ECE 532 Activity 18 Arpan Deep Hazra

Question 1

let $a = \begin{bmatrix} g_1^T \\ g_2^T \\ \vdots \\ g_{100}^T \end{bmatrix}$ $y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{100} \end{bmatrix}$

we need to build a classifier such that

$\hat{y} = \text{sign}\{g_i^T y\}$

therefore we solve, $\|aw - y\|_2^2 + \lambda \|w\|_1$

we choose $\|w\|_1$ as a regularizer because only a small number of genes are relevant (which means solution is sparse)

Question 2

a) For there to be an unique solution, rank(X) must equal dimension of w.

dim (w) = 2 but rank(X) = 1.

Thus no unique solution exists.

b) $f(w) = \|y - Xw\|_2^2$

here $y = 4$, $X = [2 \ 1]$ & $w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$

Thus $f(w) = (4 - 2w_1 - 2w_2)^2$

If $f(w) \geq 0$

$(\cancel{4} - 2w_1 - w_2)^2 \geq 0$

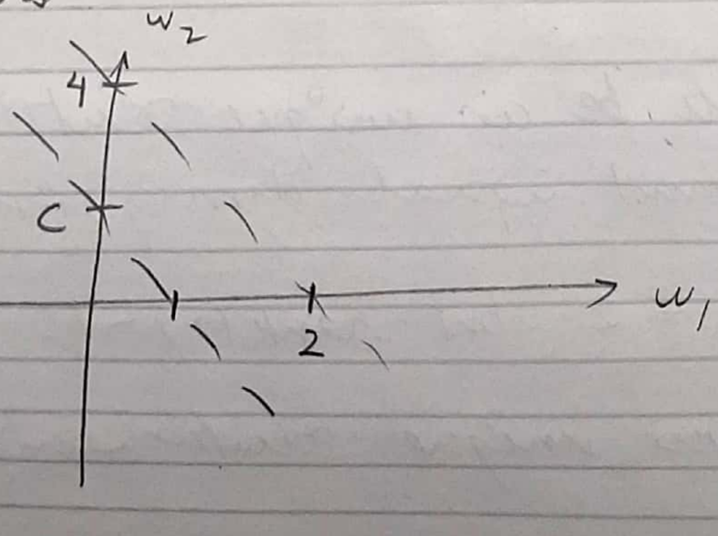
$4 \geq 2w_1 + w_2$

If we assume some arbitrary constant c for $f(w) \geq c$.

$2w_1 + w_2 = c \Rightarrow w_2 = -2w_1 + c$

So, $f(w) = (4 - c)^2$

contours



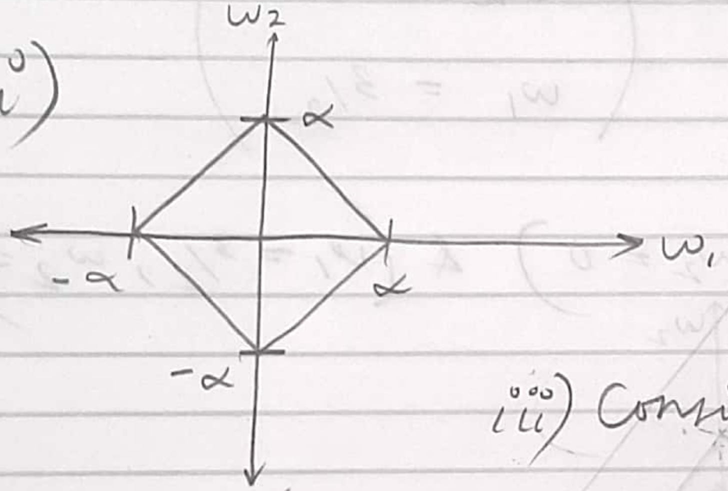
e) Considering $\|y - Xw\|_2^2 < 1$

$f(w) < 1$

let $\|w\|_1 = \alpha$

$\Rightarrow |w_1| + |w_2| = \alpha$

i)



(ii) on next page

iii) Consider $f(w) = 1$

$(4 - c)^2 = 1$

$4 - c = 1$

$c = 5$

Consider $f(w) = 0$,

$(4 - c)^2 = 0$

$4 - c = 0$

$c = 4$

iii) $2w_1 + w_2 - 4 = c$

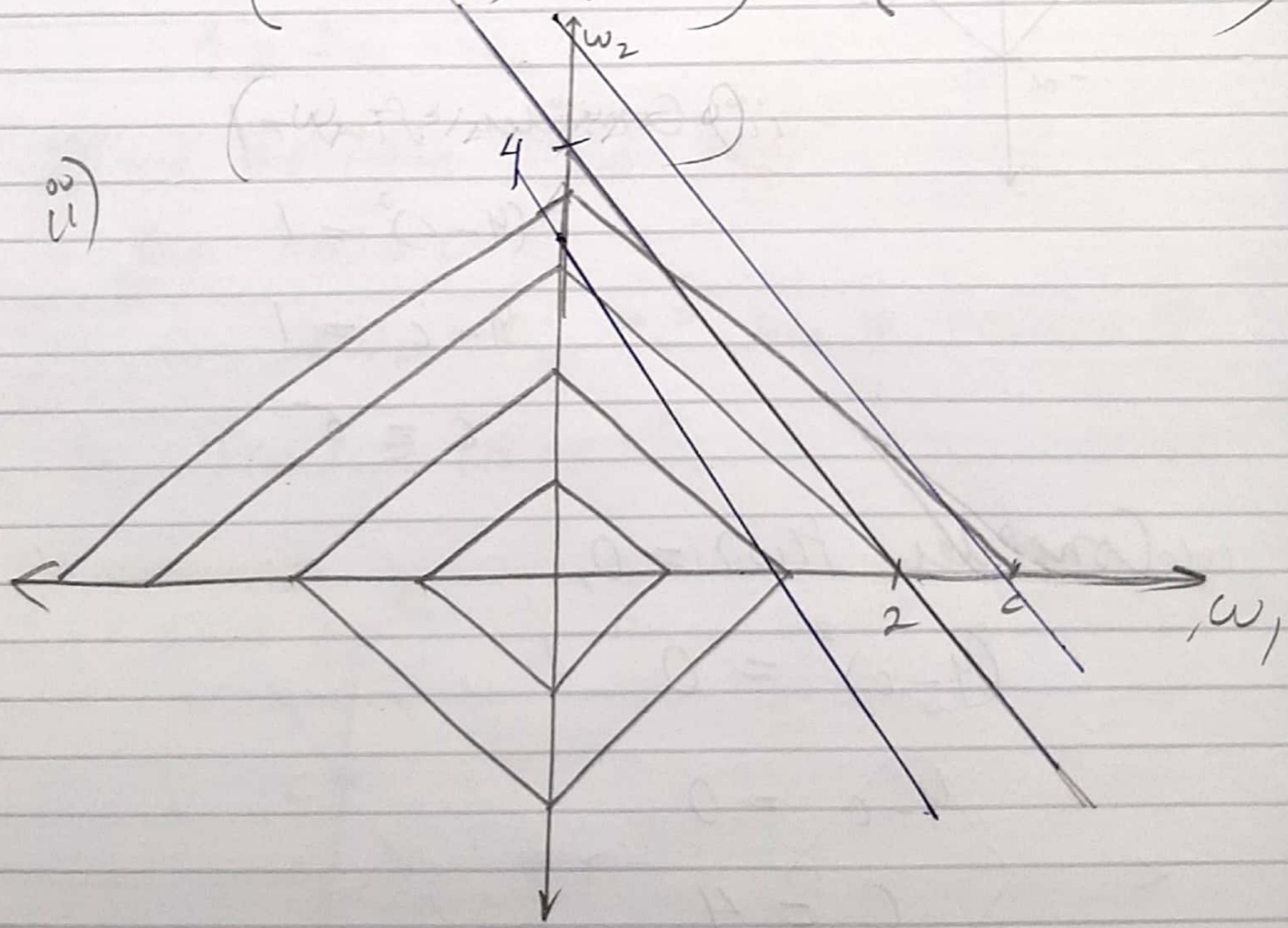
$2w_1 + w_2 - 4 = \pm 1$

$w_2 = -2w_1 + 4 \pm 1$

$\begin{pmatrix} w_2 = 0 \\ w_1 = 5/2 \end{pmatrix}$ or $\begin{pmatrix} w_2 = 0 \\ w_1 = 3/2 \end{pmatrix}$

Thus $(w_1 = 5/2, w_2 = 0)$ & $(w_1 = 3/2, w_2 = 0)$

ii)



d) $\lambda = 0, \quad w_1 = 2, \; w_2 = 0$

$\lambda \to \infty, \quad w_1 = 0, \quad w_2 = 0$