Exercise Set 2.

1. Let R be a commutative simple ring. Prove that either R is a field or $R = \mathbb{Z}(p)$ with zero multiplication.

2. Let $F = \mathbb{Z}(5)$. In $F[t]$ let $I = (x^2+x+2)$. Find the multiplicative inverse of $2x+3+I$ in $F[t]/I$.

3. Let $\mathbb{Z}[i] = \mathbb{Z} + \mathbb{Z}i$, $i^2 = -1$, be the ring of Gaussian integers. Consider the ideal $I = (2+i)\mathbb{Z}[i]$. Prove that $\mathbb{Z}[i]/I$ is a field. What are the characteristic and order of this field?

4. Find infinitely many polynomials in $\mathbb{Z}(3)[t]$ such that $f(a) = 0$ for all a in $\mathbb{Z}(3)$.

5. Construct a field of order 27.

6. Write $t^3+t^2+t+1 \in \mathbb{Z}(2)[t]$ as a product of irreducible polynomials over $\mathbb{Z}(2)$.

7. Show that for every prime p there exists a field of order p^2.

8. Show that for every prime p such that $p = 1 \bmod 3$ there exists a field of order p^3.