CS/ECE/ME532 Period 1 Activity

1) Let $\boldsymbol{x} = \begin{bmatrix} 1 \\ b \\ 3 \end{bmatrix}$ and $\boldsymbol{w} = \begin{bmatrix} c \\ 4 \\ d \end{bmatrix}$.

   a) Write out and evaluate the inner product $\boldsymbol{x}^T\boldsymbol{w}$.

   b) Now write out and evaluate the inner product $\boldsymbol{w}^T\boldsymbol{x}$.

SOLUTION:

a) $\mathbf{x}^T\mathbf{w} = \begin{bmatrix} 1 & b & 3 \end{bmatrix} \begin{bmatrix} c \\ 4 \\ d \end{bmatrix} = c + 4b + 3d$

b) $\mathbf{w}^T\mathbf{x} = \begin{bmatrix} c & 4 & d \end{bmatrix} \begin{bmatrix} 1 \\ b \\ 3 \end{bmatrix} = c + 4b + 3d$

2) Consider the second-order polynomial $y = 2(x-1)^2$.

   a) Write y as the inner product of a vector $\boldsymbol{x}$ that depends on the value x and a vector $\boldsymbol{w}$ containing the polynomial coefficients. That is, write $y = \boldsymbol{x}^T\boldsymbol{w}$. Define $\boldsymbol{x}$ and $\boldsymbol{w}$.

   b) Suppose you have five (arbitrary) values $y_i = 2(x_i - 1)^2,\ i = 1, 2, \ldots, 5$. Write the vector $\boldsymbol{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_5 \end{bmatrix} = \boldsymbol{X}\boldsymbol{w}$ and define the matrix $\boldsymbol{X}$ in terms of the x_i.

SOLUTION:

a) $y = 2x^2 - 4x + 2$, so let $\boldsymbol{x} = \begin{bmatrix} x^2 \\ x \\ 1 \end{bmatrix}$ and $\boldsymbol{w} = \begin{bmatrix} 2 \\ -4 \\ 2 \end{bmatrix}$

b) $\boldsymbol{X} = \begin{bmatrix} x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \\ x_3^2 & x_3 & 1 \\ x_4^2 & x_4 & 1 \\ x_5^2 & x_5 & 1 \end{bmatrix}$

3) Food involves fats, proteins and carbohydrates. There are 9 calories for every gram of fat, 4 calories for every gram of protein, and 4 calories for every gram of carbohydrates.

a) Define a vector $\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ where x_1 is the number of grams of fat, x_2 is the number of grams of protein, and x_3 is the number of grams of carbohydrate in a serving. Find the vector $\boldsymbol{w}$ so that the number of calories in a serving may be expressed as $\boldsymbol{x}^T\boldsymbol{w}$.

b) Write the calories per serving of four breakfast cereals in a vector $\boldsymbol{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$ as a product of a matrix $\boldsymbol{X}$ and vector $\boldsymbol{w}$ (that is, $\boldsymbol{y} = \boldsymbol{X}\boldsymbol{w}$). y_i is the number of calories per serving in cereal i where each cereal has the following data per serving:

Cereal 1: 1 gram fat, 8 grams protein, 44 grams carbohydrate
Cereal 2: 0.5 grams fat, 2 grams protein, 25 grams carbohydrate
Cereal 3: 1.3 grams fat, 2.7 grams protein, 29.3 grams carbohydrate
Cereal 4: 9 grams fat, 4 grams protein, 16 grams carbohydrate

Identify both $\boldsymbol{X}$ and $\boldsymbol{w}$.

SOLUTION:

a) calories= $9x_1 + 4x_2 + 4x_3$, so let $\boldsymbol{w} = \begin{bmatrix} 9 \\ 4 \\ 4 \end{bmatrix}$

b) Using $\boldsymbol{w}$ defined above, we obtain $\boldsymbol{X} = \begin{bmatrix} 1 & 8 & 44 \\ 0.5 & 2 & 25 \\ 1.3 & 2.7 & 29.3 \\ 9 & 4 & 16 \end{bmatrix}$