Classifying Data and Matrix Multiplication

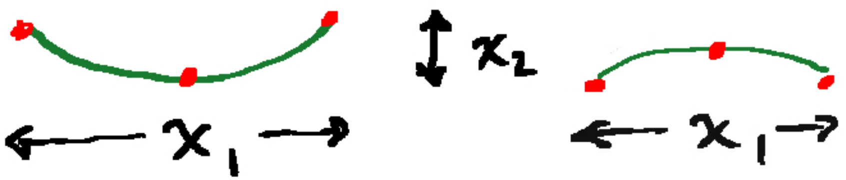
Objectives

- introduce the classification problem
- define linear classifiers
- write supervised learning of linear classifiers using matrices

Classification is assigning a category to data

Example: Decide

- extract features based on a "model"

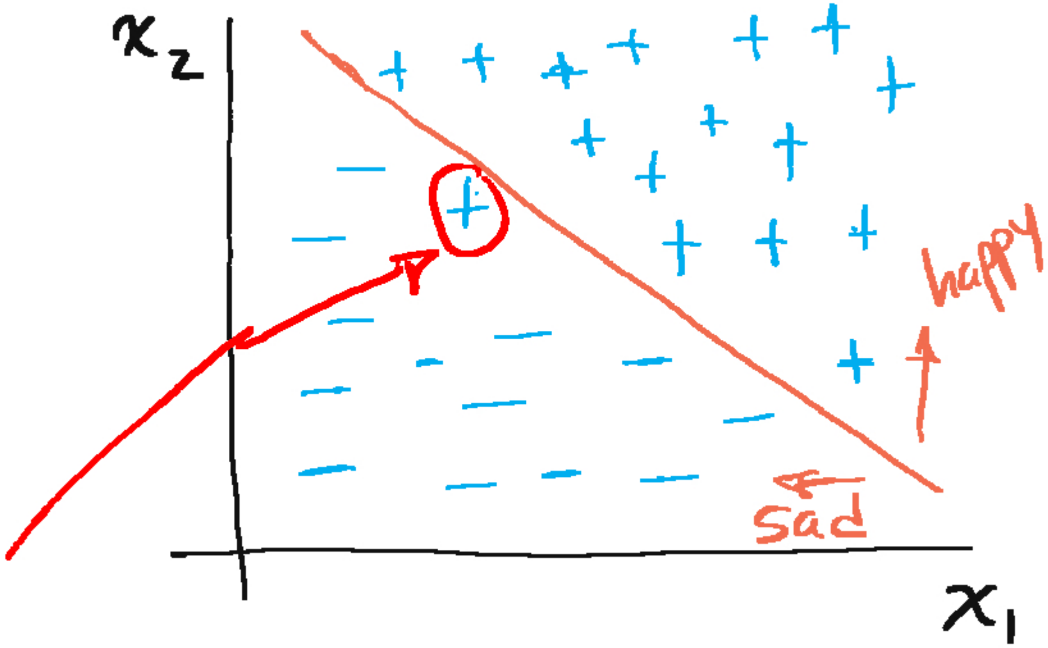


- find a decision boundary

+: happy

−: sad

misclassification



If we use a line to separate classes

$$x_2 = m x_1 + b \Rightarrow x_2 - m x_1 - b = 0$$

Find m, b

Rewrite boundary as an inner product

$$\underbrace{\begin{bmatrix} x_2 & x_1 & 1 \end{bmatrix}}_{\text{feature}} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \underline{x}^T \underline{w} = 0$$

classifier weights

Find $\underline{w}$.

Curved decision boundaries:

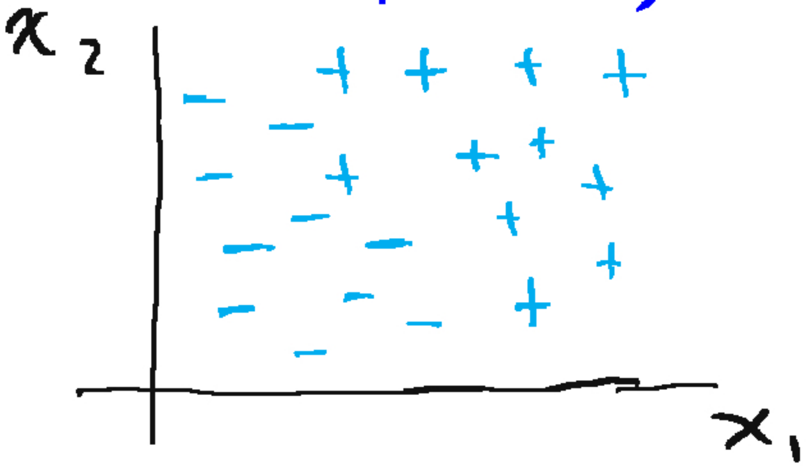
$$\underbrace{\begin{bmatrix} x_2 & x_1^3 & x_1^2 & x_1 & 1 \end{bmatrix}}_{\text{feature}} \begin{bmatrix} w_1 \\ \vdots \\ w_4 \\ w_5 \end{bmatrix} = \underline{x}^T \underline{w} = 0$$

Find $\underline{w}$

A **linear classifier** is based on a weighted sum of features $\underline{x}^T \underline{w}$

features → $\underline{x}^T$; $\underline{w}$ ← weights

Labels specify class associated with a feature



\+ happy : label "1"

− sad : label "−1"

binary classification

Supervised learning: given features/labels $(\underline{x}_i, l_i)$ find $\underline{w}$ so $\underline{x}_i^T \underline{w} \approx l_i$

Training a linear classifier involves solving a system of linear equations

$$\begin{bmatrix} \underline{x}_1^T \\ \underline{x}_2^T \\ \vdots \\ \underline{x}_N^T \end{bmatrix} \underline{w} \approx \begin{bmatrix} l_1 \\ l_2 \\ \vdots \\ l_N \end{bmatrix} \Rightarrow \underbrace{\underline{X}}_{N \times M} \; \underbrace{\underline{w}}_{M \times 1} \approx \underbrace{\underline{l}}_{N \times 1}$$

N training samples
M features

Classify candidate feature $\underline{\tilde{x}}$ using $\underline{w}$

if $\underline{\tilde{x}}^T \underline{w} > 0 \Rightarrow$ label "1", if $\underline{\tilde{x}}^T \underline{w} < 0 \Rightarrow$ label "-1"

Advanced topics

- What should we choose for "≈"? $\underline{X}\,\underline{w} \approx \underline{l}$
- Feature choice / boundary complexity
- Performance evaluation
- M-ary classification