8. a) the first principal component is u_1.

b) $Z = U \Sigma V^T$

$\hat{x}_6^1 = x_m + \sigma_1 u_1 v_1^T$ ← PC1

c) $\hat{x}_6^2 = x_m + \sigma_1 u_1 v_1^T + \sigma_2 u_2 v_2^T$

PC1 PC2

d) $\frac{1}{500} \sum_{i=1}^{500} \| \hat{x}_i^1 - x_i \|_2^2$

$= \frac{1}{500} (500 x_m)$

$= x_m$