Asgn 3 ECE 532 Gyan Deep Hazra

1. a) $w(a)$ is a degree p polynomial

$$w(a_i) = [a_i^p \;\; a_i^{p-1} \;\; a_i^{p-2} \; \dots \; a_i^2 \;\; a_i \;\; 1] \begin{bmatrix} w_p \\ w_{p-1} \\ \vdots \\ w_2 \\ w_1 \\ w_0 \end{bmatrix} = b_i$$

or $b_i = w_p a_i^p + w_{p-1} a_i^{p-1} + \dots + w_2 a_i^2 + w_1 a_i + w_0$

b) Given $Ax = d$

$$A = \begin{bmatrix} a_1^p & a_1^{p-1} & a_1^{p-2} & \dots & a_1^2 & a_1 & 1 \\ a_2^p & a_2^{p-1} & a_2^{p-2} & \dots & a_2^2 & a_2 & 1 \\ \vdots \\ a_m^p & a_m^{p-1} & a_m^{p-2} & \dots & a_m^2 & a_m & 1 \end{bmatrix}$$ is a $m \times (p+1)$ matrix

$$d = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$ is a m dimension vector

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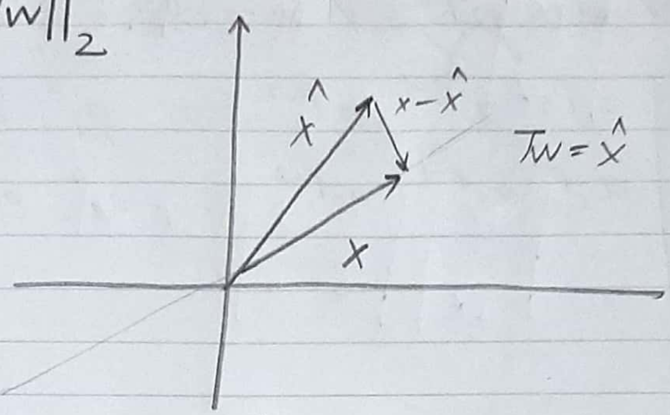
Thus we get, $Ax = d$

$$\begin{bmatrix} a_1^p & a_1^{p-1} & a_1^{p-2} & \cdots & a_1^2 & a_1 & 1 \\ a_2^p & a_2^{p-1} & a_2^{p-2} & \cdots & a_2^2 & a_2 & 1 \\ \vdots & & & & & & \\ a_m^p & a_m^{p-1} & a_m^{p-2} & \cdots & a_m^2 & a_m & 1 \end{bmatrix} \begin{bmatrix} w_p \\ w_{p-1} \\ \vdots \\ w_0 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

A is a $m \times (p+1)$ dimension matrix.

c) At the end

2. a) $\min_{w} \| x - Tw \|_2^2$



Let $\hat{x} = Tw$

Let $T = [a_1 \; a_2 \; a_3 \; \cdots \; a_n]$

$\hat{x} = Tw = a_1 w_1 + a_2 w_2 + a_3 w_3 + \cdots + a_n w_n$

for $x - \hat{x} \perp \text{span}\{a_1, a_2, \ldots a_n\}$

$\Rightarrow a_1^T (x - \hat{x}) = 0$

$\vdots$

$a_n^T (x - \hat{x}) = 0$

In general, $T^T (x - \hat{x}) = 0$

So, $T^T (x - Tw) = 0$

$T^T x - T^T T w = 0$

$T^T x = T^T T w$

$T^T x = T^T T w$, now as T is orthonormal $T^T T = I$

Thus, $\underline{w = T^T x}$

b) $X = [x_1 \quad x_2 \quad \cdots\cdots \quad x_p]$

$W = [w_1 \quad w_2 \quad \cdots\cdots \quad w_p]$ ← weights

Given

$$X \approxeq TW$$

$$\Rightarrow T^T X = T^T T W$$

$$T^T X = I W$$ (as T is orthonormal)

Thus
$$\left.\begin{aligned} w_1 &= T^T x_1 \\ w_2 &= T^T x_2 \\ &\vdots \\ w_p &= T^T x_p \end{aligned}\right\} \quad w_n = T^T x_n$$

So $W = T^T X$

3) At the end (written response too)

4. $x^T = [x_1 \quad x_2]$ as $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

Thus $x^T Q x = [x_1 \; x_2] \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$= [x_1 \;\; 2x_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$= x_1^2 + 2x_2^2$

As, $y = x^T Q x$

we have $y = x_1^2 + 2x_2^2$

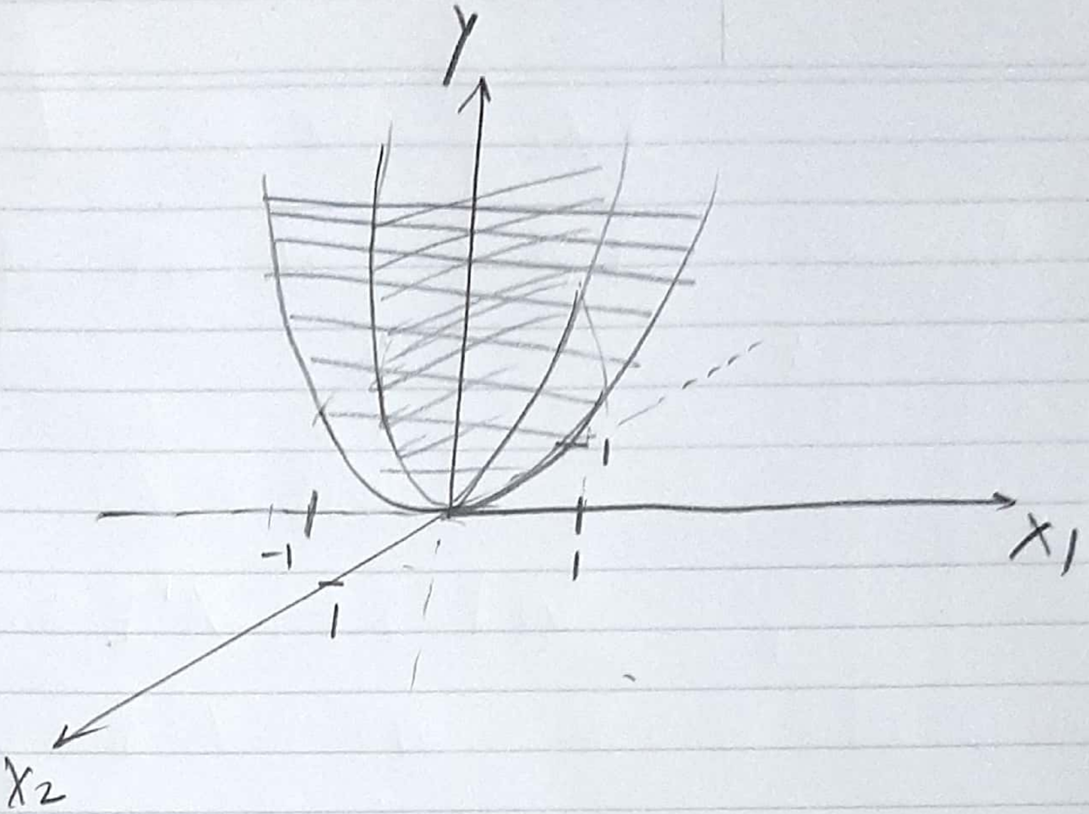
a) since y is the sum of square terms $y = 0$ iff $x_1 = x_2 = 0$.

Thus $x^T Q x > 0 \quad \forall x \neq 0$ thus Q is positive definite.

$\downarrow$

or $Q \succ 0$

b)



5. consider some $x^T Q P Q x$

let $x^T Q = v^T$

thus $v = Q^T x = Qx$

thus $x^T Q P Q x = v^T P v$

since $P > 0$, $v^T P v > 0$

Hence, $x^T Q P Q x > 0$

or $QPQ > 0$ (positive definite