Activity 12 – Regularization of LS and Principal Component Analysis

SVD for regularizing least squares:

$$\min_{\boldsymbol{w}} ||\boldsymbol{A}\boldsymbol{w} - \boldsymbol{y}||^2 \qquad \boldsymbol{w}^* = (\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T\boldsymbol{y}$$

What if $(\boldsymbol{A}^T\boldsymbol{A})$ is not invertible?

```
>> inv(A'*A)
Warning: Matrix is close to singular or badly scaled.
Results may be inaccurate. RCOND =  1.769972e-18.
```

Regularize!

Before: ridge regression: $(\boldsymbol{A}^T\boldsymbol{A} + \lambda\boldsymbol{I})^{-1}\boldsymbol{A}^T\boldsymbol{y}$

Today: truncate the SVD.

$$\boldsymbol{w}^* = \boldsymbol{V}\Sigma^{-1}\boldsymbol{U}^T\boldsymbol{y}$$

$$\begin{bmatrix} \frac{1}{\sigma_1} & & & 0 \\ & \frac{1}{\sigma_2} & & \\ & & \ddots & \\ 0 & & & \frac{1}{\sigma_p} \end{bmatrix}$$

problem: $\frac{1}{\sigma_p}$ huge if columns of A are close to linearly dependent

idea: set $\frac{1}{\sigma_p}, \ldots$ to zero, i.e, truncate the SVD.

$$\text{psuedoinverse}(\boldsymbol{A}) = \boldsymbol{V}\begin{bmatrix} \frac{1}{\sigma_1} & & & 0 \\ & \frac{1}{\sigma_2} & & \\ & & \ddots & \\ 0 & & & 0 \end{bmatrix}\boldsymbol{U}^T$$

numpy.linalg.pinv

numpy.linalg.pinv(*a, rcond=1e-15, hern*

Compute the (Moore-Penrose) pseu

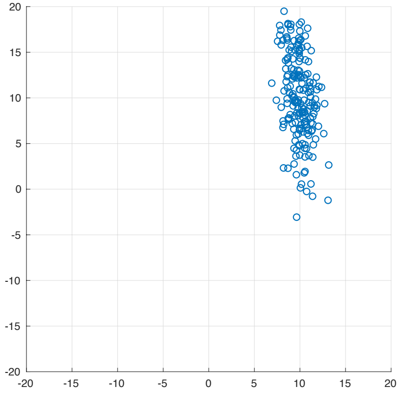
```
>> pinv(A)

ans =

   -0.0183    0.0005   -0
    0.0307    0.1449   -0
```

Principal Component Analysis:

input: data $\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots \in \mathbb{R}^2$

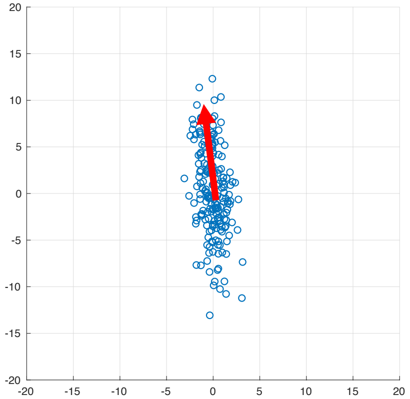


step 1: center data by removing mean

step 2: stack data as columns of matrix $\boldsymbol{X} \in \mathbb{R}^{2\times n}$

step 3: compute SVD of $\boldsymbol{X} = \boldsymbol{U}\Sigma\boldsymbol{V}^T$

1st principal component is first column of $\boldsymbol{U}$



PCA can be used to fit a line (or subspace) to data.

- PCA minimizes diagonal distance to line
- regression minimizes vertical