Asgn 6 ECE 532 Ajar Deep Hayla

1. Since A is rank 1 and symmetric, its singular value decomp is

$$A = \lambda_1 v_1 v_1^T$$

$$b_1 = \frac{Ab_0}{\|Ab_0\|_2} = \frac{\lambda v_1 v_1^T b_0}{\|\lambda_1 v_1 v_1^T b_0\|_2}$$

as $\|\lambda_1 v_1 v_1^T b_0\| = |\lambda v_1^T b_0|$ so

$$b_1 = v_1 \operatorname{sign}\{v_1^T b_0\}$$

Thus in one iteration power method converges to the correct singular vector. The sign doesn't matter, since if $v_1$ is a singular vec, then $-v_1$ is also a singular vector.

2. a) Data appears to be concentrated along a line, as even more so, in a plane but since it does not include the origin it's not a subspace.

b) We can recenter the data by removing the mean value from every datapoint. This will center the cloud on the origin & a line/plane approximation will then include the origin.

c) Yes, a line through the origin captures the majority of the variability in data. A plane captures even more.

d) a = V(:, 1) or a = np.transpose(vT)[:, 0]]

see plot.

e) $x_{xi}$ is the ith row of matrix $X_z$.
The rank-1 approx to $X_z$ is $X_z \approx U_1 * \sigma_1 * V_1^T$
where $U_1$ & $V_1$ are the left & right singular vectors associated with the largest singular value $\sigma_1$.
Thus $w_i = [U_1]_i \sigma_1$ where $[U_1]_i$ is the ith entry in $U_1$.

f) $b$ is the mean that was removed from original data. Note $x_i = x_{xi} + b$

g) We have $X = \sum_{i=1}^{3} \sigma_i u_i v_i^T$ and $X_1 = \sigma_1 u_1 v_1^T$
so $E = \sum_{i=2}^{3} \sigma_i u_i v_i^T$.

Also, $\|E\|_F^2 = \sigma_2^2 + \sigma_3^2$.

h) following prev. steps gives us $\|E\|_F^2 = \sigma_3^2$.

i) We can write $X_z \approx U_1 * \sigma_1 * V_1^T + U_2 * \sigma_2 * V_2^T$
Thus extracting the ith row of $X_z$ and rewriting it as a column vector we have

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$X_{x_i} = v_1 \times \sigma_1 \times |U_1|_i + v_2 \times \sigma_2 \times |U_2|_i$

where $|U_1|_i$ is the ith entry in $U_1$ &

$|U_2|_i$ is the ith entry in $U_2$.

Hence $a_1 = v_1$ & $a_2 = v_2$

we get $w_{1i} = |U_1|_i \sigma_1$ and

$w_{2i} = |U_2|_i \sigma_2$

i) $||E||_F^2 = \sigma_3^2$ from before.

where $\sigma_3$ is the smallest singular value of given 3-1000 matrix $X_2$.

k) Rank-1 squared error is 626.69.

Rank-2 Absolute error is 152.95.

Normalizing gives us relative squared errors of 0.023 & 0.006 respectively.

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3. We get average error rate of 0.1116 for SVD (truncated).

We get average error rate of 0.048 for ridge regression

Thus ridge regression is better here.