1) a) $X = \begin{bmatrix} 36 & 72 & 90 & 54 \\ 40 & 80 & 100 & 60 \\ 20 & 40 & 50 & 30 \end{bmatrix}$

b) Let $w = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix}$

Thus, $\begin{bmatrix} 36 & 72 & 90 & 54 \\ 40 & 80 & 100 & 60 \\ 20 & 40 & 50 & 30 \end{bmatrix} = \begin{bmatrix} 9 \\ 10 \\ 5 \end{bmatrix} \begin{bmatrix} w_1 & w_2 & w_3 & w_4 \end{bmatrix}$

$\begin{bmatrix} 36 & 72 & 90 & 54 \\ 40 & 80 & 100 & 60 \\ 20 & 40 & 50 & 30 \end{bmatrix} = \begin{bmatrix} 9w_1 & 9w_2 & 9w_3 & 9w_4 \\ 10w_1 & 10w_2 & 10w_3 & 10w_4 \\ 5w_1 & 5w_2 & 5w_3 & 5w_4 \end{bmatrix}$

from analysis,

$9w_1 = 36 \Rightarrow w_1 = 4$ $\quad 9w_3 = 90 \Rightarrow w_3 = 10$

$9w_2 = 72 \Rightarrow w_2 = 8$ $\quad 9w_4 = 54 \Rightarrow w_4 = 6$

Thus $w = \begin{bmatrix} 9 \\ 8 \\ 10 \\ 6 \end{bmatrix}$

c) Let Brianne's rating matrix be $\begin{bmatrix} x \\ 30 \\ y \end{bmatrix}$

Thus $\begin{bmatrix} x \\ 30 \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ 10 \\ 5 \end{bmatrix} \begin{bmatrix} w_B \end{bmatrix}$

$x = 9w_B$ Thus $w_B = 3$

$30 = 10w_B$

$y = 5w_B$

Thus, $x = 9(3) = 27$

$y = 5(3) = 15$

Brianna's rating for CS760 would be 27

" " for Math 521 would be 15

2.a) $T = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & -1 \\ 1 & 1 \end{bmatrix}$ $W = \begin{bmatrix} 8 & 6 & 4 & 5 & 4 \\ 2 & 3 & 3 & -2 & -1 \end{bmatrix}$

$X = TW$

$= \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 8 & 6 & 4 & 5 & 4 \\ 2 & 3 & 3 & -2 & -1 \end{bmatrix}$

$= \begin{bmatrix} 10 & 9 & 7 & 3 & 3 \\ 6 & 3 & 1 & 7 & 5 \\ 6 & 3 & 1 & 7 & 5 \\ 10 & 9 & 7 & 3 & 3 \end{bmatrix}$

b) $X = t_1 w_1^T + t_2 w_2^T$

$t_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ $t_2 = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$

$w_1 = \begin{bmatrix} 8 \\ 6 \\ 4 \\ 5 \\ 4 \end{bmatrix}$ $w_2 = \begin{bmatrix} 2 \\ 3 \\ 3 \\ -2 \\ -1 \end{bmatrix}$

3. a) $X = \begin{bmatrix} A & B \\ C^T & d^T \end{bmatrix}$

A is 2×3 $\quad A^T$ is 3×2

B is 2×2 $\quad B^T$ is 2×2

C^T is 1×3 $\quad c$ is 3×1

d^T is 1×2 $\quad d$ is 2×1

$X^T = \begin{bmatrix} A^T & c \\ B^T & d \end{bmatrix}$ (we transpose each block & then take transpose of each sub-matrix)

$R = XX^T$

$= \begin{bmatrix} A & B \\ C^T & d^T \end{bmatrix} \begin{bmatrix} A^T & c \\ B^T & d \end{bmatrix}$

$= \begin{bmatrix} AA^T + BB^T & Ac + Bd \\ C^TA^T + d^TB^T & C^TC + d^Td \end{bmatrix}$

b) Since $(AA^T + BB^T)$ is 2×2, $(Ac + Bd)$ is 2×1, $(C^TA^T + d^TB^T)$ is 1×2 and $(C^TC + d^Td)$ is 1×1 in dimensions

we get that R has dimension 3×3.

4. a) $y = Ax$

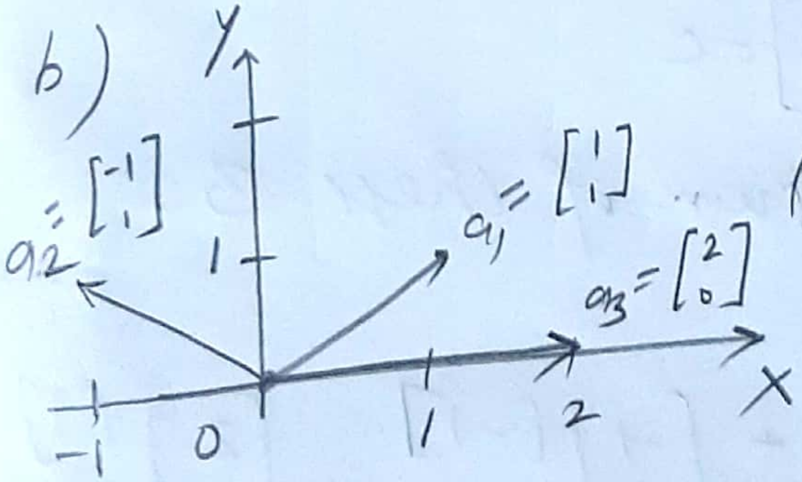
$A = [a_1 \ a_2 \ \cdots \ a_n]$

let $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$

$y = Ax = [a_1 \ a_2 \ \cdots \ a_n] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$

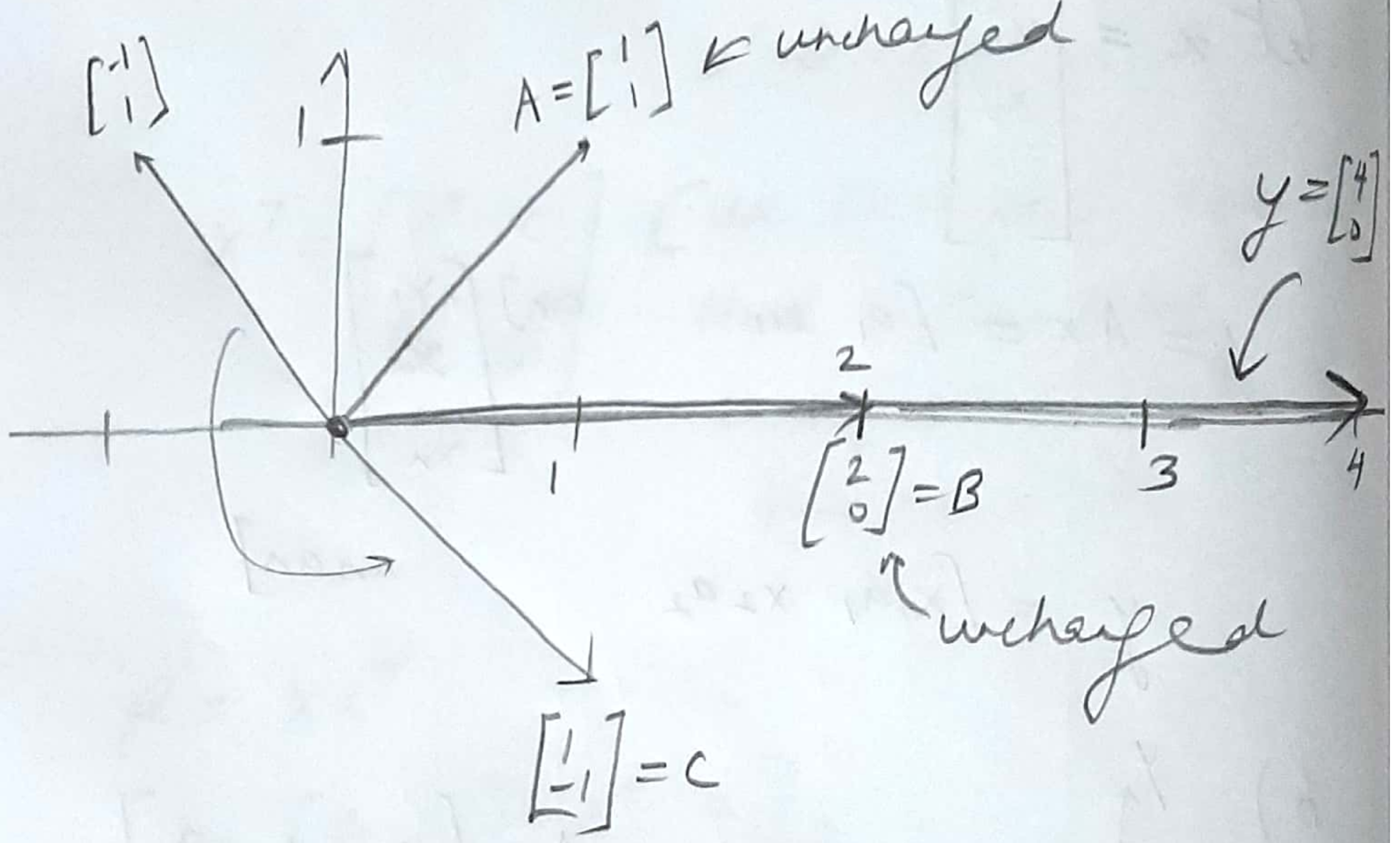
$y = [x_1 a_1 \ x_2 a_2 \ \cdots \ x_n a_n]$

b)



$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 1 & 0 \end{bmatrix}$

c) $A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 1 & 0 \end{bmatrix}$ $\quad x = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$



$y = Ax$ is the sum of these 3 vectors

$$y = \begin{bmatrix} 1 \\ 1 \end{bmatrix}[1] + \begin{bmatrix} -1 \\ 1 \end{bmatrix}[-1] + \begin{bmatrix} 2 \\ 0 \end{bmatrix}[1]$$

$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$