

CS/ECE/ME 532

Unit 1 Example Word Problems

1. Madison Outdoors LLC hires you to analyze customer preferences for hiking and biking trips in southern Wisconsin. You are to analyze four trips: Governor Dodge Hiking, Devil's Lake Hiking, Dane County Ironman Bike, and New Glarus Bike. Each is rated on a scale of 1 to 5 and the ratings for user i are stored in a 4-by-1 vector as follows

$$\mathbf{x}_i = \begin{bmatrix} \text{Gov Dodge} \\ \text{Devil's Lake} \\ \text{Ironman} \\ \text{New Glarus} \end{bmatrix}$$

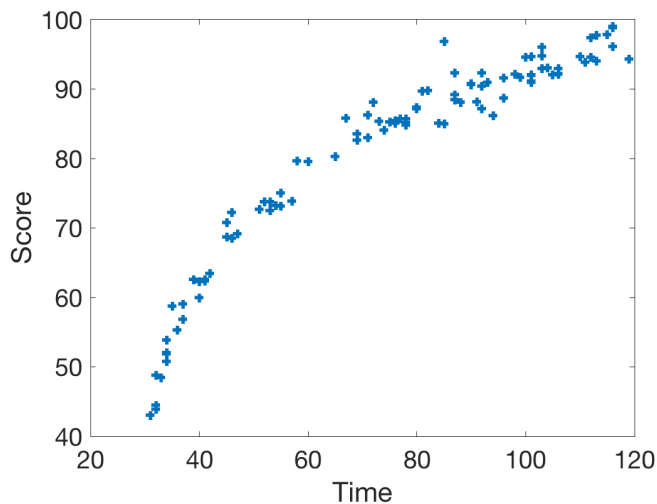
- a) Susan gives ratings: Gov Dodge = 3, Devil's Lake = 2, Ironman = 5, New Glarus = 4; Sally gives ratings: Gov Dodge = 4, Devil's Lake = 5, Ironman = 1, New Glarus = 2; and Sam gives ratings: Gov Dodge = 4, Devil's Lake = 4, Ironman = 3, New Glarus = 2. Put these ratings into a matrix \mathbf{R} where each column of \mathbf{R} represents the ratings \mathbf{x}_i for one person.

- b) Madison Outdoors want to describe their customer ratings in terms of the tastes of two representative customers. Customer H prefers hiking trips, and customer B prefers biking trips. The ratings or tastes of these representative customers are

$$\mathbf{x}_H = \begin{bmatrix} 4 \\ 4 \\ 2 \\ 2 \end{bmatrix}, \quad \mathbf{x}_B = \begin{bmatrix} 2 \\ 2 \\ 4 \\ 4 \end{bmatrix}$$

Let a ratings matrix \mathbf{R} for 10 customers be expressed in terms of these two taste profiles as $\mathbf{R} = \mathbf{TS}$ where \mathbf{T} is a matrix formed by the taste profiles. Define \mathbf{T} and the dimensions of \mathbf{R} , \mathbf{T} , \mathbf{S} .

2. You observe 100 exam scores and the time each student spent in minutes. Let $t_i, s_i, i = 1, 2, \dots, 100$ be the times and data, respectively, as shown in the figure below:



You hypothesize a model that the score is a logarithmic function of the time spent on the exam, that is, $s \approx \alpha \log_{10}(t) + \beta$ where $\mathbf{w} = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$ are unknown.

Form a column vector $\mathbf{s} = [s_1 \ s_2 \ \dots \ s_{100}]^T$. Express \mathbf{s} as the product of a matrix that depends on $t_i, i = 1, 2, \dots, 100$ and the model parameters \mathbf{w} . Indicate the dimensions of all matrices and vectors.

3. You are given a set of features $\mathbf{x}_i = (x_{1i}, x_{2i})$ and corresponding labels $y_i, i = 1, 2, \dots, 6$

$$\mathbf{x}_1 = (-2, 1), y_1 = 1 \quad \mathbf{x}_2 = (-1, 2), y_2 = -1$$

$$\mathbf{x}_3 = (1, 4), y_3 = -1 \quad \mathbf{x}_4 = (0, 3), y_4 = -1$$

$$\mathbf{x}_5 = (-3, 2), y_5 = 1 \quad \mathbf{x}_6 = (2, 4), y_6 = -1$$

Your boss tells you to use the linear classifier

$$\text{sign}(w_1x_1 + w_2x_2 + w_3)$$

Collect the classifier parameters into a column vector $\mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$ and use the given features and labels to write a system of linear equations that could be used to train the classifier.