Period 13 Activity ECE 532
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1. a) $X = \sum_{i=1}^{n} \sigma_i u_i v_i^T$

$$X_r = \sum_{i=1}^{r} \sigma_i u_i v_i^T$$

$$E_r = X - X_r$$

$$= \sum_{r+1}^{n} \sigma_i u_i v_i^T$$

SVD of $E_r = \sum_{r+1}^{n} \sigma_i u_i v_i^T$

b) $X$ is full rank, rank of $E_r = n - r$.

c) Operator norm is largest singular value

$$E_r = \sum_{r+1}^{n} \sigma_i u_i v_i^T$$

$\therefore, \|E_r\|_{op} = \sigma_{r+1}$

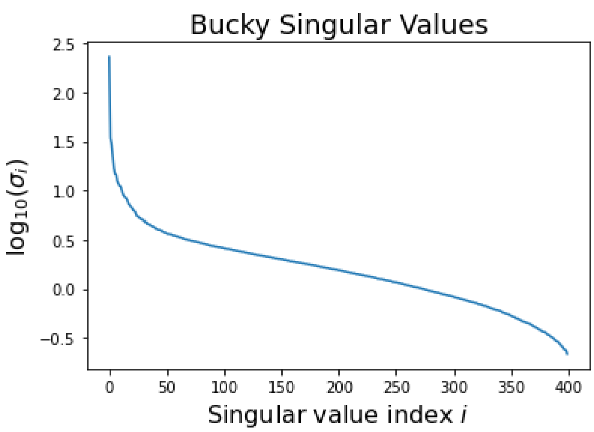
d) $X_r$ will thus be a good approx. to $X$ when $\sigma_r >> \sigma_{r+1}$

Scanned with CamScanner

In [1]:
```python
import numpy as np
from scipy.io import loadmat
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

# Load data for activity
#
in_data = loadmat('bucky.mat')
A = in_data['A']

##

# Load data for activity: Another option
# A = imageio.imread("Whateveryoulike.png")
# A = np.average(A[:,:,0:3], axis=2)/256

rows, cols = np.array(A.shape)
```

In [2]:
```python
# Display image
fig = plt.figure()
ax = fig.add_subplot(111)

ax.imshow(A,cmap='gray')
ax.set_axis_off()
plt.show()
```

In [3]:
```python
# Bucky's singular values

# Complete and uncomment line below
U,s,VT = np.linalg.svd(A)

fig = plt.figure()
ax = fig.add_subplot(111)
ax.plot(np.log10(s))
ax.set_xlabel('Singular value index $i$', fontsize=16)
ax.set_ylabel('$\log_{10}(\sigma_i)$', fontsize=16)
ax.set_title('Bucky Singular Values', fontsize=18)
plt.show()
```

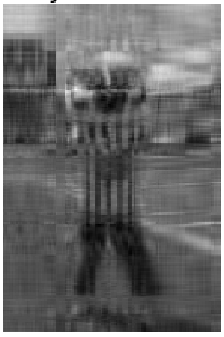



Q2 a) The approximate rank of A is 400 as s is a diagonal matrix with nearly 400 features as is seen from the graph above. Log is useful here as the first few singular values are clearly much larger than the ones that come after and as such, it would be difficult to plot them on a graph.

In [4]:

```python
# Find and display low-rank approximations

r_vals = np.array([10, 20, 50, 100 ])
err_fro = np.zeros(len(r_vals))

# display images of various rank approximations
for i, r in enumerate(r_vals):

    # Complete and uncomment two lines below
    Ur = U[:,:r]
    sr = np.diag(s[0:r])
    VTr = VT[:r]
    Ar = Ur@sr@VTr
    Er = A-Ar
    err_fro[i] = np.linalg.norm(Er,ord='fro')

    fig = plt.figure()
    ax = fig.add_subplot(111)
    ax.imshow(Ar,cmap='gray',interpolation='none')
    ax.set_axis_off()
    ax.set_title(['Bucky Rank =', str(r_vals[i])], fontsize=18)
    plt.show()
    print("Rank ", r, "approximation size: ", len(Ur)*len(Ur[0])+len(sr)*len(sr[0])+len(VTr)*len(VTr[0]))

# plot normalized error versus rank
norm_err = err_fro/np.linalg.norm(A,ord='fro')
print("Norm error:", norm_err)

fig = plt.figure()
ax = fig.add_subplot(111)
ax.stem(r_vals,norm_err)
ax.set_xlabel('Rank', fontsize=16)
ax.set_ylabel('Normalized error', fontsize=16)
plt.show()
```

['Bucky Rank =', '10']



```
Rank  10 approximation size:  10100
```

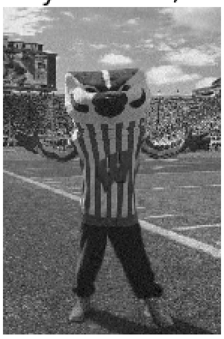
['Bucky Rank =', '20']

```
Rank  20 approximation size:  20400
```

['Bucky Rank =', '50']

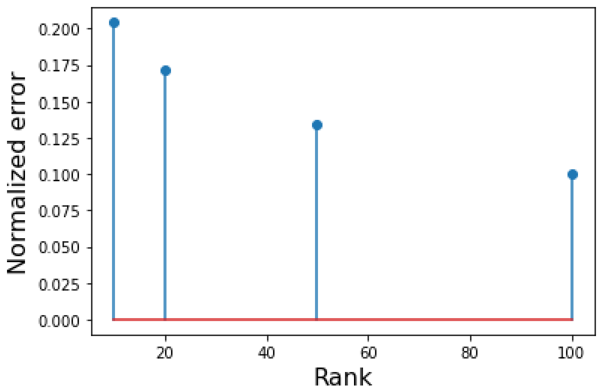
```
Rank  50 approximation size:  52500
```

['Bucky Rank =', '100']



```
Rank  100 approximation size:  110000
Norm error: [0.20435519 0.17175359 0.13413041 0.09988287]
```

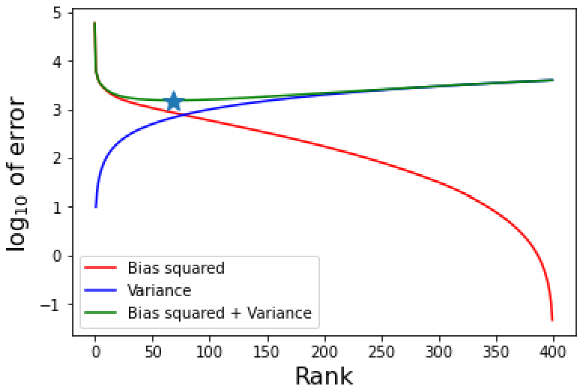
```
<ipython-input-4-806b14876249>:31: UserWarning: In Matplotlib 3.3 individual lines on a stem plot will be a
dded as a LineCollection instead of individual lines. This significantly improves the performance of a stem
plot. To remove this warning and switch to the new behaviour, set the "use_line_collection" keyword argumen
t to True.
  ax.stem(r_vals,norm_err)
```




Q2 b) As r goes up, the approximation becomes better in quality. This is intuitive as a higher r means the matrix is closer to the original matrix and is able to catch more of it's features

Q2 c) The sizes of each of the approximations can be found below their respective images.

In [5]:
```
# bias-variance tradeoff
num_sv = min(rows, cols)
bias_2 = np.zeros(num_sv)
ranks = np.arange(num_sv)

for r in range(num_sv):
    bias_2[r] = np.linalg.norm(s[r:num_sv])**2

sigma2 = 10
var = sigma2*ranks
#print(var)

fig = plt.figure()
ax = fig.add_subplot(111)
ax.plot(ranks,np.log10(bias_2),'r',label='Bias squared')
ax.plot(ranks[1:],np.log10(var[1:]),'b', label = 'Variance')
ax.plot(ranks,np.log10(bias_2+var),'g', label='Bias squared + Variance')
min_bias_plus_variance_index = np.argmin(np.log10(bias_2+var))
ax.plot(ranks[min_bias_plus_variance_index], np.log10(bias_2+var)[min_bias_plus_variance_index], '*', markers
ax.set_xlabel('Rank', fontsize=16)
ax.set_ylabel('$\log_{10}$ of error', fontsize=16)
ax.legend()
plt.show()

print("rank minimizing bias square and variance: ", ranks[min_bias_plus_variance_index])
```

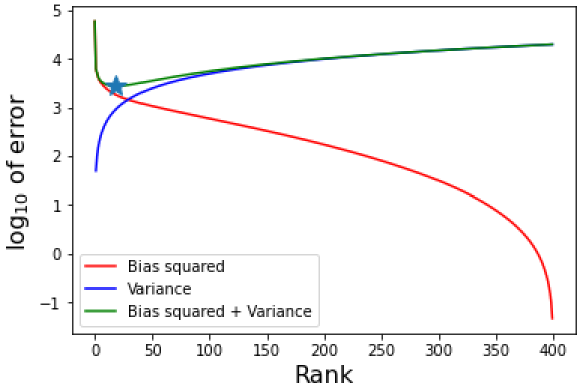



```
rank minimizing bias square and variance:  68
```

Q2 d) 1) when sigma2 = 10, rank = 68 minimizes (bias sqaure + variance)

In [6]:

```python
# bias-variance tradeoff
num_sv = min(rows, cols)
bias_2 = np.zeros(num_sv)
ranks = np.arange(num_sv)

for r in range(num_sv):
    bias_2[r] = np.linalg.norm(s[r:num_sv])**2

sigma2 = 50
var = sigma2*ranks
#print(var)

fig = plt.figure()
ax = fig.add_subplot(111)
ax.plot(ranks,np.log10(bias_2),'r',label='Bias squared')
ax.plot(ranks[1:],np.log10(var[1:]),'b', label = 'Variance')
ax.plot(ranks,np.log10(bias_2+var),'g', label='Bias squared + Variance')
min_bias_plus_variance_index = np.argmin(np.log10(bias_2+var))
ax.plot(ranks[min_bias_plus_variance_index], np.log10(bias_2+var)[min_bias_plus_variance_index], '*', markers
ax.set_xlabel('Rank', fontsize=16)
ax.set_ylabel('$\log_{10}$ of error', fontsize=16)
ax.legend()
plt.show()

print("rank minimizing bias square and variance: ", ranks[min_bias_plus_variance_index])
```




```
rank minimizing bias square and variance:  18
```

Q2 d) 2) when sigma2 = 50, rank = 18 minimizes (bias sqaure + variance)

In [ ]: