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9:15 AM Wed Jun 2

Assignment Details

DIGITAL SIGNAL PROCESSING

Description

Answer the following TEN questions. All questions are from the assigned textbook. Use MATLAB if needed

Completing the five Practice Quizzes as well as participating in this week's discussion is a prerequisite for this homework

NOTE: The little arrow you see in the sequences of some of the problems shows the value at the zeroth location such as x(0)

(1) Page 292, Problem 4.2

(2) Page 292, Problem 4.4 (x(0) value is 3)

(3) Page 293, Problem 4.6 only (e) (x(0) value is 1)

(4) Page 293, Problem 4.7, only (b)

(5) Page 294, Problem 4.9 only (b)

(6) Page 294, Problem 4.10 only (a). You need to find x(n)

(7) Page 295, Problem 4.12 only (b)

(8) Page 295 Problem 4.13

(9) Page 298 Problem 4.22 only (a)

(10) Page 298 Problem 4.22 only (d)

PROBLEM 1)

a) $$x(t) = \begin{cases} Ae^{-at}, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Signal is aperiodic → Fourier Transform

Mag. spectra = $|X(F)|$

$$X(F) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi Ft}\, dt = \int_{0}^{\infty} Ae^{-at} \cdot e^{-j2\pi Ft}\, dt$$

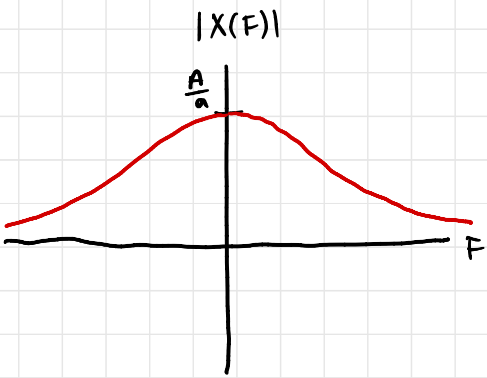
$$= A\int_{0}^{\infty} e^{-(a+j2\pi F)t}\, dt = A\left[\frac{e^{-(a+j2\pi F)t}}{-(a+j2\pi F)}\right]_{0}^{\infty}$$

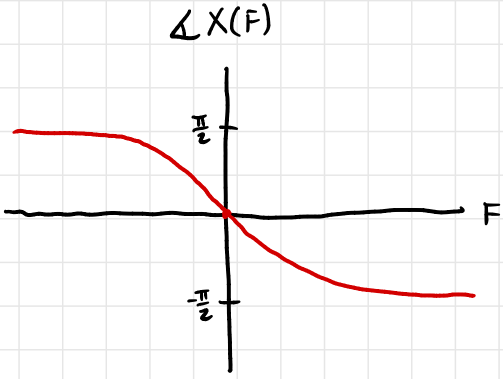
$$= \frac{A}{a+2\pi Fj} = A\left[\frac{a}{a^2+4\pi^2F^2} - \frac{2\pi Fj}{a^2+4\pi^2F^2}\right]$$

$$|X(F)| = \left|\frac{A}{a+2\pi Fj}\right| = \boxed{\frac{A}{\sqrt{a^2+4\pi^2F^2}}}$$

Phase Spectra = $\measuredangle X(F)$

$$\tan\theta = \frac{y}{x} = -\frac{2\pi F}{a}, \quad \boxed{\theta = \arctan\left(-\frac{2\pi F}{a}\right)}$$





b)

$$x(t) = Ae^{-a|t|} = \begin{cases} Ae^{-at}, & t \geq 0 \\ Ae^{at}, & t < 0 \end{cases}$$

Signal is aperiodic $\rightarrow$ Fourier Transform

$$X(F) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi Ft} dt$$

$$= \int_{-\infty}^{0} Ae^{at} e^{-j2\pi Ft} dt + \int_{0}^{\infty} Ae^{-at} e^{-j2\pi Ft} dt$$

$$= A\int_{-\infty}^{0} e^{(a-j2\pi F)t} dt + A\int_{0}^{\infty} e^{-(a+j2\pi F)t} dt$$

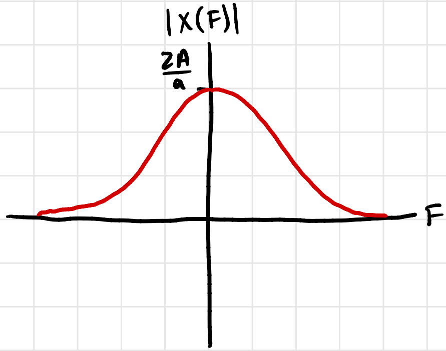
$$= A\left[\frac{e^{(a-j2\pi F)t}}{a-j2\pi F}\right]_{-\infty}^{0} + A\left[\frac{e^{-(a+j2\pi F)t}}{-(a+j2\pi F)}\right]_{0}^{\infty}$$

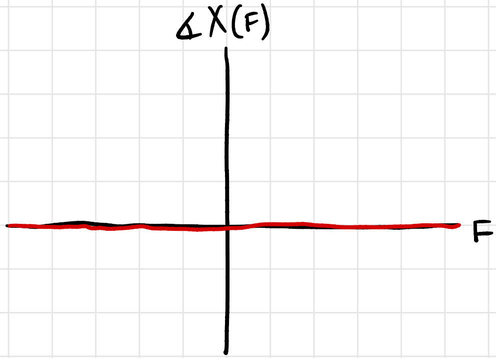
$$= \frac{A}{a-2\pi Fj} + \frac{A}{a+2\pi Fj} = \frac{A(a+2\pi Fj)}{a^2+4\pi^2F^2} + \frac{A(a-2\pi Fj)}{a^2+4\pi^2F^2}$$

$$= \frac{2Aa}{a^2+4\pi^2F^2}$$

$$|X(F)| = \boxed{\frac{2Aa}{a^2+4\pi^2F^2}}$$

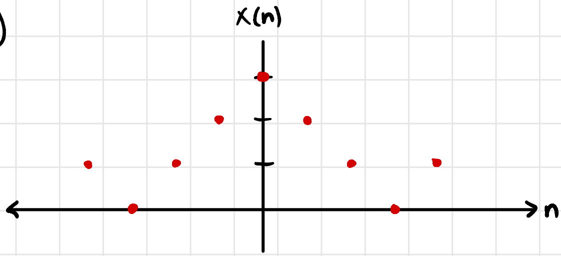
$$\measuredangle X(F) = \arctan(0) = \boxed{0}$$





PROBLEM 2) $x(n) = \{\ldots, 1, 0, 1, 2, 3, 2, 1, 0, 1, \ldots\}$

a)



periodic → Discrete Fourier Series $(N=6)$

$$x[n] = \sum_{k=0}^{N-1} c_k e^{j\frac{2\pi}{N}kn}$$

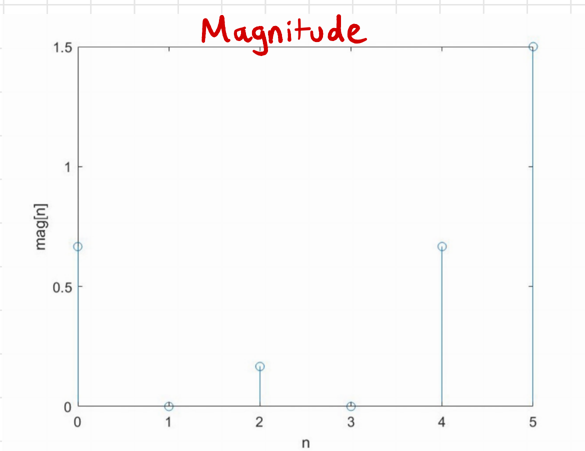
where $$c_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}kn}$$

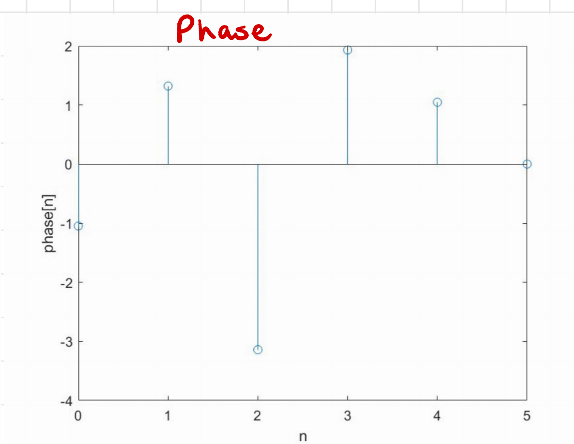
$$= \frac{1}{6}\Big[x[0]e^{-j\frac{2\pi}{6}\cdot k\cdot 0} + x[1]e^{-j\frac{2\pi}{6}\cdot k\cdot 1} + x[2]e^{-j\frac{2\pi}{6}\cdot k\cdot 2} +$$

$$x[3]e^{-j\frac{2\pi}{6}\cdot k\cdot 3} + x[4]e^{-j\frac{2\pi}{6}\cdot k\cdot 4} + x[5]e^{-j\frac{2\pi}{6}\cdot k\cdot 5}\Big]$$

$$= \frac{1}{6}\Big[3 + 2e^{-j\frac{2\pi}{6}k} + e^{-j\frac{2\pi}{3}k} + 0 + e^{-j\frac{2\pi}{6}\cdot 4k} + 2e^{-j\frac{2\pi}{6}\cdot 5k}\Big]$$

Using Matlab,





b)

Parseval's Relation

$$P_x = \sum_{k=0}^{N-1} |c_k|^2 = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2$$

Using Matlab, both power expressions equated to 3.1667, thus verifying Parseval's Relation.

PROBLEM 3) $x(n) = \{\ldots, -1, 2, 1, 2, -1, 0, -1, 2, 1, 2, \ldots\}$

Periodic Signal → DT Fourier Series $(N=6)$

$$C_k = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}$$

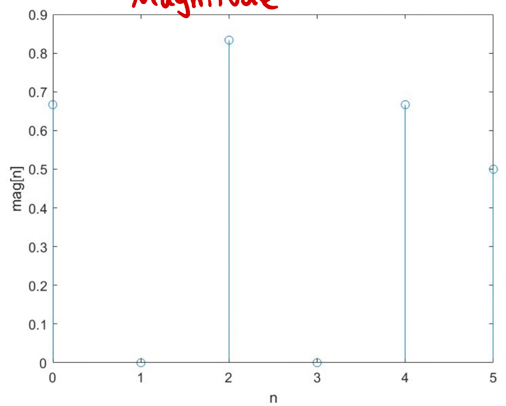
$$= \frac{1}{6} \sum_{n=0}^{5} x[n] e^{-j\frac{2\pi}{6}kn}$$

$$= \frac{1}{6}\Big[1 \cdot e^{-j\frac{2\pi}{6}k\cdot 0} + 2 \cdot e^{-j\frac{2\pi}{6}k\cdot 1} - 1 \cdot e^{-j\frac{2\pi}{6}k\cdot 2} + 0 \cdot e^{-j\frac{2\pi}{6}k\cdot 3}$$

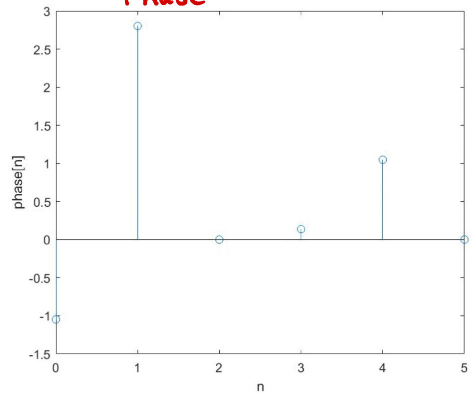
$$- 1 \cdot e^{-j\frac{2\pi}{6}k\cdot 4} + 2 \cdot e^{-j\frac{2\pi}{6}k\cdot 5}\Big]$$

using Matlab, the spectras are determined to be:

Magnitude



Phase



PROBLEM 4)

Determine the periodic signals $x(n)$, with fundamental period $N = 8$, if their Fourier coefficients are given by:

~~(a) $c_k = \cos\frac{k\pi}{4} + \sin\frac{3k\pi}{4}$~~

(b) $c_k = \begin{cases} \sin\frac{k\pi}{3}, & 0 \le k \le 6 \\ 0, & k = 7 \end{cases}$

$$C_K = \begin{cases} \sin\frac{K\pi}{3}, & 0 \le K \le 6 \\ 0, & K = 7 \end{cases}$$

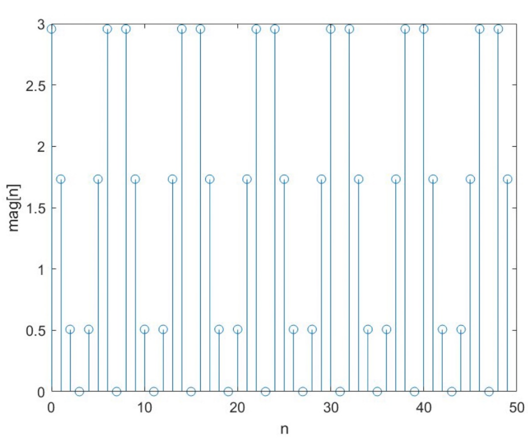
$$x[n] = \sum_{K=0}^{N-1} C_K e^{j\frac{2\pi}{N}Kn}, \quad N = 8$$

$$= \sum_{K=0}^{7} C_K e^{j\frac{2\pi}{8}Kn}$$

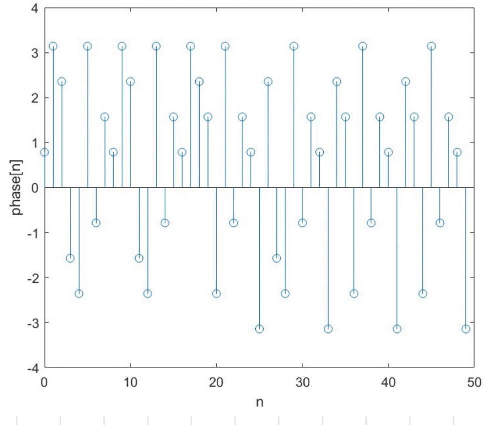
$$= \sum_{K=0}^{7} C_K \left[\cos\left[\frac{2\pi}{8}Kn\right] + j\sin\left[\frac{2\pi}{8}Kn\right]\right]$$

using Matlab, the periodic signal is determined to be:

Magnitude



Phase



PROBLEM 5)

4.9 Compute the Fourier transform of the following signals.

~~**(a)** $x(n) = u(n) - u(n-6)$~~

(b) $x(n) = 2^n u(-n)$

$$x(n) = 2^n u(-n)$$

$$X(F) = \int_{-\infty}^{\infty} x(n) e^{-j2\pi Ft} dt = \int_{-\infty}^{\infty} 2^t u(-t) e^{-j2\pi Ft} dt$$

$$= \int_{-\infty}^{0} 2^t e^{-j2\pi Ft} dt = \int_{-\infty}^{0} e^{t\ln 2} e^{-j2\pi Ft} dt$$

$$= \int_{-\infty}^{0} e^{(\ln 2 - j2\pi F)t} dt = \left[\frac{e^{(\ln 2 - j2\pi F)t}}{(\ln 2 - j2\pi F)} \right]_{-\infty}^{0}$$

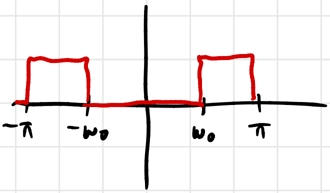
$$= \boxed{\frac{1}{(\ln 2 - j2\pi F)}}$$

PROBLEM 6)

4.10 Determine the signals having the following Fourier transforms.

(a) $X(\omega) = \begin{cases} 0, & 0 \le |\omega| \le \omega_0 \\ 1, & \omega_0 < |\omega| \le \pi \end{cases}$

$$x(n) = \int_{-\infty}^{\infty} X(F)\, e^{j2\pi F n}\, dF$$



$$= \int_{-\pi}^{-\omega_0} 1 \cdot e^{j2\pi F n}\, dF + \int_{\omega_0}^{\pi} 1 \cdot e^{j2\pi F n}\, dF$$

$$= \left[\frac{e^{j2\pi F n}}{j2\pi n}\right]_{-\pi}^{-\omega_0} + \left[\frac{e^{j2\pi F n}}{j2\pi n}\right]_{\omega_0}^{\pi}$$

$$= \left(\frac{e^{-j2\pi\omega_0 n}}{j2\pi n} - \frac{e^{-j2\pi^2 n}}{j2\pi n}\right) + \left(\frac{e^{j2\pi^2 n}}{j2\pi n} - \frac{e^{j2\pi\omega_0 n}}{j2\pi n}\right)$$

$$= -\left[\frac{e^{j2\pi\omega_0 n} - e^{-j2\pi\omega_0 n}}{j2\pi n}\right] + \left[\frac{e^{j2\pi^2 n} - e^{-j2\pi^2 n}}{j2\pi n}\right]$$

$$= -\frac{\sin(2\pi\omega_0 n)}{\pi n} + \frac{\sin(2\pi^2 n)}{\pi n}$$

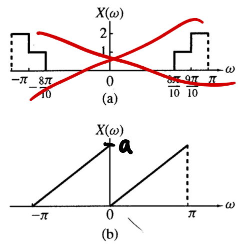
$$= \frac{\sin(2\pi^2 n) - \sin(2\pi\omega_0 n)}{\pi n}$$

$$|x(n)| = \frac{\sin(2\pi^2 n) - \sin(2\pi\omega_0 n)}{\pi n}$$

$$\angle x(n) = 0^\circ$$

PROBLEM 7)

4.12 Determine the signal $x(n)$ if its Fourier transform is as given in Fig. P4.12.



$$X(\omega) = \begin{cases} \omega + a, & -\pi \le \omega < 0 \\ \omega, & 0 \le \omega < \pi \end{cases}$$

$$x(t) = \int_{-\infty}^{\infty} X(F) e^{j2\pi F t} \, dF$$

$$= \int_{-\pi}^{0} a e^{j2\pi F t} \, dF + \int_{-\pi}^{\pi} F e^{j2\pi F t} \, dF$$

$$= a\left[\frac{e^{j2\pi F t}}{j2\pi t}\right]_{-\pi}^{0} + \left[\frac{F e^{j2\pi F t}}{j2\pi t}\right]_{-\pi}^{\pi} - \int_{-\pi}^{\pi} \frac{e^{j2\pi F t}}{j2\pi t} \, dF$$

$$= a\left[\frac{1}{j2\pi t} - \frac{e^{-j2\pi^2 t}}{j2\pi t}\right] + \left[\frac{\pi e^{j2\pi^2 t}}{j2\pi t} + \frac{\pi e^{-j2\pi^2 t}}{j2\pi t}\right]$$

$$+ \left[\frac{e^{j2\pi^2 t}}{4\pi^2 t^2} - \frac{e^{-j2\pi^2 t}}{4\pi^2 t^2}\right]$$

$$= a\left[\frac{1}{j2\pi t} - \frac{\cos(2\pi^2 t)}{j2\pi t} + \frac{\sin(2\pi^2 t)}{j2\pi t}\right] + \frac{\cos(2\pi^2 t)}{jt}$$

$$+ \frac{j\sin(2\pi^2 t)}{2\pi^2 t^2}$$

$$= -j\frac{a}{2\pi t} + j\frac{a\cos(2\pi^2 t)}{2\pi t} - j\frac{a\sin(2\pi^2 t)}{2\pi t} - j\frac{2\pi\cos(2\pi^2 t)}{2\pi t} + \frac{j\sin(2\pi^2 t)}{2\pi^2 t^2}$$

$$= j\left[\frac{-a\pi t + \pi t(a-2\pi)\cos(2\pi^2 t) + (1-a\pi t)\sin(2\pi^2 t)}{2\pi^2 t^2}\right]$$

$$\boxed{\begin{aligned} |X(n)| &= \frac{-a\pi t + \pi t(a-2\pi)\cos(2\pi^2 t) + (1-a\pi t)\sin(2\pi^2 t)}{2\pi^2 t^2} \\ \measuredangle X(n) &= \frac{\pi}{2} \end{aligned}}$$

PROBLEM 8)

$$x_1(n) = \begin{cases} 1, & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases} \quad \text{Use DFT}$$

$$X_1(\omega) = \sum_{n=-\infty}^{\infty} x_1(n) e^{-j\omega n} = \sum_{n=0}^{M} 1 \cdot e^{-j\omega n}$$

$$= \sum_{n=0}^{M} (e^{-j\omega})^n \qquad \sum_{n=0}^{n} ar^n = \frac{a_0(1-r^{(n+1)})}{1-r}$$

$$= \frac{1 \cdot (1-(e^{-j\omega})^{M+1})}{1-e^{-j\omega}}$$

$$= \frac{1-e^{-j\omega(M+1)}}{1-e^{-j\omega}}$$

$$x_2(n) = \begin{cases} 1, & -M \le n \le -1 \\ 0, & \text{otherwise} \end{cases} \quad \text{use DFT}$$

$$X_2(\omega) = \sum_{n=-M}^{-1} x_2(n) e^{-j\omega n}, \quad k = n+M$$

$$= \sum_{k=0}^{M-1} x_2(k) e^{-j\omega(k-M)} = \sum_{k=0}^{M-1} (e^{-j\omega})^{k-M}$$

$$= e^{j\omega M} \sum_{k=0}^{M-1} (e^{-j\omega})^k = e^{j\omega M}\left[\frac{1 \cdot (1-e^{-j\omega M})}{1-e^{-j\omega}}\right]$$

$$= \frac{e^{j\omega M}-1}{1-e^{-j\omega}} \cdot \frac{e^{j\omega}}{e^{j\omega}}$$

$$= \frac{e^{j\omega(M+1)}-e^{j\omega}}{e^{j\omega}-1} = \frac{e^{j\omega}-e^{j\omega(M+1)}}{1-e^{j\omega}}$$

$$X(w) = X_1(w) + X_2(w)$$

$$= \frac{1 - e^{-jw(M+1)}}{1 - e^{-jw}} + \frac{e^{jw} - e^{jw(M+1)}}{1 - e^{jw}}$$

$$= \frac{1 - e^{-jw(M+1)} - e^{jw} + e^{-jwM} + e^{jw} - e^{jw(M+1)} + e^{jwM} - 1}{2 - (e^{jw} + e^{-jw})}$$

$$= \frac{\cancel{1} - e^{-jw(M+1)} \cancel{- e^{jw}} + e^{-jwM} \cancel{+ e^{jw}} - e^{jw(M+1)} + e^{jwM} \cancel{- 1}}{2 - (e^{jw} + e^{-jw})}$$

$$= \frac{\cos(Mw) - \cos(Mw + w)}{1 - \cos(w)} = \frac{\cos(Mw) - \cos(Mw + w)}{2\sin^2(\frac{w}{2})}$$

$$= \cos\left[\left(Mw + \frac{w}{2}\right) - \frac{w}{2}\right] - \cos\left[\left(Mw + \frac{w}{2}\right) + \frac{w}{2}\right]$$

* Note : $\cos(Mw) - \cos(Mw + w)$

$$= \cos\left(Mw + \frac{w}{2} - \frac{w}{2}\right) - \cos\left(Mw + \frac{w}{2} + \frac{w}{2}\right)$$

$$= \frac{\cos(Mw) - \cos(Mw + w)}{2\sin^2(\frac{w}{2})} = \frac{\cancel{2}\sin\left(Mw + \frac{w}{2}\right)\cancel{\sin\left(\frac{w}{2}\right)}}{\cancel{2}\sin^{\cancel{2}}\left(\frac{w}{2}\right)}$$

$$= \boxed{\frac{\sin\left(Mw + \frac{w}{2}\right)}{\sin\left(\frac{w}{2}\right)}} \quad \blacksquare$$

PROBLEM 9)

$$x(n) \xrightarrow{\mathcal{F}} X(\omega) = \frac{1}{1-ae^{-j\omega}}$$

$$x(2n) \xrightarrow{\mathcal{F}} \frac{1}{|a|}X\left(\frac{\omega}{a}\right) = \frac{1/|a|}{1-ae^{-\frac{\omega}{a}}}$$

time scale property

$$x(2(n+\tfrac{1}{2})) \xrightarrow{\mathcal{F}} e^{j\omega\cdot\frac{1}{2}}X(\omega)$$

time-shift property

$$\rightarrow \boxed{\frac{1}{|a|}\cdot\frac{e^{j\frac{\omega}{2}}}{1-ae^{-\frac{\omega}{a}}}}$$

PROBLEM 10)

$$x(n) \xrightarrow{\mathcal{F}} X(\omega) = \frac{1}{1-ae^{-j\omega}}$$

Note: $\mathcal{F}\{\cos(2\pi At)\} = \frac{1}{2}[\delta(f-A)+\delta(f+A)]$

$$\cos(0.3\pi n) = \cos(2\pi(0.15)n) \xrightarrow{\mathcal{F}} \frac{1}{2}[\delta(f-0.15)+\delta(f+0.15)]$$

$$\mathcal{F}\{x(n)\cos(0.3\pi n)\} = \left(\frac{1}{1-ae^{-j\omega}}\right) * \left(\frac{1}{2}[\delta(\omega-0.15)+\delta(\omega+0.15)]\right)$$

$$= \frac{1}{2}\int_{-\infty}^{\infty}\frac{1}{1-ae^{-j\tau}}\cdot[\delta(\omega-\tau-0.15)+\delta(\omega-\tau+0.15)]\,d\tau$$

$$= \frac{1}{2}\int_{-\infty}^{\infty}\frac{1}{1-a^{-j\tau}}\cdot\delta(\omega-\tau-0.15)\,d\tau + \frac{1}{2}\int_{-\infty}^{\infty}\frac{1}{1-ae^{-j\tau}}\cdot\delta(\omega-\tau+0.15)\,d\tau$$

$$= \frac{1}{2}\left[\frac{1}{1-ae^{-j(0.15-\omega)}}\right] + \frac{1}{2}\left[\frac{1}{1-ae^{-j(-0.15-\omega)}}\right]$$

$$\boxed{= \frac{1}{2}\left[\frac{1}{1-ae^{-j(0.15-\omega)}} + \frac{1}{1-ae^{-j(-0.15-\omega)}}\right]}$$