

Lecture 7: Revisiting DFA & NFA

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Outline

- Today: Revisiting RE & NFA & DFA
- High-level story: RegEx \rightarrow NFA \rightarrow DFA \rightarrow Table

Finite automata

- Regular Expressions \Leftrightarrow Specification
- Finite Automata \Leftrightarrow Implementation
- A finite automata formally consists of:
 - An input alphabet Σ
 - A set of states S
 - A start state n
 - A set of accepting states $F \subseteq S$
 - A set of transitions $\text{state} \xrightarrow{\text{input}} \text{state}$

Finite automata

- Transition $S_1 \xrightarrow{\alpha} S_2$
- This means: In state S_1 and input character α , go to state S_2
- If end of input and in accepting state \Rightarrow accept
- Otherwise \Rightarrow reject

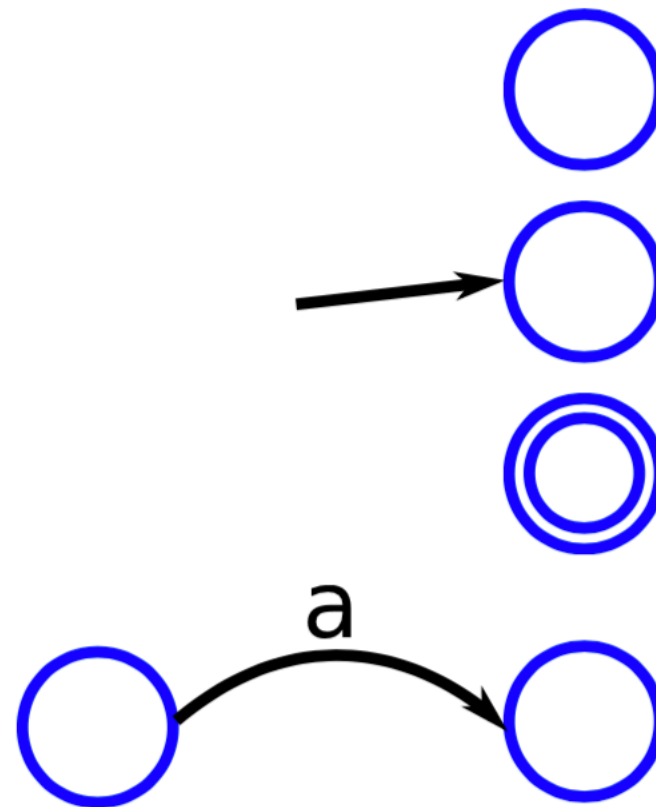
Finite Automata as State Graphs

A state:

The start state:

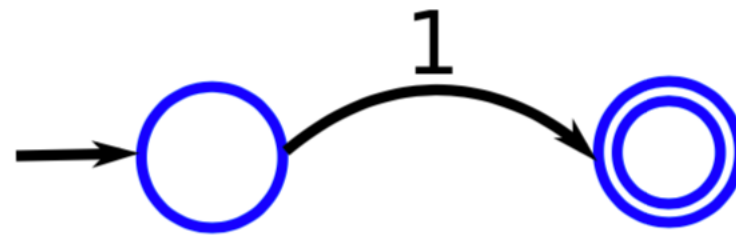
An accepting state:

A transition:



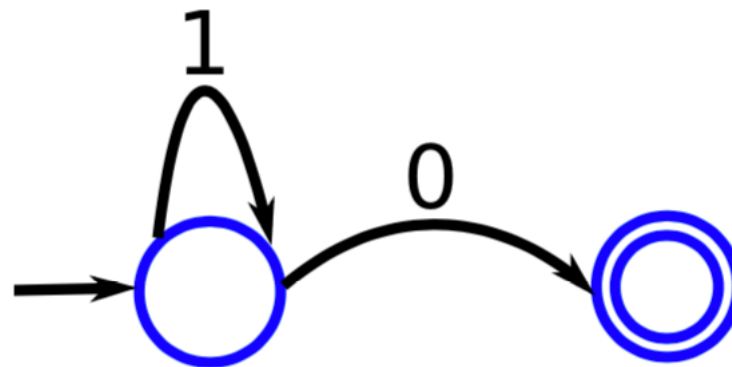
A simple example

- Here is an automaton that only accepts the string "1":



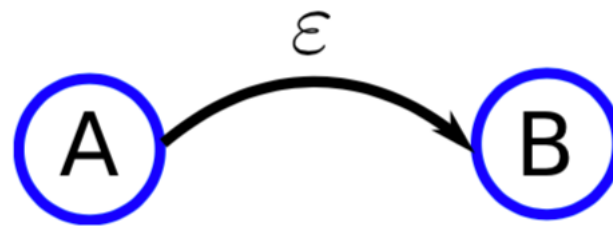
Another simple example

- A finite automaton accepting any number of 1's followed by a single 0
- Alphabet: $\{0,1\}$



Epsilon transitions

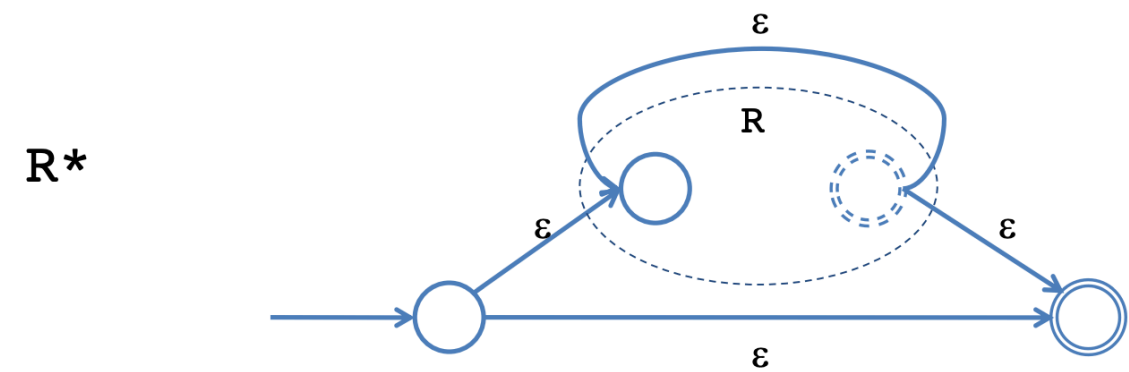
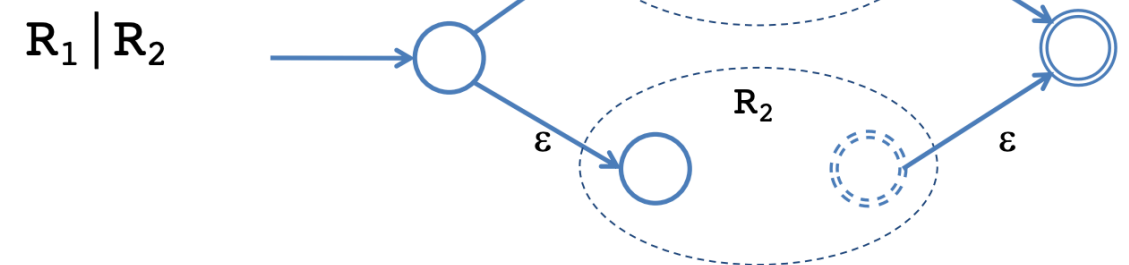
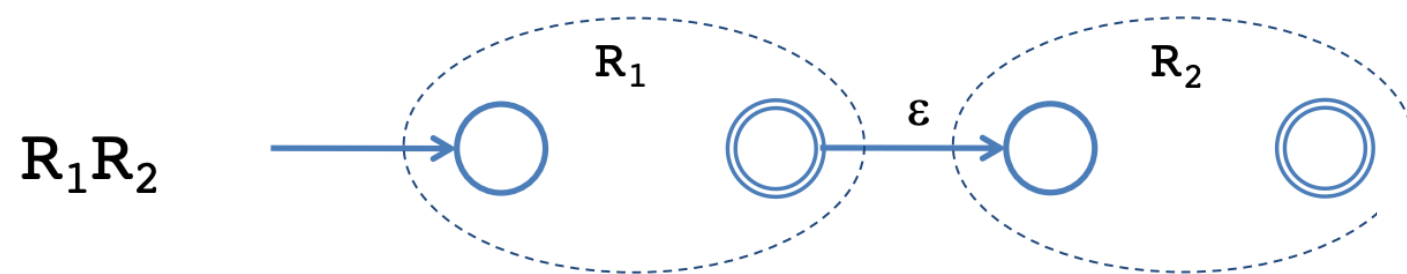
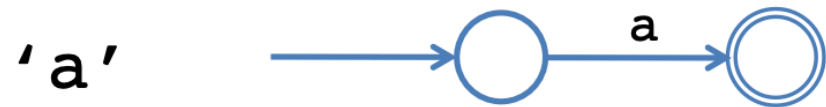
- A special kind of transition: ε -transitions
- Machine can move from state A to B without reading any input



Deterministic and Nondeterministic Automata

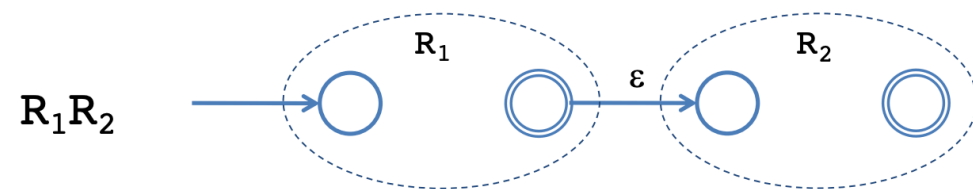
- Deterministic Finite Automata (DFA)
 - At most one transition per input on any state
 - No ϵ moves
- Nondeterministic Finite Automate (NFA)
 - Can have multiple transitions for one input in a given state
 - Can have ϵ -moves

RE to NFA

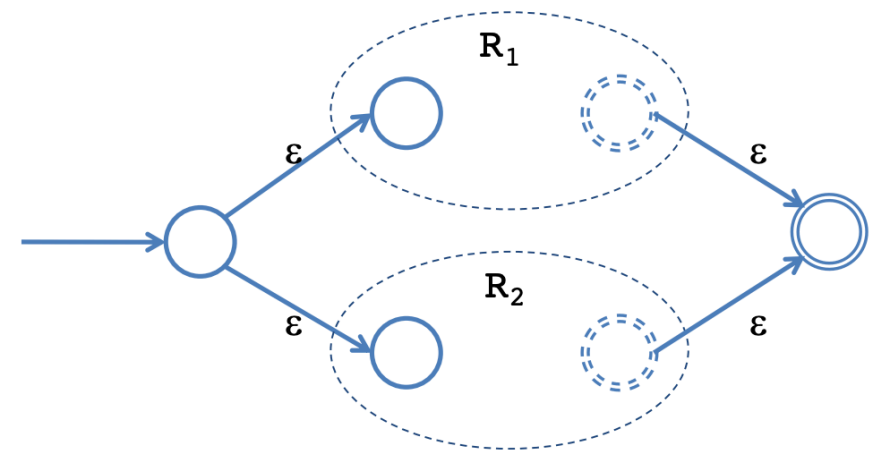


In-class exercise

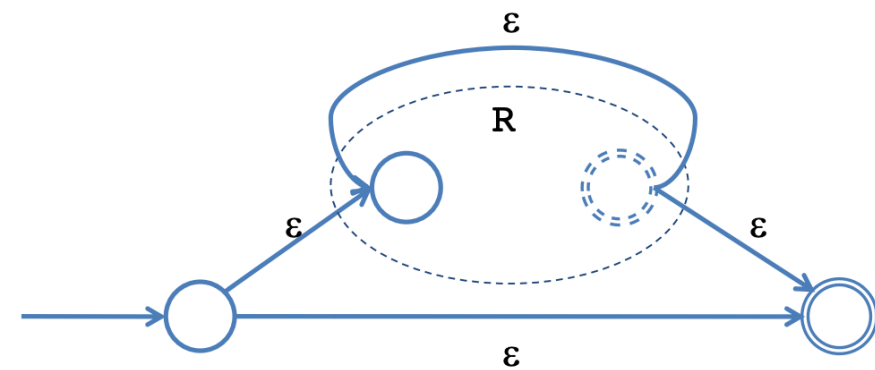
- Please draw the NFA for: $a(b \mid c)^*$



$R_1 \mid R_2$



R^*



NFA to DFA: The Algorithm

$q_0 \leftarrow \epsilon\text{-closure}(\{n_0\});$

$Q \leftarrow q_0;$

$WorkList \leftarrow \{q_0\};$

while ($WorkList \neq \emptyset$) *do*

remove q *from* $WorkList$;

for each character $c \in \Sigma$ *do*

$t \leftarrow \epsilon\text{-closure}(\Delta(q, c));$

$T[q, c] \leftarrow t;$

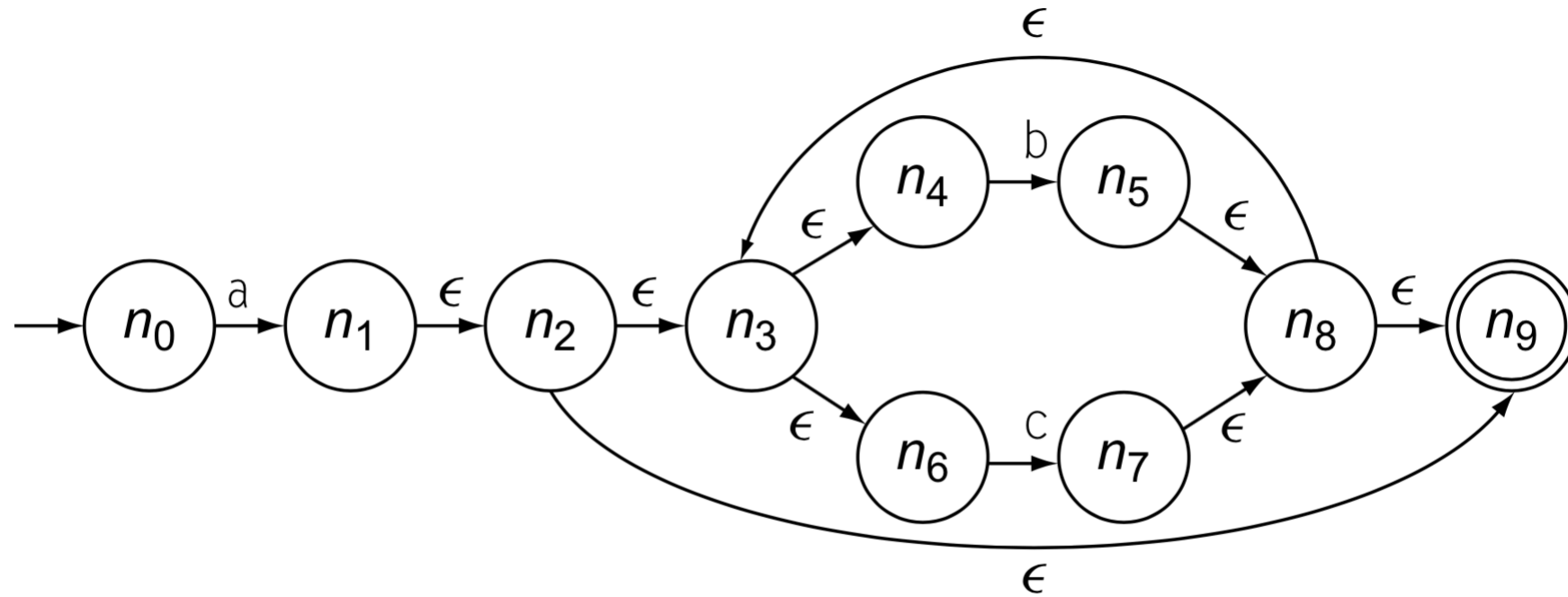
if $t \notin Q$ *then*

add t *to* Q *and to* $WorkList$;

end;

end;

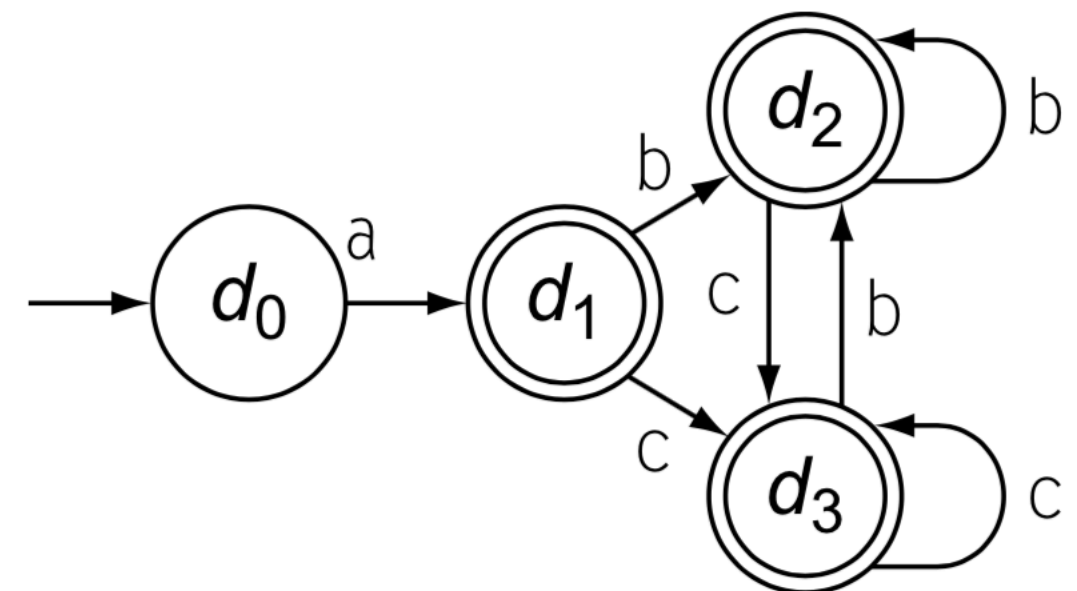
Apply NFA's
transition function to
each element of q



(a) NFA for “ $a(b \mid c)^*$ ” (With States Renumbered)

Set Name	DFA States	NFA States	$\epsilon\text{-closure}(\text{Delta}(q, *))$		
			a	b	c
q_0	d_0	n_0	$\{n_1, n_2, n_3, n_4, n_6, n_9\}$	– none –	– none –
q_1	d_1	$\{n_1, n_2, n_3, n_4, n_6, n_9\}$	– none –	$\{n_5, n_8, n_9, n_3, n_4, n_6\}$	$\{n_7, n_8, n_9, n_3, n_4, n_6\}$
q_2	d_2	$\{n_5, n_8, n_9, n_3, n_4, n_6\}$	– none –	q_2	q_3
q_3	d_3	$\{n_7, n_8, n_9, n_3, n_4, n_6\}$	– none –	q_2	q_3

(b) Iterations of the Subset Construction



(a) Resulting DFA

TODOs by next lecture

- Hw2 will be out. Get familiar with the Patina language
- Come to the discussion session if you have questions