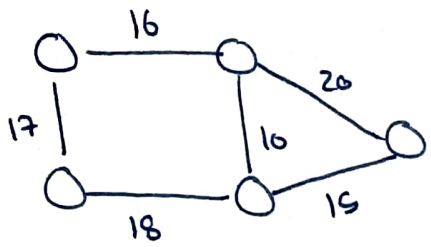
Tutorial 6

Ayank Gupta
Section - B

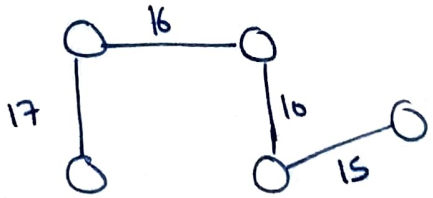
Ans1 Min. Spanning Tree :

⇒ A spanning tree of undirected graph is a subgraph that is a tree and joined all vertices. One of those tree will be having max. total cost would be its minimum spanning tree.

⊙ Eg. ⇒



for above connected / undirected graph ⇒ min. cost spanning tree would be ⇒



* Applications of MST ⇒

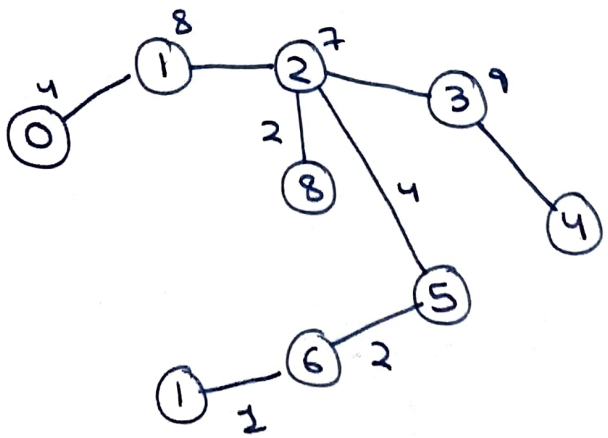
⇒ MST have direct application in design of networks including computer networks, tele. networks etc.

Ans2

	Prims. Algo.	Kruskal's Algo.	Dijsktras Algo.	Bellman Ford Algo.
Time Complexity	$O(V^2)$	$O(E \log V)$	$O(V+E \log V)$	$O(VE)$
Space Complexity	$O(V+E)$	$O(E+V)$	$O(V^2)$	$O(V^2)$

Ans3 ⇒ Prims Algorithm ⇒

0	1	2	3	4	5	6	7	5
0	∞	∞	∞	∞	∞	∞	∞	∞
	4	8	7		4	8	8	2
				16		2	7	
				9			1	

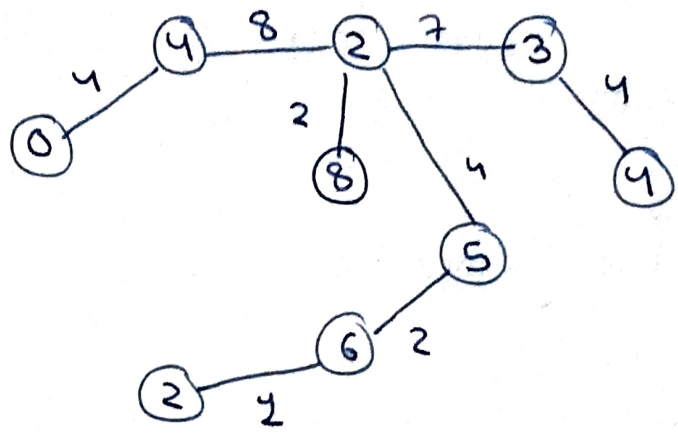


Parent ⇒

0	1	2	3	4	5	6	7	8
-1	✗	✗	✗	-1	✗	✗	✗	✗
	0	1	2		2		0	2
						5	6	

Ans3 ⑥ => Kruskal's Algorithm =>

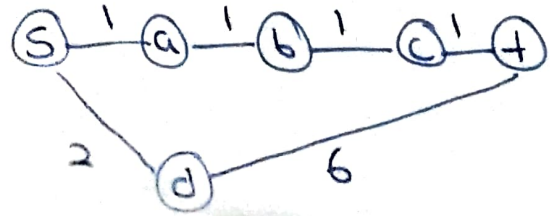
u	v	w	
7	6	1	✓
6	5	2	✓
2	8	2	✓
2	5	4	✓
0	1	4	✓
8	6	6	X
7	8	7	X
2	3	7	✓
1	2	8	✓
0	7	8	X
3	4	9	✓
5	4	10	X
1	7	11	X
3	5	14	X



weight = 37

Ans4 @ If 10 units is added to each edge, the overall wt. of path may change.

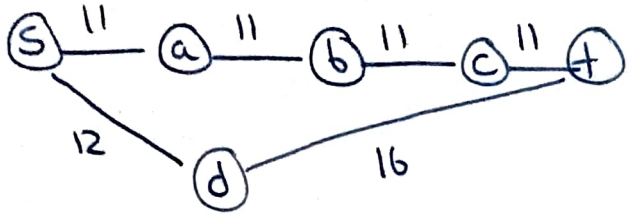
⊙ for Eg. =>



Shortest path ⇒ $s \to a \to b \to c \to t$

weight ⇒ $1+1+1+1=4$

now, if 10 unit wt. is added to each edge ⇒



shortest path ⇒ $s \to d \to t$

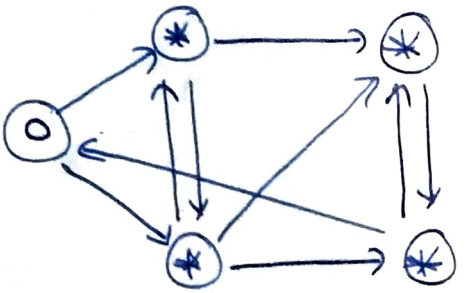
wt. ⇒ $12+16=28$

⇒ multiplying the wt. of each edge by 10 will have no impact on shortest path.

Ans5 ⇒ Dijsktra's Algorithm ⇒

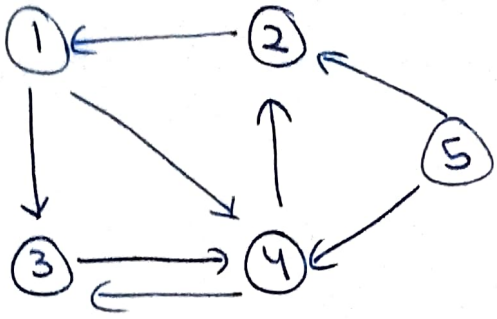
Queue: ~~s~~ ~~u~~ ~~v~~ ~~x~~ ~~y~~

visited: s u v x y



s	u	v	x	y
0	∞	∞	∞	∞
0	10	~~∞~~	5	∞
0	10	11	5	~~∞~~
0	10	11	5	7

Ans 6 ⇒ All pair shortest Path Algo. ⇒ Floyd Warshall ⇒



A^0 ⇒

	1	2	3	4	5
1	∞	∞	∞	∞	∞
2	∞	0	∞	∞	∞
3	∞	∞	0	2	∞
4	∞	1	1	0	∞
5	∞	4	∞	2	0

A^1 ⇒

	1	2	3	4	5
1	0	∞	6	3	∞
2	3	0	9	6	∞
3	∞	∞	0	2	∞
4	∞	1	1	0	∞
5	∞	4	∞	2	0

$A^0[2,3] = \infty$

$A^0[2,1] + A^1[1,3] = 3+6 = 9$

Similarly. $A^0[2,4] = \infty$

$A^0[2,1] + A^0[1,4] = 3+3 = 6 < \infty$

$\Rightarrow A^0[2,5] = \infty$

$A^0[2,1] + A^0[1,5] = 3+\infty$

$A^2 =$

	1	2	3	4	5
1	0	∞	(6)	3	(∞)
2	3	0	(9)	6	∞
3	∞	∞	0	2	∞
4	∞	1	(1)	0	∞
5	7	4	13	2	0

$A^1[1,3] = 6$

$A^1[1,2] + A^1[2,3] = \infty + 9 > 6$

$A^3 =$

	1	2	3	4	5
1	0	∞	6	(3)	∞
2	3	0	9	(6)	∞
3	∞	∞	0	(2)	∞
4	∞	1	1	0	∞
5	7	4	13	(2)	0

$\Rightarrow$ Solution