Tutorial 4 - Autoencoders

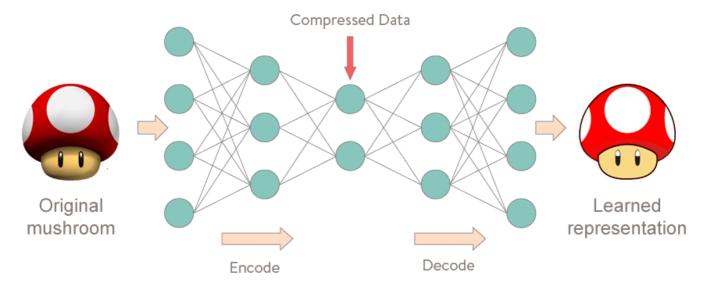
Autoencoder

An autoencoder is not used for *supervised learning*. We will no longer try to *predict* something about our input. Instead, an autoencoder is considered a **generative model**: it learns a distributed *representation* of our training data, and can even be used to generate new instances of the training data.

An autoencoder model contains two components:

- An encoder that takes an image as input, and outputs a low-dimensional embedding (representation) of the image.
- A decoder that takes the low-dimensional embedding, and reconstructs the image.

An autoencoder is typically shown like below:



Autoencoders contain an encode stage which is similar to what we have seen with our ANNs and CNNs, followed by a decode stage which is just the reverse of the encode stage. In presenting the architecture of autoencoders we will try to use code that we have seen when working with MNIST data.

```
import torch
import torch.nn as nn
import torch.nn.functional as F
import torch.optim as optim
import matplotlib.pyplot as plt
from torchvision import datasets, transforms

mnist_data = datasets.MNIST('data', train=True, download=True, transform=transforms.ToTensor(
mnist_data = list(mnist_data)[:4096]
```

```
Downloading <a href="http://yann.lecun.com/exdb/mnist/train-images-idx3-ubyte.gz">http://yann.lecun.com/exdb/mnist/train-images-idx3-ubyte.gz</a>
Downloading http://yann.lecun.com/exdb/mnist/train-images-idx3-ubyte.gz to data/MNIST/raw/tra
                                                      9913344/? [00:01<00:00, 11512510.78it/s]
Extracting data/MNIST/raw/train-images-idx3-ubyte.gz to data/MNIST/raw
Downloading http://yann.lecun.com/exdb/mnist/train-labels-idx1-ubyte.gz
Downloading <a href="http://yann.lecun.com/exdb/mnist/train-labels-idx1-ubyte.gz">http://yann.lecun.com/exdb/mnist/train-labels-idx1-ubyte.gz</a> to data/MNIST/raw/tra
                                                      29696/? [00:00<00:00, 312195.28it/s]
Extracting data/MNIST/raw/train-labels-idx1-ubyte.gz to data/MNIST/raw
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Downloading <a href="http://yann.lecun.com/exdb/mnist/t10k-images-idx3-ubyte.gz">http://yann.lecun.com/exdb/mnist/t10k-images-idx3-ubyte.gz</a> to data/MNIST/raw/t10k
                                                      1649664/? [00:00<00:00, 3299352.36it/s]
Extracting data/MNIST/raw/t10k-images-idx3-ubyte.gz to data/MNIST/raw
Downloading http://yann.lecun.com/exdb/mnist/t10k-labels-idx1-ubyte.gz
Downloading http://yann.lecun.com/exdb/mnist/t10k-labels-idx1-ubyte.gz to data/MNIST/raw/t10k
                                                      5120/? [00:00<00:00, 59726.98it/s]
Extracting data/MNIST/raw/t10k-labels-idx1-ubvte.gz to data/MNIST/raw
```

Architecture

The architecture is very similar to what we have seen in the past, except now the output will be the same size as the input. Notice also that we apply a sigmoid on the output data, this is to scale the output from 0 to 1.

```
class Autoencoder(nn.Module):
    def __init__(self):
        super(Autoencoder, self).__init__()
        encoding_dim = 32
        # encoder
        self.fc1 = nn.Linear(28 * 28, encoding_dim)
        # decoder
        self.fc2 = nn.Linear(encoding_dim, 28*28)

def forward(self, img):
    flattened = img.view(-1, 28 * 28)
        x = F.relu(self.fc1(flattened))
        # sigmoid for scaling output from 0 to 1
        x = F.sigmoid(self.fc2(x))
        return x
```

Training an Autoencoder

How do we train an autoencoder? How do we know what kind of "encoder" and "decoder" we want?

One observation is that if we pass an image through the encoder, then pass the result through the decoder, we should get roughly the same image back. Ideally, reducing the dimensionality and then generating the image should give us the same result.

This observation provides us a training strategy: we will minimize the reconstruction error of the autoencoder across our training data. We use a loss function called 'MSELoss', which computes the square error at every pixel.

Beyond using a different loss function, the training scheme is roughly the same. Note that in the code below, we are using a the optimizer called 'Adam'.

We switched to this optimizer not because it is specifically used for autoencoders, but because this is the optimizer that people tend to use in practice. Feel free to use Adam for your other neural networks.

```
def train(model, num epochs=5, batch size=64, learning rate=1e-3):
       torch. manual seed (42)
       criterion = nn. MSELoss() # mean square error loss
       optimizer = torch. optim. Adam (model. parameters (),
                                                                  lr=learning_rate,
                                                                  weight_decay=1e-5) # <--
       train loader = torch.utils.data.DataLoader(mnist data,
                                                                                              batc
                                                                                              shuf
       outputs = []
       for epoch in range (num_epochs):
               for data in train loader:
                       img, _ = data
                       recon = model(img)
                        img = img. view(-1, 28 * 28)
                       loss = criterion(recon, img)
                       loss.backward()
                       optimizer.step()
                       optimizer.zero_grad()
                print('Epoch:{}, Loss:{:.4f}'.format(epoch+1, float(loss)))
               outputs.append((epoch, img, recon),)
       return outputs
model = Autoencoder()
\max \text{ epochs} = 20
outputs = train(model, num epochs=max epochs)
     /usr/local/lib/python3.7/dist-packages/torch/nn/functional.py:1944: UserWarning: nn.functiona
       warnings.warn("nn.functional.sigmoid is deprecated. Use torch.sigmoid instead.")
     Epoch: 1, Loss: 0.0685
     Epoch: 2, Loss: 0.0608
     Epoch: 3, Loss: 0.0496
     Epoch: 4, Loss: 0.0459
     Epoch: 5, Loss: 0.0420
     Epoch: 6, Loss: 0.0372
     Epoch: 7, Loss: 0.0383
```

```
Epoch:8, Loss:0.0348
Epoch:9, Loss:0.0325
Epoch:10, Loss:0.0301
Epoch:11, Loss:0.0247
Epoch:12, Loss:0.0259
Epoch:13, Loss:0.0232
Epoch:14, Loss:0.0232
Epoch:15, Loss:0.0211
Epoch:16, Loss:0.0206
Epoch:17, Loss:0.0196
Epoch:18, Loss:0.0185
Epoch:19, Loss:0.0188
Epoch:20, Loss:0.0174
```

Just like with our ANN we can have additional layers to make a deep autoencoder, also known as a stacked autoencoder.

Convolutional Autoencoder

When working with image data it is often better to use a convolutional neural network and take advantage of the spatial relationships. The architecture for the encoder stage of a convolutional autoencoder will consist of standard convolutional layers that we have seen in our previous architectures. The decoder step will be a bit more tricky since we need a way to increase the resolution.

We need something akin to convolution, but that goes in the *opposite* direction. We will use something called a **transpose convolution**. Transpose convolutions were first called *deconvolutions*, since it is the ``inverse" of a convolution operation. However, the terminology was confusing since it has nothing to do with the mathematical notion of deconvolution.

Convolution Transpose

First, let's illustrate how convolution transposes can be "inverses" of convolution layers. We begin by creating a convolutional layer in PyTorch. This is the convolution that we will try to find an "inverse" for.

To illustrate how convolutional layers work, we'll create a random tensor and see how the convolution acts on that tensor:

```
x = torch.randn(2, 8, 64, 64)
y = conv(x)
y.shape
torch.Size([2, 8, 60, 60])
```

A convolution transpose layer with the exact same specifications as above would have the "reverse" effect on the shape.

And it does! Notice that the weights of this convolution transpose layer are all random, and are unrelated to the weights of the original $\operatorname{Conv2d}$. So, the layer convt is not the mathematical inverse of the layer conv . However, with training, the convolution transpose has the potential to learn to act as an approximate inverse to conv .

Here is another example of convt in action:

```
x = torch.randn(32, 8, 64, 64)
y = convt(x)
y.shape
torch.Size([32, 8, 68, 68])
```

Notice that the width and height of y is 68x68, because the $kernel_size$ is 5 and we have not added any padding. You can verify that if we start with a tensor with resolution 68x68 and applied a 5x5 convolution, we would end up with a tensor with resolution 64x64.

As before, we can add a padding to our convolution transpose, just like we added padding to our convolution operations:

More interestingly, we can add a stride to the convolution to increase our resolution!

Our resolution has doubled.

But what is actually happening? Essentially, we are adding a padding of zeros in between every row and every column of $\, x$.

Implementation of a Convolutional Autoencoder

To demonstrate the use of convolution transpose operations, we will build a **convolutional autoencoder**. Below is an example of a *convolutional* autoencoder that uses solely convolutional layers:

```
class Autoencoder (nn. Module):
       def __init__(self):
               super (Autoencoder, self). init ()
               self.encoder = nn.Sequential( # like the Composition layer you built
                       nn. Conv2d(1, 16, 3, stride=2,
                                                        padding=1),
                       nn. ReLU(),
                       nn. Conv2d(16, 32, 3, stride=2, padding=1),
                       nn. ReLU(),
                       nn. Conv2d (32, 64, 7)
               self.decoder = nn.Sequential(
                       nn. ConvTranspose2d (64,
                                              32, 7),
                       nn. ReLU(),
                       nn. ConvTranspose2d (32, 16, 3,
                                                      stride=2, padding=1, output_padding=1),
                       nn. ReLU(),
                       nn. ConvTranspose2d(16, 1, 3, stride=2, padding=1, output padding=1),
                       nn. Sigmoid()
               )
       def forward(self, x):
               x = self.encoder(x)
               x = self. decoder(x)
               return x
#encoder
nn. Conv2d(1, 16, 3, stride=2, padding=1),
nn. Conv2d(16, 32, 3, stride=2, padding=1),
nn. Conv2d (32, 64,
                  7)
#decoder
nn. ConvTranspose2d (64,
                       32, 7),
nn. ConvTranspose2d(32, 16, 3, stride=2, padding=1, output padding=1),
nn. ConvTranspose2d(16, 1, 3, stride=2, padding=1, output padding=1),
     (ConvTranspose2d(16, 1, kernel size=(3, 3), stride=(2, 2), padding=(1, 1), output padding=(1,
     1)),)
```

Training a Convolutional Autoencoder

The training of the convolutional autoencoder will be the same as with the fully-connected autoencoder architecture we introduced in the beginning.

```
def train(model, num_epochs=5, batch_size=64, learning_rate=1e-3):
       torch.manual_seed(42)
       criterion = nn. MSELoss() # mean square error loss
       optimizer = torch.optim.Adam(model.parameters(),
                                                               1r=learning_rate,
                                                               weight decay=1e-5)
       train loader = torch.utils.data.DataLoader(mnist data,
                                                                                           batc
                                                                                           shuf
       outputs = []
       for epoch in range (num_epochs):
               for data in train_loader:
                       img, _ = data
                      recon = model(img)
                       loss = criterion(recon, img)
                       loss.backward()
                       optimizer.step()
                       optimizer.zero_grad()
               print('Epoch:{}, Loss:{:.4f}'.format(epoch+1, float(loss)))
               outputs.append((epoch, img, recon),)
       return outputs
```

Now, we can train this network.

```
model = Autoencoder()
max_epochs = 20
outputs = train(model, num_epochs=max_epochs)
     Epoch: 1, Loss: 0.0686
     Epoch: 2, Loss: 0.0675
     Epoch: 3, Loss: 0.0578
     Epoch: 4, Loss: 0.0424
     Epoch: 5, Loss: 0.0318
     Epoch: 6, Loss: 0.0235
     Epoch: 7, Loss: 0.0191
      Epoch: 8, Loss: 0.0173
     Epoch: 9, Loss: 0.0159
     Epoch: 10, Loss: 0.0124
     Epoch:11, Loss:0.0099
     Epoch: 12, Loss: 0.0098
     Epoch: 13, Loss: 0.0113
     Epoch: 14, Loss: 0.0090
     Epoch: 15, Loss: 0.0081
      Epoch: 16, Loss: 0.0085
     Epoch: 17, Loss: 0.0076
     Epoch: 18, Loss: 0.0070
     Epoch: 19, Loss: 0.0067
      Epoch: 20, Loss: 0.0068
```

The loss goes down as we train, meaning that our reconstructed images look more and more like the actual images!

Let's look at the training progression: that is, the reconstructed images at various points of training:

```
for k in range(0, max_epochs,
       plt.figure(figsize=(9,
                             2))
       imgs = outputs[k][1].detach().numpy()
       recon = outputs[k][2].detach().numpy()
       for i, item in enumerate(imgs):
               if i >= 9: break
               plt. subplot (2, 9, i+1)
               plt.imshow(item[0])
       for i, item in enumerate (recon):
               if i >= 9: break
               plt. subplot (2, 9, 9+i+1)
               plt.imshow(item[0])
       0
      20
       0
      20
      20
       0
      20
       0
      20
       0
```

At first, the reconstructed images look nothing like the originals. Rather, the reconstructions look more like the average of some training images. As training progresses, our reconstructions are clearer.

25 0

25 0

Denoising Autoencoder

We can add noise to our data and see if we can train an autoencoder to clean out the noise added to our images.

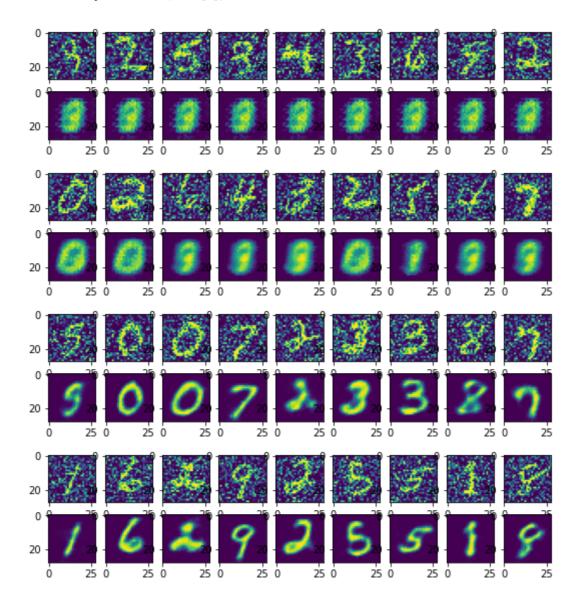
```
def train(model, num epochs=5, batch size=64, learning rate=1e-3):
       torch. manual seed (42)
       criterion = nn.MSELoss() # mean square error loss
       optimizer = torch. optim. Adam (model. parameters (),
                                                                 1r=learning rate,
                                                                 weight_decay=1e-5)
       train_loader = torch.utils.data.DataLoader(mnist_data,
       noise = 0.5
       outputs = []
       for epoch in range (num epochs):
               for data in train_loader:
                       img, = data
                       img_noisy = img + noise * torch.randn(*img.shape)
                       img_noisy = np.clip(img_noisy, 0., 1.)
                       recon = model(img_noisy)
                       \#img = img. view(-1, 28 * 28)
                       loss = criterion(recon, img)
                       loss.backward()
                       optimizer. step()
                       optimizer.zero grad()
               print('Epoch:{}, Loss:{:.4f}'.format(epoch+1, float(loss)))
               outputs.append((epoch, img noisy, recon),)
       return outputs
import numpy as np
# train denoising autoencoder
model = Autoencoder()
max epochs = 20
outputs = train(model, num epochs=max epochs)
     Epoch: 1, Loss: 0.0679
     Epoch: 2, Loss: 0.0610
     Epoch: 3, Loss: 0.0696
     Epoch: 4, Loss: 0.0690
     Epoch: 5, Loss: 0.0666
     Epoch: 6, Loss: 0.0618
     Epoch: 7, Loss: 0.0512
     Epoch: 8, Loss: 0.0378
     Epoch: 9, Loss: 0.0298
     Epoch: 10, Loss: 0.0292
     Epoch:11, Loss:0.0244
     Epoch: 12, Loss: 0.0238
```

batc shuf

```
Epoch:13, Loss:0.0219
Epoch:14, Loss:0.0195
Epoch:15, Loss:0.0174
Epoch:16, Loss:0.0197
Epoch:17, Loss:0.0172
Epoch:18, Loss:0.0180
Epoch:19, Loss:0.0166
Epoch:20, Loss:0.0173
```

reconstructed images at various parts of training
for k in range(0, max_epochs, 5):
 plt.figure(figsize=(9, 2))
 imgs = outputs[k][1].detach().numpy()
 recon = outputs[k][2].detach().numpy()
 for i, item in enumerate(imgs):
 if i >= 9: break
 plt.subplot(2, 9, i+1)
 plt.imshow(item[0])

for i, item in enumerate(recon):
 if i >= 9: break
 plt.subplot(2, 9, 9+i+1)
 plt.imshow(item[0])



Testing on new images

```
batch size = 64
mnist_data = datasets.MNIST('data', train=True, download=True, transform=transforms.ToTensor()
mnist_data = list(mnist_data)[4096:4160]
test loader = torch.utils.data.DataLoader(mnist data,
# obtain one batch of test images
dataiter = iter(test loader)
img, labels = dataiter.next()
# add noise to the test images
noise = 0.5
img_noisy = img + noise * torch.randn(*img.shape)
img_noisy = np.clip(img_noisy, 0., 1.)
# get sample outputs
recon = model(img_noisy)
# prep images for display
img noisy = img noisy.numpy()
recon = recon. detach(). numpy()
# reconstructed images at various parts of training
for k in range(1):
       plt.figure(figsize=(9, 2))
       for i, item in enumerate(img noisy):
               if i \ge 9: break
               plt. subplot (2, 9, i+1)
               plt.imshow(item[0])
       for i, item in enumerate (recon):
               if i \ge 9: break
               plt. subplot (2, 9, 9+i+1)
               plt.imshow(item[0])
```

Autoencoders are well suited for extracting compressed representations of images and can correct things that don't match the expectation. This approach can be extended to other applications such as handling object occlusion, or filling in missing segments of an image.

batch size=k shuffle=Tru€

Structure in the Embeddings

Since we are drastically reducing the dimensionality of the image, there has to be some kind of structure in the embedding space. That is, the network should be able to "save" space by mapping similar images to similar embeddings.

We will demonstrate the structure of the embedding space by having some fun with our autoencoders. Let's begin with two images in our training set. For now, we'll choose images of the same digit.

First load pre-denoising autoencoder architecture and training although you could also do this with the denosing autoencoder.

```
import torch
import torch.nn as nn
import torch.nn.functional as F
import torch.optim as optim
import matplotlib.pyplot as plt
from torchvision import datasets, transforms
mnist_data = datasets.MNIST('data', train=True, download=True, transform=transforms.ToTensor(
mnist data = list(mnist data)[:4096]
class Autoencoder (nn. Module):
       def __init__(self):
               super(Autoencoder, self).__init__()
               self.encoder = nn.Sequential(
                       nn. Conv2d(1, 16, 3, stride=2, padding=1),
                       nn. ReLU(),
                       nn. Conv2d(16, 32, 3, stride=2, padding=1),
                       nn. ReLU(),
                       nn. Conv2d (32, 64, 7)
               )
               self.decoder = nn.Sequential(
                       nn. ConvTranspose2d(64, 32,
                                                  7),
                       nn. ReLU(),
                       nn. ConvTranspose2d(32, 16, 3,
                                                      stride=2, padding=1, output_padding=1),
                       nn. ReLU(),
                       nn.ConvTranspose2d(16, 1, 3, stride=2, padding=1, output_padding=1),
                       nn.Sigmoid()
               )
       def forward(self, x):
               x = self.encoder(x)
               x = self. decoder(x)
               return x
def train(model, num epochs=5, batch size=64, learning rate=1e-3):
       torch. manual seed (42)
       criterion = nn.MSELoss() # mean square error loss
       optimizer = torch. optim. Adam (model. parameters (),
                                                                1r=learning rate,
                                                                weight decay=1e-5) # <--
       train loader = torch.utils.data.DataLoader(mnist data,
                                                                                           batc
                                                                                           shuf
       outputs = []
       for epoch in range (num epochs):
               for data in train loader:
                       img, _{-} = data
                       recon = model(img)
                       loss = criterion(recon, img)
                       loss.backward()
                       optimizer.step()
                       optimizer.zero grad()
               print('Epoch:{}, Loss:{:.4f}'.format(epoch+1, float(loss)))
```

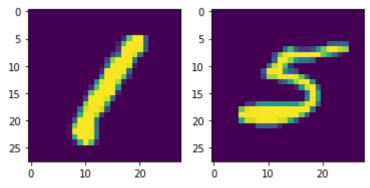
```
outputs. append ((epoch,
                                           img, recon),)
        return outputs
model = Autoencoder()
\max \text{ epochs} = 20
outputs = train(model,
                            num epochs=max epochs)
      Epoch: 1, Loss: 0.0670
     Epoch: 2, Loss: 0.0668
     Epoch: 3, Loss: 0.0492
      Epoch: 4, Loss: 0.0329
      Epoch: 5, Loss: 0.0249
     Epoch: 6, Loss: 0.0189
     Epoch: 7, Loss: 0.0162
     Epoch: 8, Loss: 0.0161
     Epoch: 9, Loss: 0.0150
     Epoch: 10, Loss: 0.0118
      Epoch: 11, Loss: 0, 0094
     Epoch: 12, Loss: 0.0098
      Epoch: 13, Loss: 0.0110
     Epoch: 14, Loss: 0.0087
     Epoch: 15, Loss: 0.0078
     Epoch: 16, Loss: 0.0085
      Epoch: 17, Loss: 0.0076
      Epoch: 18, Loss: 0.0071
      Epoch: 19, Loss: 0.0064
```

Output two sample images

Epoch: 20, Loss: 0.0066

```
imgs = outputs[max_epochs-1][1].detach().numpy()
plt.subplot(1, 2, 1)
plt.imshow(imgs[0][0])
plt.subplot(1, 2, 2)
plt.imshow(imgs[8][0])
```





We will then compute the **low-dimensional embeddings** of both images, by applying the **encoder**:

```
x1 = outputs[max_epochs-1][1][0,:,:,:] # first image
x2 = outputs[max_epochs-1][1][8,:,:,:] # second image
x = torch.stack([x1,x2]) # stack them together so we only call `encoder` once
embedding = model.encoder(x)
e1 = embedding[0] # embedding of first image
e2 = embedding[1] # embedding of second image
```

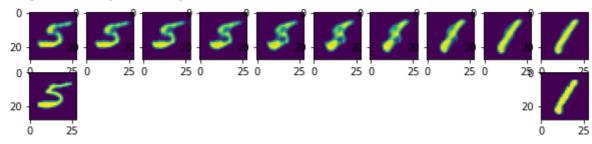
Now we will do something interesting. Not only are we going to run the decoder on those two embeddings $\,\mathrm{e}1\,$ and $\,\mathrm{e}2$, we are also going to **interpolate** between the two embeddings and decode those as well!

```
embedding_values = []
for i in range(0, 10):
        e = e1 * (i/10) + e2 * (10-i)/10
        embedding_values.append(e)
embedding_values = torch.stack(embedding_values)
recons = model.decoder(embedding_values)
```

Let's plot the reconstructions of each interpolated values. The original images are shown below too:

```
plt.figure(figsize=(10, 2))
for i, recon in enumerate(recons.detach().numpy()):
        plt.subplot(2, 10, i+1)
        plt.imshow(recon[0])
plt.subplot(2, 10, 11)
plt.imshow(imgs[8][0])
plt.subplot(2, 10, 20)
plt.subplot(2, 10, 20)
plt.imshow(imgs[0][0])
```



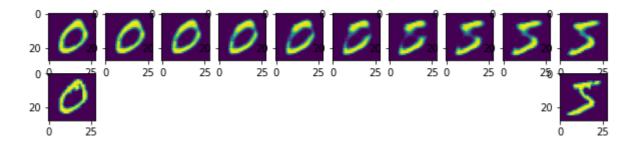


Notice that there is a smooth transition between the two images! The middle images are likely new, in that there are no training images that are exactly like any of the generated images.

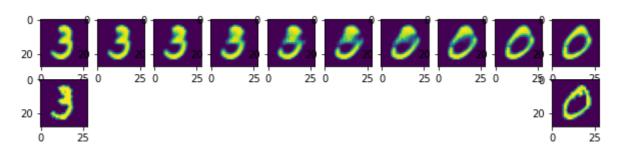
As promised, we can do the same thing with two images containing different digits. There should be a smooth transition between the two digits.

```
def interpolate(index1, index2):
       x1 = mnist_data[index1][0]
       x2 = mnist data[index2][0]
       x = torch. stack([x1, x2])
       embedding = model.encoder(x)
       e1 = embedding[0] # embedding of first image
              embedding[1] # embedding of second image
       embedding values = []
       for i in range (0, 10):
               e = e1 * (i/10) + e2 * (10-i)/10
               embedding_values.append(e)
       embedding_values = torch.stack(embedding_values)
       recons = model.decoder(embedding_values)
       plt.figure(figsize=(10, 2))
       for i, recon in enumerate (recons. detach(). numpy()):
               plt. subplot (2, 10, i+1)
               plt.imshow(recon[0])
       plt. subplot (2, 10, 11)
       plt. imshow(x2[0])
       plt. subplot (2, 10, 20)
       plt. imshow(x1[0])
```

interpolate(0, 1)



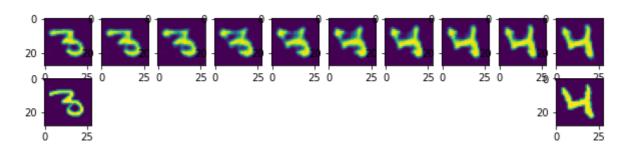
interpolate(1, 10)



interpolate(4, 5)



interpolate(20, 30)



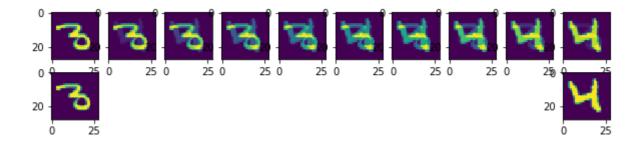
```
def interpolate_pixel(index1, index2):
    x1 = mnist_data[index1][0]
    x2 = mnist_data[index2][0]

interpolated_values = []
for i in range(0, 10):
    e = x1 * (i/10) + x2 * (10-i)/10
    interpolated_values.append(e)

plt.figure(figsize=(10, 2))
for i, recon in enumerate(interpolated_values):
    plt.subplot(2, 10, i+1)
    plt.imshow(recon[0])

plt.subplot(2, 10, 11)
plt.imshow(x2[0])
plt.subplot(2, 10, 20)
plt.imshow(x1[0])
```

interpolate pixel (20, 30)



What happens if we randomly initialize in the embedding space?

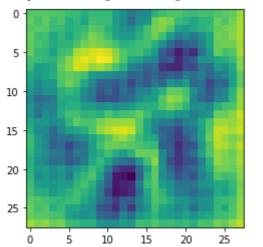
```
d = model.decoder(torch.randn(1, 64, 1, 1)).detach().numpy()
```

d. shape

(1, 1, 28, 28)

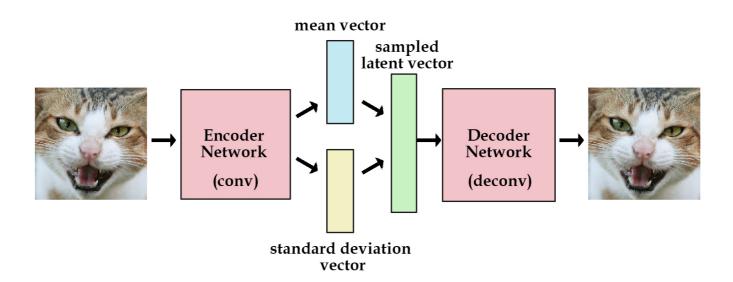
plt. imshow(d[0][0])

<matplotlib.image.AxesImage at 0x7fb29110c450>



The variational autoencoder will allow us to randomly initialize in the embedding space to generate new MNIST-like samples. Provided below is sample code showing how to do that.

Variational Autoencoder



To allow us to sample from the embedding space and generate new images, we add a constraint on the encoding network that forces it to generate latent vectors that roughly follow a unit Gaussian distribution. This constraint is what separates a variational autoencoder from the ones we've seen up until now.

Now generating new images requires that we sample a latent vector from the unit Gaussian and pass it into the decoder.

As shown in the figure, we will have encoding and decoding networks similar to what we used before, whether with fully-connected or convolutional layers. Then we add two additional linear

layers to hold the mean and standard deviation vectors of the embedding space. We will need some way to generate a sampled latent space which will act as input to the decoding network.

We will also need to update our loss function to use Kullback-Leibler divergence to constrain the embedding space to follow a unit Gaussian distribution. You will not be required to know the math behind this.

A demonstration of the variational autoencoder is provided below.

```
# dimensions of latent space
zdim = 25
# Variational Autoencoder
class Autoencoder (nn. Module):
       def init (self):
               super (Autoencoder, self). init ()
               # encoder
               self. fc1 = nn. Linear (28 * 28, 350)
               self.relu = nn.ReLU()
               self. fc2m = nn. Linear (350, zdim)
                                                   # mu layer
               self.fc2s = nn.Linear(350, zdim) # sd laver
               # decoder
               self. fc3 = nn. Linear(zdim, 350)
               self. fc4 = nn. Linear (350, 28 * 28)
               self.sigmoid = nn.Sigmoid()
       def encode(self, x):
              h1 = self. relu(self. fc1(x))
               return self. fc2m(h1), self. fc2s(h1)
       # reparameterize
       def reparameterize (self, mu, logvar):
               if self. training:
                      std = logvar.mul(0.5).exp_()
                      eps = std. data. new(std. size()). normal_()
                      return eps. mul(std).add (mu)
               else:
                      return mu
       def decode(self, z):
              h3 = self. relu(self. fc3(z))
               return self.sigmoid(self.fc4(h3))
       def forward(self, x):
               mu, logvar = self.encode(x.view(-1, 28 * 28))
               z = self.reparameterize(mu, logvar)
              return self.decode(z), mu, logvar
  loss function for VAE are unique and use Kullback-Leibler
  divergence measure to force distribution to match unit Gaussian
def loss_function(recon_x, x, mu, logvar):
       bce = F.binary_cross_entropy(recon_x, x.view(-1, 28 * 28))
       kld = -0.5 * torch.sum(1 + logvar - mu.pow(2) - logvar.exp())
       kld /= batch size * 28 * 28
       return bce + kld
```

```
def train(model, num_epochs = 1, batch_size = 64, learning_rate = 1e-3):
       model.train() #train mode so that we do reparameterization
       torch. manual seed (42)
       train_loader = torch.utils.data.DataLoader(datasets.MNIST('data',
                             train=True, download=True, transform=transforms. ToTensor()),
                             batch size = batch size, shuffle = True)
       optimizer = optim. Adam (model. parameters (), learning rate)
       for epoch in range (num_epochs):
            for data in train_loader:
                                           # load batch
                   img, _{-} = data
                   recon, mu, logvar = model(img)
                    loss = loss function(recon, img, mu, logvar) # calculate loss
                    loss.backward()
                     batch\_size = 64
model = Autoencoder()
train(model, num_epochs = 30, batch_size = batch_size)
     Epoch: 1, Loss: 0.1586
     Epoch: 2, Loss: 0.1418
     Epoch: 3, Loss: 0.1228
     Epoch: 4, Loss: 0.1224
     Epoch: 5, Loss: 0.1187
     Epoch: 6, Loss: 0.1104
     Epoch: 7, Loss: 0.1276
     Epoch: 8, Loss: 0.1221
     Epoch: 9, Loss: 0.1163
     Epoch: 10, Loss: 0.1218
     Epoch: 11, Loss: 0.1257
     Epoch: 12, Loss: 0.1127
     Epoch:13, Loss:0.1087
     Epoch: 14, Loss: 0.1192
     Epoch: 15, Loss: 0.1122
     Epoch:16, Loss:0.1083
     Epoch: 17, Loss: 0.1105
     Epoch: 18, Loss: 0.1139
     Epoch: 19, Loss: 0.1097
     Epoch: 20, Loss: 0.1169
     Epoch: 21, Loss: 0.1041
```