A Appendix

A.1 Related Work

We discuss prior work related to combinatorial resource allocation under uncertainty for achieving health outcomes. The optimization of resources can be either done in a *single-shot* manner or by considering the sequential nature of the decision-making problem. The specific paradigm for resource allocation depends on the specific problem domain. For example, the optimization of patient admissions in hospitals [Hulshof *et al.*, 2013], allocating home healthcare services [Aiane *et al.*, 2015], and redistribution of patients among hospitals in case of a surge in demand (e.g., in case of a pandemic) [Parker *et al.*, 2020] have been modeled as single-shot optimization problems. Specifically, Parker *et al.* solve the problem of finding optimal demand and resource transfers to minimize the surge capacity and resource shortage during a period of heightened demand due to COVID-19 [Parker *et al.*, 2020]. Aiane *et al.* work on allocating services such as medical, paramedical, and social services delivered to patients in their homes modeled using an MILP formulation [Aiane *et al.*, 2015]. Our problem setting and formulation is most similar to theirs in principle; but the problem setting considered by Aiane *et al.* only accounts for travel times by resources as part of the objective and does not account for uncertainty in the outcomes after resource allocation [Aiane *et al.*, 2015]. Moreover, their approach is not scalable to our setting; the number of decision variables and constraints in our problem is 10^6 times higher.

Prior work has also explored performing sequential decision-making in the context of resource allocation in healthcare settings. For example, Mate *et al.* [Mate *et al.*, 2021] and Nishtala *et al.* [Mate *et al.*, 2021] model the allocation of targeted phone calls as a restless multi-armed bandit problem (RMAB). In such an approach, historical data is used to estimate the effect of interventions (similar to our approach). Then, the RMAB model is used for planning interventions over multiple decision epochs with limited resources. Tsoukalas *et al.* present a data-driven probabilistic framework for clinical decision support by using partially observable Markov decision processes (POMDP) [Tsoukalas *et al.*, 2015]. The POMDP model is based on clinical practice, expert knowledge and data representations in emergency healthcare settings.

A.2 Proof of Proposition 1

Let v_{wH} be the vaccine drive determined by the heuristic procedure at the w-th iteration for $w \in [1, k]$. Also, arbitrarily order the k vaccine drives that the mothers in M_{VI} are part of. In particular, let v_{wI} be the w-th vaccine drive of such a chosen order among the k vaccine drives that are part of the optimal ILP solution, for $w \in [1, k]$. Further, let $M^{(w)}_{VH}$ be the set of mothers targeted by the w vaccination drives $v_{1H}, \dots, v_{wH}$. Similarly, let $M^{(w)}_{VI}$ be the set of mothers targeted by the w vaccination drives $v_{1I}, \dots, v_{wI}$. Since at every iteration $w' \leq w$ in the greedy pruning we choose to have a vaccine drive at grid $g^*_{w'}$ and time $t^*_{w'}$ with the highest value of $H^{w'}_{gt}$, we have for every $w \in [1, k]$

$$\sum_{m \in M^{(w)}_{VH}} p_{mv} - p_{mn} \geq \sum_{m \in M^{(w)}_{VI}} p_{mv} - p_{mn} \, .$$

Finally, it is easily seen that $M^{(k)}_{VH} = M_{VH}$ and $M^{(k)}_{VI} = M_{VI}$, and as the above equation holds for $w = k$, this completes the proof of the proposition.

A.3 Proof of Theorem 1

From Proposition 1, we have

$$\sum_{m \in M_{VH}} p_{mv} - p_{mn} \geq \sum_{m \in M_{VI}} p_{mv} - p_{mn}$$

Rearranging the above expression we have

$$\sum_{m \in M_{VH}} p_{mv} + \sum_{m \in M_{VI} \setminus M_{VH}} p_{mn} \geq \sum_{m \in M_{VI}} p_{mv} + \sum_{m \in M_{VH} \setminus M_{VI}} p_{mn}$$

We add $\sum_{m \in M \setminus M_{VI}} p_{mi^*_m}$ to both sides of the above expression.

$$\sum_{m \in M_{VH}} p_{mv} + \sum_{m \in M_{VI} \setminus M_{VH}} p_{mn} + \sum_{m \in M \setminus M_{VI}} p_{mi^*_m} \geq \sum_{m \in M_{VI}} p_{mv} + \sum_{m \in M_{VH} \setminus M_{VI}} p_{mn} + \sum_{m \in M \setminus M_{VI}} p_{mi^*_m}$$

Now observe that $O^* = \sum_{m \in M_{VI}} p_{mv} + \sum_{m \in M \setminus M_{VI}} p_{mi^*_m}$, and $M \setminus M_{VI} = (M_{VH} \setminus M_{VI}) \uplus (M \setminus (M_{VH} \cup M_{VI}))$. Hence, substituting for O^* in RHS and partitioning $M \setminus M_{VI}$ in the LHS of the above expression we have

$$\sum_{m \in M_{VH}} p_{mv} + \sum_{m \in M_{VI} \setminus M_{VH}} p_{mn} + \sum_{m \in M_{VH} \setminus M_{VI}} p_{mi^*_m} + \sum_{m \in M \setminus (M_{VH} \cup M_{VI})} p_{mi^*_m} \geq O^* + \sum_{m \in M_{VH} \setminus M_{VI}} p_{mn}$$

Rearranging the above expression, we have

$$\sum_{m \in M_{VH}} p_{mv} + \sum_{m \in M_{VI} \setminus M_{VH}} p_{mn} + \sum_{m \in M \setminus (M_{VH} \cup M_{VI})} p_{mi^*_m} \geq O^* - \Big(\sum_{m \in M_{VH} \setminus M_{VI}} p_{mi^*_m} - p_{mn} \Big) \tag{2}$$

It is easy to see that $M \setminus M_{VH} = (M_{VI} \setminus M_{VH}) \uplus (M \setminus (M_{VH} \cup M_{VI}))$, and recall that ADVISER finally runs the ILP on $M \setminus M_{VH}$ mothers with budget $b - k \cdot e_v$. Now since the number of vaccine drives in the optimal solution is at least k, the cost of providing interventions i^*_m to mothers $m \in M \setminus (M_{VH} \cup M_{VI})$ is at most $b - k \cdot e_v$. Hence, providing no interventions to mothers in $M_{VI} \setminus M_{VH}$ and intervention i^*_m to mother $m \in M \setminus (M_{VH} \cup M_{VI})$ is a feasible solution of the ILP run on the remaining mothers. This implies the ILP on the remaining mothers returns an intervention allocation on $M \setminus M_{VH}$ which has objective value at least $\sum_{m \in M_{VI} \setminus M_{VH}} p_{mn} + \sum_{m \in M \setminus (M_{VH} \cup M_{VI})} p_{mi_m *}$. Hence the objective value of the heuristic procedure is

$$O_H \geq \sum_{m \in M_{VH}} p_{mv} + \sum_{m \in M_{VI} \setminus M_{VH}} p_{mn} + \sum_{m \in M \setminus (M_{VH} \cup M_{VI})} p_{mi_m *} \tag{3}$$

Finally, using Equation 3 in Equation 2 we have

$$O_H \geq O^* - \Big(\sum_{m \in M_{VH} \setminus M_{VI}} p_{mi^*_m} - p_{mn} \Big) \, .$$

Table 1: Description of the features used to learn the probability of success for the interventions

Feature	Type	Description
Vaccination Status	Binary	A variable denoting whether the mother took her child for vaccination
Income Level	Binary	A variable denoting whether the family earns more than $25 or not.
Message Status	Binary	A binary variable that denotes whether the mother received a message about the upcoming vaccination appointment.
Age of the mother	Integer	The age of the mother in years.
Age of the child	Integer	The age of the child in months.
Number of children	Integer	The number of children the mother has.
Address	String	The neighborhood that the mother lives in.
Vaccination Center	String	The address of the nearest vaccination center from the mother's house.

A.4 Data

A description of the features we collect from HelpMum is presented in Table 1. The data was collected from the vaccination tracking system developed by HelpMum; a consent was obtained from every participant registered on the system about sharing of anonymous data. The locations of the rented parking depots and the vaccination sites are shown in Figure 4. We train the logistic regression model on 1000 mothers generated from real data in the same way the data D2 is generated, and associate the vaccination labels of these 1000 mothers randomly according to Bernoulli distribution with parameter being the average vaccination rate. The error of our model on the test set is 4%.

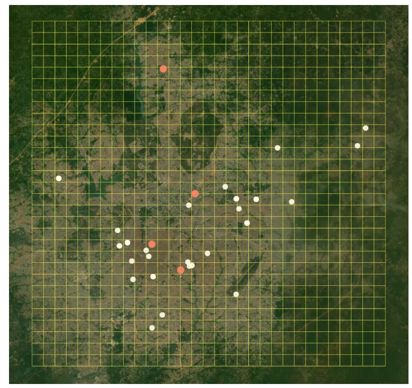


Figure 4: Locations of the rented parking locations (in orange) and the vaccination centers (in white). The yellow lines represent the grid G. We see that there the distribution of the vaccination centers is not uniform; however, HelpMum chose to rent a parking location towards the north of the city to ensure that mothers have access to vaccination centers.

A.5 Parameter Estimation

We assume that a set of features W can be used to represent each individual. The set W can encode prior information about interventions, income levels, and geographic location. However, estimating the probabilities presents a challenge—the interventions of conducting vaccine drives, operating vehicle routes, and providing travel vouchers are designed as part of this research; as a result, we lack exact historical data about the interventions. HelpMum reached out to the community to gather feedback about the potential benefit of such interventions. We use feedback from the community outreach to compute the probability of their success. HelpMum does have data on its routine phone call operation, as part of which it calls every mother to remind them about upcoming vaccination. We use the historical data about phone calls to estimate the success of making additional phone calls by training a logistic regression model.

Computing the effect of untested interventions
To aid the estimation of probability of success for interventions that have not been tested, i.e., travel vouchers, bus pickups, and vaccination drives, HelpMum asked the beneficiaries for feedback. All mothers reported that they would welcome healthcare officials when they conduct door-to-door vaccination campaigns. They also reported that transportation costs were a major barrier for accessibility to health centers, and that pickup service or travel vouchers will be of immense value. Based on the feedback, we assume that the probability of a successful vaccination for a mother given vaccine drive or picked up by a van equal to 1. HelpMum reported that in practice, the efficacy of travel vouchers is slightly lower as the intervention lacks direct monitoring (for example, the travel voucher might not be used or can be used for some other purpose). As a result, for the assignment of a travel voucher, we consider the probability of success to be lower than 1, but higher than the probability of success through a phone call alone or the probability of success in the absence of any interventions[3].

Estimating the effect of making additional phone calls
We want to estimate the probability that a mother takes a child for vaccination given the intervention of making additional phone calls to remind her about an upcoming vaccination ("additional" phone calls refer to targeted calls made after all mothers have been called, which HelpMum already does). Estimating the effect of phone calls is somewhat different than the other interventions; we do have historical data from phone calls made by HelpMum. However, note that the probability we seek to estimate is different from $P(\text{mother going to vaccination} \mid \text{phone call is made})$ as we do not want to estimate the empirical conditional

[3]In practice, we anticipate the probability of success to be slightly lower for bus pickups as well. HelpMum reports that in practice, a health center can exhaust its stock of vaccines. Our estimates can be improved as data is generated through deployment.

probability by restricting attention to the sub-population for which phone calls were made; rather, the phone call is an *intervention*, meaning that we perform the action of making phone calls, which in turn fixes the value of the random variable. Formally, we are interested in estimating $P(\text{mother going for vaccination} \mid do(\text{phone call}))$. However, we point out that a) the marginal distribution of W is invariant under the intervention of making phone calls, and b) the manner in which an individual reacts to a phone call regarding vaccination uptake is the same irrespective of whether the action of making a phone call is through a targeted intervention or not. As a result, the probability of success given the intervention can be directly estimated from historical data by simply calculating the empirical conditional distribution $P(\text{mother going to vaccination} \mid \text{phone call is made})$.

A.6 Pseudo Code

We present the pseudo code for the ADVISER framework in Algorithm 1 and the pseudo code of the heuristic pruning procedure that ADVISER uses in Algorithm 2.

Algorithm 1 The ADVISER Framework

Input M: set of mothers, a: availability matrix, d distance matrix, T: number of days in program, G: grid, b: budget, r: radius

Output I: intervention alloc. array

1: $M', C, b' \leftarrow \text{Heuristic}(M, a, d, G, b, r)$
/* Calls the Heuristic algorithm (Algorithm 2) which returns M': the remaining set of mothers, C: the vaccine drive allocation matrix indicating where the vaccine drives will be conducted, and the left over budget b'. */

2: $I(m) \leftarrow$ Vaccine Drive, for $m \in M \setminus M'$
/* Assigns the vaccine drive intervention to all mothers in M but not in M'. */

3: $R \leftarrow VRP(M', \text{Time windows})$
/* Calls the Vehicle Routing Algorithm (Sections 3.1, 4) with inputs M', time windows for each mother, and immunization centres. It returns R: the set of optimal routes for each (bus depot, centre) pair on each day. */

4: $I(M') \leftarrow \text{ILP}(M', R, a, d, C, G, b', r)$
/* Calls the ILP (Section 3) on the remaining mothers M' with routes R, leftover budget b', and other relevant parameters. The vaccine drive allocation matrix C helps the ILP ensure that there are no vaccination drives at grid $g \in G$ on day t, if the Heuristic has already decided to conduct a drive there on that day. The ILP returns the optimal intervention allocation on remaining mothers with budget b'. */

5: Return the intervention allocation array I.

Algorithm 2 Heuristic Pruning

Input M: set of mothers, a: availability matrix, d: distance matrix, T: number of days in program, G: grid, b: budget, r: radius

Output M': set of remaining mothers, C: vaccine drive allocation matrix, b': leftover budget

1: Let C, K, H be matrices of size $|G| \times T$.
/* $C_{gt} = 1$ if the Heuristic decides to conduct a vaccine drive at grid g on day t and otherwise 0, $K_{gt} = 1$ if at step 10 g, t is computed as g^*, t^* and 0 otherwise, H_{gt} is the value of computing the vaccine drive at cell g, t (refer Section 4) */
2: Initialize: $C_{gt} = 0$, $H_{gt} = 0$, and $K_{gt} = 0$ for all $g \in G$ and $t \in [T]$
3: Set Count $\leftarrow 0$
4: **while** $b \geq b'$ and count $\leq |G| \times T$ **do**
5: Set Count $\leftarrow$ Count + 1
6: **for** $g \in [|G|]$ and $t \in [T]$ **do**
7: $M_{gt} \leftarrow \{m \in M \mid a_{mt} = 1, d_{mg} \leq r\}$
8: **if** $K_{gt} = 1$ **then**
9: $H_{gt} \leftarrow -1$
10: **else if** $K_{gt} = 0$ **then**
11: $H_{gt} = \max_{S \subseteq M_{gt}, |S| \leq \gamma_v} \sum_{m \in S} (p_{mv} - p_{mn})$
12: **end if**
13: **end for**
14: $g^*, t^* = \arg\max_{g,t} H_{gt}$
15: $S_{g^*t^*} = \arg\max_{S \subseteq M_{g^*t^*}, |S| \leq \gamma_v} \sum_{m \in S} (p_{mv} - p_{mn})$
16: $K_{g^*t^*} \leftarrow 1$
17: **if** $e_t \cdot |S_{g^*t^*}| \geq e_v$ **then**
18: $C_{g^*t^*} \leftarrow 1$
19: $M \leftarrow M \setminus S_{g^*t^*}$
20: $b \leftarrow b - e_v$
21: **end if**
22: **end while**
23: $b' \leftarrow b$
24: Return M', C, b'

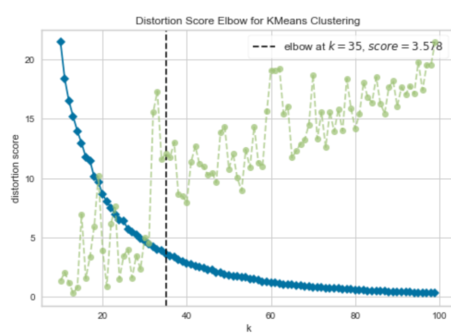
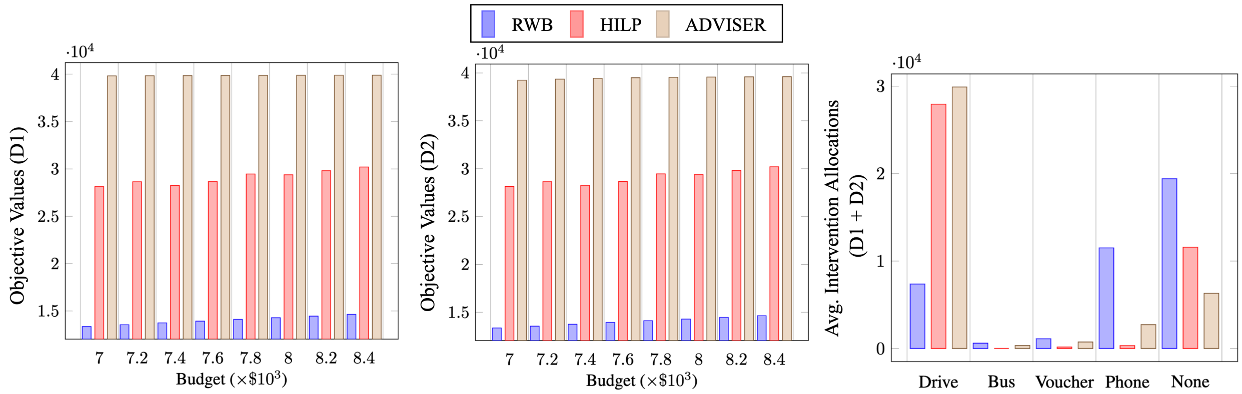


Figure 5: Elbow Curve for D1 with x axis representing the number of clusters and y axis representing the distortion score

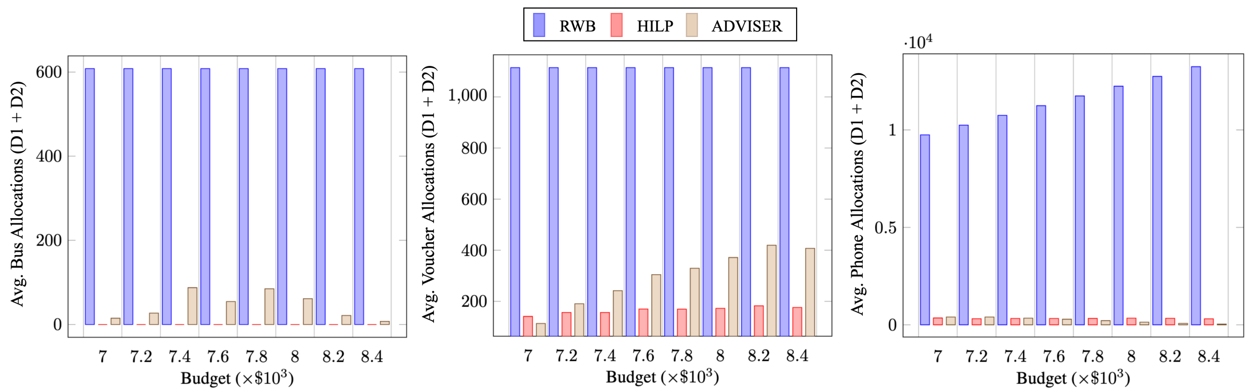
A.7 Baselines

Real-world Baseline: This baseline was designed by HelpMum in the absence of the ADVISER framework. Essentially, it allocates the four interventions sequentially in four stages. HelpMum identified 33 fixed neighbourhoods, one in each local government to conduct vaccination drives on alternate days. Bus routes are operated each day using all the F vehicles from the existing depots. Each vehicle serves one vaccination center each day in a round-robin manner. For each trip, mothers who are within some predefined distance from the routes are considered. Note that since the routes de-



(a) (b) (c)

Figure 6: The output of ADVISER with number of vaccination drives capped to 400. (a) Objective values for D1. (b) Objective values for D2. (b) Number of interventions allocated averaged across D1 and D2.



(a) (b) (c)

Figure 7: Average Intervention Allocations (D1 + D2) for (a) Vehicle Routes, (b) Travel Vouchers, and (c) Phone Calls

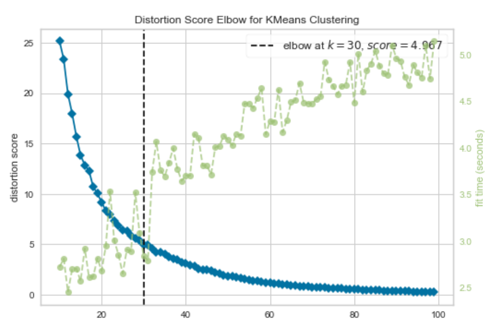


Figure 8: Elbow Curve for D2 with x axis representing the number of clusters and y axis representing the distortion score

termined by HelpMum is fixed, mothers must walk to the bus route; the predefined distance is a check on the distance that a mother can walk with a child to get on a bus. Then, travel vouchers are distributed to mothers who live more than 10 kms. away from a vaccination center. The vouchers are distributed according to income levels, i.e., mothers who have relatively lower income are targeted first. Finally, the remaining budget is used to make targeted phone calls. HelpMum decided to target mothers based on the age of their child—the younger the child, the higher the priority. This decision is motivated by the fact that an infant requires more vaccination doses, and missing one dose hampers the schedule of upcoming vaccine doses.

Hierarchical Integer Linear Programming (HILP): We start by dividing the entire set of mothers into different clusters via k-means clustering on their geographic locations. Our goal is to create smaller ILP formulations per cluster. Naturally, the abstraction introduced by hierarchical planning also induces a trade-off between scalability and utility. In order to select the optimal number of clusters, we use the *elbow method* [Bholowalia and Kumar, 2014] based on the inertia of the clusters (the sum of squared distances of samples to their closest cluster center). The clusters were initialized by sampling the locations of the mothers uniformly at random. The number of clusters for datasets D1 and D2 are 35 and 30 respectively (see Figures 5 and 8).

A.8 Additional Results

In order to evaluate the performance of all the approaches under different parameters, we repeat the experiments by cap-

Method	Average Computation Time (in sec)
ADVISER	254
HILP	4387
RWB	8

Table 2: Computation time for the proposed approaches. The computation time does not consider the routing solver. Routes are generated separately and are provided as input to each algorithm.

ping the maximum number of vaccination drives to 400, i.e., about 13 vaccination drives each day. Essentially, we want to test the robustness of the approaches when sufficient healthcare workers are not available to perform door-to-door vaccination delivery. We show the results in Figure 6. We observe that in comparison to the our original setting (shown in the main body of the paper in Figure 2, the number of mothers picked up by the bus service more than triples. This observation highlights the need for a heterogeneous set of interventions. Also, the objective value attained by capping the number of vaccination drives is lower than without the existence of such a bound; however, ADVISER significantly outperforms the baseline approaches in both the settings.

We also show the average distribution of travel vouchers, bus pickups, and phone calls in Figure 7. We observe that RWB, due to its fixed nature of resource allocation, results in the distribution of a large number of travel vouchers and bus pickups in comparison to ADVISER and HILP (Figure 7 (a)). While travel vouchers and bus pickups are effective modes of intervention, they are relatively more expensive than conducting vaccination drives. The ILP-based approaches (ADVISER and HELP) search the decision-space better an only use such interventions where (intuitively) conducting vaccination drives is not feasible. Finally, we show the average running time of our algorithm and the Hierarchical ILP in Table 2.

A.9 Solving the Vehicle Routing Problem

The route generation process requires solving a VRP with the goal of picking up sets of mothers for a day that maximize the cumulative probability of successful vaccination. For each day, there is a set of mothers available for vaccination on that day. There is also a set of vehicles available for that day, each of which will start at one of the four depots and will end at a vaccination site. Each site is assigned a depot and the capacity of each vehicle is 30 mothers. Therefore, for a given day we solve a VRP for each site. In this case, datasets D1 and D2 both consist of 30 days worth of data and there are 32 vaccination sites which results in 960 generated routes per day.

Given a set of mothers available for vaccination, a vaccination site and a depot the VRP first generates an initial solution with a parallel cheapest insertion heuristic [Chapleau *et al.*, 1984] which iteratively selects the next mother with the highest objective score until there are no more mothers to add without violating the VRP constraints. We then use Guided Local Search [Kilby *et al.*, 1999] to improve upon the solution. For a given site we only consider mothers within 40 kilometers of the vaccination site. Each VRP has a timeout of 30 seconds at which then the best solution found is returned. The VRP solver was implemented with Google OR-Tools [Perron and Furnon, 2019]. The routes were generated on a 48 core virtual machine with 128 GB RAM using the Chameleon testbed supported by the National Science Foundation [Keahey *et al.*, 2020].