

Lecture 12: Type Checking II

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Outline

- We will talk about types in Patina

Motivation

- When writing programs, everything is great as long as the program works.
- Unfortunately, this is usually not the case
- Programs crash, don't compute what we want them to compute, etc.
- This is arguably the **biggest problem** software faces today

Software correctness

- Problem: Rice's theorem. Any non-trivial property about a Turing machine is undecidable
- This means that we can never give an algorithm, that for all programs can decide if this program has an error on some inputs.
- What can we do?

Big idea

- Big Idea: Just because we cannot prove something about the original program does not mean we cannot prove something about an *abstraction of the program*.
- Strategy: In addition to the operational semantics, we will also define *abstract semantics* that will overapproximate the states a program is in.
- Example: In λ^+ , the operational semantics compute a concrete integer or list, while our abstract semantics only compute the if the result is of kind integer or list.

Abstraction

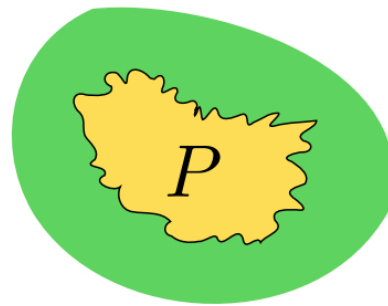
- Of course, any abstraction will be less precise than the program
- One popular abstraction: types
- Let's assume we have types Int and List
- Example: let $x = 10$ in x
- Operational semantics yield concrete value 10
- Abstract semantics that only differentiate the kind (or type) of the expression yield: Integer

Abstraction

- But we don't just want any abstraction, we need abstractions that *overapproximate* the result of the concrete program
- Recall the example: let $x = 10$ in x
- Abstract value *Integer* overapproximates 10 since 10 is a kind of integer
- On the other hand, abstract value *List* does not overapproximate 10.

Soundness

- The reason we only care about sound abstract semantics is the following:
- Theorem: If some abstract semantics are sound and an expression is of abstract value x , then its concrete value y is always part of the abstract value x .



- Why is this useful?
- This means that if a program has no error in the abstract semantics, it is guaranteed not to have an error in the concrete semantics.
- ASTREE tools: <http://www.astree.ens.fr/>



Types

- In this class, we will focus on one kind of abstraction: types
- This means abstract values are the types in the language
- What is a type? An abstract value representing an (usually) infinite set of concrete values
- Question: For proving what kind of properties are types as abstract values useful?
- Answer: To avoid run-time type errors!

Inference rules

$$\frac{\begin{array}{c} \text{Hypothesis 1} \\ \dots \\ \text{Hypothesis N} \end{array}}{\vdash \text{Conclusion}}$$

- This means “given hypothesis 1,...N, the conclusion is provable”

$$\frac{\begin{array}{c} \text{Mitem 1 grade} \geq 70 \\ \dots \\ \text{Final grade} \geq 140 \end{array}}{\vdash \text{Final grade: A}}$$

TODOs by next lecture

- HW3 is out today