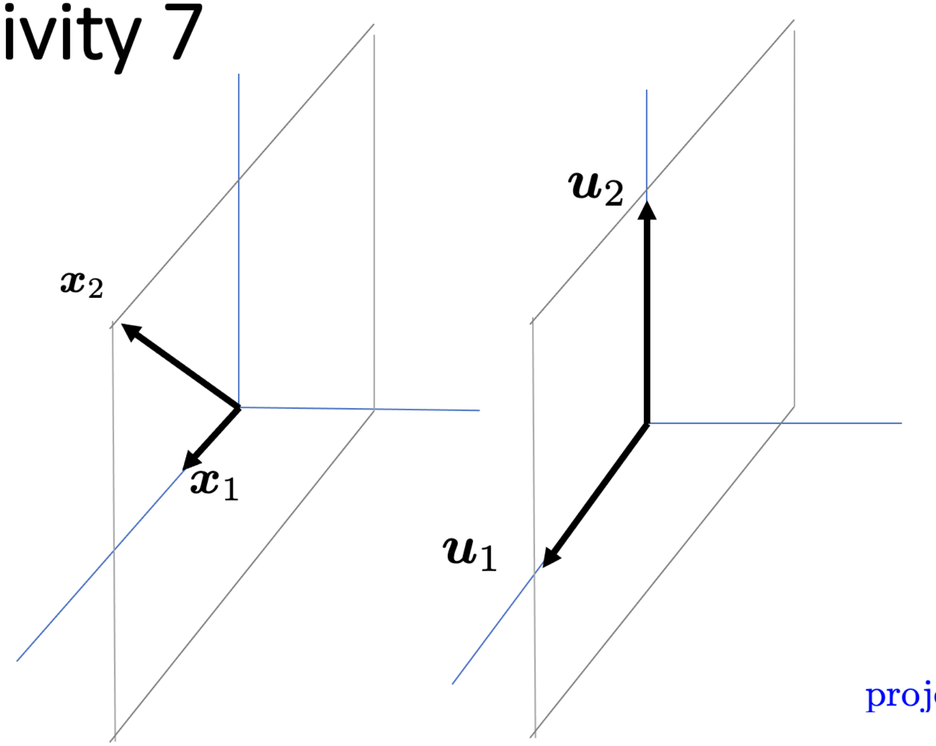
CS/ECE/ME 532 Activity 7

Gram-Schmidt Orthogonalization and Projections



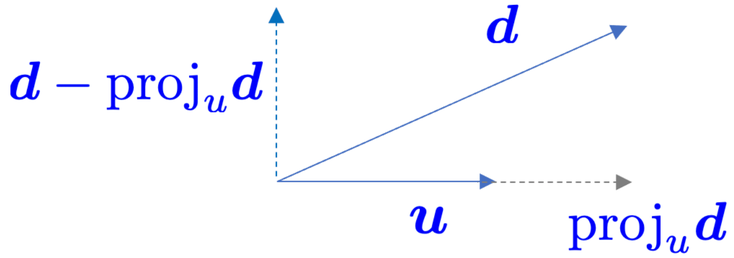
1. set $\tilde{\boldsymbol{u}}_1 = \boldsymbol{x}_1$
2. normalize: $\boldsymbol{u}_1 = \frac{\tilde{\boldsymbol{u}}_1}{||\tilde{\boldsymbol{u}}_1||_2}$
3. set $\tilde{\boldsymbol{u}}_2 = \boldsymbol{x}_2 - \text{proj}_{u_1} \boldsymbol{x}_2$
4. normalize: $\boldsymbol{u}_2 = \frac{\tilde{\boldsymbol{u}}_2}{||\tilde{\boldsymbol{u}}_2||_2}$
5. set $\tilde{\boldsymbol{u}}_3 = \boldsymbol{x}_3 - \text{proj}_{u_1} \boldsymbol{x}_3 - \text{proj}_{u_2} \boldsymbol{x}_3$

$\vdots$

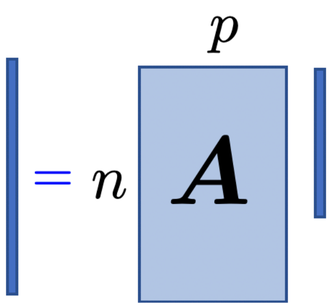
$$\text{proj}_u \boldsymbol{d} = \boldsymbol{u}(\boldsymbol{u}^T \boldsymbol{d})$$

projection of $\boldsymbol{d}$ onto $\boldsymbol{u}$

amount of $\boldsymbol{d}$ in the direction of $\boldsymbol{u}$



$$\mathbf{d} = \mathbf{A}\mathbf{w}$$



$$\hat{\mathbf{d}} = \mathbf{A}\mathbf{w} = \mathbf{A}(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{d} = \mathbf{P_A}\mathbf{d}$$
$$= \mathbf{U}\mathbf{U}^T\mathbf{d} = \mathbf{P_U}\mathbf{d}$$

$$\mathbf{e} = \mathbf{d} - \hat{\mathbf{d}} = \mathbf{d} - \mathbf{P_A}\mathbf{d}$$
$$= (\mathbf{I} - \mathbf{P_A})\,\mathbf{d}$$