

CS/ECE/ME532 Assignment 2

1. Answer the following questions. Justify your answers.

a) Are the columns of the following matrix linearly independent?

$$\mathbf{A} = \begin{bmatrix} +0.92 & +0.92 \\ -0.92 & +0.92 \\ +0.92 & -0.92 \\ -0.92 & -0.92 \end{bmatrix}$$

b) Are the columns of the following matrix linearly independent?

$$\mathbf{A} = \begin{bmatrix} +1 & +1 & +1 \\ -1 & +1 & -1 \\ +1 & -1 & -1 \end{bmatrix}$$

c) Are the columns of the following matrix linearly independent?

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 4 & 5 \\ 5 & 6 & 8 \end{bmatrix}$$

d) What is the rank of the following matrix?

$$\mathbf{A} = \begin{bmatrix} +5 & +2 \\ -5 & +2 \\ +5 & -2 \end{bmatrix}$$

e) Suppose the matrix in part d is used in to solve the system of linear equations $\mathbf{A}^T \mathbf{A} \mathbf{w} = \mathbf{d}$. Does a unique solution exist? Explain why.

2. *Norm additivity.* Suppose $\|\cdot\|_a$ and $\|\cdot\|_b$ are norms on \mathbb{R}^n .

a) Prove that $f(\mathbf{x}) = \|\mathbf{x}\|_a + \|\mathbf{x}\|_b$ is also a norm on \mathbb{R}^n .

b) The “norm ball” is defined as the set of \mathbf{x} for which an (arbitrary) norm $f(\mathbf{x}) = 1$. Sketch the norm ball in \mathbb{R}^2 for the norm $f(\mathbf{x}) = \|\mathbf{x}\|_1 + \|\mathbf{x}\|_\infty$.