Q2) Unit 6 Exam

Let $x_0^i = 1$ for all i.

$$\frac{df^{i_t}}{dw_k} = (\hat{y}^{i_t} - y^{i_t}) \frac{d\hat{y}^{i_t}}{dw_k} + 2\lambda w_k$$

as $f^i(w) = \frac{1}{2}(\hat{y}^i - y^i)^2 + \lambda \sum_{j=0}^{p} w_j^2$

Now, $$\frac{d\hat{y}^{i_t}}{dw_k} = \frac{d\hat{y}^{i_t}}{dz^{i_t}} \frac{dz^{i_t}}{dw_k}$$

where $z^{i_t} = \sum_{j=0}^{p} w_j x_j^{i_t}$ & $\hat{y}_i^i = \sigma\left(\sum_{j=0}^{p} w_j x_j^i\right)$

as it is a single neuron.

Thus $$\frac{d\hat{y}^{i_t}}{dz^{i_t}} = \begin{cases} 0, & z^{i_t} < -1/2 \\ 1, & -\frac{1}{2} \le z^{i_t} \le \frac{1}{2} \\ 0, & z^{i_t} > 1/2 \end{cases}$$

Also, $\frac{dz^{i_t}}{dw_k} = x_k^{i_t}$ clearly

Thus $\frac{d\hat{y}^{i_t}}{dw_k} = x_k^{i_t} \cdot 1\left\{-\frac{1}{2} \leq \sum_{j=0}^{p} w_j x_j^{i_t} \leq \frac{1}{2}\right\}$

If we define some

$$\delta^{i_t} = (\hat{y}^{i_t} - y^{i_t}) \cdot 1\left\{-\frac{1}{2} \leq \sum_{j=0}^{p} w_j x_j^{i_t} \leq \frac{1}{2}\right\}$$

Thus we can write

$$\nabla f^{i_t}(w^{(t)}) = \delta^{i_t} x^{i_t} + 2\lambda w^{(t)}$$