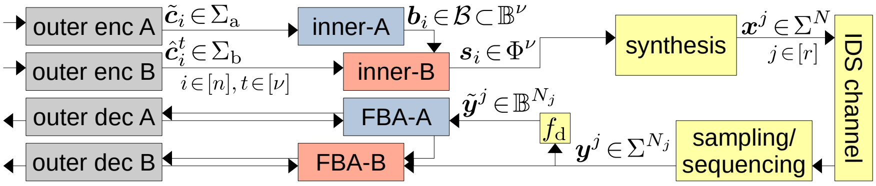


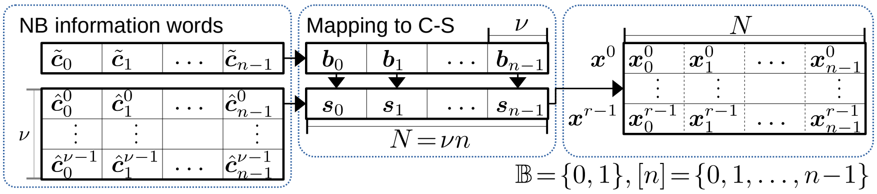
Alphabet: $\Sigma = \{0,1,2,3\} \leftrightarrow \{A, T, G, C\}$
Code length: ν symbol (even)
Block length: $N = \nu n$ symbol
(block = n inner codewords)

Constraints (binary):
run length: RL-ρ
local balance: LB-(ℓ,δ) $|w(\boldsymbol{b}_i^{i+\ell-1}) - \frac{\ell}{2}| \le \delta$
indicator func: $\mathbb{1}_{\rho,\ell,\delta}(\boldsymbol{b})$

NB-IDS channel:
error prob: p_i, p_d, p_s

$$p(y|x) = \begin{cases} 1-p_s & (y=x) \\ p_s/3 & (y \ne x) \end{cases}$$

drift vector:
$\boldsymbol{d}^j = (d_0^j, \dots, d_N^j)$ (symbol level)
$\tilde{\boldsymbol{d}}^j = (\tilde{d}_0^j, \dots, \tilde{d}_n^j)$ (word level) $\tilde{d}_i^j = d_{i\nu}^j$
$d_i^j \in \mathcal{D} = \{D_{\min}, \dots, -1, 0, 1, \dots, D_{\max}\}$



$\mathbb{B} = \{0,1\}, [n] = \{0,1,\dots,n-1\}$
$|\Sigma_a| = q_a = |\mathcal{B}|$
$|\Sigma_b| = q_b \le k+1$

Binary constraint vector set [SITA2024]

$\mathcal{B} \subset \mathbb{B}^\nu \quad (|\mathcal{B}| = 2^{k_a})$

$\forall \boldsymbol{b} \in \mathcal{B}, w(\boldsymbol{b}) = \nu/2$	(LB: inside CW)
$\forall \boldsymbol{b} \in \mathcal{B}, \forall i \in [\nu]_{\text{even}}, \lvert\tilde{w}(\boldsymbol{b}_i^{\nu-1})\rvert \le 2\delta$	(LB: CW boundary)
$\forall \boldsymbol{b} \in \mathcal{B}, \forall i \in [\nu]_{\text{odd}}, \lvert\tilde{w}(\boldsymbol{b}_i^{\nu-1})\rvert \le 2\delta - 1$	(LB: CW boundary)
$\forall \boldsymbol{b} \in \mathcal{B}, \lambda(\boldsymbol{b}) \le \rho$	(RL: inside CW)
$\forall \boldsymbol{b} \in \mathcal{B}, \lambda(\boldsymbol{b}, \nu-1) \le \rho - 1$	(RL: CW boundary)
$\boldsymbol{b} \in \mathcal{B} \to \overline{\boldsymbol{b}} \in \mathcal{B}$	(re-balance)
$(01)^{\frac{\nu}{2}}, (10)^{\frac{\nu}{2}} \in \mathcal{B}$	(erasure symbol)

Composite symbol sets

$\Phi_0 \subseteq \left\{(\sigma_0,\sigma_1,\sigma_2,\sigma_3) \in \mathbb{Z}_{k+1}^4 \,\middle|\, \sigma_0+\sigma_1 = k, \sigma_2 = \sigma_3 = 0\right\}$ AT
$\Phi_1 \subseteq \left\{(\sigma_0,\sigma_1,\sigma_2,\sigma_3) \in \mathbb{Z}_{k+1}^4 \,\middle|\, \sigma_0 = \sigma_1 = 0, \sigma_2+\sigma_3 = k\right\}$ GC
$|\Phi_0| = |\Phi_1| = q_b \le k+1 \qquad \Phi = \Phi_0 \cup \Phi_1$

Inner codebook

$\mathcal{C} = \left\{(c_0,\dots,c_{\nu-1}) \,\middle|\, c_j \in \Phi_{b_j}, (b_0,\dots,b_{\nu-1}) \in \mathcal{B}\right\}$

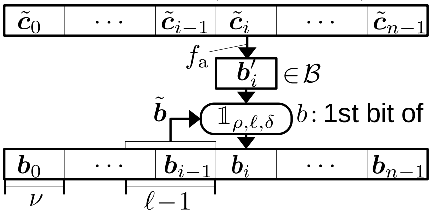
Mapping

$f_a : \Sigma_a \to \mathcal{B}$ (bijection)
$f_b^i : \Sigma_b \to \Phi_i \ (i \in \mathbb{B})$ (bijection)
$f_d : \Sigma \to \mathbb{B} \qquad f_d(x) = \mathbb{1}[x \in \{2,3\}]$

Encoding

input: $\tilde{\boldsymbol{c}} = (\tilde{\boldsymbol{c}}_0, \dots, \tilde{\boldsymbol{c}}_{n-1}) \qquad \tilde{\boldsymbol{c}}_i \in \Sigma_a$
$\hat{\boldsymbol{c}}^t = (\hat{\boldsymbol{c}}_0^t, \dots, \hat{\boldsymbol{c}}_{n-1}^t) \qquad \hat{\boldsymbol{c}}_i^t \in \Sigma_b, t \in [\nu]$
output: $\boldsymbol{s} = (\boldsymbol{s}_0, \dots, \boldsymbol{s}_{n-1}) \qquad \boldsymbol{s}_i \in \Phi^\nu$

(1) encode $\tilde{\boldsymbol{c}}$ to $\boldsymbol{b} = (\boldsymbol{b}_0, \dots, \boldsymbol{b}_{n-1}) \in \mathcal{B}^n$



$$\boldsymbol{b}_i = \begin{cases} \boldsymbol{b}_i' & (\mathbb{1}_{\rho,\ell,\delta}(\tilde{\boldsymbol{b}}, \boldsymbol{b}_i') = 1) & \text{(unchanged)} \\ \overline{\boldsymbol{b}}_i' & (\mathbb{1}_{\rho,\ell,\delta}(\tilde{\boldsymbol{b}}, \overline{\boldsymbol{b}}_i') = 1) & \text{(invert)} \\ (\overline{b}b)^{\frac{\nu}{2}} & (\mathbb{1}_{\rho,\ell,\delta}(\tilde{\boldsymbol{b}}, (\overline{b}b)^{\frac{\nu}{2}}) = 1) & \text{(rebalance)} \\ (b\overline{b})^{\frac{\nu}{2}} & (\text{otherwise}) & \text{(rebalance)} \end{cases}$$

(2) encode $(\boldsymbol{b}_i, \hat{\boldsymbol{c}}_i^0, \dots, \hat{\boldsymbol{c}}_i^{\nu-1})$ to $\boldsymbol{s}_i \ (i \in [n])$

$s_i^t = f_b^{b_i^t}(\hat{\boldsymbol{c}}_i^t) \quad (t \in [\nu]) \quad \boldsymbol{s}_i = (s_i^0, \dots, s_i^{\nu-1}) \in \Phi^\nu$
$\boldsymbol{b}_i = (b_i^0, \dots, b_i^{\nu-1}) \in \mathcal{B}$

(ex) $\nu = 6, |\mathcal{B}| = 18, k = 7, \Sigma_a = [18], \Sigma_b = [8]$

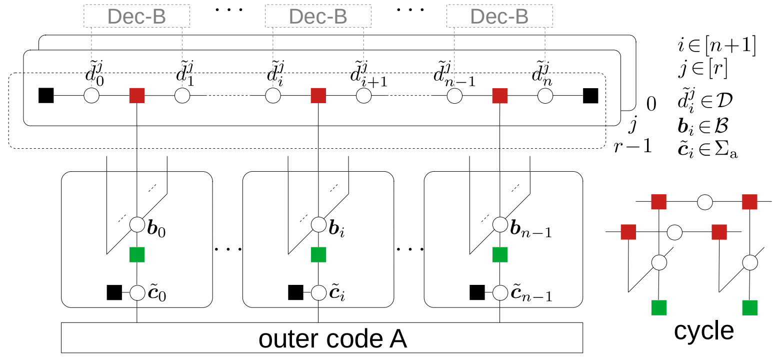
```
B:00:001011  09:110100   Φ0:0:0700  Φ1: 0:0007
  01:001101  10:110010      1:1600      1:0016
  02:001110  11:110001      2:2500      2:0025
  03:010011  12:101100      3:3400      3:0034
  04:010101  13:101010      4:4300      4:0043
  05:010110  14:101001      5:5200      5:0052
  06:011001  15:100110      6:6100      6:0061
  07:011010  16:100101      7:7000      7:0070
  08:011100  17:100011
```

(Lee distance?)

$\tilde{\boldsymbol{c}}_i$: 6 $\xrightarrow{f_a}$ 011010
$(\hat{\boldsymbol{c}}_i^0, \dots, \hat{\boldsymbol{c}}_i^5)$: 342364
f_b ↓↓↓↓↓↓
300304
400403
042060
035010
$\boldsymbol{s}_i = s_i^0 \cdots s_i^5$

non-uniform k

Decoding-A



$$p\left(\tilde{\boldsymbol{y}}^0,\dots,\tilde{\boldsymbol{y}}^{r-1},\tilde{\boldsymbol{d}}^0,\dots,\tilde{\boldsymbol{d}}^{r-1},\boldsymbol{b},\tilde{\boldsymbol{c}}\right) \qquad \left(\tilde{\boldsymbol{y}}^j=f_{\mathrm{d}}(\boldsymbol{y}^j)\in\mathbb{B}^{N_j}\right)$$

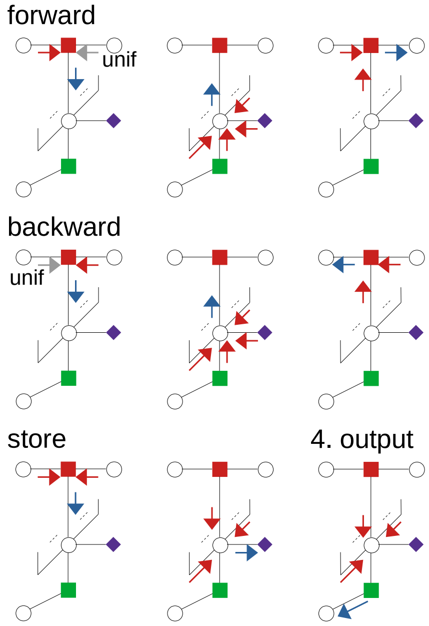
$$=p\left(\tilde{\boldsymbol{y}}^0,\dots,\tilde{\boldsymbol{y}}^{r-1}\middle|\tilde{\boldsymbol{d}}^0,\dots,\tilde{\boldsymbol{d}}^{r-1},\boldsymbol{b},\tilde{\boldsymbol{c}}\right)p\left(\tilde{\boldsymbol{d}}^0,\dots,\tilde{\boldsymbol{d}}^{r-1}\right)p\left(\boldsymbol{b}|\tilde{\boldsymbol{c}}\right)p\left(\tilde{\boldsymbol{c}}\right)$$

$$=p\left(\boldsymbol{b}|\tilde{\boldsymbol{c}}\right)p\left(\tilde{\boldsymbol{c}}\right)\prod_{j=0}^{r-1}p\left(\tilde{\boldsymbol{y}}^j\middle|\boldsymbol{b},\tilde{\boldsymbol{d}}^j\right)p\left(\tilde{\boldsymbol{d}}^j\right)$$

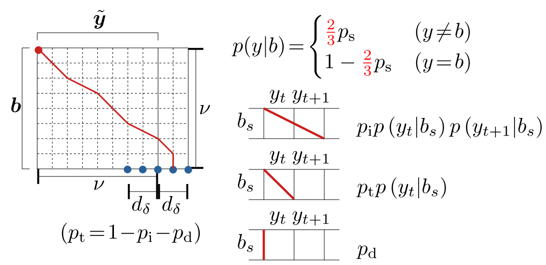
$$\simeq\left(\prod_{i=0}^{n-1}\underbrace{p\left(\boldsymbol{b}_i|\tilde{\boldsymbol{c}}_i\right)}_{\text{(sim)}}p\left(\tilde{\boldsymbol{c}}_i\right)\right)\prod_{j=0}^{r-1}\left(p(\tilde{d}^j_0)\prod_{i=0}^{n-1}p\left([\tilde{\boldsymbol{y}}^j]^{i\nu+\nu-1+\tilde{d}^j_{i+1}}_{i\nu+\tilde{d}^j_i}\middle|\boldsymbol{b}_i,\tilde{d}^j_i,\tilde{d}^j_{i+1}\right)p\left(\tilde{d}^j_{i+1}\middle|\tilde{d}^j_i\right)\right)$$

```
PF[Nseq][Ns+1][D]
PB[Nseq][Ns+1][D]
PD[Nseq][Ns][Q]
PU[Nseq][Ns][Q]
PI[Ns][Q]
PO[Ns][Q]
PM[Ns][Q]
PU0[Ns][Q]
```

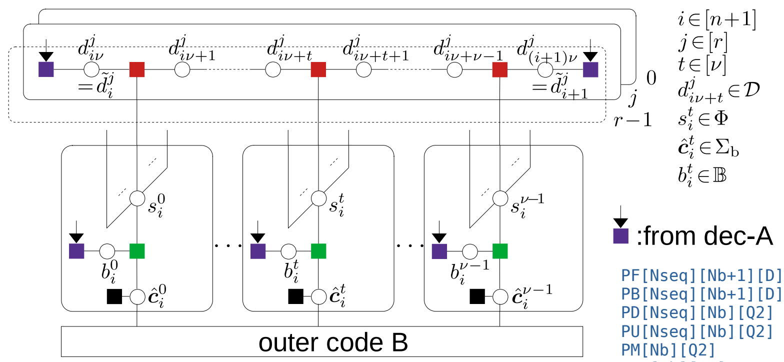
Scheduling



LUT



Decoding-B



:from dec-A

```
PF[Nseq][Nb+1][D]
PB[Nseq][Nb+1][D]
PD[Nseq][Nb][Q2]
PU[Nseq][Nb][Q2]
PM[Nb][Q2]
PXU[Nb][Q2]
PXD[Nb][Q2]
PIB[Nb][2]
PIC[Nb][Q2]
POC[Nb][Q2]

GS1[y][s]      //sub
GS2[y0][y1][s] //ins
CSL[Q2][4]
```

$$p\left(\boldsymbol{y}^0,\dots,\boldsymbol{y}^{r-1},[\boldsymbol{d}^0]^{(i+1)\nu}_{i\nu},\dots,[\boldsymbol{d}^{r-1}]^{(i+1)\nu}_{i\nu},\boldsymbol{s}_i,\boldsymbol{b}_i,\hat{\boldsymbol{c}}_i\right)$$

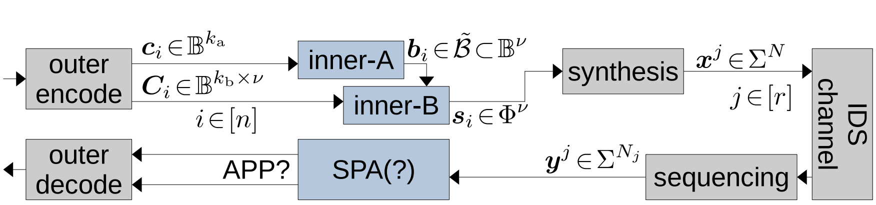
$$=p\left(\boldsymbol{b}_i\right)p\left(\hat{\boldsymbol{c}}_i\right)p\left(\boldsymbol{s}_i|\boldsymbol{b}_i,\hat{\boldsymbol{c}}_i\right)\prod_{j=0}^{r-1}p\left(\boldsymbol{y}^j\middle|\boldsymbol{s}_i,[\boldsymbol{d}^j]^{(i+1)\nu}_{i\nu}\right)p\left([\boldsymbol{d}^j]^{(i+1)\nu}_{i\nu}\right)$$

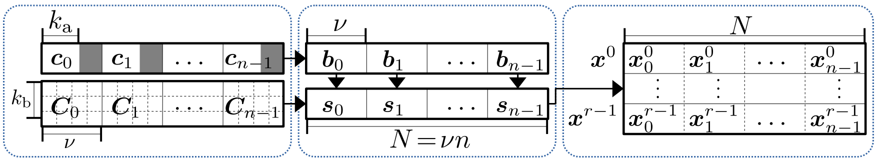
$$\simeq\left(\prod_{t=0}^{\nu-1}p\left(b^t_i\right)p\left(\hat{\boldsymbol{c}}^t_i\right)p\left(s^t_i|b^t_i,\hat{\boldsymbol{c}}^t_i\right)\right)\times$$

$$\left(\prod_{j=0}^{r-1}p(d^j_{i\nu})\prod_{t=0}^{\nu-1}p\left([\boldsymbol{y}^j]^{i\nu+t+d^j_{i\nu+t+1}}_{i\nu+t+d^j_{i\nu+t}}\middle|s^t_i,d^j_{i\nu+t},d^j_{i\nu+t+1}\right)p\left(d^j_{i\nu+t+1}\middle|d^j_{i\nu+t}\right)\right)$$

$$p\left([\boldsymbol{y}^j]^{i\nu+t+d^j_{i\nu+t+1}}_{i\nu+t+d^j_{i\nu+t}}\middle|s^t_i,d^j_{i\nu+t},d^j_{i\nu+t+1}\right)=$$

$$\sum_{x^t_{i,j}\in\Sigma}p\left([\boldsymbol{y}^j]^{i\nu+t+d^j_{i\nu+t+1}}_{i\nu+t+d^j_{i\nu+t}}\middle|x^t_{i,j},d^j_{i\nu+t},d^j_{i\nu+t+1}\right)p\left(x^t_{i,j}|s^t_i\right)$$





$\mathbb{B} = \{0,1\}, [n] = \{0,1,\dots,n-1\}$

Alphabet: $\Sigma = \{0,1,2,3\} \leftrightarrow \{\text{A, T, G, C}\}$
Code length: ν symbol (even)
Block length: $N = \nu n$ symbol
(block = n inner codewords)

Constraints (binary):
- run length: RL-ρ
- local balance: LB-(ℓ, δ) $\left|w(\boldsymbol{b}_i^{i+\ell-1}) - \frac{\ell}{2}\right| \le \delta$
- indicator func: $\mathbb{1}_{\rho,\ell,\delta}(\boldsymbol{b})$

IDS channel:
- error prob: $p_{\mathrm{i}}, p_{\mathrm{d}}, p_{\mathrm{s}}$
- drift vector:
  - $\boldsymbol{d}^j = (d_0^j, \dots, d_N^j)$ (symbol level)
  - $\tilde{\boldsymbol{d}}^j = (\tilde{d}_0^j, \dots, \tilde{d}_n^j)$ (word level) $\tilde{d}_i^j = d_{i\nu}^j$

Binary constraint vector set [SITA2024]

$\mathcal{B} \subset \mathbb{B}^{\nu} \quad (|\mathcal{B}| = 2^{k_a})$

$\forall \boldsymbol{b} \in \mathcal{B}, w(\boldsymbol{b}) = \nu/2$	(LB: inside CW)
$\forall \boldsymbol{b} \in \mathcal{B}, \forall i \in [\nu]_{\mathrm{even}}, \left|\tilde{w}(\boldsymbol{b}_i^{\nu-1})\right| \le 2\delta$	(LB: CW boundary)
$\forall \boldsymbol{b} \in \mathcal{B}, \forall i \in [\nu]_{\mathrm{odd}}, \left|\tilde{w}(\boldsymbol{b}_i^{\nu-1})\right| \le 2\delta - 1$	(LB: CW boundary)
$\forall \boldsymbol{b} \in \mathcal{B}, \lambda(\boldsymbol{b}) \le \rho$	(RL: inside CW)
$\forall \boldsymbol{b} \in \mathcal{B}, \lambda(\boldsymbol{b}, \nu-1) \le \rho - 1$	(RL: CW boundary)
$\boldsymbol{b} \in \mathcal{B} \to \bar{\boldsymbol{b}} \in \mathcal{B}$	(re-balance)
$(01)^{\frac{\nu}{2}}, (10)^{\frac{\nu}{2}} \notin \mathcal{B}$	(erasure symbol)

$\tilde{\mathcal{B}} = \mathcal{B} \cup \{(01)^{\frac{\nu}{2}}, (10)^{\frac{\nu}{2}}\}$ (re-balance)

Composite symbol sets

$\Phi_0 \subseteq \left\{(\sigma_0,\sigma_1,\sigma_2,\sigma_3) \in \mathbb{Z}_{k+1}^4 \,\middle|\, \sigma_0+\sigma_1 = k, \sigma_2=\sigma_3=0\right\}$ AT

$\Phi_1 \subseteq \left\{(\sigma_0,\sigma_1,\sigma_2,\sigma_3) \in \mathbb{Z}_{k+1}^4 \,\middle|\, \sigma_0=\sigma_1 = 0, \sigma_2+\sigma_3=k\right\}$ GC

$|\Phi_0| = |\Phi_1| = 2^{k_b} \le k+1 \qquad \Phi = \Phi_0 \cup \Phi_1$

Inner codebook

$\mathcal{C} = \left\{(c_0,\dots,c_{\nu-1}) \,\middle|\, c_j \in \Phi_{b_j}, (b_0,\dots,b_{\nu-1}) \in \mathcal{B}\right\}$

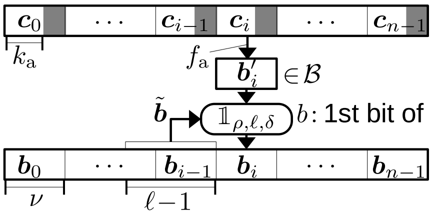
Mapping

$f_a : \mathbb{B}^{k_a} \to \mathcal{B}$ (bijection)
$f_i : \mathbb{B}^{k_b} \to \Phi_i \ (i \in \mathbb{B})$ (bijection)
$f_{\mathrm{d}} : \Sigma \to \mathbb{B} \qquad f_{\mathrm{d}}(x) = \mathbb{1}[x \in \{2,3\}]$

Encoding

input: $\boldsymbol{c} = (\boldsymbol{c}_0, \dots, \boldsymbol{c}_{n-1}) \quad \boldsymbol{c}_i \in \mathbb{B}^{k_a}$
$\boldsymbol{C} = (\boldsymbol{C}_0, \dots, \boldsymbol{C}_{n-1}) \quad \boldsymbol{C}_i \in \mathbb{B}^{k_b \times \nu}$
output: $\boldsymbol{s} = (\boldsymbol{s}_0, \dots, \boldsymbol{s}_{n-1}) \quad \boldsymbol{s}_i \in \Phi^{\nu}$

(1) encode $\boldsymbol{c}$ to $\boldsymbol{b} = (\boldsymbol{b}_0, \dots, \boldsymbol{b}_{n-1}) \in \tilde{\mathcal{B}}^n$



$$\boldsymbol{b}_i = \begin{cases} \boldsymbol{b}_i' & (\mathbb{1}_{\rho,\ell,\delta}(\tilde{\boldsymbol{b}}, \boldsymbol{b}_i') = 1) & \text{(unchanged)} \\ \bar{\boldsymbol{b}}_i' & (\mathbb{1}_{\rho,\ell,\delta}(\tilde{\boldsymbol{b}}, \bar{\boldsymbol{b}}_i') = 1) & \text{(invert)} \\ (\bar{b}b)^{\frac{\nu}{2}} & (\mathbb{1}_{\rho,\ell,\delta}(\tilde{\boldsymbol{b}}, (\bar{b}b)^{\frac{\nu}{2}}) = 1) & \text{(rebalance)} \\ (b\bar{b})^{\frac{\nu}{2}} & (\text{otherwise}) & \text{(rebalance)} \end{cases}$$

(2) encode $(\boldsymbol{C}_i, \boldsymbol{b}_i)$ to $\boldsymbol{s}_i \ (i \in [n])$

$s_{i,t} = f_{b_{i,t}}(\boldsymbol{C}_{i,t}^{\mathsf{T}}) \quad (t \in [\nu])$

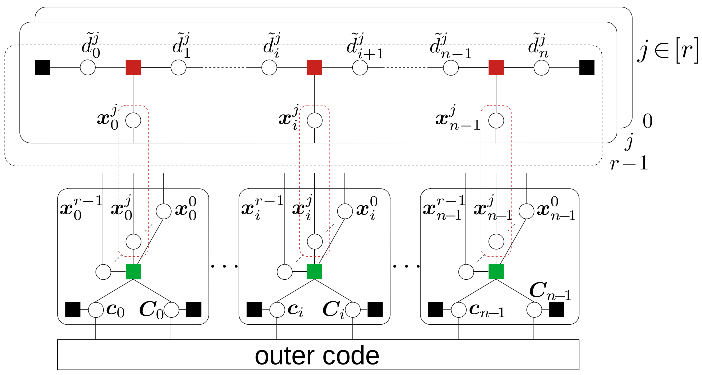
$\boldsymbol{s}_i = (s_{i,0}, \dots, s_{i,\nu-1}) \in \Phi^{\nu}$
$\boldsymbol{b}_i = (b_{i,0}, \dots, b_{i,\nu-1}) \in \tilde{\mathcal{B}}$
$\boldsymbol{C}_{i,t}$: t th column of $\boldsymbol{C}_i$

(ex) $\nu = 6, |\mathcal{B}| = 16, k = 7, k_a = 4, k_b = 3$

```
B:0:001011  8:110100  Φ0:0:0700  Φ1: 0:0007
  1:001101  9:110010     1:1600      1:0016
  2:001110  A:110001     2:2500      2:0025
  3:010011  B:101100     3:3400      3:0034
  -:010101  -:101010     4:4300      4:0043
  4:010110  C:101001     5:5200      5:0052
  5:011001  D:100110     6:6100      6:0061
  6:011010  E:100101     7:7000      7:0070
  7:011100  F:100011           (Lee distance?)
```

```
c_i :   0110(6)=011010
C_i^T : 011(3)           s_i : (0,3)=3400
        100(4)                 (1,4)=0043
        010(2)                 (1,2)=0025
        011(3)                 (0,3)=3400
        110(6)                 (1,6)=0061
        100(4)                 (0,4)=4300
```

~~Decoding~~



$$p\left(\boldsymbol{y}^0,\dots,\boldsymbol{y}^{r-1},\boldsymbol{x}^0,\dots,\boldsymbol{x}^{r-1},\tilde{\boldsymbol{d}}^0,\dots,\tilde{\boldsymbol{d}}^{r-1},\boldsymbol{c},\boldsymbol{C}\right)$$

$$= p\left(\boldsymbol{y}^0,\dots,\boldsymbol{y}^{r-1}\,\middle|\,\boldsymbol{x}^0,\dots,\boldsymbol{x}^{r-1},\tilde{\boldsymbol{d}}^0,\dots,\tilde{\boldsymbol{d}}^{r-1},\boldsymbol{c},\boldsymbol{C}\right) \times$$
$$p\left(\tilde{\boldsymbol{d}}^0,\dots,\tilde{\boldsymbol{d}}^{r-1}\right) p\left(\boldsymbol{x}^0,\dots,\boldsymbol{x}^{r-1}\middle|\boldsymbol{c},\boldsymbol{C}\right) p\left(\boldsymbol{c},\boldsymbol{C}\right)$$

$$= p(\boldsymbol{c})\, p(\boldsymbol{C}) \prod_{j=0}^{r-1} p\left(\boldsymbol{y}^j\middle|\boldsymbol{x}^j,\tilde{\boldsymbol{d}}^j\right) p\left(\tilde{\boldsymbol{d}}^j\right) p\left(\boldsymbol{x}^j\middle|\boldsymbol{c},\boldsymbol{C}\right)$$

$$\simeq \left(\prod_{i=0}^{n-1} p(\boldsymbol{c}_i)\, p(\boldsymbol{C}_i)\right) \times$$
$$\prod_{j=0}^{r-1}\left(p(\tilde{d}_0^j) \prod_{i=0}^{n-1} \underbrace{p\left([\boldsymbol{y}^j]_{i\nu+\tilde{d}_i^j}^{i\nu+\nu-1+\tilde{d}_{i+1}^j}\,\middle|\,\boldsymbol{x}_i^j,\tilde{d}_i^j,\tilde{d}_{i+1}^j,\right) p\left(\tilde{d}_{i+1}^j\middle|\tilde{d}_i^j\right)} \underbrace{p\left(\boldsymbol{x}_i^j\middle|\boldsymbol{c}_i,\boldsymbol{C}_i\right)}\right)$$

$$\underbrace{\prod_{j=0}^{r-1} p\left(\boldsymbol{x}_i^j\middle|\boldsymbol{c}_i,\boldsymbol{C}_i\right)} = \prod_{j=0}^{r-1} p\left(\boldsymbol{b}_i^j\middle|\boldsymbol{c}_i\right) p\left(\boldsymbol{x}_i^j\middle|\boldsymbol{b}_i^j,\boldsymbol{C}_i\right) \underset{\text{(sim)}}{=} \prod_{j=0}^{r-1} p\left(\boldsymbol{b}_i^j\middle|\boldsymbol{c}_i\right) \prod_{t=0}^{\nu-1} p\left(x_{i,t}^j\middle|b_{i,t}^j,\boldsymbol{C}_{i,t}\right)$$

[approximation?]

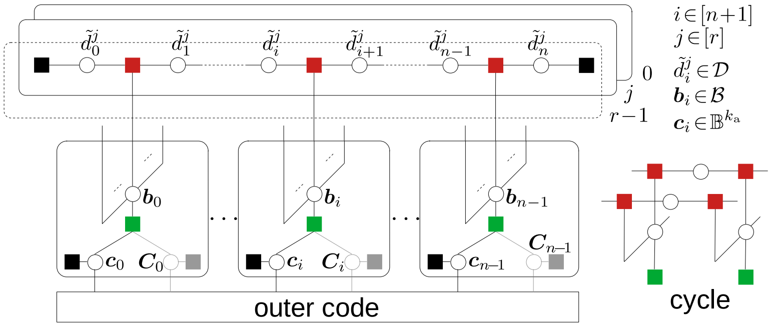
$\boldsymbol{b}_i^j = \left(f_{\mathrm{d}}(x_{i,0}^j),\dots,f_{\mathrm{d}}(x_{i,\nu-1}^j)\right)$

$p\left(x_{i,t}^j\,\middle|\,b_{i,t}^j,\boldsymbol{C}_{i,t}\right) = \sigma_{x_{i,t}^j}/k \quad \left(x_{i,t}^j \in \Sigma, b_{i,t}^j \in \mathbb{B}, \boldsymbol{C}_{i,t}^{\mathsf{T}} \in \mathbb{B}^{k_b}\right)$

$(\sigma_0,\sigma_1,\sigma_2,\sigma_3) = f_{b_{i,t}^j}(\boldsymbol{C}_{i,t}^{\mathsf{T}})$

non-uniform k

Decoding (separate)



$$p\left(\boldsymbol{y}^0,\dots,\boldsymbol{y}^{r-1},\tilde{\boldsymbol{d}}^0,\dots,\tilde{\boldsymbol{d}}^{r-1},\boldsymbol{b},\boldsymbol{c}\right)$$

$$=p\left(\boldsymbol{y}^0,\dots,\boldsymbol{y}^{r-1}\middle|\tilde{\boldsymbol{d}}^0,\dots,\tilde{\boldsymbol{d}}^{r-1},\boldsymbol{b},\boldsymbol{c}\right)p\left(\tilde{\boldsymbol{d}}^0,\dots,\tilde{\boldsymbol{d}}^{r-1}\right)p\left(\boldsymbol{b}|\boldsymbol{c}\right)p\left(\boldsymbol{c}\right)$$

$$=p\left(\boldsymbol{b}|\boldsymbol{c}\right)p\left(\boldsymbol{c}\right)\prod_{j=0}^{r-1}p\left(\boldsymbol{y}^j\middle|\boldsymbol{b}^j,\tilde{\boldsymbol{d}}^j\right)p\left(\tilde{\boldsymbol{d}}^j\right)$$

$$\simeq\left(\prod_{i=0}^{n-1}p\left(\boldsymbol{b}_i|\boldsymbol{c}_i\right)p\left(\boldsymbol{c}_i\right)\right)\prod_{j=0}^{r-1}\left(p(\tilde{d}_0^j)\prod_{i=0}^{n-1}p\left([\boldsymbol{y}^j]_{i\nu+\tilde{d}_i^j}^{i\nu+\nu-1+\tilde{d}_{i+1}^j}\middle|\boldsymbol{b}_i,\tilde{d}_i^j,\tilde{d}_{i+1}^j,\right)p\left(\tilde{d}_{i+1}^j\middle|\tilde{d}_i^j\right)\right)$$

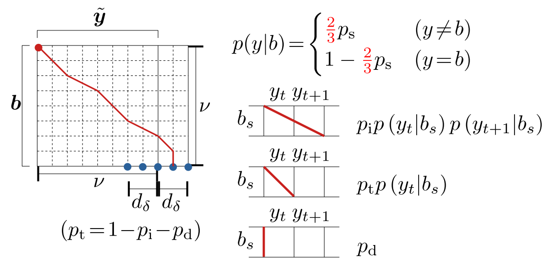
$$\propto\left(\prod_{i=0}^{n-1}p\left(\boldsymbol{b}_i|\boldsymbol{c}_i\right)p\left(\boldsymbol{c}_i\right)\right)\prod_{j=0}^{r-1}\left(p(\tilde{d}_0^j)\prod_{i=0}^{n-1}p\left([\tilde{\boldsymbol{y}}^j]_{i\nu+\tilde{d}_i^j}^{i\nu+\nu-1+\tilde{d}_{i+1}^j}\middle|\boldsymbol{b}_i,\tilde{d}_i^j,\tilde{d}_{i+1}^j,\right)p\left(\tilde{d}_{i+1}^j\middle|\tilde{d}_i^j\right)\right)$$

(sim)

$$(\tilde{\boldsymbol{y}}^j=f_{\mathrm{d}}(\boldsymbol{y}^j)\in\mathbb{B}^{N_j})$$

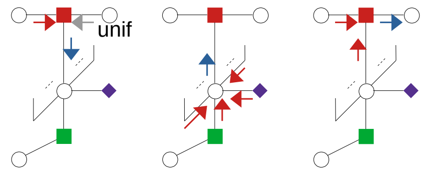
```
PF[Nseq][Ns+1][D]
PB[Nseq][Ns+1][D]
PD[Nseq][Ns][Q]
PU[Nseq][Ns][Q]
PI[Ns][Q]
PO[Ns][Q]
PM[Ns][Q]
PU0[Ns][Q]
```

LUT

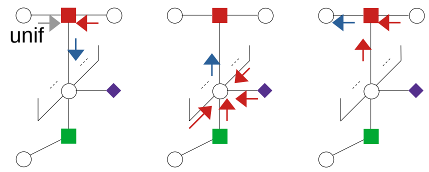


Scheduling

1. forward

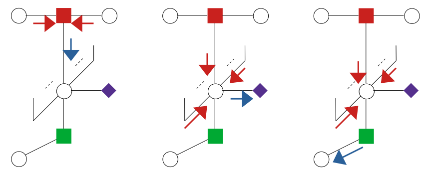


2. backward

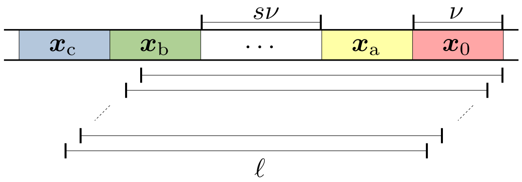


3. store

4. output



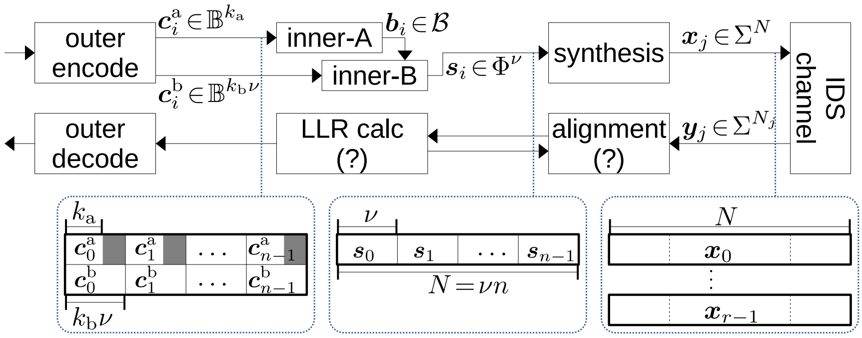
Encoding channel matrix (binary)



(1) $\ell = \nu$

8 10

(A,T,G,C)
(0,1,2,3)



$i \in [n]$
$j \in [r]$
$N = \nu n$

Code length: ν (even)
Binary constraint vector set: $\mathcal{B} \subset \mathbb{B}^\nu$ $\quad (|\mathcal{B}| \geq 2^{k_{\rm a}})$
Composite symbol set:

$$\Phi_0 \subseteq \left\{(\sigma_0,\sigma_1,\sigma_2,\sigma_3) \in \mathbb{Z}_{k+1}^4 \,\middle|\, \sigma_0+\sigma_1=k, \sigma_2=\sigma_3=0\right\} \text{ AT}$$

$$\Phi_1 \subseteq \left\{(\sigma_0,\sigma_1,\sigma_2,\sigma_3) \in \mathbb{Z}_{k+1}^4 \,\middle|\, \sigma_0=\sigma_1=0, \sigma_2+\sigma_3=k\right\} \text{ GC}$$

$$|\Phi_0| = |\Phi_1| = 2^{k_{\rm b}} \leq k+1 \qquad \Phi = \Phi_0 \cup \Phi_1$$

Inner codebook:

$$\mathcal{C} \subseteq \left\{(c_0,\ldots,c_{\nu-1}) \,\middle|\, c_j \in \Phi_{b_j}, (b_0,\ldots,b_{\nu-1}) \in \mathcal{B}\right\}$$

Encode:

$f_{\rm e}^{\rm a} : \mathbb{B}^{k_{\rm a}} \to \mathcal{B}$ (lossy)

$f_{\rm e}^{i} : \mathbb{B}^{k_{\rm b}} \to \Phi_i \;(i \in \mathbb{B})$ (bijection)

$$f_{\rm e}(c_i^{\rm a}, c_i^{\rm b}) = s_i = (s_{i,0},\ldots,s_{i,\nu-1}) : \mathbb{B}^{k_{\rm a}} \times \mathbb{B}^{k_{\rm b}\nu} \to \Phi^\nu$$

$$s_{i,j} = f_{\rm e}^{b_i}(c_{i,j}^{\rm b}) \qquad (b_0,\ldots,b_{\nu-1}) = f_{\rm e}^{\rm a}(c_i^{\rm a})$$

$$c_{i,j}^{\rm b} = [c_i^{\rm b}]_{j\nu}^{j\nu+\nu-1}$$

(example) $\nu=6, |\mathcal{B}|=16, k=7, k_{\rm a}=4, k_{\rm b}=3,$

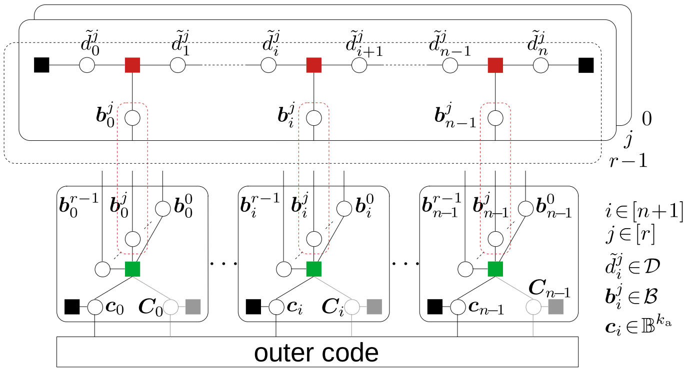
$\mathcal{B}$:	0:001011	8:110100	Φ_0: 0:0700	Φ_1: 0:0007
	1:001101	9:110010	1:1600	1:0016
	2:001110	A:110001	2:2500	2:0025
	3:010011	B:101100	3:3400	3:0034
	-:010101	-:101010	4:4300	4:0043
	4:010110	C:101001	5:5200	5:0052
	5:011001	D:100110	6:6100	6:0061
	6:011010	E:100101	7:7000	7:0070
	7:011100	F:100011		

Lee dist?

Decode:

Decoding (separate)

[approximation?]



$$p\left(\boldsymbol{y}^0,\dots,\boldsymbol{y}^{r-1},\boldsymbol{b}^0,\dots,\boldsymbol{b}^{r-1},\tilde{\boldsymbol{d}}^0,\dots,\tilde{\boldsymbol{d}}^{r-1},\boldsymbol{c}\right)$$

$$= p\left(\boldsymbol{y}^0,\dots,\boldsymbol{y}^{r-1}\middle|\boldsymbol{b}^0,\dots,\boldsymbol{b}^{r-1},\tilde{\boldsymbol{d}}^0,\dots,\tilde{\boldsymbol{d}}^{r-1},\boldsymbol{c}\right) p\left(\tilde{\boldsymbol{d}}^0,\dots,\tilde{\boldsymbol{d}}^{r-1}\right) p\left(\boldsymbol{b}^0,\dots,\boldsymbol{b}^{r-1}\middle|\boldsymbol{c}\right) p\left(\boldsymbol{c}\right)$$

$$= p\left(\boldsymbol{c}\right)\prod_{j=0}^{r-1} p\left(\boldsymbol{y}^j\middle|\boldsymbol{b}^j,\tilde{\boldsymbol{d}}^j\right) p\left(\tilde{\boldsymbol{d}}^j\right) p\left(\boldsymbol{b}^j\middle|\boldsymbol{c}\right)$$

$$\simeq \left(\prod_{i=0}^{n-1} p\left(\boldsymbol{c}_i\right)\right)\prod_{j=0}^{r-1}\left(p(\tilde{d}_0^j)\prod_{i=0}^{n-1} p\left(\left[\boldsymbol{y}^j\right]_{i\nu+\tilde{d}_i^j}^{i\nu+\nu-1+\tilde{d}_{i+1}^j}\middle|\boldsymbol{b}_i^j,\tilde{d}_i^j,\tilde{d}_{i+1}^j,\right) p\left(\tilde{d}_{i+1}^j\middle|\tilde{d}_i^j\right) p\left(\boldsymbol{b}_i^j\middle|\boldsymbol{c}_i\right)\right)$$

$$\propto \left(\prod_{i=0}^{n-1} p\left(\boldsymbol{c}_i\right)\right)\underline{\prod_{j=0}^{r-1}}\left(p(\tilde{d}_0^j)\prod_{i=0}^{n-1} \underline{p\left(\left[\tilde{\boldsymbol{y}}^j\right]_{i\nu+\tilde{d}_i^j}^{i\nu+\nu-1+\tilde{d}_{i+1}^j}\middle|\boldsymbol{b}_i^j,\tilde{d}_i^j,\tilde{d}_{i+1}^j,\right) p\left(\tilde{d}_{i+1}^j\middle|\tilde{d}_i^j\right)}\, \underline{p\left(\boldsymbol{b}_i^j\middle|\boldsymbol{c}_i\right)}\right)$$

$\underline{\prod_{j=0}^{r-1} p\left(\boldsymbol{b}_i^j\middle|\boldsymbol{c}_i\right)}$:sim $\qquad \tilde{\boldsymbol{y}}^j = f_{\mathrm{d}}(\boldsymbol{y}^j)\in\mathbb{B}^{N_j}$