

Problem 1:

CS 325 - Homework 2

a)  $T(n) = 3T(n-1) + 1$  (Master Method)  $\rightarrow$  Case 3

$a=3, b=1, f(n)=1$   $a > 1$  so  $O(n^d a^{n/b})$

$n^d \Rightarrow 1^0 \Rightarrow O(1 \cdot 3^{n/1}) \Rightarrow \Theta(3^n)$  Asymptotic upper bound

b)  $T(n) = T(n-2) + 2$  (Master Method)

$a=1, b=2, f(n)=2$   $a=1$  so  $O(n^{d+1})$

$f(n) = n^{d+1}$   $d=0 \rightarrow f(n) = O(n^{0+1}) \rightarrow O(n)$

$f(n)$  is  $\Theta(n^{d+1})$  so we can conclude  $T(n)$  is  $\Theta(n)$

c)  $T(n) = 9T(\frac{n}{3}) + 6n^2$

$a=9, b=3, f(n)=6n^2$   
 $= \Theta(n^2)$

$\log_b a = \log_3 9 = 2$

$f(n) = \Theta(n^2)$

Case 2  $T(n) = \Theta(n^c \log n)$   
 $= \Theta(n^2 \log n)$

d)  $T(n) = 2T(\frac{n}{4}) + 2n^2$

$a=2, b=4, \log_b a = \frac{1}{2}$   $f(n) = 2n^2 \rightarrow \Theta(n^2)$

$n^{\log_b a} = n^{-0.5} \leq n^2$

Case 3  $T(n) = \Theta(n^{\log_b a + \epsilon})$

Regularity Condition check:

$2 \cdot \left(\frac{n^{1.5}}{4}\right) \leq \frac{1}{2} n^2$   $c = \frac{1}{2}$   $(c \leq \frac{1}{2}) \checkmark$

$\frac{n^{1.5}}{2} \leq \frac{n^2}{2} \rightarrow T(n) = \Theta(f(n)) \rightarrow \Theta(n^2)$

**Problem 2:**

- A) A ternary search is essentially a binary search but instead of splitting into 2 search groups, it uses 3 search groups (divides the array into 3 instead of 2). The pseudo-code would look like the following:

```
ternary(array, search)
    If (length < 1)
        return error
    If (length == 1)
        return array[0] == search ? true : false
    If (length == 2)
        For in loop
            If array[i] == search
                return true
            else
                return false

    T1 = n/3
    T2 = 2n/3
    if(array[0] >= search AND search < array[T1])
        return ternary(array[0:T1], search)
    if (array[T1] <= search AND search < array[T2])
        return ternary(array[T1:T2], search)
    if (array[T2] <= search AND search < A[n-1])
        return ternary(array[T2: n], search)
    return false
```

2b) The recurrence of ternary search is

$$T(n) = T\left(\frac{2n}{3}\right) + C(1) \quad \leftarrow 1 \text{ operation for division}$$

Worst case we make  $\frac{2n}{3}$  recursive calls.

Best case we make  $\frac{n}{3}$  recursive calls

2c)  $T(n) = T\left(\frac{2n}{3}\right) + 1$

$$a = 1 \quad b = \frac{3}{2} \quad c = 0$$

$$aT\left(\frac{n}{b}\right) + n^c$$

$$\log_b^a = \log_{(3/2)}^1 = 0 = c$$

Case 2:

$$T(n) = \Theta(n^0 \log n)$$

$$= \Theta(\log n)$$



Problem 3:

3a) Recurrence is  $3T\left(\frac{2n}{3}\right) + 1$

3b)  $T(n) = 3T\left(\frac{2n}{3}\right) + 1$

$a = 3$     $b = \frac{3}{2}$     $c = 1$

$\log_{(3/2)} 3 = \sim 2.709$

$2.709 > 1$

case I:  $T(n) = \Theta(n^{\log_{3/2} 3})$

$= \Theta(n^{2.709})$

#### Problem 4:

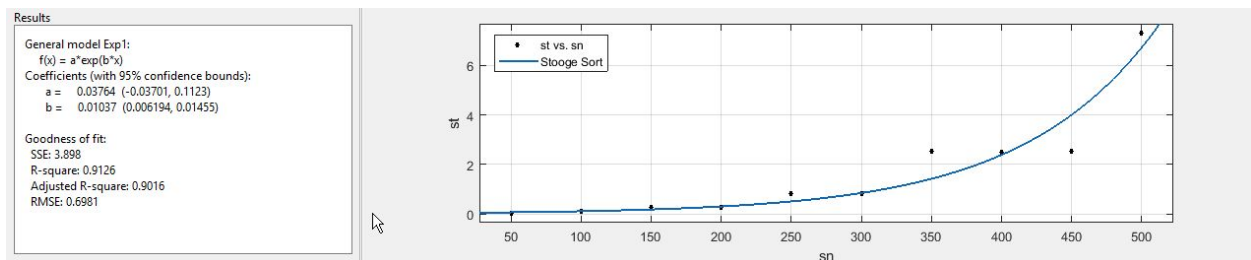
##### C) Run times on the FLIP school server

stoogeTime.py

N	T
50	0.0106010437
100	0.09317207336
150	0.2707030773
200	0.2714161873
250	0.8372240067
300	0.8203611374
350	2.522836208
400	2.479743004
450	2.523155928
500	7.320059061

##### D) Plot data and fit curve

Zoom in to view the details



##### E) Comparison

Stoogesort's experimental running time complexity is about  $n^{2.709}$  and the graph above is fit using an exponential curve ( $a \cdot \exp(b^n)$ ). There are no anomalies in comparison between the experimental running time and the tightest fitting equation/curve.

## F) Combine

insertTime (blue) vs mergeTime(red) vs stoogeTime(yellow)

