7. Given $\sigma_1 = 20$, $\sigma_2 = 4$, $\sigma_3 = 0.6$

& the remaining σ values are 0.

a) we have, $\min_w \|Aw - d\|_2^2 \Rightarrow w = (A^TA)^{-1}A^Td$

Thus, for $A = U\Sigma V^T$ as given

we have $\left((U\Sigma V^T)^T U\Sigma V^T\right)^{-1}(U\Sigma V^T)^T d$

$$= (V\Sigma^T \cancel{U^T}\cancel{U}\Sigma V^T)^{-1} V\Sigma^T U^T d$$

$$= (V\Sigma^2 V^T)^{-1} V\Sigma^T U^T d$$ as

$\Sigma = \Sigma^T$ as it is a diagonal matrix

$$= V^{T^{-1}}\Sigma^{-2}\cancel{V^{-1}}\cancel{V}\Sigma^T U^T d$$

$$= V\Sigma^{-1}U^T d$$

$$= \sum_{i=1}^{D} \frac{1}{\sigma_i} v_i (u_i^T d)$$

Thus, a rank 3 approx is

$$\sum_{i=1}^{3} \frac{1}{\sigma_i} v_i (u_i^T d)$$ for given definition of u_i & v_i.

That $$w_{t_1\text{-}SVD} = \frac{1}{\sigma_1} v_1 u_1^T d + \frac{1}{\sigma_2} v_2 u_2^T d + \frac{1}{\sigma_3} v_3 u_3^T d$$

$$= d\left(\frac{1}{20} v_1 u_1^T + \frac{1}{4} v_2 u_2^T + \frac{1}{0.6} v_3 u_3^T\right)$$

$$= (4u_2 - 3u_1)\left(0.05\, v_1 u_1^T + 0.25\, v_2 u_2^T + \frac{5}{3} v_3 u_3^T\right)$$

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b) $\| A w_{TR-SVD} - d \|_2^2$

$$= \left\| \left(4u_2 - 3v_T - 1\right)\left(0.05\, v_1 v_1^T + 0.25\, v_2 v_2^T + \frac{5}{3} v_3 u_3^T\right)^T \right\|_2$$

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