CS/ECE/ME532 Period 5 Activity

1. Let $\boldsymbol{z} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\boldsymbol{w} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

a) Sketch the subspace spanned by $\boldsymbol{z}$ in $\mathbb{R}^2$.

b) Sketch the subspace spanned by $\boldsymbol{w}$ in $\mathbb{R}^2$.

c) Sketch span $\{\boldsymbol{z}, \boldsymbol{w}\}$ in $\mathbb{R}^2$.

d) Are $\boldsymbol{z}$ and $\boldsymbol{w}$ orthogonal? Why or why not?

e) Do $\{\boldsymbol{z}, \boldsymbol{w}\}$ form an orthonormal basis? Why or why not? If not, can you modify $\boldsymbol{z}$ and $\boldsymbol{w}$ to form an orthonormal basis?

2. Consider the line in $\mathbb{R}^2$ defined by the equation $x_2 = x_1 + 1$.

a) Sketch the line in $\mathbb{R}^2$.

b) Does this line define a subspace of $\mathbb{R}^2$? Why or why not?

3. You collect ratings of three space-related science fiction movies and two romance movies from seven friends on a scale of 1-10.

Movie	Jake	Jennifer	Jada	Theo	Ioan	Bo	Juanita
Star Trek	4	7	2	8	7	4	2
Pride and Prejudice	9	3	5	6	10	5	5
The Martian	4	8	3	7	6	4	1
Sense and Sensibility	9	2	6	5	9	5	4
Star Wars: Empire Strikes	4	9	2	8	7	4	1

You put this data into a matrix $\boldsymbol{X}$ (available in the file `movie.mat`) and decide to model (approximate) as the product of a rank-r taste matrix with orthonormal columns and a weight matrix. That is, $\boldsymbol{X} \approx \boldsymbol{T}\boldsymbol{W}$.

a) What is the rank of $\boldsymbol{X}$? Relevant Python commands are `numpy.linalg.matrix_rank()`.

b) What are the dimensions of $\boldsymbol{T}$ and $\boldsymbol{W}$ (in terms of r)?

c) You know that each user's ratings have an average value that is greater than zero because the scale is 1-10. And you suspect the baseline (average) rating may differ from user to user. To account for this you decide your first basis vector in the taste matrix should be

$$\boldsymbol{t}_1 = \frac{1}{\sqrt{5}} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

Choose w_{1j} so that each element of the vector $\boldsymbol{t}_1 w_{1j}$ equals the average value j^{th} column of $\boldsymbol{X}$, denoted as $\boldsymbol{X}_{:,j}$. Find an expression for w_{1j} that depends on $\boldsymbol{t}_1$ and $\boldsymbol{X}_{:,j}$.

d) Define $\boldsymbol{w}_1^T = \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{17} \end{bmatrix}$ and find the rank-1 approximation to $\boldsymbol{X}$ that reflects the baseline ratings of each friend, $\boldsymbol{t}_1 \boldsymbol{w}_1^T$.

e) Which friend has the highest baseline rating? Which friend has the lowest baseline rating?

f) Find the residual not modeled by $\boldsymbol{t}_1 \boldsymbol{w}_1^T$, that is, $\boldsymbol{X} - \boldsymbol{t}_1 \boldsymbol{w}_1^T$. Do you see any patterns in the residual? Briefly describe them qualitatively.

This problem is continued in a homework assignment.