AMAN DEEP HAZRA

Activity 15 ECE 532

1\. a) A rank 1 matrix has columns (and rows) such that each can be written as a multiple of the other. i.e. you can obtain another row/column by multiplying a row/column by a number.

Given, $V = \begin{bmatrix} 1 & x & x \\ x & 2 & 4 \\ -1 & 2 & x \\ x & -2 & x \end{bmatrix}$

Between column 1 & 2,

we see $\frac{V_{31}}{V_{32}} = \frac{-1}{2}$

thus every unknown can be found in these 2 columns.

$\frac{V_{11}}{V_{12}} = \frac{-1}{2} \Rightarrow \frac{1}{V_{12}} = \frac{-1}{2} \Rightarrow V_{12} = -2$

$\frac{V_{21}}{V_{22}} = \frac{-1}{2} \Rightarrow V_{21} = \frac{-1}{2}(2) \Rightarrow V_{21} = -1$

$\frac{V_{41}}{V_{42}} = \frac{-1}{2} \Rightarrow V_{41} = \frac{-1}{2}(-2) \Rightarrow V_{41} = 1$

$\therefore V = \begin{bmatrix} 1 & -2 & x \\ -1 & 2 & 4 \\ -1 & 2 & x \\ 1 & -2 & x \end{bmatrix}$

If we do the same for columns 2, 3,

we see we have

$$\frac{V_{22}}{V_{23}} = \frac{2}{4} = \frac{1}{2}$$

Thus

$$\frac{V_{12}}{V_{13}} = \frac{1}{2} \Rightarrow V_{13} = (-2)(2) = -4$$

$$\frac{V_{32}}{V_{33}} = \frac{1}{2} \Rightarrow V_{33} = (2)(2) = 4$$

$$\frac{V_{42}}{V_{43}} = \frac{1}{2} \Rightarrow V_{43} = (-2)(2) = -4.$$

Thus, we get,

$$V = \begin{bmatrix} 1 & -2 & -4 \\ -1 & 2 & 4 \\ -1 & 2 & 4 \\ 1 & -2 & -4 \end{bmatrix}$$

b) The minimum number of missing entries for which you cannot complete a 4-by-3 rank 1 matrix is $\boxed{3}$.

The missing entries will be an entire row that's missing (in general, entire lowest dimension)

Because in this case there is no way to discern any of the relations between the columns as you have no value to start with. That row can have any set of 3 values which satisfy the ratio values of the other rows.