

2a)

```
In [1]: import numpy as np
import matplotlib.pyplot as plt
from scipy.io import loadmat
dataset = loadmat('face_emotion_data.mat')

X, y = dataset['X'], dataset['y']
n, p = np.shape(X)

y[y==1] = 0 # use 0/1 for Labels instead of -1/+1
X = np.hstack((np.ones((n,1)), X)) # append a column of ones

def kernel(X, sigma):
    n, p = np.shape(X)

    distsquared = np.zeros((n,n), dtype=float)

    for i in range(n):
        for j in range(n):
            dist = np.linalg.norm(X[i,:]-X[j,:])
            distsquared[i,j]=dist*dist

    K = np.exp(-1*distsquared/(2*sigma**2))
    return K

def train(X, y, lam, sigma):

    K = kernel(X,sigma)

    return np.linalg.inv(K+lam*np.identity(len(K)))@y

def errorCrossValidation(Xb, yb, W, V):
    H =logsig(np.hstack((np.ones((np.shape(Xb)[0],1)), Xb@W)))
    Yhat = logsig(H@V)
    # ERROR CALC

    error = np.mean(abs(np.round(Yhat[:,0])-yb[:,0])))
    return error

sigmas = np.linspace(0, 50, 100) print(sigmas) errors = []

for sigma in sigmas: train_ind = np.arange(128) alpha = train(X, y, 0.5, sigma) errors.append(error) print(error)

plt.figure() plt.plot(sigmas, errors)
```

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In [2]: def test(X, y, Xv, yv, alpha):
    numAlpha = len(alpha)

    K = kernel(X,sigma)

    y_hat = K@alpha
    y_hat_thresh = np.sign(y_hat)
    y_hat_thresh[y_hat_thresh>0.5] = 1
    y_hat_thresh[y_hat_thresh<=0.5] = 0

    error = np.sum(np.abs(y_hat_thresh-yv))

    return error/n
```

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2b)

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In [3]: sigmas = np.linspace(0.1, 50, 100)

errors = []
accuracies = []
lamda = 0.5
for sigma in sigmas:
    K = kernel(X,sigma)
    alpha = train(X,y, lamda, sigma)
    Yhat = abs(np.round(K@alpha))

    correct = [1 if m[0]==m[1] else 0 for m in np.hstack((Yhat, y))]

    accuracy = sum(correct)/len(correct)

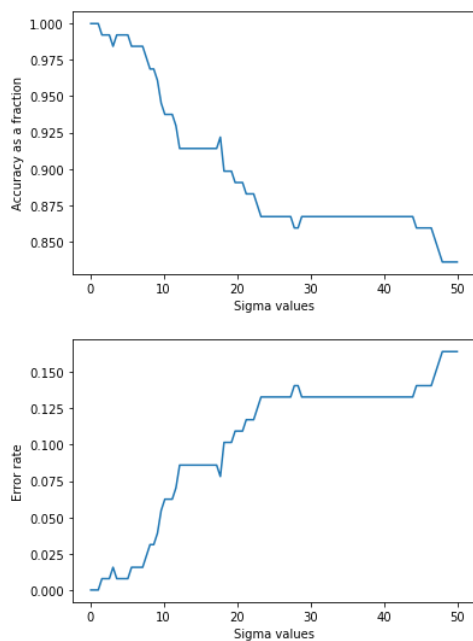
    accuracies.append(accuracy)
    errors.append(1-accuracy)

plt.figure()
plt.xlabel("Sigma values")
plt.ylabel("Accuracy as a fraction")
plt.plot(sigmas, accuracies)

plt.figure()
plt.xlabel("Sigma values")
plt.ylabel("Error rate")
plt.plot(sigmas, errors)

```

Out[3]: [



2c)

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In [4]: L = 100
lamda = 0.5
sigma = 0.1
err_total = []
sigmas = np.linspace(0.1, 50, 100)
for sigma in sigmas:
    err_somerun = []
    err_somerun_Train = []
    err_somerun_Test = []
    for i in range(7):
        start = i*16
        end = (i+1)*16

        X_train = np.vstack((X[0:start, :], X[end:-1, :]))
        y_train = np.vstack((y[0:start, :], y[end:-1, :]))

        X_test = X[start:end, :]
        y_test = y[start:end, :]

        alpha1 = train(X_train, y_train, lamda, sigma)
        alpha2 = train(X_test, y_test, lamda, sigma)

        K = kernel(X, sigma)

        Yhat1 = abs(np.round(K@np.vstack((alpha1, np.ones((17, alpha1.shape[1]))))))

        correct_vec = [1 if m[0]==m[1] else 0 for m in np.hstack((X.T@Yhat1, X_test.T@y_test))]
        accuracy = sum(correct_vec)/len(correct_vec)

        accuracies.append(accuracy)
        errors.append(1-accuracy)

    err_somerun.append(errors)

print(" run with sigma: ", sigma, "error rate: ", np.mean(err_somerun), " and accuracy rate: ", 1-np.mean(err_somerun))
err_total.append(err_somerun)

print("mean error rate over 100 iterations", np.mean(err_total))

```

```

run with sigma: 0.1 error rate: 0.16004672897196262 and accuracy rate: 0.8399532710280374
run with sigma: 0.604040404040404 error rate: 0.21162280701754385 and accuracy rate: 0.7883771929824561
run with sigma: 1.1080808080808082 error rate: 0.2572314049586777 and accuracy rate: 0.7427685950413223
run with sigma: 1.6121212121212123 error rate: 0.2978515625 and accuracy rate: 0.7021484375
run with sigma: 2.1161616161616164 error rate: 0.33425925925925926 and accuracy rate: 0.6657407407407407
run with sigma: 2.6202020202020204 error rate: 0.3670774647887324 and accuracy rate: 0.6329225352112676
run with sigma: 3.1242424242424245 error rate: 0.39681208053691275 and accuracy rate: 0.6031879194630873
run with sigma: 3.6282828282828286 error rate: 0.4238782051282051 and accuracy rate: 0.5761217948717949
run with sigma: 4.132323232323232 error rate: 0.4486196319018405 and accuracy rate: 0.5513803680981595
run with sigma: 4.636363636363637 error rate: 0.4713235294117647 and accuracy rate: 0.5286764705882353
run with sigma: 5.1404040404040404 error rate: 0.4922316384180791 and accuracy rate: 0.5077683615819208
run with sigma: 5.6444444444444444 error rate: 0.5115489130434783 and accuracy rate: 0.48845108695652173
run with sigma: 6.148484848484848 error rate: 0.5294502617801047 and accuracy rate: 0.4705497382198953
run with sigma: 6.652525252525253 error rate: 0.5460858585858586 and accuracy rate: 0.45391414141414144
run with sigma: 7.156565656565657 error rate: 0.5615853658536586 and accuracy rate: 0.4384146341463414
run with sigma: 7.660606060606060 error rate: 0.5760613207547169 and accuracy rate: 0.42393867924528306
run with sigma: 8.164646464646465 error rate: 0.5896118721461188 and accuracy rate: 0.41038812785388123
run with sigma: 8.668686868686868 error rate: 0.6023230088495575 and accuracy rate: 0.3976769911504425
run with sigma: 9.172727272727272 error rate: 0.6142703862660944 and accuracy rate: 0.3857296137339056
run with sigma: 9.676767676767676 error rate: 0.6255208333333333 and accuracy rate: 0.3744791666666667
run with sigma: 10.180808080808081 error rate: 0.6361336032388664 and accuracy rate: 0.36386639676113364
run with sigma: 10.684848484848485 error rate: 0.6461614173228346 and accuracy rate: 0.3538385826771654
run with sigma: 11.188888888888888 error rate: 0.6556513409961686 and accuracy rate: 0.3443486590038314
run with sigma: 11.692929292929293 error rate: 0.6646455223880597 and accuracy rate: 0.33535447761194026
run with sigma: 12.196969696969697 error rate: 0.6731818181818182 and accuracy rate: 0.3268181818181818
run with sigma: 12.701010101010102 error rate: 0.6812943262411347 and accuracy rate: 0.31870567375886527
run with sigma: 13.205050505050506 error rate: 0.6890138408304498 and accuracy rate: 0.3109861591695502
run with sigma: 13.709090909090909 error rate: 0.696368243242432 and accuracy rate: 0.3036317567567568
run with sigma: 14.213131313131314 error rate: 0.7033828382838284 and accuracy rate: 0.2966171617161716
run with sigma: 14.717171717171718 error rate: 0.7100806451612903 and accuracy rate: 0.2899193548387097
run with sigma: 15.221212121212122 error rate: 0.7164826498422713 and accuracy rate: 0.2835173501577287
run with sigma: 15.725252525252525 error rate: 0.722608024691358 and accuracy rate: 0.277391975308642
run with sigma: 16.229292929292929 error rate: 0.7284743202416919 and accuracy rate: 0.2715256797583081
run with sigma: 16.733333333333334 error rate: 0.7340976331360947 and accuracy rate: 0.26590236686390534
run with sigma: 17.237373737373737 error rate: 0.7394927536231884 and accuracy rate: 0.26050724637681155
run with sigma: 17.741414141414143 error rate: 0.7446732954545454 and accuracy rate: 0.2553267045454546
run with sigma: 18.245454545454545 error rate: 0.7496518105849582 and accuracy rate: 0.2503481894150418
run with sigma: 18.749494949494952 error rate: 0.7544398907103825 and accuracy rate: 0.24556010928961747
run with sigma: 19.253535353535355 error rate: 0.7590482573726541 and accuracy rate: 0.24095174262734587
run with sigma: 19.757575757575757 error rate: 0.7634868421052632 and accuracy rate: 0.23651315789473681
run with sigma: 20.261616161616164 error rate: 0.7677648578811369 and accuracy rate: 0.2322351421188631
run with sigma: 20.765656565656567 error rate: 0.7718908629441624 and accuracy rate: 0.22810913705583757
run with sigma: 21.269696969696973 error rate: 0.7758728179551122 and accuracy rate: 0.22412718204488878
run with sigma: 21.773737373737376 error rate: 0.7797181372549019 and accuracy rate: 0.2202818627450981
run with sigma: 22.277777777777778 error rate: 0.783433734939759 and accuracy rate: 0.21656626506024101
run with sigma: 22.781818181818185 error rate: 0.7870260663507109 and accuracy rate: 0.21297393364928907
run with sigma: 23.285858585858588 error rate: 0.7905011655011654 and accuracy rate: 0.20949883449883455
run with sigma: 23.789898989898994 error rate: 0.7938646788990825 and accuracy rate: 0.20613532110091748
run with sigma: 24.293939393939397 error rate: 0.7971218961625283 and accuracy rate: 0.20287810383747173
run with sigma: 24.797979797979798 error rate: 0.8002777777777778 and accuracy rate: 0.19972222222222225
run with sigma: 25.302020202020206 error rate: 0.803369803063457 and accuracy rate: 0.1966630196936543
run with sigma: 25.806060606060606 error rate: 0.8063038793103449 and accuracy rate: 0.19369612068965514
run with sigma: 26.310101010101015 error rate: 0.8091825902335457 and accuracy rate: 0.19081740976645434
run with sigma: 26.814141414141417 error rate: 0.8119769874476988 and accuracy rate: 0.18802301255230125
run with sigma: 27.318181818181818 error rate: 0.8146907216494845 and accuracy rate: 0.18538927835051547
run with sigma: 27.822222222222226 error rate: 0.8173272357723578 and accuracy rate: 0.18267276422764223
run with sigma: 28.326262626262626 error rate: 0.8198897795591182 and accuracy rate: 0.18011022044088176
run with sigma: 28.830303030303032 error rate: 0.8223814229249012 and accuracy rate: 0.17761857707509876
run with sigma: 29.334343434343438 error rate: 0.8248050682261209 and accuracy rate: 0.17519493177387913
run with sigma: 29.83838383838384 error rate: 0.8271634615384615 and accuracy rate: 0.17283653846153846
run with sigma: 30.342424242424244 error rate: 0.8294592030360531 and accuracy rate: 0.17054079696394686
run with sigma: 30.846464646464646 error rate: 0.8316947565543071 and accuracy rate: 0.16830524344569286

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run with sigma: 31.350505050505053 error rate: 0.8338724584103512 and accuracy rate: 0.16612754158964882
run with sigma: 31.854545454545456 error rate: 0.8359945255474452 and accuracy rate: 0.16400547445255476
run with sigma: 32.35858585858586 error rate: 0.8380630630630631 and accuracy rate: 0.1619369369369369
run with sigma: 32.862626262626264 error rate: 0.8400800711743772 and accuracy rate: 0.15991992882562278
run with sigma: 33.36666666666667 error rate: 0.8420474516695958 and accuracy rate: 0.15795254833040417
run with sigma: 33.87070707070708 error rate: 0.8439670138888888 and accuracy rate: 0.15603298611111116
run with sigma: 34.37474747474748 error rate: 0.8458404802744426 and accuracy rate: 0.15415951972555741
run with sigma: 34.87878787878788 error rate: 0.8476694915254237 and accuracy rate: 0.1523305084745763
run with sigma: 35.382828282828285 error rate: 0.8494556113902848 and accuracy rate: 0.1505443886097152
run with sigma: 35.88686868686869 error rate: 0.8512003311258278 and accuracy rate: 0.14879966887417218
run with sigma: 36.390909090909091 error rate: 0.8529050736497545 and accuracy rate: 0.14709492635024546
run with sigma: 36.894949494949495 error rate: 0.8545711974110033 and accuracy rate: 0.1454288025889967
run with sigma: 37.398989898989899 error rate: 0.8562 and accuracy rate: 0.14380000000000004
run with sigma: 37.903030303030306 error rate: 0.8577927215189873 and accuracy rate: 0.14220727848101267

run with sigma: 38.40707070707071 error rate: 0.8593505477308294 and accuracy rate: 0.14064945226917058
run with sigma: 38.911111111111111 error rate: 0.860874613003096 and accuracy rate: 0.13912538699690402
run with sigma: 39.41515151515152 error rate: 0.8623660030627871 and accuracy rate: 0.1376339969372129
run with sigma: 39.919191919191924 error rate: 0.8638257575757575 and accuracy rate: 0.13617424242424248
run with sigma: 40.42323232323233 error rate: 0.8652548725637181 and accuracy rate: 0.13474512743628186
run with sigma: 40.92727272727273 error rate: 0.8666543026706232 and accuracy rate: 0.13334569732937684
run with sigma: 41.43131313131313 error rate: 0.8680249632892805 and accuracy rate: 0.13197503671071953
run with sigma: 41.93535353535354 error rate: 0.8693677325581395 and accuracy rate: 0.13063226744186052
run with sigma: 42.439393939393945 error rate: 0.87068345323741 and accuracy rate: 0.12931654676258997
run with sigma: 42.94343434343435 error rate: 0.8719729344729344 and accuracy rate: 0.12802706552706555
run with sigma: 43.44747474747475 error rate: 0.8732369534555712 and accuracy rate: 0.12676304654442883
run with sigma: 43.95151515151515 error rate: 0.8744762569832403 and accuracy rate: 0.12552374301675973
run with sigma: 44.45555555555556 error rate: 0.8756915629322268 and accuracy rate: 0.12430843706777317
run with sigma: 44.95959595959596 error rate: 0.8768835616438356 and accuracy rate: 0.12311643835616437
run with sigma: 45.46363636363637 error rate: 0.8780529172320217 and accuracy rate: 0.12194708276797828
run with sigma: 45.96767676767677 error rate: 0.8792002688172043 and accuracy rate: 0.12079973118279574
run with sigma: 46.471717171717174 error rate: 0.8803262316910786 and accuracy rate: 0.1196737683089214
run with sigma: 46.97575757575758 error rate: 0.8814313984168866 and accuracy rate: 0.11856860158311344
run with sigma: 47.479797979797986 error rate: 0.882516339869281 and accuracy rate: 0.11748366013071898
run with sigma: 47.98383838383839 error rate: 0.8835816062176166 and accuracy rate: 0.11641839378238339
run with sigma: 48.48787878787879 error rate: 0.8846277278562259 and accuracy rate: 0.1153722721437741
run with sigma: 48.991919191919195 error rate: 0.8856552162849872 and accuracy rate: 0.11434478371501278
run with sigma: 49.4959595959596 error rate: 0.8866645649432535 and accuracy rate: 0.11333543505674648
run with sigma: 50.0 error rate: 0.88765625 and accuracy rate: 0.11234374999999996
mean error rate over 100 iterations 0.88765625

```

We cannot achieve perfect accuracy. Even with 100 trials, the error rate was not 0. Thus we infer that the error rate for cross validation does not go to 0. This makes sense intuitively as the hold out set of the data could have features that the test set does not, thus leading to errors.

In []: