CS/ECE/ME 532

Unit 1 Example Word Problems

1. Madison Outdoors LLC hires you to analyze customer preferences for hiking and biking trips in southern Wisconsin. You are to analyze four trips: Governor Dodge Hiking, Devil's Lake Hiking, Dane County Ironman Bike, and New Glarus Bike. Each is rated on a scale of 1 to 5 and the ratings for user i are stored in a 4-by-1 vector as follows

$$\boldsymbol{x}_i = \begin{bmatrix} \text{Gov Dodge} \\ \text{Devil's Lake} \\ \text{Ironman} \\ \text{New Glarus} \end{bmatrix}$$

a) Susan gives ratings: Gov Dodge = 3, Devil's Lake = 2, Ironman = 5, New Glarus = 4; Sally gives ratings: Gov Dodge = 4, Devil's Lake = 5, Ironman = 1, New Glarus = 2; and Sam gives ratings: Gov Dodge = 4, Devil's Lake = 4, Ironman = 3, New Glarus = 2. Put these ratings into a matrix **R** where each column of **R** represents the ratings $\boldsymbol{x}_i$ for one person.

$$\underline{R} = \left[\text{susan} \;\vdots\; \text{Sally} \;\vdots\; \text{Sam} \right]$$

$$= \begin{bmatrix} 3 & 4 & 4 \\ 2 & 5 & 4 \\ 5 & 1 & 3 \\ 4 & 2 & 2 \end{bmatrix}$$

b) Madison Outdoors want to describe their customer ratings in terms of the tastes of two representative customers. Customer H prefers hiking trips, and customer B prefers biking trips. The ratings or tastes of these representative customers are

$$\boldsymbol{x}_H = \begin{bmatrix} 4 \\ 4 \\ 2 \\ 2 \end{bmatrix}, \quad \boldsymbol{x}_B = \begin{bmatrix} 2 \\ 2 \\ 4 \\ 4 \end{bmatrix}$$

Let a ratings matrix $\mathbf{R}$ for 10 customers be expressed in terms of these two taste profiles as $\mathbf{R} = \mathbf{TS}$ where $\mathbf{T}$ is a matrix formed by the taste profiles. Define $\mathbf{T}$ and the dimensions of $\mathbf{R}$, $\mathbf{T}$, $\mathbf{S}$.

Let $\underline{T} = [\underline{x}_H \vdots \underline{x}_B] = \begin{bmatrix} 4 & 2 \\ 4 & 2 \\ 2 & 4 \\ 2 & 4 \end{bmatrix}$

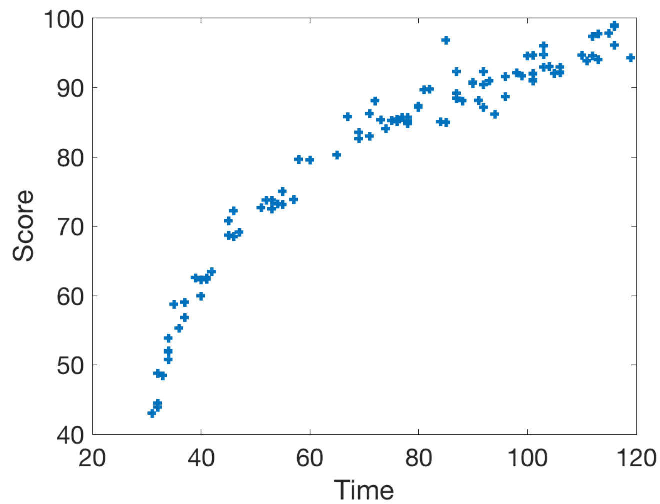
$\underline{R} = \underline{T}\,\underline{S}$

$\underline{R} : 4 \times 10$

$\underline{T} : 4 \times 2$

$\underline{S} : 2 \times 10$

2. You observe 100 exam scores and the time each student spent in minutes. Let $t_i, s_i, i = 1, 2, \ldots, 100$ be the times and data, respectively, as shown in the figure below:



You hypothesize a model that the score is a logarithmic function of the time spent on the exam, that is, $s \approx \alpha \log_{10}(t) + \beta$ where $\boldsymbol{w} = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$ are unknown.

Form a column vector $\mathbf{s} = \begin{bmatrix} s_1 & s_2 & \cdots & s_{100} \end{bmatrix}^T$. Express $\mathbf{s}$ as the product of a matrix that depends on $t_i, i = 1, 2, \ldots, 100$ and the model parameters $\boldsymbol{w}$. Indicate the dimensions of all matrices and vectors.

$$s_i = \begin{bmatrix} \log_{10}(t_i) & 1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

$$\underline{s} = \underline{X}\,\underline{w} \qquad \underline{X} = \begin{bmatrix} \log_{10}(t_1) & 1 \\ \log_{10}(t_2) & 1 \\ \vdots & \\ \log_{10}(t_{100}) & 1 \end{bmatrix}$$

$\underline{s}$: 100x1
$\underline{X}$: 100x2
$\underline{w}$: 2x1

3. You are given a set of features $\boldsymbol{x}_i = (x_{1i}, x_{2i})$ and corresponding labels $y_i, i = 1, 2, \ldots, 6$

$$\boldsymbol{x}_1 = (-2, 1),\ y_1 = 1 \quad \boldsymbol{x}_2 = (-1, 2),\ y_2 = -1$$

$$\boldsymbol{x}_3 = (1, 4),\ y_3 = -1 \quad \boldsymbol{x}_4 = (0, 3),\ y_4 = -1$$

$$\boldsymbol{x}_5 = (-3, 2),\ y_5 = 1 \quad \boldsymbol{x}_6 = (2, 4),\ y_6 = -1$$

Your boss tells you to use the linear classifier

$$\text{sign}(w_1 x_1 + w_2 x_2 + w_3)$$

Collect the classifier parameters into a column vector $\boldsymbol{w} = \begin{bmatrix} w_1 & w_2 & w_3 \end{bmatrix}^T$ and use the given features and labels to write a system of linear equations that could be used to train the classifier.

$$\underline{X}\,\underline{w} = \underline{y}$$

$$\underline{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_6 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$$

6x1

$$\underline{X} = \begin{bmatrix} -2 & 1 & 1 \\ -1 & 2 & 1 \\ 1 & 4 & 1 \\ 0 & 3 & 1 \\ -3 & 2 & 1 \\ 2 & 4 & 1 \end{bmatrix}$$

6x3