Tutorial 4

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Section B

① $T(n) = 3T(n/2) + n^2$

$T(n) = aT(n/b) + f(n)$, $a = 3$, $b = 2$

$c = \log_2 3 = 1.58$

$n^c = n^{1.58} \Rightarrow f(n) = n^2$

- By Case 3 $\Rightarrow f(n) > n^c \Rightarrow T(n) = \Theta(f(n)) = \Theta(n^2)$

② $T(n) = 4T(n/2) + n^2$

$T(n) = aT(n/b) + f(n)$, $a = 4$, $b = 3$

$c = \log_2 4 = 2$

$f(n) = n^2$

- By case 2 $\Rightarrow f(n) = n^c \Rightarrow T(n) = \Theta(n^c \log n)$

$\Rightarrow \Theta(n^2 \log n)$

③ $T(n) = T(n/2) + 2^n$

$T(n) = aT(n/b) + f(n)$, $a = 1$, $b = 2$

$c = \log_2 1 = 0$

$n^c = n^0 = 1$, $f^n = 2^n$

$\Rightarrow T(n) = \Theta(f(n))$

$\Rightarrow \Theta(2^n)$

$T(n) = 2^n T(n/2) + n^n$

$T(n) = aT(n/b) + f(n)$

$a = 2^n,\ b = 2$

$c = \log_2 2^n = n$

$n^c = n^n,\ f(n) = n^n$

$\Rightarrow f(n) = n^c$

$T(n) = \theta(n^c \log n)$

$\Rightarrow \theta(n^n \log n)$

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(5) $T(n) = 16T(n/4) + n$

$T(n) = aT(n/b) + f(n)$

$a = 16,\ b = 4$

$c = \log_4 16 = 2$

$n^c = n^2 = f(n) = n^2$

$\Rightarrow f(n) < n^c$

$T(n) = \theta(n^c)$

$T(n) = \theta(n^2)$

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(6) $T(n) = 2T(n/2) + n\log n$

$T(n) = aT(n/b) + f(n)$

$a = 2,\ b = 2$

$c = \log_2 2 = 1$

$n^c = n,\ f(n) = n\log n$

$\Rightarrow f(n) > n^c$

$T(n) = \theta(n\log n)$

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(7) $T(n) = 2T(n/2) + n/\log n$

$a = 2,\ b = 2$

$c = \log_2 2 = 1$

$n^c = n^1,\ f(n) = n/\log n$

$\Rightarrow n^c > f(n)$

$T(n) = \theta(n^c)$

$T(n) = \theta(n)$

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(8) $T(n) = 2T(n/4) + n^{0.51}$

$a = 2,\ b = 4$

$c = 0.5 \Rightarrow n^c = n^{0.5}$

$\Rightarrow f(n) > n^c$

$T(n) = \theta(n^{0.51})$

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(9) $T(n) = 0.5T(n/2) + 1/n$

$a = 0.5,\ b = 2$

$c = -1,\ n^c = 1/n$

$\Rightarrow f(n) = n^c$

$T(n) = \theta(\log n / n)$

(10) $T(n) = 16T(n/4) + n!$

$a = 16,\ b = 4$

$c = 2 \Rightarrow n^c = n^2$

$\Rightarrow f(n) > n^c$

$\Rightarrow T(n) = \Theta(n!)$

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(11) $T(n) = 4T(n/2) + \log n$

$a = 4,\ b = 2$

$c = 2 \Rightarrow n^c = n^2$

$\Rightarrow f(n) < n^c$

$T(n) = \Theta(n^2)$

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(12) $T(n) = 3T(n/2) + n$

$a = 3,\ b = 2$

$c = 1.58 \Rightarrow n^c = n^{1.58}$

$\Rightarrow f(n) < n^c$

$T(n) = \Theta(n^{1.58})$

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(14) $T(n) = 3T(n/3) + sqrt(n)$

$a = 3,\ b = 3$

$n^c = n > n^{1/2}$

$\Rightarrow T(n) = \Theta(n)$

(15) $T(n) = 4T(n/2) + cn$

$a = 4,\ b = 2$

$c = 2 \Rightarrow n^c = n^2$

$\Rightarrow f(n) < n^c$

$T(n) = \Theta(n^2)$

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(16) $T(n) = 3T(n/4) + n\log n$

$a = 3,\ b = 4$

$c = 0.79 \Rightarrow n^c = n^{0.79}$

$\Rightarrow f(n) < n^c$

$T(n) = \Theta(n\log n)$

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(17) $T(n) = 3T(n/3) + n/2$

$a = 3,\ b = 3$

$c = 1 \Rightarrow n^c = n$

$\Rightarrow f(n) < n^c$

$T(n) = \Theta(n)$

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(18) $T(n) = 6T(n/3) + n^2\log n$

$a = 6,\ b = 3$

$c = 2 \Rightarrow n^c = n^{1.63}$

$\Rightarrow T(n) = \Theta(n^2\log n)$

(19) $T(n) = 4T(n/2) + n/\log n$

$a = 4, b = 2$

$\Rightarrow c = 2 \Rightarrow n^c = n^2$

$\Rightarrow f(n) < n^c$

$T(n) = \Theta(n^c) = \Theta(n^2)$

(20) $T(n) = 64T(n/8) + n^2 \log n$

$a = 64, b = 8$

$c = 2 \Rightarrow n^c = n^2$

$\Rightarrow f(n) > n^c$

$T(n) = \Theta(f(n))$

$\Rightarrow \Theta(n^2 \log^1 n)$

(21) $T(n) = 7T(n/3) + n^2$

$a = 7, b = 3$

$c = 1.77 \Rightarrow n^c = n^{1.77}$

$\Rightarrow f(n) > n^c$

$T(n) = \Theta(f(n)) \Rightarrow \Theta(n^2)$

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