1) $X = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & -2 & -2 \\ 3 & -3 & 3 & -3 \end{bmatrix}$ $\quad w = \begin{bmatrix} 1 \\ b \\ 1 \\ c \end{bmatrix}$

a) $y = Xw$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & -2 & -2 \\ 3 & -3 & 3 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ b \\ 1 \\ c \end{bmatrix}$$

$$= \begin{bmatrix} (1\times1 + 1\times b + 1\times1 + 1\times c) \\ (2\times1 + 2\times b + -2\times1 + -2\times c) \\ (3\times1 + -3\times b + 3\times1 + -3\times c) \end{bmatrix} = \begin{bmatrix} (b+c+2) \\ (2b-2c) \\ (-3b-3c+6) \end{bmatrix}$$

b) from y above & equation $y = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$,

$b + c + 2 = 4$ — ① from row 1

$2b - 2c = 0$ — ② from row 2

$b = c$

from ②,

$2b + 2 = 4$

$2b = 2$

$b = 1$

Thus $b = c = 1$

c) from y above & equation $y = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}$

$2b - 2c = 0$ — ① from row 1

$b = c$

$-3b - 3c + 6 = 12$ — ② from row 3

$-6b = 6$

$b = -1$

Thus, $b = c = -1$

2) a) We know that for a food to be low-carb $z < \frac{1}{4}$

Thus, $\frac{\text{carb calories}}{\text{total calories}} < \frac{1}{4}$

We know that total calories count is $y = x^T w$

Since x_3 is number of gram of carbs, from $x^T w$, we know total calories from carbs $= 4x_3$.

Thus, $\frac{4x_3}{9x_1 + 4x_2 + 4x_3} < \frac{1}{4}$

$16x_3 < 9x_1 + 4x_2 + 4x_3$

$12x_3 - 9x_1 - 4x_2 < 0$

$$-9x_1 - 4x_2 + 12x_3 < 0$$

$$\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} -9 \\ -4 \\ 12 \end{bmatrix} < 0$$

$$\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} 9 \\ 4 \\ -12 \end{bmatrix} > 0$$

$$\bar{\omega} = \begin{bmatrix} 9 \\ 4 \\ 12 \end{bmatrix}$$

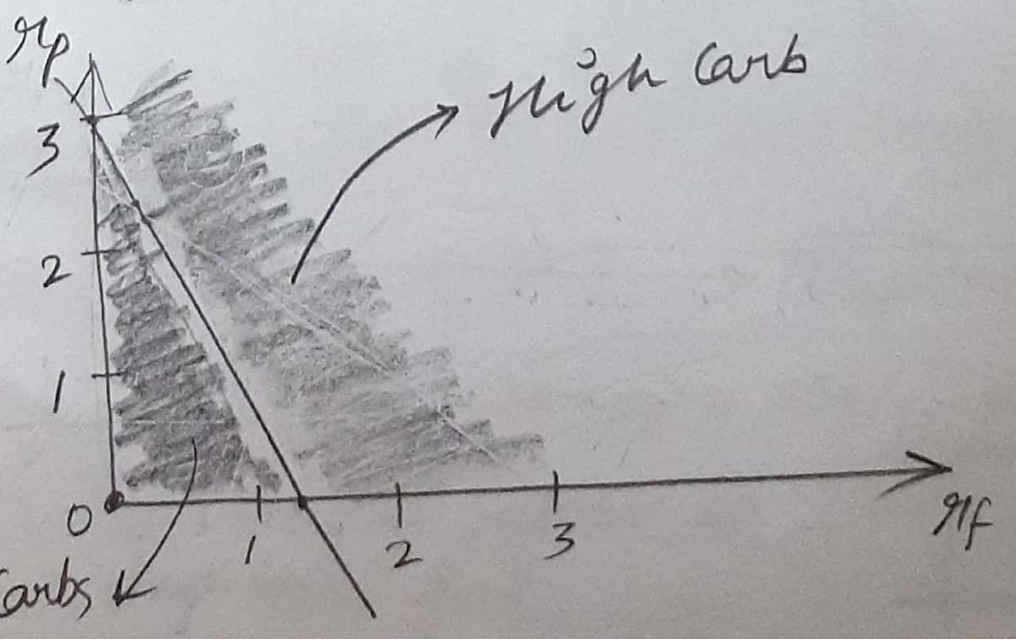
b) $$-9x_1 - 4x_2 + 12x_3 < 0$$

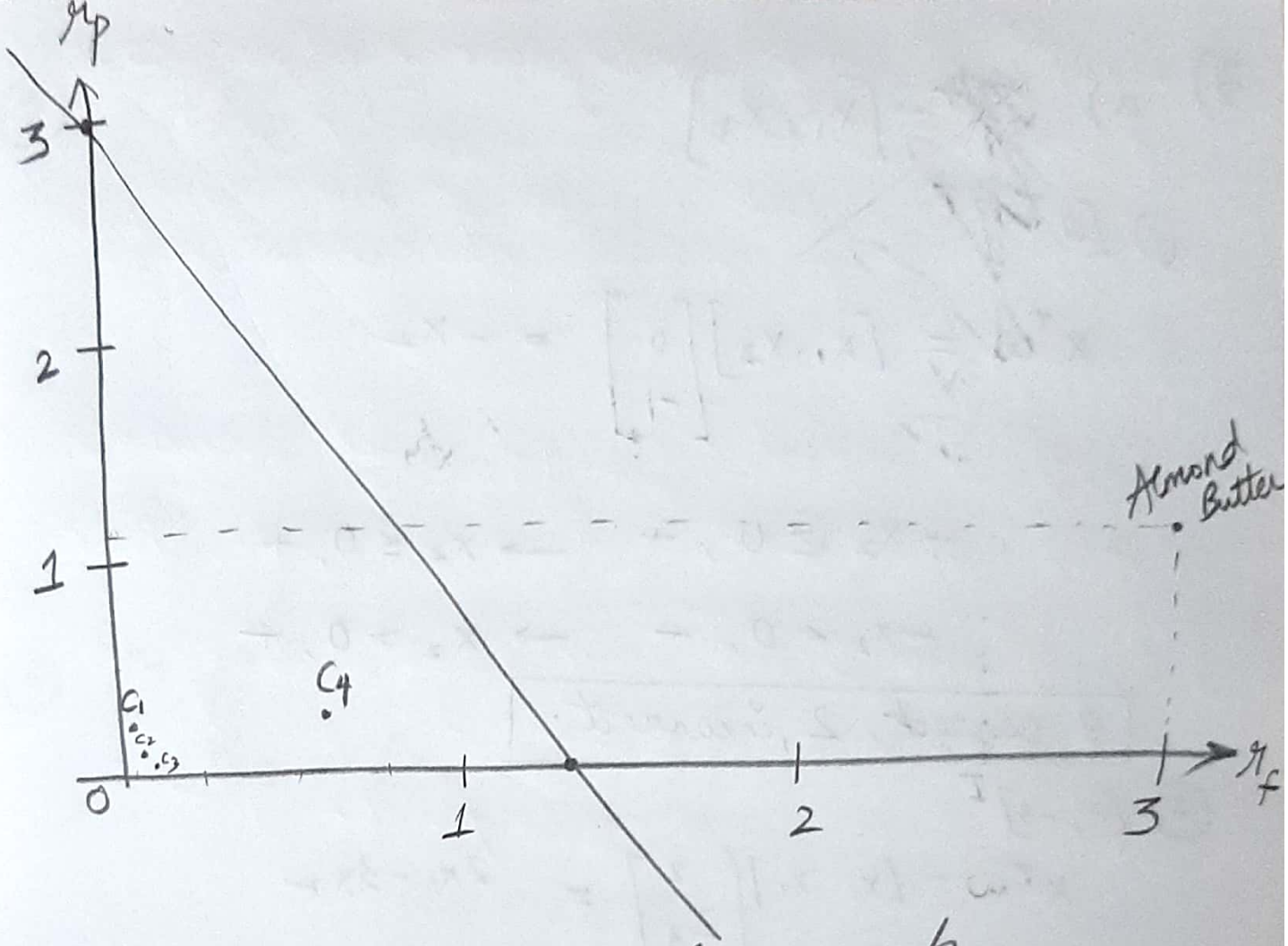
$$-9\frac{x_1}{x_3} - 4\frac{x_2}{x_3} + 12\frac{x_3}{x_3} < 0$$

$$-9\pi_f - 4\pi_p + 12 < 0$$

$$\boxed{9\pi_f + 4\pi_p > 12}$$

c)





All of the cereals are low carb.

e) $f \rightarrow 9g \quad p \rightarrow 3.4g \quad c \rightarrow 3g$

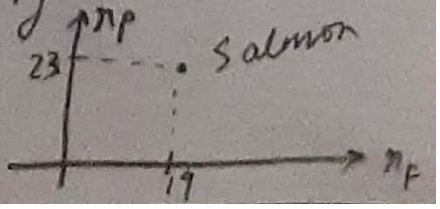
$n_f = \frac{f}{c} = \frac{9g}{3g} = 3 \qquad n_p = \frac{3.4g}{3g} = 1.13$

Since the value of $n_f = 3 > 4/3$,
Almond butter is not low carb food.

f) $f \rightarrow 19g \quad p \rightarrow 23g \quad c \rightarrow 1g$

$n_f = f/c = 19g/1g = 19 \qquad n_p = 23g/1g = 23$

Marinated Grilled Salmon is not low carb food.



3) a) $x^T = [x_1, x_2]$

① $[0, -1]^T$

$$x^T w = [x_1, x_2]\begin{bmatrix} 0 \\ -1 \end{bmatrix} = -x_2$$

$-x_2 \geq 0, + \longrightarrow x_2 \leq 0, +$

$-x_2 < 0, - \longrightarrow x_2 > 0, -$

4 correct, 2 incorrect.

② $[2, -3]^T$

$$x^T w = [x_1 \;\; x_2]\begin{bmatrix} 2 \\ -3 \end{bmatrix} = 2x_1 - 3x_2$$

$2x_1 - 3x_2 \geq 0, +$

$2x_1 - 3x_2 < 0, -$

5 correct, 1 incorrect

③ $[3, -2]^T$

$$x^T w = [x_1 \;\; x_2]\begin{bmatrix} 3 \\ -2 \end{bmatrix} = 3x_1 - 2x_2$$

$3x_1 - 2x_2 \geq 0, +$

$3x_1 - 2x_2 < 0, -$

4 correct, 2 incorrect.

④ Same process. 5 correct, 1 incorrect.

$[1, 0]^T$

⑤ $[3,2]^T$

$$x^T w = [x_1 \quad x_2] \begin{bmatrix} 3 \\ 2 \end{bmatrix} = 3x_1 + 2x_2$$

$3x_1 + 2x_2 \geq 0$, +

$3x_1 + 2x_2 < 0$, −

4 correct; 2 incorrect

⑥ $[2,3]^T$

$$x^T w = [x_1 \quad x_2] \begin{bmatrix} 2 \\ 3 \end{bmatrix} = 2x_1 + 3x_2$$

$2x_1 + 3x_2 \geq 0$, +

$2x_1 + 3x_2 < 0$, −

3 correct, 3 incorrect

$w = [2, -3]^T$ & $w = [1, 0]^T$ are the ones that minimize the number of miscalculation.

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b) $w = [0, -1]^T$ is fair as 2/6 was the error rate in both subgroups.

$w = [2, -3]^T$ is fair as 1/6 was the error rate in both.

$w = [3, -2]^T$ is unfair as blue had error rate of 3/6 while red had error rate of 1/6.

$w = [1, 0]^T$ is unfair as blue had error rate of 2/6 & red had error rate of 0/6.

$w = [3, 2]^T$ is fair as both had error rate of 2/6.

$w = [2, 3]^T$ is unfair as red had error rate of 4/6 while blue had error rate of 2/6.

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c) $[2, -3]^T = w$ is the only fair classifier that minimizes # of misclassification.
Not all classifiers are fair.