+w-1}\right) \right| \le \delta$
       $(\forall i \in [n - w\rangle)$
 * synchronization: MIR?

vector over Σ : $\boldsymbol{u} = (u_0, \ldots, u_{n-1}) \in \Sigma^n$
weight: $w(\boldsymbol{u})_x = |\{i \in [n\rangle \mid u_i = x\}|$ $(x \in \Sigma)$
balance: $f_\mathrm{B}(\boldsymbol{u}) = w_\mathsf{G}(\boldsymbol{u}) + w_\mathsf{C}(\boldsymbol{u}) - w_\mathsf{A}(\boldsymbol{u}) - w_\mathsf{T}(\boldsymbol{u}) \in [-n, n]$
max run length: $f_\mathrm{R}(\boldsymbol{u}) \in [1, n]$

Inner codebook

$\mathcal{C}_i = \left\{ (\phi(x_0 y_0), \ldots, \phi(x_{\beta-1} y_{\beta-1})) \,\middle|\, \boldsymbol{x} \in \mathcal{B}_i, \boldsymbol{y} \in \mathbb{B}^\beta \right\}$
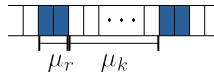$\boldsymbol{x} = (x_0, \ldots, x_{\beta-1})$ $(i \in [\nu\rangle)$
$\boldsymbol{y} = (y_0, \ldots, y_{\beta-1})$
$\mathcal{B}_i \subset \mathbb{B}^\beta$
$|\mathcal{C}_i| = 2^\beta |\mathcal{B}_i| = 2^b,\ |\mathcal{B}_i| = 2^{b-\beta}$

v	$\phi(v)$
00	A
01	T
10	G
11	C

Marker

rate: $R_\mathrm{i} = \frac{\mu_k}{\mu_k + \mu_r}$

  μ_r μ_k

Rate

$R = R_\mathrm{o} R_\mathrm{i} R_\mathrm{c}$ [bits/symbol]

  $R_\mathrm{o} \le$ 1-cross entropy: inner APP
  $R_\mathrm{i} =$ inner code rate
  $R_\mathrm{c} \le$ bound of $(\ell; w, \delta)$ constraint
        bound for given $P(X)$

$R_\mathrm{o} R_\mathrm{i} \le I(X; Y)$:

(example)

□ : 0
■ : 1

		β	ν	R_I	ℓ	δ	b_i	$\mathcal{B}_i$
1	3.4.2	3	2	4/6	2	2	4	$\mathcal{B}_0$
							4	$\mathcal{B}_1$
2	3.5.1	3	1	5/6	3	*	5	$\mathcal{B}_0$
3	4.5.2	4	2	5/8	2	2	5	$\mathcal{B}_0$
							5	$\mathcal{B}_1$
4	4.6.1a	4	1	6/8	2	2	6	$\mathcal{B}_0$
5	4.6.1b	4	1	6/8	4	*	6	$\mathcal{B}_0$
6	4.7.1	4	1	7/8	4	*	7	$\mathcal{B}_0$
7	5.7.2	5	2	7/10	5	5	7	$\mathcal{B}_0$
							7	$\mathcal{B}_1$
9	5.8.2	5	2	8/10	3	3	8	$\mathcal{B}_0$
							8	$\mathcal{B}_1$
10	5.9.1	5	1	9/10	4	*	9	$\mathcal{B}_0$
8	5.a.2	5	2	15/20	3	3	8	$\mathcal{B}_0$
							7	$\mathcal{B}_1$

(ex2b)

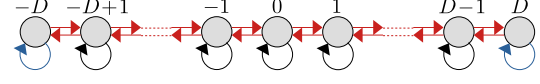| β	3	3	4	4	4	4	5	5	6
b	4	5	5	6	6	7	8	9	10
ν	2	1	2	1	1	1	2	1	1
R_i	0.67	0.83	0.63	0.75	0.75	0.88	0.80	0.90	0.83
					a	b			
$(\ell; w, \delta)$	$(2; 10, 2)$	$(3; *, *)$	$(2; 10, 2)$	$(2; 10, 2)$	$(4; *, *)$	$(4; *, *)$	$(3; 10, 2)$	$(4; *, *)$	

* (multi-read)

NB-IDS channel

block length:	n
alphabet:	Σ $(|\Sigma| = q)$
error prob.:	p_{id} $(< \frac{1}{2})$ (ins/del)
	p_{s} (sub)
input:	$\boldsymbol{x} = (x_0, \ldots, x_{n-1}) = \Sigma^n$
output:	$\boldsymbol{y} = (y_0, \ldots, y_{n'-1}) = \Sigma^{n'}$
	$(n - D \leq n' \leq n + D)$

[transmission]

1) drift vector: $\boldsymbol{d} = (d_0, \ldots, d_{n-1}, d_n) \in \mathcal{D}^{n+1}$

2) intermediate vector: $\boldsymbol{z} = (z_0, \ldots, z_{n'-1}) \in \Sigma^{n'}$

$$z_j = x_i \ (j \in [i + d_i, i + d_{i+1}])$$
$$i \in [n\rangle, \ n' = n + d_n$$

3) output vector:
$$p(y_i | z_i) = \begin{cases} 1 - p_{\mathrm{s}} & (y_i = z_i) \\ \frac{p_{\mathrm{s}}}{q-1} & (y_i \neq z_i) \end{cases}$$

max drift:	D
set of drift values:	$\mathcal{D} = \{-D, \ldots, 0, \ldots, D\}$
drift vector:	$\boldsymbol{d} = (d_0 = 0, \ldots, d_{n-1}, d_n) \in \mathcal{D}^{n+1}$

$$p(d_{i+1} | d_i) = \begin{cases} 1 - 2p_{\mathrm{id}} & (d_{i+1} = d_i, |d_i| < D) & \longrightarrow \\ 1 - p_{\mathrm{id}} & (d_{i+1} = d_i, |d_i| = D) & \longrightarrow \\ p_{\mathrm{id}} & (|d_{i+1} - d_i| = 1, |d_{i+1}| \leq D) & \longrightarrow \\ 0 & (\mathrm{otherwise}) \end{cases}$$

APP by FBA

$$p(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{d}) = p(\boldsymbol{y} | \boldsymbol{x}, \boldsymbol{d}) p(\boldsymbol{x}, \boldsymbol{d}) = p(\boldsymbol{y} | \boldsymbol{x}, \boldsymbol{d}) p(\boldsymbol{x}) p(\boldsymbol{d})$$

$$= p(d_0) \prod_{i=0}^{n-1} p\left(\boldsymbol{y}_{i+d_i}^{i+d_{i+1}} \Big| x_i, d_i, d_{i+1} \right) p(d_{i+1} | d_i) p(x_i)$$

segment level synchronization: $n = n_\beta \beta$

$$= p(d_0) \prod_{i=0}^{n_\beta - 1} \frac{p\left(\boldsymbol{y}_{I+d_I}^{I'-1+d_{I'}} \Big| \boldsymbol{x}_I^{I'-1}, d_I, d_{I'} \right) p\left(d_{I'} | d_I \right) p\left(\boldsymbol{x}_I^{I'-1} \right)}{(I = i\beta, I' = i\beta + \beta)}$$
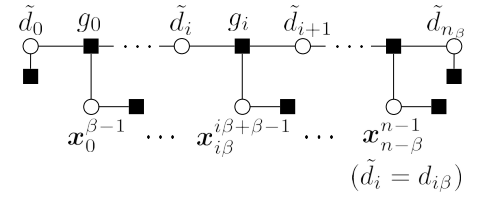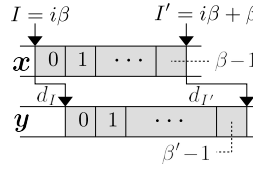
[LUT] (for a large D)

$$G\left(\beta', \boldsymbol{y}_0^{\beta'-1} \right) \text{ list of } \left[\boldsymbol{x}_0^{\beta-1}, \boldsymbol{p}_0\left(\boldsymbol{y}_0^{\beta'-1} \Big| \boldsymbol{x}_0^{\beta-1}, 0, d' \right) \boldsymbol{p}_0(d'|0) \ (> p_{\mathrm{th}}) \right]$$

$$\boldsymbol{p}_\alpha \left(\boldsymbol{y}_{\alpha'}^{\beta'-1} \Big| \boldsymbol{x}_\alpha^{\beta-1}, d, d' \right) \boldsymbol{p}_\alpha(d'|d) = \qquad \begin{aligned} (d' = \beta' - \beta) \\ (d = \alpha' - \alpha) \end{aligned}$$

$$\sum_{\delta \in \{-1, 0, 1\}} p\left(\boldsymbol{y}_{\alpha'}^{\alpha'+\delta} \Big| x_\alpha, d, d+\delta \right) p(d+\delta | d)$$

$$\times \boldsymbol{p}_{\alpha+1} \left(\boldsymbol{y}_{\alpha'+\delta+1}^{\beta'} \Big| \boldsymbol{x}_{\alpha+1}^{\beta-1}, d+\delta, d' \right) \boldsymbol{p}_{\alpha+1}(d'|d+\delta)$$

Gtable:
  int beta
  double Pid
  double Ps
  double Pth
  LIST[len][y]

LIST:
  int num
  XP[num]

XP:
  int x
  float p

[run-length of random sequence]

prob. of run-length ℓ: $p(\ell) = \left(\frac{1}{q}\right)^{\ell-1}\left(\frac{q-1}{q}\right) = \frac{q-1}{q^\ell}$

expectation: $E(\ell) = \sum_{\ell=1}^{n} \ell p(\ell) = \sum_{\ell=1}^{n} \frac{(q-1)\ell}{q^\ell}$

$$= (q-1)\sum_{\ell=1}^{n}\frac{\ell}{q^\ell}$$

$$= \frac{q}{q-1}\left(1 - \frac{1}{q^n}\right) - \frac{n}{q^n}$$

[local GC-balance of random sequence]

window size: w

prob. binary vector having weight t: $p_w(t) = \binom{w}{t}/2^w$

absolute GC-balance: $\sum_{t=0}^{w} p_w(t)\,|w-2t|$

[rate upper bound]

run length: constraint graph(?)

local GCB: $\frac{1}{w}\log_2\left(2^w\sum_{t\in\mathcal{T}}\binom{w}{t}\right) = 1 + \frac{1}{w}\log_2\left(\sum_{t\in\mathcal{T}}\binom{w}{t}\right)$

$$(\mathcal{T} = \{t\,|\,|w-2t| \le \delta\})$$

RL+LGCB: $\frac{1}{w}\log_2\left|\left\{\boldsymbol{u}\in\Sigma^w\,\middle|\,|f_{\mathrm{B}}(\boldsymbol{u})| \le \delta, f_{\mathrm{R}}(\boldsymbol{u}) \le \ell\right\}\right|$
$(\ell;w,\delta)$

Concatenated coding

channel alphabet: $\Sigma = \{A, T, G, C\}$
outer code: $\mathcal{C}_o \subset \mathbb{F}_{2^b}^{n_o}$
  n_o : code length
  b :   symbol size (bits)

inner code: $\mathcal{C}_i \subset \Sigma^\beta$ $(i \in [\nu\rangle, b < 2\beta)$

  β :   code length (symbols)
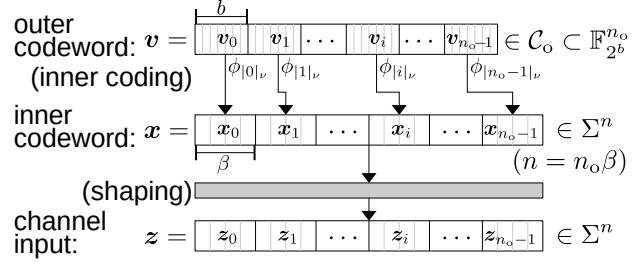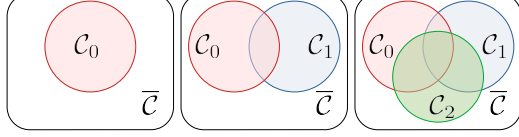 ν :   number of code books
  $|\mathcal{C}_i| = 2^b$: number of codewords
  $R = b/2\beta$: inner code rate
  $\phi_i : \mathbb{F}_{2^b} \to \mathcal{C}_i$: encoding function

Σ^β $(\nu = 1)$ $(\nu = 2)$ $(\nu = 3)$

outer codeword: $\boldsymbol{v} = (\boldsymbol{v}_0, \boldsymbol{v}_1, \ldots, \boldsymbol{v}_i, \ldots \boldsymbol{v}_{n_o-1}) \in \mathcal{C}_o \subset \mathbb{F}_{2^b}^{n_o}$
  (inner coding) $\phi_{|0|_\nu}\ \phi_{|1|_\nu}\quad\ \phi_{|i|_\nu}\quad\ \phi_{|n_o-1|_\nu}$

inner codeword: $\boldsymbol{x} = (\boldsymbol{x}_0, \boldsymbol{x}_1, \ldots, \boldsymbol{x}_i, \ldots \boldsymbol{x}_{n_o-1}) \in \Sigma^n$
                                        $(n = n_o\beta)$

(shaping)

channel input: $\boldsymbol{z} = (\boldsymbol{z}_0, \boldsymbol{z}_1, \ldots, \boldsymbol{z}_i, \ldots \boldsymbol{z}_{n_o-1}) \in \Sigma^n$

prior: $p(\boldsymbol{x}_i) = \begin{cases} 1/2^b & (\boldsymbol{x}_i \in \mathcal{C}_{|i|_\nu}) \\ 0 & (\boldsymbol{x}_i \notin \mathcal{C}_{|i|_\nu}) \end{cases}$

params:

β	b
3	4, 5
4	6, 7
5	8, 9
6	10, 11

vector over Σ : $\boldsymbol{u} = (u_0, \ldots, u_{n-1}) \in \Sigma^n$
weight: $w(\boldsymbol{u})_x = |\{i \in [n\rangle \,|\, u_i = x\}|$ $(x \in \Sigma)$
balance: $f_B(\boldsymbol{u}) = w_G(\boldsymbol{u}) + w_C(\boldsymbol{u}) - w_A(\boldsymbol{u}) - w_T(\boldsymbol{u}) \in [-n, n]$
max run length: $f_R(\boldsymbol{u}) \in [1, n]$

Constraint

RL-LB constrained coding $(\ell; w, \delta)$:

 * run length coding: $f_R(\boldsymbol{z}) \leq \ell$
 * local GC-balance: $\left| f_B\left(\boldsymbol{z}_i^{i+w-1} \right) \right| \leq \delta$
                        $(\forall i \in [n - w\rangle)$
 * synchronization: MIR?

[example 1]
  forbidden set $\overline{\mathcal{C}} \subseteq \Sigma^\beta \setminus \cup_{i \in [\nu\rangle} \mathcal{C}_i$ $(|\overline{\mathcal{C}}| \leq 2^{2\beta} - 2^b)$

  ϕ (empty)
  $\overline{\mathcal{C}}_{*,\lambda} = \left\{ \boldsymbol{u} \in \Sigma^\beta \,\big|\, f_R(\boldsymbol{u}) \geq \lambda \right\}$ (RL)
  $\overline{\mathcal{C}}_{\omega,*} = \left\{ \boldsymbol{u} \in \Sigma^\beta \,\big|\, |f_B(\boldsymbol{u})| \geq \omega \right\}$ (LB)
  $\overline{\mathcal{C}}_{\omega,\lambda} = \overline{\mathcal{C}}_{\omega,*} \cup \overline{\mathcal{C}}_{*,\lambda}$ (both)

[example 2a] $\beta = 4, b = 7$
  $\mathcal{C}_i = \left\{ (\phi(x_0 y_0), \ldots, \phi(x_3 y_3)) \big| (x_0, \ldots, x_3) \in \mathcal{B}_i, (y_0, \ldots, y_3) \in \mathbb{B}^4 \right\}$

    $\mathcal{B}_0$: 0011, 0101, 0110, 1001, 1010, 1100, 0010, 0100
    $\mathcal{B}_1$: 0011, 0101, 0110, 1001, 1010, 1100, 1101, 1011
    $|\mathcal{C}_i| = 8 \times 16 = 128 = 2^b$

$\boldsymbol{v}$	$\phi(\boldsymbol{v})$
00	A
01	T
10	G
11	C

[example 2b] $\beta = 5, b = 8$

  $\mathcal{C}_i = \left\{ (\phi(x_0 y_0), \ldots, \phi(x_4 y_4)) \big| (x_0, \ldots, x_4) \in \mathcal{B}_i, (y_0, \ldots, y_4) \in \mathbb{B}^5 \right\}$

    $\mathcal{B}_0$: 00011, 00101, 00110, 01001, 01010, 01100, 10001, 10010, 10100, 11000
    $\mathcal{B}_1$: 11100, 11010, 11001, 10110, 10101, 10011, 01110, 01101, 01011, 00111
    $|\mathcal{C}_i| = 8 \times 32 = 256 = 2^b$

    $\boldsymbol{a} = (a_0, \ldots, a_4), \boldsymbol{c} = (c_0, \ldots, c_4) \in \mathcal{B}_0$
    $\boldsymbol{b} = (b_0, \ldots, b_4), \boldsymbol{d} = (d_0, \ldots, d_4) \in \mathcal{B}_1$

  RL: max RL in $\boldsymbol{a}$ and $\boldsymbol{b}$: 3 (by definition)
       max RL in $(\boldsymbol{a}, \boldsymbol{b})$:   3 (proof 1)
       max RL in $(\boldsymbol{b}, \boldsymbol{c})$:   3 (proof 2)

  (proof 1) length of the last run of $\boldsymbol{a}$: 1 (run of 1)
                                        2 (run of 0s)
          length of the first run of $\boldsymbol{b}$: 2 (run of 1s)
                                        1 (run of 0)
  (proof 2) length of the last run of $\boldsymbol{b}$: 1 (run of 0)
                                        2 (run of 1s)
          length of the first run of $\boldsymbol{c}$: 2 (run of 0s)
                                        1 (run of 1)

  LB: even $w(\geq 10)$
  $(\boldsymbol{a}_t^4, \boldsymbol{b}, \boldsymbol{c}_0^{t-1})$:

* (multi-read)

$$\frac{1}{w} \log_2 \left(\sum_{t \in \mathcal{T}} \left(2^w \binom{w}{t} - \rho_{\ell,w,t} \right) \right)$$

$$\rho_{\ell,w,t} = |\{\boldsymbol{u} \in \Sigma^w | w_{\mathsf{G}}(\boldsymbol{u}) + w_{\mathsf{C}}(\boldsymbol{u}) = t, f_{\mathsf{R}}(\boldsymbol{u}) > \ell\}|$$

$(\beta = 3, b = 4)$
$\mathcal{B}_0 : 010,100$
$\mathcal{B}_1 : 101,110$

$(\beta = 4, b = 6)$
$\mathcal{B}_0 : 0101,0110,1001,1010$
$\mathcal{B}_1 : 0101,0110,1001,1010$

$(\beta = 5, b = 8)$ [ex2b]
$\mathcal{B}_0 : 00101,00110,01001,01010,01100,10001,10010,10100$
$\mathcal{B}_1 : 11010,11001,10110,10101,10011,01110,01101,01011$

$(\beta = 5, b = 9)$
$\mathcal{B}_0 : 00101,00110,01001,01010,01100,10001,10010,10100,...$
$\mathcal{B}_1 : 11010,11001,10110,10101,10011,01110,01101,01011,...$

$(\beta = 3, b = 5)$
$\mathcal{B}_0 : 001,010,100,101$
$\mathcal{B}_1 : 110,101,011,010$

$(\beta = 4, b = 7)$ [ex2a]
$\mathcal{B}_0 : 0011,0101,0110,1001,1010,1100,0010,0100$
$\mathcal{B}_1 : 0011,0101,0110,1001,1010,1100,1101,1011$

position	max RL	pattern
$(a_0, a_1, a_2, a_3, a_4)$	3	10001
$(a_1, a_2, a_3, a_4; b_0)$	3	$*100; 0$
$(a_2, a_3, a_4; b_0, b_1)$		
$(a_3, a_4; b_0, b_1, b_2)$		
$(a_4; b_0, b_1, b_2, b_3)$		
$(b_0, b_1, b_2, b_3, b_4)$	3	01110
$(b_1, b_2, b_3, b_4; c_0)$		
$(b_2, b_3, b_4; c_0, c_1)$		
$(b_3, b_4; c_0, c_1, c_2)$		
$(b_4; c_0, c_1, c_2, c_3)$		