CS/ECE/ME 532
Activity 14

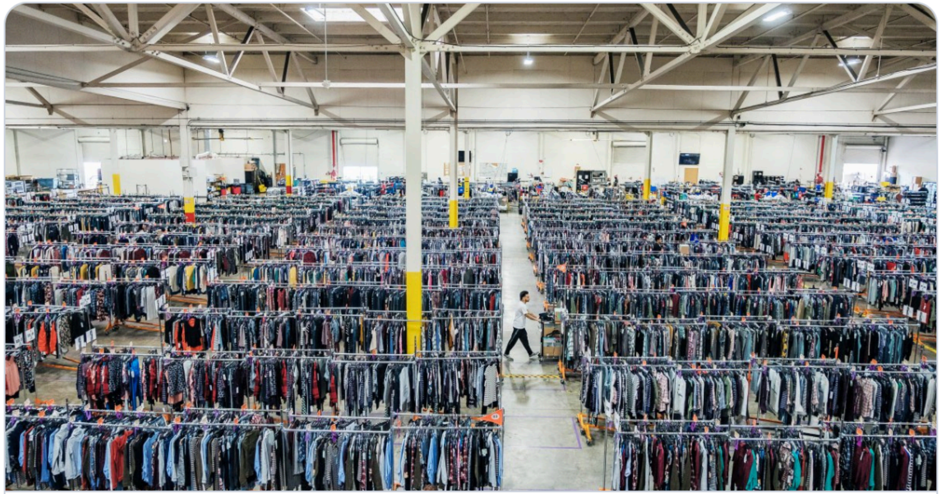
- Unit 3 + 4 Quiz, next week Thursday
- Unit 3 and 4 practice problems are posted
- Homework 6 is longer than homework 5, start early

- Today - Unit 4 continued
  - Eigenvectors and Eigenvalues, Power iterations
  - Graphs, adjacency matrices, and PageRank
- Today's activity
  - PageRank

Tweet

WIRED @WIRED

Stitch Fix is using something called eigenvector decomposition, a concept from quantum mechanics, to tease apart the overlapping "notes" in an individual's style. Using physics, the team can better understand the complexities of the clients' style minds.



The Style Maven Astrophysicists of Silicon Valley
You know who knows machine learning? People who look at the stars all day. And when it comes to what constellations of clothes and shows and music yo...
wired.com

12:12 AM · Oct 8, 2019 · SocialFlow

206 Retweets **943** Likes

Nic Dalmasso @Mr8ND · Oct 8
Just to clarify, "eigenvector decomposition" is from ~1880, quantum physics (arguably) started with Einsten's photoeletric effect discovery in 1905.

6 7 236

2 more replies

specmanic 🕴 ⌛ @SpecManic · Oct 9
with scipy library... scipy.linalg.svd(user_likes_matrix)

1 19

me, a ghost: @benadamx · Oct 8
Replying to @WIRED
eigenvouch that they are not

Today – Eigen decomposition, power iteration, adjacency matrices

Eigen decomposition

$\boldsymbol{B}$ (square) symmetric positive definite matrix

diagonal

orthonormal rows, cols

$$\boldsymbol{B} = \boldsymbol{E}\boldsymbol{\Lambda}\boldsymbol{E}^T$$
$$\boldsymbol{B} = \boldsymbol{U}\Sigma\boldsymbol{V}^T$$

What if matrix isn't symmetric?

$$\boldsymbol{A}\boldsymbol{A}^T = \boldsymbol{U}\Sigma^2\boldsymbol{U}^T$$

Eigenvectors of $\boldsymbol{A}\boldsymbol{A}^T$ are left singular vectors of $\boldsymbol{A}$

$$\boldsymbol{A}^T\boldsymbol{A} = \boldsymbol{V}\Sigma^2\boldsymbol{V}^T$$

Eigenvectors of $\boldsymbol{A}^T\boldsymbol{A}$ are right singular vectors of $\boldsymbol{A}$

Singular values of $\boldsymbol{A}$: $\sigma_i = \sqrt{\lambda_i}$

Power iteration (main idea)

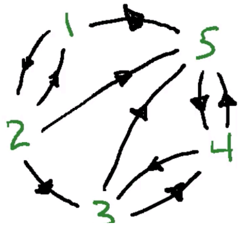
$$(\boldsymbol{A}\boldsymbol{A}^T)^k = \boldsymbol{U}\Sigma^2\boldsymbol{U}^T\boldsymbol{U}\Sigma^2\boldsymbol{U}^T \ldots \boldsymbol{U}\Sigma^2\boldsymbol{U}^T$$
$$= \boldsymbol{U}\Sigma^{2k}\boldsymbol{U}^T$$

imagine that $\sigma_1 = 1$, others less than 1

$$= \boldsymbol{U}\Sigma^{2k}\boldsymbol{U}^T \rightarrow \boldsymbol{u}_1\boldsymbol{u}_1^T$$

Adjacency matrix

- Graph: nodes with edges between them
- Adjacency matrix: non-zero entry $A_{i,j}$ if edge from *j* to *i*
- *Transition probability matrix: normalize columns of adjacency matrix to 1*



$$\tilde{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

- Normalize columns to get Q, transition probability matrix

$$\boldsymbol{Q}^k\boldsymbol{b} \rightarrow \text{direction of first eigenvector of } \boldsymbol{Q}.$$

- represents steady state probability