Exercises. Set 1.

1. List the cosets of the subgroups

(a) $8\mathbb{Z}(24) < \mathbb{Z}(24)$,

(b) $\langle 3 \rangle$ in $U(8)$

2. Let H and K be subgroups in a group G. Prove that $gH \cap gK$ is a coset of $H \cap K$ in G.

3. Suppose that G is a finite group with all nonidentical elements having orders 5 or 7. Prove that $|G| > 35$.

4. Prove or disprove: $U(8) \cong \mathbb{Z}(4)$.

5. Which of the groups $U(5), U(10), U(12)$ are isomorphic?

6. Prove that the groups $(\mathbb{Q}, +)$ and $(\mathbb{Z}, +)$ are not isomorphic.

7. Let $m, n \in \mathbb{Z}$. Prove that $\langle m, n \rangle = d\mathbb{Z}$, where $d = \gcd(m, n)$.

8. Find $\mathrm{Aut}\, \mathbb{Z}(6)$.

9. Find an order of the element $(3,4)$ in $\mathbb{Z}(4) \oplus \mathbb{Z}(6)$.

10. Prove that a group of order 12 must have an element of order 2.