Activity 10, ECE 532, Ayandeep Hazra

1. a) (In the pdf)

b) (In the pdf)

c) We see that the Rank 1 approximation is not only is it very far-off from the value of the centroid, but also does not capture the correct sign.

[For columns (1 to 3) the true value of the centroid is 2, but the Rank-1 approx gives us 1.]

[For columns (4 to 6), the true value of the centroid is -2, but the Rank-1 approx gives us 1 again (WRONG SIGN)]

d) We see that the Rank 2 approximation is a little better as it includes the correct sign of the centroid for all columns in A, even the ones that were missed.

2. a) A is a 4×6 matrix

Thus as $A = USV^T$

U has dimensions 4×4

S has dimensions 4×6

V has dimensions 6×6.

b) In the skinny case of SVD,

for $A = USV^T$

U has dimensions 4×4

S has dimensions 4×4

V has dimensions 6×4

c) i) We see from the code that

$A = USV^T$

ii) we see that $U^TU = I$, thus cols of U is orthonormal by definition

Also, $V^TV = I$, thus V's cols. orthonormal by definition.

iii) Again $UU^T = I$ and $VV^T = 1$, thus the rows of U & V are orthonormal by definition.

iv) First left singular vector

$$\begin{bmatrix} -0.5 \\ -0.5 \\ -0.5 \\ -0.5 \end{bmatrix}$$

Largest singular value

$= 9.7979$

v) Rank $\{A\} = 2$

d) i) From the code, we see that $A = USV^T$ holds.

ii) As before $U^TU = I$ & $V^TU = I$, thus the columns of U, V are orthonormal.

iii) $UU^T = I$ & $VV^T = I$, thus the rows of U, V are orthonormal.

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e) we see that even for the skinny case,

first left singular vector = $\begin{bmatrix} -0.5 \\ -0.5 \\ -0.5 \\ -0.5 \end{bmatrix}$

& largest singular value = 9.7979

Thus the values are the same. We expect this as theres only one unique way to $\overset{\text{eigen}}{\text{decompose}}$ a matrix, regardless of the approach taken.

f) In economy SVD, $A = USV^T$, where $A \in \mathbb{R}^{m \times n}$, $U \in \mathbb{R}^{m \times r}$, $S \in \mathbb{R}^{r \times r}$, $V^T \in \mathbb{R}^{r \times m}$, and r is the rank of A.

If we let $B = SV^T$

Thus $A = UB$ so that each column of A is a linear combination of the rows of U.

cols(A) is in span of cols(U)

since U is orthonormal.
thus the first r columns of U form an orthonormal basis for the space spanned by the cols of A, r depends on rank approximation. thus orthonormal bas is

$$= \begin{bmatrix} -0.5 & -0.5 \\ -0.5 & 0.5 \\ -0.5 & 0.5 \\ -0.5 & -0.5 \end{bmatrix}$$

9) similarly for $\boxed{A = USV^T}$

let $B = US$

thus $A = BV^T$

& since V is orthonormal, V^T is orthonormal by definition.

Thus each row of A is a linear comb. of the columns of V^T.

Thus given some B, first r rows of V^T acts also the orthonormal basis for the space spanned by the rows of A. r depends on rank approx.

thus, orthonormal basis is $\begin{bmatrix} -0.5 & -0.5 \\ -0.5 & 0.5 \\ -0.71 & -0.03 \\ -0.03 & 0.71 \end{bmatrix}$

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h) i) The rank 1 approx generally gets the notion of a line that divides the +ve values from the -ve values in the matrix.. But it gets all the values as the same absolute value, + or -.

ii) The rank 2 approximation correctly defines A exactly. (All values in all rows & columns).

i) Since A has dimensions 4x6

S can have minimum dimensions of 4x4

where $\left\{ A_{4\times6} = U_{4\times4} \; S_{4\times4} \; V^T_{4\times6} \right\}$

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