

LSU Algebra Question Bank Solution

Ayanava Mandal

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Chapter 1

Group Theory

G1: Let H be a normal subgroup of a group G , and let K be a subgroup of H .

- (a) Give an example of this situation where K is not a normal subgroup of G ,
- (b) Prove that if the normal subgroup H is cyclic, then K is normal in G .

Solution 1. (a) Let $G = S_4$, $H = A_4$, and $K = \{e, (123), (132)\}$.

- (b) Let $H = \langle h \rangle$ be cyclic. Let $K = \langle k \rangle$ where $k = h^a$ for some $a \in \mathbb{N}$.

Since H is normal, $ghg^{-1} = h^b \in H$ for some b .

$gkg^{-1} = gh^a g^{-1} = (ghg^{-1})^a = h^{ba} = k^b \in K$. So, K is normal in G .

Chapter 2

Ring Theory

