CS/ECE/ME 532

Unit 4 Practice Problems

1. In two dimensions, PCA with one component is equivalent to linear regression for fitting a line to data points. Explain why or why not.

2. Consider the low-rank approximation to a matrix $\boldsymbol{A} \approx \sum_{i=1}^{r} \sigma_i \boldsymbol{u}_i \boldsymbol{v}_i^T$. The term bias refers to $\sum_{i=1}^{r} \sigma_i$.

3. What algorithm is used for ranking pages on the internet. Select all that apply.

a) PageRank

b) Iterative Singular Value Thresholding

c) K-means

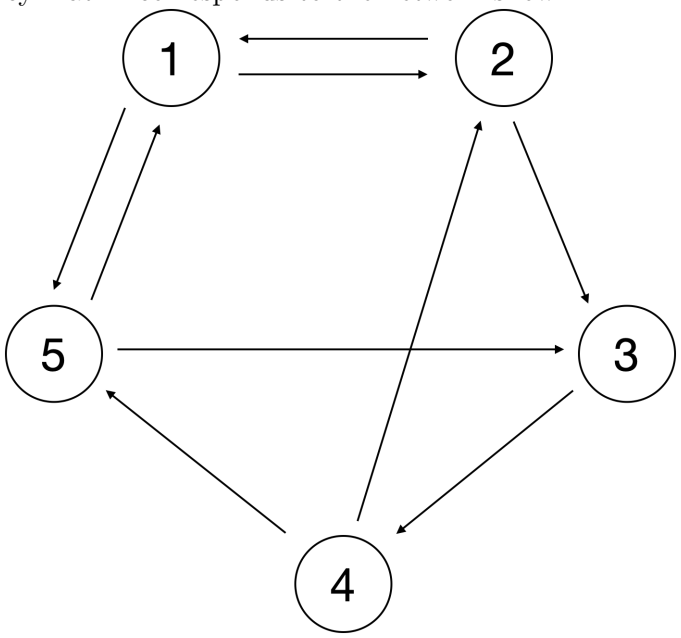
d) Power iterations

e) Spectral methods

4. Suppose $\boldsymbol{A}$ is a square, symmetric, positive definite matrix and has eigendecomposition $\boldsymbol{A} = \boldsymbol{E}\boldsymbol{\Lambda}\boldsymbol{E}^T$ and singular value decomposition $\boldsymbol{A} = \sum_{i=1}^{M} \sigma_i \boldsymbol{u}_i \boldsymbol{v}_i^T$. What is the relationship between the largest eigenvalue and the singular value decomposition?

5. You measure 500 three-dimensional vectors $\boldsymbol{x}_i, i = 1, 2, \ldots 500$. Explain how to find the best fitting line to the data.

6. Which normalized adjacency matrix corresponds to the network shown?



a) $\mathbf{A} = \begin{bmatrix} 0 & 0 & 1/2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1/3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 0 & 0 \\ 1/3 & 0 & 0 & 0 & 0 \end{bmatrix}$

b) $\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$

c) $\mathbf{A} = \begin{bmatrix} 0 & 1/2 & 1/2 & 0 & 1 \\ 0 & 0 & 1/2 & 1 & 0 \\ 1/2 & 0 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & 0 \end{bmatrix}$

d) $\mathbf{A} = \begin{bmatrix} 0 & 1/2 & 0 & 0 & 1/2 \\ 1/2 & 0 & 0 & 1/2 & 0 \\ 0 & 1/2 & 0 & 0 & 1/2 \\ 0 & 0 & 1 & 0 & 0 \\ 1/2 & 0 & 0 & 1/2 & 0 \end{bmatrix}$