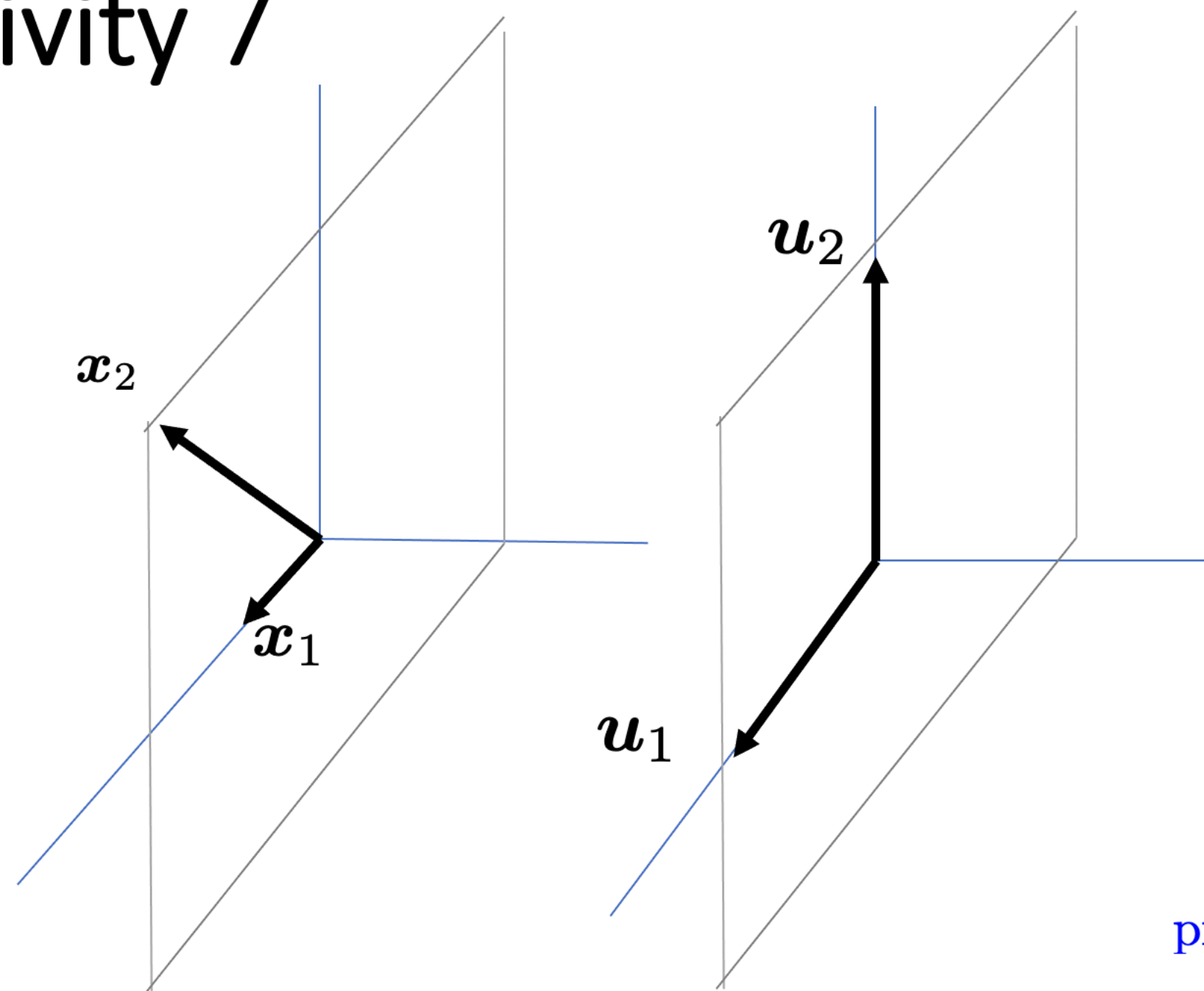


CS/ECE/ME 532 Activity 7

Gram-Schmidt Orthogonalization and Projections



1. set $\tilde{\mathbf{u}}_1 = \mathbf{x}_1$
2. normalize: $\mathbf{u}_1 = \frac{\tilde{\mathbf{u}}_1}{\|\tilde{\mathbf{u}}_1\|_2}$
3. set $\tilde{\mathbf{u}}_2 = \mathbf{x}_2 - \text{proj}_{\mathbf{u}_1} \mathbf{x}_2$
4. normalize: $\mathbf{u}_2 = \frac{\tilde{\mathbf{u}}_2}{\|\tilde{\mathbf{u}}_2\|_2}$
5. set $\tilde{\mathbf{u}}_3 = \mathbf{x}_3 - \text{proj}_{\mathbf{u}_1} \mathbf{x}_3 - \text{proj}_{\mathbf{u}_2} \mathbf{x}_3$
- \vdots

$$\text{proj}_{\mathbf{u}} \mathbf{d} = \mathbf{u}(\mathbf{u}^T \mathbf{d})$$

projection of \mathbf{d} onto \mathbf{u} amount of \mathbf{d} in the direction of \mathbf{u}

$$\mathbf{d} = \mathbf{A}\mathbf{w}$$

$$\begin{bmatrix} \vdots \\ \vdots \\ \vdots \end{bmatrix} = n \begin{bmatrix} \vdots \\ \vdots \\ \vdots \end{bmatrix} \mathbf{A} \begin{bmatrix} \vdots \\ \vdots \\ \vdots \end{bmatrix}$$

$$\begin{aligned} \hat{\mathbf{d}} &= \mathbf{A}\mathbf{w} = \mathbf{A}(\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{d} = \mathbf{P}_{\mathbf{A}} \mathbf{d} \\ &= \mathbf{U}\mathbf{U}^T \mathbf{d} = \mathbf{P}_{\mathbf{U}} \mathbf{d} \end{aligned}$$

$$\begin{aligned} \mathbf{e} &= \mathbf{d} - \hat{\mathbf{d}} = \mathbf{d} - \mathbf{P}_{\mathbf{A}} \mathbf{d} \\ &= (\mathbf{I} - \mathbf{P}_{\mathbf{A}}) \mathbf{d} \end{aligned}$$

