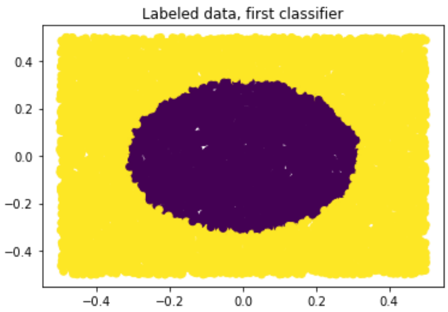
# Neural network example

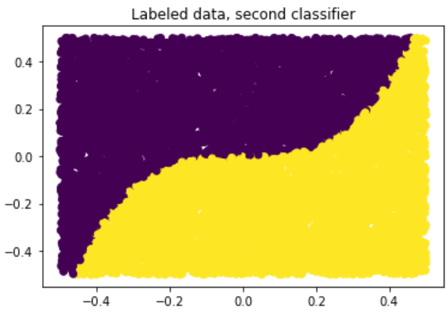
In [1]:
```
import numpy as np
import matplotlib.pyplot as plt

p = int(2) #features
n = int(10000) #examples

## generate training data
X = np.random.rand(n,p)-0.5
Y1 = np.sign(np.sum(X**2,1)-.1).reshape((-1, 1))/2+.5
Y2 = np.sign(5*X[:,[0]]**3-X[:,[1]])/2+.5
Y = np.hstack((Y1, Y2))
```

# 2 a)

In [2]:
```
# Plot training data for first classification problem
plt.scatter(X[:,0], X[:,1], c=Y1.flatten())
plt.title('Labeled data, first classifier')
plt.show()
```




In [3]:
```
# Plot training data for second classification problem
plt.scatter(X[:,0], X[:,1], c=Y2.flatten())
plt.title('Labeled data, second classifier')
plt.show()
```

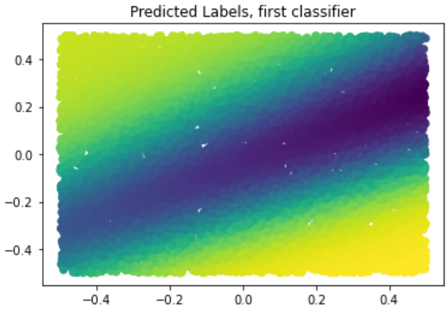


In [4]:
```python
## Train NN
Xb = np.hstack((np.ones((n,1)), X))
q = np.shape(Y)[1] #number of classification problems
M = 2 #number of hidden nodes

## initial weights
V = np.random.randn(M+1, q);
W = np.random.randn(p+1, M);

alpha = 0.1 #step size
L = 10 #number of epochs

def logsig(_x):
    return 1/(1+np.exp(-_x))

for epoch in range(L):
    ind = np.random.permutation(n)
    for i in ind:
        # Forward-propagate
        H = logsig(np.hstack((np.ones((1,1)), Xb[[i],:]@W)))
        Yhat = logsig(H@V)
         # Backpropagate
        delta = (Yhat-Y[[i],:])*Yhat*(1-Yhat)
        Vnew = V-alpha*H.T@delta
        gamma = delta@V[1:,:].T*H[:,1:]*(1-H[:,1:])
        Wnew = W - alpha*Xb[[i],:].T@gamma
        V = Vnew
        W = Wnew
    print('epoch: ', epoch)
```

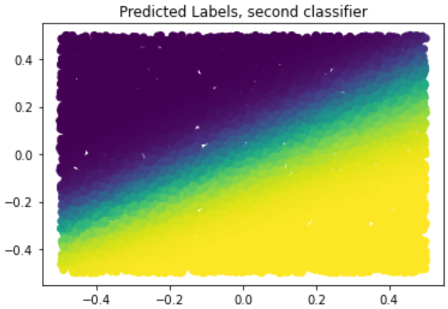
```
epoch:  0
epoch:  1
epoch:  2
epoch:  3
epoch:  4
epoch:  5
epoch:  6
epoch:  7
epoch:  8
epoch:  9
```

In [5]:
```python
## Final predicted labels (on training data)
H = logsig(np.hstack((np.ones((n,1)), Xb@W)))
Yhat = logsig(H@V)
```

In [6]:
```python
plt.scatter(X[:,0], X[:,1], c=Yhat[:,0])
plt.title('Predicted Labels, first classifier')
plt.show()
```




In [7]:
```python
plt.scatter(X[:,0], X[:,1], c=Yhat[:,1])
plt.title('Predicted Labels, second classifier')
plt.show()
```




In [8]:
```python
err_c1 = np.sum(abs(np.round(Yhat[:,0])-Y[:,0]))
print('Errors, first classifier:', err_c1)

err_c2 = np.sum(abs(np.round(Yhat[:,1])-Y[:,1]))
print('Errors, second classifier:', err_c2)
```

```
Errors, first classifier: 2780.0
Errors, second classifier: 786.0
```

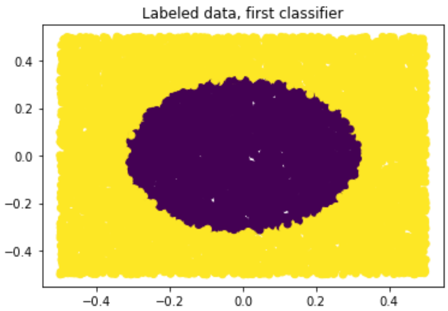
We see on running the M=2 10 epoch code, for different runs of the same code, we have different error counts for both the 2 classifiers.

## 2b)

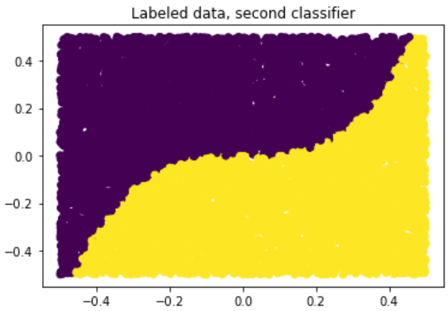
,

In [9]:
```
import numpy as np
import matplotlib.pyplot as plt

p = int(2) #features
n = int(10000) #examples

## generate training data
X = np.random.rand(n,p)-0.5
Y1 = np.sign(np.sum(X**2,1)-.1).reshape((-1, 1))/2+.5
Y2 = np.sign(5*X[:,[0]]**3-X[:,[1]])/2+.5
Y = np.hstack((Y1, Y2))
```

In [10]:
```
# Plot training data for first classification problem
plt.scatter(X[:,0], X[:,1], c=Y1.flatten())
plt.title('Labeled data, first classifier')
plt.show()
```




In [11]:
```
# Plot training data for second classification problem
plt.scatter(X[:,0], X[:,1], c=Y2.flatten())
plt.title('Labeled data, second classifier')
plt.show()
```



In [12]:
```python
## Train NN
Xb = np.hstack((np.ones((n,1)), X))
q = np.shape(Y)[1] #number of classification problems
M = 2 #number of hidden nodes

## initial weights
V = np.random.randn(M+1, q);
W = np.random.randn(p+1, M);

alpha = 0.1 #step size
L = 100 #number of epochs

def logsig(_x):
    return 1/(1+np.exp(-_x))

for epoch in range(L):
    ind = np.random.permutation(n)
    for i in ind:
        # Forward-propagate
        H = logsig(np.hstack((np.ones((1,1)), Xb[[i],:]@W)))
        Yhat = logsig(H@V)
         # Backpropagate
        delta = (Yhat-Y[[i],:])*Yhat*(1-Yhat)
        Vnew = V-alpha*H.T@delta
        gamma = delta@V[1:,:].T*H[:,1:]*(1-H[:,1:])
        Wnew = W - alpha*Xb[[i],:].T@gamma
        V = Vnew
        W = Wnew
    print('epoch: ', epoch)
```

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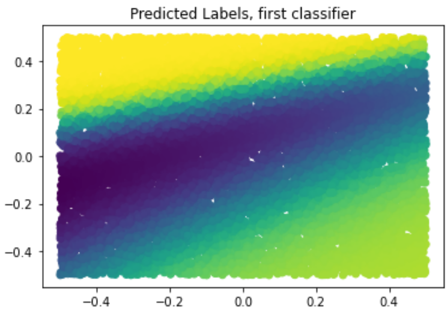
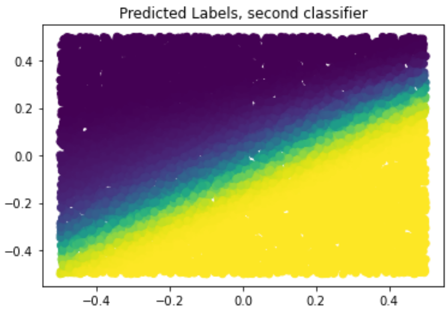
epoch:  99
```

In [13]:
```
## Final predicted labels (on training data)
H = logsig(np.hstack((np.ones((n,1)), Xb@W)))
Yhat = logsig(H@V)
```

In [14]:
```
plt.scatter(X[:,0], X[:,1], c=Yhat[:,0])
plt.title('Predicted Labels, first classifier')
plt.show()
```




In [15]:
```
plt.scatter(X[:,0], X[:,1], c=Yhat[:,1])
plt.title('Predicted Labels, second classifier')
plt.show()
```




In [16]:
```
err_c1 = np.sum(abs(np.round(Yhat[:,0])-Y[:,0]))
print('Errors, first classifier:', err_c1)

err_c2 = np.sum(abs(np.round(Yhat[:,1])-Y[:,1]))
print('Errors, second classifier:', err_c2)
```

```
Errors, first classifier: 2677.0
Errors, second classifier: 774.0
```

We see on running the M=2 100 epoch code, for different runs of the same code, we have different error counts for both the 2 classifiers.

## 2c )

More layers the neural network has, the more we can approximate higher order polynomials. This means the decision boundaries can become curved and non-linear. They are no longer limited to linear straight lines.

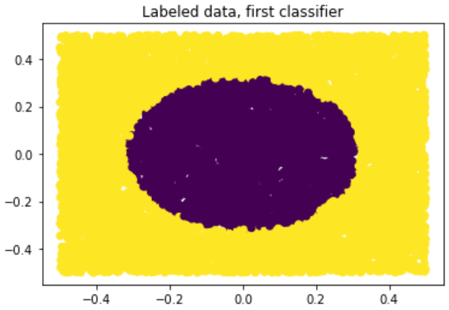
## 2d )

In [17]:
```
import numpy as np
import matplotlib.pyplot as plt

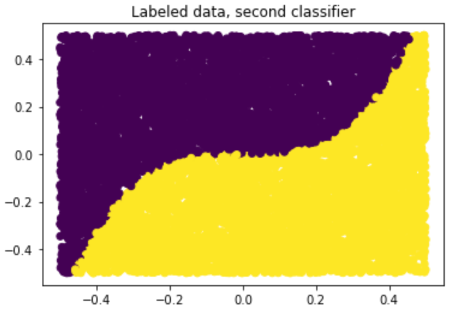
p = int(2) #features
n = int(10000) #examples

## generate training data
X = np.random.rand(n,p)-0.5
Y1 = np.sign(np.sum(X**2,1)-.1).reshape((-1, 1))/2+.5
Y2 = np.sign(5*X[:,[0]]**3-X[:,[1]])/2+.5
Y = np.hstack((Y1, Y2))
```

In [18]:
```
# Plot training data for first classification problem
plt.scatter(X[:,0], X[:,1], c=Y1.flatten())
plt.title('Labeled data, first classifier')
plt.show()
```




In [19]:
```
# Plot training data for second classification problem
plt.scatter(X[:,0], X[:,1], c=Y2.flatten())
plt.title('Labeled data, second classifier')
plt.show()
```



In [20]:
```python
## Train NN
Xb = np.hstack((np.ones((n,1)), X))
q = np.shape(Y)[1] #number of classification problems
M = 3 #number of hidden nodes

## initial weights
V = np.random.randn(M+1, q);
W = np.random.randn(p+1, M);

alpha = 0.1 #step size
L = 100 #number of epochs

def logsig(_x):
    return 1/(1+np.exp(-_x))

for epoch in range(L):
    ind = np.random.permutation(n)
    for i in ind:
        # Forward-propagate
        H = logsig(np.hstack((np.ones((1,1)), Xb[[i],:]@W)))
        Yhat = logsig(H@V)
         # Backpropagate
        delta = (Yhat-Y[[i],:])*Yhat*(1-Yhat)
        Vnew = V-alpha*H.T@delta
        gamma = delta@V[1:,:].T*H[:,1:]*(1-H[:,1:])
        Wnew = W - alpha*Xb[[i],:].T@gamma
        V = Vnew
        W = Wnew
    print('epoch: ', epoch)
```

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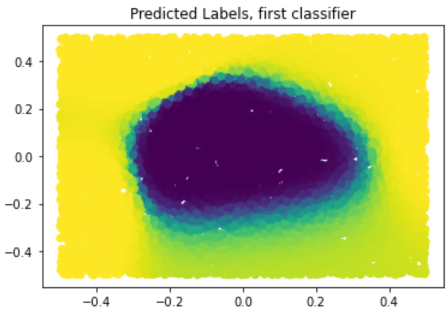
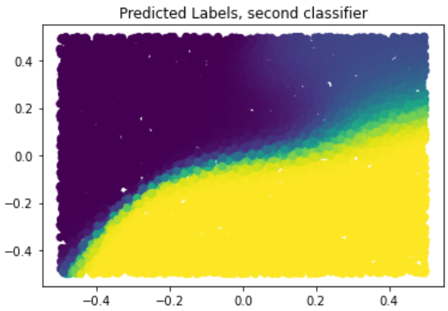
```

In [21]:
```
## Final predicted labels (on training data)
H = logsig(np.hstack((np.ones((n,1)), Xb@W)))
Yhat = logsig(H@V)
```

In [22]:
```
plt.scatter(X[:,0], X[:,1], c=Yhat[:,0])
plt.title('Predicted Labels, first classifier')
plt.show()
```




In [23]:
```
plt.scatter(X[:,0], X[:,1], c=Yhat[:,1])
plt.title('Predicted Labels, second classifier')
plt.show()
```




In [24]:
```
err_c1 = np.sum(abs(np.round(Yhat[:,0])-Y[:,0]))
print('Errors, first classifier:', err_c1)

err_c2 = np.sum(abs(np.round(Yhat[:,1])-Y[:,1]))
print('Errors, second classifier:', err_c2)
```

```
Errors, first classifier: 276.0
Errors, second classifier: 464.0
```

Going from M = 2 to M = 3 increases classifier performance as the number of errors goes down. This makes intuitive sense as the classifier is able to train itself much better due to the increased number of hidden nodes.

## 2e)

In [25]:

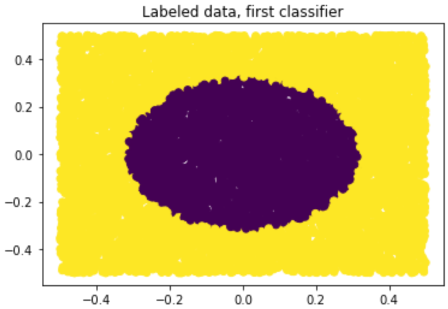
```
import numpy as np
import matplotlib.pyplot as plt

p = int(2) #features
n = int(10000) #examples

## generate training data
X = np.random.rand(n,p)-0.5
Y1 = np.sign(np.sum(X**2,1)-.1).reshape((-1, 1))/2+.5
Y2 = np.sign(5*X[:,[0]]**3-X[:,[1]])/2+.5
Y = np.hstack((Y1, Y2))
```

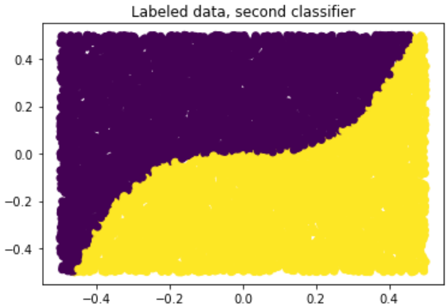
In [26]:

```
# Plot training data for first classification problem
plt.scatter(X[:,0], X[:,1], c=Y1.flatten())
plt.title('Labeled data, first classifier')
plt.show()
```




In [27]:

```
# Plot training data for second classification problem
plt.scatter(X[:,0], X[:,1], c=Y2.flatten())
plt.title('Labeled data, second classifier')
plt.show()
```



In [28]:
```
## Train NN
Xb = np.hstack((np.ones((n,1)), X))
q = np.shape(Y)[1] #number of classification problems
M = 4 #number of hidden nodes

## initial weights
V = np.random.randn(M+1, q);
W = np.random.randn(p+1, M);

alpha = 0.1 #step size
L = 100 #number of epochs

def logsig(_x):
    return 1/(1+np.exp(-_x))

for epoch in range(L):
    ind = np.random.permutation(n)
    for i in ind:
        # Forward-propagate
        H = logsig(np.hstack((np.ones((1,1)), Xb[[i],:]@W)))
        Yhat = logsig(H@V)
         # Backpropagate
        delta = (Yhat-Y[[i],:])*Yhat*(1-Yhat)
        Vnew = V-alpha*H.T@delta
        gamma = delta@V[1:,:].T*H[:,1:]*(1-H[:,1:])
        Wnew = W - alpha*Xb[[i],:].T@gamma
        V = Vnew
        W = Wnew
    print('epoch: ', epoch)
```

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```
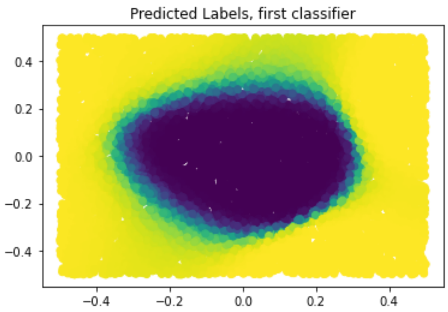
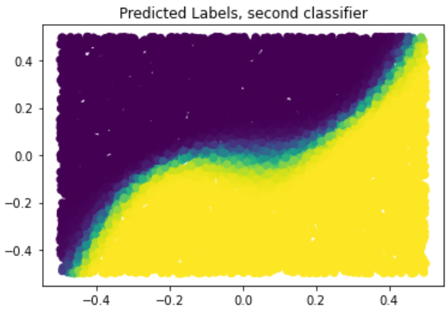

In [29]:
```
## Final predicted labels (on training data)
H = logsig(np.hstack((np.ones((n,1)), Xb@W)))
Yhat = logsig(H@V)
```

In [30]:
```
plt.scatter(X[:,0], X[:,1], c=Yhat[:,0])
plt.title('Predicted Labels, first classifier')
plt.show()
```




In [31]:
```
plt.scatter(X[:,0], X[:,1], c=Yhat[:,1])
plt.title('Predicted Labels, second classifier')
plt.show()
```




In [32]:
```
err_c1 = np.sum(abs(np.round(Yhat[:,0])-Y[:,0]))
print('Errors, first classifier:', err_c1)

err_c2 = np.sum(abs(np.round(Yhat[:,1])-Y[:,1]))
print('Errors, second classifier:', err_c2)
```

```
Errors, first classifier: 202.0
Errors, second classifier: 157.0
```

Going from M = 3 to M = 4 as expected increases performance. The number of errors goes down. This makes intuitive sense as the classifier is able to train itself much better due to the increased number of hidden nodes.

In [ ]: